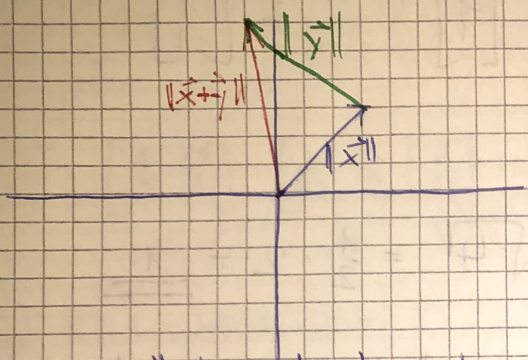
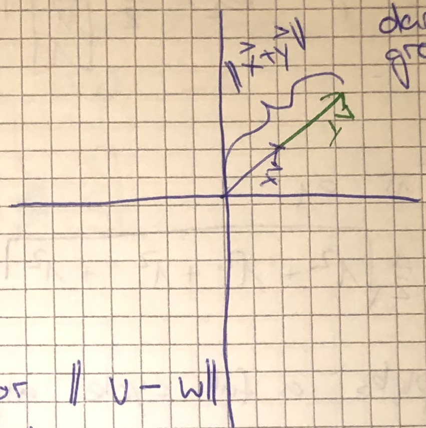
IMath Woche 1 – übungen

Seite 1

1) $\|V\| = 5$ $\|W\| = 3$





dann sind sie gleich gross / lang

(i) → smallest and largest value for $\| v - w \|$

Die Triangle Inequality besagt, dass

$\| v - w \| \leq \|v\| + \|w\| \rightarrow 5 + 3 = \underline{\underline{8}}$

Wenn zwei Vektoren in dieselbe Richtung zeigen, dann ist der Unterschied am kleinsten.

$5 - 3 = \underline{\underline{2}}$

(ii) → smallest and largest value for $v \cdot w$

We use the schwarz inequality and transform it

$|v \cdot w| \leq \|v\| \|w\| \cdot \cos\varphi \leq \|v\| \cdot \|w\|$

So it must be : $-15 \leq v \cdot w \leq 15$

2) Skalarprodukt muss $= 0$ sein

$$e_1 = \frac{1}{2}\begin{bmatrix}1\\1\\1\\1\end{bmatrix}, \quad e_2 = \frac{1}{2}\begin{bmatrix}-1\\-1\\1\\1\end{bmatrix}, \quad e_3 = \frac{1}{2}\begin{bmatrix}-1\\1\\-1\\1\end{bmatrix}, \quad e_4 = \frac{1}{2}\begin{bmatrix}-1\\1\\1\\-1\end{bmatrix}$$

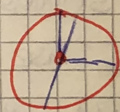
Beispiel für e_1

$$\frac{1}{2}\begin{bmatrix}1\\1\\1\\1\end{bmatrix} = \frac{1}{2}\sqrt{1^2+1^2+1^2+1^2} = \frac{1}{2}\sqrt{4} = \frac{1}{2}\cdot 2 = \underline{\underline{1}}$$

3) The minute completes a full cycle at $t=1$ and the hour hand moves 1/12 as fast.

$$m(t) = \begin{bmatrix}\sin 2\pi \cdot t\\ \cos 2\cdot\pi\cdot t\end{bmatrix}$$

$$h(t) = \begin{bmatrix}\frac{\sin 2\cdot\pi\cdot t}{12}\\ \frac{\cos 2\cdot\pi\cdot t}{12}\end{bmatrix} = \begin{bmatrix}\frac{\sin \pi\cdot t}{6}\\ \frac{\cos \pi\cdot t}{6}\end{bmatrix}$$



$\Rightarrow$ Vectors will be perpendicular when dot product $= 0$

$$m \cdot h = (\sin 2\pi t)\cdot\left(\sin\left(\frac{\pi t}{6}\right)\right) + (\cos 2\pi t)\cdot\left(\cos\left(\frac{\pi t}{6}\right)\right) = 0$$

$$= \cos\left(2\pi t - \frac{\pi t}{6}\right) = \cos\left(\frac{12\pi t}{6} - \frac{\pi t}{6}\right)$$

$$= \cos\left(\frac{11\pi\cdot t}{6}\right)$$

$\rightarrow$ wenn $\cos\left(\frac{\pi}{2}\right) = 0$, also folgende Gleichung lösen

$$\frac{11\pi\cdot t}{6} = \frac{\pi}{2} \Rightarrow 22\pi\cdot t = 6\pi$$

$$= t = \frac{3}{11} \text{ hours or } 16.36 \text{ min}$$

$\rightarrow$ should be a straight line when $\frac{11\pi t}{6} = \pi$

$$11\pi t = 6\pi \Rightarrow t = \frac{6}{11} \text{ hours or } 32.72 \text{ min}$$

