Invest vs. Prepay Under Interest Rate Uncertainty

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Abstract

This project extends my previous comparison of loan amortization methods by introducing variable interest rates and investment alternatives. Using Monte Carlo simulations with historical and stochastic rate modeling, I will determine when investing the payment differential from the annuity method outperforms straight-line prepayment under realistic economic conditions.

1 Introduction

Traditional loan comparisons assume fixed rates and ignore investment opportunities. In reality, borrowers face fluctuating interest rates and must decide between debt reduction and wealth accumulation. This research addresses: Should you prepay a variable-rate loan or invest the savings? Building on my MATH 2030 work, I will model rate uncertainty and investment returns to provide data-driven guidance for this common financial dilemma.

2 Research Questions

1. Under variable rates, when does investing beat prepayment?
2. How does rate volatility affect optimal strategy choice?
3. What break-even investment return justifies choosing annuity+invest?
4. How do loan parameters (term, reset frequency) influence decisions?

3 Methodology

3.1 Mathematical Framework

Variable-Rate Loans:

- **Annuity:** $M_t = B_{t-1} \times \frac{r_t(1+r_t)^{n-t+1}}{(1+r_t)^{n-t+1}-1}$
- **Straight-Line:** Constant principal P/n, interest $= B_{t-1} \times r_t$
- **Differential:** $D_t = \text{Payment}_{\text{SL},t} - M_t$ (investment amount)

Interest Rate Models:

- Historical: Bank of Canada data (1990–present)
- Stochastic: Vasicek model $dr_t = a(b - r_t)dt + \sigma dW_t$
- Scenarios: Rising, falling, stable rate regimes

Investment Returns:

- S&P 500: Historical mean $\approx 10\%$, volatility 15%
- Cryptocurrency: Higher risk/return profiles

3.2 Computational Approach

Monte Carlo simulation of:

1. 10,000+ interest rate paths
2. Loan payments under both methods
3. Investment growth of differential amounts
4. Net worth comparison with risk metrics

3.2.1 Performance Metrics

- **Wealth Metrics:** Final net worth, internal rate of return, time to reach financial goals
- **Risk Metrics:** Variance of outcomes, Value at Risk (VaR), Conditional VaR, maximum drawdown
- **Risk-Adjusted Returns:** Sharpe ratio, Sortino ratio, probability of negative outcomes

4 Version Control

GitHub: `https://github.com/Ad862002/Math-3030-Module-1.git`
Regular commits will show organic development of code, analysis, and report.