$\sin(150^\circ) = \sin(180-30^\circ) = \sin 30^\circ = 1/2$.

$\cos 150^\circ = \cos(180-30^\circ) = -\cos 30^\circ = -\sqrt{3}/2$.

$$\sin(180^\circ) = \left.\begin{array}{l} \nearrow \sin(\underbrace{180-0}_{2^{nd}}) = +\sin 0 = 0 \\ \searrow \sin(\underbrace{180+0}_{3^{rd}}) = -\sin 0 = 0 \\ \searrow \sin(\underbrace{90+90^\circ}_{2^{nd}}) = +\cos 90^\circ = 0 \end{array}\right\}$$

$\cos(180^\circ) = \cos(\underbrace{180-0^\circ}_{2^{nd}}) = -\cos 0 = -1$

$\tan(180^\circ) = 0$

$\sin 270^\circ = \sin(\underbrace{180+90^\circ}_{3^{rd}})$

$= -\sin 90^\circ$

$= -1$

$\cos 270 = \cos(180+90^\circ)$

$= -\cos 90^\circ = 0$

Question

$$\begin{array}{r} 510 \\ 360 \\ \hline 150 \end{array}$$

$\boxed{\cos 570°}$ sin 510° – sin 330° cos 390° equals:

A 0 ✓

B 1

C –1

D 2

$\cos 570 = \cos(210°) = -\frac{\sqrt{3}}{2}$

$\sin 510° = \sin[150°] = +\frac{1}{2}$

$\sin 330° = -\frac{1}{2}$

$\cos 390° = \cos 30° = \sqrt{3}/2$

$-\sqrt{3}/2 \times 1/2 - \left(-\frac{1}{2}\right) \times \sqrt{3}/2$

$\cancel{-\frac{\sqrt{3}}{4}} + \cancel{\frac{\sqrt{3}}{4}}$

$= 0$

Remember

If $A + B = 180°$

$\sin A = \sin B$

& $\cos A + \cos B = 0$

$B = 180 - A \longrightarrow$ cos both

Sin both sides

$\sin B = \sin(180° - A)$

$\sin B = \sin A$

$\cos B = \cos(180 - A)$

$\cos B = -\cos A$

$\cos A + \cos B = 0$

Ex: $\rightarrow$ $\sin 10 = \sin 170°$

$\cos 10 + \cos 170° = 0$

$\cos 15° + \cos 165° = 0$

Question

$$\cos 0 + \cancel{\cos\frac{\pi}{7}} + \cos\frac{2\pi}{7} + \cos\frac{3\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{5\pi}{7} + \cancel{\cos\frac{6\pi}{7}} =$$

A 1/2

B -1/2

C 0

D 1

$A = \pi/7$

$B = 6\pi/7$

$A + B = 180^\circ$

$\Rightarrow \cos A + \cos B = 0$

Question

$\tan\frac{\pi}{11}+\tan\frac{2\pi}{11}+\tan\frac{4\pi}{11}+\tan\frac{7\pi}{11}+\tan\frac{9\pi}{11}+\tan\frac{10\pi}{11}$ equals:

A 0

B 1

C –1

D 2

$\frac{\pi}{11}+\frac{10\pi}{11}=\pi$

$A+B=180^\circ$

$B=180-A$

$\tan B=-\tan A$

$\tan A+\tan B=0$

Question

The value of $\cos^2\frac{\pi}{16}+\cos^2\frac{3\pi}{16}+\cos^2\frac{5\pi}{16}+\cos^2\frac{7\pi}{16}$

A 2 ✓

B 1

C 0

D None of these

$\sin^2\frac{3\pi}{16}$

B

$\cos^2 B$

$\downarrow$

$\sin^2 A$

$\sin^2\frac{3\pi}{16}$

$\sin^2\frac{\pi}{16}$

1

1

$\cos^2 30^\circ + \cos^2 60$

$\cos^2(30+60^\circ)$

$\cos^2(90^\circ) = 0$

$\cos^2\frac{7\pi}{16} = \sin^2\frac{\pi}{16}$

$\frac{\pi}{16} + \frac{7\pi}{16} = \frac{8\pi}{16} = \frac{\pi}{2}$

$A + B \quad = 90^\circ$

$A+B = 90^\circ$

$B = 90^\circ - A$

$\cos B = \cos(90-A)$

$\boxed{\cos B = \sin A}$

$\cos^2 B = \sin^2 A$

$\cos 80 = \sin 10$

Question

$\sin^2\frac{\pi}{18}+\sin^2\frac{\pi}{9}+\sin^2\frac{7\pi}{18}+\underbrace{\sin^2\frac{4\pi}{9}}_{\cos^2\pi/18.}$ equal

A 0

B 1

C -1

D 2

$\frac{\pi}{18}+\frac{4\pi}{9}$

$\frac{\pi}{18}+\frac{8\pi}{18}=\frac{9\pi}{18}=\pi/2=90^\circ$

$\boxed{\sin^2\theta+\cos^2\theta=1}$

$\sin^2 10+\underbrace{\sin^2 80^\circ}_{\cos^2 10^\circ}$

Question

Evaluate

A. $\dfrac{\sin\frac{11\pi}{17}\cos\cancel{\frac{10\pi}{13}}\,\cancel{\tan\frac{\pi}{7}}}{\cancel{\cos\frac{3\pi}{13}}\sin\frac{6\pi}{17}\,\cancel{\tan\frac{6\pi}{7}}}$ -1 -1 $= 1$

B. $\dfrac{\cancel{\sin\frac{\pi}{2023}}\,\cancel{\cos\frac{7\pi}{90}}\,\cancel{\tan\frac{6\pi}{11}}}{\cancel{\cos\frac{83\pi}{90}}\,\cancel{\sin\frac{2022\pi}{2023}}\,\cancel{\tan\frac{5\pi}{11}}}$ -1 -1 $= 1$

$\frac{83\pi}{90} + \frac{7\pi}{90} = \frac{\cancel{90}\pi}{\cancel{90}}$

$\frac{\pi}{2023} + \frac{2022\pi}{2023}$

$= \pi$

$\frac{11\pi}{17} + \frac{6\pi}{17} = \frac{\cancel{17}\pi}{\cancel{17}} = 180^\circ$

$A + B = 180^\circ$

$\Rightarrow \sin A = \sin B$

$\frac{10\pi}{13} + \frac{3\pi}{13} = \pi$

$\cos B = -\cos A$

$\pi/7 + 6\pi/7 = 180^\circ$

Question

Find the value:

A. $\cos\left(\frac{19\pi}{3}\right)$

B. $\tan\left(\frac{-2023\pi}{4}\right)$

We can Remove $2\pi, 4\pi, 6\pi, \ldots$ from any angle.

$\frac{19}{3} \approx 6.33$

$\boxed{\frac{19\pi}{3} \approx 6.33\pi}$

$\cos\left(\frac{19\pi}{3} - 6\pi\right)$

$= \cos\left(\frac{19\pi - 18\pi}{3}\right)$

$= \cos\pi/3 = \frac{1}{2}$ Ans

$180^\circ \rightarrow \pi$ ✓

$360^\circ \rightarrow 2\pi$ ✓

$720^\circ \rightarrow 4\pi$ ✓

$\frac{2023}{4}\pi \approx 505.75\pi$

$= -\tan\left(\frac{2023\pi}{4}\right)$

$= -\tan\left(\frac{2023\pi}{4} - 506\pi\right)$

$= -\tan\left(\frac{2023\pi - 2024\pi}{4}\right) = \tan\pi/4 = 1$

$= -\tan(-\pi/4)$

Compound Angle Formulas

- $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

$B \rightarrow -B$

- $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

- $\cot(A+B) = \dfrac{\cot A \cot B - 1}{\cot A + \cot B}$
- $\cot(A-B) = \dfrac{\cot A \cot B + 1}{\cot B - \cot A}$

optional

$$\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)}$$

$$\frac{\dfrac{\sin A \cancel{\cos B}}{\cos A \cancel{\cos B}} + \dfrac{\cancel{\cos A} \sin B}{\cancel{\cos A} \cos B}}{\dfrac{\cancel{\cos A} \cancel{\cos B}}{\cancel{\cos A} \cancel{\cos B}} - \dfrac{\sin A \sin B}{\cos A \cos B}}$$

$$\frac{\tan A + \tan B}{1 - \tan A \tan B}$$

Q JEE Main 2023 (I) (30 Jan.)

HW

If $\tan 15^\circ + \frac{1}{\tan 75^\circ} + \frac{1}{\tan 105^\circ} + \tan 195^\circ = 2a$, then the value of $\left(a + \frac{1}{a}\right)$ is:

A 4

B $4 - 2\sqrt{3}$

C 2

D $5 - \frac{3}{2}\sqrt{3}$

Homework

- $\sin(405^\circ)$ [Ans. $\frac{1}{\sqrt{2}}$]
- $\sec(-1470^\circ)$ [Ans. $\frac{2}{\sqrt{3}}$]
- $\tan(-300^\circ)$ [Ans. $\sqrt{3}$]
- $\cot(585^\circ)$ [Ans. 1]
- $\text{cosec}(-750^\circ)$ [Ans. -2]
- $\cos(-2220^\circ)$ [Ans. 1/2]

Find the value:

A. $\sin\left(\frac{25\pi}{3}\right)$

B. $\cos\left(\frac{41\pi}{4}\right)$

C. $\tan\left(-\frac{16\pi}{3}\right)$

D. $\cot\left(\frac{29\pi}{4}\right)$

A. $\frac{\sqrt{3}}{2}$

B. $\frac{1}{\sqrt{2}}$

C. $-\sqrt{3}$

D. 1

Homework

- $\cos(45^\circ - A)\cos(45^\circ - B) - \sin(45^\circ - A)\sin(45^\circ - B) = \sin(A + B)$
- $\sin(45^\circ + A)\cos(45^\circ - B) + \cos(45^\circ + A)\sin(45^\circ - B) = \cos(A - B)$
- $\dfrac{\sin(A-B)}{\cos A\cos B} + \dfrac{\sin(B-C)}{\cos B\cos C} + \dfrac{\sin(C-A)}{\cos C\cos A} = 0$
- $\cos a\cos(\gamma - a) - \sin a\sin(\gamma - a) = \cos\gamma$
- $\tan\left(\dfrac{\pi}{4} + \theta\right) \times \tan\left(\dfrac{3\pi}{4} + \theta\right) = -1$
- $1 + \tan A\tan\dfrac{A}{2} = \tan A\cot\dfrac{A}{2} - 1 = \sec A$

Question

The value of $\frac{(1+\tan 11^\circ)(1+\tan 34^\circ)}{(1+\tan 17^\circ)(1+\tan 28^\circ)}$ is equal to

A 1

B 2

C 4

D None of these

Question

If $A + B = 45°$, then prove that: $(\cot A - 1)(\cot B - 1) = 2$

Question

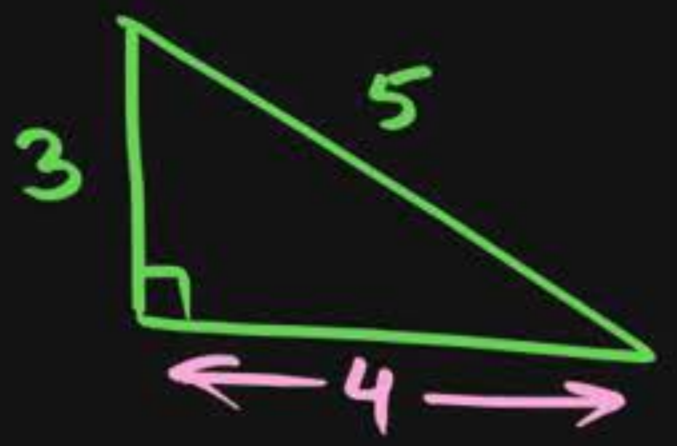
If $\underbrace{\sin A = 3/5}$ and $A \underbrace{\in (90^0, 180^0)}_{2^{nd}\ \text{Quad}}$ then find the value of

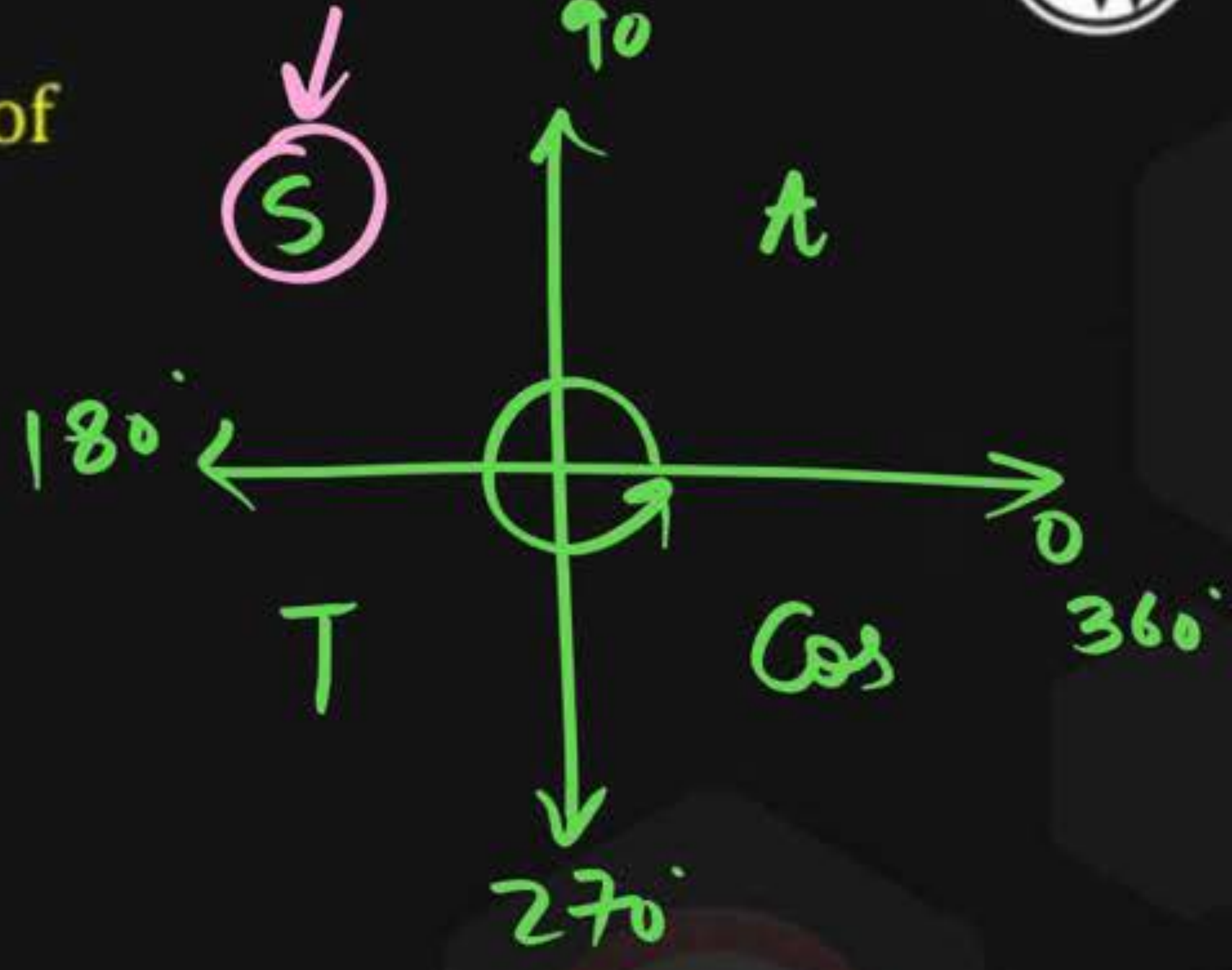
1) tan A

2) cos A

$\tan A = -\frac{3}{4}$

$\cos A = -4/5$





Question

Convert $\underbrace{10°30'}$ into $\underbrace{\text{radians}}$.

A $\frac{5\pi}{12}$

B $\frac{7\pi}{120}$ ✓

C $\frac{11\pi}{12}$

D $\frac{\pi}{12}$

Ex→2

$40° 15'$

$60' = 1°$

$1' = \frac{1°}{60}$

$15' = \frac{15}{60} = \frac{1}{4}° = 0.25°$

$\underline{40.25°}$

$30'$ is same as $0.5°$

$\Rightarrow 10° 30' = 10.5°$

$180° \longrightarrow \pi$ Radians

$1° \longrightarrow \frac{\pi}{180}$ Rad

$10.5 \longrightarrow \frac{\pi}{180} \times 10.5$

$= \frac{\pi}{18 \times 1\cancel{00}_{20}} \times \cancel{105}^{21}$

$= \frac{\cancel{21}\pi}{18 \times 20} = \frac{7\pi}{6 \times 20} = \frac{7\pi}{120}$

Question

$45°15'30''$ change into degrees.

A $\left(\frac{5431}{120}\right)^{\circ}$

B $\left(\frac{5400}{120}\right)^{\circ}$

C 45°

D $\left(\frac{543}{120}\right)^{\circ}$

$60'' = 1'$

$30'' = \left(\frac{1}{2}\right)' = (0.5)'$

$45° 15' 30'' = 45° (15.5)'$

$60' = 1°$

$1' = \frac{1}{60}$

$(15.5)' = \frac{1}{60} \times 15.5$

$= \frac{1}{600} \times 155$

$= \left(\frac{31}{120}\right)^{\circ}$

$$\text{Ans} = 45 + \frac{31}{120}$$

$$= \frac{45 \times 120 + 31}{120}$$

$$= \frac{90 \times 60 + 31}{120} = \frac{5400 + 31}{120}$$

$$= \left(\frac{5431}{120}\right)^{\circ}$$

Question

Convert 3 radians into degrees, use $\pi = 22/7$

A $171°49'6''$

B $171°39'12''$

C $171°49'0''$

D $170°3'12''$

$1° \longrightarrow 60'$

$0.818° \rightarrow 60 \times 0.818$

$= (49.08)'$

$1' \longrightarrow 60''$

$0.08' \longrightarrow (60 \times 0.08)$

$(4.8)'' = 5''$

$\pi \text{ Rad} = 180°$

$1 \text{ Rad} = \frac{180}{\pi}$

$3 \text{ Rad} = \frac{180}{\pi} \times 3$

$= \frac{180 \times 3}{22} \times 7$

$= \frac{90 \times 21}{11}$

$= 171.818°$

Question

The value of $\underbrace{\cos(270° + \theta)}_{\sin\theta} \cdot \underbrace{\cos(90° - \theta)}_{\sin\theta} - \underbrace{\sin(270° - \theta)}_{(-\cos\theta)} \underbrace{\cos\theta}_{\cos\theta}$ is

A 0

B −1

C ½

D 1

$$\sin\theta \; \sin\theta - (-\cos\theta)\cos\theta$$

$$\sin^2\theta + \cos^2\theta$$

$$= 1$$

Question

$$\frac{\boxed{\tan(90^\circ - \theta)}\,\boxed{\sec(180^\circ - \theta)}\,\boxed{\sin(-\theta)}}{\boxed{\sin(180^\circ + \theta)}\,\boxed{\cot(3\cancel{6}0 + \theta)}\,\boxed{\text{cosec}(90^\circ - \theta)}}$$ is equals:

$\cot\theta$ $\quad -\sec\theta$ $\quad -\sin\theta$

$-\sin\theta$ $\quad \cot\theta$ $\quad \sec\theta$

A 0

B 1

C –1 ✓

D 2

$$\frac{\cancel{\cot\theta}\,\cancel{\sec\theta}\,\cancel{\sin\theta}}{-\cancel{\sin\theta}\,\cancel{\cot\theta}\,\cancel{\sec\theta}}$$

$= -1$

Question

(HW)

Simplify: $\dfrac{\operatorname{cosec}(90^\circ+\theta)+\cot(450^\circ+\theta)}{\operatorname{cosec}(450^\circ-\theta)-\tan(180^\circ+\theta)}+\dfrac{\tan(180^\circ+\theta)+\sec(180^\circ-\theta)}{\tan(360^\circ+\theta)-\sec(-\theta)}$

$450^\circ-\theta$

$= \underbrace{450-\theta-360^\circ}$

$= 90-\theta$

Question

HW

Prove that: $\cos 24° + \cos 55° + \cos 125° + \cos 204° + \cos 300° = 1/2$

Homework

HW

- $\sec(270^\circ - A)\sec(90^\circ - A) - \tan(270^\circ - A)\tan(90^\circ + A)$ [Ans. -1]

- $\cot A + \tan(180^\circ + A) + \tan(90^\circ + A) + \tan(360^\circ - A)$ [Ans. 0]

- $\dfrac{\tan(180^\circ-\theta)\cos(180^\circ-\theta)\tan(90^\circ-\theta)}{\sin(90^\circ+\theta)\cot(90^\circ-\theta)\tan(90^\circ+\theta)}$ [Ans. -1]

- If $\tan A = 2$ and $A \in \left(\pi, \frac{3\pi}{2}\right)$ then the expression $\frac{\cos A}{\sin^3 A+\cos^3 A}$ is equal to [Ans. 5/9]

Question

Prove that, $\underbrace{\sin^3 A + \cos^3}\ A = (\sin A + \cos A)(1 - \sin A \cos A)$

LHS:

$(\sin A + \cos A)(\underbrace{\sin^2 A + \cos^2 A}_{1} - \sin A \cos A)$

$(\sin A + \cos A)(1 - \sin A \cos A)$

$\underline{\underline{RHS}}$.

#

$$\boxed{a^3 + b^3 = (a+b)(a^2 + b^2 - ab)}$$

$a \to \sin A$

$b \to \cos A$

$$\boxed{a^3 - b^3 = (a-b)(a^2 + b^2 + ab)}$$

Question

Prove that, $\cos^6 A + \sin^6 A = \underbrace{1 - 3\sin^2 A \cos^2 A}$

LHS:

$(\cos^2 A)^3 + (\sin^2 A)^3$

$(\underbrace{\cos^2 A + \sin^2 A}_{1})(\underbrace{\cos^4 A + \sin^4 A} - \cos^2 A \sin^2 A)$

$[1 - \underbrace{2\sin^2 A \cos^2 A - \cos^2 A \sin^2 A}]$

$[1 - 3\sin^2 A \cos^2 A]$

RHS

$a^3 + b^3 = (a+b)(a^2 + b^2 - ab)$

$a \to \cos^2 A$

$b \to \sin^2 A$

$\cos^6 A = (\cos^2 A)^3$

Question

Prove that, $\sin^8 A - \cos^8 A = (\underbrace{\sin^2 A - \cos^2 A})(1 \underbrace{- 2\sin^2 A\cos^2 A})$

LHS: $(\sin^4 A)^2 - (\cos^4 A)^2$

$(\underbrace{\sin^4 A + \cos^4 A})(\sin^4 A - \cos^4 A)$

$(1 \underbrace{- 2\sin^2 A \cos^2 A}) \left[(\sin^2 A)^2 - (\cos^2 A)^2\right]$

$\underbrace{(1 - 2\sin^2 A \cos^2 A)[\underbrace{\sin^2 A - \cos^2 A}][\underbrace{\sin^2 A + \cos^2 A}_{1}]}_{\text{RHS}}$

Question

$\tan\theta \cot\theta = 1$

$\cot\theta = \frac{1}{\tan\theta}$

$\sin^2 20^\circ$

$$2\left(\frac{\cos^2 20^\circ + \cos^2 70^\circ}{\sin^2 25^\circ + \sin^2 65^\circ}\right) - \boxed{\tan 45^\circ} + \tan 13^\circ \tan 23^\circ \tan 30^\circ \tan 67^\circ \tan 77^\circ$$

$\tan 45^\circ = 1$

$\cos^2 25^\circ$

$\cot 23^\circ$ $\quad$ $\cot 13$

$\frac{1}{\sqrt{3}}$

$$2\left[\frac{\cos^2 20^\circ + \sin^2 20^\circ}{\sin^2 25^\circ + \cos^2 25^\circ}\right]$$

$$\frac{2\times 1}{1} = 2$$

Ans: $2 - 1 + \frac{1}{\sqrt{3}} = 1 + \frac{1}{\sqrt{3}}$ Ans

$\cos 70 = \sin 20$

$\cos^2 70 = \sin^2 20^\circ$

$\sin^2 65^\circ = \cos^2 25^\circ$

$\tan 77^\circ = \cot 13^\circ$

$\tan 30^\circ = \frac{1}{\sqrt{3}}$

Question

$$\left(\frac{\tan 20^\circ}{\operatorname{cosec} 70^\circ}\right)^2 + \left(\frac{\cot 20^\circ}{\sec 70^\circ}\right)^2 + 2\cancel{\tan 15^\circ}\cancel{\tan 37^\circ}\cancel{\tan 53^\circ}\underbrace{\tan 60^\circ}_{\sqrt{3}}\tan 75^\circ$$

$\tan 75^\circ \downarrow \cot 15^\circ$

$\operatorname{cosec} 70^\circ = \sec 20^\circ$

$\sec 70^\circ = \operatorname{cosec} 20^\circ$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$$\left(\frac{\tan 20}{\sec 20}\right)^2 + \left(\frac{\cot 20^\circ}{\operatorname{cosec} 20}\right)^2$$

$$\left(\frac{\sin 20}{\cancel{\cos 20}\,\cancel{\sec 20}}\right)^2 + \left(\frac{\cos 20^\circ}{\cancel{\sin 20^\circ}\,\cancel{\operatorname{cosec} 20}}\right)^2$$

$$\underbrace{\sin^2 20^\circ + \cos^2 20^\circ}$$

$= 1$

Ans $= 1 + 2\sqrt{3}$

Question

Prove

$\tan 1^\circ \tan 2^\circ \tan 3^\circ \ldots\ldots\ldots \tan 89^\circ = 1$

$\cancel{\tan 44}\ \tan 45\ \cancel{\tan 46}$

$\cancel{\tan 1}\ \cancel{\tan 2}\ \tan 3\ \tan 4^\circ \ldots\ldots\ldots \underbrace{\tan 88^\circ}_{\cancel{\cot 2}}\ \underbrace{\tan 89^\circ}_{\cancel{\cot 1^\circ}} = ?$

$= \tan 45^\circ$

$= 1$

Question

$$\sin^2 5^\circ + \underbrace{\sin^2 10^\circ + \cdots +}_{1} \underbrace{\sin^2 85^\circ}_{\cos^2 5^\circ} + \underbrace{\sin^2 90^\circ}_{1} = 9\frac{1}{2}$$

1

$\sin^2 85^\circ = \cos^2 5^\circ$

$\sin^2 80^\circ = \cos^2 10^\circ$

$$\sin^2 5 + \sin^2 10^\circ + \sin^2 15^\circ + \cdots \sin^2 40 + \sin^2 45^\circ + \underbrace{\sin^2 50}_{\cos^2 40^\circ} + \cdots \sin^2 85 + \underbrace{\sin^2 90^\circ}$$

1

$$\underbrace{8} + \sin^2 45^\circ + \sin^2 90^\circ$$

$$8 + \left(\frac{1}{\sqrt{2}}\right)^2 + (1)^2$$

$$= 9 + \frac{1}{2} = 9.5 \text{ Ans}$$

Question

If θ is an acute angle such that $5\cot\theta = 3$, then find the value of $\dfrac{(5\sin\theta - 3\cos\theta)}{(4\sin\theta + 3\cos\theta)}$

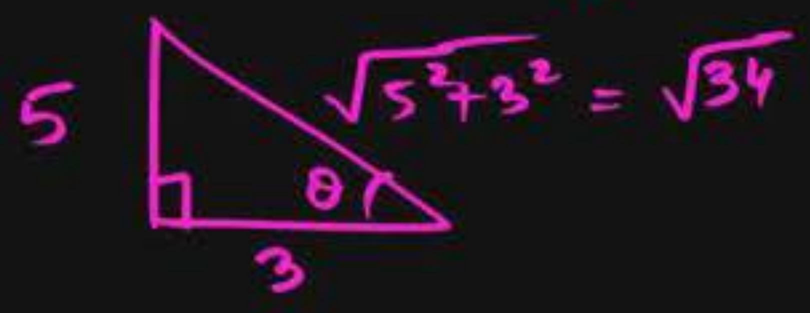
A $\dfrac{11}{18}$

B $\dfrac{16}{29}$ ✓

C $\dfrac{14}{27}$

D none of these

$\cot\theta = 3/5 = b/p$ ✓



Method → 2 $\qquad \cot\theta = \dfrac{\cos\theta}{\sin\theta}$

divide N^r & D^r with $\sin\theta$

$$\frac{5 - 3\dfrac{\cos\theta}{\sin\theta}}{4 + 3\dfrac{\cos\theta}{\sin\theta}}$$

$$= \frac{5 - 3\cot\theta}{4 + 3\cot\theta} = \frac{5 - \dfrac{3\times 3}{5}}{4 + 3\times 3/5}$$

$$= \frac{5 - 9/5}{4 + 9/5} = \frac{25-9}{20+9} = \frac{16}{29}$$

Question

$(\underbrace{\operatorname{cosec}^2 72^\circ} - \tan^2 18^\circ) =$

$\rightarrow \sec^2 18^\circ - \tan^2 18^\circ$

$= 1$

A 0

B 1

C $\frac{3}{2}$

D none of these

Q. [Main Jan. 11, 2019 (I)]

Let $\boxed{f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)}$ for $k = 1,2,3, \ldots$ Then for all $x \in R$, the value of $f_4(x) - f_6(x)$ is equal to:

A $\frac{1}{12}$ ✓

B $\frac{1}{4}$

C $\frac{-1}{12}$

D $\frac{5}{12}$

$$f_4(x) = \frac{1}{4}[\sin^4 x + \cos^4 x] = \frac{1}{4}[1 - 2\sin^2 x\cos^2 x]$$
$$= \frac{1}{4} - \frac{1}{2}\sin^2 x\cos^2 x$$

$$f_6(x) = \frac{1}{6}[\underbrace{\sin^6 x + \cos^6 x}] = \frac{1}{6}[1 - 3\sin^2 x\cos^2 x]$$
$$= \frac{1}{6} - \frac{1}{2}\sin^2 x\cos^2 x$$

(1) − (2)

$$\left(\frac{1}{4} - \cancel{\frac{1}{2}\sin^2 x\cos^2 x}\right) - \left[\frac{1}{6} - \cancel{\frac{1}{2}\sin^2 x\cos^2 x}\right]$$

$$= \frac{1}{4} - \frac{1}{6} = \frac{3-2}{12} = \frac{1}{12}.$$

Question

If $\sec\theta + \tan\theta = p$, then $\cos\theta =$

A $\dfrac{2p}{1-p^2}$

B $\dfrac{2p}{1+p^2}$

C $\dfrac{2p}{p^2-1}$

D $\dfrac{p}{p^2+1}$

If $\sec\theta + \tan\theta = P$ — (1)

$\sec\theta - \tan\theta = 1/P$ — (2)

(+)

$2\sec\theta = P + 1/P$

$2\sec\theta = \dfrac{P^2+1}{P}$

$\sec\theta = \dfrac{P^2+1}{2P}$

$\cos\theta = \dfrac{2P}{P^2+1}$

Homework

Prove that

- $\cos^4 A - \sin^4 A + 1 = 2\cos^2 A$
- $(\sin A + \cos A)(1 - \sin A\cos A) = \sin^3 A + \cos^3 A$
- $\dfrac{\sin A}{1+\cos A} + \dfrac{1+\cos A}{\sin A} = 2\operatorname{cosec} A$
- $\cos^6 A + \sin^6 A = 1 - 3\sin^2 A\cos^2 A$
- $\sqrt{\dfrac{1-\sin A}{1+\sin A}} = \sec A - \tan A$
- $\dfrac{\operatorname{cosec} A}{\operatorname{cosec} A - 1} + \dfrac{\operatorname{cosec} A}{\operatorname{cosec} A + 1} = 2\sec^2 A$

Homework

Prove that

$$\frac{\operatorname{cosec} A}{\cot A + \tan A} = \cos A$$

$$\frac{1}{\cot A + \tan A} = \sin A \cos A$$

$$\frac{1}{\sec A - \tan A} = \sec A + \tan A$$

$$\frac{1 - \tan A}{1 + \tan A} = \frac{\cot A - 1}{\cot A + 1}$$

$$\frac{\sec A - \tan A}{\sec A + \tan A} = 1 - 2\sec A \tan A + 2\tan^2 A$$

$$\frac{\cos A}{1 - \tan A} + \frac{\sin A}{1 - \cot A} = \sin A + \cos A$$

Q JEE Main 2023 (I) (30 Jan.)

If $\tan 15^\circ + \frac{1}{\tan 75^\circ} + \underbrace{\frac{1}{\tan 105^\circ}} + \underbrace{\tan 195^\circ}_{\tan 15^\circ} = 2a$, then the value of $\left(a + \frac{1}{a}\right)$ is:

A 4 ✓

B $4 - 2\sqrt{3}$

C 2

D $5 - \frac{3}{2}\sqrt{3}$

$\tan 195^\circ = \tan(180 + 15^\circ)$
$= \tan 15^\circ$

$\tan 105^\circ = \tan(180 - 75^\circ)$
$= -\tan 75^\circ$

$\tan 15^\circ + \cancel{\frac{1}{\tan 75^\circ}} - \cancel{\frac{1}{\tan 75^\circ}} + \tan 15^\circ$

$2\tan 15^\circ = 2a$

$\Rightarrow a = \tan 15^\circ$

Ans $= a + \frac{1}{a} = \tan 15^\circ + \cot 15^\circ$
$= 2 - \cancel{\sqrt{3}} + 2 + \cancel{\sqrt{3}}$
$= 4$

Question

Prove that: $\tan 80^\circ = \tan 10^\circ + 2\tan 70^\circ$

Consider $80^\circ = 10^\circ + 70^\circ$

$$\tan 80^\circ = \tan(10^\circ + 70^\circ)$$

$$\tan 80^\circ = \frac{\tan 10^\circ + \tan 70^\circ}{1 - \tan 10^\circ \tan 70^\circ}$$

$$\tan 80^\circ - \underbrace{\tan 80^\circ}_{\cancel{\cot 10^\circ}} \cancel{\tan 10^\circ} \tan 70^\circ = \tan 10^\circ + \tan 70^\circ$$

$$\tan 80^\circ - \tan 70^\circ = \tan 10^\circ + \tan 70^\circ$$

$$\tan 80^\circ = \tan 10^\circ + 2\tan 70^\circ$$

Question

Find the value of $\underbrace{\tan 125^0 + \tan 100^0} + \underbrace{\tan 100^0 \tan 125^0} = 1$

$$225^\circ = 125^\circ + 100^\circ$$

$$\underbrace{\tan(225^\circ)} = \tan(125^\circ + 100^\circ)$$

$$1 = \frac{\tan 125^\circ + \tan 100^\circ}{1 - \tan 125^\circ \tan 100^\circ}$$

$$1 - \tan 125^\circ \tan 100 = \tan 125^\circ + \tan 100^\circ$$

$$1 = \tan 125 + \tan 100^\circ + \tan 125^\circ \tan 100^\circ$$

$\boxed{Ans = 1}$ ✓

$\tan 225^\circ = 1$

$\tan(180 + 45^\circ) = +\tan 45^\circ$

$= 1$

Question

Prove that : $\boxed{\tan 3A \cdot \tan 2A \cdot \tan A = \tan 3A - \tan 2A - \tan A}$

Consider $3A = 2A + A$.

$$\tan 3A = \tan(2A + A).$$

$$\tan 3A = \frac{\tan 2A + \tan A}{1 - \tan 2A \tan A}$$

HW

Question

[Ans.]

If $\boxed{\tan A \text{ and } \tan B}$ are roots of the quadratic equation $\boxed{x^2 - ax + b = 0}$, then the value of $\underbrace{\sin^2(A+B)}$ is:

(A) $\dfrac{a^2}{a^2+(1-b)^2}$ ✓

(B) $\dfrac{a^2}{a^2+b^2}$

(C) $\dfrac{a^2}{(b+a)^2}$

(D) $\dfrac{a^2}{b^2(1-a)^2}$

$$x^2 - ax + b = 0 \;\; \begin{cases} \tan A \\ \tan B \end{cases}$$

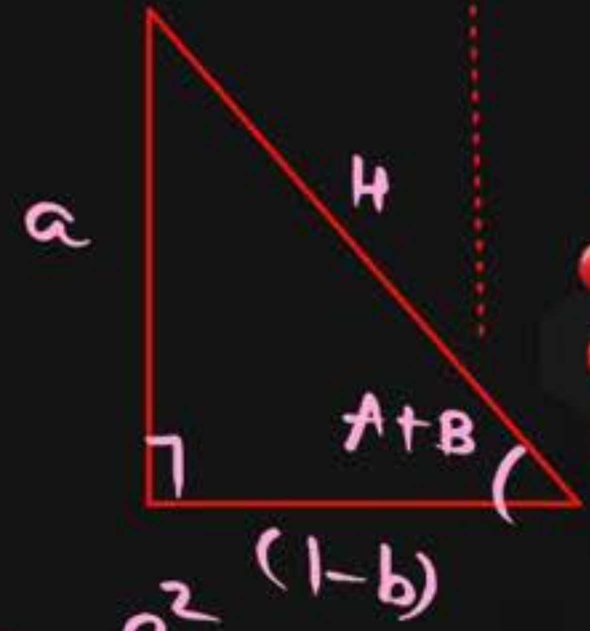
$$\tan A + \tan B = \frac{-(-a)}{1} = a$$

$$\tan A \cdot \tan B = \frac{b}{1} = b$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A+B) = \frac{a}{1-b}$$

$$H^2 = a^2 + (1-b)^2$$



$$\sin(A+B) = \frac{a}{H}$$

$$\sin^2(A+B) = \frac{a^2}{H^2} = \frac{a^2}{a^2+(1-b)^2}$$

$$ax^2 + bx + c = 0 \;\; \begin{cases} \alpha \\ \beta \end{cases}$$

$$\alpha + \beta = -b/a = -\frac{\text{Coeff of } x}{\text{Coeff of } x^2}$$

$$\alpha\beta = c/a = \frac{\text{Const' term}}{\text{Coeff of } x^2}$$

Question

Prove that $1+\tan A\tan\frac{A}{2}=\sec A$ ✓

$A-A/2=\frac{2A-A}{2}=A/2$

$\tan(A/2)\neq\frac{\tan A}{2}$

LHS: $1+\frac{\sin A}{\cos A}\cdot\frac{\sin A/2}{\cos A/2}$

$\cos A\cos B+\sin A\sin B=\cos(A-B)$

$B\to A/2$

$$\frac{\boxed{\cos A\cos A/2+\sin A\sin A/2}}{\cos A\cos A/2}$$

$$\frac{\cos(\underbrace{A-A/2})}{\cos A\cos A/2}=\frac{\cancel{\cos A/2}}{\cos A\cancel{\cos(A/2)}}=\frac{1}{\cos A}=\sec A.$$

Q. [Main Apr. 8, 2019 (I)]

If $\boxed{\cos(\underbrace{\alpha+\beta}_{P}) = \frac{3}{5}}$, $\boxed{\sin(\underbrace{\alpha-\beta}_{Q}) = \frac{5}{13}}$ and $0 < \alpha, \beta < \frac{\pi}{4}$, then $\boxed{\tan(2\alpha)}$ is equal to :

A) $\frac{63}{52}$

B) $\frac{63}{16}$ ✓

C) $\frac{21}{16}$

D) $\frac{33}{52}$

$\alpha + \beta = P$

$\alpha - \beta = Q$

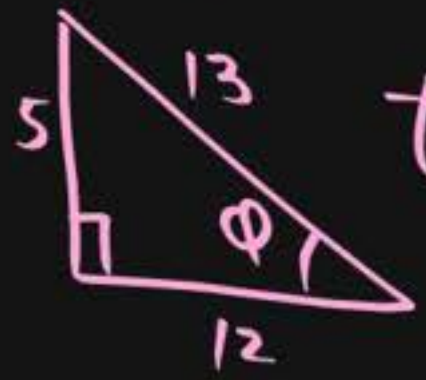
\# Add

$2\alpha = P + Q$

$\cos P = 3/5$

$\tan P = 4/3$

$\sin Q = 5/13$



$\tan Q = \frac{5}{12}$

$\tan(P+Q) = ?$

$= \frac{\tan P + \tan Q}{1 - \tan P \tan Q}$

$= \frac{4/3 + \frac{5}{12}}{1 - 4/3 \cdot 5/12} = \frac{\frac{16+5}{12}}{\frac{36-20}{36}}$

$= \frac{21}{\cancel{12}\times 16} \times \cancel{36}^{\,3} = \frac{63}{16}$.

Question

$\cos A = \frac{1}{3}$, find $\cos 2A$ $= 2\cos^2 A - 1$

$= 2\left(\frac{1}{3}\right)^2 - 1$

$= \frac{2}{9} - 1$

$= \frac{2-9}{9} = -\frac{7}{9}$.

A 7/9

B -7/9

C 1/9

D None Of these

Question

$\sin A = \frac{2}{5}$, find $\cos 2A = 1 - 2\sin^2 A$

$= 1 - 2\left(2/5\right)^2$

$1 - 2 \times \frac{4}{25}$

$1 - 8/25 = \frac{25-8}{25} = \frac{17}{25}$.

A 24/25

B 17/25 ✓

C -17/25

D None Of these

Question

$\sin A = \frac{3}{5}$, find $\sin 2A = 2\sin A\cos A$

$= 2 \times \frac{3}{5}\cos A$ $\nearrow 4/5$, $\searrow -4/5$

A 24/25

B 4/25

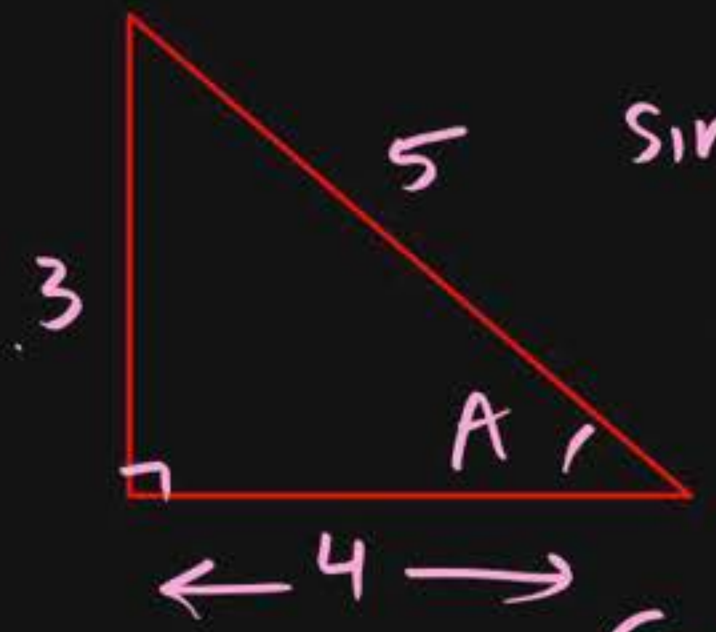
C -24/25

D 3/5

Ans $= 2 \times 3/5 \times 4/5 = \frac{24}{25}$

or

$2 \times 3/5 \times (-4/5) = -\frac{24}{25}$

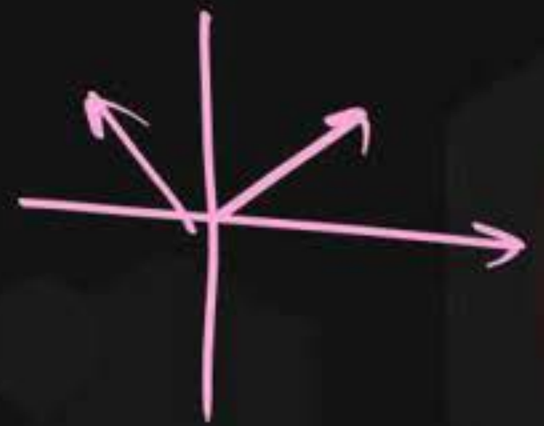


$\sin A = 3/5$

$\Downarrow$

$\cos A = 4/5$

or $-4/5$



Question

$\boxed{\tan A = \frac{1}{3}}$, find $\boxed{\tan 2A}$ $= \dfrac{2\tan A}{1-\tan^2 A}$

$= \dfrac{2 \times 1/3}{1 - 1/9}$

A 3/5

B 2/5

C 3/4

D None Of these

$= \dfrac{2/3}{8/9} = \dfrac{2}{8 \times 3} \times 9$

$= \dfrac{1}{4} \times 3 = 3/4$

Question

[Ans.]

Exact value of tan 200°(cot 10° – tan 10°) is

$\tan 20°(\cot 10° - \tan 10°)$

$\frac{2\tan 10°}{1-\tan^2 10°}\left[\frac{1}{\tan 10°} - \tan 10°\right]$

$\frac{2\cancel{\tan 10°}}{\cancel{1-\tan^2 10°}}\left(\frac{\cancel{1-\tan^2 10°}}{\cancel{\tan 10°}}\right)$

$= 2$

$\tan 200° = \tan(180 + 20°)$

$= \tan 20$

$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$\theta = 10°$

$\tan 20° = \frac{2\tan 10°}{1-\tan^2 10°}$

Brain Teaser

Simplify $\boxed{\boxed{\sqrt{3}}\ \text{cosec}\, 20° - \sec 20°}$ $= 4$ Ans

$\sin 2\theta = 2\sin\theta\cos\theta$

$\theta = 20°$

$\sin 40° = 2\sin 20° \cos 20°$

$$\frac{\sin 60°}{\cos 60°}\frac{1}{\sin 20°} - \frac{1}{\cos 20°}$$

$$= \frac{\boxed{\sin 60° \cos 20 - \cos 60 \sin 20°}}{\cos 60° \sin 20° \cos 20°} \rightarrow \sin(A-B)$$

$$= \frac{\sin(60° - 20°)}{\cos 60 \sin 20 \cos 20} = \frac{\sin 40°}{\underbrace{\cos 60}\sin 20 \cos 20°} = \frac{2\sin 20° \cos 20°}{\cos 60 \sin 20 \cos 20°}$$

$$= \frac{2}{\cos 60°} = \frac{2}{1/2} = 4$$

Question

[Ans.]

The tangents of two acute angles are 3 and 2. The sine of twice their difference is

A 7/24

B 7/48

C 7/50

D 7/25 ✓

Given $\tan A = 3$

$\tan B = 2$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$= \frac{3-2}{1+3\times 2} = \frac{1}{7}$$

let $A - B = \theta$

$$\underbrace{\tan\theta = \frac{1}{7}}$$

$$\sin\Big(2(\underbrace{A-B}_{\theta})\Big) = ?$$

$$\sin 2\theta = ?$$

$$= \frac{2\tan\theta}{1+\tan^2\theta}$$

$$= \frac{2\times\frac{1}{7}}{1+\frac{1}{49}} = \frac{2/7}{\frac{50}{49}}$$

$$= \frac{2\times 7}{50}$$

$$= 7/25$$

Question

Prove that $\left(\frac{\sin 3\theta}{\sin\theta}\right)^2-\left(\frac{\cos 3\theta}{\cos\theta}\right)^2=8\cos 2\theta$

$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

$\frac{\sin 3\theta}{\sin\theta} = 3-4\sin^2\theta$

$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

$\frac{\cos 3\theta}{\cos\theta} = 4\cos^2\theta - 3$

LHS:

$(3-4\sin^2\theta)^2 - (4\cos^2\theta-3)^2$

$[(\cancel{3}-4\sin^2\theta)+(4\cos^2\theta-\cancel{3})][3-4\sin^2\theta-4\cos^2\theta+3]$

$4(\cos^2\theta-\sin^2\theta)[\underbrace{6-4}]$

$8(\underbrace{\cos^2\theta-\sin^2\theta})$

$8\cos 2\theta = \text{RHS}$

Question

$\sqrt{8} = 2\sqrt{2}$

[Ans. ( )]

Prove that: $(4\cos^2 9° - 3)(4\cos^2 27° - 3) = \tan 9°$

$$\frac{(4\cos^3 9° - 3\cos 9°)}{\cos 9°} \cdot \frac{(4\cos^3 27° - 3\cos 27)}{\cos 27°}$$

$$\frac{\cos(3\times 9°)}{\cos 9} \cdot \frac{\cos(3\times 27°)}{\cos 27°}$$

$$= \frac{\cancel{\cos 27}}{\cos 9} \cdot \frac{\cos 81° \to \sin 9°}{\cancel{\cos 27°}}$$

$$= \frac{\sin 9°}{\cos 9°} = \tan 9°$$

$$\boxed{\tan(22.5°) = \sqrt{2} - 1}$$

Let $\theta = 22.5°$

$2\theta = 45°$ $\quad \tan\theta = ?$

$\tan 2\theta = 1$

$$\frac{2\tan\theta}{1-\tan^2\theta} = 1$$

Let $\tan\theta = x$

$2\tan\theta = 1 - \tan^2\theta$

$2x = 1 - x^2$

$$\boxed{x^2 + 2x - 1 = 0}$$

$$x = \frac{-2 \pm \sqrt{8}}{2\times 1} = \frac{-\cancel{2} \pm \cancel{2}\sqrt{2}}{\cancel{2}} = -1 \pm \sqrt{2}$$

$x = \tan 22.5^\circ > 0$

$\Rightarrow x = -1 + \sqrt{2}$ or $\underbrace{-1-\sqrt{2}}_{\text{Reject}}$

$\pi/4 = 45^\circ$

$\pi/8 = 22.5^\circ$

$x = \sqrt{2} - 1$

$\tan \pi/8 = \boxed{\tan 22.5^\circ = \sqrt{2} - 1}$ ✓

$\boxed{\cot 22.5^\circ = \sqrt{2} + 1}$ ✓

$$\cot 22.5^\circ = \frac{1}{\tan 22.5} = \frac{1}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} = \frac{\sqrt{2}+1}{2-1} = \sqrt{2}+1$$

Q. 2023 Main [29 January]

The domain of $f(x)=\dfrac{\log_{(x+1)}(x-2)}{e^{2\log_e x}-(2x+3)}, x\in R$ is

A $\mathbb{R}-\{1-3\}$

B $(2,\infty)-\{3\}$

C $(-1,\infty)-\{3\}$

D $\mathbb{R}-\{3\}$

x^2

$$\begin{cases} x-2>0 \Rightarrow x>2 \\ \& \\ x+1>0 \Rightarrow x>-1 \\ x+1\neq 1 \Rightarrow x\neq 0 \end{cases}$$

2

$$f(x)=\frac{\log_{x+1}(x-2)}{x^2-2x-3}$$

$x^2-2x-3\neq 0$

$(x-3)(x+1)\neq 0$

$x\neq 3$. $x\neq -1$

$e^{2\log_e x}$

$=e^{\log_e x^2}$

$=x^2$

2 3

$(2,\infty)-\{3\}$

Range of a Function

Q : $y = 7 - |x|$ Range ?

A) $[7, \infty)$ B) $(-\infty, 7]$ $y = 7 - (?)$

C) $[0, \infty)$ D) None of these

$y_{max} \Rightarrow |x|$ min

$|x|_{min} = 0$

$\Rightarrow \boxed{y_{max} = 7}$

$y_{min} \Rightarrow |x|_{max} \to \infty$

$y_{min} \to -\infty$

$y \in (-\infty, 7]$

Range of a Function

Find Range of Following

$y_{min} = 3$
$y_{max} \to \infty$

$x^2 \in [0, \infty)$
$5x^2 \in [0, \infty)$
add 3
$\underbrace{3+5x^2}_{} \in [3, \infty)$.
$y \in [3, \infty)$

1) $f(x) = 3 + 5x^2$ A) $[0, \infty)$ B) $(5, \infty)$ C) $[3, \infty)$ ✓ D) None

2) $f(x) = 7 - x^2$ A) $[7, \infty)$ B) $(-\infty, -7]$ C) $(-\infty, 7)$ D) None ✓

$y = 7 - x^2$
$y_{max} = 7$. $(-\infty, 7]$
$y_{min} \to -\infty$

3) $f(x) = \dfrac{2}{3+x^2}$

4) $f(x) = \sqrt{x}$

3) $y = \dfrac{2}{3+x^2}$

Hint: for y_{min} $3+x^2$ must be max $\Rightarrow 3+x^2 \to \infty \Rightarrow y_{min} \to \frac{2}{\infty}$

For y_{max} $3+x^2$ must be min

$y_{min} \to 0$

$(3+x^2)_{min} = 3$

$y_{max} = 2/3$

A) $[0, 2/3]$

B) $(0, 2/3)$

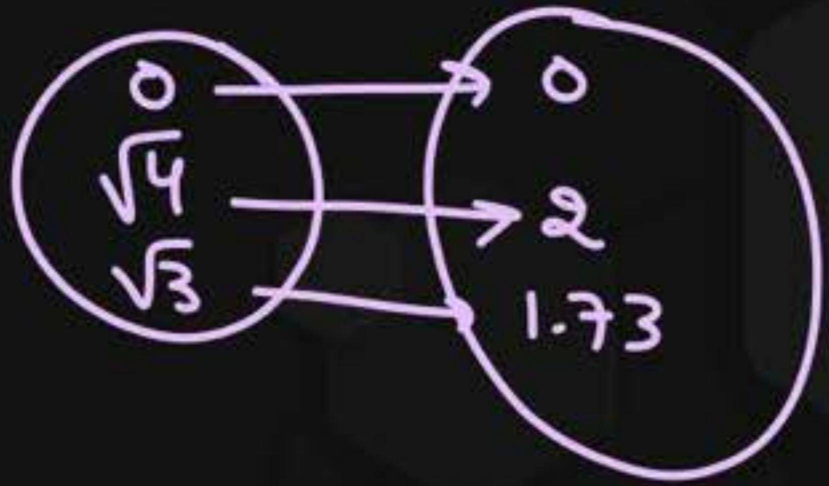
C) $(0, 2/3]$

D) None

$y \in (0, 2/3]$

4) $y = \sqrt{x}$

$\sqrt{4} = +2$



$y \in [0, \infty)$

Question

Find Range of Following

1) $f(x) = \sqrt{x-4}$

2) $f(x) = \sqrt{25 - x^2}$.

3) $f(x) = \sqrt{x^2 - 9}$.

9

A) [.

B) $[3, \infty)$

C) $[0, \infty)$

D) None

$y_{min} = \sqrt{9-9} = 0$

$y_{max} \to \infty$

$y \in [0, \infty)$

$y = \sqrt{x-4}$

Domain: $x - 4 \geq 0$

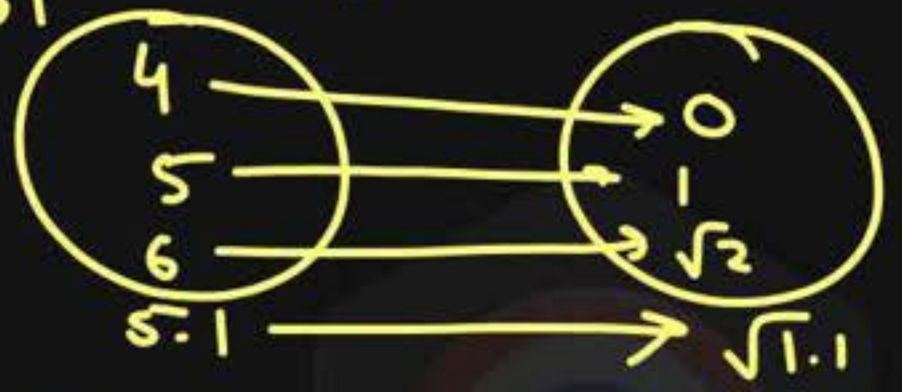
$\Rightarrow x \geq 4$

Domain: $[4, \infty)$

Range: A) $[4, \infty)$ B) $[2, \infty)$

C) R D) $[0, \infty)$

Method → 1



Method → 2

$0 \leq x - 4 < \infty$

Square root

$0 \leq \underbrace{\sqrt{x-4}}\ < \infty$

$0 \leq y < \infty$

2) $y = \sqrt{25 - x^2}$

Ans : $y \in [0, 5]$

3) $y = \sqrt{3 - x^2}$

A) $y \in [0, \infty)$

B) $y \in [0, 3]$

C) $y \in [0, \sqrt{3}]$

D) None

$y_{min} = 0$ when $x^2 = 3$

$y_{max} = \sqrt{3}$ $\quad y \in [0, \sqrt{3}]$

Domain: $25 - x^2 \geq 0$

$x^2 - 25 \leq 0$

$(x-5)(x+5) \leq 0$

$+ \quad -5 \quad 5 \quad +$

$x \in [-5, 5]$

$t = 25 - x^2$

$t_{max} = 25$

$t_{min} = 0$

$0 \leq 25 - x^2 \leq 25$

$$\boxed{0 \leq \sqrt{25 - x^2} \leq 5}$$

$0 \leq y \leq 5$

Find the range of

1) $y = 3|x| - 7$

2) $y = -3x^2 + 7$

3) $y = \sqrt{x} + 3$

4) $y = \sqrt{x} - 3$ $\quad y \in [-3, \infty)$

5) $y = \sqrt{x - 3}$

$y_{min} = 0$, $y_{max} \to \infty$

$y \in [0, \infty)$

(1) $|x|$

y_{min} when $\underbrace{3|x|}_{\Downarrow\ 0}$ is min

$\Rightarrow y_{min} = 0 - 7 = -7$

$y_{max} \to \infty$

$y \in [-7, \infty)$

(2) $y = 7 - 3x^2$

$y_{min} \Rightarrow 3x^2 \to max$

$y_{min} \to -\infty$

$y_{max} = 7 - 0 = 7$

$y \in (-\infty, 7]$

(3) A) $[3, \infty)$ B) $[0, \infty)$ C) R D) None

$y_{min} = 3$

$y_{max} \to \infty$

Question

Find the domain and range of $f(x) = \dfrac{3-x}{\boxed{3x+4}} = y$

Domain: $3x+4 \neq 0$

$3x \neq -4$

$x \neq -4/3$

✓ $x \in R - \{-4/3\}$

$\boxed{\text{For Range: Find } x \text{ in terms of } y}$

$\dfrac{3-x}{3x+4} = y$

$3-x = y(3x+4)$

$3-x = 3xy+4y$

$-x-3xy = 4y-3$

$-x(1+3y) = 4y-3$

$y = \dfrac{\text{linear}}{\text{linear}}$

$\boxed{x = \dfrac{-(4y-3)}{\boxed{1+3y}}}$

$1+3y \neq 0$

$y \neq -\frac{1}{3}$

$\boxed{y \in R - \{-\frac{1}{3}\}}$

Range

Question

Find the Range of $f(x) = \frac{x-3}{2x+1}$

A $R - 2$

B $R - \{-\frac{1}{2}\}$

C $R - \{\frac{1}{2}\}$

D *None of These*

$$y = \frac{x-3}{2x+1}$$

$$2xy + y = x - 3$$

$$2xy - x = -3 - y$$

$$x(2y-1) = -3 - y$$

$$x = \frac{-(3+y)}{2y-1}$$

$$2y - 1 \neq 0$$

$$y \neq \frac{1}{2}$$

HW-BT-13

HW

If (x_1, y_1) and (x_2, y_2) are the solution of the system of equation

$\log_{225}(x) + \log_{64}(y) = 4$

$\log_x(225) - \log_y(64) = 1$, then find the value of $\log_{30}(x_1 y_1 x_2 y_2)$

Integer

Question

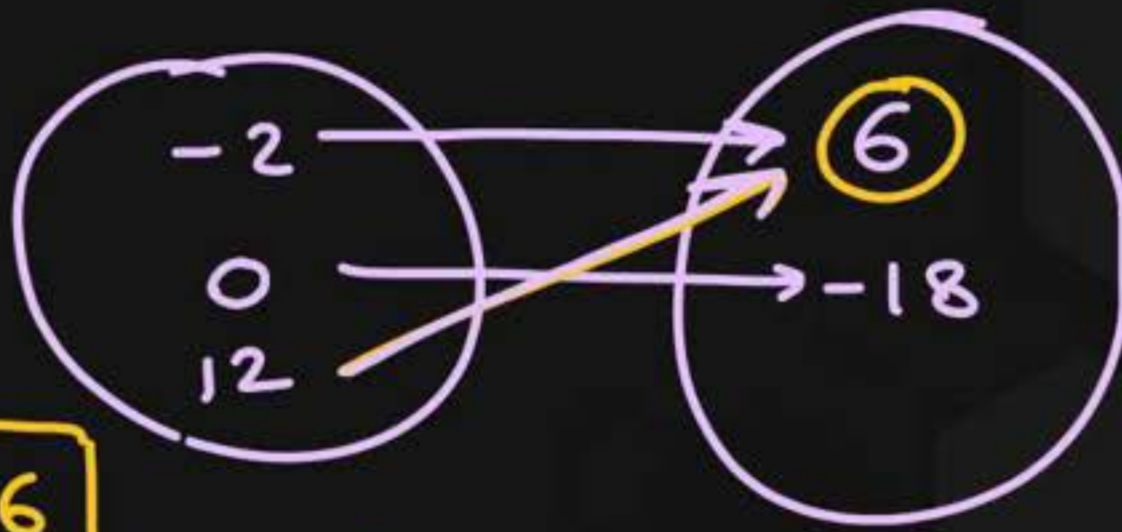
If $g(x) = x^2 - 10x - 18$. Determine the image of -2 and the pre-image of 6.

$x = -2$

$g(-2) = (-2)^2 - 10(-2) - 18$

$= 4 + 20 - 18$

$g(-2) = 6$



$\underbrace{g(x)}_{6} = \boxed{x^2 - 10x - 18 = 6}$

$x^2 - 10x - 18 = 6$

$x^2 - 10x - 24 = 0$

$x^2 - 12x + 2x - 24 = 0$

$(x-12)(x+2) = 0$

$x = 12$ or $x = -2$

Ans Pre image of 6 is $\{-2$ and $12\}$

$\boxed{f(x) = 1/x}$

(3) $f(x) = \sqrt{x-3}$

A) $[0, \infty)$

B) $[3, \infty)$ ✓

C) $(3, \infty)$

D) None

here $x - 3 \geq 0$

$\Rightarrow \boxed{x \geq 3}$

$[3, \infty)$.

$0 \longrightarrow -1/3$

(4) $f(x) = \log_7(x-3)$

$x - 3 > 0$

$\Rightarrow x > 3$

$(3, \infty)$

(C)

$\sqrt{4} = 2$

$\frac{1}{\sqrt{0}} = N.D.$

5) $f(x) = \frac{1}{\sqrt{x-3}}$ $\rightarrow 0$ ×

$x - 3 > 0$

$\boxed{x > 3}$ $(3, \infty)$

6) $f(x) = \frac{1}{x-3}$ $\Rightarrow$ not zero

$x - 3 \neq 0$

$\boxed{x \neq 3}$

$x \in R - \{3\}$.

1) $f(x) = \dfrac{1}{x+7} \neq 0 \Rightarrow x+7 \neq 0 \Rightarrow \boxed{x \neq -7}$

$x \in R - \{-7\}$.

2) $f(x) = \log_{x}(3x-1)$

A) $[1/3, \infty)$

B) $(1/3, \infty)$

C) $[1/3, \infty) - \{1\}$

D) $(1/3, \infty) - \{1\}$ ✓

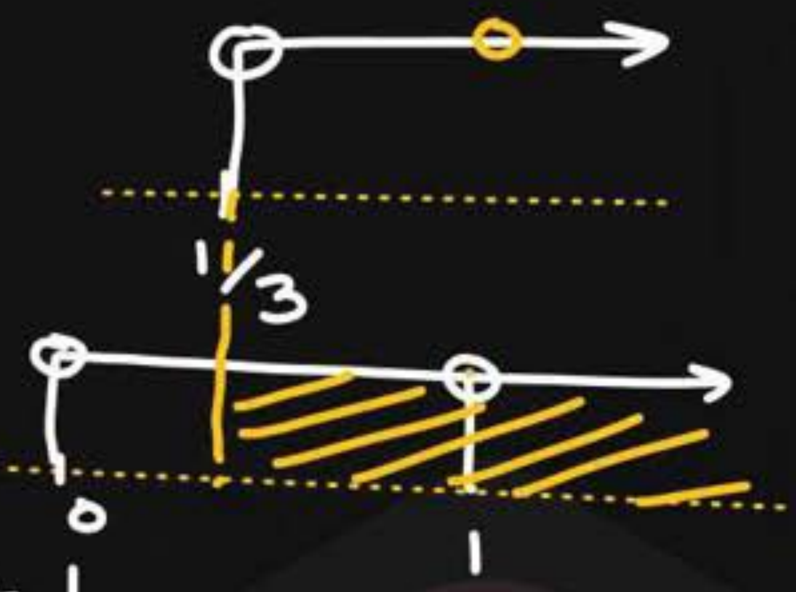
$3x - 1 > 0$

$3x > 1$

$\Rightarrow \boxed{x > 1/3}$

&

$x > 0,\ x \neq 1$



Ans:

$x \in (1/3, \infty) - \{1\}$

Q→

3) $f(x) = \log_{(x-5)}(2x-7)$

1) $\left(\frac{7}{2}, \infty\right)$

2) $\left(\frac{7}{2}, \infty\right) - \{5\}$

3) $\left[\frac{7}{2}, \infty\right) - \{5\}$

4) $(5, \infty) - \{6\}$

$2x - 7 > 0 \Rightarrow \boxed{x > 7/2}$

& $x - 5 > 0$, $x - 5 \neq 1$

$x > 5$, $x \neq 6$.

