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## Hoja de trabajo #3

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### Ejercicio #1

Utilizando la definicion de suma ( $\oplus$ ) para los numeros naturales unarios, llevar a cabo la suma entre tres  $[s(s(s(0)))]$  y cuatro  $[s(s(s(s(0))))]$ .

$$s(s(s(0))) \oplus s(s(s(s(0))))$$

a=s(x)  
x=s(s(0))  
b=s(s(s(s(0))))

$$s[s(s(0)) \oplus b]$$

a=s(x)  
x=s(0)  
b=s(s(s(s(0))))

$$s[s(s(0) \oplus b)]$$

a=s(x)  
x=0  
b=s(s(s(s(0))))

$$s[s(s(0 \oplus b))]$$

0+b=b

$$\begin{aligned} & s[(s(s(b)))] \\ & [s(s(s(s(s(s(s(0)))))))] \\ & = 7 \end{aligned}$$

## Ejercicio #2

Definir inductivamente una función para multiplicar ( $\otimes$ ) numeros naturales unarios.

$$a \otimes b := \begin{cases} 0 & \text{si } a = o \\ 0 & \text{si } b = o \\ a & \text{si } b = 1 \\ b & \text{si } a = 1 \\ b \oplus (x \otimes b) & \text{si } a = \sigma(x) \end{cases}$$

## Ejercicio #3

Verifique que su definición de multiplicación es correcta multiplicando los siguientes valores:

- $\sigma(\sigma(\sigma(0))) \otimes 0$

$$\sigma(\sigma(\sigma(0))) \otimes 0$$

$$a = \sigma(\sigma(\sigma(0)))$$

$$a \otimes 0 = 0$$

- $\sigma(\sigma(\sigma(0))) \otimes \sigma(0)$

$$\sigma(\sigma(\sigma(0))) \otimes \sigma(0)$$

$$\begin{aligned} a &= \sigma(x) \\ x &= \sigma(\sigma(0)) \\ b &= \sigma(0) \end{aligned}$$

$$\begin{aligned} &b \oplus (x \otimes b) \\ &= b \oplus (\sigma(\sigma(0)) \otimes b) \end{aligned}$$

$$\begin{aligned} a &= \sigma(x) \\ x &= \sigma(0) \end{aligned}$$

$$= b \oplus b \oplus (\sigma(0) \otimes b)$$

$$\begin{aligned} a &= \sigma(x) \\ x &= 0 \end{aligned}$$

$$= b \oplus b \oplus b \oplus (0 \otimes b)$$

$$0 \otimes b = 0$$

$$= b \oplus b \oplus b \oplus (0)$$

$$= b \oplus b \oplus b$$

$$= \sigma(0) \oplus \sigma(0) \oplus \sigma(0)$$

$$= \sigma(\sigma(\sigma(0)))$$

$$= 3$$

- $\sigma(\sigma(\sigma(0))) \otimes \sigma(\sigma(0))$

$$\sigma(\sigma(\sigma(0))) \otimes \sigma(\sigma(0))$$

$$\begin{aligned} a &= \sigma(x) \\ x &= \sigma(\sigma(0)) \\ b &= \sigma(\sigma(0)) \end{aligned}$$

$$b \oplus (x \otimes b)$$

$$\begin{aligned}
&= b \oplus (\sigma(\sigma(0)) \otimes b) \\
\text{a} &= \sigma(x) \\
\text{x} &= \sigma(0) \\
&= b \oplus b \oplus (\sigma(0) \otimes b) \\
\text{a} &= \sigma(x) \\
\text{x} &= 0 \\
&= b \oplus b \oplus b \oplus (0 \otimes b) \\
0 \otimes b &= 0 \\
&= b \oplus b \oplus b \oplus (0) \\
&= b \oplus b \oplus b \\
&= \sigma(\sigma(0)) \oplus \sigma(\sigma(0)) \oplus \sigma(\sigma(0)) \\
&= \sigma(\sigma(\sigma(\sigma(\sigma(0))))) \\
&= 6
\end{aligned}$$

## Ejercicio #4

Demostrar utilizando inducción:

- $a \oplus \sigma(\sigma(0)) = \sigma(\sigma(a))$   
Caso Base:  $a=0$

$$\begin{aligned}
0 + \sigma(\sigma(0)) &= \sigma(\sigma(0)) \\
\sigma(\sigma(0)) &= \sigma(\sigma(0))
\end{aligned}$$

Caso Inductivo:  $a=\sigma(x)$

Hipótesis Inductiva:  $x + \sigma(\sigma(0)) = \sigma(\sigma(x))$

$$\begin{aligned}
\sigma(x) + \sigma(\sigma(0)) &= \sigma(\sigma(\sigma(0))) \\
\sigma(x + \sigma(\sigma(0))) &= \sigma(\sigma(\sigma(0))) \\
\sigma(\sigma(\sigma(0)) + x) &= \sigma(\sigma(\sigma(0))) \\
\sigma(\sigma(\sigma(0) + x)) &= \sigma(\sigma(\sigma(0))) \\
\sigma(\sigma(\sigma(0 + x))) &= \sigma(\sigma(\sigma(0))) \\
\sigma(\sigma(\sigma(x))) &= \sigma(\sigma(\sigma(0)))
\end{aligned}$$