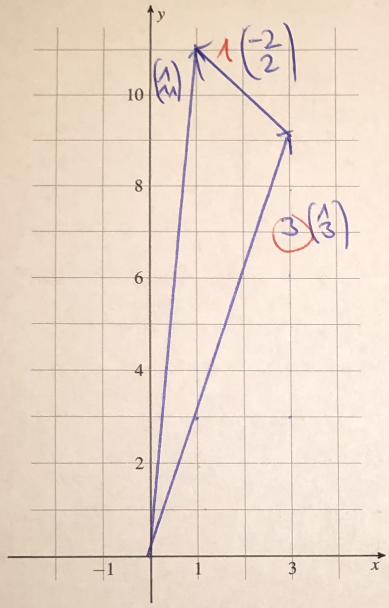
4) siehe Blatt

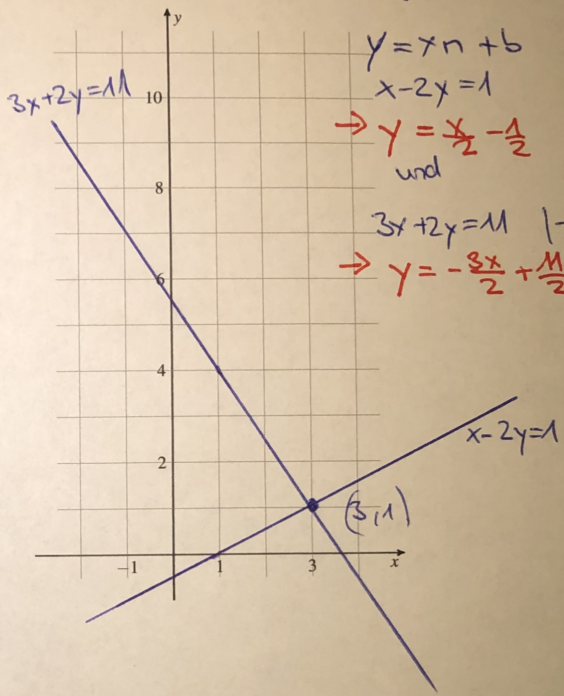
$$3\binom{1}{3}+1\binom{-2}{2}=\binom{1}{11}$$

in the two unknowns x and y using the (i) the row picture and (ii) the column picture. Depict both methods in the (x, y)-plane. column picture row picture (i)

(ii)

$x=3$
$y=1$





$y = xn + b$
$x-2y=1$
$\rightarrow y = \frac{x}{2}-\frac{1}{2}$
und
$3x+2y=11 \quad |-3x \quad |:2$
$\rightarrow y = -\frac{3x}{2}+\frac{11}{2}$

5 Elimination

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What multiple l_{21} of equation 1 should be subtracted from equation 2 such that x disappears from this equation? 5

$$\begin{aligned} 2x+3y &= 1 \\ 10x+9y &= 11 \end{aligned}$$

After this elimination step, write down the upper triangular system and circle the two pivots. Note: the number on the right hand side have no influence on those pivots.

6 Back substitution

Solve the triangular system of the previous problem by back substitution, y before x. Verify that x times the first column of the original coefficient matrix plus y times the second column equals the right hand side. If the right hand side changes to $[4, 44]$, what is the new solution?

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7 No or infinitely many solutions

In the following system choose a right side which gives (i) no solution and another right side which gives (ii) infinitely many solutions:

$$\begin{aligned} 2x+2y &= 10 \\ 6x+4y &= \end{aligned}$$

Draw the corresponding geometrical interpretation in (i) the row picture and in (ii) the column picture.

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ToDo

5) $2x + 3y = 1 \quad | \cdot (-5)$
$10x + 9y = 11$

$2x + 3y = 1$
$-6y = 6$

Die Pivot-Elemente sind 2 und -6

$$\underbrace{\begin{bmatrix} 1 & 0 \\ -5 & 1 \end{bmatrix}}_{E} \underbrace{\begin{bmatrix} 2 & 3 \\ 10 & 9 \end{bmatrix}}_{A} \left| \underbrace{\begin{bmatrix} 1 \\ 11 \end{bmatrix}}_{b} \right. = \begin{bmatrix} 2 & 3 \\ 0 & -6 \end{bmatrix} \left| \begin{bmatrix} 1 \\ c \end{bmatrix} \right.$$

6) Rücksubstitution

$-6y = 6 \Rightarrow y = -1$

$2x + 3 \cdot (-1) \overset{!}{=} 1$
$2x - 3 \overset{!}{=} 1 \quad | +3$
$2x \overset{!}{=} 4 \quad | :2$
$x \overset{!}{=} 2$

$L = \{2, -1\}$

→ testen $\begin{pmatrix} 2 \\ 10 \end{pmatrix} \cdot \overset{x}{2} + \begin{pmatrix} 3 \\ 9 \end{pmatrix} \cdot \overset{y}{(-1)} = \begin{pmatrix} 1 \\ 11 \end{pmatrix}$ ✓

Wenn die rechte Seite $\begin{pmatrix} 4 \\ 44 \end{pmatrix}$ ist, dann ist $L = \{8, -4)$, da es gerade der Faktor 4 ist.