$(5, \infty) - \{6\}$

Q→ $f(x) = \underbrace{\sqrt{x-3}}_{\downarrow} + \underbrace{\sqrt{7-x}}_{\downarrow}$

$D_1 \cap D_2 =$ Ans

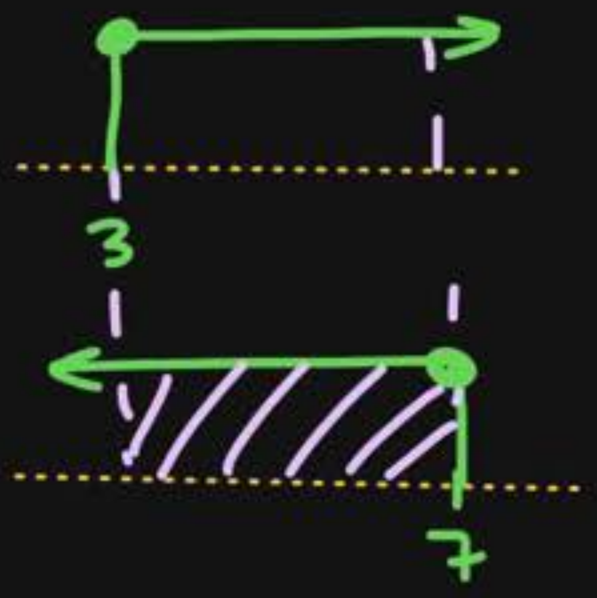
$D_1: \ x-3 \geq 0$

$\boxed{x \geq 3.}$

&

$D_2: \ 7-x \geq 0$

$7 \geq x \Rightarrow \boxed{x \leq 7}$



$D: [3,7]$ Ans

Question

Find the domain of function : $f(x)=\frac{\log_2(x+3)}{x^2+3x+2}$

$x+3>0$

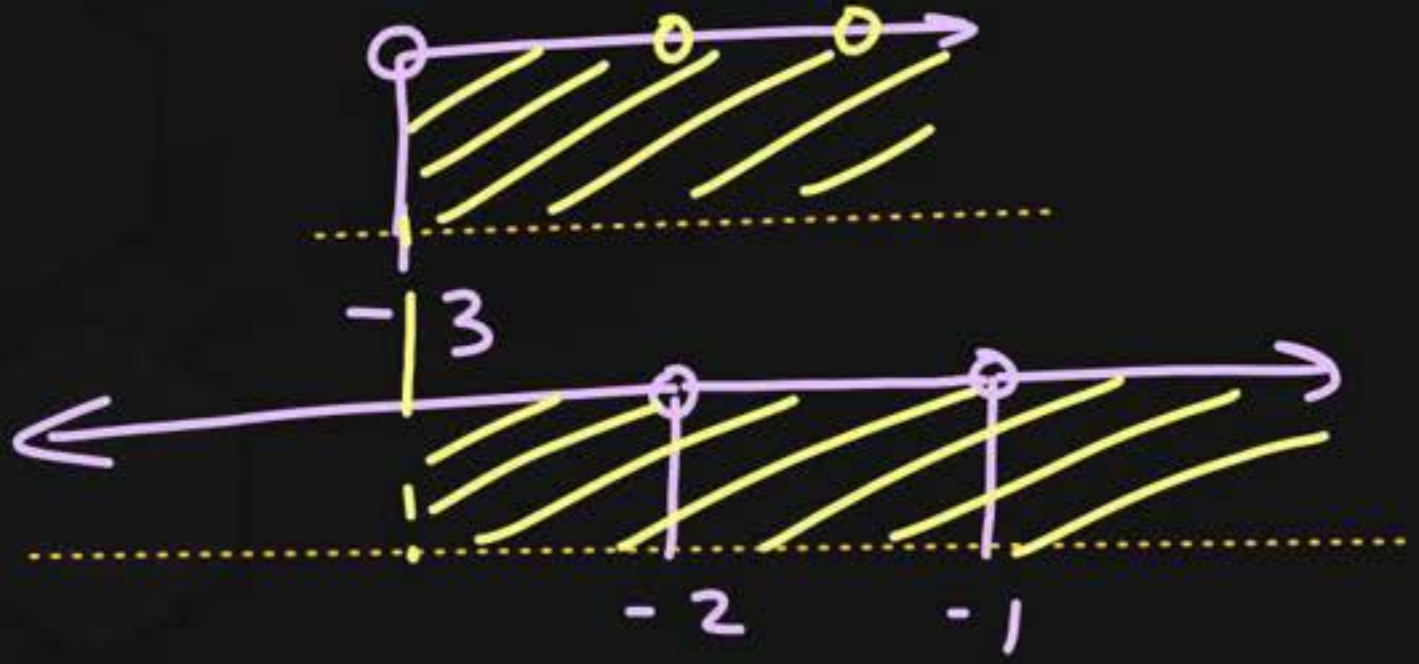
D_1 $\boxed{x>-3}$

D_2: $x^2+3x+2\neq 0$

$(x+2)(x+1)\neq 0$

$x\neq -2, -1$

D_2: $R-\{-2,-1\}$



Ans: $(-3,\infty)-\{-2,-1\}$

Question

The domain of the function $f(x) = \sqrt{(x-1)} + \sqrt{(2-x)} + \frac{1}{2x-3}$ is:

A. $[1,2]$

B. $[1,2] - \{\frac{3}{2}\}$

C. $(1,2)$

D. $(1,\frac{3}{2})$

$x - 1 \geq 0$

$\boxed{x \geq 1}$

$2 - x \geq 0$

$\boxed{x \leq 2}$

$2x - 3 \neq 0$

$x \neq 3/2$

Ans: $x \in [1, 2] - \{3/2\}$

Question

The domain of the function $f(x) = \sqrt{(x^2 - 1)} + \sqrt{(9 - x^2)}$ is:

A $(-3, -1) \cup (1,3)$

B $[-3, -1] \cup [1,3]$

C $[-1,1]$

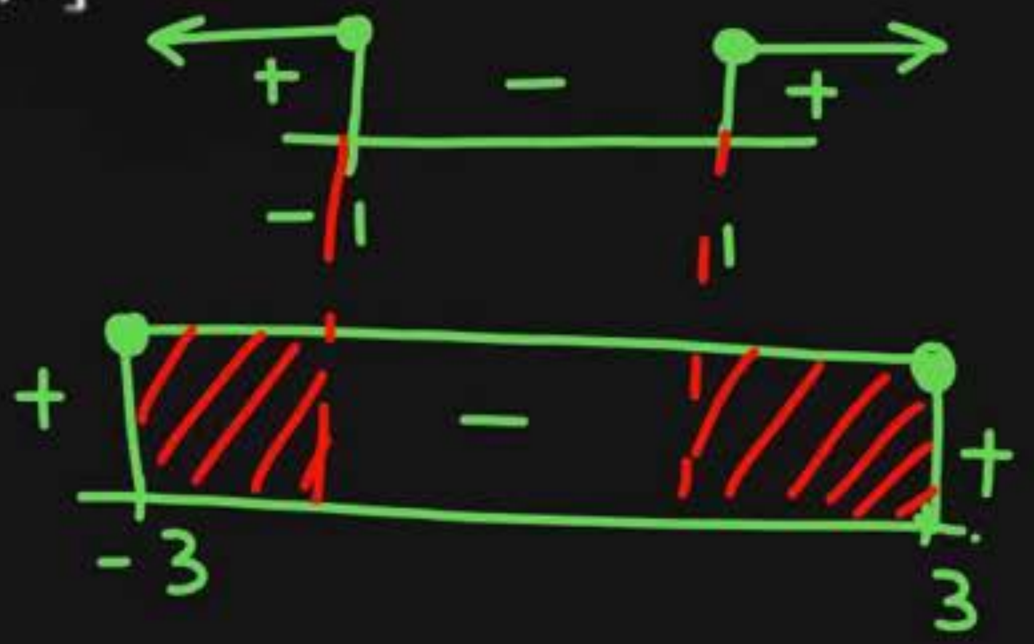
D $[-3,3]$

$x^2 - 1 \geq 0$ & $9 - x^2 \geq 0$

$(x-1)(x+1) \geq 0$

$x^2 - 9 \leq 0$

$(x-3)(x+3) \leq 0$



Question

Find the domain of function : $f(x) = \dfrac{1}{4-x^2} + \log_{10}(\underbrace{x^3 - x})$

$4 - x^2 \neq 0$

$4 \neq x^2 \Rightarrow x \neq \pm 2$

A) $x \in (-\infty, -1) \cup (1, \infty) - \{2, -2\}$

B) $x \in (-1, 0) \cup (1, \infty) - \{2\}$ ✓

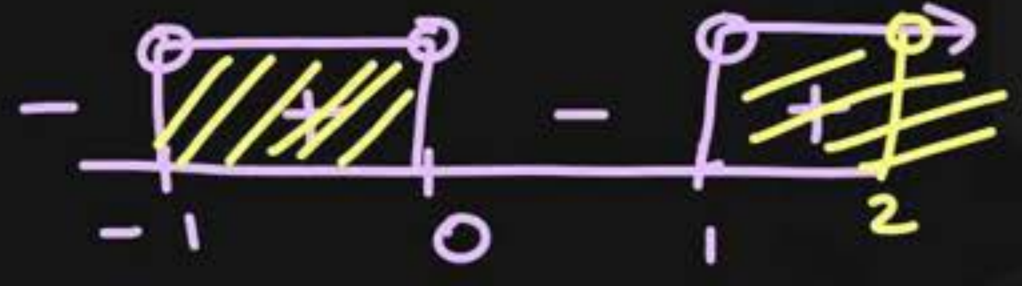
C) $x \in (-\infty, 0) \cup (0, 1) - \{2, -2\}$

D) None

$x^3 - x > 0$

$x(x^2 - 1) > 0$

$x(x-1)(x+1) > 0$



$(-1, 0) \cup (1, \infty) - \{2\}$

Question

Domain of $f(x) = \frac{1}{\sqrt{13x - x^2 - 36}}$ is:

A $x \in [4, 9]$

B $x \in (-\infty, 4) \cup (9, \infty)$

C $x \in (4, 9)$

D $x \in R$

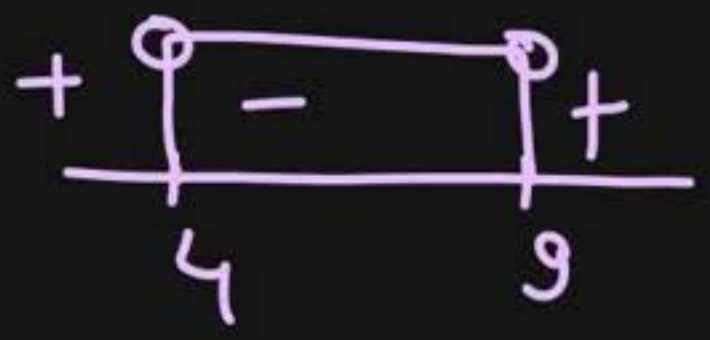
$\boxed{13x - x^2 - 36 > 0}$

$-13x + x^2 + 36 < 0$

$x^2 - 13x + 36 < 0$

$x^2 - 9x - 4x + 36 < 0$

$(x-9)(x-4) < 0$



$(4, 9)$

Question

Complete set of permissible values of x for which $f(x) = \sqrt{x-2} \cdot \sqrt{x-5}$ is defined

A $x \in (-\infty, 2] \cup [5, \infty)$

B $x \in [2, \infty)$

C $x \in [5, \infty)$

D $x \in [5, \infty) \cup \{2\}$

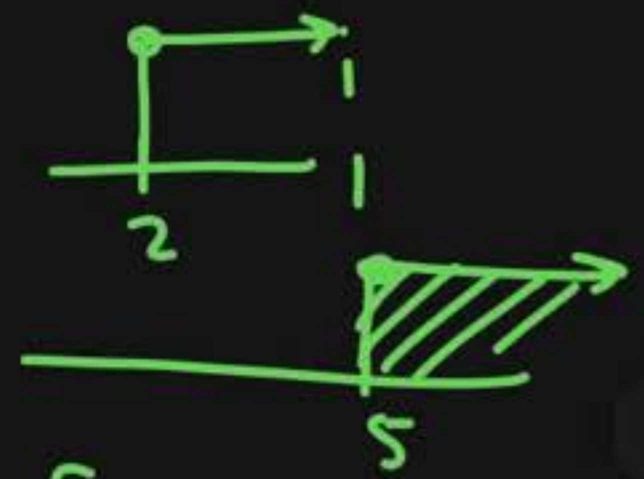
$x - 2 \geq 0$

$x \geq 2$ & $x \geq 5$

$f(2) = \sqrt{0}\sqrt{2-5}$

$= 0 \times$ anything

$= \underline{\underline{0}}$



$[5, \infty) \cup \{2\}$

Question

A relation is defined on set of Natural Numbers such that $R = \{(x, x^3) : x \text{ is a prime number less than } 10\}$. Write this relation in roaster form.

Roster form

$R = \{(2, 8), (3, 27), (5, 125), (7, 343)\}$

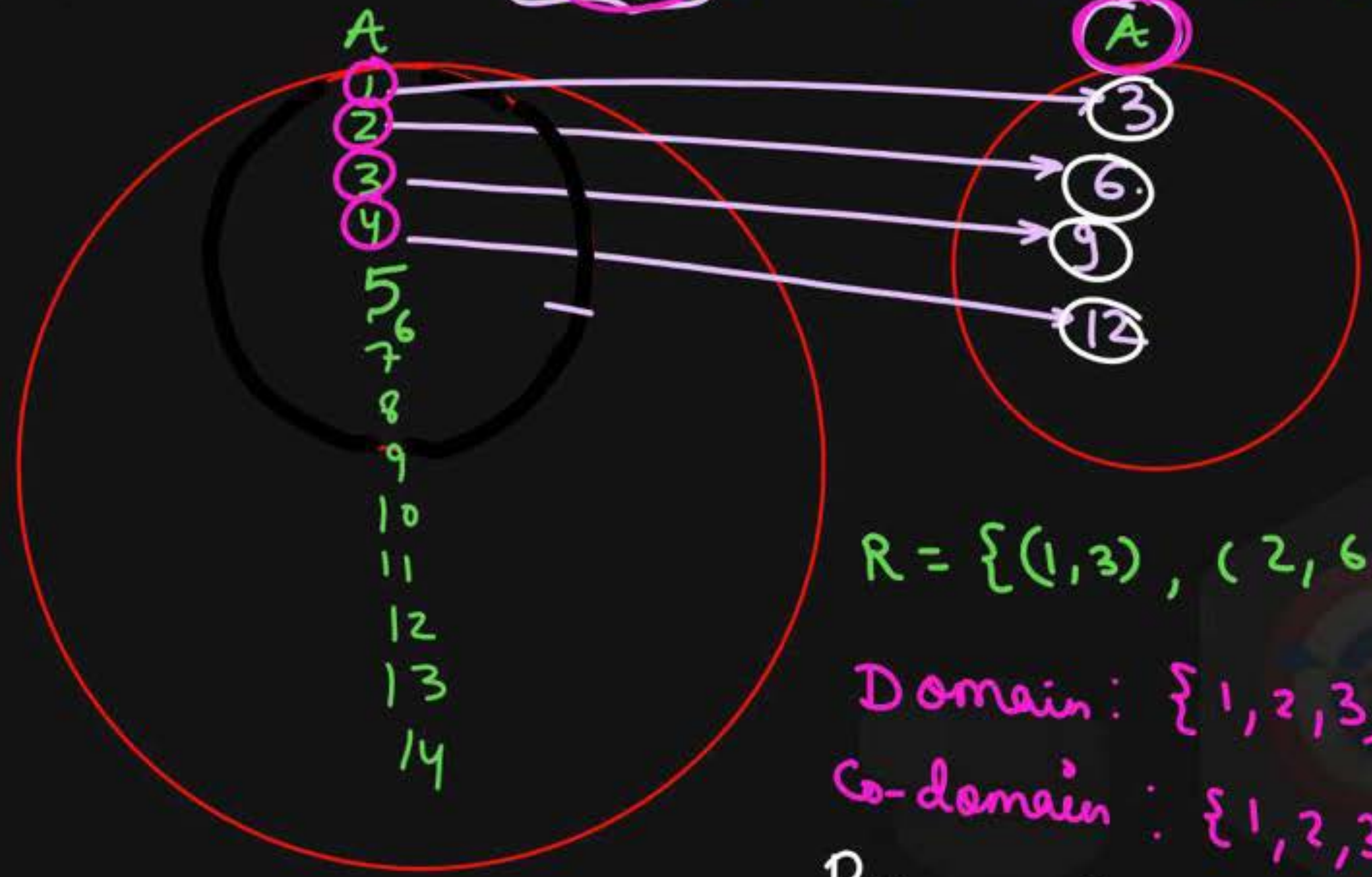
Domain : $\{2, 3, 5, 7\}$

Co-domain : Natural nos.

Range : $\{8, 27, 125, 343\}$

Question

Let A = {1,2,3,......, 14}. Define a relation R on set A given by $R = \{(x, y) : 3x = y$



$R = \{(1,3), (2,6), (3,9), (4,12)\}$

Domain: $\{1, 2, 3, 4\}$

Co-domain: $\{1, 2, 3, 4, \ldots 14\}$

Range: $\{3, 6, 9, 12\}$

Question

codomain

The relation R from $A \rightarrow B$ is given as $R = \{(5, 3), (2, 7), (8, 5)\}$. The range of R is

A $\{5, 2, 8\}$

B $\{3, 7, 5\}$

C $\{2, 3, 5, 7, 8\}$

D $\{2, 3, 5, 7\}$

Question

Let $A = \{1, 2, 3, 4, 5, 6\}$. Define a relation R from $\boxed{A \text{ to } A}$ by $R = \{(x, y) : y = x + 1\}$

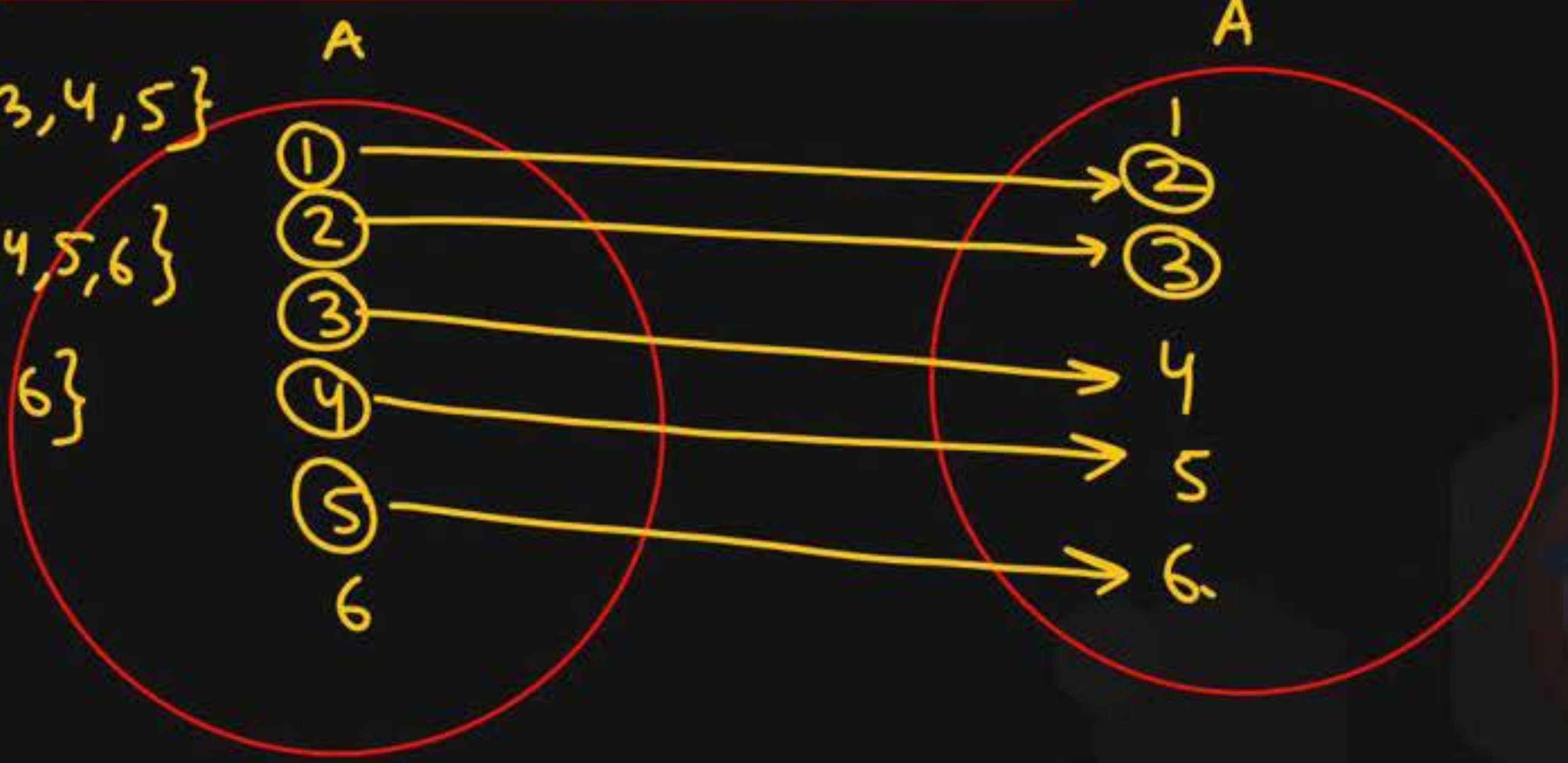
(i) Depict this relation using an arrow diagram.

(ii) Write down the $\boxed{\text{domain, codomain and range of } R.}$

Domain : $\{1, 2, 3, 4, 5\}$

Co-domain : $\{1, 2, 3, 4, 5, 6\}$

Range $\{2, 3, 4, 5, 6\}$



Question

Let $A = \underbrace{\{1,2\}}$ and $B = \{3,4,7\}$, then the number of relations from A to B is:

A 2^2

B 2^3

C 2^4

D 2^6 ✓

$n = 2$

$m = 3$

$n(A \times B) = 6.$

$\Rightarrow$ Subsets = Relations = 2^6

Question

Consider set $A = \{-1,0,1,2,3\}$ and $B = \{1,2,4,5,0\} \cdot R = \{(a,b): b - a = 1, a \in A, b \in B\}$, then domain of relation R is:

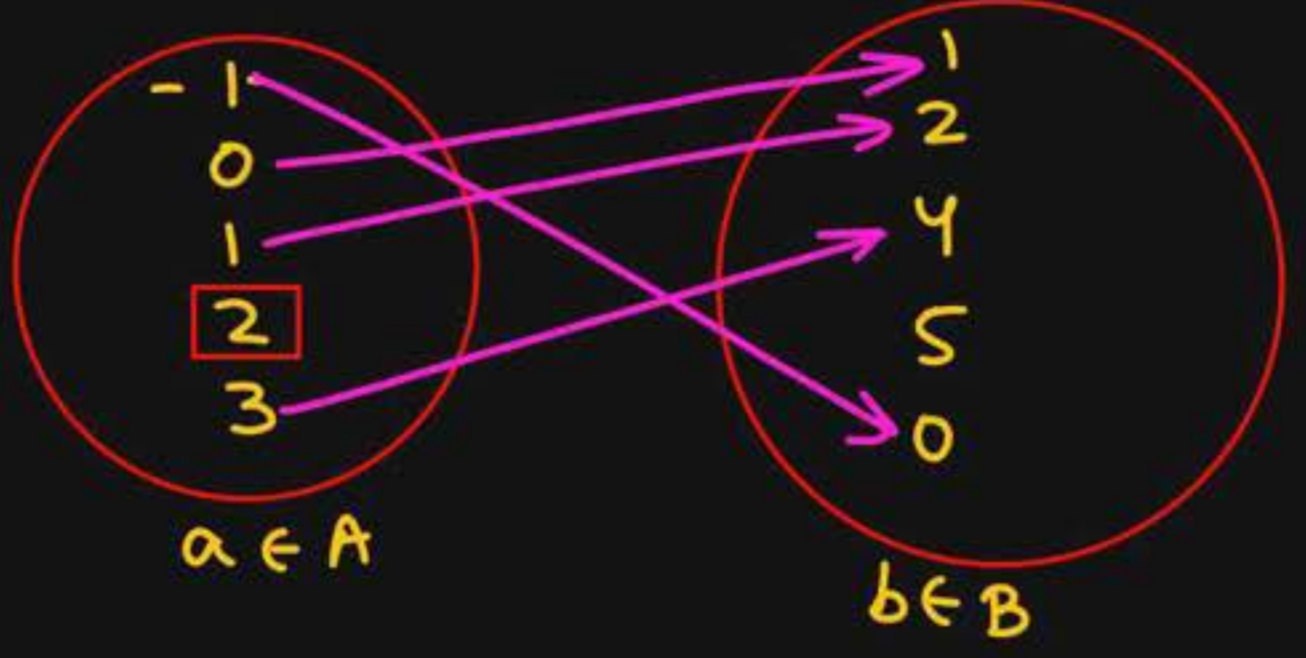
$b = a + 1$

A $\{0,1,3\}$

B $\{-1,0,1,3\}$

C $\{1,0,-1,2,3\}$

D $\{1,0,2,3\}$



Domain :

$\{-1, 0, 1, 3\}$.

Question

[Ans. [redacted]

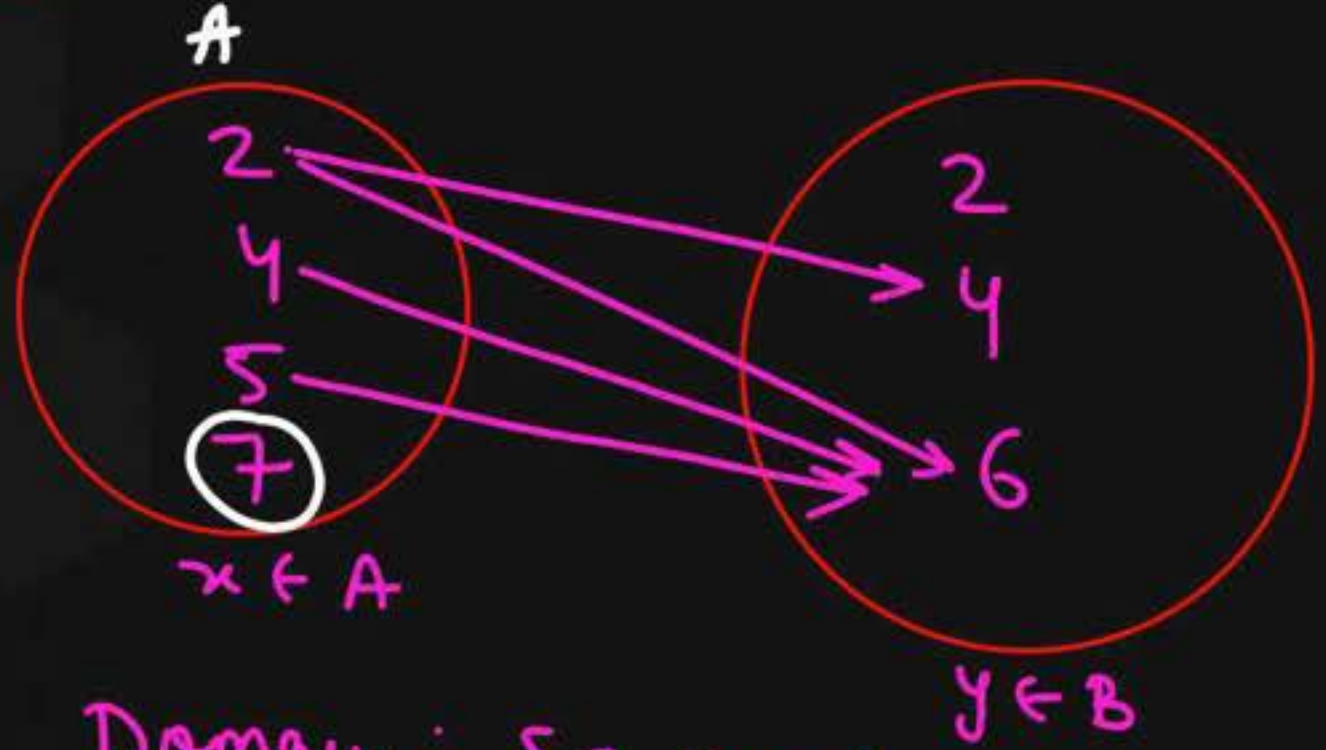
Let $A = \{2, 4, 5, 7\}$ and $B = \{2, 4, 6\}$ be two sets and let a relation R be a relation from A to B is defined as $R = \{(x, y) : x < y, x \in A, y \in B\}$, then the difference between the sum of elements of domain and range of R is-

A 2

B 3

C 4

D 1



Domain : $\{2, 4, 5\} \Rightarrow$ Sum $= 11$

Range $\{4, 6\} \Rightarrow$ Sum $= 10$

Difference $= 11 - 10 = 1$

Question

Plot the Graph of $f(x)$ & Hence Find the Range.

$$\boxed{f(x) = \frac{|x|}{\boxed{x}}}$$

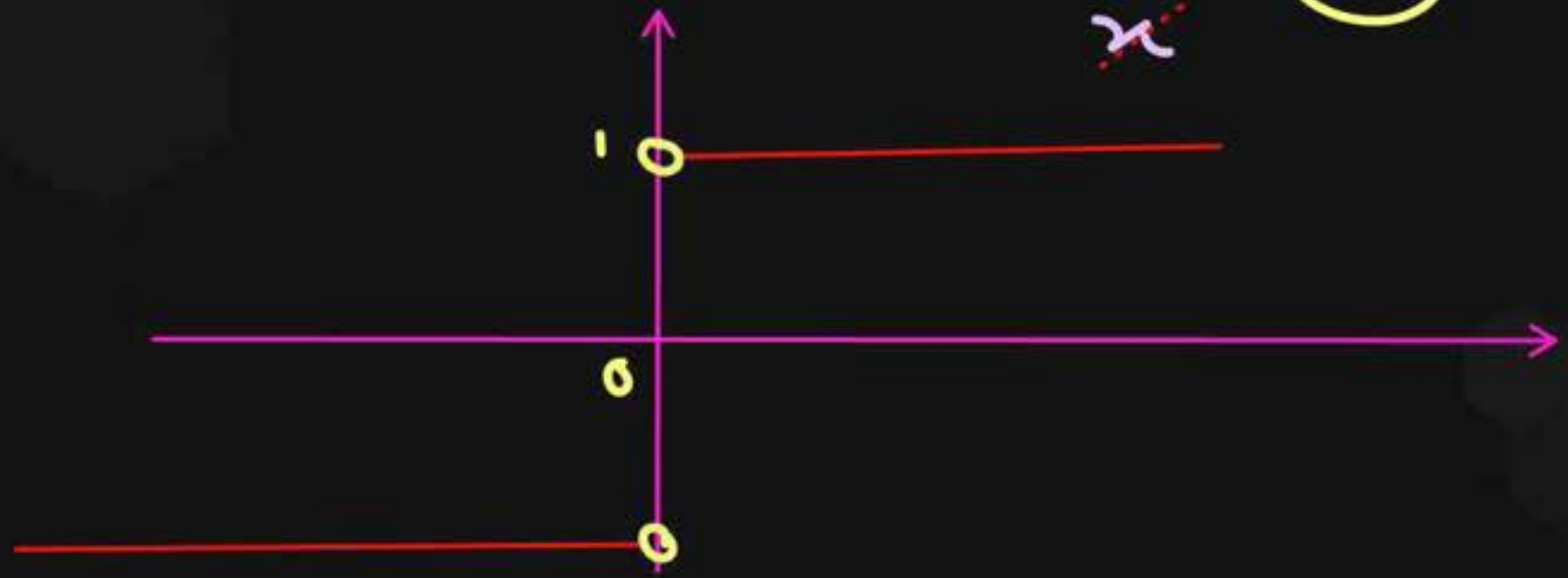
Domain $x \in R - \{0\}$

Case → 1 $\quad x > 0$ ✓

$$\Rightarrow f(x) = \frac{x}{x} = 1$$

Case → 2 $\quad x < 0$

$$\Rightarrow f(x) = \frac{-\cancel{x}}{\cancel{x}} = -1$$



$|x| = +x$ if $x \geq 0$

$|x| = -x$ if $x < 0$

Range : $\{1, -1\}$

Question

Plot the Graph of $f(x)$ & Hence Find the Range.

$$f(x) = \frac{x^2-25}{\boxed{x-5}} = \frac{\cancel{(x-5)}(x+5)}{\cancel{(x-5)}}$$

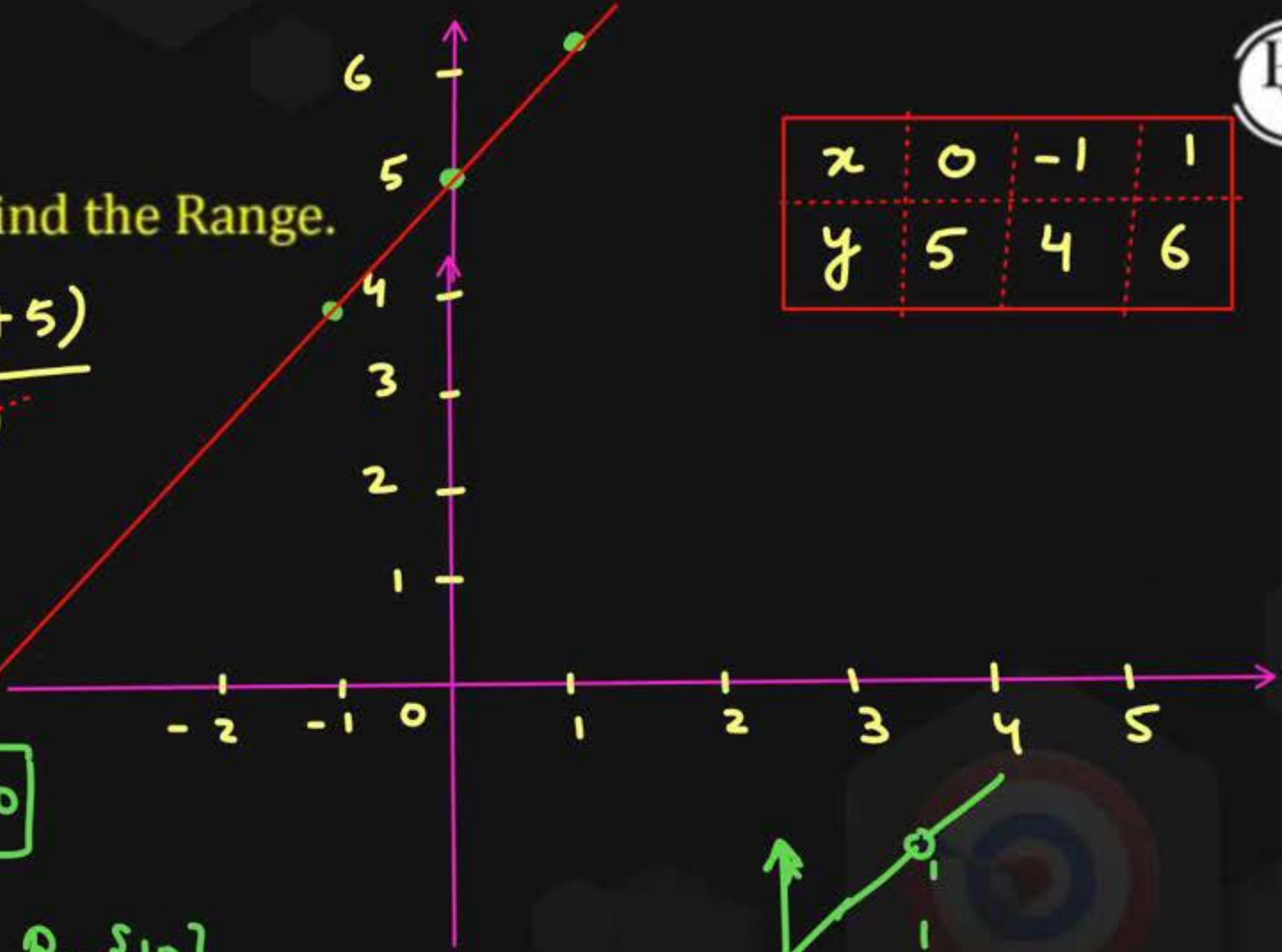
x	0	-1	1
y	5	4	6

$$\boxed{f(x) = x+5}$$

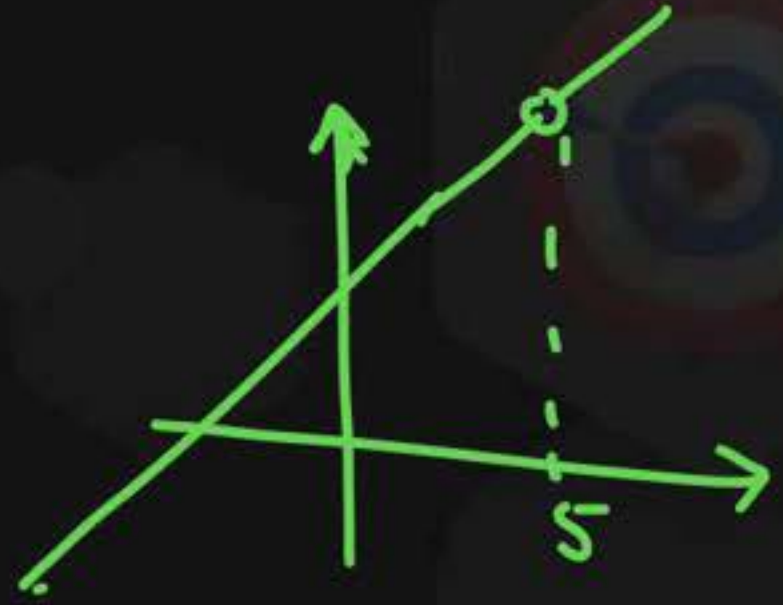
$$\Rightarrow y = x+5$$

Domain:
$x \in R - 5$

$$\boxed{x \neq 5,\ y \neq 10}$$



Range: $R - \{10\}$



9. Greatest Integer Function
(Step Function / Box Function)

$y = [x] \rightarrow$ represents greatest integer less than or equal to 'x'

Domain $x \in R$

Range : $y \in I$

1) $y = [x] = 0 \Rightarrow x \in [0, 1)$
2) $y = [x] = 1 \Rightarrow x \in [1, 2)$
3) $y = [x] = 2 \Rightarrow x \in [2, 3)$
4) $y = [x] = 3 \Rightarrow x \in [3, 4)$
5) $y = [x] = -1 \Rightarrow x \in [-1, 0)$
6) $y = [x] = -2 \Rightarrow x \in [-2, -1)$

$[\ ] \rightarrow$ square bracket

$[2.1] = 2$

$[10.9] = 10$ $\quad [1] = 1$

$[5] = 5$

$[-1.9] = -2$ ✓

$[100] = 100$

$[\pi] = 3$

$[e] = 2$

$[-100.1] = -101$

$[2.5] = 2$

$[0] = 0$

$[-10] = -10$

Question

Solve for x :

(i) $\boxed{[x] = 1}$ $\Rightarrow x \in [1, 2)$

(ii) $[x] = -4$ $\Rightarrow x \in [-4, -3)$

(iii) $\boxed{[x] = \sqrt{3}}$ $\Rightarrow x \in \phi$

(iv) $\boxed{[x-2] = 3}$ $\Rightarrow 3 \le x-2 < 4 \Rightarrow \boxed{5 \le x < 6}$

(v) $\left[x + \frac{3}{4}\right] = 1$ $\Rightarrow 1 \le x + 3/4 < 2 \Rightarrow 1 - 3/4 \le x < 2 - 3/4$

$1/4 \le x < 5/4$

$x \in [1/4, 5/4)$

(vi) $[|x|] = 5$

$|x| = t$

$[t] = 5 \Rightarrow t \in [5, 6)$

$|x| \in [5, 6) \Rightarrow x \in [5, 6) \cup (-6, -5]$

Question

Draw graph of $f(x) = |x-1| + |x-2|$

Case → 1

$\boxed{x \le 1}$

$y = -(x-1) - (x-2)$

$y = -x + 1 - x + 2$

$\boxed{y = 3 - 2x}$

x	1	0	−1
y	1	3	5

Case → 2

$\boxed{1 < x < 2}$

$y = x - 1 - (x-2)$

$y = \cancel{x} - 1 - \cancel{x} + 2$

$\boxed{y = 1}$

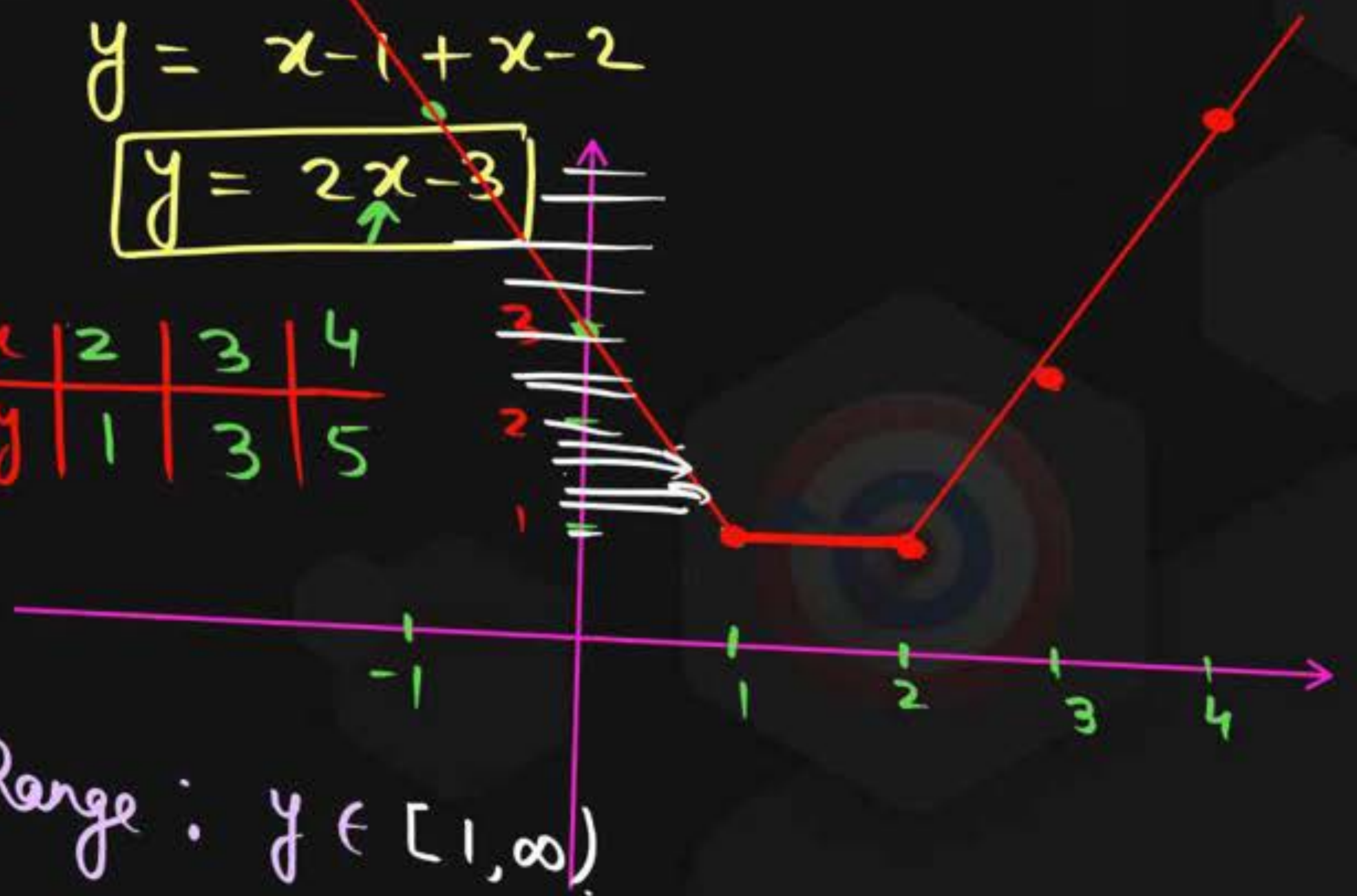
Case → 3

$\boxed{x \geq 2}$

$y = x - 1 + x - 2$

$\boxed{y = 2x - 3}$

x	2	3	4
y	1	3	5



Range : $y \in [1, \infty)$

Question

$$x^2 - 5x \times 0 + 6 = 0$$
$$x^2 - 0 + 6 = 0 \Rightarrow x^2 + 6 = 0$$

The equation $x^2 - 5x\,\text{sgn}(x^2 - 4) + 6 = 0$ has _ number of Solutions

A 1 ✓

B 0

C 2

D 3

Case → 1

$\text{sgn}(x^2-4) = 0$

$x^2 + 6 = 0$

$x^2 = -6$

Not possible

Case → 2

$\text{sgn}(x^2-4) = 1$

$x^2 - 5x + 6 = 0$

$(x-2)(x-3) = 0$

$\Rightarrow x = 2$ or $x = 3$

$\text{sgn}(4-4)$

$\text{sgn}(0) = 0$

$\Rightarrow x = 2$ is rejected

$\text{sgn}(9-4)$

$\text{sgn}(5) = 1$

accept $\boxed{x = 3}$

Case → 3

$\text{sgn}(x^2-4) = -1$

$x^2 + 5x + 6 = 0$

$(x+2)(x+3) = 0$

$x = -2$ or $x = -3$

$\text{sgn}(0) = 0$

$x = -2$ is rejected

$\text{sgn}(9-4)$

$\text{sgn}(5) = 1$

Reject

$\text{sgn}(x) = 1$ if $x > 0$

$\text{sgn}(0) = 0$

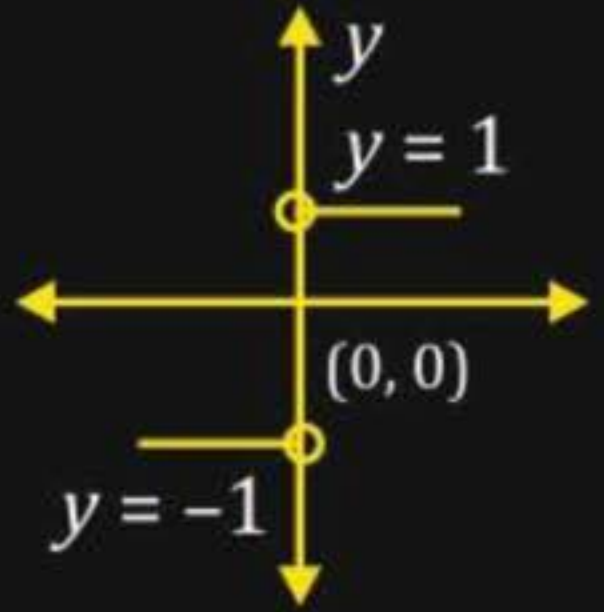
$\text{sgn}(-ve) = -1$

Question

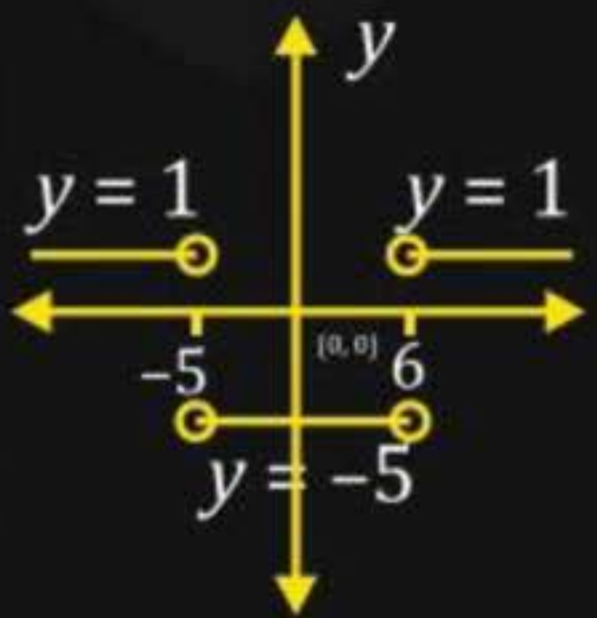
Which one of the following best represents the graph of function $f(x) = \frac{|x^2-x-30|}{x^2-x-30}$?

A

B



C



D

HW-BT-14

HW

[Ans.]

The number of ordered pairs (x, z) satisfying the equation $|x+2| = \sqrt{9-z^2} - 3$.

A 1

B 0

C 2

D 3

HW-BT-15

HW

Solve for real x, $\sqrt{\underbrace{x+2\sqrt{x-1}}_{\substack{\text{whole sq} \\ (a+b)^2}}}+\sqrt{\underbrace{x-2\sqrt{x-1}}_{(a-b)^2}}=2.$

HW-BT-16

[Ans. 0.80]

JEE ADVANCED 2022

In a study about a pandemic, data of 900 persons was collected. It was found that

190 persons had symptom of fever,

220 persons had symptom of cough,

220 persons had symptom of breathing problem,

330 persons had symptom of fever or cough or both,

350 persons had symptom of cough or breathing problem or both,

340 persons had symptom of fever or breathing problem or both,

30 persons had all three symptoms (fever, cough and breathing problem).

If a person is chosen randomly from these 900 persons, then the probability that the person has at most one symptom is ________

$$P = \frac{\text{Fav}}{\text{Total}}$$

Question

$-\frac{5/3}{2} = -\frac{5}{6}$

Find the range of

(i) $y = 2x^2 - 4x + 5$

$$y/2 = x^2 - 2x + 5/2$$

$$y/2 = (x-1)^2 - 1 + 5/2$$

$$y/2 = (x-1)^2 + 3/2$$

$$y = 2(x-1)^2 + 3$$

$$\boxed{y = 3 + 2(x-1)^2}$$

$$y \in [3, \infty)$$

(ii) $\boxed{y = \boxed{3}x^2 - 5x + 7}$

divide by 3

$$y/3 = x^2 - \frac{5}{3}x + 7/3$$

$$y/3 = \left(x - 5/6\right)^2 - \frac{25}{36} + \frac{7}{3} \times \frac{12}{12}$$

$$y/3 = \left(x - 5/6\right)^2 \frac{-25 + 84}{36}$$

$$y/3 = \left(x - 5/6\right)^2 + \frac{59}{36}$$

$$y = \frac{59}{12} + 3\underbrace{\left(x - 5/6\right)^2}$$

$y_{min} = 59/12$, $y_{max} \to \infty$

Question

$\boxed{x \in R}$

If x be real then the minimum value of $\underbrace{40 - 12x + x^2}$ is -

A 28

B 4 ✓

C –4

D 0

$y = x^2 - 12x + 40$

$y = (x-6)^2 - 36 + 40$

$y = (x-6)^2 + 4$

$\boxed{y = 4 + \underbrace{(x-6)^2}_{\min}}$

$y_{min} = 4 + 0$

$= 4$

Finding Range of a Quadratic using Completing the square method

$-2/2$

Case 2: $a < 0$

Question

Find the range of

(i) $\boxed{y = -2x^2 + 4x + 3}$

$$\frac{y}{-2} = x^2 + \frac{4x}{-2} + \frac{3}{-2}$$

$$\frac{y}{-2} = x^2 - 2x - 3/2$$

$$\frac{y}{-2} = (x-1)^2 - 1 - 3/2$$

$$\frac{y}{-2} = (x-1)^2 - 5/2$$

$$\boxed{y = 5 - 2(x-1)^2}$$

for y_{max} $(x-1)^2$ must be min

$\Rightarrow y_{max} = 5$

for $y_{min} \to -\infty$

$y \in (-\infty, 5]$

$a^{\log_a N} = N$

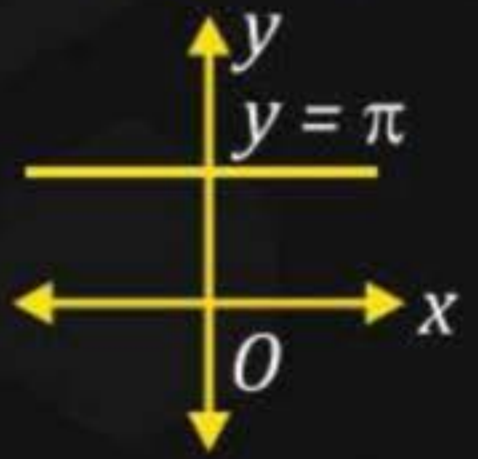
Question

Which one of function best represent the graph of $y = x^{\log_x \pi}$? $\Rightarrow \boxed{y = \pi}$

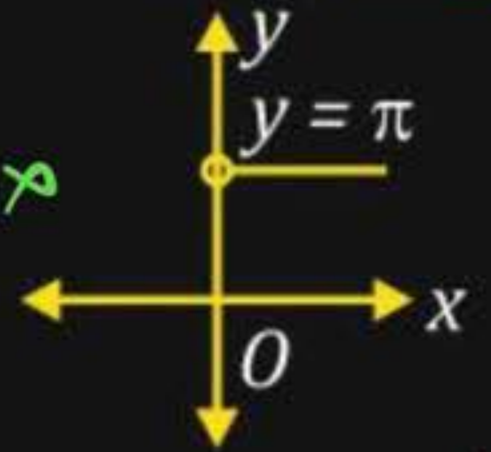
π

base $\rightarrow$ $\boxed{x > 0,\ x \neq 1}$

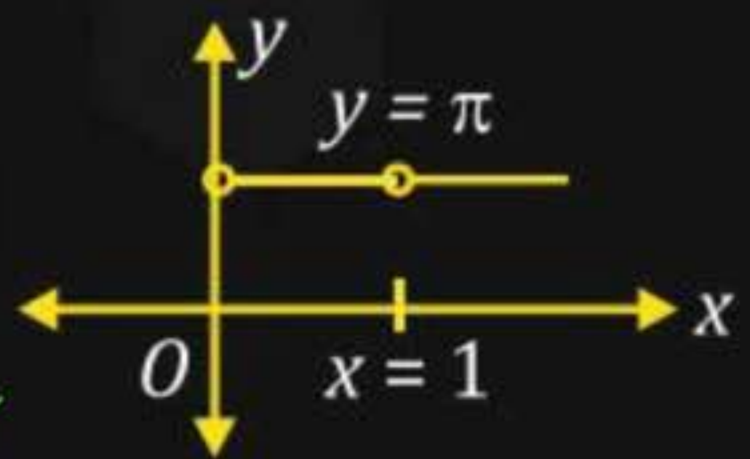
A



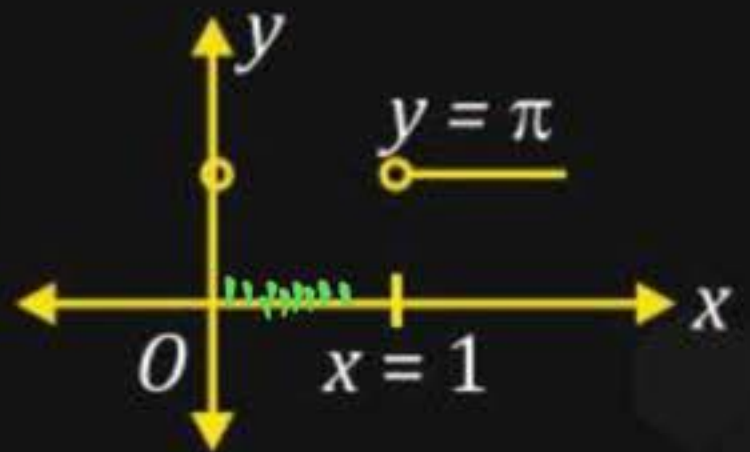
B



C



D



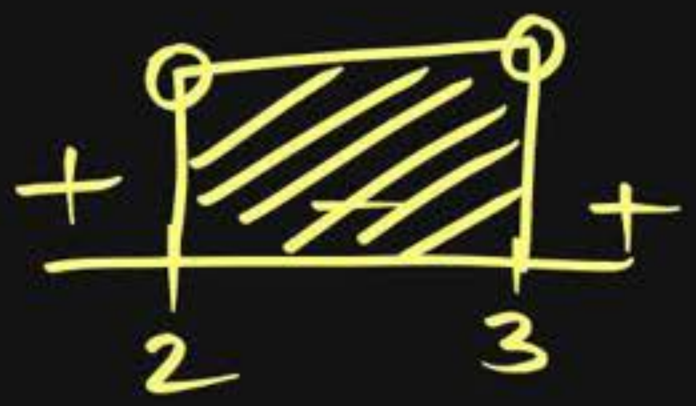
Q→ $\text{sgn}(\underbrace{x-3}) = 1$. Solve for x:

$x-3$ must be +ve

$x - 3 > 0$

$\Rightarrow x > 3$

$\Rightarrow x \in (3, \infty)$



Q→2 $\text{sgn}(\underbrace{x^2-5x+6}) = -1$.

$\Rightarrow x^2 - 5x + 6 < 0$

$(x-2)(x-3) < 0$ $\quad x \in (2,3)$ Ans

Homework

Find the range of the following function:

(i) $2x^2 + 12x - 9$

(ii) $3x^2 + 12x + 15$

(iii) $4x^2 + 16x - 3$

(iv) $2x^2 - 8x + 5$

(v) $3x^2 - 18x + 29$

(vi) $4x^2 - 16x + 12$

(vii) $-2x^2 + 4x - 11$

(viii) $-3x^2 + 18x - 20$

(ix) $-5x^2 - 60x - 75$

[Ans. (i) $[-27, \infty)$,
(ii) $[3, \infty)$,
(iii) $[-19, \infty)$,
(iv) $[-3, \infty)$,
(v) $[-38, \infty)$,
(vi) $[-9.75, \infty)$,
(vii) $(-\infty, -9]$,
(viii) $(-\infty, 7]$,
(ix) $(-\infty, 105]$]

Question

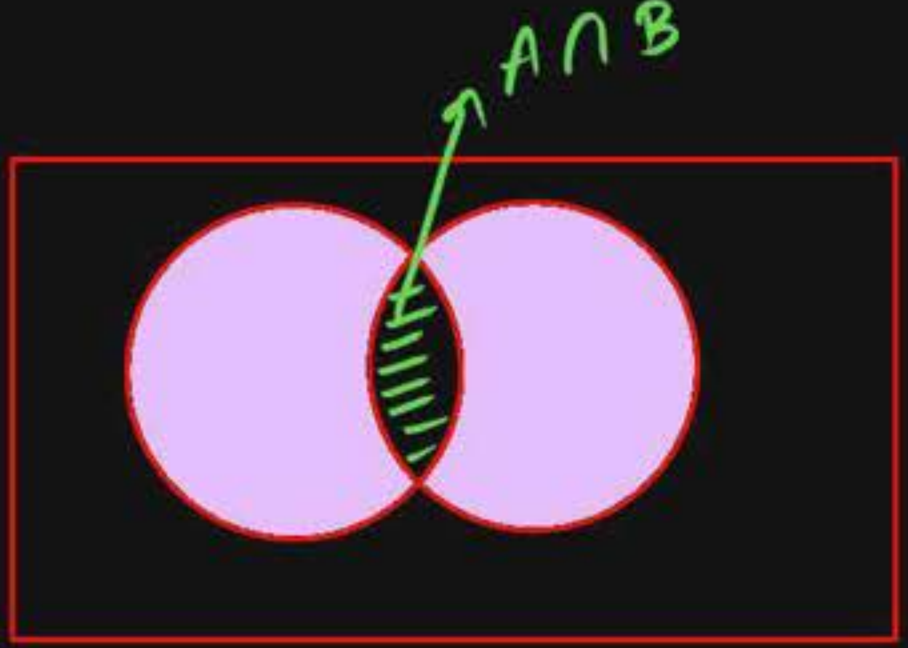
Which is equal to $(\underbrace{A - B}) \cup (\underbrace{B - A})$?

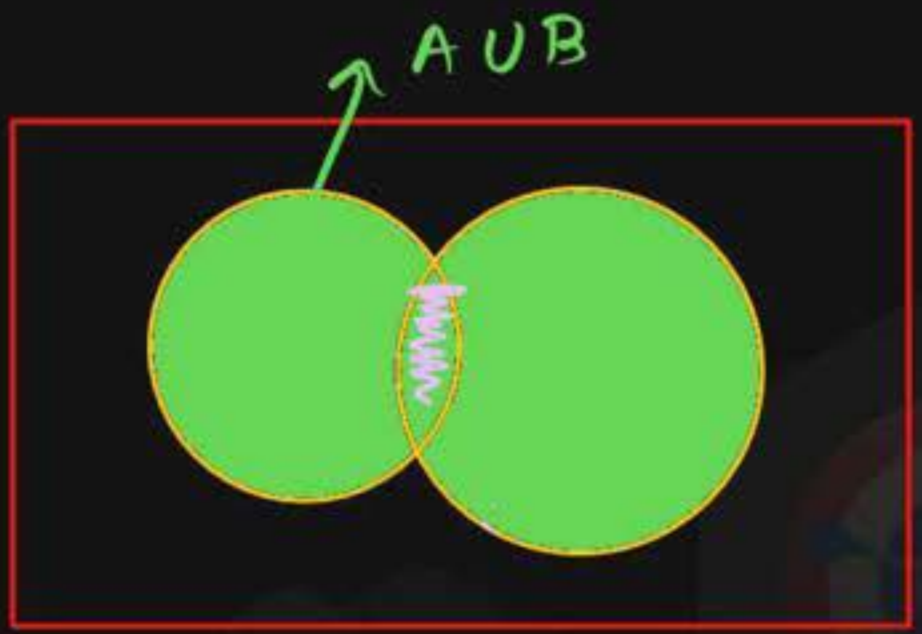
A $(A \cup B) \cup (A - B)$

B $(A \cup B) \cup (A \cap B)$

C $(\underbrace{A \cup B}) - (\underbrace{A \cap B})$

D $A - B$





Question

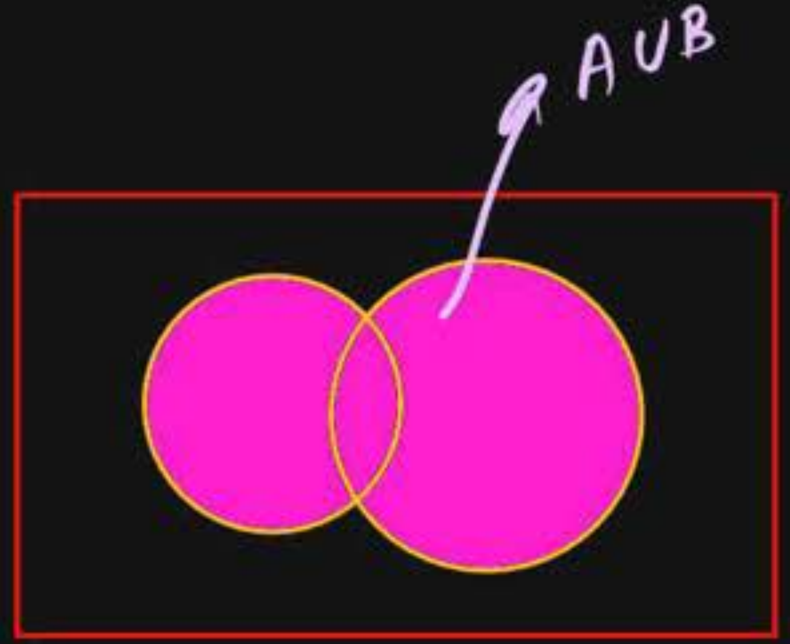
If A and B are two sets then $\underbrace{A \cap (A \cup B)}_{A}$ equals

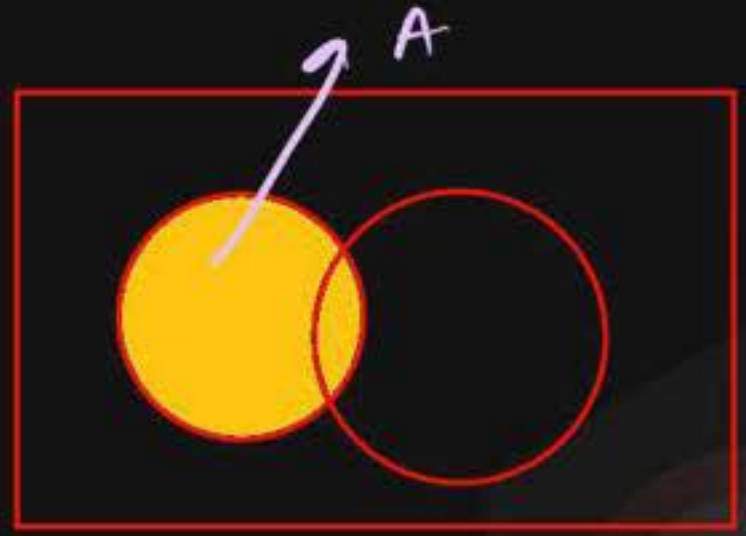
A. A ✓

B. B

C. ϕ

D. $A \cup B$





Question

Let A and B be two sets such that $n(A) = 24$, $n(A \cup B) = 46$ and $n(A \cap B) = 8$. Find

(i) $n(B)$ $= 30$

(ii) $n(A - B)$ $= 16$

(iii) $n(B - A)$ $= 22$

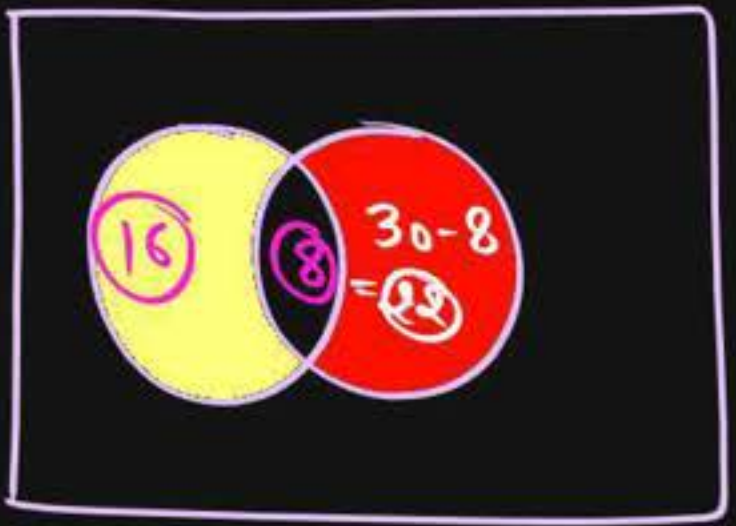
(iv) $n(A \triangle B) = 16 + 22$

$= \boxed{38}$

$$\underbrace{n(A \cup B)}_{46} = \underbrace{n(A)}_{24} + \underbrace{n(B)}_{x} - \underbrace{n(A \cap B)}_{8}$$

$$46 = 16 + x$$

$$\Rightarrow \boxed{x = 30}$$

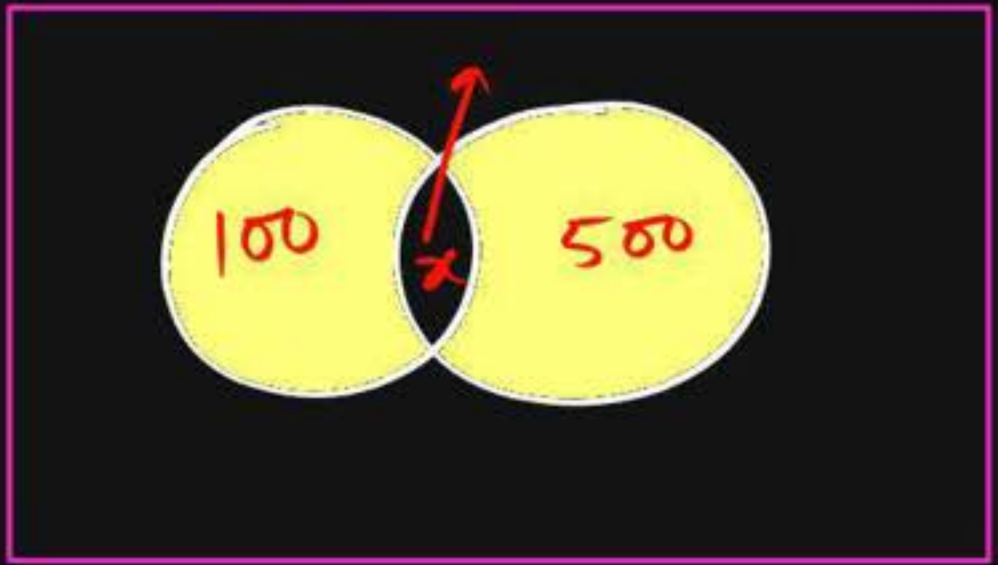


Question

If $n(A \cup B) = 1000$, $n(A - B) = 100$ & $n(B - A) = 500$ then what is the value of $n(A \cap B)$?