- $a \otimes b = b \otimes a$   
Caso Base:  $a=0$

$$\begin{aligned}
0 \otimes b &= b \otimes 0 \\
0 &= 0
\end{aligned}$$

Caso Inductivo:  $a=\sigma(x)$

Hipótesis Inductiva:  $x \otimes b = b \otimes x$

$$\sigma(x) \otimes b = b \otimes \sigma(x)$$

$$b = \sigma(i)$$

$$b \oplus (x \otimes b) = b \otimes \sigma(x)$$

$$\sigma(i) \oplus (x \otimes \sigma(i)) = \sigma(i) \otimes \sigma(x)$$

$$\sigma(i) \oplus (x \otimes \sigma(i)) = \sigma(x) \oplus (i \otimes \sigma(x))$$

$$\sigma(i \oplus (x \otimes \sigma(i))) = \sigma(x \oplus (i \otimes \sigma(x)))$$

$$\sigma(i \oplus (x \otimes b)) = \sigma(x \oplus (i \otimes a))$$

Por Hipótesis inductiva:

$$x \otimes b = b \otimes x$$

$$i \otimes a = a \otimes i$$

$$\sigma(i \oplus (b \otimes x)) = \sigma(x \oplus (a \otimes i))$$

$$\sigma(i \oplus (\sigma(i) \otimes x)) = \sigma(x \oplus (\sigma(x) \otimes i))$$

$$\sigma(i \oplus x \oplus (i \otimes x)) = \sigma(x \oplus i \oplus (x \otimes i))$$

$$\sigma(x \oplus i \oplus (i \otimes x)) = \sigma(x \oplus i \oplus (x \otimes i))$$

$$x \otimes i = i \otimes x$$

$$\sigma(x \oplus i \oplus (x \otimes i)) = \sigma(x \oplus i \oplus (x \otimes i))$$

$$k=x \oplus i \oplus (x \otimes i)$$

$$\sigma(k) = \sigma(k)$$

- $a \otimes (b \otimes c) = (a \otimes b) \otimes c$

Caso Base:  $a=0$

$$0 \otimes (b \otimes c) = (0 \otimes b) \otimes c$$

$$0 \otimes (b \otimes c) = 0 \otimes c$$

$$0 = 0$$

Caso Inductivo:  $a=\sigma(x)$

Hipótesis Inductiva:  $x \otimes (b \otimes c) = (x \otimes b) \otimes c$

$$\sigma(x) \otimes (b \otimes c) = (\sigma(x) \otimes b) \otimes c$$

$$(b \otimes c) \oplus (x \otimes (b \otimes c)) = (\sigma(x) \otimes b) \otimes c$$

$$(b \otimes c) \oplus (x \otimes (b \otimes c)) = (b \oplus (x \otimes b)) \otimes c$$

Distribuir la multiplicación  $b \oplus (x \otimes b) \otimes c$  (ver siguiente demostración  $(a \oplus b) \otimes c = (a \otimes c) \oplus (b \otimes c)$ ):

$$(b \otimes c) \oplus (x \otimes (b \otimes c)) = (b \otimes c) \oplus ((x \otimes b) \otimes c)$$

$$(b \otimes c) \oplus [(x \otimes (b \otimes c)) = ((x \otimes b) \otimes c)] \oplus (b \otimes c)$$

Asumiendo que  $[(x \otimes (b \otimes c)) = ((x \otimes b) \otimes c)]$  es verdadero (Hipótesis Inductiva):

$$(b \otimes c) = (b \otimes c)$$

- $(a \oplus b) \otimes c = (a \otimes c) \oplus (b \otimes c)$

Caso Base:  $a=0$

$$(0 \oplus b) \otimes c = (0 \otimes c) \oplus (b \otimes c)$$

$$(b) \otimes c = (0) \oplus (b \otimes c)$$

$$b \otimes c = b \otimes c$$

Caso Inductivo:  $a=\sigma(x)$

Hipótesis Inductiva:  $(x \oplus b) \otimes c = (x \otimes c) \oplus (b \otimes c)$

$$(\sigma(x) \oplus b) \otimes c = (\sigma(x) \otimes c) \oplus (b \otimes c)$$

$$\sigma(x \oplus b) \otimes c = (\sigma(x) \otimes c) \oplus (b \otimes c)$$

$$c \oplus ((x \oplus b) \otimes c) = (\sigma(x) \otimes c) \oplus (b \otimes c)$$

$$c \oplus ((x \oplus b) \otimes c) = (c \oplus (x \otimes c)) \oplus (b \otimes c)$$

$$c \oplus [(x \oplus b) \otimes c] = (x \otimes c) \oplus (b \otimes c) \oplus c$$

Asumiendo que  $[(x \oplus b) \otimes c = (x \otimes c) \oplus (b \otimes c)]$  es verdadero (Hipótesis Inductiva):

$$c = c$$