Deep Dive: Invest vs. Prepay Under Rate Uncertainty

MATH 3030 Technical Presentation

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GitHub: `https://github.com/Ad862002/Math-3030-Module-1`

The Core Question

Should you prepay your loan or invest the savings?

Traditional (Fixed Rate) Analysis

Straight-line saves **19.4%** interest

The Real World is Different

- **Rates change**: Canada prime: 2.25% (2009) $\rightarrow$ 7.2% (2023)
- **Investment options**: S&P 500 (10%), Crypto (15%+), Bonds (3-5%)
- **Risk matters**: Volatility, crashes, uncertainty

Mathematical Framework

Annuity (Variable Rate)

$$M_t = B_{t-1} \times \frac{r_t(1+r_t)^{n-t+1}}{(1+r_t)^{n-t+1}-1}$$

- M_t: Payment month t
- B_{t-1}: Previous balance
- r_t: Current rate (varies!)

Straight-Line

$$\text{Principal} = P/n$$
$$\text{Interest}_t = B_{t-1} \times r_t$$
$$\text{Payment}_t = P/n + B_{t-1} \times r_t$$

Payment Differential (Investment Amount)

$$D_t = \text{Payment}_{\text{SL},t} - M_t$$

Early: $D_t > 0$ (invest extra)
Late: $D_t < 0$ (sell investments)

Modeling Rate Uncertainty

Historical

- Bank of Canada data
- 1990–present
- Real patterns

Vasicek Model

$$dr_t = a(b - r_t)dt + \sigma dW_t$$

- a: Reversion speed
- b: Long-term mean
- σ: Volatility

Scenarios

- Rising (+0.25%/yr)
- Stable ($\pm$0.1%)
- Falling (-0.15%/yr)

10,000+ simulated rate paths

Investment Returns: Three Risk Levels

	S&P 500	**Crypto**	**Bonds**
Return	8-10%	12-20%+	3-5%
Volatility	15-20%	60-100%+	3-5%
Max Loss	-50%	-80%	-10%

Geometric Brownian Motion

$$\frac{dS_t}{S_t} = \mu dt + \sigma dW_t$$

μ: Mean return, σ: Volatility, dW_t: Random shock

Monte Carlo Simulation Flow

6-Step Process:

1. Generate 10,000 interest rate paths
2. Calculate payments (both methods)
3. Compute differentials D_t
4. Simulate investment growth
5. Track portfolio value
6. Compare final net worth

Scale

10,000 sims $\times$ 360 months $=$ 3.6M calculations

Performance Metrics

Wealth

- Final Net Worth
- Internal Rate of Return
- Time to Goals

Risk

- Value at Risk (VaR)
- Maximum Drawdown
- Downside Deviation

Risk-Adjusted

- Sharpe Ratio

$$\frac{\text{Return} - \text{Risk-free}}{\text{Volatility}}$$

- Sortino Ratio
- Win Rate (% winning sims)

Expected Findings

Invest Wins When:

- Market return $>$ Loan rate $+$ 3%
- Time horizon $>$ 15 years
- Rates stable/falling
- High risk tolerance

Potential gain: 20-50% more wealth

Prepay Wins When:

- Risk aversion high
- Rates rising
- Time horizon $<$10 years
- Market uncertainty

Guaranteed return = loan rate (5%)

No single best answer - depends on personal factors!

Simple Decision Rules

For Borrowers

1. **If** (Investment return - Loan rate) $> 3\%$ **and** time > 15 years $\rightarrow$ INVEST
2. **If** rates expected to rise $\rightarrow$ LEAN TOWARD PREPAYMENT
3. **If** risk averse $\rightarrow$ PREPAY (guaranteed return)
4. **If** young with growing income $\rightarrow$ INVEST

Example: Age 30

- High risk tolerance
- Long horizon
- Growing income
- Verdict: Invest!

Example: Age 55

- Low risk tolerance
- Short horizon
- Near retirement
- Verdict: Prepay!

Summary & Deliverables

What This Project Delivers

- **Realistic model** beyond fixed-rate assumptions
- **Quantitative framework** using Monte Carlo simulation
- **Risk analysis** (not just expected value)
- **Practical guidance** for real borrowers

Final Deliverables

- Python simulation package (GitHub)
- Technical report (LaTeX PDF)
- Visualizations & analysis
- This presentation