$100 + x + 500 = 1000$

$x = 400$



A 600

B 400

C 500

D 900

Question

If A, B and C are three sets and U is the universal set such that $n(U) = 700, n(A) = 200, n(B) = 300$ and $n(A \cap B) = 100$. Find $n(A' \cap B')$. $= n(A \cup B)'$

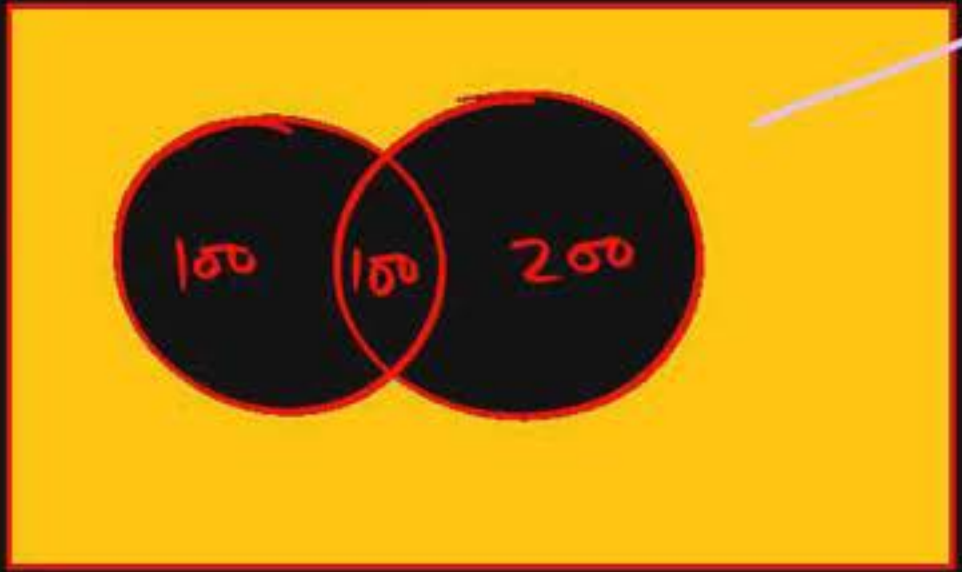
D.M.L

A 600

B 400

C 500

D None



$(A \cup B)' = 700 - 400$

$= 300$

Ans.

$n(A \cup B) = 100 + 100 + 200$

$= 400$

Question

OR → $\cup$ and → $\cap$

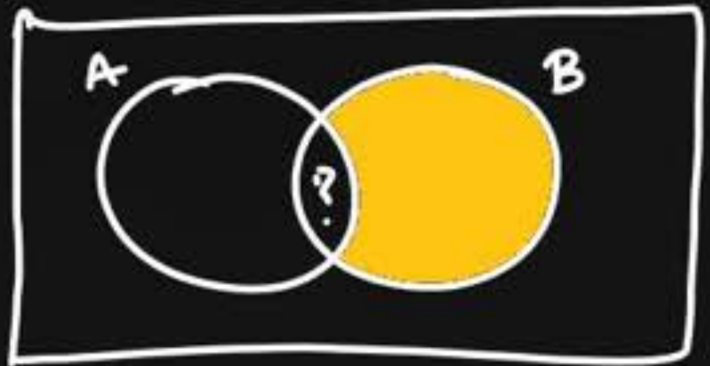
In a school there are 20 teachers who teach math's or chemistry. Out of these 12 teach math and 4 teach both math and chemistry. How many teach only chemistry?

A 10

B 12

C 8

D None of these



Maths → A
chem → B

$n(A \cup B) = 20$

$n(A) = 12$

$n(A \cap B) = 4$

$\underbrace{n(A \cup B)} = n(A) + n(B) - n(A \cap B)$

$20 = 12 + n(B) - 4$

$\Rightarrow 20 = 8 + n(B)$

$\Rightarrow \boxed{n(B) = 12}$

$\Rightarrow$ only chemistry $= 12 - 4 = 8$

Question

There are 200 individuals with a skin disorder, out of them 120 have been exposed to chemical C_1, 50 to chemical C_2 and 30 to both chemical C_1 and C_2 find the number of individuals exposed to neither C_1 nor C_2.

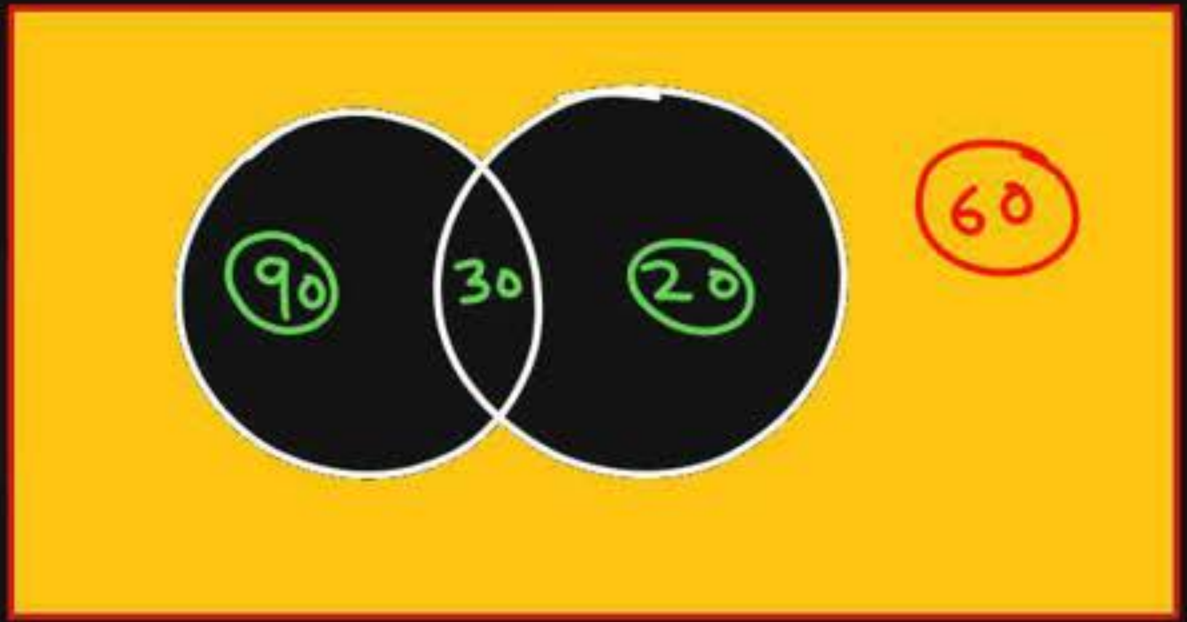
A: C_1 B: C_2

A 60

B 40

C 50

D 80



$n(A) = 120$, $n(B) = 50$

$n(A \cap B) = 30$

$n(A \cup B) = 90 + 50 = 140$

method → 2

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 120 + 50 - 30$

$n(A \cup B) = 140$

$n(A \cup B)' = ?$

$= 200 - 140$

$= 60$

Question

In a survey of 60 people, it was found that 25 people read newspaper H, 26 read newspaper T, 26 read newspaper I, 9 read both H and I, 11 read both H and T, 8 read both T and I, 3 read all three newspapers. Find:

(i) the number of people who read at least one of the newspapers.

$n(H \cup T \cup I) = ?$

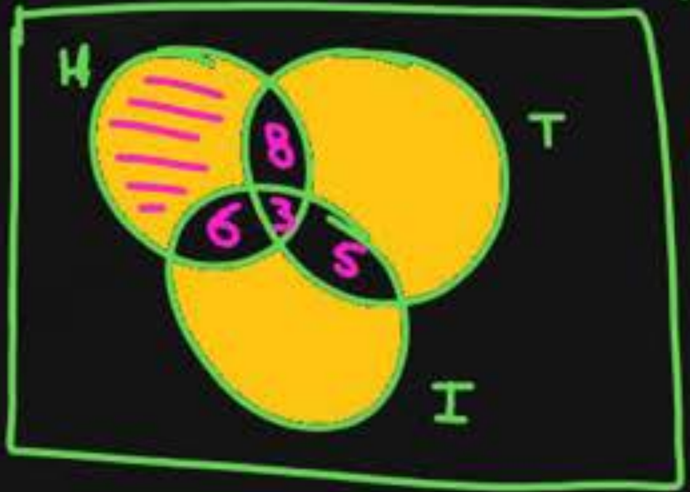
(ii) the number of people who read exactly one newspaper.

$n(H) = 25$
$n(T) = 26$
$n(I) = 26$
$n(H \cap I) = 9$
$n(H \cap T) = 11$
$n(I \cap T) = 8$
$n(H \cap T \cap I) = 3$

(i) $n(H \cup T \cup I) = (25+26+26) - (\underbrace{9+11+8}) + 3$

$= \cancel{25} + 52 - \cancel{25} = 52$

(ii)

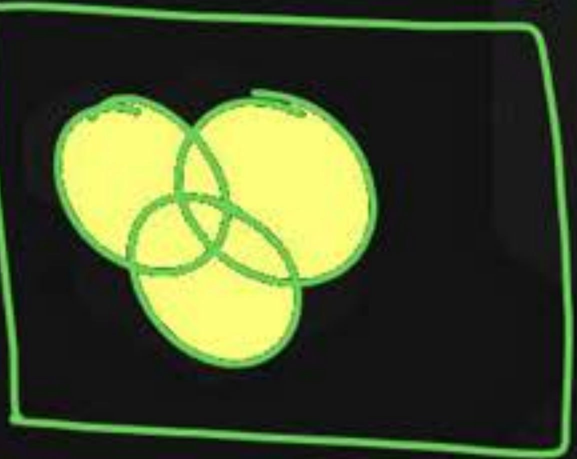


Exactly one Newspaper

$= 52 - [8+3+6+5]$

$= 52 - [22]$

$= 30$



$n(M \cup E \cup G) = 70$

Question

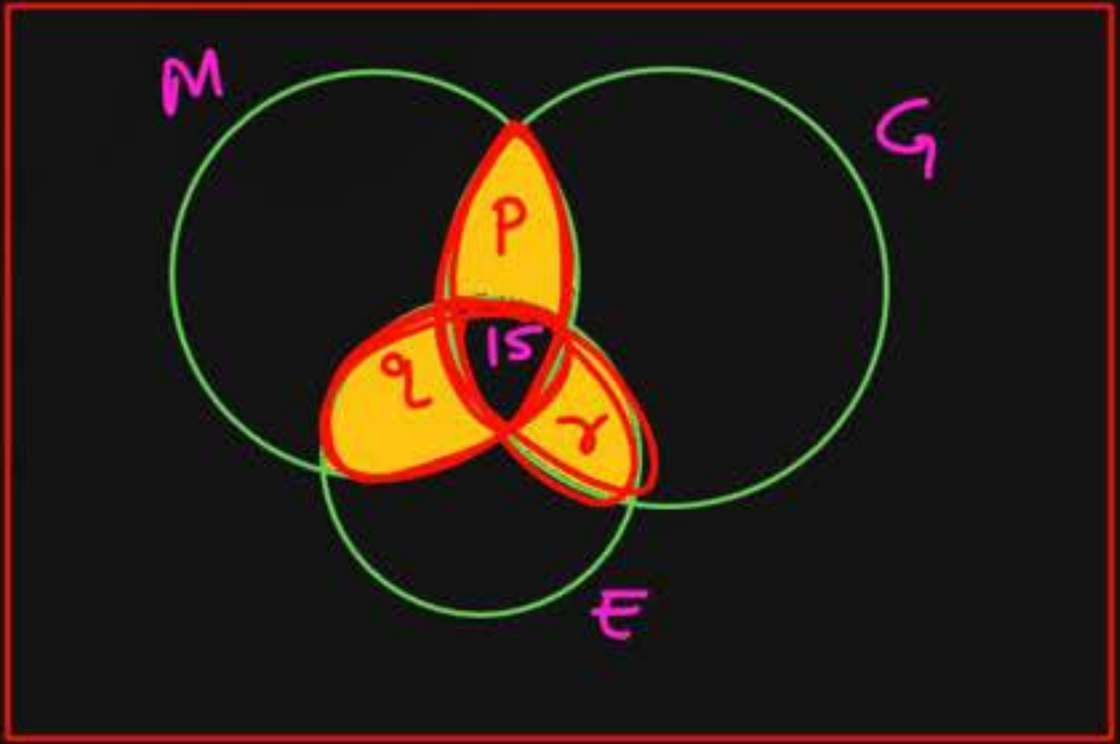
There are 70 students studying Math or English or German. 40 are studying Math, 30 studying German, 35 studying English and 15 in all three course. How many students are enrolled in exactly two of the courses ?

A 10

B 5

C 15

D 20



$n(M \cup G \cup E) = n(M) + n(G) + n(E) - n(M \cap G) - n(M \cap E) - n(E \cap G) + 15$

$70 = (40 + 30 + 35) - (x) + 15$

$x = 35 + 15 = 50$

$P + q + r = ?$

$x = 50 = P + 15 + q + 15 + r + 15$

$50 = 45 + P + q + r$

$\boxed{P + q + r = 5}$

Q JEE Main 2015 (Online)

$\underbrace{n(P\cap C)} = 2000$

In a certain town, 25% of the families own a phone and 15% own a car; 65% families own neither a phone nor a car and $\underbrace{2{,}000}$ families own both a car and a phone. Consider the following three statements :

(A) 5% families own both a car and a phone

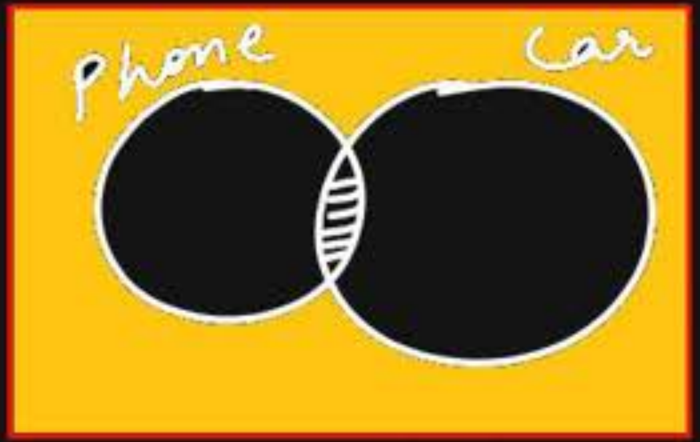
(B) 35% families own either a car or a phone

(C) 40,000 families live in the town

Then,

$n(P) = 25$

$n(C) = 15$



$n(P\cup C) = 35$

A Only (A) and (C) are correct.

B Only (B) and (C) are correct.

C All (A), (B) and (C) are correct.

D Only (A) and (B) are correct.

$\underbrace{n(P\cup C)} = \underbrace{n(P)} + n(C) - \underbrace{n(P\cap C)}_{x}$

$35 = 25 + 15 - x.$

$\Rightarrow x = 40 - 35 = 5\%$

Total Families $= y.$

5% of $y = 2000$

$\frac{5}{100} \times y = 2000$

$\Rightarrow y = 40{,}000$

Q Main April, 2019

The are two newspapers which are published in a city, namely A and B. It is known that 25% of the city population reads A and 20% reads B while 8% reads both A and B. Further 30% of those who read A but not B, read advertisement and 40% of those who read B but not A also read advertisements, while 50% of those who read both A and B read advertisements. What is the percentage of the population who read the advertisements?

$n(A) = 25, \quad n(B) = 20$

$n(A \cap B) = 8$

$n(A \cup B) = 25 + 20 - 8$

(A) 13.7 %

(B) 14.9 %

(C) 14.7 %

(D) 13.9 %

Total % Readings ads =

$$\frac{30}{100} \times 17 + \frac{40}{100} \times 12 + \frac{50}{100} \times 8$$

$$= 5.1 + 4.8 + 4$$

$$= 13.9\%$$

Brain Teaser

In a survey it was found that, 63% Indians like mangoes whereas 76% Like apples. If x % of Indians like both Apples and Mangoes then identify the correct Statement

A $x = 39$

B $x = 63$

C $39 \le x \le 63$ ✓

D Cannot be determined

$n(M) = 63$

$n(A) = 76$

$n(M \cap A) = x$

$\underbrace{n(M \cup A)} = \underbrace{63+76} - x$

$x = 139 - \underbrace{n(M \cup A)}$

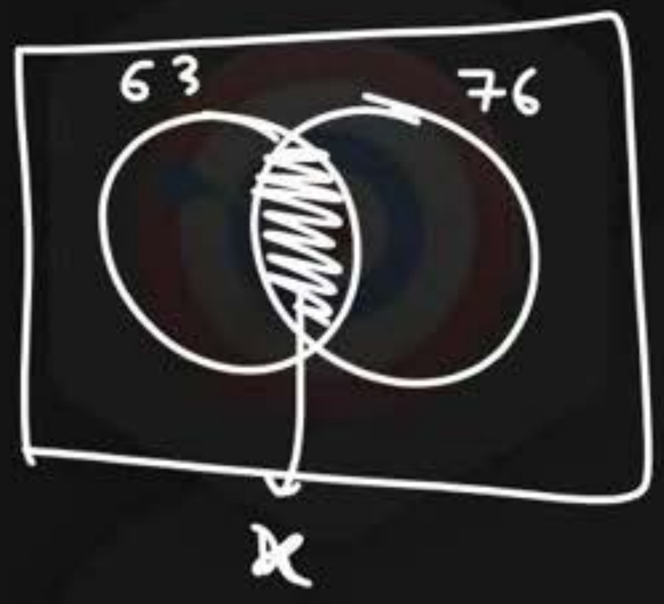
$x_{min} = 139 - 100$

$x_{min} = \boxed{39}$

$x_{max} = 63$

$n(M \cup A) = 100$ $(\times)$

$n(M \cup A)_{max} = 100$



Q Main Sep.04, 2020 (I)

[Ans.]

A survey shows that 63% of the people in a city read newspaper A whereas 76% read newspaper B. If x% of the people read both the newspapers, then a possible value of x can be:

$$\boxed{39 \le x \le 63}$$

A 29

B 37

C 65

D 55 ✓

HW-BT-9

HW

Find the product of roots of the equation

$\sqrt{2023}\, x^{\log_{2023} x} = x^2$

Ans. $(2023)^2$

HW-BT-10

HW

If A and B be two sets such that $n(A) = 15$, $n(B) = 25$, then number of elements in the range of $n(A \Delta B)$ (symmetric difference of A and B) is:

A 31

B 16

C 26

D 40

Question

If $A = \{a, b, c, d, e\}$ then the number of proper subsets are?

A 32

B 33

C 31

D None of These

$2^n - 1 = 2^5 - 1 = 32 - 1 = 31$

Question

NCERT:

Let $A = \{1,2,\{3,4\},5\}$. State TRUE or FALSE?

(i) $\{3,4\} \subset A \longrightarrow$ False

(ii) $\{3,4\} \in A \longrightarrow$ True

(iii) $\{\{3,4\}\} \subset A \longrightarrow$ True.

(iv) $1 \in A \longrightarrow$ True

(v) $1 \subset A \longrightarrow$ False.

(vi) $\{1,2,5\} \subset A \longrightarrow$ True.

(vii) $\{1,2,5\} \in A \longrightarrow$ False.

*(viii) $\{1,2,3\} \subset A \longrightarrow$ False.

(ix) $\phi \in A \longrightarrow$ False

(x) $\phi \subset A \longrightarrow$ True

(xi) $\{\phi\} \subset A \longrightarrow$ False

$A = \{1, 2, \underline{\{3,4\}}, 5\}$

$\{\{3,4\}\}$

XI^{th}

$A = \{\text{Ajay, Vijay, Mona, } \underline{XI^{th}(C)}\}$

$\{1, 2, \{3,4\}\} \subset A \rightarrow$ True

Power Set

Set of all subsets of A is called power set of A, It is denoted by P(A)

Question

Find power set of the following sets

1) $A = \{a, b\} \rightarrow P(A) = \{\phi, \{a\}, \{b\}, \{a, b\}\}$

2) $A = \{a, \{b, c\}\}$ $P(A) = \{\phi, \{a\}\ \{\{b,c\}\}, \{a, \{b,c\}\}\}$

$\{a, x\} \rightarrow \{\phi, \{a\}, \{x\}, \{a, x\}\}$

Question

no of subsets $= \underbrace{2^n} = 2^3 = \underline{\underline{8}}$.

$B \to P(A)$

If $A = \{1, 2, 4\}$ How many elements are present in

$n(B) = 8$

1) $P(A) =$ 8

$P(P(P(A))) = 2^{2^8}$

$P(B) = ?$

2) $P(P(A)) =$ 2^8

$\underline{\underline{2^8}}$.

A 9

B 8

C 2^8 ✓

D 81

Question

$\boxed{n=0}$

$n(P(A)) = 2^n = 2^0 = 1$

$A = \phi$

How many element has $P(A)$, if $A = \{\}$

$P(A) = \{\phi\}$

A 0

B 1 ✓

C 2

D 4

Question

$A = \{\phi, \{\phi\}\}$, then $P(A)$ is

(a) $\{\phi, \{\phi\}\}$

(b) $\{\phi, \{\phi\}, \{\{\phi\}\}\}$

(c) $\{\phi, \{\phi\}, \{\{\phi\}\}, \{\{\phi\}, \{\phi\}\}\}$

(d) $\{\phi, \{\phi\}, \{\{\phi\}\}, \{\phi, \{\phi\}\}\}$

$A = \{\phi, \{\phi\}\}$

$\phi \to 1$

$A = \{1, \{1\}\}$

$P(A) = \{\phi, \{1\}, \{\{1\}\}, \{1, \{1\}\}\}$

$1 \longrightarrow \phi$

$\{\phi, \{\phi\}, \{\{\phi\}\}, \{\phi, \{\phi\}\}\}$

Question

What is $P(\underbrace{P(\phi)}_{B})$

A $\{\phi, \{\phi\}\}$

B $\{\phi\}$

C $\{\phi, \{\phi\}, \{\{\phi\}\}\}$

D $\{\{\phi\}\}$

Power set of $\phi = \{\phi\}$.

$\phi \to 1$

$B = \{1\}$

$P(B) = \{\phi, \{1\}\}$

$P(B) = \{\phi, \{\phi\}\}$

$P(\phi) = 2^0 = 1$

$P(P(\phi)) = 2^n = 2^1 = 2$

Question

What is n(P(*P*(*P*(ϕ)))

A 1

B 2

C 4

D 8

$n\,P(\phi) = 2^0 = 1$

$n\,PP(\phi)) = 2^1$

$n\,PPP(\phi) = 2^n = 2^{2^1} = 2^2 = 4$

Question

If $X = \{4^n - 3n - 1; n \in \mathbb{N}\}$ and $Y = \{9(n-1): n \in \mathbb{N}\}$ Where $\mathbb{N}$ is a set of natural numbers. Then $X \cup Y$ is equal to:

A X

B Y

C N

D $Y - X$

$16-7$

$$\begin{array}{r} 64 \\ 10 \\ \hline 54 \end{array} \qquad \begin{array}{r} 256 \\ 13 \\ \hline 243 \end{array}$$

$X = \{1, 2, 3, 4\}$
$Y = \{1, 2\}$
$X \cup Y = X$

$Y = \{9(1-1), 9(2-1), 9(3-1), 9(4-1), \ldots\}$

$Y = \{0, 9, 18, 27, \ldots 54 \ldots 243\}$

$X = \{(4^1 - 3 - 1), (4^2 - 6 - 1), (4^3 - 9 - 1), (4^4 - 13), \ldots\}$

$X = \{0, 9, 54, 243 \ldots\}$

$\Rightarrow X \cup Y = \{0, 9, 18, 27, \ldots\}$

$X \cup Y = Y$

Question

$A = \{x : x = 2n, \underbrace{n \leq 3}, \underbrace{n \in \mathbb{N}}\}$ $A = \{2, 4, 6\}$.

$B = \{x : x = 3n, n \leq 3, n \in \mathbb{N}\}$ $B: \{3, 6, 9\}$.

Find $A \cup B$ and $A - B$:

$A \cup B: \{2, 4, 6, 3, 9\}$

$A - B: \{2, 4\}$.

Question

Find $A - (A - B)$
if $A = \{1, 4, 7, 10, 13\}$ and
$B = \{2, 4, 6, 8, 10, 12, 14, 16\}$

Let $C = A - B$

$C = \{\underbrace{1, 7, 13}\}$

$A - (\underbrace{A - B}_{C})$

$(A - C) = \{4, 10\}$ Ans.

Question

If universal set $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and $A = \{1, 2, 3, 4\}$, $B = \{2, 3, 5, 6\}$, $C = \{2, 3, 7\}$, then find

(i) $A' \longrightarrow \underbrace{\{0, 5, 6, 7, 8, 9\}}$

(ii) $(A')' \longrightarrow A = \{1, 2, 3, 4\}$

(iii) $\underbrace{B' - A'}$ $\quad B' = \{0, 1, 4, 7, 8, 9\}$ $\quad A' = \{0, 5, 6, 7, 8, 9\}$

$B' - A' = \{1, 4\}$

(iv) $A' \cap C = \{7\}$

common

Q [JEE Main 2020 (Jan.)] [Ans.]

Let $X = \{n \in N: 1 \le n \le 50\}$. If $A = \{n \in X: n$ is a multiple of 2); $B = \{n \in X: n$ is a multiple of 7}, then the number of elements in the smallest subset of X containing both A and B is

$$X = \{1, 2, 3, 4, 5, \ldots 50\}$$
$$A = \{2, 4, 6, 8, \ldots 50\}$$
$$B = \{7, 14, 21, 28, 35, 42, 49\}$$

$$A \cup B = \{\underbrace{2, 4, 6, 8, \ldots 50}_{25 \text{ elements}}, \underbrace{7, 21, 35, 49}_{4}\}$$

$$25 + 4 = \boxed{29}$$

Ans = 29

$X = \{1, 2, 3, 4, 5\}$
$A = \{1, 2, 3\}$
$B = \{2, 3, 4\}$
$A \cup B : \{1, 2, 3, 4\}$

HW-BT-7

Solve for x :

$$\log 4+\left(1+\frac{1}{2x}\right)\log 3=\log\left(\sqrt[x]{3}+27\right)$$

HW-BT-8

HW

Find x satisfying the equation

$$\log^2\left(1+\frac{4}{x}\right)+\log^2\left(1-\frac{4}{x+4}\right)=2\log^2\left(\frac{2}{x-1}-1\right).$$

Ans. $x=\sqrt{2}$ or $\sqrt{6}$

Question

$2^x \rightarrow$ always Even

$2^x - 1 \rightarrow$ always odd.

Write the following sets in the roster form.

(i) $A = \{x \mid x$ is a positive integer less than 10 and $2^x - 1$ is an odd number$\}$

$A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

(ii) $B = \{x : x^2 + 7x - 8 = 0, x \in R\}$

$x^2 + 8x - x - 8 = 0$

$(x+8)(x-1) = 0$

$x = -8$ or $x = 1$

$B = \{-8, 1\}$

Question

The set of the Natural numbers which are multiples of 6 and less than 50 is written in set builder form as

$A = \{6, 12, 18, 24, 30, 36, 42, 48\}$

(A) $\{x: x \in \mathrm{N}$ and x is a multiple of $6\}$

(B) $\{x: x = 6$ and $0 < x < 50\}$

(C) $\{x: x$ is a multiple of 6 and $x \leq 50\}$

(D) $\{x: x$ is a multiple of 6 and $0 < x < 50, x \in \mathrm{N}\}$

Question

Which of the following pairs of sets are equal? Given reasons.

Equal → Yes

not equal → No.

(i) $A = \{\underbrace{0, 2}\}$ and $B = \{x: x \text{ is root of the equation } \underbrace{x^2 - 2x} = 0\}$

Yes

$x(x-2) = 0$

$x = 0$ or $x - 2 = 0$

$\Rightarrow x = 2$ $\{0, 2\}$

(ii) $A = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$ and $B = \{x: x \in Z \text{ and } |x| < 4\}$

$B = \Rightarrow -4 < x < 4$

-4 4

$B = \{-3, -2, -1, 0, 1, 2, 3\}$

not equal

Question

Which of the following may be considered as universal set(s) for all the three sets

$X = \{a, c\}, Y = \{a, e, i, o, u\}, Z = \{i, j, k, l\}$

(A) $\{a, b, c, d, \ldots\ldots m\}$ ✗

(B) $\{a, e, i, o, u, k, l\}$ ✗

(C) $\{x : x$ is a letter of English alphabet$\}$

{a to z}

Question

Examine whether the following statements are true or false:

(i) $\{a, b\} \not\subset \{b, c, a\}$ → False.

(ii) $\{a, e\} \subset \{x : x$ is a vowel in the English alphabet$\}$ $\{a, e, i, o, u\}$. True.

(iii) $\{1, 2, 3\} \subset \{1, 3, 5\}$ → False.

(iv) $\{a\} \subset \{a, b, c\}$ → True.

(v) $\{a\} \in \{a, b, c\}$ → False.

(vi) $\{x : x$ is an even natural number less than $6\} \subset \{x : x$ is a natural number which divides $36\}$

$\{2, 4\} \subset \{1, 2, 3, 4, 6, 12, 18, 36\}$

True

Question

$n(A)=3$

$n(B)=6$

If A and B be two sets containing 3 and 6 elements respectively, what can be the maximum number of elements in $A \cup B$?

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$= 3 + 6 - n(A \cap B)$

$\Rightarrow \underbrace{n(A \cup B)}_{\text{maximum}} = 9 - n(A \cap B)$

min $= 0$

A 9

B 6

C 3

D Cant say

$\Rightarrow n(A \cup B)_{max} = 9 - 0$
$= 9$

Find also, the minimum number of elements in $A \cup B$.

$(A \cup B)$ is min when $A \cap B$ is max

$(A \cap B)_{max} = 3$

$(A \cup B)_{min} = 9 - 3 = 6$

A 9

B 6

C 3

D Cant say

Q [JEE Main (March)-I]

[Ans.]

In a school, there are three types of games to be played. Some of the students play two types of games, but none play all the three games. Which Venn diagrams can justify the above statement?

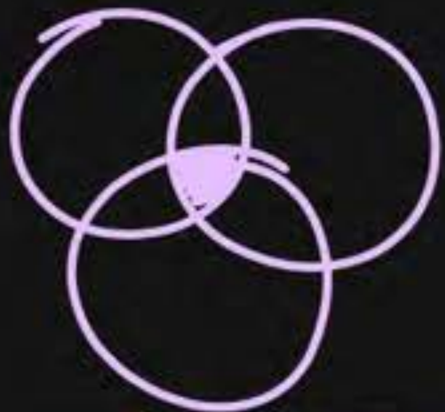
P Q R

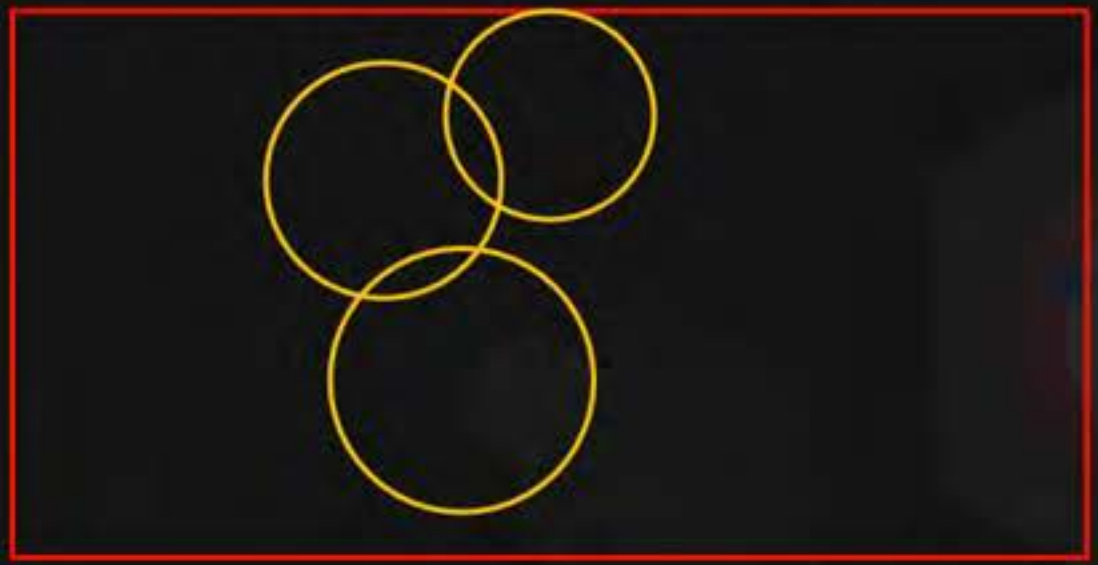
(A) P and Q

(B) P and R

(C) Q and R

(D) None of these





Question

Let $A = \{1,2,3\}$, $B = \{3,4\}$ and $C = \{4,5,6\}$. Find the set

$(A \times B) \cap (A \times C)$

Ans $= \{(1,4), (2,4), (3,4)\}$

A: 1, 2, 3 B: 3, 4

$A \times B = \{(1,3), (1,4), (2,3), (2,4), (3,3), (3,4)\}$

A: 1, 2, 3 C: 4, 5, 6

$A \times C = \{(1,4), (1,5), (1,6), (2,4), (2,5), (2,6), (3,4), (3,5), (3,6)\}$

Question

$x^2 - 5x + 6 = 0$

$(x-3)(x-2) = 0 \Rightarrow x = 2$ or 3

$A = \{2, 3\}$

If $A = \{x : x^2 - 5x + 6 = 0\}$, $\boxed{B = \{2, 4\}, C = \{4, 5\}}$ then $A \times \underbrace{(B \cap C)}$ is

A $\{(2, 4), (3, 4)\}$

B $\{(4, 2), (4, 3)\}$

C $\{(2, 4), (3, 4), (4, 4)\}$

D $\{(2, 2), (3, 3), (4, 4), (5, 5)\}$

A: 2, 3

$B \cap C$: 4

Ans $\{(2, 4), (3, 4)\}$

Question

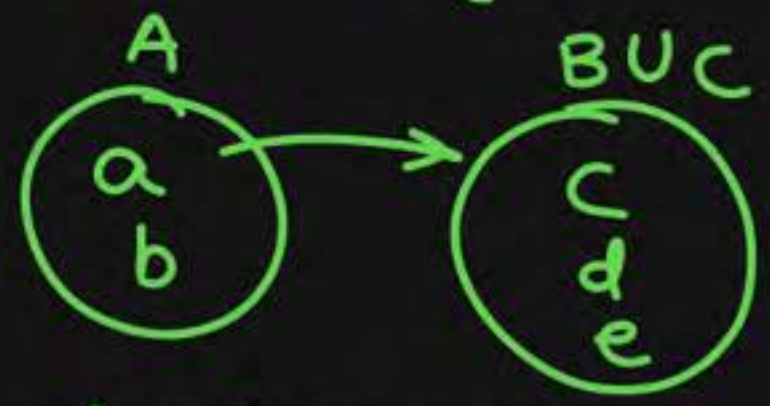
If $A = \{a, b\}$, $B = \{c, d\}$, $C = \{d, e\}$ then $\{(a, c), (a, d), (a, e), (b, c), (b, d), (b, e)\}$ is equal to

A. $A \cap (B \cup C)$

B. $A \cup (B \cap C)$

C. $A \times (B \cup C)$

D. $A \times (B \cap C)$



A×(B∪C)

Q JEE Main 2019

$n(A\times B) = n(A)\cdot n(B)$
$= 3\times 5 = 15$

Subset $= 2^{15}$

Let Z be the set of integers. If $A = \{x \in Z : 2^{(x+2)(x^2-5x+6)} = 1\}$ and $B = \{x \in Z : -3 < 2x - 1 < 9\}$, then the number of subsets of the set $A \times B$, is:

A 2^{18}

B 2^{15}

C 2^{12}

D 2^{10}

A: $(x+2)(x^2-5x+6)=0$

$(x+2)(x-2)(x-3)=0$

A: $x = -2, 2, 3$ $\quad n(A)=3$

$A\ \{-2, 2, 3\}$

If $a^b = 1$
$b=0$ or $a=1$

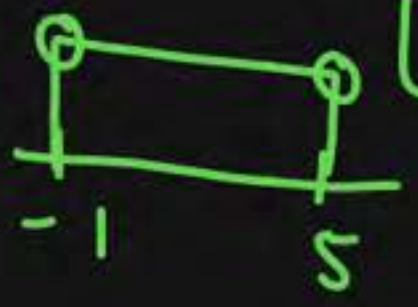
B: $-3 < 2x-1 < 9$

add 1

$-2 < 2x < 10$

divide by 2

$\boxed{-1 < x < 5}$



$n(B)=5$

$B = \{0, 1, 2, 3, 4\}$

Brain Teaser

Two finite sets have m and n elements respectively. The total number of subsets of first set is 992 more than the total number of subsets of the second set. Find the value of m & n.

Subsets of I^{st} set $= 2^m$

Subsets of 2^{nd} set $= 2^n$

$$\boxed{2^m = 992 + 2^n}$$

Method → 1

$2^m - 2^n = 992$, $m \& n \in N$

$2^{10} - 2^5 = 992$

$$\boxed{m = 10 \ \& \ n = 5}$$

$2^1 = 2$
$2^2 = 4$
$2^3 = 8$
$2^4 = 16$
$2^5 = 32$
$2^6 = 64$
$2^7 = 128$
$2^8 = 256$
$2^9 = 512$
$2^{10} = 1024$

$$\begin{array}{r} 1024 \\ 32 \\ \hline 992 \end{array}$$

PYQ [JEE Main 2020 (Sep.)] [Ans. [redacted]

Set A has m elements and Set B has n elements. If the total number of subsets of A is 112 more than total number of subsets of B, then the value of $\underbrace{m \cdot n}_{28}$ is

$$2^m = 112 + 2^n$$

$$2^m - 2^n = 112$$

$$2^7 - 2^4 = 112$$

$$\begin{array}{r} 128 \\ 16 \\ \hline 112 \\ \hline \end{array}$$

$$m = 7, \; n = 4$$

PYQ [JEE Main 2022 (June)-II]

[Ans. B]

Let $A = \{x \in R : |x+1| < 2\}$ and $B = \{x \in R : |x-1| \geq 2\}$. Then which one of the following statements is **NOT true**?

(A) $A - B = (-1, 1)$

(B) $B - A = R - (-3, 1)$

(C) $A \cap B = (-3, -1]$

(D) $A \cup B = R - [1, 3)$

A: $|x+1| < 2$

$-2 < x+1 < 2$

$\boxed{A: \ -3 < x < 1}$

B: $|x-1| \geq 2$

$x-1 \geq 2$ or $x-1 \leq -2$

$x \geq 3$ or $x \leq -1$

$\boxed{B: \ (-\infty, -1] \cup [3, \infty)}$

(1) $|x| < a$

$\Rightarrow -a < x < a$

(2) $|x| \geq a$

$x \geq a$ or $x \leq -a$

PYQ [JEE Main 2020 (Jan.)] HW [Ans. [redacted]]

If $A = \{x \in R : |x| < 2\}$ and $B = \{|x - 2| \geq 3\}$: then:

A $A - B = [-1,2)$

B $A \cup B = R - (2,5)$

C $B - A = R - (-2, 5)$

D $A \cap B = (-2, -1)$

PYQ [JEE Main 2020]

$3^x > 0$

[Ans.]

Solution set of $3^x(3^x - 1) + 2 = |3^x - 1| + |3^x - 2|$ contains

A Singleton set

B Two elements

C At least four elements

D Infinite elements

Let $3^x = t > 0$

$t(t-1) + 2 = |t-1| + |t-2|$

$t^2 - t + 2 = |t-1| + |t-2|$

1 2

Case → 1: $t < 1$

$t^2 - t + 2 = 1 - t + 2 - t$

$t^2 - t + 2 = 3 - 2t$

$t^2 + t - 1 = 0$

$t = \frac{-1 \pm \sqrt{1+4}}{2\times 1} = \frac{-1 \pm \sqrt{5}}{2}$

$\boxed{t = \frac{-1+\sqrt{5}}{2}}$

Case → 2: $1 \le t < 2$

$t^2 - t + 2 = \cancel{t} - 1 + 2 - \cancel{t}$

$t^2 - t + 2 = 1$

$t^2 - t + 1 = 0$

$D < 0$

no solⁿ

Case → 3: $t \geq 2$

$t^2 - t + 2 = t - 1 + t - 2$

$t^2 - t + 2 = 2t - 3$

$t^2 - 3t + 5 = 0$

$D = 9 - 4\times 5 = -ve$

no solⁿ

HW-BT-11

HW

From 50 students taking examinations in Mathematics, Physics and Chemistry, each of the student has passed in at least one of the subject, 37 passed Mathematics, 24 Physics and 43 Chemistry. At most 19 passed Mathematics and Physics, at most 29 Mathematics and Chemistry and at most 20 Physics and Chemistry. What is the largest possible number that could have passed all three examination?

A 600

B 400

C 500

D 900

HW-BT-12

HW

Out of 14 people, twelve said that it was not the case that they watched T.V. but did not listen to the radio. Also, for nine people it is not the case that they do not watch T.V. and do not listen to the radio. Finally, seven people either watch T.V. or listen to the radio but do not do both. Now let A be number of people that watch T.V. and B be the number of people that listen radio. Then

A $A = 2$

B $B = 5$

C $A = 4$

D $B = 7$

Brain Teaser

Solve $|\underbrace{x^2 + 2x - 3}_{\alpha}| + |\underbrace{1 - 2x}_{\beta}| = |x^2 - 2|$

$$\alpha = x^2 + 2x - 3$$

$$\oplus \quad \beta = 1 - 2x$$

$$\alpha + \beta = x^2 - 2$$

$\Rightarrow$ Given $\boxed{|\alpha| + |\beta| = |\alpha + \beta|}$

$\Rightarrow \alpha\beta \geq 0$

$\Rightarrow (x^2 + 2x - 3)(1 - 2x) \geq 0$

$(x+3)(x-1)(1-2x) \geq 0$

$-(x+3)(x-1)(2x-1) \geq 0$

$\Rightarrow (x+3)(x-1)(2x-1) \leq 0$

Ans: $x \in (-\infty, -3] \cup [\tfrac{1}{2}, 1]$.

Question

$\sin^3\theta = (\sin\theta)^3$

$\log_b^3 x = (\log_b x)^3$

$\log_b^2 x = (\log_b x)^2$

Find the value of the following

1) $\log_2 16 =$ 4

2) $\log_3 81 =$ 4

3) $\log_{1000} 100 =$ $y \Rightarrow 100 = (1000)^y \Rightarrow (10)^2 = [(10)^3]^y$

$\Rightarrow (10)^2 = (10)^{3y} \Rightarrow 2 = 3y$

$\Rightarrow y = 2/3$.

4) $2\underbrace{\log_{10} 100}_{2} =$ 4 $\quad \log_{10} 100 = y \Rightarrow 100 = 10^y \Rightarrow y = 2$

5) $\log_{10} 10^3 =$ $y \Rightarrow (10)^3 = (10)^y \Rightarrow y = 3$

6) $\log^3_{10} 10 =$ $(\underbrace{\log_{10} 10}) ^3 = (1)^3 = 1$

let $\log_{10} 10 = y \Rightarrow 10^1 = 10^y \Rightarrow y = 1$

(7) $\log^2_3 27 = (\underbrace{\log_3 27})^2 = (3)^2$
$= 9$

$\log_3 27 = x$

$\Rightarrow 27 = 3^x \Rightarrow \boxed{x = 3}$

Question

Simplify the Following

i) $\log_{10} 2 + \log_{10} 5$ → A) 2 → B) 1

$\log_{10}(2\times5) = \log_{10} 10 = 1$

(ii) $\log_{10} 25 + \log_{10} 4 - \log_{10} 10$ → A = 1 → B = 3

$\Rightarrow \log_{10}\left(\frac{25\times4}{10}\right) = \log_{10} 10 = 1$

(iii) $\log_{39}\frac{15}{7} + \log_{39}\frac{13}{3} - \log_{39}\frac{5}{21}$ → A = 3 → B = 2 → C = None

$\log_{39}\left(\frac{\frac{15\times13}{7\times3}}{5/21}\right)$

$\log_{39}\left[\frac{\cancel{15}^{3}\times13}{\cancel{7\times3}}\times\frac{\cancel{21}}{\cancel{5}}\right]$

$\log_{39} 39 = 1$

(iv) $\log_3 243$ → A = 4 → B = 5 → C = None

$\log_3 243 = y$

$243 = (3)^y$

$(3)^5 = (3)^y$

Question

Simplify the Following

(v) $\log_5(\log_3 3)$

$\log_5 1 = 0$

$\rightarrow A = 1$

$\rightarrow B = 0$ ✓

$\rightarrow C = \text{None}.$

(vi) $2\log_3 81 + \log_5 125$

$\underbrace{2\log_3 81}_{8} + \underbrace{\log_5 125}_{3}$

$\rightarrow A = 3$

$\rightarrow B = 4$

$\rightarrow C = 7$

$\rightarrow D = 11$ ✓

(HW)

(vii) $2\log_6 2 + 3\log_6 3 + \log_6 12$

$2\log_3 (3)^4$

$8\log_3 3$

$= 8 \times 1 = 8$

$\log_5 125 = \log_5 (5)^3$

$= 3\log_5 5$

$= 3$

Question (HW)

Find the value of :

$$\log_2 [\log_2 \{ \log_3 (\underbrace{\log_3 27^3}) \}]$$

A 1

B 2

C 3

D 0

Question (HW)

	Column I		Column II
(A)	$\dfrac{\log_3 243}{\log_2 \sqrt{32}}$	(p)	positive integer
(B)	$\dfrac{2\log 6}{(\log 12 + \log 3)}$	(q)	negative integer
(C)	$\log_{1/3}\left(\dfrac{1}{9}\right)^{-2}$	(r)	rational but not integer
(D)	$\dfrac{\log_5 16 - \log_5 4}{\log_5 128}$	(s)	prime

PYQ [JEE Mains 2020]

General Method

Better method

$|x|^2 = x^2$

The product of the roots of the equation $9x^2 - 18|x| + 5 = 0$, is :

A 25/9

B 5/27

C 25/81

D 8/9

Case → 1 $x \geq 0$

$9x^2 - 18x + 5 = 0$

$9x^2 - 15x - 3x + 5 = 0$

$(3x-1)(3x-5) = 0$

$3x - 1 = 0 \Rightarrow x = 1/3$

OR

$3x - 5 = 0 \Rightarrow x = 5/3$

Case → 2 $x < 0$

$9x^2 - 18(-x) + 5 = 0$

$9x^2 + 18x + 5 = 0$

$9x^2 + 15x + 3x + 5 = 0$

$(3x+5)(3x+1) = 0$

$x = -5/3$

or

$x = -1/3$

Product $= \frac{1}{3} \cdot \frac{5}{3} \cdot \frac{5}{3} \cdot \frac{1}{3} = \frac{25}{81}$

Question 8_DPP-3

$(x-3)^2 + |x-3| - 11 = 0$, the sum of solutions of this equation is ?

A 5

B 6 ✓

C 7

D 8

$|x-3| = 2$

$x - 3 = \pm 2$

$|x-3| = -5$ ✗

Trick.

$(x-3)^2 = |x-3|^2$

$|x-3|^2 + |x-3| - 11 = 0$

Let $|x-3| = t$

$\boxed{t^2 + t - 11 = 0}$

$D = b^2 - 4ac$

$= (1)^2 - 4(1)(-11)$

$= 1 + 44 = 45$

$t = \dfrac{-b \pm \sqrt{D}}{2a}$

$t = \dfrac{-1 \pm \sqrt{45}}{2 \times 1}$

$t = \dfrac{-1 \pm 3\sqrt{5}}{2}$

$|x|^2 = x^2$

+ve

$|x-3| = t = \underbrace{\dfrac{-1 + 3\sqrt{5}}{2}}_{①}$ or $\underbrace{\dfrac{-1 - 3\sqrt{5}}{2}}_{②}$ → -ve.

Reject.

$|x-3| = \dfrac{3\sqrt{5} - 1}{2}$ ✓ → a

$|x-3| = a$

$\Rightarrow x - 3 = \pm a$

$\Rightarrow x = 3 \pm a$

$x_1 = 3 + \cancel{a}$

$\oplus \; x_2 = 3 - \cancel{a}$

$x_1 + x_2 = 6$

Question

Set of values of x satisfying $\frac{|x|-1}{|x|+2} > 0$ is

A $(-\infty,-2) \cup (2,\infty)$

B $(-2,-1) \cup (1,2)$

C $(-\infty,-1) \cup (1,\infty)$

D $(-2,2)$

$$\frac{|x|-1}{|x|+2} > 0$$

always +ve

$$\Rightarrow |x|-1 > 0$$

$$|x| > 1$$

$$\Rightarrow x \in (-\infty,-1) \cup (1,\infty)$$

$|x|+2$

$\underbrace{|x|}_{+ve} + 2 = +ve.$

$(|x|-1)$

Question

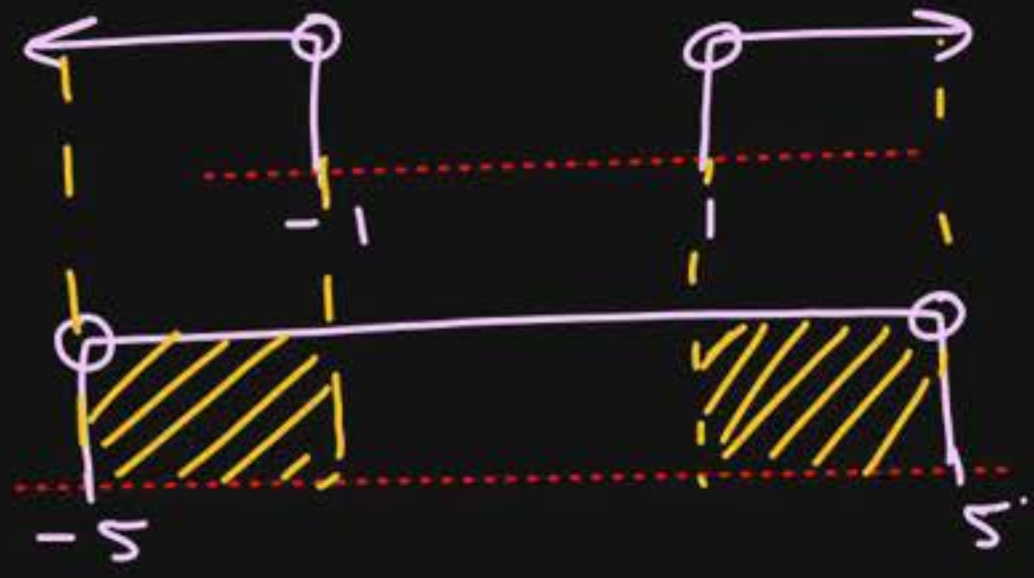
$\underbrace{1 < |x|}_{①} \underbrace{< 5}_{②}$

① $|x| > 1$

$x \in (-\infty, -1) \cup (1, \infty)$

② $|x| < 5$

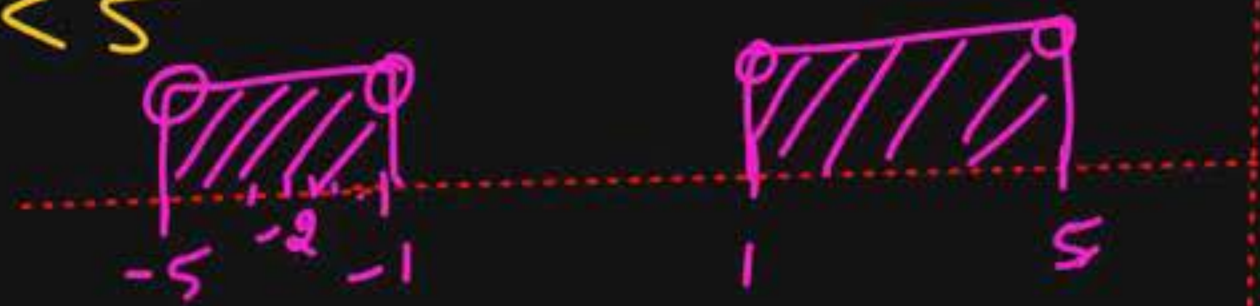
$x \in (-5, 5)$



Ans: $(-5,-1) \cup (1,5)$

2nd Method →

$1 < |x| < 5$



Q→2

$1 \le |x| \le 7$

$\Rightarrow x \in [1,7] \cup [-7,-1]$

Q→3 $\frac{3}{2} < |x| \le 9$

$x \in \left(\frac{3}{2}, 9\right] \cup \left[-9, -\frac{3}{2}\right)$

Question

$$\frac{|x|-1}{(|x|-2)\underbrace{(|x|+3)}_{\text{always +ve}}} \le 0$$

$$\frac{|x|-1}{|x|-2} \le 0$$

$|x| = t$

$$\frac{t-1}{t-2} \le 0$$

$t \in [1, 2)$

$1 \le |x| < 2$

$x \in [1, 2) \cup (-2, -1]$ Ans

Q→ $1 < |x+3| \le 10$.

$1 < \underbrace{x+3}_{} \le 10$ OR $-10 \le x+3 < -1$

subtract 3

$1-3 < x \le 10-3$ or $-10-3 \le x < -1-3$

$-2 < x \le 7$ or $-13 \le x < -4$

$x \in (-2, 7] \cup [-13, -4)$.

$1 < |t| \le 10$.

$\Rightarrow$

$t \in (1, 10] \cup [-10, -1)$

$1 < t \le 10$ or $-10 \le t < -1$

Question

If $||x-2|-3| \le 2$ then complete set of values of x is

A $[-3,7]$

B $[-3,1] \cup [3,7]$

C $[1,7]$

D $[-3,3] \cup [4,7]$

$|t| \le 2$

$-2 \le t \le 2$

$-2 \le |x-2| - \underbrace{3} \le 2$

add 3

$3-2 \le |x-2| \le 3+2$

$\underbrace{1 \le |x-2|} \underbrace{\le 5}$

$1 \le \underbrace{x-2} \le 5$ OR $\underbrace{-5 \le x-2 \le -1}$

add 2 add 2

$\boxed{3 \le x \le 7}$ OR $-3 \le x \le 1$

$[3,7] \cup [-3,1]$

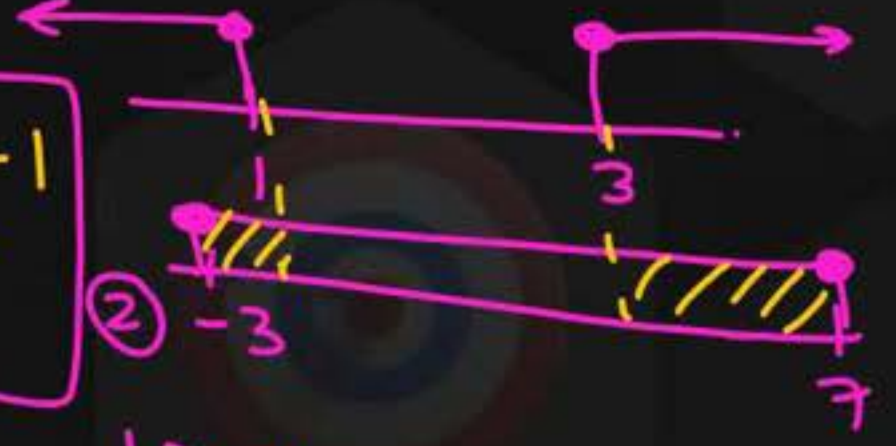
2nd Method

$\underbrace{1 \le |x-2|}_{1} \underbrace{\le 5}_{2}$

1 $|x-2| \geq 1$

$x-2 \geq 1$ or $x-2 \le -1$

$x \geq 3$ or $x \le 1$



2 $|x-2| \le 5$

$-5 \le x-2 \le 5$

$-3 \le x \le 7$

Solving Modulus Inequations -having Multiple mod Sign

Question

$$|x-1| + |x-2| < 7$$

$x - 1 = 0$

$x - 2 = 0$

①, ②, ③ ; 1, 2

Case → 1

$x \le 1$

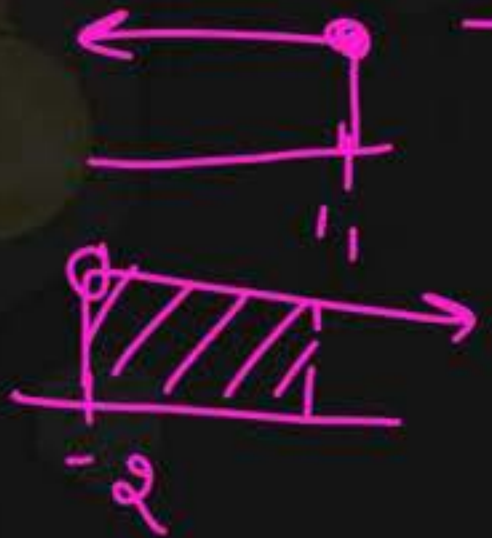
$$-(x-1) - (x-2) < 7$$

$$-x+1-x+2 < 7.$$

$$-2x+3 < 7.$$

$$-2x < 4$$

$$\boxed{x > -2}$$



I_1

$I_1 : (-2, 1]$

Case → 2

$1 < x \le 2$

$$x-1-(x-2) < 7$$

$$\cancel{x}-1-\cancel{x}+2 < 7.$$

$$\boxed{1 < 7}$$

always true.

$$\boxed{1 < x \le 2}$$

I_2

$I_2 : (1, 2]$

Case → 3.

$x > 2$

$$x-1+x-2 < 7.$$

$$2x-3 < 7$$

$$2x < 10.$$

$$\boxed{x < 5}$$

I_3

$I_3 : (2, 5)$

Solving Modulus Inequations-having Multiple mod Sign

Question

$|x-1| + 2|x+3| \geq 7$

Case → 1

$x \leq -3$

$-(x-1) - 2(x+3) \geq 7$

$-x + 1 - 2x - 6 \geq 7$

$-3x - 5 \geq 7$

$-3x \geq 12$

$\boxed{x \leq -4}$

I_1

$I_1 : (-\infty, -4]$

$-4 \quad -3$

Case → 2

$-3 < x < 1$

$-(x-1) + 2(x+3) \geq 7$

$-x + 1 + 2x + 6 \geq 7$

$x + \cancel{7} \geq \cancel{7}$

$\boxed{x \geq 0}$

I_2

$-3 \quad 1$

0

$I_2 : [0, 1)$

Case → 3

$x \geq 1$

$x - 1 + 2(x+3) \geq 7$

$x - 1 + 2x + 6 \geq 7$

$3x + 5 \geq 7$

$3x \geq 2$

$x \geq 2/3$

I_3

$2/3 \quad 1$

1

$I_3 : [1, \infty)$

Question

HW

Solution set of $|x+1|+|x-4|>7$

A $(-\infty,-2)\cup(5,\infty)$

B $(-2,5)$

C $(-\infty,-1)\cup(6,\infty)$

D None of these

REVISION

$$|x-1| - 2|x-3| + 3|x+1| = 10$$

$-1 \quad 1 \quad 3$

$x < -1$	$-1 \le x \le 1$	$1 < x \le 3$	$x > 3$
		$(x-1) + 2(x-3) + 3(x+1) = 10$ $x - 1 + 2x - 6 + 3x + 3 = 10$ $6x - 7 = 7$ $6x = 14$ $\boxed{x = 7/3}$	

Question

$3-(-100)$ $\quad$ $3-100$

$$|5x+2| + |3-x| = 7x+5$$

$5x+2=0 \Rightarrow x = -2/5$

$3-x=0 \Rightarrow x = 3$

① $\quad -2/5 \quad$ ② $\quad 3 \quad$ ③

Case → 1

$x \le -2/5$

$x \le -(0.4)$

$-(5x+2)+(3-x) = 7x+5$

$-5x-2+3-x = 7x+5$

$-6x+1 = 7x+5$

$-6x-7x = 5-1$

$-13x = 4$

$x = -4/13 \simeq \underbrace{-0.3}$

Reject.

Case → 2

$-\frac{2}{5} < x < 3$

$5x+2+3-x = 7x+5$

$4x+\cancel{5} = 7x+\cancel{5}$

$4x-7x=0$

$-3x=0$

$\boxed{x=0}$

Accept.

Ans

Case → 3

$x \geq 3$

$5x+2-(3-x) = 7x+5$

$5x+2-3+x = 7x+5$

$6x-1 = 7x+5$

$6x-7x = 6$

$-x = 6$

$\boxed{x=-6}$

Reject.

Question

$|x| + |x+2| = 2$

check

$x = 0$

$\Rightarrow LHS = 0+2$

$x = -1$

$LHS: |-1| + |1|$

$= 1 + 1$

$= 2$

Case → 1

$x < -2$

$-x-(x+2) = 2$

$-x-x-2 = 2$

$-2x = 4$

$\boxed{x = -2}$

Reject.

Case → 2

$-2 \le x < 0$

$-x + (x+2) = 2$

$-\cancel{x} + \cancel{x} + 2 = 2$

$\boxed{2 = 2}$

Always true

$-2 \le x < 0$

Case → 3

$x \geq 0$

$x + x + \cancel{2} = \cancel{2}$

$2x = 0$

$\boxed{x = 0}$

Accepted

$-2 \quad\quad 0$

Final ans $-2 \le x \le 0$ $\quad x \in [-2, 0]$

Q→ $|x-1| + |x-2| = 4$

Case → 1

$x \le 1$

$-(x-1)-(x-2) = 4$

$-x+1-x+2 = 4$

$-2x+3 = 4$

$-2x = 1$

$\boxed{x = -\frac{1}{2}}$

Accept

Case → 2

$1 < x < 2$

$x-1-(x-2) = 4$

$\cancel{x}-1-\cancel{x}+2 = 4$

$\boxed{1 = 4}$

Never true

ϕ

Case → 3

$x \ge 2$

$x-1+x-2 = 4$

$2x = 7$

$x = 7/2$

$x \in \{7/2, -1/2\}$

Check $x = 7/2$

LHS: $|7/2 - 1| + |7/2 - 2|$

$= |5/2| + |3/2|$

$= 8/2 = 4$

Yesterdays Class Example_Short Approach

$\boxed{|x+2| = 2(3-x)}$ then x is equal to

A $4/3$ ✓

B -8

C 8

D $-4/3$

$|x+2| = 2(3-x)$

Sq. both sides.

