Modeling Poly-Phase Capacitors in an Arbitrary Reference Frame

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***Abstract*—**

I. Introduction

Poly-phase systems are commonly used in power conversion applications in order to take advantage of the improved power handling capabilities for a given voltage and current limit and the ability to make a rotating flux.

II. Complex Vector Formulation

For this paper we use the complex vector formulation described in [1] which maps an arbitrary balanced poly-phase system into the complex plane. Equation 1 shows how this is done for three phases [2].

$$f_{dq} = \frac{2}{3}\left(\alpha_a f_a + \alpha_b f_b + \alpha_c f_c\right) \tag{1}$$

The factor of $2/3$ causes the magnitude of the complex vector to be the same as the magnitude of any of the terminal quantities. This choice requires a factor of $3/2$ in order perform power calculations. Other options for this factor cause other invariants when performing system calculations. The coefficients α_n are the position of the phases in the given reference frame. This can be used with differing α_n on the same phase quantities in order to work in reference frames at different spatial and temporal frequencies. [3] From this complex vector formulation, we can multiply by a fixed-speed rotating unit vector to obtain complex quantities in a rotating reference frame.

III. Capacitor Model

In this paper we are working with a three-phase star-connected capacitor with terminal quantities defined in the standard passive convention (voltages referenced phase-to-neutral, currents flowing into the phase terminals). The capacitor voltage differential equations are then

$$\frac{dV_{an}}{dt} = \frac{I_a}{C} \tag{2}$$

$$\frac{dV_{bn}}{dt} = \frac{I_b}{C} \tag{3}$$

$$\frac{dV_{cn}}{dt} = \frac{I_c}{C} \tag{4}$$

We can then translate these into the stationary refrence frame as

$$C\frac{dV_q^s}{dt} = I_q^s \tag{5}$$

$$C\frac{dV_d^s}{dt} = I_d^s \tag{6}$$

which can be translated into intstantaneous complex variables in the stationary reference frame as

$$C\frac{dV_{qd}^s}{dt} = I_{qd}^s \tag{7}$$

Now, when transforming from the stationary to arbitrary reference frame we start with a transformed voltage and current such that

$$I_{qd}^\omega = I_{qd}^s e^{j\omega t} \tag{8}$$

Substituting equation 8 into equation 7 we get

$$C\frac{d}{dt}\left(V_{qd}^\omega e^{-j\omega t}\right) = I_{qd}^\omega e^{-j\omega t} \tag{9}$$

We then apply the chain rule in order to obtain

$$C\frac{dV_{qd}^\omega}{dt} = I_{qd}^\omega - j\omega C V_{qd}^\omega \tag{10}$$

which for $\omega = 0$ simplifies down to equation 7.

The final $-j\omega C V_{qd}^\omega$ term is the speed current associated with the rotating reference frame. This is analogous to the speed voltage associated with currents found found in rotating magnetic machinery.

IV. Conclusions

References

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[3] A. Rockhill and T. Lipo, "A simplified model of a nine phase synchronous machine using vector space decomposition," in *Power Electronics and Machines in Wind Applications, 2009. PEMWA 2009. IEEE*, June 2009, pp. 1–5.