**ECE/CS 498: Mobile Computing and Applications**

# Homework 1: Math Foundations

## 1 True or False [6 X 5 = 30 points]

**Write True/False with a brief justification (around 1 sentence). Make assumptions where necessary and state them clearly.**

**Q1 :**
A is a $m \times n$ matrix with $m < n$. The null space $N(A)$ is always 0.

**Q2 :**
The matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, when left multiplied with a second matrix $B$ (i.e. $B * A$), subtracts twice of the first column of $B$ from the second column of $B$.

**Q3 :**
For an orthonormal matrix $Q$ (columns of $Q$ are orthogonal to each other and the length of each column is 1), $Q^{-1} = Q^T$.

**Q4 :**
A signal $S1$ is composed of frequencies $f_1, f_2, f_3$ and $S2$ consists of frequencies $f_2, f_3, f_4$. Then, a signal $S3 = S1 \star S2$ (convolution of $S1$ and $S2$), would consist of union of these frequencies i.e., $f_1, f_2, f_3, f_4$.

**Q5 :**
If $b_1, b_2, b_3$ form the basis of a space, then $c_1 b_1 + c_2 b_2 + c_3 b_3 = 0$ implies that all $c_1$,$c_2$ and $c_3$ are zero.

**Q6 :**
The length of the vector $\frac{V}{\|V\|}$ is 0 ($\|V\|$ is the $l_2$ norm of $V$).

## 2 Symmetric Matrices [5 points]

Prove that $A^T A$ is a symmetric matrix.

*Hint :* use the basic properties of transpose, as discussed in class.

## 3 Column Spaces [10 points]

Choose $b$ which gives no solution and another $b$ which gives infinitely many solutions. What are two of those solutions ?

$$3x + 2y = 10 \tag{1}$$
$$6x + 4y = b \tag{2}$$

# 4 Least Squares [10 points]

Consider the following system of equations.

$$x - y = 2 \tag{3}$$
$$x + y = 4 \tag{4}$$
$$2x + y = 8 \tag{5}$$

How many solutions exist for the above system of equations ? If a solution exists find one, if not, determine the least squares solution for $x$ and $y$.

# 5 Fourier Transform Matrix [10 points]

Compute the $4 \times 4$ Fourier Transform matrix $W$. Use $W$ to transform $\begin{bmatrix}1\\2\\3\\4\end{bmatrix}$ into the frequency domain.

Also, find the inverse of $W$ and use it to transform $\begin{bmatrix}0\\0\\1\\0\end{bmatrix}$ into the time domain.

*Hint :* recall the Fourier Transform equation as taught in class, and write it as a matrix form.

# 6 Fourier Transform Property [10 points]

Prove that real-input Fourier Transform obeys the symmetry : if $\mathbf{x} = \begin{bmatrix}x_0\\x_1\\\cdots\\x_{N-1}\end{bmatrix}$ are real numbers, then for its $N$-point Fourier Transform $\mathbf{X}$ we have : $X[N-k] = X[k]^*$.

($X^*$ denotes complex conjugation, e.g. $(e^{j\theta})^* = e^{-j\theta}$.)

# 7 A Real Fourier Transform [25 points]

## 7.1 Averaging Filter [5 points]

Filters such as $h = \begin{bmatrix}1/8 & 1/8 & 1/8 & 1/8 & 1/8 & 1/8 & 1/8 & 1/8\end{bmatrix}^T$ have an averaging effect, hence called an averaging filter. Use MATLAB to compute $h$'s 8-point FFT. In your solution, plot the magnitude of the FFT.

*Hint :* once you compute its FFT as $H$, you may use this MATLAB command to plot the graph :

*stem(abs(H)) ; xlabel('H[k]') ; ylabel('FFT Magnitude') ;*

## 7.2 Human Voice [10 points]

Use your phone to record your own voice, and say "My name is [Your Name]." Use MATLAB to import the saved audio file, and compute its FFT. In your solution, plot the magnitude of the FFT.

*Hint :* once you compute the voice's FFT as $X$, and get the sampling rate $Fs$ from the audio file, you may use this MATLAB command to plot the graph :

*plot((1 :length(X))/length(X)*Fs, abs(X)) ; xlabel('Frequency (Hz)') ; ylabel('FFT Magnitude') ;*

### 7.3 Convolution **[10 points]**

In MATLAB, convolve the average filter in (7.1) with your voice in (7.2). Listen to the sound before and after convolution, to feel the difference. In your solution, plot the magnitude of the FFT of the new signal after convolution.