$|x+2|^2 = 4(3-x)^2$

$(x+2)^2 = 4(3-x)^2$

$x^2+4+4x = 4(9+x^2-6x)$

$x^2+4+4x = 36+4x^2-24x$

$0 = 4x^2-x^2-24x-4x+36-4$

$0 = 3x^2-28x+32$

$3x^2-28x+32=0$

$3x^2-24x-4x+32=0$

$3x(x-8)-4(x-8)$

$(3x-4)(x-8)=0$

$x = 4/3$ or 8

$|x|^2 = x^2$

non negative

$|x| = +x$ if $x \geq 0$

$= -x$ if $x < 0$

$|x|^2 = x^2$

$|x+2|^2 = (x+2)^2$

32×3

$8 \times 4 \times 3$

24×4

$$|x+2| = 2(3-x).$$

$x = 4/3$

LHS:

$\left|\frac{4}{3}+2\right| = \frac{10}{3}$

RHS: $2\left(3-\frac{4}{3}\right)$

$2\left[\frac{9-4}{3}\right]$

$= 2 \times \frac{5}{3} = \frac{10}{3}$

$\Rightarrow$ Accept

$x = 8.$

LHS $= 10$

RHS $= 2(3-8)$

$= 2(-5) = -10$

Reject ✓.

$x = 4/3$ only.

when ever we s.b.s then always verify value of x obtained by putting in the Question

PYQ [JEE Mains 2020]

The product of the roots of the equation $9x^2 - 18|x| + 5 = 0$, is :

A 25/9

B 5/27

C 25/81 ✓

D 8/9

$9|x|^2 - 18|x| + 5 = 0$

Let $|x| = t$

$9t^2 - 18t + 5 = 0$

$9t^2 - 15t - 3t + 5 = 0$

$3t(3t-5) - 1(3t-5) = 0$

$(3t-5)(3t-1) = 0$

$3t - 5 = 0$ or $t = \frac{1}{3}$

$\Rightarrow t = 5/3$

$|x| = 5/3$ or $|x| = \frac{1}{3}$

$x = \pm 5/3$ $\quad$ $x = \pm \frac{1}{3}$

$x^2 = |x|^2$

$45 \rightarrow$

$3 \times 5 \times 3$

Product $= 5/3 \times \frac{1}{3} \times (-5/3) \times (-\frac{1}{3})$

$= \frac{25}{81}$ ✓

Question

Solve for x: $|x^2 - 1| = 2x - 4$

No Solution

Case → 1 $x^2 - 1 \geq 0$

$x^2 - 1 = 2x - 4$

$x^2 - 2x - 1 + 4 = 0$

$\boxed{x^2 - 2x + 3 = 0} \rightarrow x = \phi$

$D = b^2 - 4ac$

$= (2)^2 - 4 \times 3 \times 1$

$= 4 - 12 = -8$

$D < 0 \Rightarrow$ no real roots.

Case → 2

$x^2 - 1 < 0$

$\boxed{x^2 < 1}$

$-(x^2 - 1) = 2x - 4$

$-x^2 + 1 = 2x - 4.$

$-x^2 - 2x + 5 = 0$

$x^2 + 2x - 5 = 0$

$x = \frac{-b \pm \sqrt{D}}{2a}$

$D = (-2)^2 - 4 \times (1) \times (-5)$

$= 4 + 20 = 24$

$x = \frac{-2 \pm \sqrt{24}}{2 \times 1} = \frac{-2 \pm 2\sqrt{6}}{2} = \frac{\cancel{2}(-1 \pm \sqrt{6})}{\cancel{2}}$

$x = -1 \pm \sqrt{6}.$

Reject

$x = -1+\sqrt{6}$

$x = -1+2.4$

$x = 1.4$

$x^2 = (1.4)^2$

> 1

Reject

$x = -1-\sqrt{6}.$

$x \simeq -1-2.4$

$x = -3.4.$

$x^2 = (3.4)^2$

> 1

Reject

$x^2 < 1$

✓

$\sqrt{6} = 2.4$

Question

$||x-2|-3| \leq 0$ then number of values of x satisfy the given inequality is

A 0

B 1

C 2

D Infinite

$$\left|\boxed{|x-2|-3}\right| \leq 0$$

$$\Rightarrow \left|\underline{|x-2|-3}\right| = 0$$

$$|x-2|-3 = \pm 0$$

$$|x-2| = 3$$

$$x-2 = \pm 3$$

$$x = 2 \pm 3$$

$$x = 5 \text{ or } -1$$

Check

$x = -1$

LHS $=$

$$|\,|\underbrace{-1-2}|-3|$$

$$|\,|\underbrace{-3}|-3|$$

$$|3-3|$$

$$= \underline{\underline{0}}$$

$$|\text{Ajay}| \leq 0$$

$$\Rightarrow |\text{Ajay}| = 0$$

$$|t| \leq 0$$

$$|t| = 0$$

$Q \to 1$ $\quad |x-2| \leq 3$

$-3 \leq x-2 \leq 3$

add 2 to this inequality

$-3+2 \leq x-\cancel{2}+\cancel{2} \leq 3+2$

$-1 \leq x \leq 5$

Ans $\quad x \in [-1, 5]$.

$Q \to 2$ $\quad |3x-1| \leq 5$

$-5 \leq 3x-1 \leq 5$

add 1

$-5+1 \leq 3x-\cancel{1}+\cancel{1} \leq 5+1$

$-4 \leq 3x \leq 6.$

divide by 3

$-4/3 \leq x \leq 6/3$

$-4/3 \leq x \leq 2$

Ans $\quad x \in [-4/3, 2]$.

Question

(i) $|2x - 3| < 4$

$$-4 < 2x - 3 < 4.$$

add 3

$$-4 + 3 < 2x < 4 + 3$$

$$-1 < 2x < 7$$

$$-\frac{1}{2} < x < \frac{7}{2}$$

$$x \in \left(-\frac{1}{2}, \frac{7}{2}\right)$$

Question

If $|3-2x| \le 7$ then

A $-2 \le x \le 5$

B $-5 \le x \le 2$

C $-2 \le x \le 2$

D $-5 \le x \le -2$

$-7 \le 3-2x \le 7$

subtract 3

$-7-3 \le \cancel{3}-2x-\cancel{3} \le 7-3$

$-10 \le -2x \le 4$

divide by -2

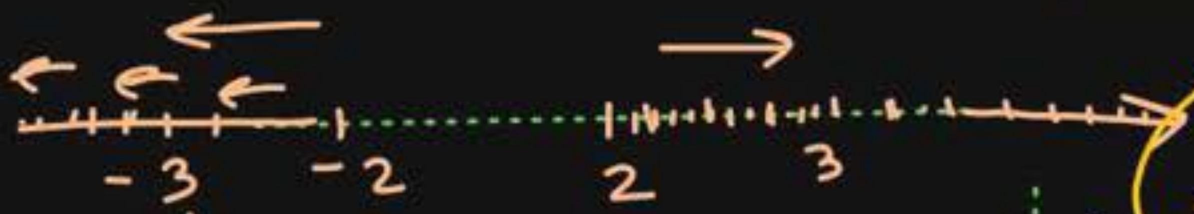
$\frac{-10}{-2} \ge \frac{\cancel{-2}x}{\cancel{-2}} \ge \frac{4}{-2}$.

$5 \ge x \ge -2$

$x \in [-2, 5]$

Solving Modulus Inequations-having Single mod Sign

① $|x| \geq 2$



$x \in (-\infty, -2] \cup [2, \infty)$

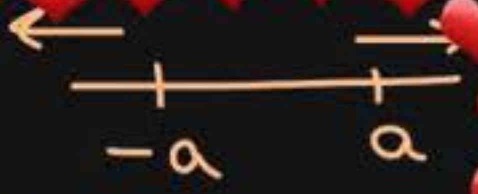
$|x| \geq 2$

$\Rightarrow$ $x \geq 2$ or $x \leq -2$

② $|x| \geq 5$

$x \in (-\infty, -5] \cup [5, \infty)$

③ $|x| \geq a$



$x \in (-\infty, -a] \cup [a, \infty)$

$|x| \geq a \Rightarrow$

$x \geq a$ or $x \leq -a$

Question

If $|x-3| \geq 2$ then

A $x \geq 5$ or $x \leq 1$ ✓

B $x \geq 5$ or $x \geq 1$

C $x \leq -5$ or $x \geq 1$

D $x \leq -1$ or $x \geq 5$

$$|x-3| \geq 2$$

$$\Rightarrow x-3 \geq 2 \quad \text{or} \quad x-3 \leq -2$$

$$x \geq 3+2 \quad \text{or} \quad x \leq 3-2$$

$$x \geq 5 \quad \text{or} \quad x \leq 1$$

$$x \in [5, \infty) \cup (-\infty, 1].$$

Question

$$\left|\frac{3x-1}{x+2}\right| \le 1$$

$$\underbrace{-1 \le \frac{3x-1}{x+2}}_{2} \ \underbrace{\le 1}_{1}$$

$\circ \cap \circ = \circ$

$\bullet \cap \bullet = \bullet$

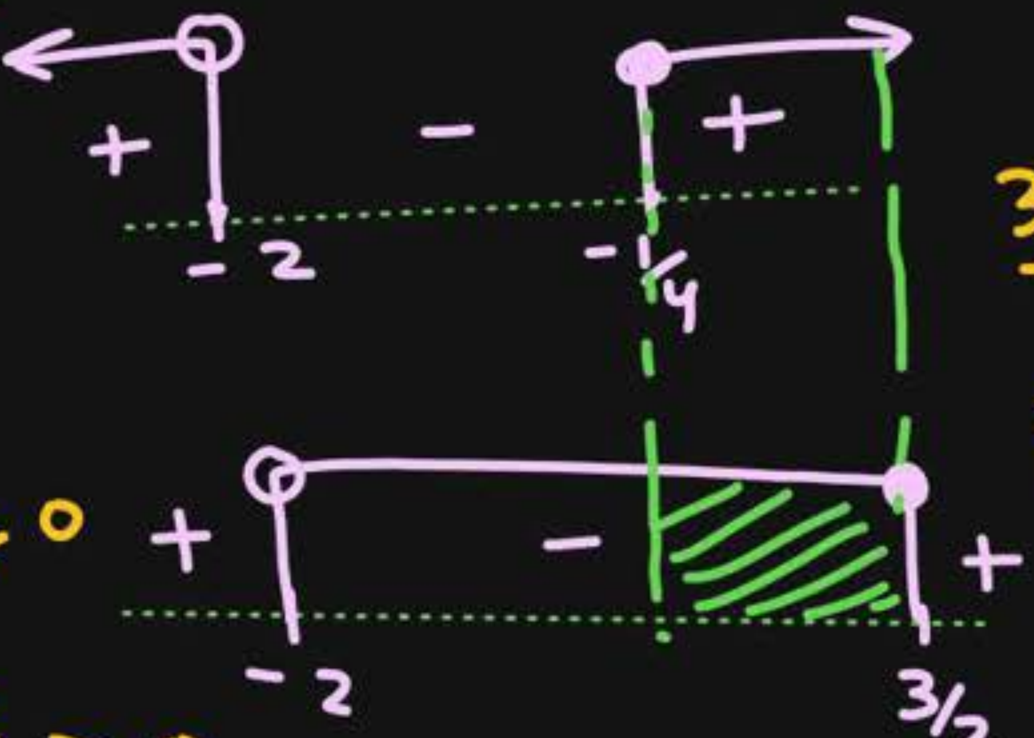
(2) $-1 \le \dfrac{3x-1}{x+2}$

$$\frac{3x-1}{x+2} \ge -1$$

$$\frac{3x-1}{x+2} + 1 \ge 0$$

$$\frac{3x-1+x+2}{x+2} \ge 0$$

$$\frac{4x+1}{x+2} \ge 0$$



(1) $\dfrac{3x-1}{x+2} \le 1$

$$\frac{3x-1}{x+2} - 1 \le 0$$

$$\frac{3x-1-(x+2)}{x+2} \le 0$$

$$\frac{3x-1-x-2}{x+2} \le 0$$

$$\frac{2x-3}{x+2} \le 0$$

$x \in \left[-\frac{1}{4}, \frac{3}{2}\right]$ Ans

Homework Discussion

1) $x^4 - 5x^2 + 4 < 0$

$\underbrace{x^2 = t} \Rightarrow x^4 = t^2$

$t^2 - 5t + 4 < 0$

$t^2 - 4t - t + 4 < 0$

$(t-1)(t-4) < 0$

$1 < t < 4$

$1 < x^2 < 4$

$x^2 > 1$

$x^2 - 1 > 0$

$(x-1)(x+1) > 0$

$x^2 < 4$

$x^2 - 4 < 0$

$(x-2)(x+2) < 0$

$x \in (-2,-1) \cup (1,2)$

$$\frac{x-2}{x^2+1} < -\frac{1}{2}$$

x^2+1 always +ve

$\Rightarrow 2(x-2) < -(x^2+1)$

$2x - 4 < -(x^2+1)$

$x^2 + 2x - 4 + 1 < 0$

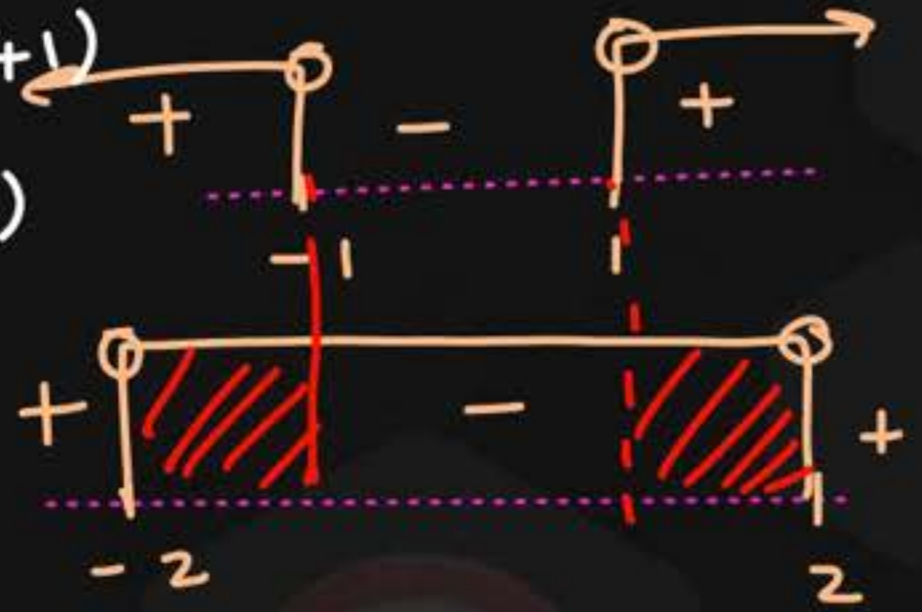
$x^2 + 2x - 3 < 0$

$x^2 + 3x - x - 3 < 0$

$x(x+3) - 1(x+3) < 0$

$(x-1)(x+3) < 0$

$x \in (-3, 1)$



Solving Modulus Equations- Having Single Mod Sign

Question

(a) $|x| = 2 \Rightarrow x = \pm 2$

(b) $|x - 2| = 3 \Rightarrow x - 2 = \pm 3$

$x - 2 = +3 \Rightarrow x = 5$ ✓

$x - 2 = -3 \Rightarrow x = 2 - 3 = -1$ ✓

(c) $\left|\frac{x-2}{2x-1}\right| = 5 \Rightarrow \frac{x-2}{2x-1} = \pm 5$

$\frac{x-2}{2x-1} = 5 \Rightarrow x - 2 = 5(2x-1)$

$x - 2 = 10x - 5$

$x - 10x = -5 + 2$

$-9x = -3$

$\boxed{x = \frac{1}{3}}$ ✓

$\frac{x-2}{2x-1} = -5 \Rightarrow x - 2 = -5(2x-1)$

$x - 2 = -10x + 5$

$11x = 7$

$x = \frac{7}{11}$

Question

$$||x-2|-3|-4| = 1$$

Check: $x=-4$

$LHS = ||| -6| -3| -4|$

$||6-3|-4|$

$|3-4| = |-1| = 1$ ✓

$\Rightarrow ||x-2|-3|-4 = \pm 1$

$\Rightarrow ||x-2|-3| = 4 \pm 1$

$|\boxed{|x-2|-3}| = 5$ or 3

$|x-2|-3 = \pm 5$ or ± 3

$\Rightarrow |x-2| = 3 \pm 5$ or 3 ± 3

$|x-2| = 8, -2, 6, 0$

$|x-2| = 8 \Rightarrow x-2 = \pm 8 \Rightarrow x = 2 \pm 8$

$x = 10, -6$

$|x-2| = -2 \rightarrow$ Rejected

$|x-2| = 6 \Rightarrow x-2 = \pm 6 \Rightarrow x = 2 \pm 6$

$x = 8, -4$ ✓

$|x-2| = 0 \Rightarrow x-2 = 0 \Rightarrow x = 2$

Solve → $\left|\,\big||x+3|-5\big|-7\,\right| = 2$

$\big||x+3|-5\big|-7 = \pm 2$

$\big||x+3|-5\big| = 9$ or 5

$\Rightarrow |x+3|-5 = \pm 9$ or ± 5

$|x+3| = 14, \cancel{-4}, 10, 0$

$x+3 = \pm 14$ or $\pm 10, 0$

$x = -3 \pm 14,$ or -3 ± 10 or $-3+0$

Ans $x = -17, 11, 7, -13, -3$

Question

$|x+2| = 2(3-x)$ then x is equal to

A 4/3

B −8

C 8

D −4/3

$|x| = x$ if $x > 0$

$|x| = -x$ if $x < 0$

$x+2=0$

$x=-2$

Case→2 Case→1

$\leftarrow x$ -2 $\rightarrow$

Case→1

$x \geq -2$

$x+2 = +ve.$

$(x+2) = 2(3-x)$

$x+2 = 6-2x$

$3x = 6-2$

$\boxed{x = 4/3.}$

Accept as it lies in the case

Case→2

$\boxed{x < -2}$

$-(x+2) = 2(3-x)$

$-x-2 = 6-2x.$

$-x+2x = 6+2.$

$\boxed{x = 8}$

Rejected as it does not lie in the case.

Check $x = 8$

LHS $= 10$

RHS $= 2(3-8) = -10$

Solving Modulus Equations-having Multiple Mod Sign

Question

$|x-2| + 2|x-3| = 8$

① ② ③

2 3

Case → 1

$x \le 2$

$-(x-2) - 2(x-3) = 8$

$-x + 2 - 2x + 6 = 8$

$-3x + 8 = 8$

$\boxed{x = 0}$

✓

Case → 2

$2 < x \le 3$

$(x-2) - 2(x-3) = 8$

$x - 2 - 2x + 6 = 8$

$-x + 4 = 8$

$\boxed{x = -4}$

Reject

Case → 3

$x > 3$

$(x-2) + 2(x-3) = 8$

$x - 2 + 2x - 6 = 8$

$3x - 8 = 8$

$3x = 16$

$\boxed{x = 16/3}$

✓

Ans $\Rightarrow x = 0$ or $16/3$

Question

$4-(-10) = 4+10 = +ve$

$|2x+1| - 3|4-x| = -3$

$2x+1=0$

$x = -\frac{1}{2}$

$4-x=0$

$x=4$

Case → 1

$x \le -\frac{1}{2}$

$-(2x+1)-3(4-x)=-3$

$-2x-1-12+3x=-3$

$x = 13-3$

$\Rightarrow \boxed{x=10}$ ✗

Reject ✗

Case → 2

$-\frac{1}{2} < x < 4$

$(2x+1)-3(4-x)=-3$

$2x+1-12+3x=-3$

$5x-11=-3$

$5x = 11-3$

$5x=8$

$\underbrace{x = 8/5}$ ✓

Case → 3

$x \ge 4$

$(2x+1)+3(4-x)=-3$

$2x+\underbrace{1+12-3x}=-3$

$-x+13=-3$

$-x=-13-3$

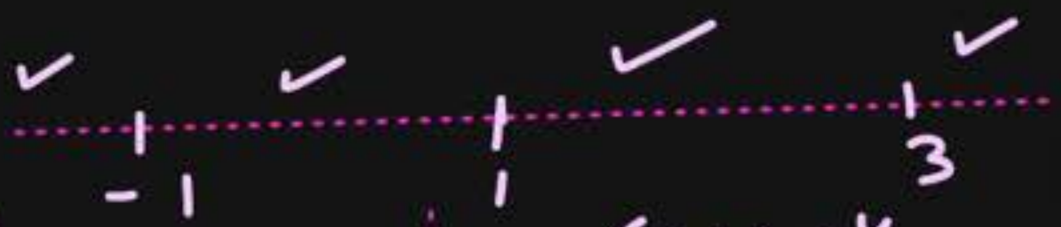
$-x=-16$

$\boxed{x=16}$ ✓

Question

[Ans. x = {−9, 7/3}]

$$|x-1| - 2|x-3| + 3|x+1| = 10$$



Case → 1

$\underbrace{x \le -1}$

$-(x-1)+2(x-3)-3(x+1)=10$

$-x+1+2x-6-3x-3=10$

$-2x-8=10$

$-2x=18$

$\boxed{x=-9}$

✓

Case → 2

$-1 < x < 1$

$-(x-1)+2(x-3)+3(x+1)=10$

$-x+1+2x-6+3x=7$

$4x-5=7$

$x=12/4$

$\boxed{x=3}$

×

Case → 3

$1 \le x < 3$

$(x-1)+2(x-3)+3(x+1)=10$

$x-1+2x-6+3x=7$

$6x-7=7$

$6x=14$

$\boxed{x=7/3}$

Case → 4

$x \geq 3$

$x-1-2(x-3)+3(x+1)=10$

$x-1-2x+6+3x=7$

$2x+5=7$

$2x=2$

$\boxed{x=1}$

×

Ans = $\{-9, 7/3\}$ ✓

Concluding Example

$$(x^2 - x - 1)(x^2 - x - 7) < -5$$

Let $\boxed{x^2 - x = t}$ ✓

$$(t-1)(t-7) < -5$$

$$t^2 - 7t - t + 7 < -5$$

$$t^2 - 8t + 7 + 5 < 0$$

$$t^2 - 8t + 12 < 0$$

$$t^2 - 6t - 2t + 12 < 0$$

$$(t-6)(t-2) < 0$$

$$2 < t < 6$$

$$\underbrace{2 < x^2 - x}_{①}\;\underbrace{< 6}_{②}$$

①

$$2 < x^2 - x$$

$$x^2 - x > 2$$

$$x^2 - x - 2 > 0$$

$$(x-2)(x+1) > 0$$

②

$$x^2 - x - 6 < 0$$

$$(x-3)(x+2) < 0$$

Ans: $x \in (-2, -1) \cup (2, 3)$

Wavy Curve Method (For Repeated Linear Factors)

2) $$\frac{(2x+1)^{3}(x-1)^{2}x^{2}}{(x+5)^{5}} \le 0$$

$2x+1=0$

$x=-\frac{1}{2}$

$x-1=0$

$x=0$

$x+5=0$

$x=-5$

$x \in \left(-5, -\frac{1}{2}\right] \cup \{0, 1\}$

1) $$\frac{(x-2)^{2}(x-3)^{3}}{(x+1)} > 0$$

Ans

$x \in (-\infty, -1) \cup (3, \infty)$

Wavy Curve Method- For Repeated Linear Factors

NOTE

Sign about that critical point will not change whose power is Even.

$(-1)^4 \rightarrow$ same

$(1)^4 \rightarrow$

Ex → 3 $\quad (x-1)^2(x-2) > 0$

$\ominus \quad \ominus \quad \oplus$

$\leftarrow 1 \quad 2$

Case → 1 $\quad x > 2$

LHS : $(3-1)^2(3-2) = 2^2 \times 1 = +ve.$

Case → 2 $\quad 1 < x < 2 \quad x = 1.1$

LHS : $(0.1)^2 \cdot (1.1 - 2) = -ve$

Case → 3 $\quad x < 1$: LHS : $(-1)^2(0-2) = 1 \times (-2) = -2 = -ve.$

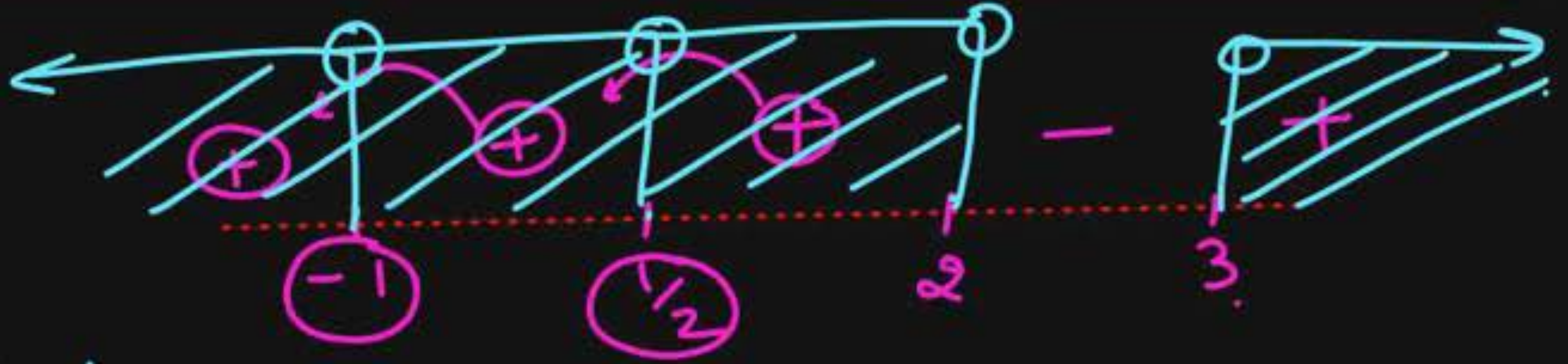
Wavy Curve Method- For Repeated Linear Factors

Question

Solve

(i) $\dfrac{(x-2)^3(x-3)(2x-1)^2}{(x+1)^4} > 0$

(ii) $\dfrac{(x-2)^3(x-3)(2x-1)^2}{(x+1)^4} \geq 0$



Ans → 1

✓ $x \in (-\infty,-1) \cup (-1, \tfrac{1}{2}) \cup (\tfrac{1}{2}, 2) \cup (3, \infty)$

✓ Better way: $x \in (-\infty, 2) \cup (3, \infty) - \{-1, \tfrac{1}{2}\}$

Ans → 2 $x \in (-\infty, 2] \cup [3, \infty) - \{-1\}$

Wavy Curve Method- For Repeated Linear Factors

Question

$$\frac{(2x+3)^3(-x-1)^2x^2}{(-x^2+5x-6)(x+6)^2}\geq 0$$

$$\frac{(2x+3)^3(-1)^2(x+1)^2x^2}{-(x^2-5x+6)(x+6)^2}\geqslant 0$$

$$\frac{(2x+3)^3(x+1)^2(x)^2}{(x-2)(x-3)(x+6)^2}\leq 0$$

$(-1)^2 = 1$

Ans: $x \in$

$(-\infty, -3/2] \cup \{-1, 0\} \cup (2, 3) - \{-6\}$

Wavy Curve when Quadratic has $D < 0$

Q→1 $x^2 - 5x + 12$ → always +ve

> 0

$\boxed{x \in R}$

$a = 1$
$b = -5$
$c = 12$
$D = b^2 - 4ac$
$= (-5)^2 - 4 \times 1 \times 12$
$= 25 - 48$
$D = -ve$

Method – 1 Complete the square

Method – 2 By Graph of Quadratic

If $a > 0$ & $D < 0$ then Quadratic is always +ve

If $a < 0$ & $D < 0$ then Quadratic is always -ve

Q→2 $x^2 - 5x + 12 < 0$

$x = \phi$

Question

$$\frac{x^2-5x+6}{x^2+x+1}<0$$

x^2+x+1 always +ve

$$\frac{(x-2)(x-3)}{x^2+x+1}<0$$

$\oplus$

$\Rightarrow (x-2)(x-3)<0$

$x \in (2,3)$

x^2+x+1

$a=1 \quad b=1, \quad c=1$

$D = b^2-4ac$

$= 1^2-4\times1\times1$

$= 1-4 = -3$

$a>0$ & $D<0$

Question

(a) $\dfrac{0.5}{x - x^2 - 1} < 0$

$\dfrac{0.5}{-x^2 + x - 1} < 0$

$\to$ -ve

$0.5 > 0$

always true.

$\boxed{x \in R}$

✓ $x \in (-\infty, \infty)$

$a = -1$

$b = 1$

$c = -1$

$D = b^2 - 4ac$

$= 1^2 - 4 \times (-1)(-1)$

$= 1 - 4$

$D = -3$

(b) $\dfrac{x^2 - 7x + 12}{2x^2 + 4x + 5} > 0$

$\to$ always +ve.

$x^2 - 7x + 12 > 0$

$(x-4)(x-3) > 0$

$2x^2 + 4x + 5$

$a = 2$, $b = 4$, $c = 5$

$D = b^2 - 4ac$

$= (4)^2 - 4 \times 2 \times 5$

$= 16 - 40$

$= -24$

Ans $x \in (-\infty, 3) \cup (4, \infty)$

Wavy Curve when Quadratic has D < 0

Question

$$\frac{5x^2-2}{4x^2-x+3} < 1$$

always +ve

$\Rightarrow 5x^2-2 < 1(4x^2-x+3)$

$5x^2-2 < 4x^2-x+3$

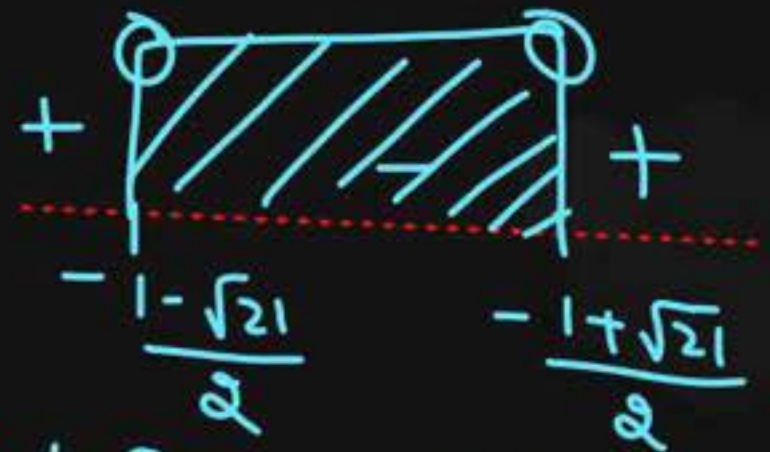
$x^2+x-2-3<0$

$\boxed{x^2+x-5<0}$

$x = \dfrac{-b \pm \sqrt{D}}{2a}$

$x = \dfrac{-1 \pm \sqrt{21}}{2\times 1}$

$x = \dfrac{-1-\sqrt{21}}{2}$ or $\dfrac{-1+\sqrt{21}}{2}$



$x \in \left(\dfrac{-1-\sqrt{21}}{2}, \dfrac{-1+\sqrt{21}}{2}\right)$. Ans

$a=4$, $b=-1$, $c=3$ $\quad ax^2+bx+c$

$D = b^2-4ac$

$= (-1)^2-4\times4\times3$

$= 1-48 = -47$

$\boxed{D<0}$

$a=1$, $b=1$, $c=-5$

$D = b^2-4ac$

$= (1)^2-4\times1(-5)$

$D = 1+20 = 21$

Wavy Curve when Quadratic has $D < 0$

Question

$$\frac{x^2+6x-7}{x^2+1} \le 2$$

always +ve

$x^2+6x-7 \le 2(x^2+1)$

$x^2+6x-7 \le 2x^2+2$

$-x^2+6x-7-2 \le 0$

$-x^2+6x-9 \le 0$

$x^2-6x+9 \ge 0$

$\boxed{(x-3)^2 \ge 0}$

$x \in (-\infty, \infty)$

$\boxed{x \in R}$ ✓

$x^2 \rightarrow$ always non-ve.

$x^2+1 \rightarrow$ always +ve

Question

$$\frac{2x}{x^2-9} \le \frac{1}{x+2}$$

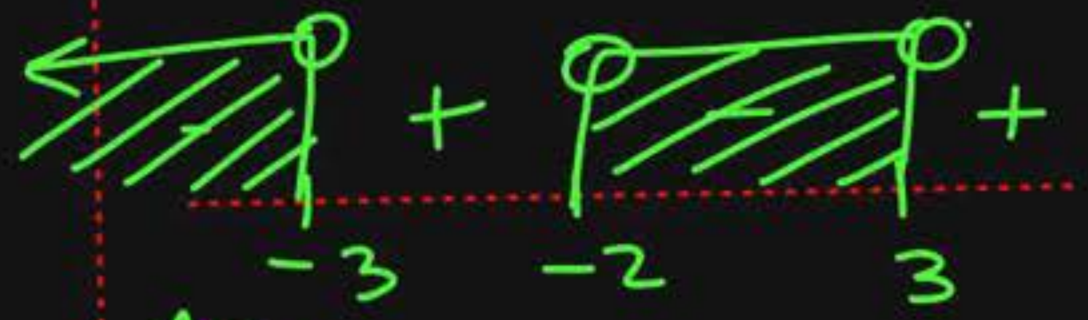
$$\frac{2x}{x^2-9} - \frac{1}{x+2} \le 0$$

$$\frac{2x(x+2)-(x^2-9)}{(x^2-9)(x+2)} \le 0$$

$$\frac{2x^2+4x-\cancel{x^2}+9}{(x-3)(x+3)(x+2)} \le 0$$

$$\frac{\underbrace{x^2+4x+9}_{\text{always } \oplus}}{(x-3)(x+3)(x+2)} \le 0$$

$$\frac{1}{(x-3)(x+3)(x+2)} \le 0$$



Ans.

$x \in (-\infty, -3) \cup (-2, 3)$

x^2+4x+9

$a=1$, $b=4$, $c=9$

$D = b^2-4ac$

$= 16-4\times1\times9$

$= 16-36$

$= -20$

$a>0$, $D<0$

$\Rightarrow$ always +ve

Concluding Example

$$\frac{(x^2-2x)(2x-2)}{1} - 9\left(\frac{2x-2}{x^2-2x}\right) \le 0$$

$$\frac{(x^2-2x)^2(2x-2) - 9(2x-2)}{(x^2-2x)} \le 0$$

$$\frac{(2x-2)\left[(x^2-2x)^2 - 9\right]}{x^2-2x} \le 0$$

always +ve

$$\frac{(2x-2)(x^2-2x+3)(x^2-2x-3)}{x(x-2)} \le 0$$

$$\frac{2(x-1)(x+1)(x-3)}{x(x-2)} \le 0$$

$$(x^2-2x)^2 - (3)^2$$

$$(x^2-2x+3)(x^2-2x-3)$$

$$x^2-2x-3.$$

$$x^2-3x+x-3$$

$$(x+1)(x-3)$$

$$x \in (-\infty,-1] \cup [1,0) \cup (2,3]$$

Homework

- $\dfrac{1-2x-3x^2}{3x-x^2-5} > 0$ [Ans. $(-\infty, -1) \cup (1/3, +\infty)$]

- $\dfrac{(x-1)(x+2)^2}{-1-x} < 0$ [Ans. $(-\infty, -2) \cup (-2, -1) \cup (1, +\infty)$]

- $x^4 - 5x^2 + 4 < 0$ [Ans. $(-2, -1) \cup (1, +2)$]

- $\dfrac{x-2}{x^2+1} < -\dfrac{1}{2}$ [Ans. $(-3, 1)$]

HW Discussion

$$\frac{1}{x-1} \le 2$$

$$\frac{1}{x-1} - 2 \le 0$$

$$\frac{1-2(x-1)}{x-1} \le 0$$

$$\frac{1-2x+2}{x-1} \le 0$$

$$\frac{3-2x}{x-1} \le 0$$

$$\therefore \frac{-(2x-3)}{x-1} \le 0$$

$$\frac{2x-3}{x-1} \ge 0$$

$$x \in (-\infty, 1) \cup \left[\tfrac{3}{2}, \infty\right)$$

$$\frac{x-1}{x} - \frac{x+1}{x-1} < 2$$

$$\frac{x-1}{x} - \frac{x+1}{x-1} - 2 < 0$$

$$\frac{(x-1)^2 - x(x+1) - 2x(x-1)}{x(x-1)} < 0$$

$$\frac{\cancel{x^2} + 1 - \cancel{2x} - \cancel{x^2} - x - 2x^2 + \cancel{2x}}{x(x-1)} < 0$$

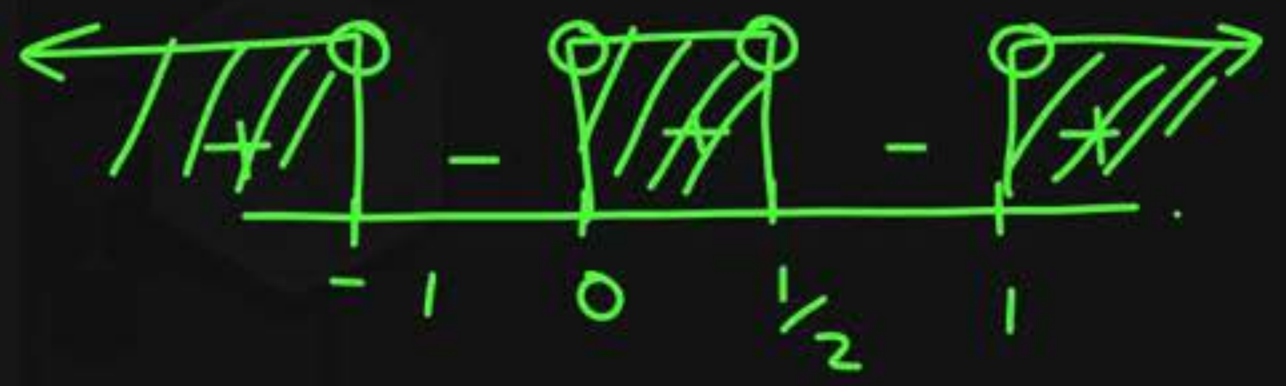
$$\frac{-2x^2 - x + 1}{x(x-1)} < 0$$

$$\therefore \frac{-(2x^2 + x - 1)}{x(x-1)} < 0$$

$$\frac{2x^2+x-1}{x(x-1)} > 0$$

$$\frac{2x^2+2x-x-1}{x(x-1)} > 0$$

$$\frac{(2x-1)(x+1)}{x(x-1)} > 0$$



RECAP_ Example

The set of all real values of x for which $\frac{8x^2+16x-51}{(2x-3)(x+4)} \le 3$, is

A. $\left(\frac{3}{2}, \frac{5}{2}\right]$

B. $(-4, -3]$

C. $(-4, -3] \cup \left(\frac{3}{2}, \frac{5}{2}\right]$ ✓

D. none of these

$$\begin{aligned} &2x^2+6x-5x-15 \\ &2x(x+3)-5(x+3) \\ &(2x-5)(x+3) \end{aligned}$$

$$\begin{array}{r} -51 \\ 36 \\ \hline -15 \end{array}$$

$$\frac{8x^2+16x-51-3(2x-3)(x+4)}{(2x-3)(x+4)} \le 0$$

$$\frac{8x^2+16x-51-3[2x^2+8x-3x-12]}{(2x-3)(x+4)} \le 0$$

$$\frac{8x^2+16x-51-6x^2-15x+36}{(2x-3)(x+4)} \le 0$$

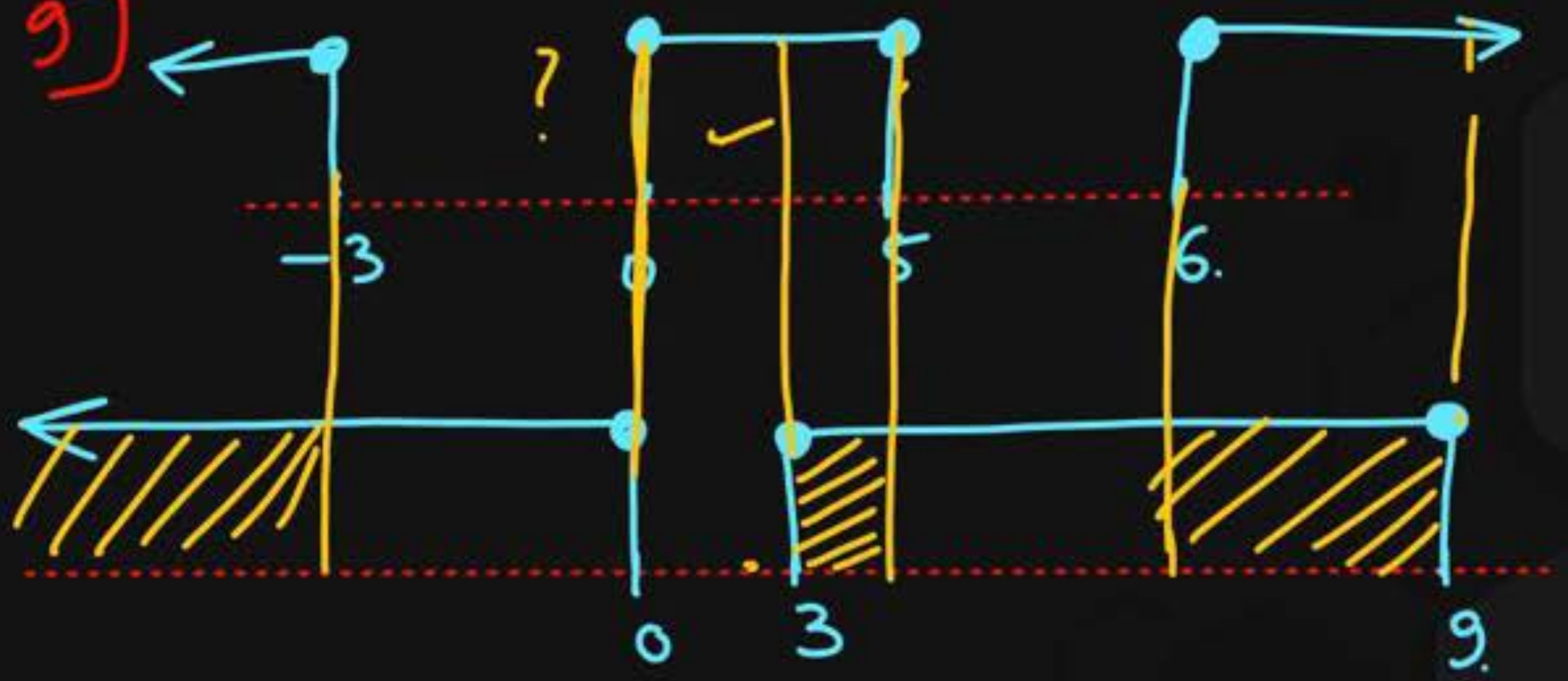
$$\frac{2x^2+x-15}{(2x-3)(x+4)} \le 0 \Rightarrow \frac{(2x-5)(x+3)}{(2x-3)(x+4)} \le 0$$

Q→

A : $(-\infty, -3] \cup [0, 5] \cup [6, \infty)$

B : $(-\infty, 0] \cup [3, 9]$

Find $A \cap B$.



$A \cap B = (-\infty, -3] \cup \{0\} \cup [3, 5] \cup [6, 9]$

$A : (-\infty, -4) \cup (-3, 2] \cup (5, \infty)$

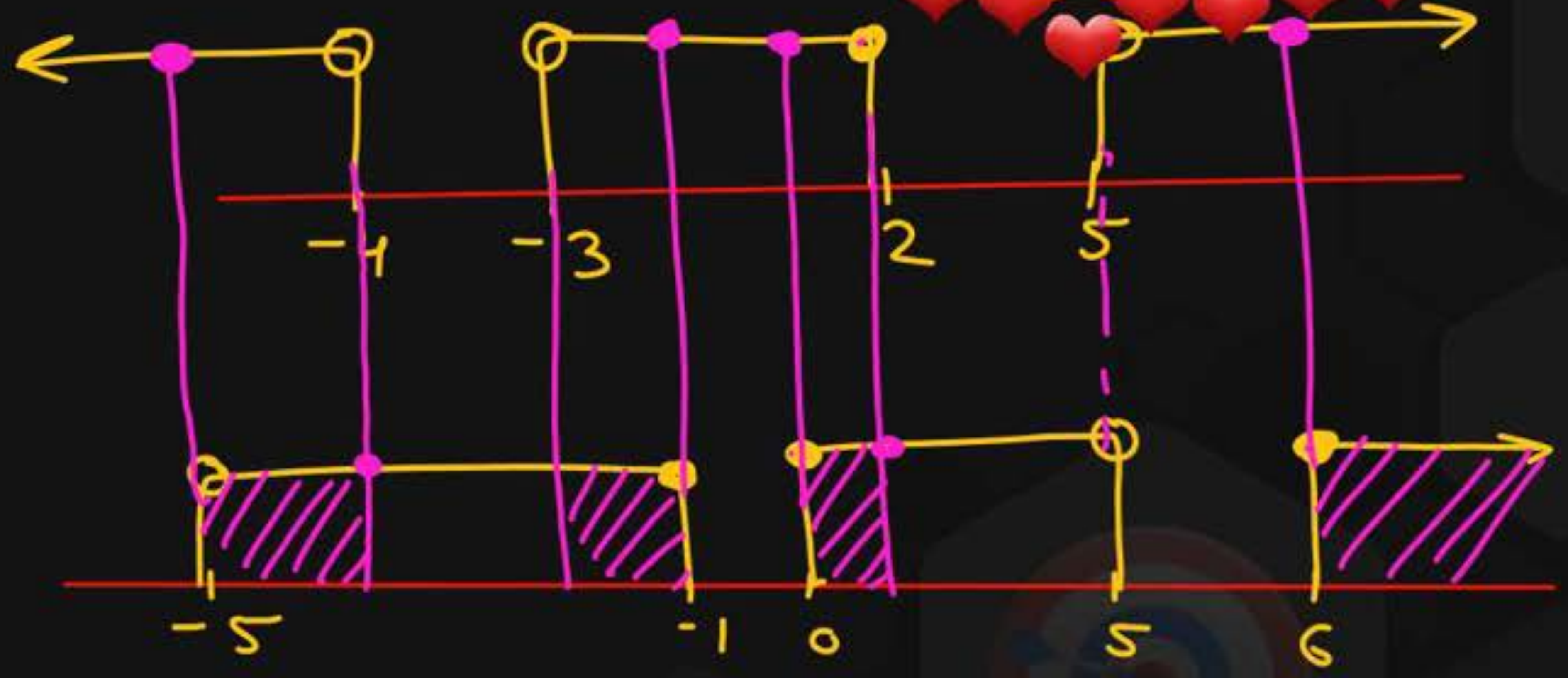
$B : (-5, -1] \cup [0, 5) \cup [6, \infty)$

$\circ \cap \circ = \circ$

$\bullet \cap \circ = \circ$

$\bullet \cap \bullet = \bullet$

$A \cap B = ?$



$A \cap B = (-5, -4) \cup (-3, -1] \cup [0, 2] \cup [6, \infty)$

A : $(-\infty, -3] \cup [0, 2) \cup [10/3, \infty)$

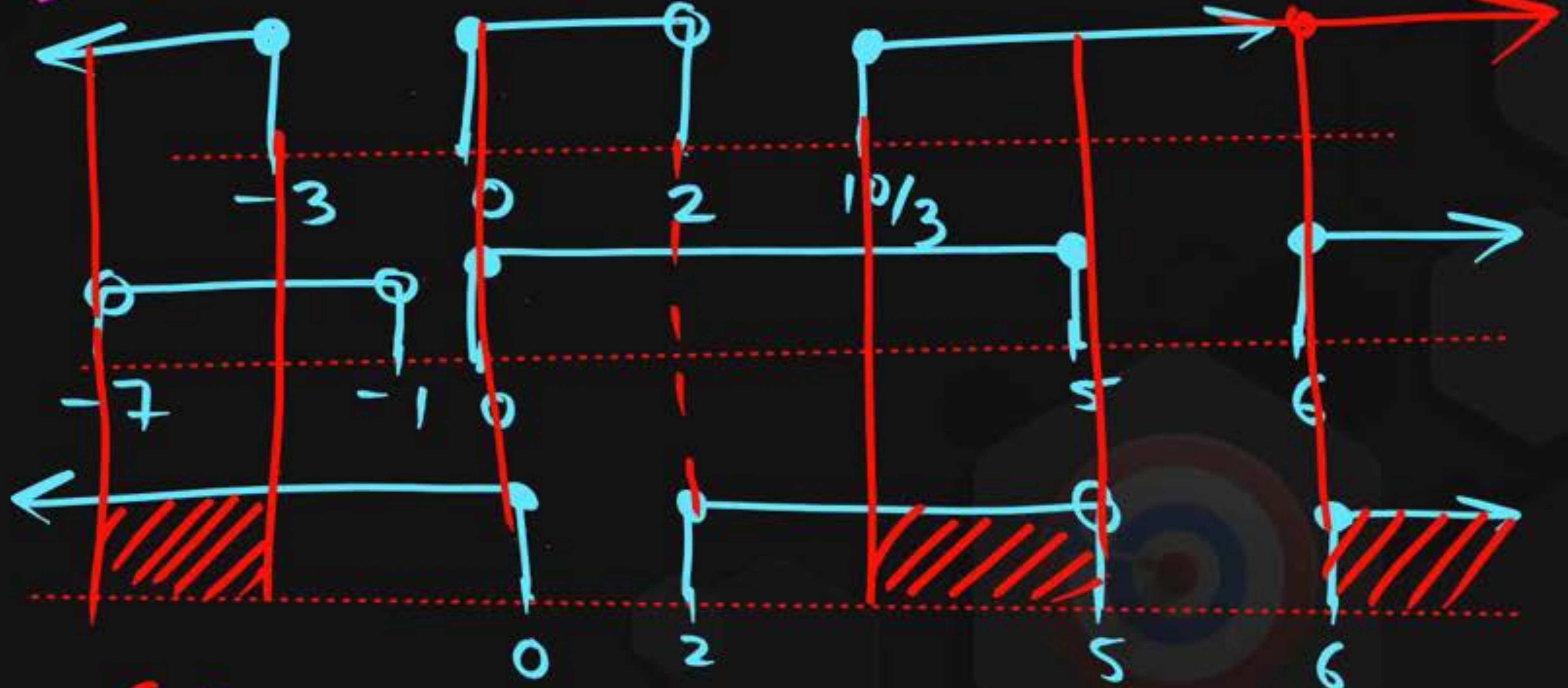
B : $(-7, -1) \cup [0, 5] \cup [6, \infty)$

C : $(-\infty, 0] \cup (2, 5) \cup [6, \infty)$

$10/3 = 3.3$

$\bullet \cap \bullet \cap \bullet = \bullet$

$A \cap B \cap C = ?$



$(-7, -3] \cup \{0\} \cup (10/3, 5) \cup [6, \infty)$

Question

$$-1 < \frac{3x-1}{x-2} \le 4$$

> or <

And → ∩

① ∩ ②

$a < b$

$\Rightarrow b > a$

① $-1 < \frac{3x-1}{x-2}$

$\frac{3x-1}{x-2} > -1$

$\frac{3x-1}{x-2} + 1 > 0$

$\frac{3x-1+x-2}{x-2} > 0$

$\boxed{\frac{4x-3}{x-2} > 0}$

② $\frac{3x-1}{x-2} \le 4$

$\frac{3x-1}{x-2} - 4 \le 0$

$\frac{3x-1-4(x-2)}{x-2} \le 0$

$\frac{3x-1-4x+8}{x-2} \le 0$

$\frac{-x+7}{x-2} \le 0$

$-\frac{(x-7)}{x-2} \le 0$

$\boxed{\frac{x-7}{x-2} \ge 0}$

+ − + ; 3/4, 2

+ − ; 2, 7

Ans $x \in (-\infty, 3/4) \cup [7, \infty)$

Question

$>$ or $<$ $\quad (0, 10)$

The greatest negative integer satisfying $\underbrace{x^2 - 4x - 77 < 0}$ and $\underbrace{x^2 > 4}$, is

A. -4

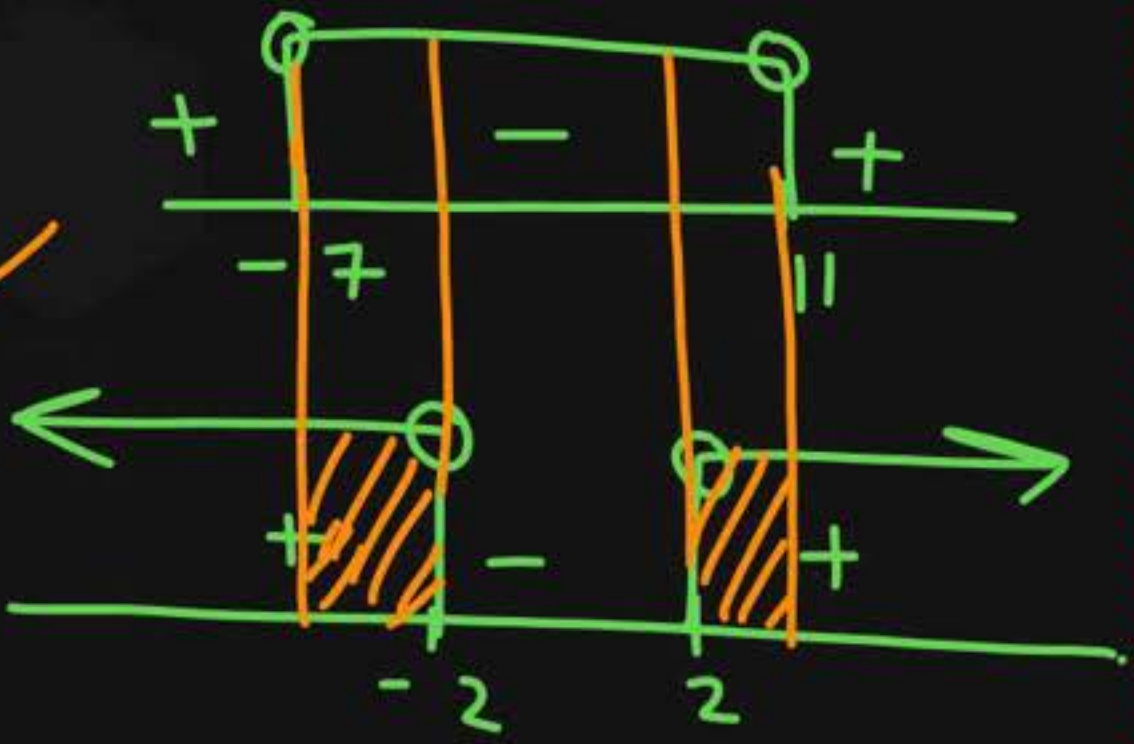
B. -6

C. -7

D. -3

$x^2 - 4x - 77 < 0$

$(x-11)(x+7) < 0$



$x^2 > 4$

$x^2 - 4 > 0$

$(x-2)(x+2) > 0$

Ans: $x \in (-7, -2) \cup (2, 11)$

$-6, -5, -4, -3$

greatest negative

Question

The solution set of $\underbrace{\boxed{x^2 + 2 \le 3x}}_{(1)} \le 2x^2 - 5$, is

A ϕ → Phi

B $[1,2]$

C $(-\infty, -1] \cup [5/2, \infty)$

D none of these

(1) $x^2 + 2 \le 3x$

$x^2 - 3x + 2 \le 0$

$(x-2)(x-1) \le 0$

$3x \le 2x^2 - 5$

$2x^2 - 5 \ge 3x$

$2x^2 - 3x - 5 \ge 0$

$2x^2 - 5x + 2x - 5 \ge 0$

$x(2x-5) + 1(2x-5) \ge 0$

$(2x-5)(x+1) \ge 0$

No solⁿ ϕ → Nothing.

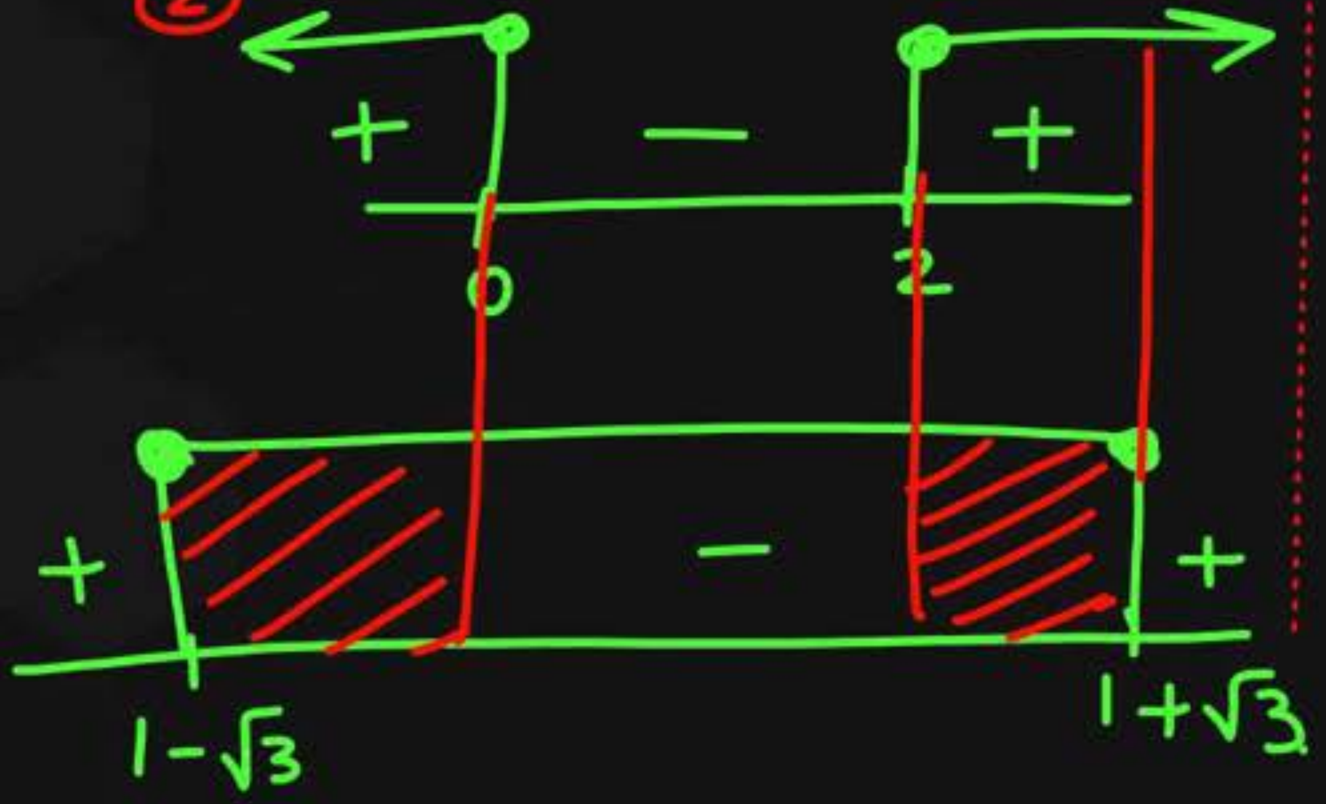
Example

$$\underbrace{0 \le x^2 - 2x}_{①} \le 2 \qquad \text{(② : } x^2 - 2x \le 2\text{)}$$

$0 \le x^2 - 2x$

$x^2 - 2x \geq 0$

$x(x-2) \geq 0$



Ans $x \in [1-\sqrt{3}, 0] \cup [2, 1+\sqrt{3}]$

$x^2 - 2x \le 2$

$x^2 - 2x - 2 \le 0$

$x = \dfrac{2 \pm \sqrt{12}}{2 \times 1}$

$x = \dfrac{2 \pm 2\sqrt{3}}{2}$

$= \dfrac{\cancel{2}(1 \pm \sqrt{3})}{\cancel{2}}$

$x = 1+\sqrt{3}$ or $1-\sqrt{3}$

$\sqrt{3} = 1.73$

$\sqrt{12} = 2\sqrt{3}$

$a = 1$
$b = -2$
$c = -2$

$D = b^2 - 4ac$
$= (-2)^2 - 4 \times 1(-2)$
$= 4 + 8 = 12$

$x = \dfrac{-b \pm \sqrt{D}}{2a}$

$D = b^2 - 4ac$

Inequations / Inequality

Any Expression involving $\boxed{>, <, \geq \text{ OR } \leq}$ sign is known as Inequality.

$>, <, \geq$ or $\leq$

Equation

$\underline{\text{Ex} \to 1}$ $3x+7 > 2$

$2x-3=7$

$\text{Ex} \to 2$ $\dfrac{3x-1}{2x+5} \leq 3$

$x > 7 \to 1$

$x \geq 7 \to 2$

$7 \geq 7 \to \text{True}$

$\left.\begin{array}{l} x < 6 \\ x \leq 6 \end{array}\right\}$

Question

$9/2 = 4.5$

Find the number of Integers in the following Intervals-

$7/3 = 2.33$

a) (1, 7]   2, 3, 4, 5, 6, 7   6

b) (- 2,10)   −1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9   11

Same

c) (1, 7]

d) [- 3, 7] ⟶ −3, −2, −1, 0, 1, 2, 3, ... 7   11

e) [- 3, 9/2] ⟶ [−3, 4.5]   −3, −2, −1, 0, 1, 2, 3, 4   8

f) [- 3, 7/3) ⟶ [−3, 2.33) ⟶ −3, −2, −1, 0, 1, 2   6

g) { 1 , 1/2 } $x = 1$ 1

Question

$\cup \rightarrow$ Union

$\pm\infty$ are always open bracket

$\epsilon \rightarrow$ Epsylon

$\in \rightarrow$ belongs to

$A \cup B$
$B \cup A$ ✓

Write the following in interval form-

1) $-1 \leq x \leq 5 \longrightarrow x \in [-1, 5]$

2) $-1 < x \leq 6 \longrightarrow x \in (-1, 6]$

3) $x > 3 \longrightarrow x \in (3, \infty)$

3 ∞

4) $x > 5$ or $x < -3$ $\quad x \in (5, \infty) \cup (-\infty, -3)$

$\cup$

5) $x \geqslant 6$ or $x \leq 0$ $\quad x \in [6, \infty) \cup (-\infty, 0]$

Q→ Represent the following Intervals in number line

1) $x \in [2, 5)$ ✓

2) $x \in (-3, 5] \cup [6, \infty)$.

closed → ● → solid
open → o → hollow.

3) $x \in \left(-\frac{1}{2}, \frac{3}{2}\right) \cup (5, \infty)$.

4) $x \in (-\infty, -1] \cup [2, 5] \cup \left(\frac{11}{2}, \infty\right)$

5) $x \in (-5, 8] \cup \{10\}$.

6) $x \in (-5, 1] - \{0\}$.

