Comparative Analysis of Loan Amortization Methods

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Research Question

Primary Question:

How do different amortization methods impact the total financial cost and repayment structure of a loan?

Specific Investigations:

- Which method results in lower total interest payments?
- How does payment composition differ over time?
- What are the practical implications for borrowers?
- How does equity building compare between methods?

Mathematical Framework: Annuity Method

Declining Balance (Annuity) Method

Foundation: Time Value of Money

- Money today is worth more than the same amount in the future
- Interest represents the cost of borrowing over time

Annuity Formula Derivation:

$$\text{Present Value of All Payments} = \text{Loan Principal}$$

$$P = \frac{M}{1+r} + \frac{M}{(1+r)^2} + \cdots + \frac{M}{(1+r)^n}$$

$$P = M \times \frac{1-(1+r)^{-n}}{r}$$

$$M = P \times \frac{r(1+r)^n}{(1+r)^n - 1}$$

M = Monthly payment
P = Principal loan amount
r = Monthly interest rate
n = Total number of payments

Mathematical Framework: Straight-Line Method

Straight-Line Amortization

Core Principle: Constant Principal Payments

Monthly Calculations:

$$\begin{aligned}
\text{Principal Payment} &= \frac{P}{n} \\
\text{Interest Payment}_t &= \text{Remaining Balance}_{t-1} \times r \\
\text{Total Payment}_t &= \text{Principal Payment} + \text{Interest Payment}_t
\end{aligned}$$

Key Characteristics:

- Principal portion remains constant
- Interest portion decreases over time
- Total payment decreases each period

Comparison of Payment Structures

Fundamental Differences:

Characteristic	Annuity Method	Straight-Line Method
Total Payment	Constant	Decreasing
Principal Payment	Increasing	Constant
Interest Payment	Decreasing	Decreasing
Early Payments	Lower	Higher
Late Payments	Higher	Lower

Visualization:

- Annuity: Smooth payment curve
- Straight-line: Declining staircase pattern

Python Implementation Overview

Core Algorithm Structure:

```python
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd

# Set style for better looking plots
plt.style.use('seaborn-v0_8-whitegrid')

def annuity_amortization(principal, annual_rate, years):
    monthly_rate = annual_rate / 12
    periods = years * 12
    payment = principal * (monthly_rate * (1 + monthly_rate)**periods) / ((1 + monthly_rate)**periods - 1)
    return generate_annuity_schedule(principal, monthly_rate, periods, payment)

def straightline_amortization(principal, annual_rate, years):
    monthly_rate = annual_rate / 12
    periods = years * 12
    principal_payment = principal / periods
    return generate_straightline_schedule(principal, monthly_rate, periods, principal_payment)
```

Figure: Evolution of Principal vs. Interest Payments Over Time

Key Python Packages:

- `NumPy`: Financial calculations
- `Pandas`: Amortization table management
- `Matplotlib`: Visualization and comparative graphs

Computational Implementation Details

```python
def generate_annuity_schedule(principal, rate, periods, payment):
    schedule = []
    balance = principal

    for month in range(1, periods + 1):
        interest = balance * rate
        principal_pmt = payment - interest
        balance -= principal_pmt
        if balance < 0:
            balance = 0

        schedule.append({
            'month': month,
            'payment': payment,
            'principal': principal_pmt,
            'interest': interest,
            'balance': balance
        })

    return pd.DataFrame(schedule)

def generate_straightline_schedule(principal, rate, periods, principal_payment):
    schedule = []
    balance = principal

    for month in range(1, periods + 1):
        interest = balance * rate
        total_payment = principal_payment + interest
        balance -= principal_payment
        if balance < 0:
            balance = 0

        schedule.append({
            'month': month,
            'payment': total_payment,
            'principal': principal_payment,
            'interest': interest,
            'balance': balance
        })
```

Key Results: Payment Comparison

Scenario: $200,000 Loan, 5% Interest, 30 Years

Metric	Annuity Method	Straight-Line Method
Monthly Payment (Start)	$1,073.64	$1,388.89
Monthly Payment (End)	$1,073.64	$558.06
Total Interest	$186,511.57	$150,416.67
Interest Savings	-	$36,094.90
Savings Percentage	-	19.4%

Key Insight:

Straight-line method saves $36,094.90 (19.4%) in total interest costs

Key Results: Equity Building Patterns

Time to Reach Equity Milestones:

Equity Level	Annuity Method	Straight-Line Method
25% Equity	11 years	7.5 years
50% Equity	24 years	15 years
75% Equity	29 years	22.5 years

Visualization Insights:

- Straight-line builds equity 2x faster in early years
- Annuity method front-loads interest payments
- Crossover point around year 15-20

Graphical Analysis: Payment Composition

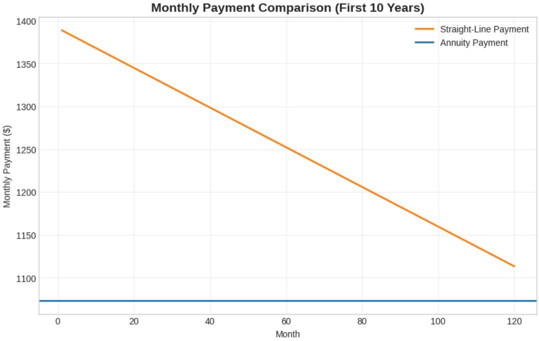

Figure: Evolution of Principal vs. Interest Payments Over Time

Observations:

- Annuity: Gradual shift from interest to principal
- Straight-line: Constant principal, declining interest

Graphical Analysis: Remaining Balance

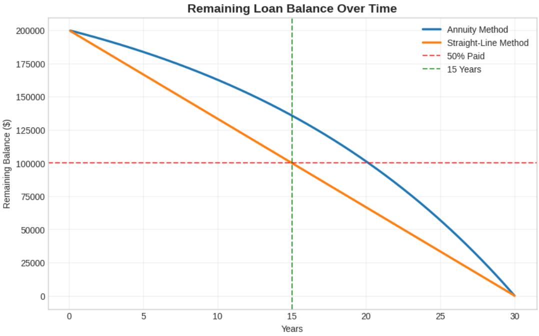

Figure: Remaining Loan Balance Over Time

Key Finding:

Straight-line method achieves zero balance in same time but with different payment pattern and significantly less total interest

Financial Implications for Borrowers

Decision Framework:

Choose Straight-Line If:

- Can handle higher initial payments
- Want to minimize total interest
- Plan to build equity quickly
- Expect future income decreases

Choose Annuity If:

- Need payment stability
- Have tight initial budget
- Prefer predictable cash flow
- Expect stable long-term income

Mathematical Insight:

No "best" method - optimal choice depends on individual financial circumstances and preferences

Conclusions

Key Findings Summary:

- **Total Cost**: Straight-line saves 19.4% in total interest
- **Payment Pattern**: Annuity offers stability, straight-line offers decreasing payments
- **Equity Building**: Straight-line builds equity 2x faster in early years
- **Risk Profile**: Annuity has predictable payments, straight-line has higher initial burden

Mathematical Contribution:

Demonstrated how time value of money and payment timing fundamentally shape loan structures and total borrowing costs

Limitations & Future Work

Current Project Limitations:

- Fixed interest rate assumption
- No prepayment penalties considered
- Simplified credit risk model
- No inflation adjustment

Potential Extensions:

- Variable interest rate scenarios
- Early repayment impact analysis
- Different loan types (ARM, interest-only)
- Risk-adjusted return comparisons
- Tax implication considerations

Thank You

Repository: `https://github.com/Ad862002/Math---2030-Module-2.git`