Variable-Rate Loan Strategies: Invest vs. Prepay
A Monte Carlo Simulation Approach

[Adarsh Dubey (202152195)]

Department of Mathematics and Statistics
Memorial University of Newfoundland

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Research Question

Should you prepay your loan or invest the savings?

From Previous Research (MATH 2030)

- **Loan**: $200,000, 30 years, 5% fixed rate
- **Straight-line**: Saves $36,095 (19.4%) in total interest
- **But**: Assumes fixed rates, no investment alternatives

Key Questions

1. When does investing beat prepayment under variable rates?
2. How does rate volatility affect strategy choice?
3. What break-even investment return justifies investing?

Mathematical Framework: Two Repayment Methods

Annuity Method

$$M_t = B_{t-1} \times \frac{r_t(1 + r_t)^{n-t+1}}{(1 + r_t)^{n-t+1} - 1}$$

- M_t: Payment month t
- B_{t-1}: Previous balance
- r_t: **Current variable rate**

Straight-Line Method

$$\text{Payment}_t = \frac{P}{n} + B_{t-1} \times r_t$$

- Constant principal: $\frac{P}{n}$
- Variable interest: $B_{t-1} \times r_t$

Payment Differential (Investment Amount)

$$D_t = \text{Payment}_{\text{SL},t} - M_t$$

- $D_t > 0$: Early months $\rightarrow$ **invest extra**
- $D_t < 0$: Later months $\rightarrow$ **sell investments**

Interest Rate Modeling

Vasicek Model

$$dr_t = a(b - r_t)dt + \sigma dW_t$$

- $a = 0.3$: Reversion speed
- $b = 0.05$: Long-term mean (5%)
- $\sigma = 0.01$: Volatility

Scenarios

- **Rising**: $+0.25\%/\text{yr}$
- **Stable**: $\pm 0.1\%$
- **Falling**: $-0.15\%/\text{yr}$

Simulation Scale

- **10,000+** rate paths
- **360** months (30 years)
- **3.6M+** calculations
- Historical data: Bank of Canada 1990-2024

Practical Example

- Initial rate: 5%
- Loan: $500,000, 30-year term
- Reset: Every 5 years

Investment Returns: Practical Numbers

Asset	Return	Volatility	Max Loss	Potential Profit*
S&P 500	8-10%	15-20%	-50%	$250K-$400K
Crypto	12-20%	60-100%	-80%	$400K-$800K
Bonds	3-5%	3-5%	-10%	$80K-$150K

Table: *

*Estimated on $500K loan over 30 years, assuming 5% loan rate

GBM Model

$$\frac{dS_t}{S_t} = \mu dt + \sigma dW_t$$

- S&P: $\mu = 0.09$, $\sigma = 0.18$
- Crypto: $\mu = 0.16$, $\sigma = 0.80$
- Bonds: $\mu = 0.04$, $\sigma = 0.04$

6-Step Simulation Process

1. **Generate** 10,000 interest rate paths
2. **Calculate** payments (both methods)
3. **Compute** differentials D_t
4. **Simulate** investment growth
5. **Track** portfolio value
6. **Compare** final net worth

S&P Example

$10,000 invested at 9% for 30 years = $132,677 (13x growth)

Computational Load

10,000 sims $\times$ 360 months $\times$ 2 methods $\times$ 3 asset classes = **21.6M calculations**

Performance Metrics

Wealth Metrics

- **Final Net Worth**
- IRR (Internal Rate of Return)
- Time to goals

Risk-Adjusted

- **Sharpe Ratio**: $\frac{E[R] - R_f}{\sigma}$
- **Sortino Ratio**: $\frac{E[R] - R_f}{\sigma_{\text{down}}}$
- **Win Rate**: % winning sims

Risk Metrics

- **VaR (95%)**: Max loss in worst 5% cases
- **Max Drawdown**: Largest peak-to-trough drop
- Downside deviation

Practical Example

S&P 500 (30 years):

- Sharpe: 0.4-0.6
- Win Rate: 65-75%
- Avg. gain: 25-35% vs prepay

Expected Findings: Practical Outcomes

Invest Wins When:
- Market return > Loan rate + 3%
- Time > 15 years
- Rates stable/falling
- High risk tolerance
- **Profit**: 20-50% more wealth

Prepay Wins When:
- Risk aversion high
- Rates rising
- Time < 10 years
- Market uncertainty
- **Guaranteed**: 5% return

S&P 500 (30 years)
- Avg. extra wealth: $125K-$250K
- IRR: 7-9% vs 5% prepay
- Best case: +$400K
- Worst case: -$50K

Crypto (High Risk)
- Avg. extra: $300K-$600K
- Best case: +$1.2M
- Worst case: -$200K
- Win rate: 40-50%

Decision Rules & Recommendations

Simple Rules

- If (Investment return - Loan rate) > 3% & time > 15 years → **INVEST**
- If rates rising → **PREPAY**
- If risk averse → **PREPAY** (guaranteed 5%)
- If young → **INVEST**

By Age Group

- **20-35**: Invest in S&P 500 (70% probability better)
- **35-50**: Mix S&P 500 + some prepayment
- **50+**: Prepay (guaranteed return safer)

Break-even Analysis

- Need 8%+ return to beat 5% prepay over 30 years
- S&P historical: 9-10% → **Favors investing**
- Time horizon critical: <10 years needs 10%+

Thank You

Contact & Resources

GitHub: `https://github.com/Ad862002/Math-3030-Module-1`

Email: [adubey@mun.ca]

Key Takeaways

1. S&P investing beats prepayment 65-75% of the time
2. Need 8%+ returns over 30 years to justify risk
3. Age and risk tolerance are critical factors