(3) $\dfrac{-3x+5}{7} > 2$

$-3x+5 > 7\times 2$

$-3x+5 > 14$

$-3x > 14-5$

$-3x > 9$

$x < \dfrac{9}{-3}$

$\boxed{x < -3}$

Ans: $x \in (-\infty, -3)$

Rational Inequalities

$x-3=0 \Rightarrow x=3$

$x-6=0 \Rightarrow x=6$

$$\boxed{\frac{x-3}{x-6}<0}$$

LHS < 0

⇒ LHS is -ve

Case→1 $\boxed{x>6}$ LHS $=\frac{7-3}{7-6}=$ +ve

Case→2 $3<x<6$ LHS: $\frac{5-3}{5-6}=\frac{2}{-1}=$ -ve

Case→3 $x<3$ LHS: $\frac{2-3}{2-6}=\frac{-1}{-4}=$ +ve

Ans: $x \in (3,6)$

Rational Inequalities

$$\frac{(x-3)(x-1)}{(x-6)} < 0$$

Case → 1 $x > 6$

$x = 10$

Case → 2 $\boxed{x = 5}$

Ans. $x \in (-\infty, 1) \cup (3, 6)$.

$$\left.\begin{array}{l} x-3=0 \\ x-1=0 \\ x-6=0 \end{array}\right\}$$

Question

$$\frac{2x-3}{x+4} < 4$$

$$\frac{2x-3}{x+4} - \frac{4}{1} < 0$$

$$\frac{(2x-3)-4(x+4)}{(x+4)} < 0$$

$$\frac{2x-3-4x-16}{(x+4)} < 0$$

$$\frac{-2x-19}{x+4} < 0$$

$$\frac{-(2x+19)}{(x+4)} < 0$$

$$\frac{2x+19}{x+4} > 0$$

$2x+19=0$

$x = -\frac{19}{2}$

$x+4=0$

$x=-4$

Ans

$x \in \left(-\infty, -\frac{19}{2}\right) \cup (-4, \infty)$

Question

$$\frac{2x+7}{3x+1} > 2$$

$$\frac{2x+7}{3x+1} - \frac{2}{1} > 0$$

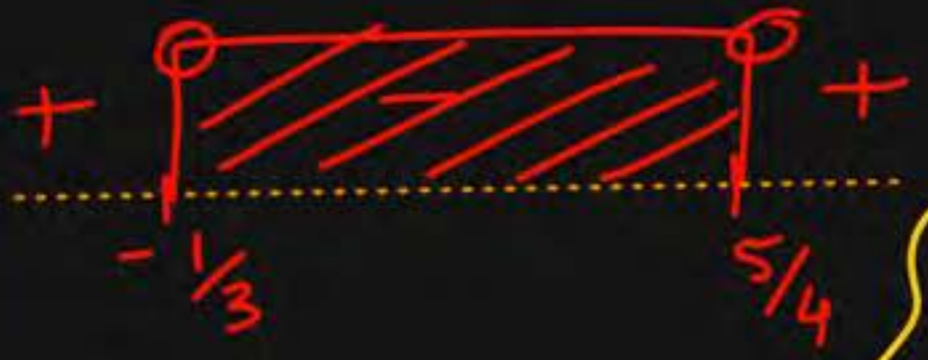
$$\frac{(2x+7)-2(3x+1)}{(3x+1)} > 0$$

$$\frac{2x+7-6x-2}{(3x+1)} > 0$$

$$\frac{-4x+5}{3x+1} > 0$$

$$\frac{-[4x-5]}{3x+1} > 0$$

$$\frac{4x-5}{3x+1} < 0$$



Ans

$x \in \left(-\frac{1}{3}, \frac{5}{4}\right)$

$4x-5=0$

$x=5/4$

$3x+1=0$

$x=-\frac{1}{3}$

Question

The set of real values of x for which $\frac{10x^2+17x-34}{x^2+2x-3} < 8$ is

A $\left(-\frac{5}{2}, 2\right)$

B $\left(-3, -\frac{5}{2}\right) \cup (1, 2)$ ✓

C $(-3, 1)$

D none of these

$$\frac{10x^2+17x-34}{x^2+2x-3} - \frac{8}{1} < 0$$

$$\frac{10x^2+17x-34-8(x^2+2x-3)}{(x^2+2x-3)} < 0$$

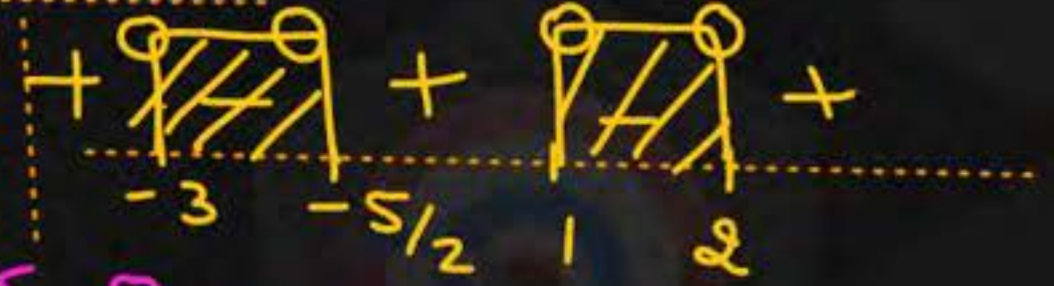
$$\frac{10x^2+17x-34-\cancel{8x^2}-16x+24}{x^2+2x-3} < 0$$

$$\frac{2x^2+x-10}{x^2+2x-3} < 0$$

$$\frac{2x^2+5x-4x-10}{x^2+3x-x-3} < 0$$

$$\frac{x(2x+5)-2(2x+5)}{x(x+3)-1(x+3)} < 0$$

$$\frac{(x-2)(2x+5)}{(x-1)(x+3)} < 0$$ ✓



Question

If S is the set of all real x such that $\frac{2x}{2x^2+5x+2} > \frac{1}{x+1}$, then S is equal to

A $(-2,-1)$

B $(-2/3, 0)$

C $(-2/3, -1/2)$

D $(-2,-1) \cup (-2/3, -1/2)$ ✓

$-2/3,\ -1/2,\ -2,\ -1$

$2x^2+5x+2$

$2x^2+4x+x+2$

$(2x+1)(x+2)$

$$\frac{2x}{2x^2+5x+2} - \frac{1}{x+1} > 0$$

$$\frac{2x(x+1) - (2x^2+5x+2)}{(2x^2+5x+2)(x+1)} > 0$$

$$\frac{\cancel{2x^2} + 2x - \cancel{2x^2} - 5x - 2}{(2x^2+5x+2)(x+1)} > 0$$

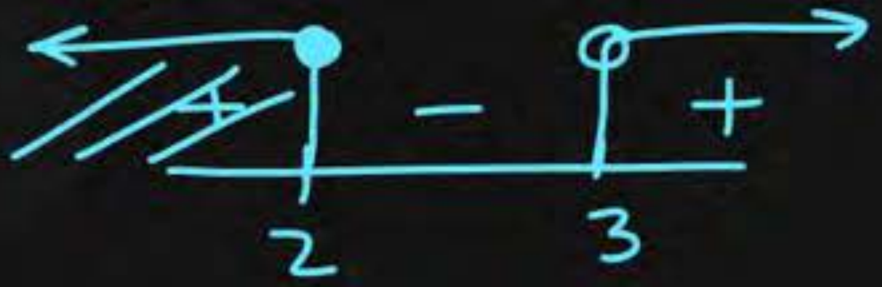
$$\frac{-3x-2}{(2x^2+5x+2)(x+1)} > 0$$

$$\frac{-(3x+2)}{(2x+1)(x+2)(x+1)} > 0$$

$$\boxed{\frac{3x+2}{(2x+1)(x+2)(x+1)} < 0}$$

For Inequalities Involving ≤ oR ≥ Sign

Q → $\frac{x-2}{x-3} \geqslant 0$



$x \in (-\infty, 2] \cup (3, \infty)$

$\boxed{\frac{x-2}{x-3} = 0}$

$\Rightarrow x - 2 = 0$

$\boxed{x = 2}$

$>$ or $<$

$\geqslant$ or $\leq$

$>$ or $\geqslant$

$p/q = 0$

$\Rightarrow p = 0$

STEP-7 For > or < sign- all critical points are open bracket.
For ≥ or ≤ Sign, numerator critical points are closed bracket whereas denominator critical points are open bracket.

$$\frac{(x-2)(x-5)}{x-7} \le 0$$

$$x \in (-\infty, 2] \cup [5, 7)$$

$$\frac{(x-2)(x-5)}{x-7} = 0$$

$$\boxed{x = 2 \text{ or } x = 5}$$

Homework

$$\frac{6x-5}{4x+1}<0$$ [Ans. $(-1/4, 5/6)$

$$\frac{3}{x-2}<1$$ [Ans. $(-\infty, 2) \cup (5, +\infty)$

$$\frac{1}{x-1}\le 2$$ [Ans. $(-\infty, 1) \cup [3/2, +\infty)$

$$\frac{x-1}{x}-\frac{x+1}{x-1}<2$$ [Ans. $(-\infty, -1)\left(0, \frac{1}{2}\right) \cup (1, +\infty)$

$$1+\frac{2}{x-1}\ge\frac{6}{x}$$ [Ans. $(-\infty, 0) \cup (1,2] \cup [3, +\infty)$

Question

Evaluate : $\left[\left\{(729)^{-3}\right\}^{-\frac{1}{9}}\right]^{-3}$

A 729

B 1/729

C $(729)^{1/3}$

D $(729)^{2/3}$

$(729)^{\overbrace{-3\left(-\frac{1}{9}\right)(-3)}}$

$(729)^{\frac{1}{9}(-9)}$

$(729)^{-1} = \frac{1}{729}$.

Question

Simplify : $\left(\frac{x^a}{x^b}\right)^{a+b} \cdot \left(\frac{x^b}{x^c}\right)^{b+c} \cdot \left(\frac{x^c}{x^a}\right)^{c+a}$

$(a+b)(a-b) = a^2 - b^2$

A $x^{a^2-b^2}$

B $x^{b^2-c^2}$

C $x^{a^2-c^2}$

D 1

$\left(x^{a-b}\right)^{a+b}$

$x^{(a-b)(a+b)}$

$x^{a^2-b^2} \cdot x^{b^2-c^2} \cdot x^{c^2-a^2}$

$x^{\cancel{a^2}-\cancel{b^2}+\cancel{b^2}-\cancel{c^2}+\cancel{c^2}-\cancel{a^2}}$

$x^0 = 1$

Brain Teaser-1_HW

If $\frac{9^n \times 3^2 \times (3^{-n/2})^{-2} - (27)^n}{3^{3m} \times 2^m} = \frac{1}{27}$, where m & n are natural Numbers then find the value of $(m-n)$

A) -1

B) 1

C) 2

D) -2

$2^x \cdot 3^y = 2 \times 3 \Rightarrow x=1, \; y=1$

$$\frac{3^{2n} \cdot 3^2 \cdot 3^{-n/2(-2)} - 3^{3n}}{3^{3m} \cdot 2^m} = \frac{1}{27}$$

$9^n = (3^2)^n = 3^{2n}$

$(27)^n = (3^3)^n = 3^{3n}$

$$\frac{3^{2n} \cdot 3^2 \cdot 3^n - 3^{3n}}{3^{3m} \cdot 2^m} = \frac{1}{27}$$

$2^3 \quad \underbrace{3^{2n} \cdot 3^n} = 3^{3n}$

$$\frac{3^{3n} \cdot 3^2 - 3^{3n}}{3^{3m} \cdot 2^m} = \frac{1}{27}$$

$$\frac{3^{3n}[3^2 - 1]}{3^{3m} \cdot 2^m} = \frac{1}{27}$$

$$\frac{3^{3n} \cdot 2^3}{3^{3m} \cdot 2^m} = \frac{1}{(3)^3} \rightarrow 3^{-3}$$

Question

If $x^2 - 5x - 1 = 0$, then the value of $x^2 + \frac{1}{x^2}$ is

A 20

B 27

C 25

D –25

$x^2 - 1 = 5x$

divide by x.

$\frac{x^2 - 1}{x} = 5$

$\boxed{x - \frac{1}{x} = 5}$

S.B.S

$\left(x - \frac{1}{x}\right)^2 = 25$

$x^2 + \frac{1}{x^2} - 2 \cdot \cancel{x} \cdot \frac{1}{\cancel{x}} = 25.$

$x^2 + \frac{1}{x^2} - 2 = 25$

$(a-b)^2$

$a = x$

$b = \frac{1}{x}$

$\boxed{x^2 + \frac{1}{x^2} = 27}$

$(a-b)^2 = a^2 + b^2 - 2ab$

$a = x$

$b = \frac{1}{x}$.

$\left(x - \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} - 2 \cdot \cancel{x} \cdot \frac{1}{\cancel{x}}$

Question

If $x + \frac{1}{x} = \sqrt{7}$ find the value of $x^4 + \frac{1}{x^4}$

S.B.S.

A 23

B 25

C 27

D 29

$$\left(x + \frac{1}{x}\right)^2 = 7$$

$a \to x$

$b \to \frac{1}{x}$.

$(a+b)^2 = a^2 + b^2 + 2ab$

$$x^2 + \frac{1}{x^2} + 2\cancel{x}\cdot\frac{1}{\cancel{x}} = 7$$

$$x^2 + \frac{1}{x^2} + 2 = 7.$$

$$\boxed{x^2 + \frac{1}{x^2} = 5}$$

S.B.S.

$$\left(x^2 + \frac{1}{x^2}\right)^2 = 25$$

$$(x^2)^2 + \left(\frac{1}{x^2}\right)^2 + 2\cancel{x^2}\cdot\frac{1}{\cancel{x^2}} = 25$$

$$x^4 + \frac{1}{x^4} + 2 = 25.$$

$$\boxed{x^4 + \frac{1}{x^4} = 23}$$

Question

If $x + x^{-1} = 4$ Then $\boxed{x^3 + \frac{1}{x^3}}$ would be

A 64

B 52 ✓

C 32

D 12

$x^{-1} = 1/x$.

$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

$a \to x$

$b \to 1/x$

Given $\boxed{x + 1/x = 4}$

Cube both sides.

$$\left(x + \frac{1}{x}\right)^3 = (4)^3$$

$$\underbrace{x^3 + \frac{1}{x^3}} + 3 \cdot \cancel{x} \cdot \frac{1}{\cancel{x}} \underbrace{\left(x + \frac{1}{x}\right)}_{4} = 64$$

$$x^3 + \frac{1}{x^3} + 3 \times 4 = 64$$

$$x^3 + \frac{1}{x^3} = 64 - 12$$

$$= 52.$$

Question

If $x = 2^{2/3} + 2^{1/3}$, then

A $x^3 - 6x - 6 = 0$

B $x^3 + 6x - 6 = 0$

C $x^3 - 6x + 6 = 0$

D $x^3 + 6x + 6 = 0$

$x = 2^{2/3} + 2^{1/3}$

Cube both sides.

$2^{2/3} \cdot 2^{1/3} = 2^{2/3 + 1/3} = 2^{3/3} = 2^1$

$x^3 = \left(2^{2/3} + 2^{1/3}\right)^3$

$x^3 = \left(2^{2/3}\right)^3 + \left(2^{1/3}\right)^3 + 3 \cdot 2^{2/3} \cdot 2^{1/3} \left(\underbrace{2^{2/3} + 2^{1/3}}_{x}\right)$

$x^3 = 2^2 + 2^1 + 3 \times 2\,(x)$

$x^3 = 6 + 6x$

$\boxed{x^3 - 6x - 6 = 0}$

Ex→ $5^3 - 3^3 = (5-3)\left[5^2 + 3^2 + 5\times 3\right]$

$$= (5-3)\left[25 + 9 + 15\right]$$

$$7^3 + 3^3 = (7+3)\left[7^2 + 3^2 - 7\times 3\right]$$

Question

If $x^2 + \frac{1}{x^2} = 51$, then what is the positive value of $x^3 - \frac{1}{x^3}$

A 364

B 365

C 756

D 367

$a^3 - b^3 = (a-b)(a^2+b^2+ab)$

$a \to x, \quad b \to 1/x.$

Let $x - 1/x = y$

S.B.S.

$(x - 1/x)^2 = y^2$

$x^2 + \frac{1}{x^2} - 2 \cdot \cancel{x} \cdot \frac{1}{\cancel{x}} = y^2$

$x^2 + \frac{1}{x^2} - 2 = y^2.$

$51 - 2 = y^2.$

$49 = y^2 \Rightarrow \boxed{y = 7}$

$x^3 - \frac{1}{x^3} = (x - 1/x)\left(x^2 + \frac{1}{x^2} + x \cdot \frac{1}{x}\right)$

$= (x - 1/x)\left(x^2 + \frac{1}{x^2} + 1\right)$

$x^2 + \frac{1}{x^2} \to 51$

$(x - 1/x) \times (52)$

7×52

$= 364$

If $x+\frac{1}{x}=3$ then find the value of

1) $x^2+\frac{1}{x^2}=7$

2) $x^3+\frac{1}{x^3}=18$

3) $x^4+\frac{1}{x^4}=47$

4) $x^6+\frac{1}{x^6}$

1) SBS

$$\left(x+\frac{1}{x}\right)^2=3^2$$

$$x^2+\frac{1}{x^2}+2=9.$$

$$\boxed{x^2+\frac{1}{x^2}=7}$$

$$(x^3)^2=x^6$$

$$(x^2)^3=x^6$$

(2) $x^3+\frac{1}{x^3}=x^3+\left(\frac{1}{x}\right)^3=\left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}-\cancel{x}\cdot\frac{1}{\cancel{x}}\right)$

$$a^3+b^3=(a+b)(a^2+b^2-ab)$$

$$\left(x+\frac{1}{x}\right)(7-1)$$

$$=3(7-1)$$

$$=3\times6=18$$

$$x^2+\frac{1}{x^2}=7$$

S.B.S.

$$\left(x^2+\frac{1}{x^2}\right)^2=49$$

$$x^4+\frac{1}{x^4}+2\cdot\cancel{x^2}\cdot\frac{1}{\cancel{x^2}}=49$$

$$\boxed{x^4+\frac{1}{x^4}=47}$$

(d) $x^3 + \frac{1}{x^3} = 18$

S.B.S

$$\left(x^3 + \frac{1}{x^3}\right)^2 = (18)^2$$

$$(x^3)^2 + \left(\frac{1}{x^3}\right)^2 + 2 \cdot \cancel{x^3} \cdot \frac{1}{\cancel{x^3}} = 324$$

$$x^6 + \frac{1}{x^6} + 2 = 324$$

$$\boxed{x^6 + \frac{1}{x^6} = 322}$$

$$(a+b+c)^2 = \underbrace{a^2+b^2+c^2} + 2ab + 2bc + 2ca$$

$$(a+b-c)^2 = a^2+b^2+c^2+2ab-2bc-2ac$$

$(+)\times(+) = +$

$(-)\times(+) = -$

$(-)\times(-) = +$

$$(a-b-c)^2 = a^2+b^2+c^2-2ab+2bc-2ac \checkmark$$

$$(-a+b-c)^2 = a^2+b^2+c^2-2ab-2bc+2ac \checkmark$$

(ii) $a^3+b^3+c^3-3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$$= \frac{1}{2}(a+b+c)[2a^2+2b^2+2c^2-2ab-2bc-2ca]$$

$$= \frac{1}{2}(a+b+c)[a^2+a^2+b^2+b^2+c^2+c^2-2ab-2bc-2ca]$$

$$\frac{1}{2}(a+b+c)((a-b)^2+(a-c)^2+(b-c)^2)$$

Question

4

Find the value of $\dfrac{a^3+b^3+c^3-3abc}{ab+bc+ca-a^2-b^2-c^2}$, when $a=-5, b=-6, c=10$

A 1

B -1

C 2

D -2

$$\frac{(a+b+c)\cancel{(a^2+b^2+c^2-ab-bc-ca)}}{-\cancel{(a^2+b^2+c^2-ab-bc-ca)}}$$

$$-(a+b+c)$$

$$-[-5-6+10]$$

$$-[-11+10]$$

$$=-(-1)$$

$$=1$$

School → Marks $= 3/4$

IIT Marks $= -\frac{1}{4}$

Question

The value of $\dfrac{(1.5)^3+(4.7)^3+(3.8)^3-3\times1.5\times4.7\times3.8}{(1.5)^2+(4.7)^2+(3.8)^2-1.5\times4.7-1.5\times3.8-4.7\times3.8}$ is

A 8

B 9

C 10

D 11

$$= \frac{a^3+b^3+c^3-3abc}{a^2+b^2+c^2-ab-bc-ca}$$

$$= a+b+c$$

$$1.5+4.7+3.8$$

$$= 6.2+3.8$$

$$= 10$$

Brain Teaser-2

HW

Simplify: $\dfrac{(a^2-b^2)^3+(b^2-c^2)^3+(c^2-a^2)^3}{(a-b)^3+(b-c)^3+(c-a)^3}$

Brain Teaser-3_HW

HW

If $x = 3 + 3^{1/3} + 3^{2/3}$, then the value of $x^3 - 9x^2 + 18x - 12$ is

A 0

B -1

C 1

D 2

Homework

Find x such that :

(i) $\left(\frac{7}{4}\right)^{-3} \times \left(\frac{7}{4}\right)^{-5} = \left(\frac{7}{4}\right)^{x-2}$ [Ans. $x = -6$]

(ii) $\left(\frac{125}{8}\right) \times \left(\frac{125}{8}\right)^{x} = \left(\frac{5}{2}\right)^{18}$ [Ans. $x = 5$]

(iii) $\left(\frac{35}{11}\right)^{4} \times \left(\frac{11}{7}\right)^{4} = 5^{x}$ [Ans. $x = 4$]

(iv) $\left(\left(\frac{-3}{7}\right)^{4} \times \left(\frac{-3}{7}\right)^{8}\right)^{-5} = \left(\left(\frac{-3}{7}\right) \times \left(\frac{-3}{7}\right)^{5}\right)^{x}$ [Ans. $x = -10$]

Question

If $5^{2x-1} - 100 = 25^{x-1}$, then the value of 6^x:

A 6

B 1/6

C 36 ✓

D 1/36

$$a^{x-y} = \frac{a^x}{a^y}$$

$$\frac{5^{2x}}{5^1} - 100 = \frac{25^x}{25^1}$$

$$\frac{(25)^x}{5} - 100 = \frac{(25)^x}{25}$$

$$\frac{(25)^x}{5} - \frac{(25)^x}{25} = 100$$

$$(25)^x\left[\frac{1}{5} - \frac{1}{25}\right] = 100$$

$$(25)^x \frac{4}{25} = 100$$

$$5^{2x} = (5^2)^x$$
$$= (25)^x$$

$$(25)^x = \frac{\cancel{100}^{25} \times 25}{\cancel{4}}$$

$$(25)^x = (25)^2$$

$$\boxed{x = 2}$$

$$6^x = 6^2 = 36$$

Question

If $\sqrt{\left(\frac{3}{5}\right)^{1-2x}} = 4\frac{17}{27}$, then the value of x is

mixed

A $-2/7$

B $2/7$

C $-7/2$

D $7/2$ ✓

$$\boxed{\left(\frac{x}{y}\right)^{m} = \left(\frac{y}{x}\right)^{-m}}$$

$$\sqrt{a} = (a)^{1/2}$$

$$\left[\left(\frac{3}{5}\right)^{(1-2x)}\right]^{1/2} = \left(\frac{5}{3}\right)^{3}$$

$$\left(\frac{3}{5}\right)^{\frac{1-2x}{2}} = \left(\frac{3}{5}\right)^{-3}$$

$$\text{R.H.S} = \frac{4\times 27 + 17}{27} = \frac{108+17}{27} = \frac{125}{27}.$$

$$= \left(\frac{5}{3}\right)^{3}$$

$$\frac{1-2x}{2} = -3.$$

$$1-2x = -6.$$

$$2x = 7 \Rightarrow x = 7/2$$

Brain Teaser-1

HW

If $\frac{9^n \times 3^2 \times (3^{-n/2})^{-2} - (27)^n}{3^{3m} \times 2^m} = \frac{1}{27}$, where m & n are natural Numbers then find the value of $(m - n)$

A) -1

B) 1

C) 2

D) -2

Q. JEE Main Online April 9, 2017

The sum of all the real values of x satisfying the equation $2^{(x-1)(x^2+5x-50)} = 1$ is :

A 16

B 14

C -4 ✓

D -5

$2^{(\quad)} = 2^{0}$

$\underbrace{(x-1)}(x^2+5x-50) = 0$

either $x-1=0$ or $x^2+5x-50=0$

$\Rightarrow \boxed{x=1}$

$\underbrace{x^2+10x}-\underbrace{5x-50} = 0$

$x(x+10)-5(x+10)$

$\underbrace{(x-5)}(x+10)=0$

$x-5=0$ or $x+10=0$

$x=5$ $\quad$ $x=-10$

Sum $= 1+5-10$

$= 6-10$

$= -4$

Question

$\log_4 x - \log_x 4 \le \dfrac{3}{2}$

$\log_4 x \nearrow t \qquad \log_x 4 \nearrow 1/t$

Domain: $x > 0,\ x \neq 1$

Let $\log_4 x = t$

$\Rightarrow \log_x 4 = 1/t$

$4^{-\infty} \to 0$

$4^{-\infty} \to \dfrac{1}{4^{\infty}} \to \dfrac{1}{\infty} \to 0$

$4^{-1/2}$

$(2)^{2\times(-1/2)}$

2^{-1}

$= \dfrac{1}{2}$

$t - \dfrac{1}{t} \le 3/2$

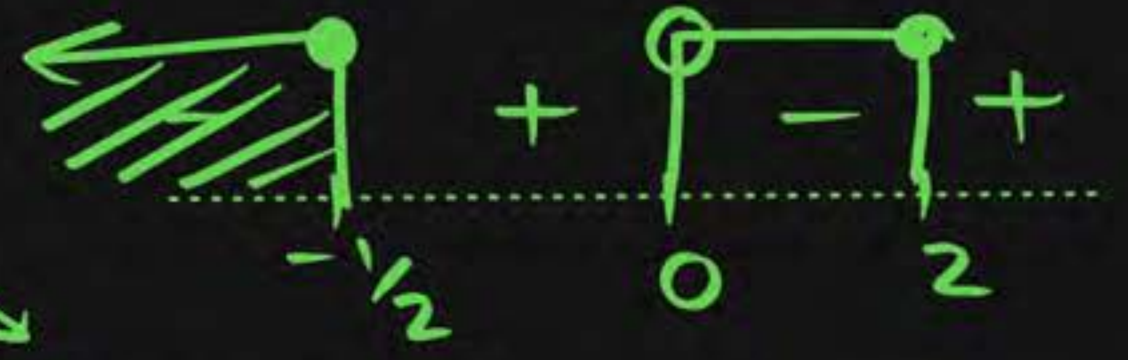
$t - \dfrac{1}{t} - \dfrac{3}{2} \le 0$

$\dfrac{2t^2 - 2 - 3t}{2t} \le 0$

$\dfrac{2t^2 - 3t - 2}{2t} \le 0$

$\dfrac{2t^2 - 4t + t - 2}{2t} \le 0$

$\dfrac{(2t+1)(t-2)}{2t} \le 0$



$t \in (-\infty, -1/2] \cup (0, 2]$

$\log_4 x \in (-\infty, -1/2] \cup (0, 2]$

$x \in (4^{-\infty}, 4^{-1/2}] \cup (4^0, 4^2]$

Ans: $x \in (0, \tfrac{1}{2}] \cup (1, 16]$ Ans

Question

Domain: $x > 0$

$$\log_2 x \le \frac{2}{\log_2 x - 1}$$

let $\log_2 x = t$

A $\left(0, \frac{1}{2}\right] \cup (2, 4)$

B $\left(0, \frac{1}{2}\right] \cup (2, 4]$ ✓

C $\left(\frac{1}{2}, 2\right) \cup [4, \infty)$

D *None of these*

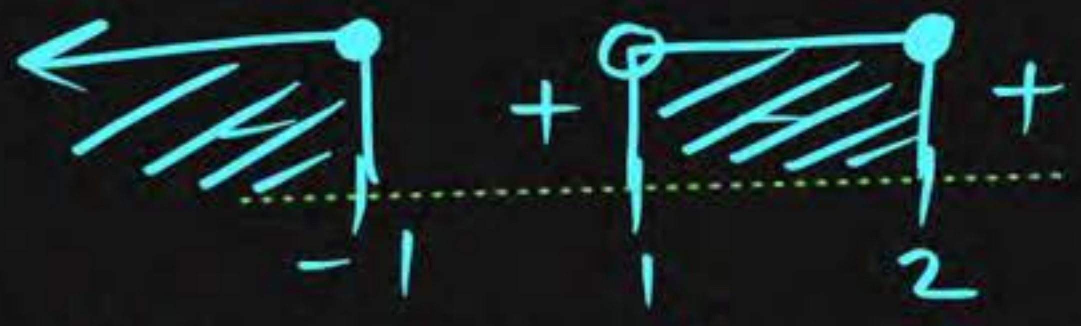
$$t \le \frac{2}{t-1}$$

$$t - \frac{2}{t-1} \le 0$$

$$\frac{t(t-1) - 2}{(t-1)} \le 0$$

$$\frac{t^2 - t - 2}{(t-1)} \le 0$$

$$\frac{(t-2)(t+1)}{(t-1)} \le 0$$



$\log_2 x \in (-\infty, -1] \cup (1, 2]$

$x \in (2^{-\infty}, 2^{-1}] \cup (2^1, 2^2]$

$x \in (0, \frac{1}{2}] \cup (2, 4]$

Question

$\sqrt{2x-5} > \sqrt{5-x}$ → non negative

$\sqrt{2x-5}$ → non negative

⇒ LHS & RHS both are non negative ∴ we can S.B.S.

→ Sq both sides

$2x-5 > 5-x$

$2x + x > 5+5$

$3x > 10$

$\boxed{x > 10/3}$ ✓

always take intersection with domain

Domain :

$2x-5 \geq 0 \Rightarrow \boxed{x \geq 5/2}$

& &

$5-x \geq 0$ $\quad -x \geq -5$

$\boxed{x \leq 5}$

Domain : $[5/2, 5]$ ✓

Ans: $(10/3, 5]$

$\boxed{\sqrt{x} \text{ if } x \geq 0}$

$\sqrt{x-2}$ if $x-2 \geq 0$

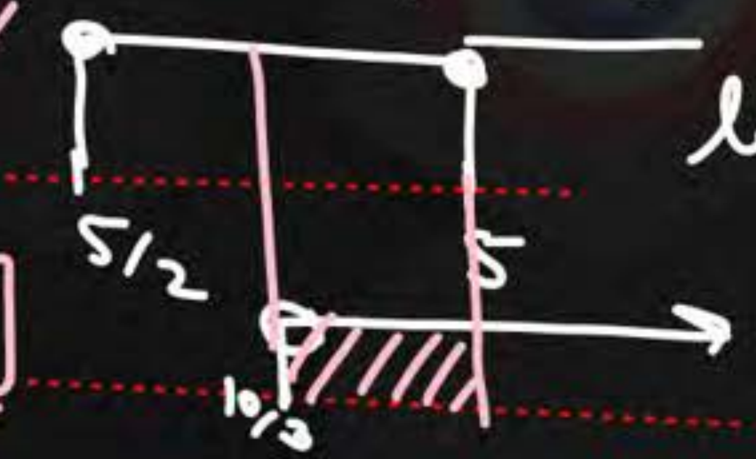
$\sqrt{4} = +2$ ✓

$\sqrt{36} = 6$

$\sqrt{x}$ → non negative

Square root is always non negative

$\sqrt{0} = 0$



log 0 = N.D.

Question

$\boxed{\sqrt{7x-6}} < x \rightarrow +ve$

Domain

$7x-6 \geq 0$

$7x \geq 6$

$\boxed{x \geq 6/7}$ ✓

$\downarrow$

LHS = non-ve

RHS: x = +ve in the domain :

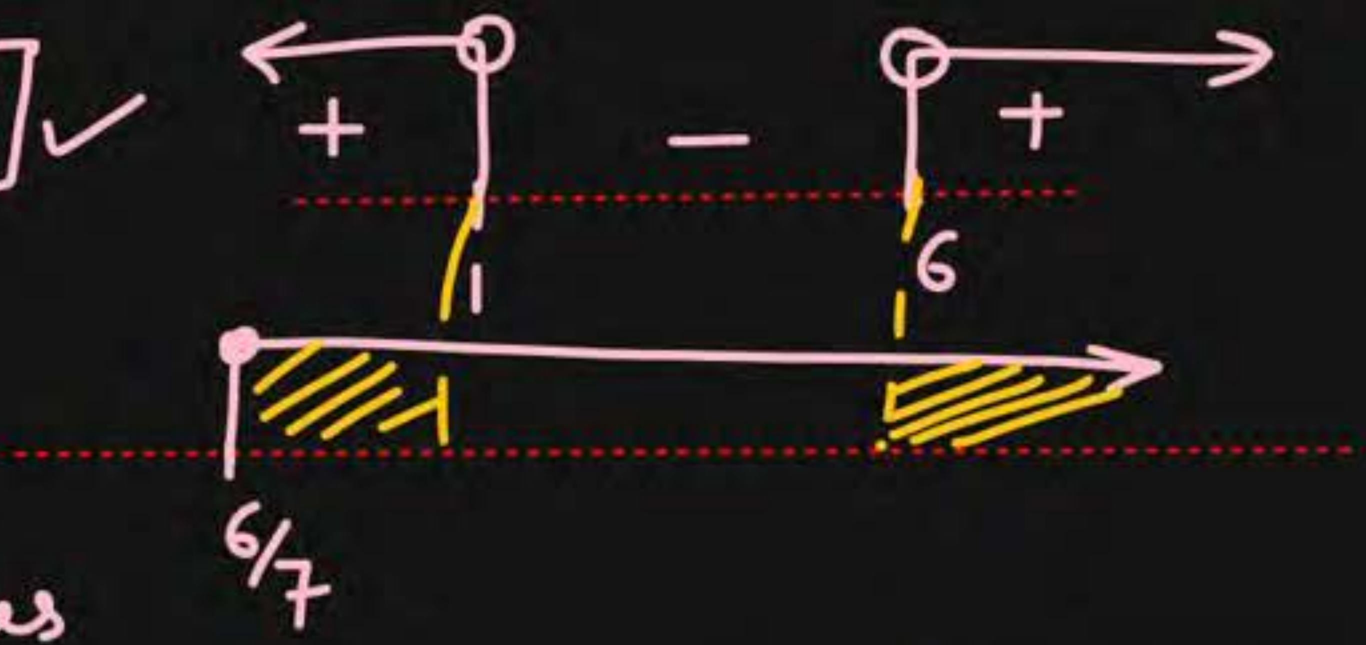
now we can square both sides

$7x-6 < x^2$

$-x^2+7x-6 < 0$

$x^2-7x+6 > 0$

$(x-6)(x-1) > 0$



Ans: $x \in [6/7, 1) \cup (6, \infty)$

Question

[Ans. [−18,−2)]

$\sqrt{x+18} < 2-x$

LHS = $\underbrace{\text{non -ve}}$

RHS : $2-x = +ve$

LHS < RHS

$\downarrow$

+ve < RHS $\Rightarrow$ RHS > 0

$\Rightarrow 2-x > 0$

$\Rightarrow 2 > x$

$\Rightarrow \boxed{x < 2}$

$-x > -2$

$x < 2$

✓ Domain :

$x + 18 \geq 0$

$\Rightarrow x \geq -18$

Domain : $[-18, \infty)$

Sq both sides

$x+18 < (2-x)^2$

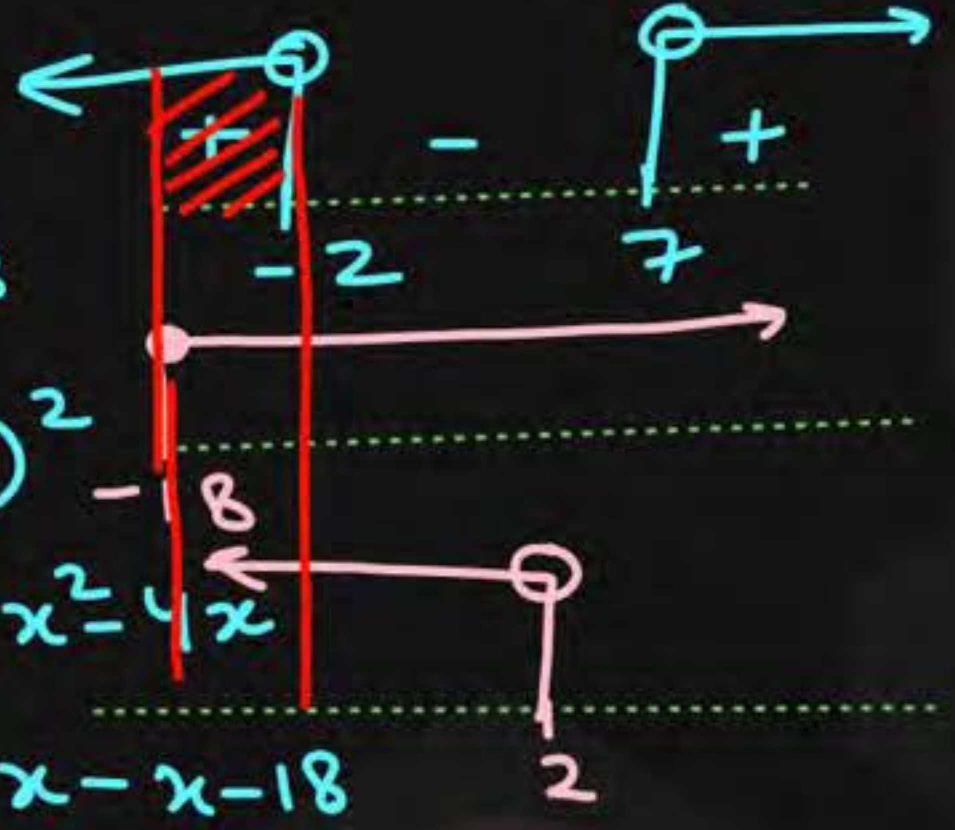
$x + 18 < 4 + x^2 - 4x$

$0 < 4 + x^2 - 4x - x - 18$

$x^2 - 5x - 14 > 0$

$x^2 - 7x + 2x - 14 > 0$

$(x+2)(x-7) > 0$



$x \in [-18, -2)$

Ans ✓

Question

[Ans. (1, 4]]

$$\sqrt{8+2x-x^2} > 6-3x$$

LHS = +ve

RHS = ?

Case → 1

RHS ≥ 0

$6-3x \geq 0$

$6 \geq 3x$

$2 \geq x \Rightarrow \boxed{x \leq 2}$

S.B.S.

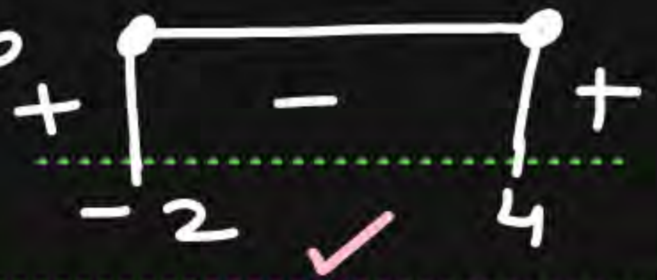
$8+2x-x^2 > (6-3x)^2$

Domain :

$8+2x-x^2 \geq 0$

$x^2-2x-8 \leq 0$

$(x-4)(x+2) \leq 0$



$8+2x-x^2 > 36+9x^2-36x$

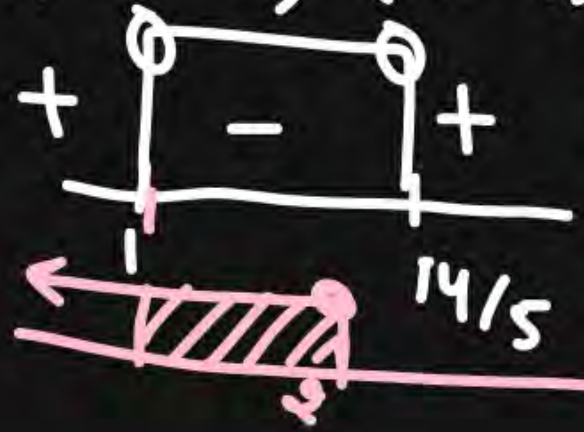
$0 > 36-8+10x^2-38x$

$10x^2-38x+28 < 0$

$5x^2-19x+14 < 0$

$5x^2-5x-14x+14 < 0$

$(5x-14)(x-1) < 0$



$I_1 : (1, 2]$

Case → 2

RHS < 0

$\underbrace{6-3x < 0}$ ⇒ $6 < 3x$ ⇒ $\boxed{x > 2}$

LHS = +ve

$\underbrace{\text{LHS}}_{+ve} > \underbrace{\text{RHS}}_{-ve}$

⇒ always true.

$x > 2$

$I_2 : x \in (2, \infty)$

Question

[Ans. [redacted]

$$\sqrt{x+3}+\sqrt{x+15}<6$$

Concluding Example

$|2x-4| < x-1$

$\text{LHS} < \text{RHS}$

$\text{LHS} = +ve$

$+ve < \text{RHS}$

$\text{RHS} = ? = +ve.$

$x-1>0$ $\boxed{x>1}$ ✓

Now we can S.B.S.

$|2x-4|^2 < (x-1)^2$

$(2x-4)^2 < x^2+1-2x$

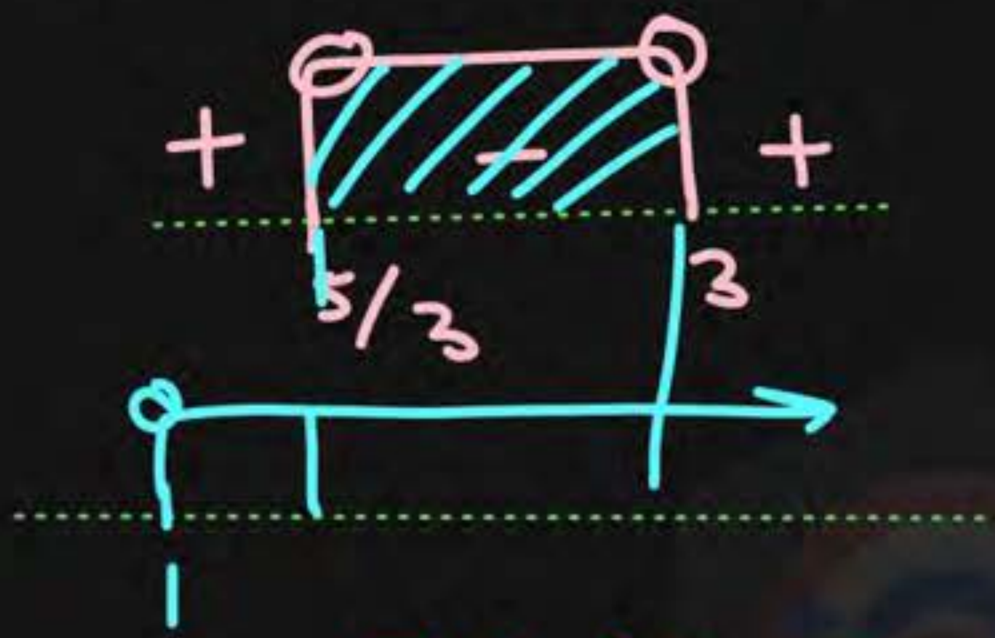
$4x^2+16-16x < x^2+1-2x$

$3x^2-14x+15<0$

$|x|^2 = x^2$

$3x^2-9x-5x+15<0$

$(3x-5)(x-3)<0$



$x \in \left(\frac{5}{3}, 3\right)$ Ans

Solving Exponential Equations

Brain Teaser

The number of solutions of the equation $|x-3|^{\frac{x^2-8x+15}{x-2}} = 1$ is/are

$x=3$	$x=2$
Question $(0)^0$	Rejected as Denominator $(x-2)$ becomes zero.

0^0 is not defined

$$\log_{10}|x-3|^{\frac{x^2-8x+15}{x-2}} = \log_{10} 1$$

$$\left(\frac{x^2-8x+15}{x-2}\right)\log|x-3| = 0$$

either $\frac{x^2-8x+15}{x-2} = 0$

$x^2-8x+15=0$

$(x-3)(x-5)=0 \Rightarrow x = 3$ or 5

OR : $\log_{10}|x-3| = 0$

$|x-3| = 1$

$x-3 = \pm 1$

$x = 3 \pm 1 \Rightarrow x = 4$ or 2

If $ab = 0$

$\Rightarrow$ either $a=0$ or $b=0$

$x=5$ or $x=4$ are the only answers

Brain Teaser

Find all the solutions of the equation $|x-1|^{(\log x)^2-\log x^2}=|x-1|^3$, where base of logarithm is 10.

Ans : $x = 1000, \frac{1}{10}$ or 2

$$\frac{|x-1|^{(\log x)^2-2\log x^2}}{|x-1|^3} = 1$$

$$|x-1|^{(\log x)^2-2\log x-3} = 1$$

$(\log x)^2-2\log x-3=0$ OR $|x-1|=1$

Let $\log x = t$

$t^2-2t-3=0$

$(t-3)(t+1)=0$

$t=3$ or $t=-1$

$\log_{10} x = 3$ or -1

$x = 10^3$ or 10^{-1}

1000 or $\frac{1}{10}$ ✓

$x-1=\pm 1$

$x = 1 \pm 1$

$x = 2$ ✓ or $x=0$ → Reject

HW-BT-5

HW

Find the real solutions to the system of equations

$$\log_{10}(2000xy) - \log_{10} x \cdot \log_{10} y = 4$$

$$\log_{10}(2yz) - \log_{10} y \cdot \log_{10} z = 1$$

and $$\log_{10}(zx) - \log_{10} z \cdot \log_{10} x = 0$$

Ans. $x = 1, y = 5,$
$z = 1$ or $x = 100,$
$y = 20, z = 100$

Q HW- BT-6

[Ans.

Let (x_0, y_0) be the solution of the following equations $(2x)^{\ln 2} = (3y)^{\ln 3}$
$3^{\ln x} = 2^{\ln y}$

HW

Then x_0 is

JEE Advanced 2011

A $1/6$

B $1/3$

C $1/2$

D 6

$\ln x = \log_e x$

Question

$$\log_{2}\left(\frac{2x^2-3x+1}{x^2-5x+6}\right) \geq 1$$

$$\Rightarrow \frac{2x^2-3x+1}{x^2-5x+6} \geqslant 2^1$$

$$\frac{2x^2-3x+1}{x^2-5x+6} - 2 \geqslant 0$$

$$\frac{2x^2-3x+1-2(x^2-5x+6)}{x^2-5x+6} \geqslant 0$$

$$\frac{\cancel{2x^2}-3x+1-\cancel{2x^2}+10x-12}{x^2-5x+6} \geqslant 0.$$

$$\frac{7x-11}{(x-2)(x-3)} \geqslant 0$$

~~Domain:~~

$$\cancel{\frac{2x^2-3x+1}{x^2-5x+6} > 0}$$

~~No need of Domain.~~

$x \in \left[\frac{11}{7}, 2\right) \cup (3, \infty)$

Question

$$\log_{1/2}(x^2 - 1) > 0$$

A $(-\sqrt{2}, -1) \cup (1, \sqrt{2})$

B $(-2, -1) \cup (1, 2)$

C $(-\infty, -1) \cup (1, \infty)$

D $(-2, 2)$

$x^2 - 1 < \left(\tfrac{1}{2}\right)^0$

$x^2 - 1 < 1$

$\Rightarrow x^2 < 2$

$1 < x^2 < 2$

$\Rightarrow x \in (1, \sqrt{2}) \cup (-\sqrt{2}, -1)$

Domain:

$x^2 - 1 > 0$

$x^2 > 1$

Question

$$\log_2(\underbrace{\log_3\log_4(x-1)})>0$$

A $(1,\infty)$

B $(1,65)$

C $(1,5)\cup(65,\infty)$

D $(65,\infty)$

$\log_3(\underbrace{\log_4(x-1)}) > 1$

$\log_4(x-1) > 3^1$

$\log_4(\underbrace{x-1}) > 3$

$(x-1) > 4^3$

$x-1 > 64$

$\boxed{x > 65}$

$x \in (65,\infty)$

Logarithm Inequalities- CASE-2 (when base is Variable)

Question

Solve : $\log_{2x}(x^2-5x+6) \geq 1.$

or $\rightarrow \cup$

Case $\rightarrow$ 1

$2x > 1 \Rightarrow \boxed{x > \frac{1}{2}}$

$\Rightarrow x^2-5x+6 \geq (2x)^1$

$x^2-7x+6 \geq 0$

$(x-6)(x-1) \geq 0$

$I_1 : \left(\frac{1}{2}, 1\right] \cup (6, \infty)$

Case $\rightarrow$ 2

$0 < 2x < 1$

$\boxed{0 < x < \frac{1}{2}}$

base is less than 1

$x^2-5x+6 \leq (2x)^1$

$I_2 = \phi$

Domain:

$x^2-5x+6 > 0$

$(x-3)(x-2) > 0$

Ans: $I_1 \cup I_2 \cap \text{Dom}$

Ans: $\left(\frac{1}{2}, 1\right] \cup [6, \infty)$

Solving Exponential Inequations

Case 1(A) When base is > 1

Question

$$5^{x+3} > \left(\frac{1}{25}\right)^{1/x}$$

$5^{x+3} > (5^{-2})^{1/x}$

$5^{x+3} > 5^{-2/x}$

$\Rightarrow x+3 > -2/x$

$x+3+2/x > 0$

$\frac{x^2+3x+2}{x} > 0$

$\frac{(x+2)(x+1)}{x} > 0$

$- \quad + \quad - \quad +$

$-2 \quad -1 \quad 0$

$x \in (-2,-1) \cup (0, \infty)$

Case 1(B) When base is < 1

Question

Solve $(0.5)^{1/x} \geq 0.0625$

$(0.5)^{1/x} \geq (0.5)^4$

$1/x \leq 4$

$\frac{1}{x} - 4 \leq 0$

$\frac{1-4x}{x} \leq 0 \Rightarrow \frac{4x-1}{x} \geq 0$

$+ \quad - \quad +$

$0 \quad 1/4$

HW-BT-3 HW

Find 'x' satisfying the equation

$$4^{\log_{10} x+1} - 6^{\log_{10} x} - 2.3^{\log_{10} x^2+2} = 0.$$

HW-BT-4

HW

If a, b, c are distinct positive numbers, each different from 1, such that
$[\log_b a \log_c a - \log_a a] + [\log_a b \log_c b - \log_b b] + [\log_a c \log_b c - \log_c c] = 0$, then $abc =$

A 1

B 2

C 3

D None of these

DPP-7

If $a = \log_{24} 12$ and $b = \log_{36} 24$ & $c = \log_{48} 36$ then the value of abc = ?

A. $2bc - 1$

B. $2bc + 1$

C. $bc - 1$

D. $bc + 1$

$$abc = \frac{\log 12}{\log 24} \times \frac{\log 24}{\log 36} \times \frac{\log 36}{\log 48}$$

$$abc = \frac{\log_{12} 12}{\log_{12} 48} = \frac{1}{\log_{12} 48} = \text{LHS}$$

$$bc = \frac{\log 24}{\cancel{\log 36}} \times \frac{\cancel{\log 36}}{\log 48} = \frac{\log_{12} 24}{\log_{12} 48} = bc$$

$$2bc = \frac{2\log_{12} 24}{\log_{12} 48} = \frac{\log (24 \times 24)}{\log 48} = \frac{\log_{12}(48 \times 12)}{\log_{12} 48}$$

$$2bc = \frac{\log_{12} 48 + \log_{12} 12}{\log_{12} 48}$$

1

$$2bc = 1 + \frac{1}{\log_{12} 48}$$

$$\boxed{\underbrace{2bc-1}_{} = \frac{1}{\log_{12} 48}}$$

Ans : $\underline{2bc-1}$

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[Ans. [redacted]

The value of $\left(\left(\log_2 9\right)^{\frac{1}{\log_2(\log_2 9)}} \times \left(\sqrt{7}\right)^{\frac{1}{\log_4 7}}\right)$ is ______.

Let $\log_2 9 = a$

$a^{\frac{1}{\log_2 a}}$

$a^{\log_a 2}$

$= 2$

$= 2$

$(\sqrt{7})^{\log_7 4}$

$\left[(7)^{1/2}\right]^{\log_7 4}$

$7^{\frac{1}{2}\log_7 4}$

$7^{\log_7 4^{1/2}}$

$4^{1/2}$

2

$$\boxed{a^{\log_a x} = x}$$

$\boxed{\text{Ans} = 4}$

Brain Teaser Solve for x:

$$25^{\log_{10}x} = 5 + 4 . x^{\log_{10}5}$$

$$5^{2\log_{10}x} = 5 + 4 . (x)^{\log_{10}5} \rightarrow \text{(P-11)}$$

$$5^{2\log x} = 5 + 4 . (5)^{\log x}$$

Let $5^{\log x} = t$

$$\boxed{t^2 = 5 + 4t}$$

$$t^2 - 4t - 5 = 0$$

$$(t-5)(t+1) = 0$$

$$t = 5 \text{ or } t = -1$$

$$5^t = 5^1$$

$$5^{\log x} = t$$

$$(5^{\log x})^2 = t^2$$

$$5^{2\log x} = t^2$$

$5^{\log x} = 5^1$ or $\underbrace{5^{\log x} = -1}$

$\Rightarrow \log_{10} x = 1$

$\Rightarrow x = 10^1$

$\boxed{x = 10}$ Ans

Reject. 5^y is non -ve.

Brain Teaser

Solve the system of equations for x , y & z :

$$(\log_a x)\boxed{\log_a(xyz)} = 48 \quad \text{--- (1)}$$

$$(\log_a y)\boxed{\log_a(xyz)} = 12 \quad \text{--- (2)}$$

$$(\log_a z)\boxed{\log_a(xyz)} = 84 \quad \text{--- (3)}$$

$a > 0, a \neq 1.$

$\sqrt{x^2} = |x|$

(1) $b^2 = 144$

$\Rightarrow b = \pm\sqrt{144}$

$b = \pm 12$

(2) If $b^2 = 3$

$\Rightarrow b = \pm\sqrt{3}$

$x^2 = 1 \Rightarrow x = \pm 1$

(1) + (2) + (3)

$$\log_a(xyz)\cdot\left[\underbrace{\log_a x + \log_a y + \log_a z}\right] = 48 + 12 + 84$$

$$= 144$$

$$\log_a(xyz)\cdot\log_a(xyz) = 144$$

$$\left[\log_a(xyz)\right]^2 = 144 \Rightarrow \log_a(xyz) = \pm 12$$

Case → 1

$$\log_a(xyz) = 12$$

$$(\log_a x)12 = 48$$

$$\log_a x = \frac{48}{12}$$

$$\log_a x = 4$$

$$\Rightarrow x = a^4$$

$$(\log_a y) \times \cancel{12} = \cancel{12}$$

$$y = a^1$$

$$(\log_a z) \times 12 = 84$$

$$\Rightarrow \log_a z = 7 \Rightarrow z = a^7$$

Case → 2

$$\log_a(xyz) = -12$$

$$\left.\begin{matrix} x = a^{-4} \\ y = a^{-1} \\ z = a^{-7} \end{matrix}\right\} \text{Ans.}$$

Brain Teaser

$\log_b N \quad N>0$

$\sqrt{4} = 2$
$\sqrt{5} = 2\cdots$

The number of real solutions of the equation $\log(-x) = 2\log(x+1)$, is

A 0

B 1

C 2

D 4

$$\log(-x) = \log(x+1)^2$$
$$\Rightarrow -x = (x+1)^2$$
$$-x = x^2+1+2x$$
$$\Rightarrow \boxed{x^2+3x+1=0}$$
$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$
$$x = \frac{-3 \pm \sqrt{9-4\times1\times1}}{2\times1}$$
$$x = \frac{-3\pm\sqrt{5}}{2}$$

$\frac{-3+\sqrt{5}}{2} \rightarrow -ve$

$\frac{-3-\sqrt{5}}{2} \rightarrow -ve$

$(-x) > 0$
$\Rightarrow \boxed{x<0}$

If $x<0$
$\Rightarrow -x>0$

If $\log a = \log b$
$\Rightarrow a = b$

$x+1>0$

$$x = \frac{-3+\sqrt{5}}{2}$$

$$\Rightarrow x+1 = \frac{-3+\sqrt{5}}{2} + \frac{1}{1}$$

$$x+1 = \frac{-3+\sqrt{5}+2}{2}$$

$$(x+1) = \frac{\sqrt{5}-1}{2} \rightarrow +ve$$

❤ ❤

only 1 solutions

$$x = \frac{-3-\sqrt{5}}{2}$$

$$x+1 = \frac{-3-\sqrt{5}}{2} + 1$$

$$x+1 = \frac{-3-\sqrt{5}+2}{2}$$

$$x+1 = \frac{-1-\sqrt{5}}{2} = -ve$$

Brain Teaser

2×70
$2^2\times 35 = 2^2\times 5\times 7$

If $a = \log_{62} 3$ and $b = \log_{62} 5$ & $c = \log_7 2$ then find the value of $\log_{63} 140$ in terms of a, b, c.

$$c = \frac{1}{\log_2 7}$$

$$\Rightarrow \log_2 7 = \frac{1}{c}$$

$$\log_{63} 140 = \frac{\log_2(140)}{\log_2 63}$$

$$= \frac{\log_2(2^2\times 5\times 7)}{\log_2(3^2\cdot 7)}$$

$$= \frac{\log_2 2^2 + \log 5 + \log 7}{\log 3^2 + \log 7}$$

$$= \frac{2 + \underbrace{\log_2 5}_{b} + \underbrace{\log_2 7}_{1/c}}{2\underbrace{\log_2 3}_{a} + \underbrace{\log_2 7}_{1/c}}$$

$$\text{Ans} = \frac{2 + b + 1/c}{2a + 1/c}$$

$$= \frac{2c + bc + 1}{2ac + 1} \quad \text{Ans}$$

Brain Teaser

HW

If $a = \log_{12} 18$ and $b = \log_{24} 54$ then find the value of $ab + 5(a - b)$.

A -1

B 1

C 0

D 3

Brain Teaser HW

Simplify : $5^{\log_{1/5}\left(\frac{1}{2}\right)} + \log_{\sqrt{2}}\frac{4}{\sqrt{7}+\sqrt{3}} + \log_{1/2}\frac{1}{10+2\sqrt{21}}$.

Brain Teaser

If $x = \log_a bc$, $y = \log_b ca$, $z = \log_c ab$, then $\frac{1}{x+1} + \frac{1}{y+1} + \frac{1}{z+1} =$

A. -1

B. 1 ✓

C. 0

D. 3

\# $y+1 = 1 + \log_b ca = \log_b b + \log_b ca$

$y+1 = \log_b bca$

$x = \log_a bc$

$x+1 = 1 + \log_a bc$

$x+1 = \log_a a + \log_a bc$

$x+1 = \log_a (abc)$

$\Rightarrow \frac{1}{x+1} = \log_{abc} a \sim (1)$

Similarly $\frac{1}{y+1} = \log_{abc} b \sim (2)$

$\frac{1}{z+1} = \log_{abc} c \sim (3)$

$(1) + (2) + (3)$

$\frac{1}{x+1} + \frac{1}{y+1} + \frac{1}{z+1} = \log a + \log b + \log c$

$= \log_{abc} abc$

$= 1$

Q JEE Advanced 2013

Multiple correct.

If $3^x = 4^{x-1}$, then $x =$

A $\dfrac{2\log_3 2}{2\log_3 2 - 1}$ ✓

B $\dfrac{2}{2 - \log_2 3}$ ✓

C $\dfrac{1}{1 - \log_4 3}$ ✓

D $\dfrac{2\log_2 3}{2\log_2 3 - 1}$ ✗

$$3^x = 4^{x-1}$$

take log both sides

$$\log_2 (3)^{x} = \log_2 (4)^{x-1}$$

$$x\log_2 3 = (x-1)\boxed{\log_2 4} \to 2$$

$$x\log_2 3 = (x-1)\times 2$$

$$x\log_2 3 = 2x - 2$$

$$x\log_2 3 - 2x = -2$$

$$x(\log_2 3 - 2) = -2 \Rightarrow x = -\frac{2}{\log_2 3 - 2} = \frac{2}{2-\log_2 3} \text{ (B)}$$

$$x = \frac{2}{2-\log_2 3}$$

$$x = \frac{2}{2 - \frac{1}{\log_3 2}}$$

$$x = \frac{2}{\frac{2\log_3 2 - 1}{\log_3 2}}$$

$$x = \frac{2\log_3 2}{2\log_3 2 - 1}$$

(A)

(c) $\dfrac{1}{1-\log_4 3} = \dfrac{1}{1-\log_{2^2} 3} = \dfrac{1}{1-\frac{1}{2}\log_2 3}$

$= \dfrac{2}{2-\log_2 3}$

$= x$

Characteristic & Mantissa of Logarithm

$\log_{10} 101 = 2. \ldots$

$\log_{10} 999 = 2.9\ldots$

$\log_5 5 = 1$

$\log_5 25 = 2$

$\log_5 125 = 3$

$\log_5 90 =$ characteristic ?
$= 2$

characteristic

If $\log_{10} N = 1.201$

Mantissa

$\log_{10} 100 = 2.0$

Mantissa

Charecterstic

$\log_{10} 1000 = 3$

Question

Find the number of positive integers which have the characteristic 3, if the base of the logarithm is 5.

Characteristic is 3 for

$\log_5 125 = 3$

$\underbrace{1, 2, 3, \ldots 124}$ $\{125, 126, 127, 128, \ldots \ldots 624\}$

$\log_5 625 = 4$

$= 624 - 124$

$= \underline{\underline{500}}$ ✓

$624 = a + (n-1)d$

$624 = 125 + (n-1) \times 1$

$624 = 124 + n$

$\boxed{n = 500}$ ✓

Characteristic & Mantissa of Logarithm

Characteristic = 2 is for

$\{\underbrace{100, 101, 102, 103, \ldots\ldots 999}\}$

Characteristic = 3 is for

$\{\underbrace{1000, 1001, 1002, \ldots\ldots 9999}\}$

$\log_{10} 10 = 1$

$\log_{10} \underbrace{100} = 2$

$\log_{10} \underbrace{1000} = 3$

$\log_{10} \underbrace{10,000} = 4$

If $\log N$ has characteristic $= 3$ then N is a 4 digit No.

If characteristic of $\log_{10} N$ is x then no of digits in N is $\underbrace{x+1}$

characteristic of $\log_{10} N$ is 20 then no of digits in N is $= 21$

Question

Number of digits in the number 12^{300} are (Given $\log_{10} 2 = 0.3010$, $\log_{10} 3 = 0.4771$)

A 323

B 324

C 325

D None of these

$$N = (12)^{300}$$

$$\log_{10} N = \log_{10} (12)^{300}$$

$$= 300 \log_{10} 12$$

$$300 \log_{10} (2^2 \cdot 3)$$

$$300 [2 \log_{10} 2 + \log_{10} 3]$$

$$= 300 [2 \times 0.301 + 0.4771]$$

$$= 300 \times [0.602 + 0.4771]$$

$$\log_{10} N = 323.7\ldots$$

Question

The value of $\log_3 4 \cdot \log_4 5 \cdot \log_5 6 \cdot \log_6 7 \cdot \log_7 8 \cdot \log_8 9$ is

A 1

B 2

C 3

D 4

$$\frac{\cancel{\log 4}}{\log 3}\cdot\frac{\cancel{\log 5}}{\cancel{\log 4}}\cdot\frac{\cancel{\log 6}}{\cancel{\log 5}}\cdot\frac{\cancel{\log 7}}{\cancel{\log 6}}\cdot\frac{\cancel{\log 8}}{\cancel{\log 7}}\cdot\frac{\log 9}{\cancel{\log 8}}$$

$$= \frac{\log 9}{\log 3} = \frac{2\cancel{\log(3)}}{\cancel{\log 3}} = 2$$

Question

Solve for x:

$$1+2\log_{(x+2)}5 = \log_5(x+2)$$

$$\log_{(x+2)}5 \to t \qquad \log_5(x+2) \to \frac{1}{t}$$

Let $\log_{(x+2)}5 = t$

$$1+2t = \frac{1}{t}$$

$$t+2t^2 = 1$$

$$\boxed{2t^2+t-1=0}$$

$$2t^2+2t-t-1=0$$

$$2t(t+1)-1(t+1)=0$$

$$(2t-1)(t+1)=0$$

$t = \frac{1}{2}$ or $t = -1$

$\Rightarrow \log_{x+2}5 = \frac{1}{2}$ or $\log_{x+2}5 = -1$

$\Rightarrow 5 = (x+2)^{1/2}$

S.B.S.