7) $\left[\begin{array}{cc|c} 2 & 2 & 10 \\ 6 & 4 & b \end{array}\right] \mid (-3) \Rightarrow \left[\begin{array}{cc|c} 2 & 2 & 10 \\ 0 & -2 & b-30 \end{array}\right]$

$-2y = b-30$

$y \stackrel{!}{=} -\frac{b}{2} + 15 \rightarrow 2x + 2(-\frac{b}{2}$

8) $\left[\begin{array}{ccc|c} 2 & -3 & 0 & 8 \\ 4 & -7 & 1 & 20 \\ 0 & -2 & 2 & 0 \end{array}\right] \mid \cdot(-2)$

$\left[\begin{array}{ccc|c} 2 & -3 & 0 & 8 \\ 0 & -1 & 1 & 4 \\ 0 & -2 & 2 & 0 \end{array}\right] \mid \cdot(-2)$

$\left[\begin{array}{ccc|c} 2 & -3 & 0 & 8 \\ 0 & -1 & 1 & 4 \\ 0 & 0 & 0 & -8 \end{array}\right]$

Pivot-Elemente sind 2, -1 und 0

Gleichungssystem hat keine Lösung, da

$0 = -8$! → Back substitution

9) $EA = U$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix}}_{E_{32}} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{E_{21}} \underbrace{\left[\begin{array}{ccc|c} 2 & -3 & 0 & 8 \\ 4 & -7 & 1 & 20 \\ 0 & -2 & 2 & 0 \end{array}\right]}_{A}$$

$$E = E_{32} E_{21} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 4 & -2 & 1 \end{bmatrix} = E$$

Prüfen: $EA = U$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 4 & -2 & 1 \end{bmatrix}}_{E} \underbrace{\left[\begin{array}{ccc|c} 2 & -3 & 0 & 8 \\ 4 & -7 & 1 & 20 \\ 0 & -2 & 2 & 0 \end{array}\right]}_{A} = \underbrace{\left[\begin{array}{ccc|c} 2 & -3 & 0 & 8 \\ 0 & -1 & 1 & 4 \\ 0 & 0 & 0 & -8 \end{array}\right]}_{U}$$

10) Change row 2 with row 3

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}}_{P_{23}} \underbrace{\left[\begin{array}{ccc|c} 2 & -3 & 0 & 8 \\ 4 & -7 & 1 & 20 \\ 0 & -2 & 2 & 0 \end{array}\right]}_{A} = \left[\begin{array}{ccc|c} 2 & -3 & 0 & 8 \\ 0 & -2 & 2 & 0 \\ 4 & -7 & 1 & 20 \end{array}\right]$$

11)

$$\begin{array}{c} \qquad A \qquad\qquad\qquad I \\ \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 \end{array}\right] \end{array} \quad Z_{12}(1)$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 \end{array}\right] \quad Z_{23}(1)$$

$$\begin{array}{c} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{array}\right] \\ \qquad I \qquad\qquad\qquad A^{-1} \end{array}$$

$$A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Prüfen:

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}}_{A^{-1}} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}}_{A} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{I}$$

→ Wenn $\det A = 0$ keine inverse und wenn nicht quadratisch

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12) $x + 2y + 2z = 1$
$4x + 8y + 9z = 3$
$3y + 2z = 1$

$$\left[\begin{array}{ccc|c} 1 & 2 & 2 & 1 \\ 4 & 8 & 9 & 3 \\ 0 & 3 & 2 & 1 \end{array}\right] Z_{12}\,(-4) \rightarrow E_{21} = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 2 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 3 & 2 & 1 \end{array}\right] \begin{matrix} Z_3 \\ Z_2 \end{matrix} \rightarrow P_{32} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 2 & 1 \\ 0 & 3 & 2 & 1 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

$$P_{32}\,E_{21} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ -4 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ -4 & 1 & 0 \end{bmatrix} \left[\begin{array}{ccc|c} 1 & 2 & 2 & 1 \\ 4 & 8 & 9 & 3 \\ 0 & 3 & 2 & 1 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 2 & 2 & 1 \\ 0 & 3 & 2 & 1 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

$z = -1$

$3y + 2\cdot(-1) = 1$

$3y - 2 = 1$

$3y = 3 \Rightarrow y = 1$ folglich $x = 1$

13) Da eine Zeilenvertauschung stattgefunden hat, multiplizieren wir die Matrix A zuerst mit der Permutationsmatrix

$$PA = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ 4 & 8 & 9 \\ 0 & 3 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 3 & 2 \\ 4 & 8 & 9 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 4 & 0 & 1 \end{bmatrix}}_{L} \underbrace{\begin{bmatrix} 1 & 2 & 2 \\ 0 & 3 & 2 \\ 0 & 0 & 1 \end{bmatrix}}_{U} = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 3 & 2 \\ 4 & 8 & 9 \end{bmatrix}$$