$25 = x+2$

$\Rightarrow \boxed{x = 23}$

Ans

$\Rightarrow 5 = (x+2)^{-1}$

$5 = \frac{1}{x+2}$

$\Rightarrow 5(x+2) = 1$

$5x+10 = 1$

$5x = -9$

$\boxed{x = -9/5}$

Ans

Question

Find the Exact value of $\frac{\log_3 135}{\log_{15} 3} - \frac{\log_3 5}{\log_{405} 3}$

$135 = 15 \times 9$
$= 3 \times 5 \times 3^2$
$= 3^3 \cdot 5$

$405 = 27 \times 15$
$= 3^3 \cdot 3 \cdot 5$
$= 3^4 \cdot 5$

$(\log_3 135)(\log_3 15) - (\log_3 5)(\log_3 405)$

$\log(3^3 \cdot 5) \quad \log(3 \times 5) - (\log 5) \log(3^4 \cdot 5)$

$(\log_3 3^3 + \log 5)(\log_3 3 + \log 5) - \log_3 5(\log_3 3^4 + \log_3 5)$

$3 \qquad 1 \qquad 4$

$(3 + \log_3 5)(1 + \log_3 5) - \log_3 5(4 + \log_3 5)$ Let $\log_3 5 = t$

$(3+t)(1+t) - t(4+t)$

$(3 + \cancel{4t} + \cancel{t^2}) - \cancel{4t} - \cancel{t^2}$

$= 3$

Question

Logarithm of $\underbrace{32\sqrt[5]{4}}$ to the base $\underbrace{2\sqrt{2}}$ is

A 3.6 ✓

B 5

C 5.6

D None of these

$$\log_{(2)^{3/2}} (2)^{27/5}$$

$$\frac{27/5}{3/2} \log_2 2 \xrightarrow{} 1$$

$$= \frac{27 \times 2}{5 \times 3}$$

$$= \frac{18}{5} = \boxed{3.6}$$

$$b = 2\sqrt{2}$$

$$b = 2^1 \cdot 2^{1/2}$$

$$2^{1+1/2}$$

$$b = 2^{3/2}$$

$$N = 32(4)^{1/5}$$

$$N = 2^5 \cdot (2^2)^{1/5}$$

$$2^5 \cdot 2^{2/5}$$

$$= 2^{5+2/5}$$

$$N = 2^{27/5}$$

Question

If $\log_{7} 2 = m$, then $\log_{49} 28$ is equal to

A $2(1+2m)$

B $\dfrac{1+2m}{2}$

C $\dfrac{2}{1+2m}$

D $1+m$

$$\log_{7^2} 28 = \frac{1}{2}\log_7(28)$$

$$= \frac{1}{2}\log_7(7\times 2^2)$$

$$= \frac{1}{2}\left[\log_7 7 + \log_7 2^2\right]$$

$$= \frac{1}{2}\left[1 + 2\underbrace{\log_7 2}_{m}\right]$$

$$= \frac{1}{2}\left[1+2m\right]$$

Question

The value of $\boxed{81^{(1/\log_5 3)}} + \boxed{27^{\log_9 36}} + \boxed{3^{4/\log_7 9}}$ is equal to

A 49

B 625

C 216

D 890

$$(81)^{\frac{1}{\log_5 3}} = (81)^{\log_3 5} = 3^{4\log_3 5} = 3^{\log_3 (5)^4}$$

$$= (5)^4 = 25^2 = 625$$

$$(27)^{\log_9 36} = (27)^{\log_{3^{\cancel{2}}} 6^{\cancel{2}}}$$

$$= (3)^{3\log_3 6} = 3^{\log_3 6^3} = (6)^3 = 216$$

$$3^{4/\log_7 9} = 3^{4\log_9 7} = 3^{4\log_{3^2} 7} = 3^{2\log_3 7} = 3^{\log_3 7^2} = 7^2 = 49$$

Question

Match the following:

Column 1		Column 2	
(1)	$\frac{1}{9}^{\log_3 7}$ → (c)	(A)	$\frac{1}{27}$
(2)	$2^{-\log_{1/2} 7}$ D)	(B)	$\frac{1}{2}$
(3)	$8^{-\frac{1}{\log_3 2}}$ A)	(C)	$\frac{1}{49}$
(4)	$3^{-\log_3 2}$ B)	(D)	7

1) $\left(\frac{1}{9}\right)^{\log_3 7} = \left(\frac{1}{3}\right)^{2\log_3 7} = (3^{-1})^{2\log_3 7} = 3^{-2\log_3 7} = (7)^{-2} = \frac{1}{7^2} = \frac{1}{49}$

2) $2^{-\log_{2^{-1}} 7} = 2^{\log_2 7} = 7.$

3) $(8)^{-\frac{1}{\log_3 2}} = 8^{-\log_2 3} = (2)^{-3\log_2 3} = (3)^{-3} = \frac{1}{3^3} = \frac{1}{27}$

$2^{-1} = \frac{1}{2}$

Question

$x^2 + 7^{\log_7 x} - 2 = 0$ then x is

x

A 1

B −2

C −1

D 2

$$x^2 + x - 2 = 0$$

$$\Rightarrow x^2 + 2x - x - 2 = 0$$

$$(x+2)(x-1) = 0$$

$$\underbrace{x = -2}_{\text{Reject}} \text{ or } \underbrace{x = 1}_{\text{Accept}}$$

Question

$$7^{\log_3 5} + 3^{\log_5 7} - 5^{\cancel{\log_3 7}} - 7^{\log_5 3} = \underline{\text{zero}}$$

$$\cancel{5^{\log_3 7}}$$

$$7^{\log_5 3}$$

Question

HW Discussion

Find the value of :

$\log_2 [\log_2 \{ \log_3 (\log_3 27^3) \}]$ → 1

A) 1

B) 2

C) 3

D) 0 ✓

$\{ \log_3 (9) \}$

$\log_3 (3)^2$

$2(\log_3 3)$

$= 2$

$\log_2 2 = 1$

$\log_2 (1) = 0$

$\log_3 (27)^3$

$\log_3 [(3)^3]^3$

$= \log_3 (3)^9$

$(a^m)^n = a^{mn}$

$= 9 \log_3 3$ → 1

$= 9$

Question

$243 = (3)^5$

$\sqrt{32} = (32)^{1/2} = (2^5)^{1/2} = 2^{5/2}$

	Column I		Column II
(A)	$\left\{\dfrac{\log_3 243}{\log_2 \sqrt{32}}\right\}$	(p)	positive integer
(B)	$\dfrac{2\log 6}{\boxed{(\log 12 + \log 3)}} = 1$	(q)	negative integer
(C)	$\log_{1/3}\left(\dfrac{1}{9}\right)^{-2}$ (q)	(r)	rational but not integer
(D)	$\dfrac{\log_5 16 - \log_5 4}{\log_5 128}$	(s)	prime

A) $\dfrac{\log_3 243}{\log_2 \sqrt{32}} = \dfrac{\log_3 (3)^5}{\log_2 (2)^{5/2}}$

$= \dfrac{5 \log_3 3}{5/2 \log_2 2}$ $\log_3 3 \rightarrow 1$, $\log_2 2 \rightarrow 1$

$= \dfrac{5}{5/2} = \dfrac{\cancel{5}}{\cancel{5}} \times 2 = 2$

$A \rightarrow p, (s)$

B) $\dfrac{2 \log 6}{\log 36} = \dfrac{\log 6^2}{\log 36}$

$= \dfrac{\cancel{\log 36}}{\cancel{\log 36}}$

(B) $\rightarrow$ (P)

(C) $\log_{1/3}\left(\frac{1}{3}\right)^{2\times(-2)} = \log_{1/3}\left(\frac{1}{3}\right)^{-4} = -4 \log_{1/3} \frac{1}{3} \rightarrow 1 = -4 \times 1 = -4$

Concuding Example

The value of $7\log\frac{16}{15}+5\log\frac{25}{24}+3\log\frac{81}{80}=?$

A log 2

B log 3

C log 5

D None of these

$5^7\cdot5^3=5^{7+3}$
$=5^{10}$

$\log x+\log y=\log(xy)$

$\log\left(\frac{16}{15}\right)^7+\log\left(\frac{25}{24}\right)^5+\log\left(\frac{81}{80}\right)^3$

$=\log\left[\left(\frac{16}{15}\right)^7\cdot\left(\frac{25}{24}\right)^5\cdot\left(\frac{81}{80}\right)^3\right]$

$\log\left[\left(\frac{2^4}{3\cdot5}\right)^7\cdot\left(\frac{5^2}{2^3\cdot3}\right)^5\cdot\left(\frac{3^4}{2^4\cdot5}\right)^3\right]$

$\log\left[\frac{2^{28}}{3^7\cdot5^7}\cdot\frac{5^{10}}{2^{15}\cdot3^5}\cdot\frac{3^{12}}{2^{12}\cdot5^3}\right]=\log\left[\frac{2^{28}\cdot\cancel{5^{10}}\cdot\cancel{3^{12}}}{\cancel{3^{12}}\cdot\cancel{5^{10}}\cdot2^{27}}\right]$

$80=16\times5$
$=2^4\times5$

$\log\frac{2^{28}}{2^{27}}=\log 2^{28-27}$
$=\log 2$

Examples on Basic Properties of Logarithm (P-1 to P-6)

Question

The value of $\sqrt{(\log_{0.5}^2 4)}$ is

A -2

B $\sqrt{(-4)}$

C 2

D None of these

$\sqrt{x^2} = |x|$

$\log^2 x = (\log x)^2$

$$= \sqrt{(\log_{0.5} 4)^2}$$

$$= |\log_{1/2} 4|$$

$$= |\log_{1/2} (2)^2|$$

$$= |2 \log_{1/2} 2| \quad \log_{1/2} 2 \to -1$$

$$= |2 \times (-1)| = |-2| = 2.$$

$\sqrt{4} = \pm 2$

Question

The value of $\log_{\sqrt{2}}\sqrt{2\sqrt{2\sqrt{2\sqrt{2}}}}$, is

A $\frac{15}{16}$

B $\frac{7}{16}$

C $\frac{15}{8}$

D $\frac{31}{32}$

N

$\underbrace{2^x \cdot 2^y = 2^{(x+y)}}$

$(a^m)^n = a^{mn}$

$$N = \sqrt{2\sqrt{2\sqrt{2\sqrt{2}}}}$$

$$\sqrt{2\sqrt{2\sqrt{2^1 \cdot 2^{1/2}}}}$$

$$\sqrt{2\sqrt{2\cdot(2^1\cdot 2^{1/2})^{1/2}}}$$

$$\sqrt{2\sqrt{2^1(2^{3/2})^{1/2}}}$$

$$N = \sqrt{2\sqrt{2^1 \cdot 2^{3/4}}}$$

$$N = \sqrt{2\sqrt{2^{1+3/4}}}$$

$$= \sqrt{2\sqrt{2^{7/4}}}$$

$$= \sqrt{2\cdot(2^{7/4})^{1/2}} = \sqrt{2^1 \cdot 2^{7/8}}$$

$$= \sqrt{2^{1+7/8}} = \sqrt{2^{15/8}} = (2^{15/8})^{1/2}$$

$$= (2)^{\frac{15}{16}}$$

$$\sqrt{2} = 2^{1/2}$$

Examples on Basic Properties of Logarithm (P-1 to P-6)

Question

Simplify : $\log_{1/3}\boxed{\sqrt[4]{729\cdot\sqrt[3]{9^{-1}\cdot 27^{-4/3}}}} \to N$

$N = \left[3^6 \cdot \left(3^{-2} \cdot 3^{\cancel{3}(-4/\cancel{3})}\right)^{1/3}\right]^{1/4}$

$\left[3^6 \left(\underbrace{3^{-2} \cdot 3^{-4}}\right)^{1/3}\right]^{1/4}$

$\left[3^6 \left(3^{-\cancel{6}}\right)^{1/\cancel{3}}\right]^{1/4}$

$\left[3^6 \cdot 3^{-2}\right]^{1/4}$

$\left[3^{6-2}\right]^{1/4}$

$\left[3^{\cancel{4}}\right]^{1/\cancel{4}}$

$N = 3$

$\log_{1/3} 3 = -1$

Ans

$\sqrt[3]{x} = (x)^{1/3}$

$\sqrt[4]{x} = (x)^{1/4}$

$729 = (3)^6$

$9 = 3^2$

$27 = 3^3$

Solving Logarithm Equations

Q→1 $\log_2 x = 5 \Rightarrow x = 2^5 = 32$

Q→2 $|\log_2 x| = 3$

$\Rightarrow \log_2 x = \pm 3$

$x = (2)^{\pm 3}$

$x = 2^3$ or $x = 2^{-3}$

$x = 8$ or $x = \frac{1}{2^3} = \frac{1}{8}$ ✓

✓

$|t| = 3$

$\Rightarrow t = \pm 3$

Question

Find the value of x

(i) $\log_2(x+1) - \log_2(2x-3) = 0$ $\Rightarrow$ $\log_2\left[\frac{(x+1)}{(2x-3)}\right] = 0$

$\Rightarrow \frac{x+1}{2x-3} = 2^0$

$\frac{x+1}{2x-3} = 1 \Rightarrow x+1 = 2x-3$

$x - 2x = -3-1$

$-x = -4$

$\Rightarrow \boxed{x=4}$

(ii) $\log_2(\log_3(\log_2 x)) = 0$

A) 1

B) 3

C) 4

D) 8 ✓

$\Rightarrow \log_3(\log_2 x) = 2^0$

$\log_3(\log_2 x) = 1$

$\log_2 x = 3^1$

$\log_2 x = 3$

$x = (2)^3$

$x = 8$

Question

Solve for x: $\log_4 \boxed{\left(2\log_3(1+\log_2(1+3\log_3 x))\right)} = 1/2$

N

$\log_4 N = \frac{1}{2}$

$\Rightarrow N = (4)^{1/2} = 2$

$\cancel{2}\log_3(1+\log_2(1+3\log_3 x)) = \cancel{2}$

$\log_3 \boxed{(1+\log_2(1+3\log_3 x))} = 1$

$\log_3 N = 1$

$\Rightarrow N = 3^1 = 3$

$1+\log_2(1+3\log_3 x) = 3$

$\Rightarrow \log_2(\underline{1+3\log_3 x}) = 2$

$1+3\log_3 x = (2)^2$

$1+3\log_3 x = 4$

$\cancel{3}\log_3 x = \cancel{3}$

$\Rightarrow \log_3 x = 1$

$\Rightarrow x = 3^1$

$\boxed{x=3}$

Question

#.

$$\log(x)^y = y\log x$$

Solve for x: $x^{\log_{10} x+1} = 10^6$

Taking log both sides.

$$\log_{10}(x)^{\log_{10}x+1} = \log_{10}(10)^6$$

$$(\log_{10}x+1)\log_{10}x = 6\log_{10}10 \rightarrow 1$$

$$\Rightarrow (\log_{10}x+1)\log_{10}x = 6$$

Let $\log_{10}x = t$

$$(t+1)t = 6$$

$$t^2+t-6=0$$

$$t^2+3t-2t-6=0$$

$$(t+3)(t-2)=0$$

either $t+3=0 \Rightarrow t=-3$

or $t-2=0 \Rightarrow t=2$

$$\log_{10}x = 2 \text{ or } \log_{10}x=-3$$

$$\Rightarrow x=10^2 \text{ or } x=10^{-3}$$

$$x=100 \text{ or } x=\frac{1}{1000}$$

Question

Solve for x: $x^{2\log x} = 10x^2$ (*where base of log is 10*)

take log both sides

$$\log_{10}(x)^{2\log_{10}x} = \log_{10}(10x^2)$$

$$(2\log_{10}x)(\log_{10}x) = \log_{10}10 + \log_{10}x^2$$

$$(2\log_{10}x)\log_{10}x = 1 + 2\log_{10}x$$

Let $\log_{10}x = t$

$$2t \cdot t = 1 + 2t$$

$$2t^2 = 1 + 2t$$

$$\boxed{2t^2 - 2t - 1 = 0}$$

$$t = \frac{-b \pm \sqrt{D}}{2 \times a}$$

$$t = \frac{2 \pm \sqrt{4 - 4(2)(-1)}}{2 \times 2}$$

$$t = \frac{2 \pm \sqrt{12}}{2 \times 2}$$

$$\frac{2 \pm 2\sqrt{3}}{2 \times 2} = \frac{\cancel{2}(1 \pm \sqrt{3})}{\cancel{2} \times 2}$$

$$\log_{10}x = \frac{1+\sqrt{3}}{2} \text{ or } \frac{1-\sqrt{3}}{2}$$

$$x = 10^{\frac{1+\sqrt{3}}{2}} \text{ or } 10^{\frac{1-\sqrt{3}}{2}} \text{ Ans}$$

Q JEE Main – 2019 (April)

HW

[Ans. (A)]

The value of $\cos^2 10° - \cos 10° \cos 50° + \cos^2 50°$ is:

A $\frac{3}{4}$

B $\frac{1}{3}$

C $\frac{3}{2}$

D 0

Question

HW

Find the value of $\sin 9^0 - \cos 9^0$

Question

If $\underbrace{\sin\theta_1}_{1} + \underbrace{\sin\theta_2}_{1} + \underbrace{\sin\theta_3}_{1} = 3$ then value of $\cos\theta_1 + \cos\theta_2 + \cos\theta_3$ is

A 0

B 1

C –1

D –2

$$\sin\theta_1 = 1 = \sin\theta_2 = \sin\theta_3.$$

$$\underbrace{\sin^2\theta}_{1} + \cos^2\theta = 1$$

$$\cancel{1} + \cos^2\theta = \cancel{1}$$

$$\boxed{\cos\theta_1 = 0}$$

$$\cos\theta_2 = 0$$

$$\cos\theta_3 = 0$$

HW-BT-23

HW

The value of the expression $\frac{2(\sin 1^\circ+\sin 2^\circ+\sin 3^\circ+......+\sin 89^\circ)}{2(\cos 1^\circ+\cos 2^\circ+........+\cos 44^\circ)+1}$ equals

A $\sqrt{2}$

B $\frac{1}{\sqrt{2}}$

C $\frac{1}{2}$

D 1

HW-BT-24

HW

The value of $2 \tan 18° + 3 \sec 18° - 4 \cos 18°$ is

A Zero

B $\sqrt{5}$

C $-\sqrt{5}$

D $\sqrt{3}$

Question

Find the value of: $\boxed{\cos^2 48° - \sin^2 12°}$

$= \cos(48+12)\cos(48-12°)$

$= \cos(60°)\cos 36°$

$= \frac{1}{2} \cdot \frac{\sqrt{5}+1}{4}$

$= \frac{\sqrt{5}+1}{8}$

Question

[Ans. ( )]

$$\frac{\sin 2A}{\boxed{1-\cos 2A}} = \cot \boxed{A}$$

$$\text{LHS: } \frac{\sin 2A}{2\sin^2 A} = \frac{\cancel{2}\sin A\cos A}{\cancel{2}\sin^2 A} = \frac{\cos A}{\sin A} = \cot A$$

$$\frac{\sin 2A}{\underbrace{1+\cos 2A}} = \tan A$$

$$\text{LHS: } \frac{\sin 2A}{2\cos^2 A} = \frac{\cancel{2}\sin A\cancel{\cos A}}{\cancel{2}\cos^{\cancel{2}} A} = \tan A.$$

Question

[Ans. ( )]

$$\frac{\sin\theta + \sin 2\theta}{1 + \cos\theta + \cos 2\theta} = \tan\theta$$

LHS: $\dfrac{\sin\theta + \sin 2\theta}{(1+\cos 2\theta) + \cos\theta}$

$= \dfrac{\sin\theta + 2\sin\theta\cos\theta}{2\cos^2\theta + \cos\theta}$

$= \dfrac{\sin\theta\,[1 + 2\cos\theta]}{\cos\theta\,[2\cos\theta + 1]}$

$= \tan\theta$

Question

[Ans. ( )]

Prove the identity $\cos^4 t = \frac{3}{8} + \frac{1}{2}\cos 2t + \frac{1}{8}\cos 4t$

$$2\cos^2 t = 1 + \cos 2t$$

$$\boxed{\cos^2 t = \frac{1+\cos 2t}{2}}$$

LHS: $\cos^4 t = \frac{(1+\cos 2t)^2}{4}$

$$= \frac{1 + \cos^2 2t + 2\cos 2t}{4}$$

$$= \frac{1}{4} + \frac{\cos^2 2t}{4} + \frac{1}{2}\cos 2t$$

$$2\cos^2 2t = 1 + \cos 4t$$

$$\cos^2 2t = \frac{1+\cos 4t}{2}$$

LHS $= \frac{1}{4} + \frac{1}{4}\left[\frac{1+\cos 4t}{2}\right] + \frac{1}{2}\cos 2t$

$$= \frac{1}{4} + \frac{1}{8}(1 + \cos 4t) + \frac{1}{2}\cos 2t$$

$$= \underbrace{\frac{1}{4} + \frac{1}{8}} + \frac{1}{8}\cos 4t + \frac{1}{2}\cos 2t$$

$$= \frac{3}{8} + \frac{1}{8}\cos 4t + \frac{1}{2}\cos 2t$$

Question

[Ans. ( )]

$$\frac{\boxed{1-\cos A}+\boxed{\cos B-\cos(A+B)}}{\boxed{1+\cos A}-\cos B-\cos(A+B)}=\tan\frac{A}{2}\cot\frac{B}{2}$$

LHS $\dfrac{2\sin^2 A/2+\boxed{\cos B-\cos(A+B)}}{2\cos^2 A/2-[\cos B+\cos(A+B)]}$

$$=\frac{2\sin^2 A/2+2\sin\left(\frac{B+A+B}{2}\right)\sin(A/2)}{2\cos^2 A/2-2\cos\left(\frac{A+B+B}{2}\right)\cos(-A/2)}$$

$$=\frac{2\sin^2 A/2+2\sin(B+A/2)\sin A/2}{2\cos^2 A/2-2\cos(B+A/2)\cos A/2}$$

$$\frac{\cancel{2}\sin A/2\,[\sin A/2+\sin(B+A/2)]}{\cancel{2}\cos A/2\,[\cos A/2-\cos(B+A/2)]}$$

$$\tan A/2\,\frac{\cancel{2}\cancel{\sin\left(\frac{B+A}{2}\right)}\cos(B/2)}{\cancel{2}\cancel{\sin\left(\frac{B+A}{2}\right)}\sin(B/2)}$$

$$\tan(A/2)\cot(B/2)$$

RHS

Question

[Ans. ( )]

If $\frac{2\sin\alpha}{\{1+\cos\alpha+\sin\alpha\}} = y$, then $\frac{\{1-\cos\alpha+\sin\alpha\}}{1+\sin\alpha} =$

A $\frac{1}{y}$

B y ✓

C $1-y$

D $1+y$

$$y = \frac{2\sin\alpha}{2\cos^2\alpha/2 + \sin\alpha}$$

$$= \frac{2\cdot 2\sin\alpha/2\cos\alpha/2}{2\cos^2\alpha/2 + 2\sin\alpha/2\cos\alpha/2}$$

$$= \frac{\cancel{2}\cdot 2\sin\alpha/2\,\cancel{\cos\alpha/2}}{\cancel{2\cos\alpha/2}\left[\cos\alpha/2 + \sin\alpha/2\right]}$$

$$y = \frac{2\sin\alpha/2}{\cos\alpha/2 + \sin\alpha/2}$$

$$= \frac{2\sin^2\alpha/2 + 2\sin\alpha/2\cos\alpha/2}{(\sin\alpha/2 + \cos\alpha/2)^2}$$

$$= \frac{2\sin\alpha/2\left[\sin\alpha/2 + \cos\alpha/2\right]}{\left[\sin\alpha/2 + \cos\alpha/2\right]^2}$$

$$= \frac{2\sin\alpha/2}{\sin\alpha/2 + \cos\alpha/2}$$

$$= y$$

Question

$\cos^2 x = \frac{1+\cos 2x}{2}$

[Ans. ( )]

Prove that $\boxed{\cos^2 x + \cos^2\left(x+\frac{\pi}{3}\right) + \cos^2\left(x-\frac{\pi}{3}\right)} = \frac{3}{2}$.

$\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\frac{C-D}{2}$

LHS: $\frac{1+\cos 2x}{2} + \frac{1+\cos 2(x+\pi/3)}{2} + \frac{1+\cos 2(x-\pi/3)}{2}$

$\frac{1}{2}\left[3 + \cos 2x + \underbrace{\cos(\underbrace{2x+2\pi/3}_{C}) + \cos(\underbrace{2x-2\pi/3}_{D})}\right]$

$\frac{1}{2}\left[3 + \cos 2x + 2\cos(2x)\boxed{\cos(2\pi/3)}\right]$ $-1/2$

$\frac{1}{2}\left[3 + \cos 2x + \cancel{2}\cos(2x) \times \left(-\frac{1}{\cancel{2}}\right)\right]$

$\frac{1}{2}\left[3 + \cancel{\cos 2x} - \cancel{\cos 2x}\right]$

$= 3/2$

$2x + 2\pi/3 = C$

$2x - 2\pi/3 = D$

$C - D = 4\pi/3$

$\frac{C-D}{2} = 2\pi/3$

~~Question~~ HW

HWB.T.

[Ans. ( )]

If $\underbrace{\cos^3\theta}+\underbrace{\cos^3(120°}+\theta)+\underbrace{\cos^3(240°+}\theta)=\frac{a}{b}\cos 3\theta$, where '$a$' and '$b$' are relative prime, then the value $a+b$ is

$$\cos^2 x=\frac{1+\cos 2x}{2}$$

$$\cos 3x=4\cos^3 x-3\cos x.$$

$$\cos^3 x=\frac{\cos 3x+3\cos x}{4}$$

$Q \rightarrow$ $\cos 20 \cos 40^\circ \underbrace{\cos 60^\circ}_{1/2} \underbrace{\cos 80}_{} = ?$

$$\underbrace{\cos 20^\circ \cos(60-20^\circ) \cos(60+20^\circ)} \quad \cos 60^\circ$$

$$\frac{1}{4} \cos 60^\circ \times \cos 60^\circ$$

$$\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$$

Question

[Ans. ( )]

$\tan 6° \tan 42° \tan 66° \tan 78° = ?$

$$\underbrace{\boxed{\tan 6° \tan 54° \tan 66°}}_{\tan 54°} \cdot \tan 42° \tan 78°$$

$\tan 66° \rightarrow \tan 18°$

$$\underbrace{\boxed{\tan 18° \tan 42° \tan 78°}}_{\tan 54°} \rightarrow \frac{\cancel{\tan 54°}}{\cancel{\tan 54°}} = 1$$

Ans

$60 - 6 = 54°$

$42 = 60 - 18°$

$78 = 60 + 18$

Question

$\pi = 180$

$\frac{\pi}{18} = 10^\circ$

[Ans. ( )]

Find the value of the continued product $\prod_{k=1}^{17} \sin\frac{k\pi}{18}$

$\sin(\pi/18)\ \sin\left(\frac{2\pi}{18}\right)\ \sin\frac{3\pi}{18} \cdots\cdots \sin\frac{17\pi}{18}$

$\left(\sin(10^\circ)\ \sin 20^\circ\ \sin 30^\circ \cdots \underbrace{\sin 160^\circ}_{\sin 20^\circ}\ \underbrace{\sin 170^\circ}_{\sin 10^\circ}\right)$

$\underbrace{\sin 10^\circ \sin 20^\circ \sin 30^\circ \cdots \sin 80^\circ}\ \underbrace{\sin 90^\circ}\ \sin 100^\circ \cdots \underbrace{\sin 170^\circ}_{\sin 10}$

$= \left[\sin 10^\circ \sin 20 \sin 30^\circ \cdots \underbrace{\sin 80}_{\cos 10}\right]^2 \underbrace{\sin 90^\circ}_{1}$

$[2\sin 10 \cos 10\ 2\sin 20 \cos 20\ 2\sin 30^\circ \cos 30^\circ\ 2\sin 40 \cos 40]^2$

$\frac{[\sin 20^\circ \sin 40^\circ \sin 60 \sin 80]^2}{(16)^2}$ $(16)^2$

$$\sum_{K=1}^{10} (K)^2 = (1)^2 + 2^2 + 3^2 + \cdots (10)^2$$

$$\prod_{K=1}^{10} (K)^2 = (1)^2 (2)^2 (3)^2 \cdots (10)^2$$

$$\prod_{K=1}^{3} K^3 = 1^3 \cdot 2^3 \cdot 3^3$$
$$= 1 \times 8 \times 27$$

$\sin 170 = \sin(180 - 10^\circ)$
$= \sin 10^\circ$

Question

[Ans. ( )]

Find the exact value of $\boxed{\text{cosec } 10° + \text{cosec } 50° - \text{cosec } 70°}$

$$\frac{1}{\sin 10°} + \frac{1}{\sin 50°} - \frac{1}{\sin 70°}$$

$$\frac{\sin 50 + \sin 10°}{\sin 10 \sin 50} - \frac{1}{\sin 70°}$$

$$\frac{\cancel{2} \cancel{\sin} 30° \cos 20°}{\sin 10 \sin 50} - \frac{1}{\sin 70°}$$

$$\frac{\cos 20°}{\sin 10° \sin 50°} - \frac{1}{\sin 70°}$$

$$\frac{\cos 20° \sin 70° - \sin 10 \sin 50°}{\boxed{\sin 10 \sin 50 \sin 70°}}$$

$\rightarrow 1/8$

$$= 8[\sin 70° \cos 20 - \sin 50 \sin 10°]$$

$$= 4[2\sin 70° \cos 20 - 2\sin 50 \sin 10°]$$

$$= 4[\sin 90° + \sin 50 - (\cos 40 - \cos 60°)]$$

$$= 4[1 + \cancel{\sin 50} - \cancel{\cos 40} + \tfrac{1}{2}]$$

$$4[1 + \tfrac{1}{2}]$$

$$4 \times 3/2 = 6$$

Question

[Ans. ( )]

The value of $\cos\frac{\pi}{11}+\cos\frac{3\pi}{11}+\cos\frac{5\pi}{11}+\cos\frac{7\pi}{11}+\cos\frac{9\pi}{11}$ is

A 0

B 1

C $\frac{1}{2}$ ✓

D None of these

$n=5,\ \alpha=\pi/11,\ \beta=\frac{2\pi}{11}\Rightarrow \beta/2=\pi/11$

$$\frac{\sin\frac{n\beta}{2}}{\sin\beta/2}\cos\left(\alpha+(n-1)\beta/2\right)$$

$$=\frac{\sin(5\cdot\pi/11)}{\sin\pi/11}\cos\left(\frac{\pi}{11}+4\pi/11\right)$$

$$=\frac{2\sin 5\pi/11\cos\frac{5\pi}{11}}{2\sin\pi/11}=\frac{\sin\left(\frac{10\pi}{11}\right)}{2\sin(\pi/11)}=\frac{\cancel{\sin\pi/11}}{2\cancel{\sin\pi/11}}=\frac{1}{2}$$

Question PYQ → 2022

[Ans. ( )]

$$\cos\frac{2\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{6\pi}{7}$$

$\frac{\beta}{2} = \pi/7$

$\alpha = 2\pi/7, \ \beta = 2\pi/7, \ n = 3$

$$= \frac{\sin\left(\frac{n\beta}{2}\right)}{\sin \beta/2} \cos\left(\alpha + (n-1)\beta/2\right)$$

$$= \frac{\sin 3.\pi/7}{\sin \pi/7} \cos\left[\frac{2\pi}{7} + 2.\pi/7\right]$$

$$= \frac{2\sin 3\pi/7 \cos 4\pi/7}{2\sin \pi/7}.$$

$$\frac{\sin\left[\frac{3\pi}{7}+\frac{4\pi}{7}\right] + \sin\left[\frac{3\pi}{7}-\frac{4\pi}{7}\right]}{2\sin \pi/7}$$

$$\frac{\overset{0}{\sin \pi} + \sin(-\pi/7)}{2\sin \pi/7}.$$

$$= \frac{-\cancel{\sin \pi/7}}{2\cancel{\sin \pi/7}}$$

$$= -\frac{1}{2} \text{ Ans}$$

Brain Teaser

$1-\cos 2^\circ = 2\sin^2 1^\circ$

$1+\cos 2^\circ = 2\cos^2 1^\circ$

[Ans. ( )]

Arithmetic mean of $2\sin 2^\circ, 4\sin 4^\circ, 6\sin 6^\circ, \ldots$ upto 90 terms

(A) $\cot 1^\circ$

(B) $\dfrac{\cot 44^\circ - 1}{\cot 44^\circ + 1}$

(C) $\dfrac{\cos^2 20^\circ - \sin^2 19^\circ}{\sin^2 26^\circ - \sin^2 25^\circ}$

(D) $\sqrt{\dfrac{1-\cos 2^\circ}{1+\cos 2^\circ}}$

$$A.M = \frac{2\sin 2^\circ + 4\sin 4^\circ + 6\sin 6^\circ + \ldots + 178\sin 178^\circ + 180\sin 180^\circ}{90}$$

$\sin 2^\circ$

$176\sin 176^\circ$

$176\sin 4^\circ$

$88\sin 88^\circ + 90\sin 90^\circ + 92\sin 92^\circ$

$$\frac{180\sin 2^\circ + 180\sin 4^\circ + 180\sin 6^\circ + \ldots 180\sin 88^\circ + 90\sin 90^\circ}{90}$$

$$AM = \frac{180[\sin 2^\circ + \sin 4^\circ + \sin 6^\circ + \ldots \sin 88^\circ] + 90}{90}$$

$\sqrt{\tan^2 1^\circ} = \tan 1^\circ$

$$AM = 2[\underbrace{\sin 2^\circ + \sin 4^\circ + \sin 6^\circ + \dots + \sin}_{\alpha=2,\ \beta=2,\ n=44} 88] + 1$$

$$\boxed{\tan(45-\theta) = \frac{1-\tan\theta}{1+\tan\theta}}$$

$$2\,\frac{\sin 44^\circ \times 1}{\sin 1^\circ}\sin\left[2 + 43\times 1\right] + 1$$

$$\frac{2\sin 44 \sin 45^\circ}{\sin 1^\circ} + 1$$

$$\frac{\cos 1^\circ - \cos 89^\circ}{\sin 1^\circ} + 1$$

$$\frac{\cos 1^\circ - \cancel{\cos 89^\circ} + \cancel{\sin 1^\circ}}{\sin 1^\circ}$$

$$AM = \cot 1^\circ$$

B) $$\frac{\cot 44^\circ - 1}{\cot 44 + 1} = \frac{1-\tan 44^\circ}{1+\tan 44^\circ}$$

$$= \tan(45^\circ - 44^\circ)$$

$$= \tan 1^\circ$$

C) $$\frac{\cos^2 20^\circ - \sin^2 19^\circ}{\sin^2 26 - \sin^2 25^\circ}$$

$$= \frac{\cancel{\cos(39^\circ)}\cos 1^\circ}{\cancel{\sin 51^\circ}\sin 1^\circ} = \cot 1^\circ.$$

HW-BT-21

Ans = $\underline{\underline{28}}$

Find the exact value of $\tan^2\frac{\pi}{16}+\tan^2\frac{3\pi}{16}+\tan^2\frac{5\pi}{16}+\tan^2\frac{7\pi}{16}$

HW-BT-22 (HW) [Ans. ( )]

$2\sin^2\beta + 4\cos(\alpha+\beta)\sin\alpha\sin\beta + \cos2(\alpha+\beta) =$

A $\sin 2\alpha$

B $\cos 2\beta$

C $\cos 2\alpha$

D $\sin 2\beta$

Question

Find the value of the expression

1. $\cot 9° + \cot 27° + \cot 63° + \cot 81°$

$$\underbrace{\cot 63°}_{\tan 27°} + \underbrace{\cot 81°}_{\tan 9°}$$

$$\left(\cot 9° + \tan 9°\right) + \left(\tan 27° + \cot 27°\right)$$

$$\left(\frac{\cos 9°}{\sin 9°} + \frac{\sin 9°}{\cos 9°}\right) + \left(\frac{\sin 27}{\cos 27} + \frac{\cos 27}{\sin 27}\right)$$

$$\left(\frac{1\times 2}{2\sin 9\cos 9}\right) + \left(\frac{1\times 2}{2\sin 27\cos 27}\right)$$

$$\frac{2}{\sin 18°} + \frac{2}{\sin 54°}$$

$$\frac{2\times 4}{\sqrt{5}-1} + \frac{2\times 4}{\sqrt{5}+1}$$

$$\boxed{\sin 18° = \frac{\sqrt{5}-1}{4} \qquad \sin 54 = \cos 36° = \frac{\sqrt{5}+1}{4}}$$

$4\sqrt{5}$

$$8\left[\frac{1}{\sqrt{5}-1} + \frac{1}{\sqrt{5}+1}\right]$$

$$8\,\frac{\left[\sqrt{5}+\cancel{1}+\sqrt{5}-\cancel{1}\right]}{5-1}$$

$$= \frac{8\times 2\sqrt{5}}{4} = 4\sqrt{5} \text{ Ans}$$

Question

[Ans. ( )]

$$\frac{\cos 17° + \sin 17°}{\cos 17° - \sin 17°} = \frac{1 + \tan 17°}{1 - \tan 17°} = \tan(45 + 17°)$$

$$= \tan(62°).$$

A. $\tan 62°$ ✓

B. $\tan 56°$

C. $\tan 54°$

D. $\tan 73°$

Question

Prove that

$$2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}=0$$

A, B

$\cos(-\theta)=\cos\theta$

$\frac{10\pi}{13}+\frac{3\pi}{13}=180^\circ$

$\Rightarrow \cos\frac{10\pi}{13}+\cos\frac{3\pi}{13}=0$

$$\cos\left(\underbrace{\frac{\pi}{13}+\frac{9\pi}{13}}\right)+\cos\left(\frac{\pi}{13}-\frac{9\pi}{13}\right)+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}$$

$$\cancel{\cos\left(\frac{10\pi}{13}\right)}+\underbrace{\cos\left(\frac{8\pi}{13}\right)+\cancel{\cos\left(\frac{3\pi}{13}\right)}+\cos\frac{5\pi}{13}}$$

$= 0$

Question

Prove that

$$\frac{\boxed{\sin 7\theta - \sin 5\theta}}{\boxed{\cos 7\theta + \cos 5\theta}} = \tan\theta$$

(vi)

(vii)

$$\frac{\cancel{2}\cos\left(\frac{7\theta+5\theta}{2}\right)\sin\left(\frac{7\theta-5\theta}{2}\right)}{\cancel{2}\cos\left(\frac{7\theta+5\theta}{2}\right)\cos\left(\frac{7\theta-5\theta}{2}\right)}$$

$$\frac{\cancel{\cos(6\theta)}\cdot\sin\theta}{\cancel{\cos(6\theta)}\cos(\theta)} = \tan\theta$$

$$\frac{\boxed{\sin 2A + \sin 2B}}{\boxed{\sin 2A - \sin 2B}} = \tan(A+B)\cot(A-B)$$

$$\frac{\cancel{2}\sin\left(\frac{\cancel{2}A+\cancel{2}B}{\cancel{2}}\right)\cos\left(\frac{\cancel{2}A-\cancel{2}B}{\cancel{2}}\right)}{\cancel{2}\cos\left(\frac{2A+2B}{2}\right)\sin\left(\frac{2A-2B}{2}\right)}$$

$$\frac{\sin(A+B)}{\cos(A+B)}\,\frac{\cos(A-B)}{\sin(A-B)}$$

$\tan(A+B)\cot(A-B)$

$= \underline{RHS}$

Question

$\cos 3A + \cos 5A + \cos 7A + \cos 15A = 4\cos 4A \cos 5A \cos 6A$ ✓

$2\cos\left(\frac{3A+5A}{2}\right)\cdot\cos\left(\frac{3A-5A}{2}\right) + 2\cos\left(\frac{7A+15A}{2}\right)\cdot\cos\left(\frac{7A-15A}{2}\right)$

$2\cos(4A)\cos(A) + 2\cos(11A)\cos(4A)$

$2\cos 4A\,[\cos A + \cos 11A]$

$2\cos 4A\; 2\cos 6A\,\cos(5A)$

$4\cos 4A \cos 6A \cos 5A$

Question

$$\frac{\boxed{\sin A-\sin 5A}+\boxed{\sin 9A-\sin 13A}}{\boxed{\cos A-\cos 5A}-\underbrace{\cos 9A+\cos 13A}}=\cot 4A$$

$$\frac{(\sin A-\sin 5A)+(\sin 9A-\sin 13A)}{(\cos A-\cos 5A)-(\cos 9A-\cos 13A)}$$

$$\frac{2\cos 3A\sin(-2A)+2\cos(11A)\sin(-2A)}{2\sin 3A\sin(2A)-2\sin(11A)\sin(2A)}$$

$$=\frac{\cancel{2}\sin\overset{-1}{\cancel{(-2A)}}\left[\cos 3A+\cos 11A\right]}{\cancel{2}\cancel{\sin 2A}\left[\sin 3A-\sin 11A\right]}$$

$$-\frac{[\cos 3A+\cos 11A]}{(\sin 3A-\sin 11A)}$$

$$-\frac{\cancel{2}\cancel{\cos 7A}\cos(4A)}{\cancel{2}\cancel{\cos 7A}\cdot\sin(-4A)}$$

$$\frac{\cos 4A}{\sin 4A}$$

$$=\cot 4A$$

$$=\text{RHS}$$

Question

$$\sin\frac{\theta}{2}\sin\frac{7\theta}{2}+\sin\frac{3\theta}{2}\sin\frac{11\theta}{2}=\sin 2\theta\,\sin 5\theta$$

LHS: $\frac{1}{2}\left[2\underbrace{\sin\theta/2\,\sin}7\theta/2 + 2\underbrace{\sin\frac{3\theta}{2}\sin}\frac{11\theta}{2}\right]$

$\frac{1}{2}\left[\cos\left(\frac{\theta}{2}-\frac{7\theta}{2}\right)-\cos\left(\frac{\theta}{2}+\frac{7\theta}{2}\right)+\cos\left(\frac{3\theta}{2}-\frac{11\theta}{2}\right)-\cos\left(\frac{3\theta}{2}+\frac{11\theta}{2}\right)\right]$

$\frac{1}{2}\left[\cos 3\theta - \cancel{\cos 4\theta} + \cancel{\cos 4\theta} - \cos 7\theta\right]$

$\frac{1}{2}\left[\underbrace{\cos 3\theta - \cos 7\theta}\right]$

$\frac{1}{\cancel{2}}\cancel{2}\sin\left(\frac{3\theta+7\theta}{2}\right)\sin\left(\frac{7\theta-3\theta}{2}\right) = \sin 5\theta\,\sin 2\theta.$

$= \text{RHS}$

Question

Find the exact value of $\cos 24° - \cos 12° + \cos 48° - \cos 84° = ?$

$$(\cos 24° - \cos 84°) + (\cos 48° - \cos 12°)$$

$$2\sin\left(\frac{108}{2}\right)\sin\left(\frac{60}{2}\right) + 2\sin\left(\frac{60}{2}\right)\sin\left(\frac{12-48}{2}\right)$$

$$2\sin(54°)\sin 30° + 2\sin 30°\sin(-18°)$$

$$\cancel{2}\cancel{\sin 30°}\left[\sin 54° - \sin 18°\right]$$

$$\left[\frac{\sqrt{5}+1}{4} - \left(\frac{\sqrt{5}-1}{4}\right)\right]$$

$$2/4 = 1/2$$

PYQ (2022)

Find the value of the expression $(2\sin 12° - \sin 72°)$

$\sin 12° + (\sin 12° - \sin 72)$

$\sin 12 + 2\cos\left(\frac{12+72}{2}\right) \cdot \sin\left(\frac{12-72}{2}\right)$

$\sin 12 + 2\cos 42° \sin(-30°)$

$\sin 12 - \cancel{2}\cos 42° \times \frac{1}{\cancel{2}}$

$\sin 12 - \underbrace{\cos 42°}$

$\sin 12 - \sin 48°$

$2\cos\left(\frac{12+48}{2}\right)\sin\left(\frac{12-48°}{2}\right)$

$2\cos 30° \cdot \sin(-18°)$

$= -\cancel{2}\frac{\sqrt{3}}{\cancel{2}} \cdot \left(\frac{\sqrt{5}-1}{4}\right)$

$= -\frac{\sqrt{3}(\sqrt{5}-1)}{4}$

Ans

Brain Teaser

#. If $3\sin\alpha = 5\sin\beta \neq 0$, then the value of $\dfrac{\tan\left(\frac{\alpha+\beta}{2}\right)}{\tan\left(\frac{\alpha-\beta}{2}\right)} = 4$ is :

$3\sin\alpha = 5\sin\beta$

$\Rightarrow \dfrac{\sin\alpha}{\sin\beta} = 5/3$

C & D

$$\frac{\sin\alpha + \sin\beta}{\sin\alpha - \sin\beta} = \frac{5+3}{5-3}$$

$$\frac{\cancel{2}\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)}{\cancel{2}\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)} = 4 \Rightarrow \frac{\tan\left(\frac{\alpha+\beta}{2}\right)}{\tan\left(\frac{\alpha-\beta}{2}\right)} = 4.$$

C & D

If $\dfrac{a}{b} = c/d$

C & D $\Rightarrow$

$$\frac{a+b}{a-b} = \frac{c+d}{c-d}$$

Proof

$a/b = c/d$

add 1 $\rightarrow$ ①

add -1 $\rightarrow$ ②

①/②

Brain Teaser

If $\cos\alpha + \cos\beta = \frac{1}{2}$ and $\sin\alpha + \sin\beta = \frac{1}{3}$

	Column-I		Column-II
(A)	$\cos\frac{\alpha+\beta}{2}$	(P)	$\pm\frac{\sqrt{13}}{12}$
(B)	$\cos\frac{\alpha-\beta}{2}$	(Q)	$\frac{2}{3}$
(C)	$\tan\left(\frac{\alpha+\beta}{2}\right)$	(R)	$\pm\frac{3}{\sqrt{13}}$
(D)	$\tan\left(\frac{\alpha-\beta}{2}\right)$	(S)	$\pm\sqrt{\frac{131}{13}}$

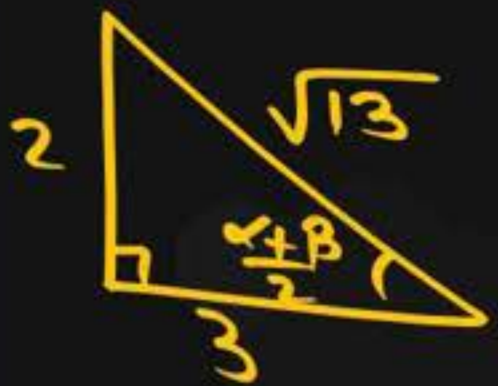
$\cancel{2}\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) = \frac{1}{2}$

$\cancel{2}\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) = \frac{1}{3}$

$\cot\left(\frac{\alpha+\beta}{2}\right) = \frac{1}{2}\times 3$

$\cot\left(\frac{\alpha+\beta}{2}\right) = 3/2$

$\tan\left(\frac{\alpha+\beta}{2}\right) = 2/3$ $\nearrow$ I, $\searrow$ III



$\cos\left(\frac{\alpha+\beta}{2}\right) = \pm\frac{3}{\sqrt{13}}$

HW-BT-19

HW

If $x = \frac{\pi}{48}$, find the value of $\dfrac{\sin 7x + \sin 5x + \sin 9x + \sin 3x}{\cos 7x + \cos 5x + \cos 9x + \cos 3x}$.

HW-BT-20

HW

The lower end, A, of a ladder AB rests on a horizontal floor; B presses on a vertical wall. In this position the ladder is inclined at an angle a with the floor. The end A now slips a distance a while the end B comes down by a distance b. The inclination of the ladder is now β. Show that $\tan\left(\frac{\alpha+\beta}{2}\right) = \frac{a}{b}$.

Question

$\tan^6\frac{\pi}{9} - 33\tan^4\frac{\pi}{9} + 27\tan^2\frac{\pi}{9}$ is equal to (3) Ans

$$\boxed{\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}}$$

$\theta = \pi/9$

$$\underbrace{\tan\left(3\pi/9\right)}_{\downarrow} = \frac{3\tan\pi/9 - \tan^3\pi/9}{1-3\tan^2\pi/9}$$

$$\sqrt{3} = \frac{3\tan\pi/9 - \tan^3\pi/9}{1-3\tan^2\pi/9}$$

$$\sqrt{3}\left(1-3\tan^2\pi/9\right) = 3\tan\pi/9 - \tan^3\pi/9$$

(S.B.S)

$$3\left(1-3\tan^2\pi/9\right)^2 = \left(3\tan\pi/9 - \tan^3\pi/9\right)^2$$

$$3\left[1+9\tan^4\pi/9 - 6\tan^2\pi/9\right] = 9\tan^2\pi/9 + \tan^6\pi/9 - 6\tan^4\pi/9$$

$$3 + 27\tan^4\pi/9 - 18\tan^2\pi/9 = 9\tan^2\pi/9 + \tan^6\pi/9 - 6\tan^4\pi/9$$

$$\boxed{3 = -33\tan^4\pi/9 + 27\tan^2\pi/9 + \tan^6\pi/9}$$

Question

To express $\cos 5A$ in term of $\cos A$

$$\cos(5A) = \cos(3A + 2A) = \cos 3A \cos 2A - \sin 3A \sin 2A$$

$$= (4\cos^3 A - 3\cos A)(2\cos^2 A - 1) - (3\sin A - 4\sin^3 A) \cdot 2\sin A \cos A$$

$$- \sin A\,[3 - 4\sin^2 A]\, 2\sin A \cos A$$

$$- 2\cos A\,(1 - \cos^2 A)\,[3 - 4(1 - \cos^2 A)]$$

PYQ

$2\sin\theta\cos\theta = \sin 2\theta$

The value of $\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right)$ is

A $\frac{1}{4}$

B $\frac{3}{4}$

C $\frac{1}{8}$

D $\frac{3}{8}$

$\left(1+\cos\pi/8\right)\left(1+\cos 3\pi/8\right)\left(1-\cos\frac{3\pi}{8}\right)\left(1-\cos\pi/8\right)$

$\left(1-\cos^2\pi/8\right)\left(1-\cos^2 3\pi/8\right)$

$= \sin^2\pi/8 \sin^2 3\pi/8$

$= \left(\sin\pi/8 \sin 3\pi/8\right)^2$

$\left(\frac{2\sin\pi/8 \cos\pi/8}{2}\right)^2 = \left(\frac{\sin(2\pi/8)}{2}\right)^2 = \left(\frac{\sin\pi/4}{2}\right)^2 = \left(\frac{1}{2\sqrt{2}}\right)^2 = \frac{1}{8}$

$7\pi/8 + \pi/8 = \pi$

$\cos 7\pi/8 = -\cos\pi/8$

$\cos 5\pi/8 = -\cos\frac{3\pi}{8}$

$\pi/8 + 3\pi/8 = \pi/2$

$3\pi/8 = 90^\circ - \pi/8$

$\sin 3\pi/8 = \cos\pi/8$

Question

Prove that:

(a) $\tan 4x = \dfrac{4\tan x(1-\tan^2 x)}{1-6\tan^2 x+\tan^4 x}$ (HW)

(b) $\boxed{\cos 4x = 1 - 8\sin^2 x\cos^2 x}$

(c) $\boxed{\cos 6x = 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1}$

$\cos 2x =$

$(a-b)^3 = a^3 - b^3 - 3ab(a-b)$

(c) $\cos 6x = 4\cos^3 2x - 3\cos 2x$ $(6x \to 2x \to x)$

$= 4[2\cos^2 x - 1]^3 - 3[2\cos^2 x - 1]$

$= 4[(2\cos^2 x)^3 - 1^3 - 6\cos^2 x(2\cos^2 x - 1)] - 6\cos^2 x + 3$

$= 4[8\cos^6 x - 1 - 12\cos^4 x + 6\cos^2 x] - 6\cos^2 x + 3$

$32\cos^6 x - 4 - 48\cos^4 x + 24\cos^2 x$

$-6\cos^2 x + 3$

$32\cos^6 x - 1 - 48\cos^4 x + 18\cos^2 x$

2nd Method. (HW)

$6x \to 3x \to x$

$\cos 6x = 2\cos^2 3x - 1$

(b) $\cos 4x = 1 - 8\sin^2 x\cos^2 x$

LHS:

$$\cos 4x = 1 - 2\underbrace{\sin^2 2x}$$

$$1 - 2\left(2\sin x\cos x\right)^2$$

$$= 1 - 8\sin^2 x\cos^2 x$$

RHS

$\cos 2\theta = 1 - 2\sin^2\theta$

$\theta \to 2x$

$\cos 4x = 1 - 2\underbrace{\sin^2 2x}$

Question

If $\tan x = \frac{3}{4}$, $\pi < x < \frac{3\pi}{2}$, find the value of $\sin\frac{x}{2}$, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$.

$\pi < x < \frac{3\pi}{2}$

$\pi/2 < x/2 < 3\pi/4$

$90^\circ < x/2 < 135^\circ$

$x/2 \rightarrow 2^{nd}$ Quad

$$\tan x = \frac{2\tan x/2}{1-\tan^2 x/2}$$

$$\Rightarrow \quad 3/4 = \frac{2\tan x/2}{1-\tan^2 x/2}$$

$$3[1-\tan^2 x/2] = 8\tan x/2.$$

$$\tan x/2 = t$$

$$3(1-t^2) = 8t$$

$$3t^2 + 8t - 3 = 0$$

$$3t^2 + 9t - t - 3 = 0$$

$$(3t-1)(t+3) = 0$$

$t = 1/3$ or $t = -3$

$\tan\frac{x}{2} = 1/3$ or $\underbrace{\tan\frac{x}{2} = \frac{-3}{1}}_{\text{Accept}}$

(X)

$\sin x/2 = +\frac{3}{\sqrt{10}}$

$\cos x/2 = -\frac{1}{\sqrt{10}}$

Question

[Ans. ( )]

If $\sin\alpha - \cos\alpha = \frac{1}{5}$, then $\tan\left(\frac{\alpha}{2}\right) =$

(A) -3 or $\frac{1}{2}$

(B) 3 or $-\frac{1}{2}$

(C) -2 or $\frac{1}{3}$

(D) 2 or $-\frac{1}{3}$

$$\frac{2\tan\alpha/2}{1+\tan^2\alpha/2} - \left(\frac{1-\tan^2\alpha/2}{1+\tan^2\alpha/2}\right) = \frac{1}{5}$$

$$\boxed{\tan\alpha/2 = t}$$

$$\frac{2t}{1+t^2} - \left(\frac{1-t^2}{1+t^2}\right) = \frac{1}{5}$$

$$\frac{2t-1+t^2}{1+t^2} = \frac{1}{5}$$

$$10t-5+5t^2 = t^2+1$$

$$4t^2+10t-6=0 \Rightarrow 2t^2+5t-3=0$$

$$2t^2+6t-t-3=0$$

$$(2t-1)(t+3)=0$$

$$t = \tfrac{1}{2} \text{ or } -3$$

$$\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$$

$$\boxed{\sin\alpha} = \frac{2\tan\alpha/2}{1+\tan^2\alpha/2}$$

$$\boxed{\cos\alpha} = \frac{1-\tan^2\alpha/2}{1+\tan^2\alpha/2}$$

$$\sin\alpha - \cos\alpha = \frac{1}{5}$$

S.B.S.

$$(\sin\alpha - \cos\alpha)^2 = \frac{1}{25}$$

$$1 - \underbrace{2\sin\alpha\cos\alpha} = \frac{1}{25}$$

$$1 - \sin 2\alpha = \frac{1}{25}$$

Q JEE Main – 2021 (July)

[Ans. ( )]

If $\sin\theta + \cos\theta = \frac{1}{2}$, then $16(\sin(2\theta) + \cos(4\theta) + \sin(6\theta)) =$

$\cos 2A = 1 - 2\sin^2 A$

$A \to 2\theta$

A 23

B 27

C −23

D 27

$\sin\theta + \cos\theta = 1/2$

SBS.

$1 + 2\sin\theta\cos\theta = 1/4$

$1 + \sin 2\theta = 1/4$

$\sin 2\theta = -3/4$

$\cos 4\theta = 1 - 2\sin^2 2\theta = 1 - 2(-3/4)^2 = 1 - \frac{2\times 9}{16} = 1 - 9/8$

$= -1/8$

$\sin 6\theta = 3\sin 2\theta - 4\sin^3 2\theta$

$= 3[-3/4] - 4(-3/4)^3 = -9/4 + \frac{27}{16}$

Ans $= 16\left[-3/4 - 1/8 - 9/4 + \frac{27}{16}\right]$

$= -12 - 2 - 36 + 27$

$= -14 - 9$

$= \boxed{-23}$

$\cos 72^\circ = \boxed{\sin 18^\circ = \frac{\sqrt{5}-1}{4}}$ ✓✓

$\theta = 18^\circ \qquad \sin\theta = ?$

$5\theta = 90^\circ$

$3\theta + 2\theta = 90^\circ$

$2\theta = 90^\circ - 3\theta$

Sin both sides.

$\sin 2\theta = \cos 3\theta$

$2\sin\theta\cos\theta = 4\cos^3\theta - 3\cos\theta$

$2\sin\theta\cancel{\cos\theta} = \cancel{\cos\theta}\left(4\cos^2\theta - 3\right)$

$2\sin\theta = 4\cos^2\theta - 3$

$2\sin\theta = 4(1-\sin^2\theta) - 3$

$2\sin\theta = 4 - 4\sin^2\theta - 3$

$4\sin^2\theta + 2\sin\theta - 1 = 0$

$\sin\theta = t \qquad \boxed{t > 0}$

$4t^2 + 2t - 1 = 0$

$t = \frac{-2 \pm \sqrt{4 + 4\times 4}}{2\times 4}$

$t = \frac{-\cancel{2} \pm \cancel{2}\sqrt{5}}{\cancel{2}\times 4}$

$t = \frac{-1 \pm \sqrt{5}}{4}$

$t = \frac{-1+\sqrt{5}}{4}$ or $t = \frac{-1-\sqrt{5}}{4}$

Accept. Reject

$$\boxed{\cos 36^\circ = \frac{\sqrt{5}+1}{4}} = \sin 54^\circ$$

✓

$$\cos(2\theta) = 1 - 2\sin^2\theta$$

$$\theta = 18^\circ$$

$$\cos 36^\circ = 1 - 2\underbrace{\sin^2 18^\circ}$$

$$\cos 36^\circ = 1 - 2\left(\frac{\sqrt{5}-1}{4}\right)^2$$

$$= 1 - \frac{2}{16}(\sqrt{5}-1)^2$$

$$= 1 - \frac{1}{8}[5+1-2\sqrt{5}]$$

$$= 1 - \frac{1}{8}[6-2\sqrt{5}]$$

$$= 1 - \frac{1}{4}[3-\sqrt{5}]$$

$$= \frac{4-(3-\sqrt{5})}{4}$$

$$= \frac{4-3+\sqrt{5}}{4}$$

$$\boxed{\cos 36^\circ = \frac{1+\sqrt{5}}{4}}$$

Question

HW

Find the value of the expression

1. $\cot 9° + \cot 27° + \cot 63° + \cot 81°$

HW-BT-17

HW

The difference $(\sin^8 75° - \cos^8 75°)$ is equal to

A 1

B $\frac{3\sqrt{3}}{8}$

C $\frac{3\sqrt{3}}{16}$

D $\frac{7\sqrt{3}}{16}$

HW-BT-18

HW

$0 < \alpha, \beta < \frac{\pi}{2}$, $\tan\alpha = \frac{1}{7}$ and $\sin\beta = \frac{1}{\sqrt{10}}$. The value of $\alpha + 2\beta$ is

A $\frac{\pi}{6}$

B $\frac{\pi}{4}$

C $\frac{\pi}{3}$

D $\frac{\pi}{2}$

Question

Find the value of $\boxed{\sin 9^0 - \cos 9^0}$

$\cos 9^\circ > \sin 9^\circ$

$\cos 9^\circ - \sin 9^\circ > 0$

$\sin 9^\circ - \cos 9^\circ < 0$

$\sin 9^\circ - \cos 9^\circ = x.$

S.B.S

$(\sin 9^\circ - \cos 9^\circ)^2 = x^2$

$1 - 2\underbrace{\sin 9^\circ \cos 9^\circ} = x^2$

$1 - \underbrace{\sin 18^\circ} = x^2$

$1 - \left(\frac{\sqrt{5}-1}{4}\right) = x^2$

$\frac{4-\sqrt{5}+1}{4} = x^2$

$\frac{5-\sqrt{5}}{4} = x^2$

☆☆

$-\sqrt{\frac{5-\sqrt{5}}{4}} = x.$

$x^2 = 5$

$x = \pm\sqrt{5}$

A) (+)

B) (−v)

C) (±)

D) None.

Question

HW

If $A + B + C = 180°$, prove that: $\sin^2 A + \underbrace{\sin^2 B - \sin^2 C}_{\text{Extra Formula}} = 2 \sin A \sin B \cos C$.

$\sin(B+C)\sin(B-C)$.

Brain Teaser

If $xy + yz + zx = 1$ where $x, y, z \in R^+$ then prove that

$$\frac{x}{1+x^2} + \frac{y}{1+y^2} + \frac{z}{1+z^2} = \frac{2}{\sqrt{(1+x^2)(1+y^2)(1+z^2)}}.$$

Let $x = \tan A/2$, $y = \tan B/2$ $z = \tan C/2$.

$$A + B + C = 180^\circ$$

LHS: $\dfrac{x}{1+x^2} = \dfrac{2\tan A/2}{2(1+\tan^2 A/2)} = \dfrac{\sin A}{2}$

LHS: $\dfrac{\sin A}{2} + \dfrac{\sin B}{2} + \dfrac{\sin C}{2}$

$= \dfrac{\sin A + \sin B + \sin C}{2}$

$= \dfrac{4\cos A/2 \cos B/2 \cos C/2}{2} \Rightarrow$ LHS: $2\cos A/2 \cos B/2 \cos C/2$

RHS: $\dfrac{2}{\sqrt{\sec^2 A/2 \sec^2 B/2 \sec^2 C/2}}$

$\dfrac{2}{\sec A/2 \sec B/2 \sec C/2}$

$= 2\cos A/2 \cos B/2 \cos C/2$

LHS = RHS

HW-BT-25

HW

If $x + y + z = xyz$

Prove that $\frac{x}{1-x^2} + \frac{y}{1-y^2} + \frac{z}{1-z^2} = \frac{4}{\sqrt{(1-x^2)(1-y^2)(1-z^2)}}$.

Maximum & Minimum Values

$y = \sin x$

$y \in [-1, 1]$

Find the range of the following

1. $\boxed{y = 3\sin x}$ $\Rightarrow y \in [-3, 3]$
2. $\boxed{y = 3\sin x + 7}$ $y_{max} = 3+7$, $y_{min} = -3+7$ $y \in [4, 10]$
3. $y = 3\sin x - 5$ $y_{max} = 3-5 = -2$, $y_{min} = -3-5 = -8$ $y \in [-8, -2]$
4. $y = 5 - 3\sin x$ $\Rightarrow y_{min} = 5-3 = 2$, $y_{max} = 5-3(-1) = 5+3 = 8$ $y \in [2, 8]$
5. $y = 3 - 8\cos x$ $y_{min} = 3-8 = -5$, $y_{max} = 3+8 = 11$
6. $y = 5 + 3\sin^2 x$ $y_{min} = 5+3\times 0 = 5$ $y_{max} = 8$
7. $y = 5 - 7\cos^2 x$

$y_{min} = 5-7\times 1 = -2$

$y_{max} = 5-7\times 0 = 5$

A) $[-5, 0]$
B) $[0, 5]$
C) $[-2, 5]$
D) $[-2, 12]$

Maximum & Minimum Values

Find the range of the following

1. $y = 5\sin^2 x + 7\cos^2 x$
2. $y = 5\sin^2 x + 8\cos^2 x - 2$
3. $y = 2\sin^2 x - 9\cos^2 x$
4. $y = 3\sin x + 4\cos z$
5. $y = 3\sin x + 4\cos x$

##

###

$y_{min} = 0$, $y_{max} = 12$ $[0, 12]$ (WRONG) !

$y = \underbrace{5\sin^2 x + 5\cos^2 x} + 2\cos^2 x$

$y = 5 + 2\cos^2 x$ $y \in [5, 7]$ (correct)

(2) $y = 5 + 3\cos^2 x - 2 = 3 + 3\cos^2 x$

$y \in [3, 6]$

(3) $y = 2(1 - \cos^2 x) - 9\cos^2 x$

$\boxed{y = 2 - 11\cos^2 x}$

$y_{max} = 2$, $y_{min} = -9$ $y \in [-9, 2]$

(4) $y_{max} = 7$

$y_{min} = -7$

Ans → $[-5, 5]$

Question

HW

Find the maximum & minimum value of:

(i) $\sqrt{3}\sin x - \cos x$

(ii) $6 \sin x \cos x - 4 \cos 2x$

(iii) $\cos^2 x - 6 \sin x \cos x + 3 \sin^2 x + 2$

$2\sin^2 x = 1-\cos 2x$

Question

Find the maximum & minimum value of:

(i) $\sqrt{3}\sin x - \cos x \longrightarrow a=\sqrt{3},\ b=-1 \Rightarrow \sqrt{a^2+b^2} = \sqrt{3+1} = 2$ $\quad y \in [-2, 2]$

(ii) $6\sin x\cos x - 4\cos 2x = 3\sin 2x - 4\cos 2x = \sqrt{3^2+4^2} = 5$ $\quad y \in [-5, 5]$

(iii) $\cos^2 x - 6\sin x\cos x + 3\sin^2 x + 2$

(iii)

$$y = 1 + 2\sin^2 x - 3\sin 2x + 2$$

$$y = 3 + 2\sin^2 x - 3\sin 2x$$

$$y = 3 + 1 - \cos 2x - 3\sin 2x$$

$$y = \underbrace{4}_{\text{Const}} + [-\underbrace{(\cos 2x - 3\sin 2x}]$$

$\sqrt{a^2+b^2} = \sqrt{1+9} = \sqrt{10}$

$y_{max} = 4+\sqrt{10}$, $y_{min} = 4-\sqrt{10}$

PYQ

HW

The maximum value of the expression $\dfrac{1}{\boxed{\sin^2\theta+3\sin\theta\cos\theta+5\cos^2\theta}}$ is

$\boxed{\text{Ans} = 2}$

Question

Prove that $5\cos\theta + 3\cos\left(\theta + \frac{\pi}{3}\right) + 3$ lies between -4 and 10.

$$5\cos\theta + 3\left[\cos\theta\cos\pi/3 - \sin\theta\sin\pi/3\right] + 3$$

$$5\cos\theta + 3\left[\frac{1}{2}\cos\theta - \frac{\sqrt{3}}{2}\sin\theta\right] + 3$$

$$\underbrace{5\cos\theta + 3/2\cos\theta}\, - \frac{3\sqrt{3}}{2}\sin\theta + 3$$

$$\cos\theta\left(5 + 3/2\right) - \frac{3\sqrt{3}}{2}\sin\theta + 3$$

$$y = \boxed{\frac{13}{2}\cos\theta - 3\frac{\sqrt{3}}{2}\sin\theta} + 3$$

$$y_{max} = 7 + 3 = 10, \quad y_{min} = -7 + 3 = -4$$

$$a = 13/2, \quad b = -\frac{3\sqrt{3}}{2}$$

$$\sqrt{a^2+b^2} = \sqrt{\left(\frac{13}{2}\right)^2 + \left(\frac{3\sqrt{3}}{2}\right)^2}$$

$$= \sqrt{\frac{169}{4} + \frac{27}{4}}$$

$$= \sqrt{\frac{196}{4}}$$

$$= \frac{14}{2} = 7$$

Question

The minimum value of $3\tan^2\theta + 12\cot^2\theta$ is

A 6

B 15

C 24

D 12

$a^2 = 3 \quad a = \sqrt{3}$

$b^2 = 12 \quad b = 2\sqrt{3}$

$y_{min} = 2ab$

$= 2 \times \sqrt{3} \times 2\sqrt{3}$

$= 12$

Q JEE Main 2020 (September)

The minimum value of $2^{\sin x} + 2^{\cos x}$ is

A $2^{-1+\sqrt{2}}$

B $2^{1-\sqrt{2}}$

C $2^{1-\frac{1}{\sqrt{2}}}$

D $2^{-1+\frac{1}{\sqrt{2}}}$

$a \geq b \geq c$

$\Rightarrow a \geq c$

$\sin x + \cos x \geq -\sqrt{2}$

$2^{\sin x}, \quad 2^{\cos x}$

$AM \geq GM$

$$\frac{2^{\sin x} + 2^{\cos x}}{2} \geq \sqrt{2^{\sin x} \cdot 2^{\cos x}}$$

$$2^{\sin x} + 2^{\cos x} \geq 2\sqrt{2^{\sin x + \cos x}} \geq 2 \cdot \sqrt{2^{-\sqrt{2}}}$$

$\sin x + \cos x \geq -\sqrt{2}$

$$\sqrt{2^{\sin x + \cos x}} \geq \sqrt{2^{-\sqrt{2}}}$$

$$2\sqrt{2^{\sin x + \cos x}} \geq 2\sqrt{2^{-\sqrt{2}}}$$

$$2^{\sin x} + 2^{\cos x} \geq 2\left(2^{-\sqrt{2}}\right)^{1/2}$$

$$\geq 2^1 \cdot 2^{-\sqrt{2}/2}$$

$$\geq 2^{1-\sqrt{2}/2}$$

Range using completing the square method

Find Range →

(1) $\boxed{y = 2\sin^2 x - 3\sin x + 4}$ ✓

$\sin x = t$

$y = 2t^2 - 3t + 4$

$\frac{y}{2} = t^2 - \frac{3}{2}t + 2$

$\frac{y}{2} = \left(t - \frac{3}{4}\right)^2 - \frac{9}{16} + 2$

$\frac{y}{2} = \left(t - \frac{3}{4}\right)^2 + \frac{23}{16}$

$$\boxed{y = \frac{23}{8} + 2\left(\sin x - \frac{3}{4}\right)^2}$$

for y_{min} $\left(\sin x - \frac{3}{4}\right)^2$ must be min $= 0$

$\Rightarrow y_{min} = \frac{23}{8}$ ✓

For y_{max} $\left(\sin x - \frac{3}{4}\right)^2$ must be max

$\Rightarrow \sin x = -1$

$y_{max} = 2(-1)^2 - 3(-1) + 4$

$= 9$

$y \in \left[\frac{23}{8}, 9\right]$

$$y = 3\sin^2 x + 4\sin x - 1$$

$\sin x = t$

$$y = 3t^2 + 4t - 1$$

$$y/3 = t^2 + 4t/3 - 1/3.$$

$$y/3 = \left(t + 2/3\right)^2 - 4/9 - \frac{1}{3}\times\frac{3}{3}$$

$$y/3 = \left(t + 2/3\right)^2 - 7/9$$

$$\boxed{y = -7/3 + 3\left(t + 2/3\right)^2}$$

$$y = -7/3 + 3\left(\sin x + 2/3\right)^2$$

$\sin x = -2/3 \rightarrow$

$y_{min} = -7/3.$

for y_{max} $\left(\sin x + 2/3\right)^2$ must be max

A) $\sin x = 1$

B) $\sin x = -1$

$\Rightarrow$ For y_{max} $\boxed{\sin x = 1}$

$$y_{max} = 3 + 4 - 1$$
$$= 6.$$

$$y \in \left[-7/3, 6\right]$$

$$y = 2\cos^2 x - 6\cos x + 1$$

$\cos x = t$

$$y = 2t^2 - 6t + 1$$

$$y/2 = t^2 - 3t + 1/2$$

$$y/2 = (t - 3/2)^2 - 9/4 + \tfrac{1}{2}\times 2/2$$

$$y/2 = (t - 3/2)^2 - 7/4$$

$$\boxed{y = -7/2 + 2(\cos x - 3/2)^2}$$

$\Downarrow$

$\cos x - 3/2 \neq 0$

$\cos x \neq 3/2$

$y_{min} = -7/2 \rightarrow$ WRONG

$y_{max} \Rightarrow \cos x = -1$

$$y = 2 + 6 + 1$$
$$= 9$$

$$y \in [-7/2, 9]$$

$\times$

For y_{min}, $\cos x = 1$

$$y_{min} = 2 - 6 + 1$$
$$= -3$$

☆ $y \in [-3, 9]$

$$y = \underbrace{2\sin^2 x + 8\sin x}$$

$$\sin x = t$$

$$y = 2t^2 + 8t$$

$$y/2 = t^2 + 4t$$

$$y/2 = (t+2)^2 - 4$$

$$y = -8 + 2(t+2)^2$$

$$\boxed{y = -8 + 2(\sin x + 2)^2}$$

$\sin x + 2 \neq 0$

$\sin x \neq -2$

For

y_{min} $\sin x = -1$

$y_{min} = 2 - 8 = -6$

For $y_{max} \Rightarrow \sin x = 1$

$y_{max} = 10$

$y \in [-6, 10]$

Question (PYQ)

[Ans. (B)]

Given $A = \sin^2\theta + \cos^4\theta$, then for all real θ.

A $1 \le A \le 2$

B $3/4 \le A \le 1$

C $13/16 \le A \le 1$

D $3/4 \le A \le 13/16$

$A = 1 - \cos^2\theta + \cos^4\theta$

$\boxed{\cos^2\theta = t}$

$A = 1 - t + t^2$

$A = t^2 - t + 1$

$A = \left(t - \frac{1}{2}\right)^2 - \frac{1}{4} + 1$

$A = \left(\cos^2\theta - \frac{1}{2}\right)^2 + 3/4$

$\boxed{A_{min} = 3/4}$

for A_{max}

$\left(\cos^2\theta - \frac{1}{2}\right)^2$ must be max

$\cos^2\theta \to 0, 1$

$\boxed{A_{max} = 1}$

Q JEE Main 2023 (January-II)

The set of all values of λ for which the equation $\boxed{\cos^2 2x - 2\sin^4 x - 2\cos^2 x} = \lambda$ has a solution

A $[-2, -1]$

B $\left[-2, -\frac{3}{2}\right]$

C $\left[-1, -\frac{1}{2}\right]$

D $\left[-\frac{3}{2}, -1\right]$ ✓

$\cos^2 2x - \frac{(1-\cos 2x)^2}{2} - (1+\cos 2x) = \lambda$

$\lambda \to$ Lambda

$\cos^2 2x - \frac{(1+\cos^2 2x - 2\cos 2x)}{2} - 1 - \cos 2x = \lambda$

$2\cos^2 x = 1 + \cos 2x$

$\frac{2\cos^2 2x - 1 - \cancel{\cos^2 2x} + 2\cancel{\cos 2x} - 2 - 2\cancel{\cos 2x}}{2} = \lambda$

$2\sin^4 x = \frac{(2\sin^2 x)^2}{2}$

$2\sin^4 x = \frac{(1-\cos 2x)^2}{2}$

$\frac{\cos^2 2x - 3}{2} = \lambda$

$\cos^2 2x = 2\lambda + 3$

$[0, 1]$

$0 \le 2\lambda + 3 \le 1$

$-3 \le 2\lambda \le -2$

$\boxed{-3/2 \le \lambda \le -1}$

HW-BT-26

[Ans. ( )]

Show that $\tan 7.5° = \sqrt{6} - \sqrt{3} + \sqrt{2} - 2$.

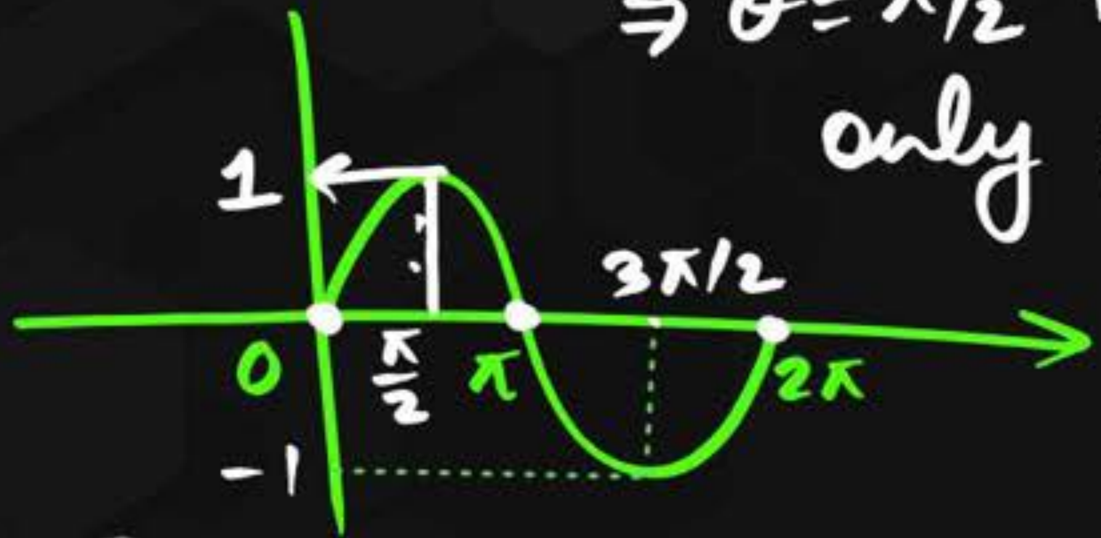
Learn

HW-BT-27

If $\frac{9x}{\cos\theta}+\frac{5y}{\sin\theta}=56$ and $\frac{9x\sin\theta}{\cos^2\theta}-\frac{5y\cos\theta}{\sin^2\theta}=0$, then find the value of $\left[(9x)^{\frac{2}{3}}+(5y)^{\frac{2}{3}}\right]^3$

⑤ $\sin\theta = 1$

$\Rightarrow \theta = \pi/2$ is the only principal solⁿ.



⑥ $\sin\theta = -1$

$\theta = 3\pi/2$

⑦ $\sin\theta = 0$

$\Rightarrow \theta = 0, \pi, \cancel{2\pi}$

Find Principal solⁿ

① $\cos 2\theta = -1/2$

$2\pi/3 = \pi - \pi/3$

$\theta \in [0, 2\pi)$
$2\theta \in [0, 4\pi)$

$\frac{4\pi}{3} = \pi + \pi/3$

$2\theta = 2\pi/3, 4\pi/3, 2\pi + 2\pi/3, 2\pi + \frac{4\pi}{3}$

$2\theta = 2\pi/3, 4\pi/3, 8\pi/3, 10\pi/3$

$\theta = \pi/3, 2\pi/3, 4\pi/3, 5\pi/3$

4 Principal solⁿ.

$\pi/2$, $\pi - \pi/2$

2) $\sin 2\theta = -\frac{1}{2}$.

$\theta \in [0, 2\pi)$

$2\theta \in [0, 4\pi)$

$\pi + \pi/6 = 7\pi/6$

$2\pi - \pi/6 = 11\pi/6$.

$2\theta = 7\pi/6$ or $11\pi/6$ or $2\pi + 7\pi/6$ or $2\pi + 11\pi/6$

$2\theta = 7\pi/6$ or $11\pi/6$ or $19\pi/6$ or $23\pi/6$.

$\theta = 7\pi/12, \ 11\pi/12, \ 19\pi/12, \ \frac{23\pi}{12}$

Q →

$\tan 3\theta = 1$

$\theta \in [0, 2\pi)$

$3\theta \in [0, 6\pi)$

6 principal sol^n

$\pi + \pi/4$

$3\theta = \pi/4$ or $5\pi/4$

$= 2\pi + \pi/4$ or $2\pi + 5\pi/4$

$= 4\pi + \pi/4$ or $4\pi + 5\pi/4$

$\theta =$ ✓

0

Question

Find the principal solutions for the following:

(1) $\sin 2\theta = -\frac{1}{2}$

(2) $\cos\theta = -\frac{\sqrt{3}}{2}$

(3) $\cos 2\theta = -\frac{1}{2}$

(4) $\sin x + \cos x = 0$

(5) $\boxed{\sin\theta = \frac{\sqrt{3}}{2} \text{ and } \cos\theta = -\frac{1}{2}}$

(6) $\sin\left(\frac{3x}{2}\right) = 0$

(7) $\cos\theta = -\frac{1}{3}$

(8) $\tan 2\theta = -\sqrt{2}$

$\sin x = -\cos x$

$\frac{\sin x}{\cos x} = -1$

$\tan x = -1$

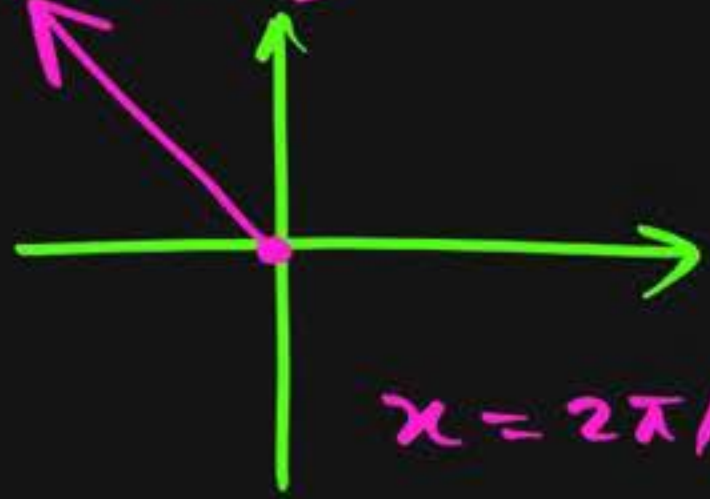
$\pi - \pi/4$

$x = 3\pi/4, 7\pi/4$

$2\pi - \pi/4.$

5) And → ∩ → common

$\pi - \pi/3 = 2\pi/3$



$x = 2\pi/3$

$\sin\left(\frac{3x}{2}\right) = 0$

$\frac{3x}{2} = 0, \pi, 2\pi,$

$x = 0, \frac{2\pi}{3}, \frac{4\pi}{3}$

Ans

$x \in [0, 2\pi)$

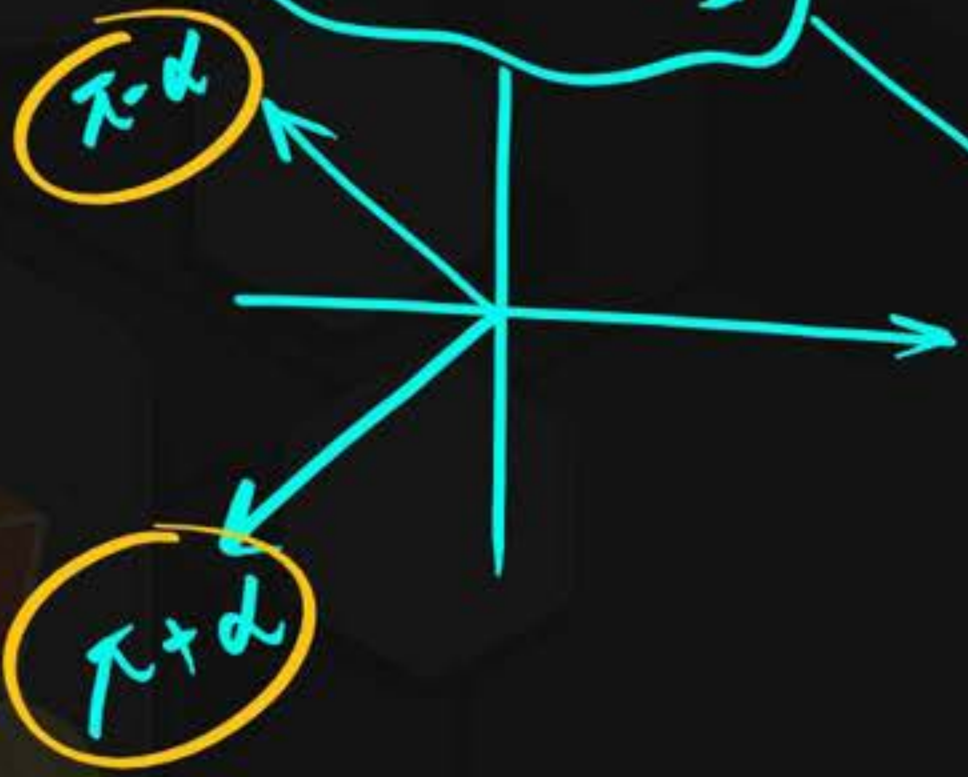
$\frac{3x}{2} \in [0, 3\pi)$

$3x/2 \in [0, 3\pi)$

$\rightarrow \boxed{\cos\theta = \frac{1}{5} \Rightarrow \theta = \cos^{-1}\left(\frac{1}{5}\right)}$

$\cos\theta = -\frac{1}{3}$

$\alpha = \cos^{-1}\left(\frac{1}{3}\right)$

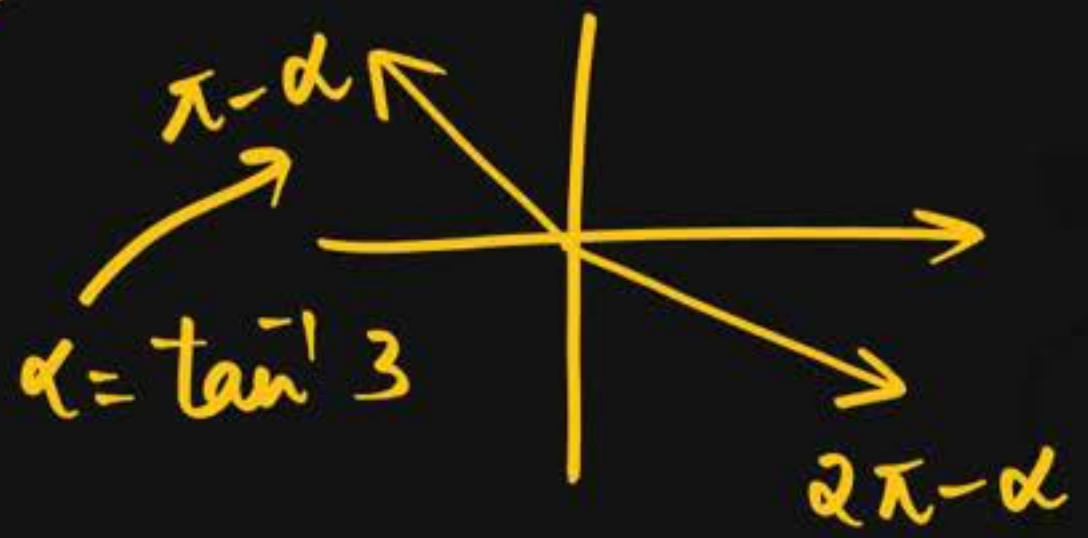


$\theta = \pi - \cos^{-1}\frac{1}{3}$

or

$\pi + \cos^{-1}\frac{1}{3}$

Q→ $\tan x = -3$



$x = \pi - \tan^{-1}3$

or $2\pi - \tan^{-1}3$.

Q→ $\sin x = -3/5$

$\alpha = \sin^{-1} 3/5$

Ans:

$\pi + \sin^{-1}3/5$ or $2\pi - \sin^{-1}3/5$

$\underline{\underline{Q}} \rightarrow \tan(2\theta) = -\sqrt{2}$

$\theta \in [0, 2\pi)$

$2\theta \in [0, 4\pi)$

$$2\theta = \pi - \alpha,\ 2\pi-\alpha,$$

$$= 3\pi-\alpha,\ 4\pi-\alpha$$

$$2\theta = \pi - \tan^{-1}\sqrt{2},\ 2\pi - \tan^{-1}\sqrt{2},\ 3\pi - \tan^{-1}\sqrt{2},\ 4\pi - \tan^{-1}\sqrt{2}$$

$$\theta = \frac{\pi}{2} - \frac{1}{2}\tan^{-1}\sqrt{2},\ \pi - \frac{1}{2}\tan^{-1}\sqrt{2},\ \frac{3\pi}{2} - \frac{1}{2}\tan^{-1}\sqrt{2},\ 2\pi - \frac{1}{2}\tan^{-1}\sqrt{2}$$

Question

The number of solutions of the equation, $\boxed{|\cot x| = \cot x + \frac{1}{\sin x}\ (0 \leq x \leq 2\pi)}$ is:

A 0

B 1 ✓

C 2 WRONG

D 3

Case → 1

$\cot x \geq 0$

$+\cancel{\cot} x = \cancel{\cot} x + \frac{1}{\sin x}$

$\frac{1}{\sin x} = 0$

$\Rightarrow$ not possible.

Case → 2

$\cot x < 0$

$-\cot x = \cot x + \frac{1}{\sin x}$

$-2\cot x = \frac{1}{\sin x}.$

$-\frac{2\cos x}{\cancel{\sin x}} = \frac{1}{\cancel{\sin x}}$

$\Rightarrow -2\cos x = 1$

$\cos x = -\frac{1}{2}$

$x = 2\pi/3$ ✓ or $4\pi/3$

Reject

$x = 4\pi/3$ is Rejected as it does not Satisfy $\cot x < 0$

Q JEE Main 2020

[Ans. (8)]

The number of distinct solutions of the equation, $\log_{1/2}|\sin x| = 2 - \log_{1/2}|\cos x|$ in the interval $[0, 2\pi]$, is

$$\log_{1/2}|\sin x| + \log_{1/2}|\cos x| = 2$$

$$\log_{1/2} m + \log_{1/2} n = \log_{1/2}(mn)$$

$$|ab| = |a||b|$$

$$|x| = 1/2$$

$$x = \pm 1/2$$

$$\log_{1/2}(|\sin x||\cos x|) = 2$$

$$|\sin x||\cos x| = \left(\tfrac{1}{2}\right)^2$$

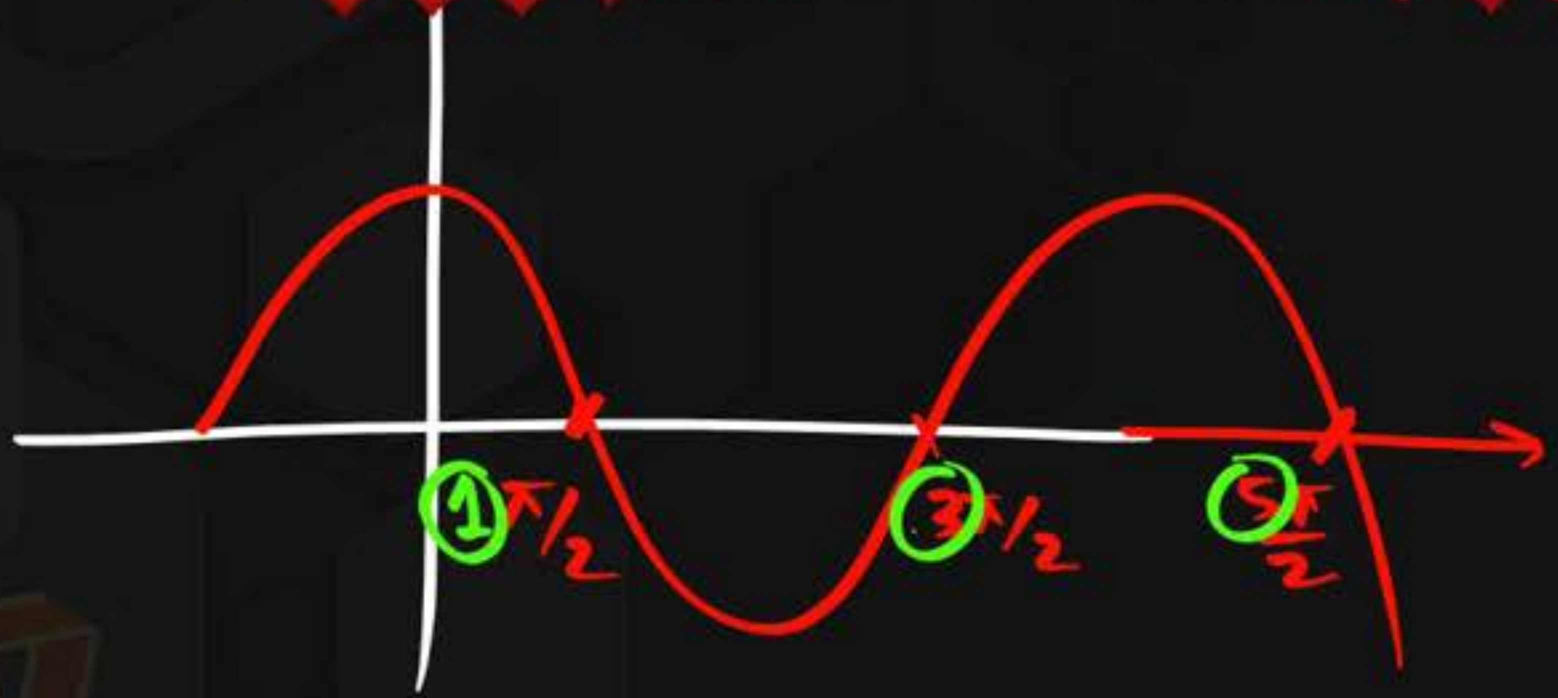
$$\tfrac{1}{2}|2\sin x \cos x| = \tfrac{1}{4}$$

$$|\sin 2x| = 2/4$$

$$|\sin 2x| = \tfrac{1}{2}$$

$$\sin 2x = \pm \tfrac{1}{2}$$

$\boxed{\cos\theta = 0} \Rightarrow \theta = (2n+1)\pi/2$



Q →

$\sin(3\theta) = -\frac{1}{2}$

General sol^n

$3\theta = n\pi + (-1)^n \cdot (-\pi/6)$

Ans $\theta = \frac{n\pi}{3} + (-1)^n \left(-\frac{\pi}{18}\right)$

$n \in I$

Q → $\operatorname{cosec}\theta = \sqrt{2}$

$\Rightarrow \sin\theta = \frac{1}{\sqrt{2}}$

$\theta = n\pi + (-1)^n \pi/4$

Question

$2\pi - \pi/3 = 5\pi/3$

Find the general solutions for the following

(1) $\cos 2\theta = \frac{1}{2}$ $\Rightarrow 2\theta = 2n\pi \pm \pi/3 \Rightarrow \theta = n\pi \pm \pi/6$

(2) $\cot\theta = -1$ $\Rightarrow \tan\theta = -1 \Rightarrow \theta = n\pi + \alpha$ (2)

$\boxed{\theta = n\pi - \pi/4}$

(3) $\sec 3\theta = -2$ $\rightarrow \cos 3\theta = -1/2$

$3\theta = 2n\pi \pm 2\pi/3$

$\boxed{\theta = \frac{2n\pi}{3} \pm 2\pi/9}$

(4) $\underbrace{\sin 2\theta + \cos 2\theta}$ $= 0$

(5) $\cos\theta = \frac{1}{2}$ & $\tan\theta = -\sqrt{3}$

(6) $\sin 3\theta = -\frac{1}{2}$

4) $\sin 2\theta = -\cos 2\theta$

$\boxed{\tan 2\theta = -1}$

$2\theta = n\pi - \pi/4$

$\theta = \frac{n\pi}{2} - \pi/8$

(5)

$\boxed{-\pi/3 + 2n\pi}$

Types of Trigonometric Equations

Type-1

Trigonometric equations which can be solved by factorization.

Solve: $\underbrace{2\sin^2 x \cos x - \cos x}{} = 0$, where $0 \leq x \leq 2\pi$

$\cos x\,[2\sin^2 x - 1] = 0$

$\cos x = 0 \Rightarrow x = (2n+1)\pi/2$ ✓

or $2\sin^2 x - 1 = 0$ OR

$\boxed{\sin^2 x = \frac{1}{2}} \Rightarrow \boxed{x = n\pi \pm \pi/4}$

$\sin^2 x = \sin^2 \alpha$

$\sin^2 x = \left(\frac{1}{\sqrt{2}}\right)^2$

$= \left(\sin \pi/4\right)^2$

$2\sin^2 x - 1 = 0$

$1 - 2\sin^2 x = 0$

$\cos 2x = 0$

$2x = (2n+1)\pi/2$

$\boxed{x = (2n+1)\pi/4}$

Q JEE Main 2019

$Ans = 7\pi/6 + 11\pi/6 + \frac{7\pi}{6} + 11\pi/6 - 2\pi - 2\pi$

$6\pi - 4\pi = 2\pi$

[Ans. (D)]

Let $S = \{\theta \in [-2\pi, 2\pi] : 2\cos^2\theta + 3\sin\theta = 0\}$, then the sum of the elements of S is:

A π

B $\frac{5\pi}{3}$

C $\frac{13\pi}{3}$

D 2π

$[0, 2\pi]$

$[-2\pi, 0]$

$2(1-\sin^2\theta) + 3\sin\theta = 0$

$\boxed{\sin\theta = t}$

$2(1-t^2) + 3t = 0$

$2 - 2t^2 + 3t = 0$

$2t^2 - 3t - 2 = 0$

$2t^2 - 4t + t - 2 = 0$

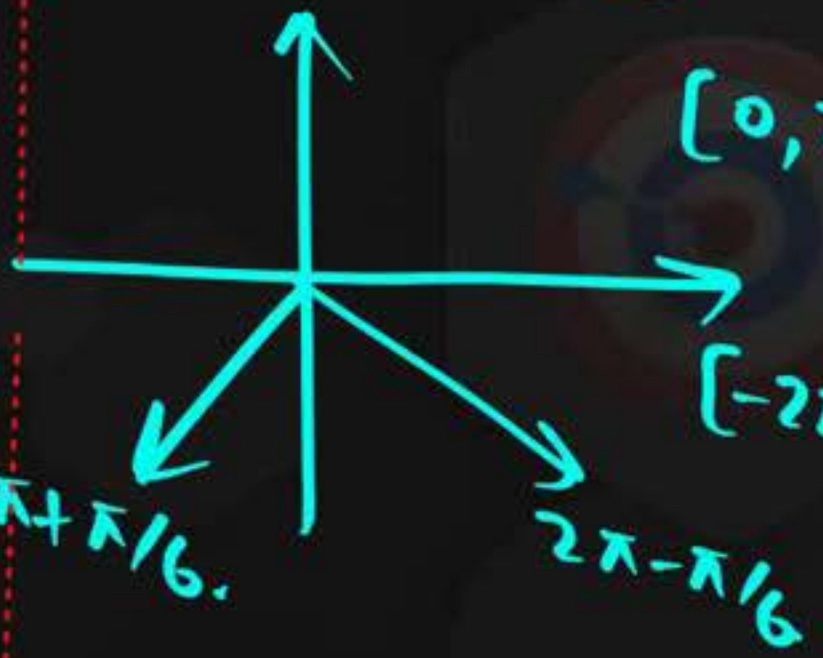
$(2t+1)(t-2) = 0$

$t = -1/2$ or $t = 2$

$\sin\theta = -1/2$ or $\sin\theta = 2$

Reject

$\sin\theta = -1/2$



$[0, 2\pi) \rightarrow 7\pi/6$ or $11\pi/6$

$[-2\pi, 0] \rightarrow \frac{7\pi}{6} - 2\pi$, $11\pi/6 - 2\pi$

Types of Trigonometric Equations

Type-2

Trigonometric equations which can be solved by reducing them in to quadratic equations.

Solve: $\boxed{16^{\sin^2 x} + 16^{\cos^2 x} = 10,}$ where $0 \le x \le 2\pi$

$$16^{\sin^2 x} + 16^{1-\sin^2 x} = 10$$

$$\underbrace{16^{\sin^2 x}}_{t} + \frac{16^1}{\underbrace{16^{\sin^2 x}}_{t}} = 10$$

$$t + \frac{16}{t} = 10$$

$$\frac{t^2+16}{t} = 10$$

$$t^2 - 10t + 16 = 0$$

$$t = 8 \text{ or } t = 2$$

$$16^{\sin^2 x} = 8 \text{ or } 2$$

$$2^{4\sin^2 x} = 2^3 \text{ or } 2^1$$

$$4\sin^2 x = 3 \text{ or } 4\sin^2 x = 1$$

$$\sin^2 x = 3/4 \text{ or } \sin^2 x = 1/4$$

Types of Trigonometric Equations

Type-3

Trigonometric equations which can be solved by transforming a sum or difference of trigonometric ratios into their product.

Example:

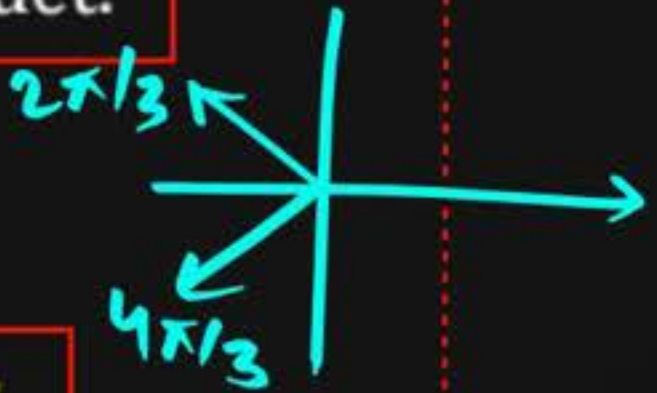
Find the values of x, $0 \leq x \leq 2\pi$, such that

$$\sin x + \sin 2x + \sin 3x = \cos x + \cos 2x + \cos 3x.$$

$2\cos x + 1 = 0$ or $\sin 2x = \cos 2x$

$\cos x = -\frac{1}{2}$ or $\tan 2x = 1$

$x = 2\pi/3, 4\pi/3$



$(\sin x + \sin 3x) + \sin 2x = (\cos x + \cos 3x) + \cos 2x$

$2\sin 2x \cos x + \sin 2x = 2\cos 2x \cos x + \cos 2x$

$\sin 2x\,[\cancel{2\cos x + 1}] = \cos 2x\,[\cancel{2\cos x + 1}]$

$2x = \pi/4, 5\pi/4$

$2\pi + \pi/4, 2\pi + \frac{5\pi}{4}$

$2x = \pi/4, 5\pi/4, 9\pi/4, \frac{13\pi}{4}$

$x = \pi/8, 5\pi/8, 9\pi/8, \frac{13\pi}{8}.$

Question

HW

The general solution of,
$\sin x - 3\sin 2x + \sin 3x = \cos x - 3\cos 2x + \cos 3x$ is

A $n\pi + \frac{\pi}{8}$

B $\frac{n\pi}{2} + \frac{\pi}{8}$

C $(-1)^n\left(\frac{n\pi}{2}\right) + \frac{\pi}{8}$

D $2n\pi + \cos^{-1}\left(\frac{3}{2}\right)$

Q JEE Main 2019

HW

If $0 \le x < \frac{\pi}{2}$, then the number of values of x for which $\sin x - \sin 2x + \sin 3x = 0$, is

A 2

B 1

C 3

D 4

Homework_NCERT- EX-3.4

EXERCISE 3.4

Find the principal and general solutions of the following equations:

1. $\tan x = \sqrt{3}$
2. $\sec x = 2$
3. $\cot x = -\sqrt{3}$
4. $\operatorname{cosec} x = -2$

Find the general solution for each of the following equations:

5. $\cos 4x = \cos 2x$
6. $\cos 3x + \cos x - \cos 2x = 0$
7. $\sin 2x + \cos x = 0$
8. $\sec^2 2x = 1 - \tan 2x$
9. $\sin x + \sin 3x + \sin 5x = 0$

Homework_NCERT- EX-3.4_Answers

EXERCISE 3.4

1. $\frac{\pi}{3}, \frac{4\pi}{3}, n\pi + \frac{\pi}{3}, n \in \mathbf{Z}$

2. $\frac{\pi}{3}, \frac{5\pi}{3}, 2n\pi \pm \frac{\pi}{3}, n \in \mathbf{Z}$

3. $\frac{5\pi}{6}, \frac{11\pi}{6}, n\pi + \frac{5\pi}{6}, n \in \mathbf{Z}$

4. $\frac{7\pi}{6}, \frac{11\pi}{6}, n\pi + (-1)^n \frac{7\pi}{6}, n \in \mathbf{Z}$

5. $x = \frac{n\pi}{3}$ or $x = n\pi, n \in \mathbf{Z}$

6. $x = (2n+1)\frac{\pi}{4}$, or $2n\pi \pm \frac{\pi}{3}, n \in \mathbf{Z}$

7. $x = n\pi + (-1)^n \frac{7\pi}{6}$ or $(2n+1)\frac{\pi}{2}, n \in \mathbf{Z}$

8. $x = \frac{n\pi}{2}$, or $\frac{n\pi}{2} + \frac{3\pi}{8}, n \in \mathbf{Z}$

9. $x = \frac{n\pi}{3}$, or $n\pi \pm \frac{\pi}{3}, n \in \mathbf{Z}$

Question

The general solution of,

$\underbrace{\sin x - 3\sin 2x + \sin}\, 3x = \cos x - 3\cos 2x + \cos 3x$ is

A $n\pi + \frac{\pi}{8}$

B $\frac{n\pi}{2} + \frac{\pi}{8}$

C $(-1)^n\left(\frac{n\pi}{2}\right) + \frac{\pi}{8}$

D $2n\pi + \cos^{-1}\left(\frac{3}{2}\right)$

$\boxed{x = (-1)^n n\pi/2 + \pi/8}$

$n = 1 \ni x = -\pi/2 + \pi/8.$

$\underbrace{\sin x + \sin}\, 3x - 3\sin 2x = \underbrace{\cos x + \cos}\, 3x - 3\cos 2x$

$2\sin 2x\cos x - 3\sin 2x = 2\cos 2x\cos x - 3\cos 2x$

$\sin 2x\,(\cancel{2\cos x - 3}) = \cos 2x\,(\cancel{2\cos x - 3})$

$\sin 2x = \cos 2x$

$\tan 2x = 1$

$2x = n\pi + \pi/4$

ect)

Q JEE Main 2022 (29 June II)

HW

[Ans. (32)]

The number of elements in the set

$S = \{\theta \in [-4\pi, 4\pi] : 3\cos^2 2\theta + 6\cos 2\theta - 10\cos^2\theta + 5 = 0\}$ is ______.

Types of Trigonometric Equations

$2\sin A\cos B =$

Type-4

Trigonometric equations which can be solved by transforming a product of trigonometric ratios into $\boxed{\text{their sum or difference.}}$

Example:

$$\boxed{2\underbrace{\sin 5x\cos 3x} = 2\sin 6x\cos 2x}$$

$$\sin(5x+3x) + \sin(5x-3x) = \cancel{\sin 8x} + \sin 4x.$$

$$\sin 2x = \sin 4x$$

M → 1 $\sin 4x = \sin 2x$

$$\sin 4x - \sin 2x = 0$$

$$2\underbrace{\cos 3x}\,\underbrace{\sin x} = 0$$

$$\underbrace{\cos 3x} = 0 \Rightarrow 3x = (2n+1)\pi/2$$

$$x = (2n+1)\pi/6$$

or OR

$$\sin x = 0 \Rightarrow \boxed{x = n\pi}$$

$(-1)^{10} = 1$

$(-1)^{3} = -1$

M-2

$$\underbrace{\sin 4x} = \sin 2x$$

$$\downarrow$$

$$2\cancel{\sin 2x}\cos 2x = \cancel{\sin 2x}$$

$$\sin 2x = 0 \Rightarrow 2x = n\pi \Rightarrow \boxed{x = \frac{n\pi}{2}}$$

or

$$2\cos 2x = 1$$

$$\cos 2x = \tfrac{1}{2}$$

$$2x = 2n\pi \pm \pi/3$$

$$\boxed{x = n\pi \pm \pi/6}$$

OR

M-3

$$\sin 4x = \sin 2x$$

$$\boxed{4x = n\pi + (-1)^{n}(2x)}$$

Case → 1 $\quad n = \text{even} = 2m$

$$4x = 2m\pi + \underbrace{(-1)^{2m}}_{1}(2x)$$

$$4x = 2m\pi + 2x$$

$$\cancel{2}x = \cancel{2}m\pi \Rightarrow \boxed{x = m\pi}$$

Case → 2 $\quad n = (2m+1)$

$$4x = (2m+1)\pi - 2x$$

$$6x = (2m+1)\pi$$

$$x = (2m+1)\pi/6$$

Q→ $\tan 7x = \cot 4x$

$\tan 7x = \tan\left(\underbrace{\pi/2}-4x\right)$

Formula

$7x = n\pi + \pi/2 - 4x$

$11x = n\pi + \pi/2$

$11x = (2n+1)\pi/2$

$x = (2n+1)\pi/22$

Types of Trigonometric Equations

Type-5

Trigonometric equations of the form $\boxed{a \sin x + b \cos x = c}$, where $a, b, c \in R$, can be solved by 2 methods-

Method-1 dividing both sides of the equation by $\boxed{\sqrt{a^2 + b^2}}$.

Method-2 by converting into $\tan(x/2)$

Example:

Solve $\sin x + \cos x = \sqrt{2}$

Ex →

$\sin x + \cos x = 1$

divide by $\sqrt{2}$

$\underbrace{\frac{1}{\sqrt{2}}}_{\cos \pi/4} \sin x + \underbrace{\frac{1}{\sqrt{2}}}_{\sin \pi/4} \cos x = \frac{1}{\sqrt{2}}$

$\cos \pi/4 \sin x + \sin \pi/4 \cos x = \frac{1}{\sqrt{2}}$

$\boxed{\sin(x + \pi/4) = \frac{1}{\sqrt{2}}}$

$x + \pi/4 = n\pi + (-1)^n \pi/4$

$\boxed{x = n\pi + (-1)^n \pi/4 - \pi/4}$

$$\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x = \frac{1}{\sqrt{2}}$$

$\sin \pi/4$ $\cos \pi/4$

$$\underbrace{\sin x \sin \pi/4 + \cos x \cos \pi/4} = \frac{1}{\sqrt{2}}$$

$$\boxed{\cos\left(x - \pi/4\right) = \frac{1}{\sqrt{2}}}$$

$$x - \pi/4 = 2n\pi \pm \pi/4$$

(+) $x - \pi/4 = 2n\pi + \pi/4$

$$\boxed{x = 2n\pi + \pi/2}$$

(−) $x - \cancel{\pi/4} = 2n\pi - \cancel{\pi/4}$

$$\boxed{x = 2n\pi}$$

Ans: $x = 2n\pi$
or
$2n\pi + \pi/2$

$$x = n\pi + (-1)^n \pi/4 - \pi/4$$

$n = \text{even} = 2m$

$$x = 2m\pi + \cancel{\pi/4} - \cancel{\pi/4}$$

$$\boxed{x = 2m\pi}$$

$n = \text{odd} = \underbrace{2m+1}$

$$x = (2m+1)\pi - \underbrace{\pi/4 - \pi/4}$$

$$x = 2m\pi + \underbrace{\pi - \pi/2}$$

$$\boxed{x = 2m\pi + \pi/2}$$

Question

Solve $3\cos x + 4\sin x = 5$

divide by $\sqrt{3^2+4^2} = 5$

$\frac{3}{5}\cos x + \frac{4}{5}\sin x = 1$

$\frac{3}{5} \to \cos\alpha$ $\quad$ $\frac{4}{5} \to \sin\alpha$

$\cos\alpha = 3/5$

$\alpha = \cos^{-1} 3/5$

$\cos(x-\alpha) = 1$

$x-\alpha = 2n\pi$

$\boxed{x = 2n\pi + \alpha}$

$x = 2n\pi + \cos^{-1} 3/5$

Q → III $\quad \sin x - \cos x = \sqrt{2}$

$\frac{\sin x}{\sqrt{2}} - \frac{\cos x}{\sqrt{2}} = \frac{\cancel{\sqrt{2}}}{\cancel{\sqrt{2}}}$

$\frac{1}{\sqrt{2}}\sin x - \cos x \times \frac{1}{\sqrt{2}} = 1$

$\frac{1}{\sqrt{2}} \to \sin\pi/4$ $\quad$ $\frac{1}{\sqrt{2}} \to \cos\pi/4$.

$\sin x \sin\pi/4 - \cos x \cos\pi/4 = 1$

$-\cos(x+\pi/4) = 1$

$\cos(x+\pi/4) = -1$

$x+\pi/4 = (2n+1)\pi \Rightarrow x = (2n+1)\pi - \pi/4$

$= 2n\pi + 3\pi/4$

M-2

$$\underbrace{\sin x} + \underbrace{\cos x} = 1$$

$$\sin x = \frac{2\tan x/2}{1+\tan^2 x/2}$$

$$\frac{2\tan x/2}{1+\tan^2 x/2} + \frac{1-\tan^2 x/2}{1+\tan^2 x/2} = 1$$

$$\tan x/2 = t$$

$$\frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2} = 1$$

$$\frac{2t+1-t^2}{1+t^2} = 1$$

$$2t + \cancel{1} - t^2 = \cancel{1} + t^2$$

$$\cancel{2}t = \cancel{2}t^2$$

$$\cancel{t} = t^{\cancel{2}}$$

$$t = 0 \text{ or } t = 1$$

$$\tan x/2 = 0 \text{ or } \tan x/2 = 1$$

$$x/2 = n\pi \text{ or } x/2 = n\pi + \pi/4$$

$$\underbrace{x = 2n\pi} \text{ or } \underbrace{x = 2n\pi + \pi/2}$$

Brain Teaser HW

Find all value of θ, between 0 and π, which satisfy the equation,

$$\cos\theta \cdot \cos 2\theta \cdot \cos 3\theta = \frac{1}{4}.$$

HW- BT-28

$\frac{\pm\pi}{3}, \frac{-\pi}{2}, \pm\pi$

Solve for x, $(-\pi \leq x \leq \pi)$ the equation,
$2(\cos x + \cos 2x) + \sin 2x(1 + 2\cos x) = 2\sin x$.

HW- BT-29

Find the general solution of the equation $\sin \pi x + \cos \pi x = 0$. Also find the sum of all solutions in [0, 100].

$x = n - \frac{1}{4}$, $n \in I$; sum = 5025

Types of Trigonometric Equations

Type-6

Trigonometric equations of the form $\boxed{P(\sin x \pm \cos x, \sin x \cos x) = 0}$ (where $P(y, z)$ is a polynomial) can be solved by using the substitution $\boxed{\sin x \pm \cos x = t.}$

$\sin x + \cos x = t$

S.B.S.

$(\sin x + \cos x)^2 = t^2$

$1 + 2\sin x \cos x = t^2$

$\sin x \cos x = \frac{t^2 - 1}{2}$

Example:

Solve $\boxed{\underbrace{\sin x + \cos x} = 1 + \underbrace{\sin x \cdot \cos x}}$

$t = 1 + \frac{t^2 - 1}{2}$

$t = \frac{2 + t^2 - 1}{2}$

$\Rightarrow 2t = t^2 + 1 \Rightarrow t^2 - 2t + 1 = 0 \Rightarrow (t-1)^2 = 0 \Rightarrow \boxed{t = 1}$

$\Rightarrow \boxed{\sin x + \cos x = 1}$

$x = 2n\pi$ or $2n\pi + \pi/2$

Types of Trigonometric Equations

$\sin x = 1 \Rightarrow x = (4n+1)\pi/2$

$\cos y = 1 \Rightarrow y = 2m\pi$

Ans

Type-7(a)

Trigonometric equations (having multiple Variables) which can be solved by the use of boundness of the trigonometric functions.

Range

$\boxed{\sin x + \cos y = 2}$, solve for x & y

$[-1, 1]$ $[-1, 1]$

$\Rightarrow \left.\begin{array}{l} LHS_{max} = 2 \\ LHS_{min} = -2 \end{array}\right\} \Rightarrow LHS \in [-2, 2]$

$\Rightarrow LHS = 2 \Rightarrow$ RHS = 2

$\sin x = 1 \Rightarrow$

$\cos y = 1$

Q→ $\sin x + \cos y = 3$

No soln

Q→ $\sin x + \cos y = 1.5$

Data Insufficient

Solve for x & y

Q→ $3\sin x + 4\cos x = y^2 - 2y + 6$

LHS: $\underbrace{[-5, 5]}$

RHS: $(y-1)^2 + 5$

RHS: $[5, \infty)$

$\Rightarrow$ LHS = RHS = 5

$3\sin x + 4\cos x = 5$ & $\Rightarrow (y-1)^2 = 0 \Rightarrow \boxed{y=1}$

$\frac{3}{5}\sin x + \frac{4}{5}\cos x = 1$

$y = a\sin x + b\cos x$

$y \leftarrow \left[-\sqrt{a^2+b^2}, \sqrt{a^2+b^2}\right]$

Question

If $\boxed{\sin x + \cos x} = \sqrt{\left(y + \frac{1}{y}\right)}, x \in [0, \pi]$, then

A $x = \frac{\pi}{4}, y = 1$

B $x \in R, y = 1$

C $x = \frac{\pi}{4}, y = 2$

D $x = \frac{3\pi}{4}, y \in R$

$\boxed{\text{LHS}: [-\sqrt{2}, \sqrt{2}]}$

RHS: $\sqrt{y + \frac{1}{y}} \geq \sqrt{2}$

$\Rightarrow \boxed{\text{RHS}: [\sqrt{2}, \infty)}$

$\Rightarrow \text{LHS} = \text{RHS} = \sqrt{2}$

$y = 1$

$\sin x + \cos x = \sqrt{2}$

$x = 45^\circ = \pi/4$

$y + 1/y = 2$

when $y = 1$

Question

If $\theta \in [0, 5\pi]$ and $r \in R$ such that $2\sin\theta = r^4 - 2r^2 + 3$ then the maximum number of values of the pair (r, θ) is

Total Pairs $= 3 \times 2 = 6$ Ans

LHS: $[-2, 2]$

$r^2 = t$

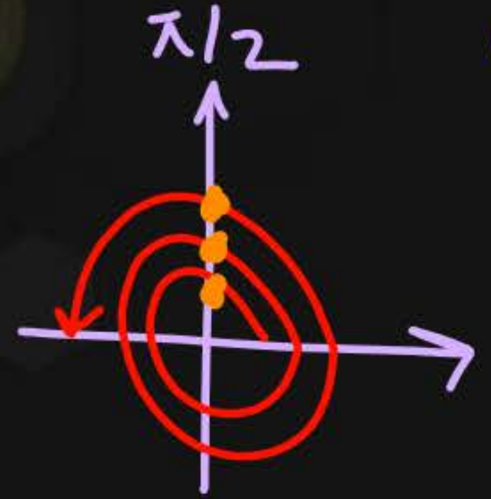
$t^2 - 2t + 3$

$(t-1)^2 + 2$

RHS $= (r^2 - 1)^2 + 2$

RHS: $[2, \infty)$

$\Rightarrow$ For LHS = RHS, only possibility is LHS = RHS = 2



$2\sin\theta = 2$

$\sin\theta = 1$

$\Rightarrow$ 3 sol^{n}s

RHS $= 2$

$r^2 - 1 = 0 \Rightarrow r^2 = 1 \Rightarrow \boxed{r = \pm 1} \rightarrow$ 2 val

Question

If $\boxed{0 \le x \le 3\pi}$, $\boxed{0 \le y \le 3\pi}$ and $\boxed{\cos x \sin y = 1}$ then the possible number of values of the ordered pair (x, y) is

$$\underbrace{\cos x}\ \underbrace{\sin y} = 1$$

Case → 1 $\cos x = 1$ & $\sin y = 1$

$x = 2n\pi$

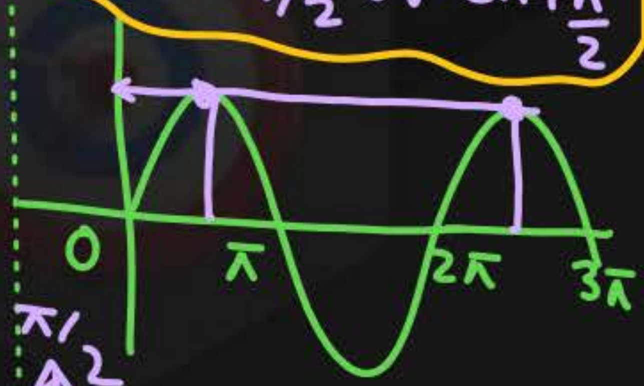
$n = 0$, $x = 0$

$n = 1$, $x = 2\pi$

~~$n = 2$, $x = 4\pi$~~

$\sin y = 1$

$y = \pi/2$ or $2\pi + \frac{\pi}{2}$



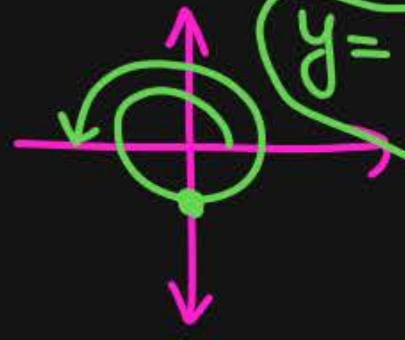
Pairs of $(x, y) = 4$

Case → 2

$\cos x = -1$ & $\sin y = -1$

$\cos x = (2n+1)\pi$

$x = \pi, 3\pi$



$y = \frac{3\pi}{2}$

Pairs $(x, y) = 2$

Ans $= 2 + 4 = 6$

V.V. Impt. Concept:

$$\boxed{y = x + \frac{1}{x}}$$

Proof →

① If $x > 0$

$x, \frac{1}{x}$

AM ≥ GM

$$\frac{x + \frac{1}{x}}{2} \geq \sqrt{x \cdot \frac{1}{x}}$$

$$\frac{x + \frac{1}{x}}{2} \geq 1$$

$$x + \frac{1}{x} \geq 2$$

② If $x < 0$

$-x, -\frac{1}{x}$

$$\frac{-x - \frac{1}{x}}{2} \geq \sqrt{1}$$

$$-\left(x + \frac{1}{x}\right) \geq 2 \Rightarrow \boxed{x + \frac{1}{x} \leq -2}$$

① $x + \frac{1}{x} \geq 2$ for $x > 0$

equality holds $\left(x + \frac{1}{x} = 2\right)$ when $x = 1$

② $x + \frac{1}{x} \leq -2$ for $x < 0$

equality holds $\left[x + \frac{1}{x} = -2\right]$ when $\boxed{x = -1}$

Question

HW

If $2\cos\theta = x + \frac{1}{x}$ where $\theta \in [0, 2\pi]$ and $x \in R$, then find the number of pairs of (θ, x)

HW-BT-30

HW

If $3\sin x + 12\sin y + 4\cos x + 5\cos y = 18$, the value of $\tan(x+y) =$

A $\frac{-63}{16}$

B $\frac{63}{16}$

C $\frac{-33}{56}$

D $\frac{33}{56}$

Types of Trigonometric Equations

Type-7(b)

Trigonometric equations (having single Variable) which can be solved by the use of boundness of the trigonometric ratios.

Example: $\cos^{50} x - \sin^{50} x = 1$

$(\cos x)^2 = 1$

$\cos x = \pm 1$ ✓

$\underbrace{(\cos x)^{50}}_{} = 1 + (\sin x)^{50} \leftarrow 0$

LHS: $[0, 1]$

only possibility is

$(\cos x)^{50} = 1$ & $(\sin x)^{50} = 0$

$(\cos x)^{50} = 1$

$(\cos x) = \pm 1$

$\Rightarrow \cos^2 x = 1$

$\cos^2 x = \cos^2 0$

$x = n\pi \pm 0$

$\boxed{x = n\pi}$

& $(\sin x)^{50} = 0$

$\sin x = 0$

$\boxed{x = n\pi}$

Intersection is

$\boxed{x = n\pi}$

Question

Find the general solution for

$\cos x + \cos 2x + \cos 3x + \cos 4x = \boxed{4}$

Intersection is
$x = 0, 2\pi, 4\pi,$
$\Rightarrow x = 2n\pi, \ n \in I$
Ans.

only possibility is

$\boxed{\cos x = 1} \Rightarrow x = 2n\pi, \Rightarrow x = 0, 2\pi, 4\pi, 6\pi, 8\pi, \ldots$

&

$\boxed{\cos 2x = 1} \Rightarrow \cancel{2}x = \cancel{2}n\pi \Rightarrow x = n\pi \Rightarrow x = 0, \pi, 2\pi, 3\pi, 4\pi, \ldots$

&

$\cos 3x = 1 \Rightarrow 3x = 2n\pi \Rightarrow x = 2n\pi/3 \Rightarrow x = 0, 2\pi/3, 4\pi/3, 2\pi, 8\pi/3, \ldots$

&

$\cos 4x = 1 \Rightarrow 4x = 2n\pi, \ x = n\pi/2 \Rightarrow x = 0, \pi/2, \pi, 3\pi/2, 2\pi, 5\pi/2, \ldots$

Question

Solve for x : $\sin x\left(\cos\frac{x}{4}-2\sin x\right)+\left(1+\sin\frac{x}{4}-2\cos x\right)\cos x=0$

Ans = $2\pi, 10\pi, 18\pi, 26\pi, 34\pi, \ldots$

$a=2\pi,\ d=8\pi \Rightarrow T_n = a+(n-1)d$

$= 2\pi+(n-1)8\pi$

$= 8n\pi-6\pi$

$\sin x\cos x/4 - 2\sin^2 x + \cos x + \cos x \sin x/4 - 2\cos^2 x = 0$

$\sin\left(x+x/4\right)+\cos x = 2\sin^2 x + 2\cos^2 x$

$\sin\left(\frac{5x}{4}\right)+\cos x = 2$

$\sin\left(\frac{5x}{4}\right)=1 \Rightarrow \frac{5x}{\cancel{4}_2} = (4n+1)\pi/\cancel{2}$

$\Rightarrow x = (4n+1)2\pi/5 = \frac{(4n+1)2\pi}{5}$

&

$\cos x = 1 \Rightarrow x = 2n\pi$

$x = 0, 2\pi, 4\pi, 6\pi, \ldots$

Intersection

$n=1 \Rightarrow x = 2\pi$

$n=6 \Rightarrow x = 10\pi$

$n=11 \Rightarrow x = 18\pi$

Final Ans:

$x = 8n\pi-6\pi$ where $n \in I$

Question

The number of solutions of the equation $\sin\left(\underbrace{\frac{\pi x}{2\sqrt{3}}}_{}\right) = x^2 - 2\sqrt{3}x + 4$ is

$x = \sqrt{3}$

(1)

$[-1, 1]$

$(x-\sqrt{3})^2 - (\sqrt{3})^2 + 4$

$(x-\sqrt{3})^2 + 1$

RHS : $[1, \infty)$

LHS = RHS = 1

$(x-\sqrt{3})^2 = 0$

$\boxed{x = \sqrt{3}}$

LHS: $\sin\left(\frac{\pi \cancel{\sqrt{3}}}{2\cancel{\sqrt{3}}}\right) = 1$

Q JEE Main 2019

[Ans. (D)]

The number of solution of the equation $1 + \sin^4 x = \underbrace{\cos^2 3x}_{[0,1]}, x \in \left[-\frac{5\pi}{2}, \frac{5\pi}{2}\right]$ is

A 7

B 4

C 3

D 5

$[-2.5\pi, 2.5\pi]$

$\sin^4 x \downarrow 0$

$\sin x = 0$ & $\cos^2 3x = 1$

$x = n\pi$ & $3x = n\pi$

$x = n\pi/3$

$x = 0, \pi, 2\pi, -\pi, \ldots$

Intersection

$x = 0, \pi, 2\pi, -\pi, -2\pi$

Brain Teaser

If $\boxed{\dfrac{\sin^4\theta}{a}+\dfrac{\cos^4\theta}{b}=\dfrac{1}{a+b}}$, then show that $\boxed{\dfrac{\sin^8\theta}{a^3}+\dfrac{\cos^8\theta}{b^3}=\dfrac{1}{(a+b)^3}}$

LHS :

$$\frac{(a+b)\sin^4\theta}{a}+\frac{(a+b)\cos^4\theta}{b}=1$$

$$\left(1+b/a\right)\sin^4\theta+\left(a/b+1\right)\cos^4\theta=1$$

$$\underbrace{\sin^4\theta+\cos^4\theta}+\frac{b}{a}\sin^4\theta+\frac{a}{b}\cos^4\theta=1$$

$$\cancel{1}-2\sin^2\theta\cos^2\theta+\frac{b\sin^4\theta}{a}+\frac{a\cos^4\theta}{b}=\cancel{1}$$

$$\frac{b^2\sin^4\theta+a^2\cos^4\theta-2ab\sin^2\theta\cos^2\theta}{ab}=0$$

$$\sin^4\theta+\cos^4\theta=1-2\sin^2\theta\cos^2\theta$$

$$b^2\sin^4\theta+a^2\cos^4\theta-2ab\sin^2\theta\cos^2\theta=0$$

$$(b\sin^2\theta-a\cos^2\theta)^2=0$$

$$\Rightarrow b\sin^2\theta-a\cos^2\theta=0$$

$$\Rightarrow b\sin^2\theta=a\cos^2\theta.$$

$$\Rightarrow \frac{\sin^2\theta}{a}=\frac{\cos^2\theta}{b}=k$$

$\sin^2\theta=ak$ & $\cos^2\theta=bk$

add

$$1=ak+bk$$

$$1=k(a+b)$$

$$k=\frac{1}{a+b}$$

$$\boxed{\sin^2\theta = \frac{a}{a+b}} \Rightarrow \sin^8\theta = \frac{a^4}{(a+b)^4}$$

$$\cos^2\theta = \frac{b}{a+b} \Rightarrow \cos^8\theta = \frac{b^4}{(a+b)^4}$$

LHS: $$\frac{\sin^8\theta}{a^3} + \frac{\cos^8\theta}{b^3}$$

$$= \frac{a^{\cancel{4}}}{(a+b)^4\,\cancel{a^3}} + \frac{b^{\cancel{4}}}{(a+b)^4\,\cancel{b^3}}$$

$$\frac{a}{(a+b)^4} + \frac{b}{(a+b)^4}$$

$$\frac{\cancel{(a+b)}^1}{(a+b)^4} = \frac{1}{(a+b)^3} = \text{RHS}$$

HW-BT-23

The value of the expression $\dfrac{2(\sin 1^\circ+\sin 2^\circ+\sin 3^\circ+\ldots\ldots+\sin 89^\circ)}{\boxed{2(\cos 1^\circ+\cos 2^\circ+\ldots\ldots+\cos 44^\circ)+1}}$ equals

A $\sqrt{2}$ ✓

B $\dfrac{1}{\sqrt{2}}$

C $\dfrac{1}{2}$

D 1

$$N^r = 2\left\{(\underbrace{\sin 1 + \sin 89}) + (\underbrace{\sin 2 + \sin 88}) + \ldots + (\underbrace{\sin 44 + \sin 46}) + \sin 45\right\}$$

$$= 2\left[2\underbrace{\sin}45\cos 44 + 2\underbrace{\sin}45\cos 43 + \ldots + 2\underbrace{\sin}45\cos 1^\circ + \underbrace{\sin}45\right]$$

$$N^r = 2\sin 45^\circ\left[\underbrace{2\cos 44 + 2\cos 43 + 2\cos 42 + \ldots + 2\cos 1^\circ} + 1\right]$$

$$= 2\sin 45\,\boxed{\left[2\{\cos 1^\circ + \cos 2^\circ + \cos 3^\circ + \ldots + \cos 44\} + 1\right]}$$

$$\text{Ans} = 2\sin 45^\circ = 2\times\frac{1}{\sqrt{2}} = \sqrt{2}$$

HW-BT-24

The value of $2 \tan 18° + 3 \sec 18° - 4 \cos 18°$ is

A Zero

B $\sqrt{5}$

C $-\sqrt{5}$

D $\sqrt{3}$

$$\frac{2\sin 18°}{\cos 18°} + \left(\frac{3}{\cos 18°} - 4\cos 18°\right)$$

$$+ \left(\frac{3 - 4\cos^2 18°}{\cos 18°}\right)$$

$$+ \left(\frac{3\cos 18 - 4\cos^3 18°}{\cos 18 \cdot \cos 18}\right)$$

$$- \frac{4\cos^3 18° - 3\cos 18°}{\cos^2 18°}$$

$$\frac{2\sin 18}{\cos 18°} - \frac{\cos 54°}{\cos^2 18°}$$

$$\frac{2\sin 18° \cos 18 - \cos 54°}{\cos^2 18°}$$

$$\frac{\cancel{\sin 36} - \cancel{\cos 54°}}{\cos^2 18°}$$

$$= \underline{\underline{0}}$$

$\tan A + \tan B + \tan C = \tan A \tan B \tan C$

HW-BT-25

If $\boxed{x + y + z = xyz}$

Prove that $\frac{x}{1-x^2} + \frac{y}{1-y^2} + \frac{z}{1-z^2} = \frac{4xyz}{(1-x^2)(1-y^2)(1-z^2)}$

Let $x = \tan A$, $y = \tan B$, $z = \tan C$

LHS: $\frac{x}{1-x^2} = \frac{2\tan A}{2(1-\tan^2 A)} = \frac{\tan 2A}{2}$

LHS: $\frac{\tan 2A + \tan 2B + \tan 2C}{2}$

RHS: $\frac{8 \tan A \tan B \tan C}{2(1-\tan^2 A)(1-\tan^2 B)(1-\tan^2 C)} = \frac{2\tan A \cdot 2\tan B \cdot 2\tan C}{2(1-\tan^2 A)(1-\tan^2 B)(1-\tan^2 C)}$

$= \frac{\tan 2A \tan 2B \tan 2C}{2}$

OR to Prove:

$\tan 2A + \tan 2B + \tan 2C = \tan 2A \tan 2B \tan 2C$

$A + B + C = 180°$

$2A + 2B + 2C = 360°$

$2A + 2B = 360 - 2C$

tan both sides

$\tan(2A + 2B) = \tan(360° - 2C)$

$\frac{\tan 2A + \tan 2B}{1 - \tan 2A \tan 2B} = -\tan 2C$

HW-BT-27

If $\frac{9x}{\cos\theta} + \frac{5y}{\sin\theta} = 56$ and $\frac{9x\sin\theta}{\cos^2\theta} - \frac{5y\cos\theta}{\sin^2\theta} = 0$, then find the value of $\left[(9x)^{\frac{2}{3}} + (5y)^{\frac{2}{3}}\right]^3$

$$\frac{9x\sin\theta}{\cos^2\theta} = \frac{5y\cos\theta}{\sin^2\theta}$$

$$\frac{9x}{\cos^3\theta} = \frac{5y}{\sin^3\theta} = K$$

$$\left\{\begin{array}{l} 9x = K\cos^3\theta \\ 5y = K\sin^3\theta \end{array}\right\}$$

$$\Rightarrow \frac{K\cos^{\cancel{3}}\theta}{\cancel{\cos\theta}} + \frac{K\sin^{\cancel{3}}\theta}{\cancel{\sin\theta}} = 56$$

$$K[1] = 56$$

$$\boxed{K = 56}$$

$$9x = 56\cos^3\theta$$

$$5y = 56\sin^3\theta$$

$$(9x)^{2/3} = (56)^{2/3}(\cos\theta)^2 \sim (1)$$

$$(5y)^{2/3} = (56)^{2/3}(\sin\theta)^2 \sim (2)$$

Add

$$(9x)^{2/3} + (5y)^{2/3} = (56)^{2/3}[1]$$

$$\left[(9x)^{2/3} + (5y)^{2/3}\right]^3 = (56)^{2/\cancel{3} \times \cancel{3}}$$

$$\text{Ans} = (56)^2$$

Continued Product of Cosine Series

$$\cos A \cos 2A \cos 4A \cos 8A \ldots\ldots\ldots \cos 2^{n-1} A = \frac{1}{2^n \sin A} \sin(2^n A)$$

K

$2 \sin A \cos A = \cancel{\sin 2A}$

$2 \cancel{\sin 2A} \cos 2A = \cancel{\sin 4A}$

$2 \cancel{\sin 4A} \cos 4A = \cancel{\sin 8A}$

$\vdots$

$2 \cancel{\sin(2^{n-1} A)} \cos(2^{n-1} A) = \sin 2^n A$

Multiply

$2^n \cdot \sin A \, (K) = \sin(2^n A)$

$K = \dfrac{\sin(2^n A)}{2^n \sin A} = \underline{\text{RHS}}$

Brain Teaser

Find the value: $\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{3\pi}{7}$? $\rightarrow -\cos 4\pi/7$.

$$\cos\frac{3\pi}{7} = \cos\left(\pi - 4\pi/7\right)$$
$$= -\cos 4\pi/7.$$

$$-\cos \pi/7 \cos 2\pi/7 \cos 4\pi/7$$

$$A = \pi/7, \quad n = 3$$

$$\sin 8\pi/7 = \sin(\pi + \pi/7)$$
$$= -\sin\pi/7.$$

$$= -\frac{\sin 2^3\pi/7}{2^3 \cdot \sin\pi/7} = -\frac{\sin 8\pi/7}{8\sin\pi/7} = \frac{\cancel{\sin\pi/7}}{8\cancel{\sin\pi/7}} = \frac{1}{8}$$

Brain Teaser

Prove that:

$$\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{3\pi}{15}\cos\frac{4\pi}{15}\cos\frac{5\pi}{15}\cos\frac{6\pi}{15}\cos\frac{7\pi}{15}=\frac{1}{128}$$

$\cos 7\pi/15 = -\cos\frac{8\pi}{15}$

$\sin\frac{16\pi}{15} = \sin(\pi + \pi/15)$

$= -\sin\pi/15$

$-\cos 8\pi/15.$

$\cos 60 = \frac{1}{2}$

LHS. $-\underbrace{\left(\cos \pi/15 \cos\frac{2\pi}{15}\cos 4\pi/15 \cos 8\pi/15\right)}\left(\cos\frac{3\pi}{15}\cos 6\pi/15\right)\left(\cos\frac{\cancel{5}\pi}{\cancel{15}_3}\right)$

$$-\frac{\sin 2^4\pi/15}{2^4\sin\pi/15}\times\frac{\sin(2^2\cdot 3\pi/15)}{2^2\sin(3\pi/15)}\times\frac{1}{2}$$

$$=\frac{-\cancel{\sin 16\pi/15}}{2^4\cancel{\sin\pi/15}}\times\frac{\cancel{\sin 12\pi/15}}{2^2\cancel{\sin 3\pi/15}}\times\frac{1}{2}=\frac{1}{2^4\cdot 2^2\cdot 2^1}=\frac{1}{2^7}=\frac{1}{128}$$

Brain Teaser

(1) $\boxed{\cos\frac{2\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{6\pi}{7}}$

M-2 $\frac{2\sin\frac{2\pi}{7}\cdot\left[\cos 2\pi/7+\cos\frac{4\pi}{7}+\cos\frac{6\pi}{7}\right]}{2\sin 2\pi/7}$ $6\pi/7+8\pi/7$

$\cos 6\pi/7=\cos\left(2\pi-\frac{8\pi}{7}\right)$

$\boxed{\cos\frac{6\pi}{7}=\cos 8\pi/7}$

(2) $\boxed{\cos\frac{2\pi}{7}\cos\frac{4\pi}{7}\cos\frac{6\pi}{7}}$ → $\cos 8\pi/7$

$=\cos 2\pi/7\cos 4\pi/7\cos 8\pi/7=\frac{\sin 2^3\cdot 2\pi/7}{2^3\sin 2\pi/7}$

$=\frac{\sin 16\pi/7}{8\sin 2\pi/7}$

$\frac{\sin\left(\cancel{2\pi}+2\pi/7\right)}{8\,\cancel{\sin 2\pi/7}}=\frac{1}{8}$

(3) $\cos\frac{2\pi}{7}\cos\frac{4\pi}{7}+\cos\frac{4\pi}{7}\cos\frac{6\pi}{7}+\cos\frac{6\pi}{7}\cos\frac{2\pi}{7}$

$\frac{1}{2}\left[2\cos 2\pi/7\cos 4\pi/7+2\cos 4\pi/7\cos 6\pi/7+2\cos 6\pi/7\cos 2\pi/7\right]$

$\frac{1}{2}\left[\cos 6\pi/7+\cos 2\pi/7+\underbrace{\cos 10\pi/7}_{\cos 4\pi/7}+\cos 2\pi/7+\underbrace{\cos 8\pi/7}_{\cos 6\pi/7}+\cos 4\pi/7\right]$

$\frac{1}{\cancel{2}}\left[\cancel{2}\cos 2\pi/7+\cancel{2}\cos 4\pi/7+\cancel{2}\cos 6\pi/7\right]=-\frac{1}{2}$

(1) $\underset{=}{M-2}$ $\dfrac{1}{2\sin\frac{2\pi}{7}}\left[2\sin 2\pi/7\cos 2\pi/7 + 2\sin 2\pi/7 \cos 4\pi/7 + 2\sin 2\pi/7 \cos 6\pi/7\right]$

$\dfrac{1}{2\sin\frac{2\pi}{7}}\left[\sin \cancel{4\pi/7} + \sin \cancel{6\pi/7} + \sin(-2\pi/7) + \sin \cancel{8\pi/7} + \sin\cancel{\left(\frac{-4\pi}{7}\right)}\right]$

$\dfrac{\sin(-2\pi/7)}{2\sin 2\pi/7} = -\dfrac{\sin \cancel{2\pi/7}}{2\sin\cancel{2\pi/7}} = -\dfrac{1}{2}$ ✓

M-2

2) $\dfrac{2\sin\frac{2\pi}{7}\cos 2\pi/7 \cos 4\pi/7 \cos 8\pi/7}{2\sin 2\pi/7}$

$= \dfrac{2\sin 4\pi/7 \cos 4\pi/7 \cos 8\pi/7}{2\times 2\sin 2\pi/7} = \dfrac{2\sin 8\pi/7 \cos 8\pi/7}{2\times 4\sin 2\pi/7} = \dfrac{\sin \cancel{16\pi/7}}{8\sin\cancel{2\pi/7}}$

$= \dfrac{1}{8}$

Brain Teaser

Method → 3

Let $\cos 2\pi/7 = \alpha$, $\cos 4\pi/7 = \beta$, $\cos 6\pi/7 = \gamma$

(1) $\boxed{\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7}} = \alpha + \beta + \gamma$

(2) $\boxed{\cos\frac{2\pi}{7}\cos\frac{4\pi}{7}\cos\frac{6\pi}{7}} = \alpha\beta\gamma$

(3) $\cos\frac{2\pi}{7}\cos\frac{4\pi}{7} + \cos\frac{4\pi}{7}\cos\frac{6\pi}{7} + \cos\frac{6\pi}{7}\cos\frac{2\pi}{7}$

$\alpha\beta + \beta\gamma + \gamma\alpha =$

$\theta = 2\pi/7,\ 4\pi/7,\ 6\pi/7$

$7\theta = 2\pi$ or 4π or 6π

$4\theta + 3\theta = 2\pi,\ 4\pi,\ 6\pi$

$3\theta = 2\pi - 4\theta,\ 4\pi - 4\theta,\ 6\pi - 4\theta$

Cos both sides

$\cos 3\theta = \cos 4\theta$

3° $2\cos^2 2\theta - 1$

$2(2\cos^2\theta - 1)^2 - 1$

4° eq^n is $\cos\theta$

WRONG APPROACH

Sin both sides

$\sin 3\theta = -\sin 4\theta$

$3\sin\theta - 4\sin^3\theta = -2\sin 2\theta \cos 2\theta$

$\cancel{\sin\theta}[3 - 4\sin^2\theta] = -4\cancel{\sin\theta}\cos\theta\,[2\cos^2\theta - 1]$

$(3 - 4(1 - \cos^2\theta)) = -4\cos\theta\,[2\cos^2\theta - 1]$

Brain Teaser

[Ans. $\sqrt{7/8}$]

Find $\sin\left(\frac{\pi}{7}\right)\sin\left(\frac{2\pi}{7}\right)\sin\left(\frac{3\pi}{7}\right) = x$

$$\boxed{\sin^2\theta = 1-\cos 2\theta}$$

$$\underbrace{\sin^2 \pi/7}\ \sin^2 2\pi/7\ \sin^2 3\pi/7 = x^2$$

$$(1-\cos \underbrace{2\pi/7})(1-\cos \underbrace{4\pi/7})(1-\cos \underbrace{6\pi/7}) = x^2$$

$$(1-\alpha)(1-\beta)(1-\gamma) = x^2$$

$$(1-\alpha-\beta+\alpha\beta)(1-\gamma) = x^2$$

$$1-\alpha-\beta+\alpha\beta-\gamma+\alpha\gamma+\beta\gamma-\alpha\beta\gamma = x^2$$

$$1-(\underbrace{\alpha+\beta+\gamma})+(\underbrace{\alpha\beta+\beta\gamma+\gamma\alpha})-\alpha\beta\gamma = x^2$$

$$1-\left(-\tfrac{1}{2}\right)-\tfrac{1}{2}-\tfrac{1}{8} = x^2$$

$$1+\cancel{\tfrac{1}{2}}-\cancel{\tfrac{1}{2}}-\tfrac{1}{8} = x^2$$

$$1-\tfrac{1}{8} = x^2$$

$$\tfrac{7}{8} = x^2$$

$$x = +\sqrt{\tfrac{7}{8}}$$

PYQ-1

If $0 \leq x < \frac{\pi}{2}$, then the number of values of x for which $\boxed{\sin x - \sin 2x + \sin 3x = 0}$, is

[JEE Main-2019]

[Ans. A]

A 2

B 1

C 3

D 4

$2\sin 2x \cos x - \sin 2x = 0$

$\sin 2x(2\cos x - 1) = 0$

$\sin 2x = 0$ or $\cos x = 1/2$.

$2x = n\pi$

$x = n\pi/2$ $\qquad x = \pi/3$.

$x = 0, \pi/2$

Reject.

PYQ-2

Let S be the set of all $\alpha \in R$ such that the equation, $\cos 2x + \alpha \sin x = 2\alpha - 7$ has a solution. Then S equal to-

[JEE Mains-2019]

A [3,7]

B [1,4]

C [2,6]

D R

[Ans. C]

$1 - 2\sin^2 x + \alpha \sin x = 2\alpha - 7.$

$\sin x = t$

$-2t^2 + \alpha t = 2\alpha - 8$

$2t^2 - \alpha t + 2\alpha - 8 = 0.$

$D = \alpha^2 - 4 \times 2(2\alpha - 8).$

$D = \alpha^2 - 16\alpha + 64 = (\alpha - 8)^2$

$\dfrac{\alpha \pm (\alpha - 8)}{2 \times 2.}$

$\dfrac{2\alpha - 8}{4}, \quad 8/4$

$\dfrac{\alpha - 4}{2}, \quad 2$

$\sin x = \dfrac{\alpha - 4}{2}$

$-1 \leq \dfrac{\alpha - 4}{2} \leq 1$

$-2 \leq \alpha - 4 \leq 2$

$2 \leq \alpha \leq 6.$

PYQ-3

The number of distinct solutions of the equation, $\log_{1/2}|\sin x| = 2 - \log_{1/2}|\cos x|$ in the interval $[0, 2\pi]$, is

[JEE Main-2020]

[Ans. 8]

$\log_{\frac{1}{2}}(|\sin x||\cos x|) = 2$

$|\sin x||\cos x| = \frac{1}{4}$

$|2\sin x \cos x| = \frac{1}{2}$

$|\sin 2x| = \frac{1}{2}$.

$\sin 2x = \pm\frac{1}{2}$.

$\boxed{8 \text{ sol}^n}$

$2x \in [0, 4\pi]$

PYQ-4

If for $x \in \left(0, \frac{\pi}{2}\right)$, $\log_{10} \sin x + \log_{10} \cos x = -1$ and $\log_{10}(\sin x + \cos x) = \frac{1}{2}(\log_{10} n - 1), n > 0$ then the value of n is equal to:

[JEE Mains March 2021]

A 20

B 12

C 9

D 16

$\log_{10}(\sin x \cos x) = -1$

$2 \sin x \cos x = \frac{1}{5}$.

$\sin x \cos x = \frac{1}{10}$.

$(\sin x + \cos x)^2 = 1 + 2 \sin x \cos x$

$= 1 + 2/10 = 1 + \frac{1}{5}$

$(\sin x + \cos x)^2 = 6/5$

$\sin x + \cos x = +\sqrt{6/5}$

$\log(\sin x + \cos x)^2 = \log n - \log$

$\log(6/5) = \log(n/10)$

$6/5 = \frac{n}{10}$

$\Rightarrow \boxed{n = 12}$.

PYQ-5

All the pairs (x, y) that satisfy the inequality, $2^{\sqrt{\sin^2 x - 2\sin x + 5}} \cdot \frac{1}{4^{\sin^2 y}} \leq 1$ also satisfy the equation.

[JEE Mains 2019]

[Ans. B]

A $\sin x = 2\sin y$

B $\sin x = |\sin y|$

C $2|\sin x| = 3\sin y$

D $2\sin x = \sin y$

$$2^{\sqrt{(\sin x - 1)^2 + 4}} \leq 2^{2\sin^2 y}.$$

$$\sqrt{(\sin x - 1)^2 + 4} \leq 2\sin^2 y.$$

LHS: $[2, 2\sqrt{2}]$ RHS: $[0, 2]$

LHS = RHS = 2

$\sin x = 1$ & $\sin^2 y = 1$

$\sin y = \pm 1$

TYQ-7

The number of solutions of the equation

$\frac{2}{2}\cos\left(x+\frac{\pi}{3}\right)\cos\left(\frac{\pi}{3}-x\right)=\frac{1}{4}\cos^2 2x, x\in[-3\pi,3\pi]$ is:

[JEE Main-2022 (24 June, Shift-II)]

[Ans. D]

A 8

B 5

C 6

D 7

$\frac{1}{\cancel{2}}\left[\cos\frac{2\pi}{3}+\cos 2x\right]=\frac{1}{4}\cos^2 2x.$

$\left[-\frac{1}{2}+\cos 2x\right]=\frac{1}{2}\cos^2 2x$

$\boxed{\cos 2x = t}$

$\left(-\frac{1}{2}+t\right)=\frac{t^2}{2}$

$-1+2t=t^2$

$t^2-2t+1=0$

$(t-1)^2=0 \Rightarrow t=1$

$\cos 2x = 1$

$\cancel{2}x=\cancel{2}n\pi$

$\boxed{x=n\pi}$

$0, \pi, 2\pi, 3\pi$
$-\pi, -2\pi, -3\pi$

TYQ-8

If $\sin^2(10°)\sin(20°)\sin(40°)\sin(50°)\sin(70°) = \alpha - \frac{1}{16}\sin(10°)$, then $16+\alpha^{-1}$ is equal to $\underline{16+64} = 80$ Ans. [JEE Main-2022 (26 June, Shift-I)]

[Ans. 80]

$\sin 10\left[\frac{2}{2}\sin 20° \sin 40°\right]\left(\sin 10° \sin 50 \sin 70°\right)$

$\sin 10° \frac{\left[\cos 20 - \cos 60°\right]}{2} \times \frac{1}{4} \times \frac{1}{2}$

$\frac{1}{16}\sin 10°\left[\cos 20 - \frac{1}{2}\right]$

$\frac{1}{16}\left[\frac{2\sin 10 \cos 20}{2} - \frac{\sin 10}{2}\right]$

$\frac{1}{16}\left[\frac{\sin 30° - \sin 10°}{2} - \frac{\sin 10}{2}\right]$

$\frac{1}{16}\left[\frac{1}{4} - \sin 10\right]$

$\left(\frac{1}{64} - \frac{1}{16}\sin 10°\right)$

$\Rightarrow \alpha = \frac{1}{64}$

TYQ-9

$T = \cos 0 + \cos 2\pi + \cos(-2\pi) + \cos 60^\circ + \cos 300^\circ$

$T = 1 + 1 + 1 + 1 = 4$

Let $S = \left\{\theta \in [-\pi, \pi] - \left\{\pm\frac{\pi}{2}\right\} : \boxed{\sin\theta\tan\theta + \tan\theta = \sin 2\theta}\right\}$.

If $T = \sum_{\boxed{\theta \in S}} \cos 2\theta$ then $T + \boxed{n(S)}$ is equal to $\rightarrow 5$

[JEE Main-2022 (24 June, Shift-I)]

[Ans. B]

A $7+\sqrt{3}$

B 9

C $8+\sqrt{3}$

D 10

$(\sin\theta + 1)\frac{\cancel{\sin\theta}}{\cos\theta} = 2\cancel{\sin\theta}\cos\theta$

$(\sin\theta + 1) = 2\cos^2\theta$

$\cancel{(\sin\theta + 1)} = 2(1-\sin\theta)\cancel{(1+\sin\theta)}$

$1 = 2(1-\sin\theta)$

$1 - \sin\theta = \frac{1}{2} \Rightarrow \sin\theta = \frac{1}{2}$

$\theta = \pi/6 = 30^\circ$

$\theta = 5\pi/6 = 150^\circ$

$\sin\theta = 0$

$\theta = n\pi$

$\boxed{\theta = 0, \pi, -\pi}$

$1 + \sin\theta = 0$

$\sin\theta = -1$

$\boxed{\theta = -\pi/2}$ $\Rightarrow$ Rejected

TYQ-10

$2\times 3\times 7\times 2.$

The number of values of 'x' in the interval $\boxed{\left(\frac{\pi}{4}, \frac{7\pi}{4}\right)}$ for which $\boxed{14\,\mathrm{cosec}^2 x - 2\sin^2 x = 21 - 4\cos^2 x}$ holds, is ________.

[JEE Main-2022 (25 June, Shift-I)]

[Ans. 04]

$\sin^2 x = t$

$\frac{14}{t} - 2t = 21 - 4(1-t)$

$\frac{14 - 2t^2}{t} = 17 + 4t$

$14 - 2t^2 = 17t + 4t^2$

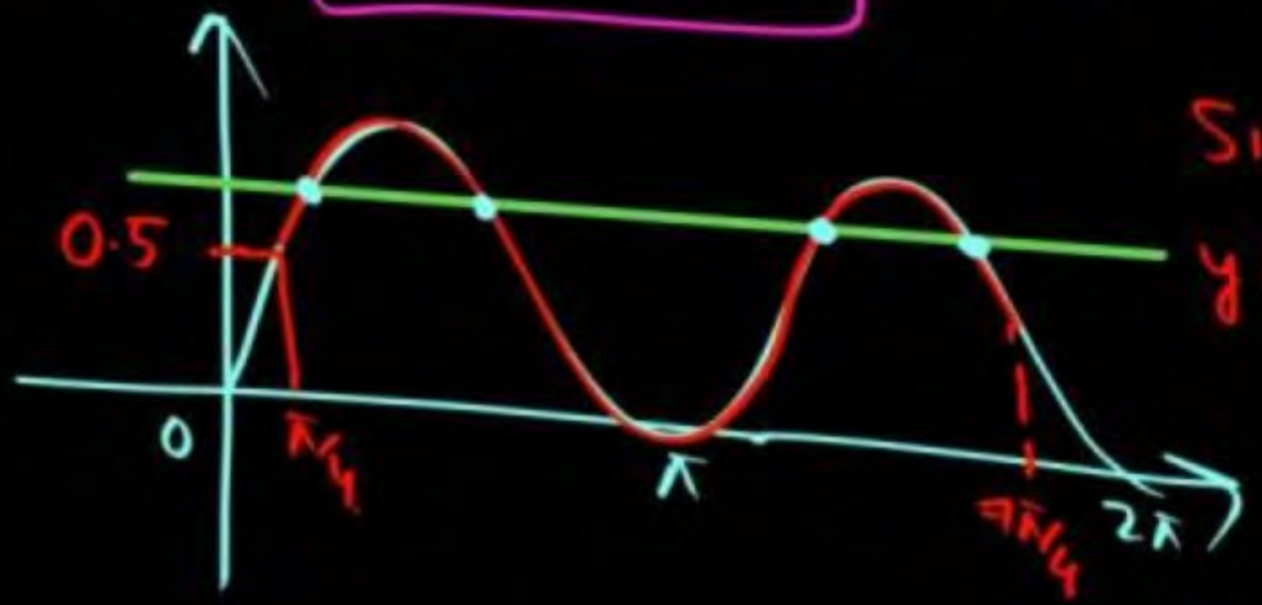
$\boxed{6t^2 + 17t - 14 = 0}$

$6t^2 + 21t - 4t - 14 = 0$

$(3t-2)(2t+7) = 0$

$t = 2/3$ or $-7/2 \rightarrow \times$

$\boxed{\sin^2 x = 2/3} = 0.67$



$\sin^2 \pi/4 = \frac{1}{2} = 0.5$

TYQ-11

The number of elements in the set

$S = \{\theta \in [\underbrace{-4\pi, 4\pi}] \boxed{3\cos^2 2\theta + 6\cos 2\theta - 10\cos^2\theta + 5 = 0}\}$ is ______.

[JEE Main-2022 (29 June, Shift-I)]

[Ans. 32]

$2\theta \in [-8\pi, 8\pi]$.

$-5(\underbrace{2\cos^2\theta - 1}_{\cos 2\theta})$

$\boxed{3\cos^2 2\theta + 6\cos 2\theta - 5\cos 2\theta = 0}$

$2\theta = x$.

$\cos 2\theta = t$.

$3t^2 + t = 0$

$t(3t+1) = 0$.

$t = 0$ or $t = -\frac{1}{3}$.

$\cos 2\theta = 0$ or $\cos 2\theta = -\frac{1}{3}$

$\cos x = 0 \Rightarrow x = \pi/2, 3\pi/2$

$\Rightarrow 8 \times 2 = 16$.

$\cos x = -\frac{1}{3}. \Rightarrow 2 \times 8 = 16$

Total no. of sol = 32

TYQ-12

The number of solutions of the equation $\boxed{2\theta - \cos^2\theta + \sqrt{2} = 0}$ is R is equal to _____. $\underline{Ans = 1}$.

[JEE Main-2022 (29 June, Shift-I)]

[Ans. 01]

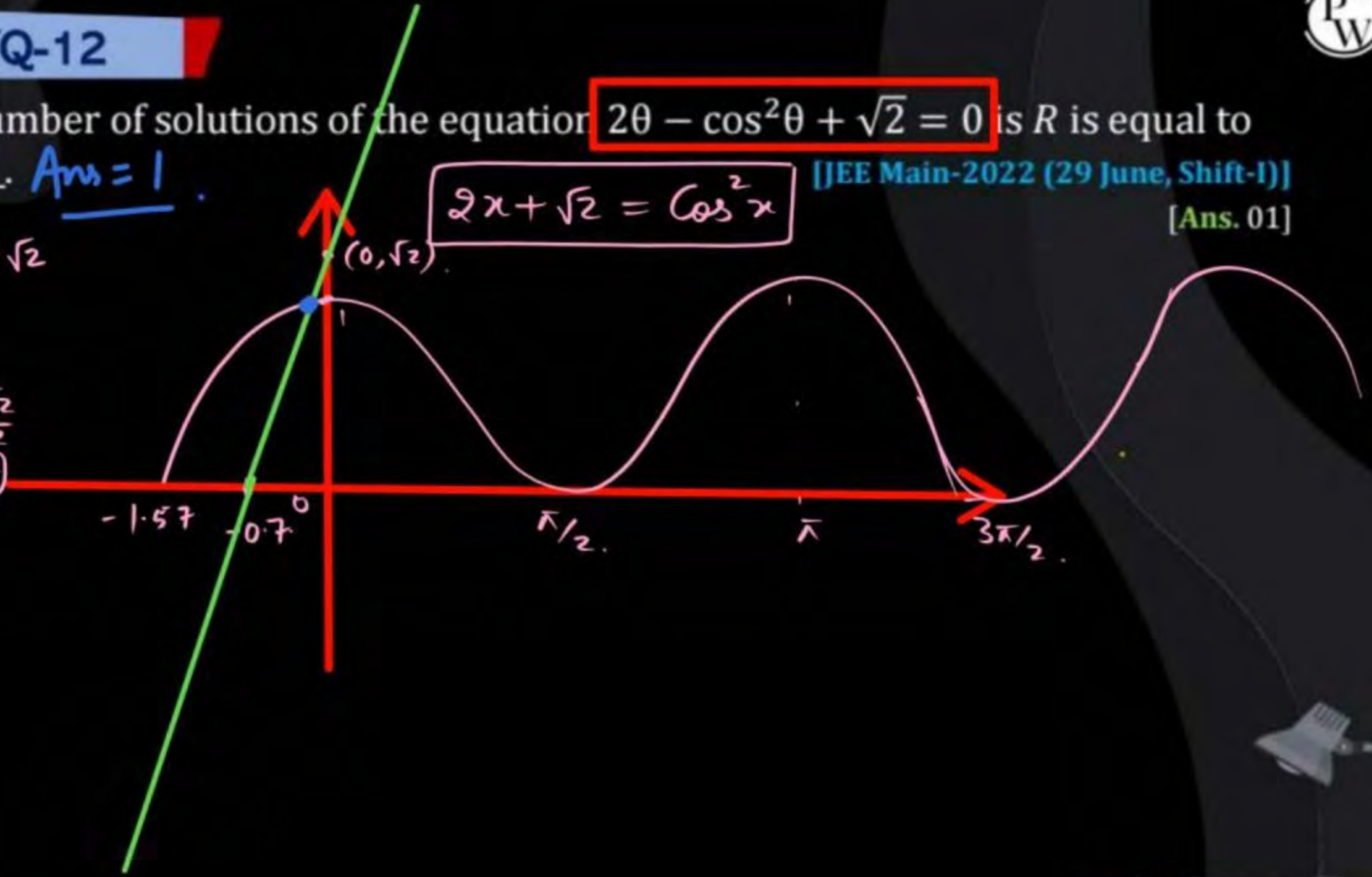
$$\boxed{2x + \sqrt{2} = \cos^2 x}$$

$y = 2x + \sqrt{2}$

$y = 0$

$\Rightarrow x = -\frac{\sqrt{2}}{2}$

$\boxed{x = -0.7}$



TYQ-13

The number of solutions of the equation $\boxed{\sin x = \cos^2 x}$ in the interval $(0, 10)$ is ______.

[JEE Main-2022 (29 June, Shift-II)]

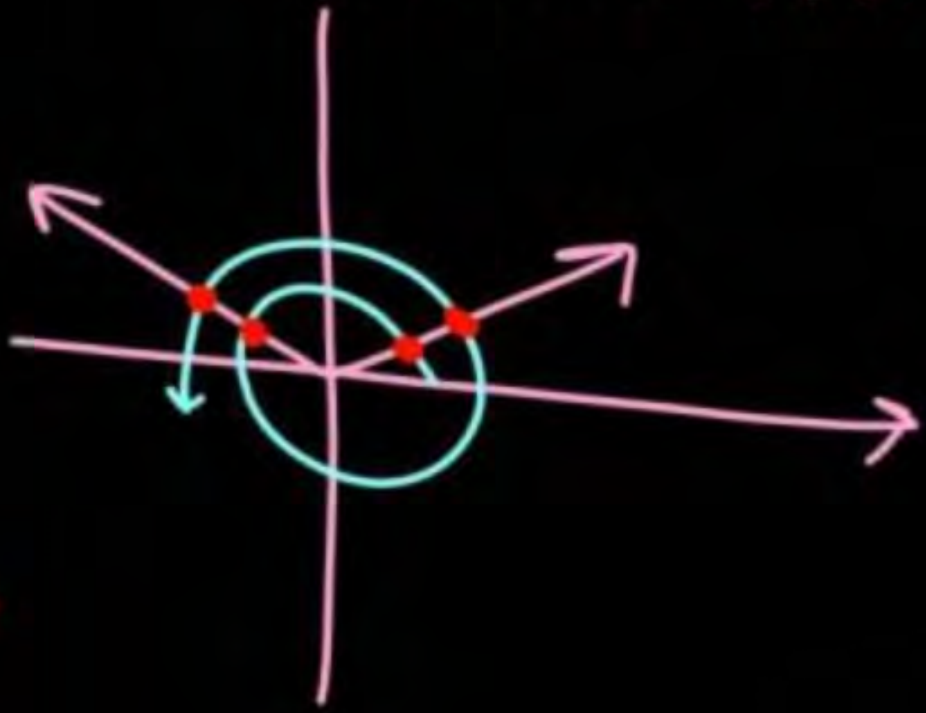
$(0, 3\pi)$

[Ans. 04]

Ans. 4 soln.

$\boxed{\sin x = \frac{\sqrt{5}-1}{2}}$

$\sin x = +ve.$



$\sin x = 1 - \sin^2 x$

$\sin x = t$

$t^2 + t - 1 = 0$

$t = \frac{-1 \pm \sqrt{1+4}}{2\times 1}$

$t = \frac{-1 \pm \sqrt{5}}{2}$

$\sin x = \frac{-1+\sqrt{5}}{2}$

TYQ-JEE Advanced

TYQ-14

$x = 2n\pi$ or $2n\pi + \pi/2$

$x = 0, \pi/2$

Consider the following lists:

	List-I		List-II
$[-120, 120]$ (I)	$\left\{x \in \left[-\frac{2\pi}{3}, \frac{2\pi}{3}\right] : \boxed{\cos x + \sin x = 1}\right\}$	(P)	has two elements
$[-50, 50]$ (II)	$\left\{x \in \left[-\frac{5\pi}{18}, \frac{5\pi}{18}\right] : \sqrt{3}\tan 3x = 1\right\}$	(Q)	has three elements
(III)	$\left\{x \in \left[-\frac{6\pi}{5}, \frac{6\pi}{5}\right] : 2\cos(2x) = \sqrt{3}\right\}$	(R)	has four elements
$[-315, 315]$ (IV)	$\left\{x \in \left[-\frac{7\pi}{4}, \frac{7\pi}{4}\right] : \sin x - \cos x = 1\right\}$	(S)	has five elements
		(T)	has six elements

[JEE ADV.-2022]

(II) $\tan 3x = \frac{1}{\sqrt{3}}$

$\Rightarrow 3x = n\pi + \pi/6.$

$x = n\pi/3 + \pi/18 = n \times 60^\circ + 10^\circ$

$x = 10^\circ, -50^\circ$

III $2\cos 2x = \sqrt{3}$.

$\cos 2x = \sqrt{3}/2$

$2x = 2n\pi \pm \pi/6$.

$\left[-\frac{6\pi}{5}, \frac{6\pi}{5}\right]$ $\quad x = n\pi \pm \pi/12$

$n=0,\ x = \pm \pi/12 = \pm 15^\circ$ ✓

$[-216^\circ, 216^\circ]$ $\quad n=1 \Rightarrow x = 180^\circ \pm 15^\circ$ ✓

$n=-1 \Rightarrow -180^\circ \pm 15^\circ =$ ✓

$\frac{12\pi}{10} = 18 \times 12 = 36\ 18 = 216$

$$\begin{array}{r} 18 \\ 12 \\ \hline 36 \\ 18\ \\ \hline 216 \end{array}$$

6 sol^n

$\frac{\sin x}{\sqrt{2}} - \frac{\cos x}{\sqrt{2}} = \frac{1}{\sqrt{2}}$.

$\frac{\cos x}{\sqrt{2}} - \frac{\sin x}{\sqrt{2}} = -1/\sqrt{2}$

$\cos(x + \pi/4) = \cos 3\pi/4$

$x + \pi/4 = 2n\pi \pm 3\pi/4$.

$\oplus$

$x = 2n\pi + \pi/2$

$\ominus$

$x = 2n\pi - \pi$

$\pi/2,\ -2\pi + \pi/2,\ -\pi,\ \pi$

PYQ

The values of x in $(0, 2\pi)$ which satisfy the equation $\sin x\sqrt{8\cos^2 x} = 1$ are in increasing A.P. with common difference ?

A $\frac{\pi}{4}$

B $\frac{\pi}{8}$

C $\frac{3\pi}{8}$

D $\frac{5\pi}{8}$

Ans $x = \frac{\pi}{8}, 3\pi/8, 5\pi/8, 7\pi/8$

$13\pi/8 > \frac{12\pi}{8} \to 270$

$\sin x \, 2\sqrt{2} \sqrt{\cos^2 x} = 1$

$\boxed{\sqrt{t^2} = |t|}$

$2\sqrt{2} \sin x |\cos x| = 1$

$x \in (0, 2\pi)$

$2x \in (0, 4\pi)$

Case → 1

$\cos x \geq 0$ ✓

$2\sqrt{2} \sin x \cos x = 1$

$\sin 2x = \frac{1}{\sqrt{2}}$

$2x = \pi/4$ or $3\pi/4$ or $2\pi + \pi/4$, $2\pi + 3\pi/4$

$2x = \pi/4, 3\pi/4, 9\pi/4, 11\pi/4$

$x = \pi/8$ (I), $3\pi/8$ (I), $9\pi/8$ (III), $11\pi/8$ (III)

Case → 2

$\cos x < 0$

$-2\sqrt{2} \sin x \cos x = 1$

$\sin 2x = -\frac{1}{\sqrt{2}}$

$2x = 5\pi/4, 7\pi/4, 5\pi/4 + 2\pi, 7\pi/4 + 2\pi$

$x = 5\pi/8$ (II) ✓, $7\pi/8$ (II) ✓, $13\pi/8$ (IV), $15\pi/8$ (IV)

$4\pi/8 > \frac{3\pi}{8}$

$\pi/2 > 3\pi/8$

$3\pi/8 < \pi/2$ 90

$9\pi/8 > 8\pi/8$

$11\pi/8 < 12\pi/8 \to 270$

0, 270

Graphical Approach

Question

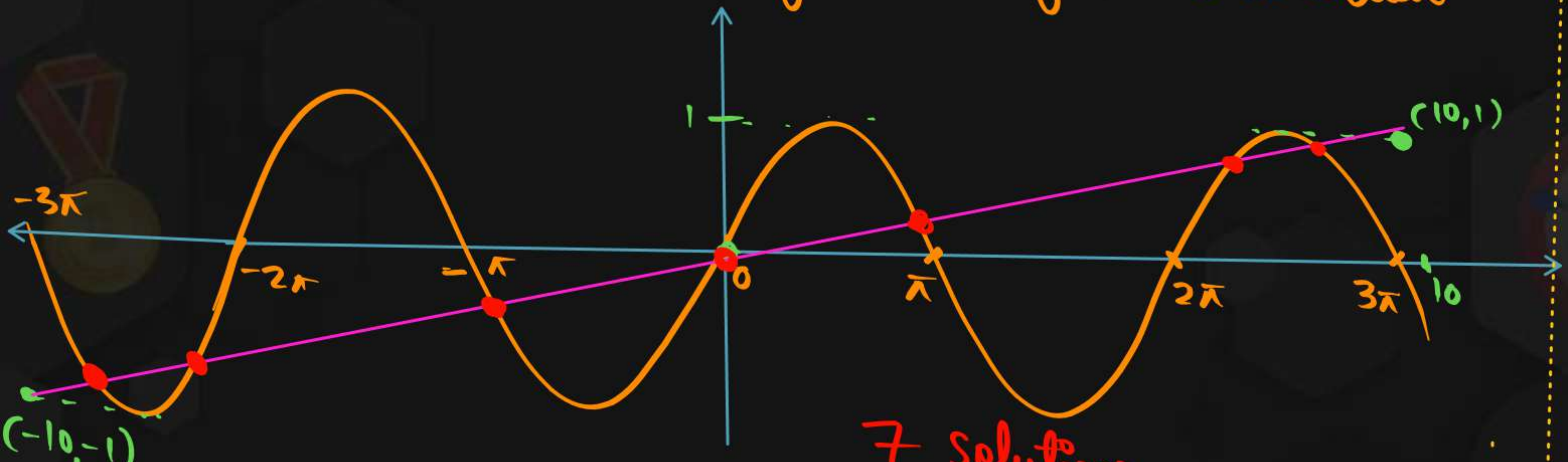
Find the number of solutions of equation $\sin x = \frac{x}{10}$

Plot the graphs of LHS & RHS & check no. of Pts of Intersection.

$$-1 \le \sin x \le 1$$
$$-1 \le \frac{x}{10} \le 1$$
$$-10 \le x \le 10$$

1) no of sol^n is asked
2) mixed fns are present in trigo eq^n.

Ex: $\tan x = x^2$



7 Solutions

$$3\pi = 3 \times 3.14 = 9.42$$

$$y = \frac{x}{10}$$

x	y
0	0
10	1
-10	-1

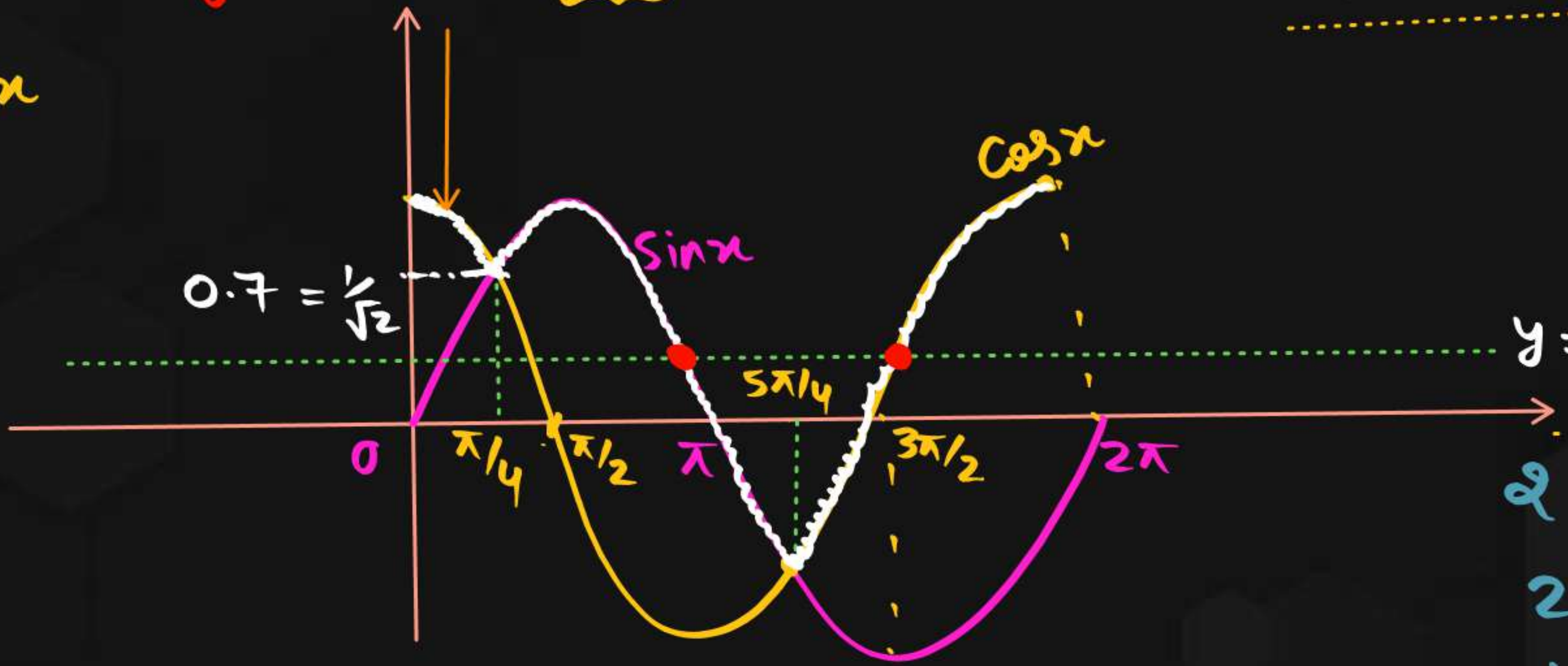
Question

Number of solutions of the equation: $\underbrace{\max\{\sin x, \cos x\}}$ $= \frac{1}{2}$ in $x \in (-2\pi, 5\pi)$, is equal to

$y = \max\{\sin x, \cos x\}$

$y = \frac{1}{2} = 0.5$

$\sin x = \cos x$
$\tan x = 1$



$\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$= \frac{1.41}{2}$

$\frac{1}{\sqrt{2}} = 0.7$

2 sol^n in $x \in [0, 2\pi)$
2 sol^n in $(2\pi, 4\pi)$
1 sol^n in $(4\pi, 5\pi)$
2 sol^n in $(-2\pi, 0)$

7 sol^n Ans

Q JEE Main March 2021

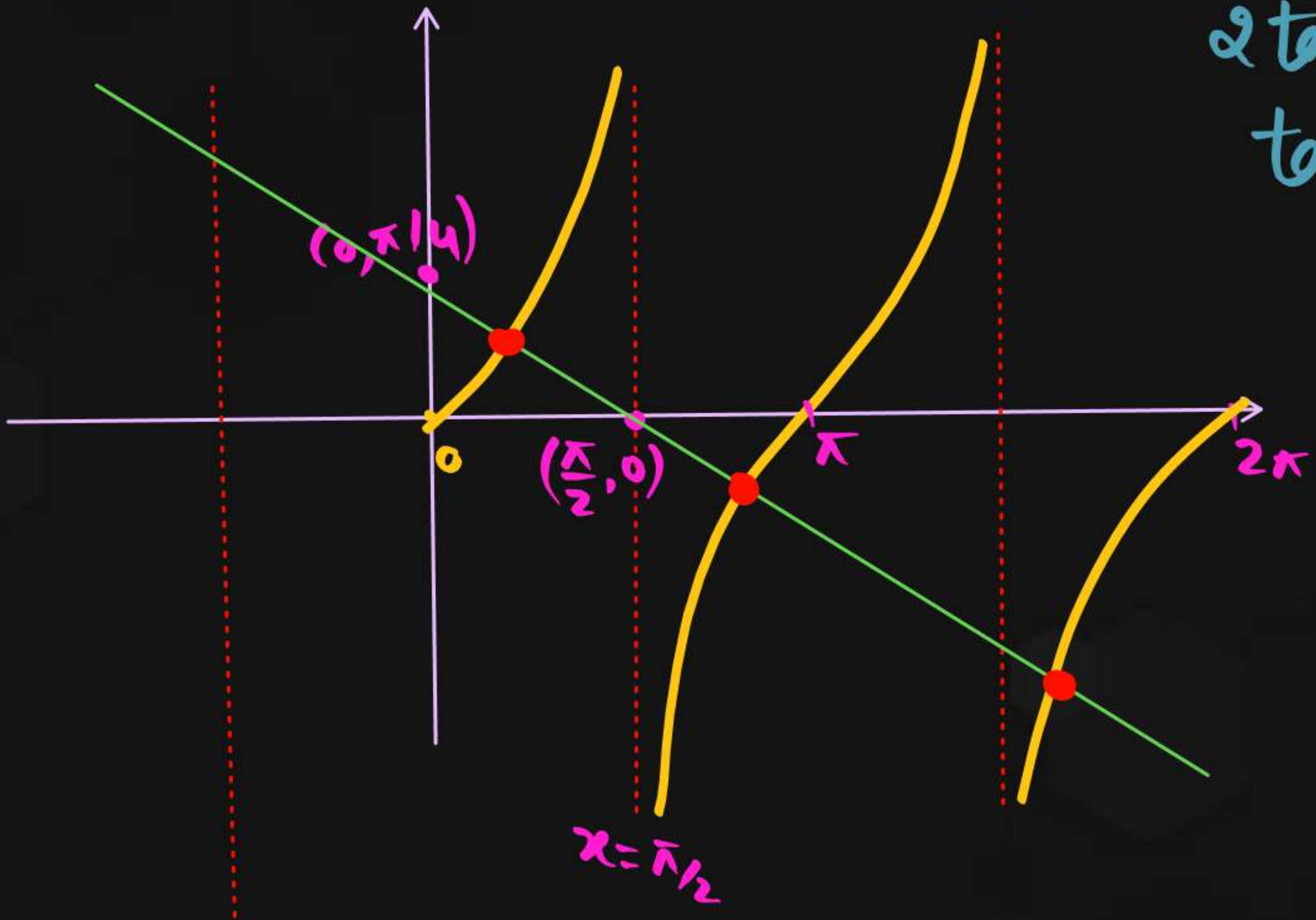
The number of solutions of the equation $x + 2\tan x = \frac{\pi}{2}$ in the interval $[0, 2\pi]$ is:

A 3 ✓

B 4

C 2

D 5



$2\tan x = \pi/2 - x$

$\tan x = \pi/4 - x/2$

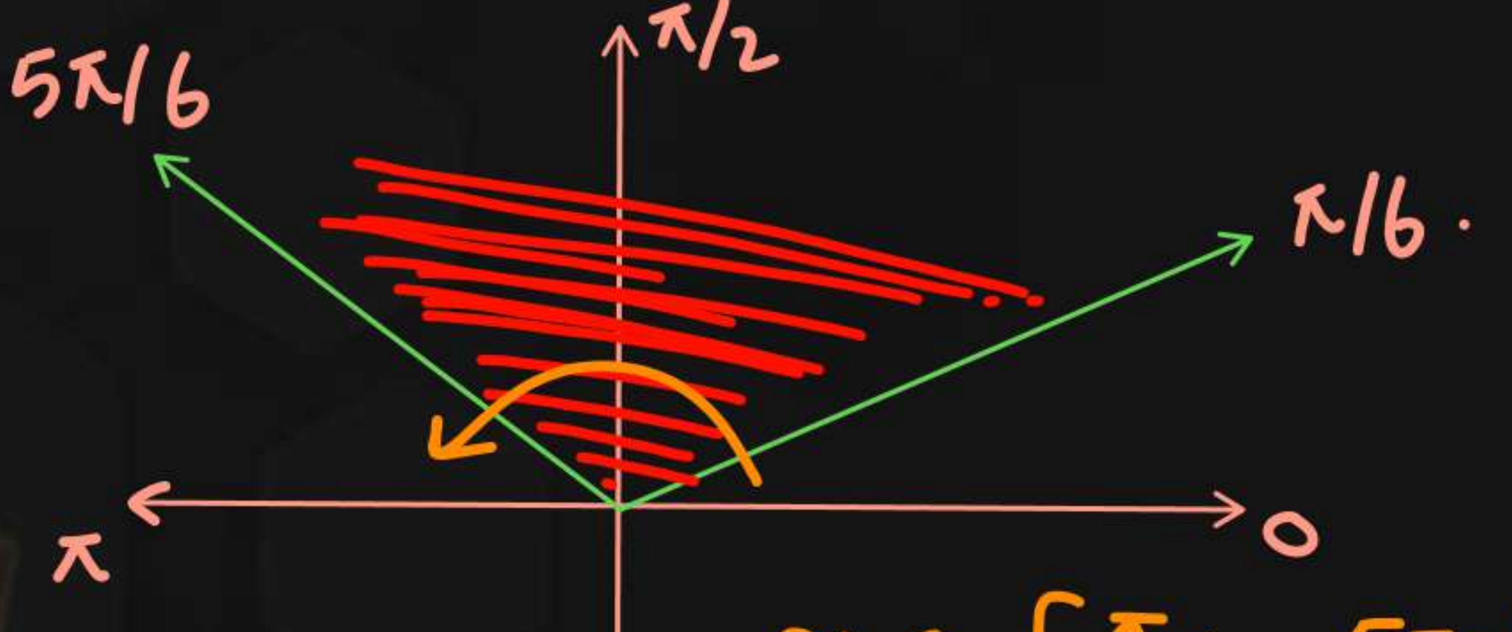
$y = \pi/4 - x/2$

x	0	$\pi/2$	
y	$\pi/4$	0	

Solving Trigonometric Inequations for sin x & cos x

(Quadrant Method).

$\sin x \geq 1/2$



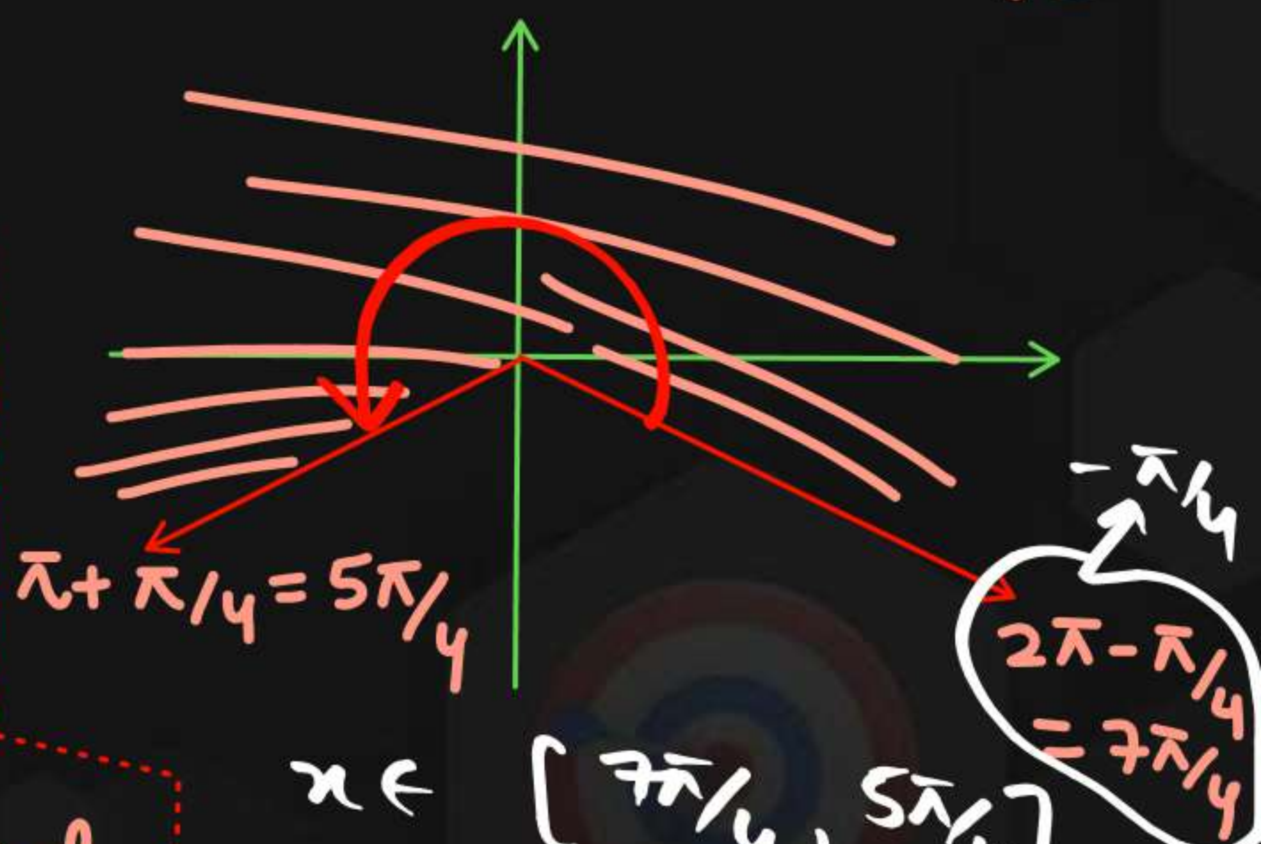
$x \in [\pi/6, 5\pi/6]$

add $2n\pi$ to get general soln.

Always write interval in anti clock wise direction

Final Ans: $x \in [2n\pi+\pi/6, 2n\pi+5\pi/6]$

② $\sin x \geq -1/\sqrt{2}$



$x \in [7\pi/4, 5\pi/4]$ WRONG

$x \in [-\pi/4, 5\pi/4]$

$x \in [2n\pi-\pi/4, 2n\pi+5\pi/4]$

1) Arrow

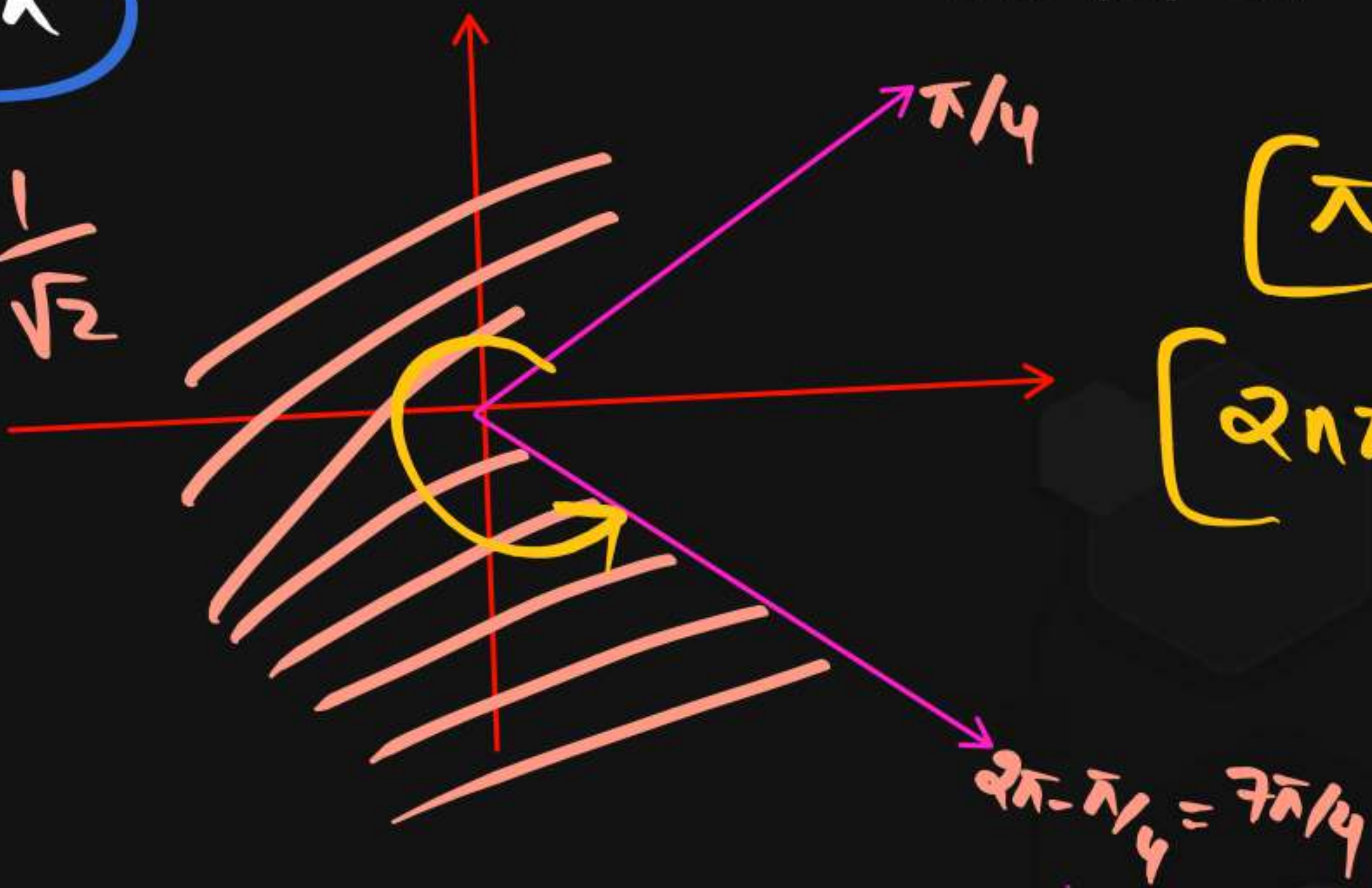
2) Shade

3) A.C.W interval (in 1 cycle)
aur agar pehla angle
bada hai
to usko −ve
bana lo

−ve < +ve

4). add $2n\pi$

Q→ $\cos x \le \frac{1}{\sqrt{2}}$



$\left[\frac{\pi}{4}, \frac{7\pi}{4}\right]$

$\left[2n\pi + \frac{\pi}{4}, 2n\pi + \frac{7\pi}{4}\right]$

$|x| < 2 \Rightarrow -2 < x < 2$

Question

Solve

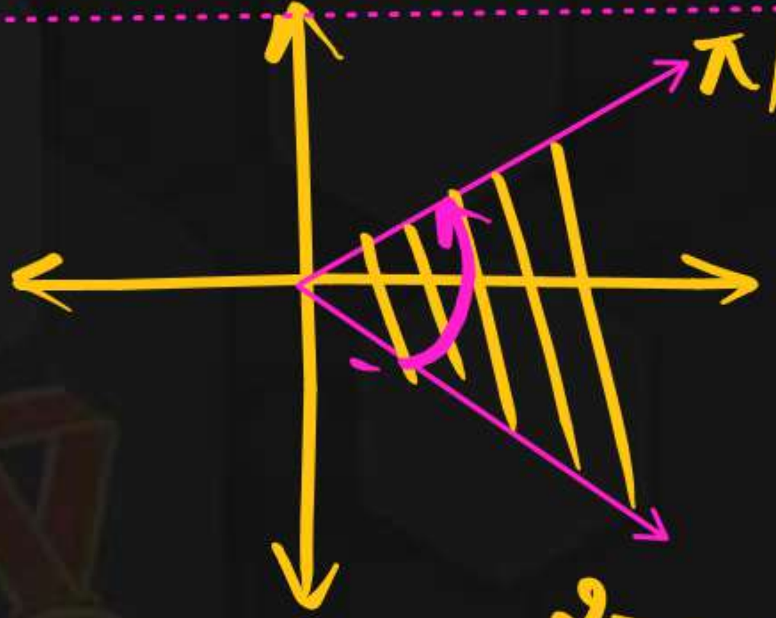
1. $\cos x \geq \frac{1}{2}$

2. $|\cos x| < \sqrt{3}/2$

3. $-\frac{\sqrt{3}}{2} < \sin x \leq \frac{1}{2}$

$\Rightarrow -\frac{\sqrt{3}}{2} < \cos x < \sqrt{3}/2$

0

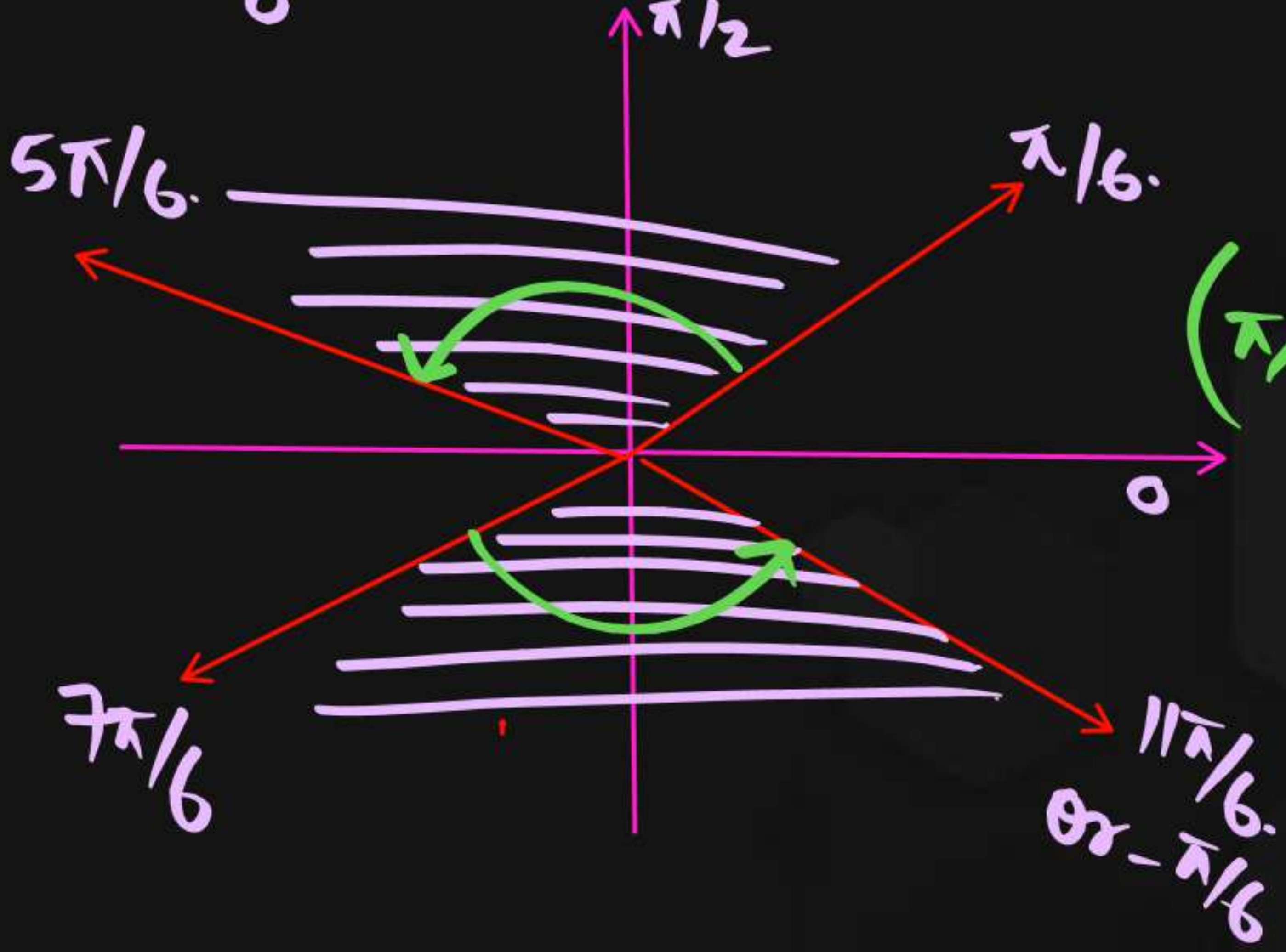


$2\pi - \pi/3 = 5\pi/3$

$-\pi/3$

$[-\pi/3, \pi/3]$

$[2n\pi - \pi/3, 2n\pi + \pi/3]$



$(\pi/6, 5\pi/6) \cup (7\pi/6, 11\pi/6)$

add $2n\pi$

Solving Trigonometric Inequations for $\tan x$

Period (π)

Q→1 $\tan x \geq 1$ -ve

$x \in [\pi/4, \pi/2)$

$n\pi$ add

Ans $x \in [n\pi + \pi/4, n\pi + \pi/2)$

1) For $\tan x$, find the solⁿ in $(\frac{1}{2})$ cycle
2) add $n\pi$ to get general solⁿ.

Question

Solve

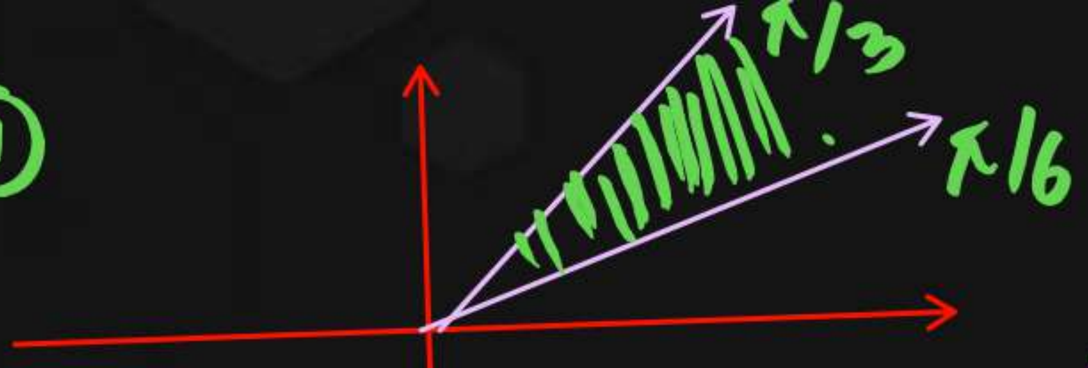
1. $\frac{1}{\sqrt{3}} < \tan x \leq \sqrt{3}$

2. $\tan x \leq 1$

3. $|\tan x| \leq 1$

4. $\frac{1}{4} \leq \sin^2 x \leq \frac{3}{4}$

(1)



$x \in \left(\pi/6, \pi/3\right]$

$x \in \left(n\pi + \pi/6, n\pi + \pi/3\right]$

(2)