$x_1 + x_2 - 1 = 1$
$x_1 - x_2 - 1 = 1$

dotted:
$f_1 \ldots = 1$
$f_2 \ldots = 1$

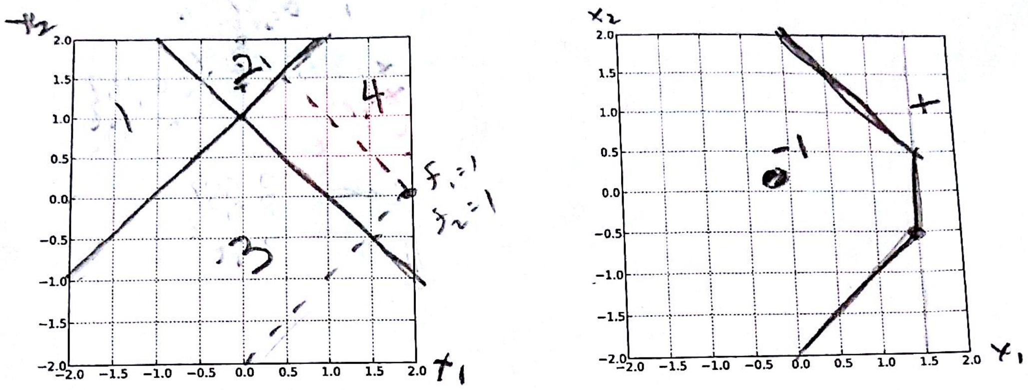


Figure 1: Empty grids of x_1 vs. x_2 for the network problem

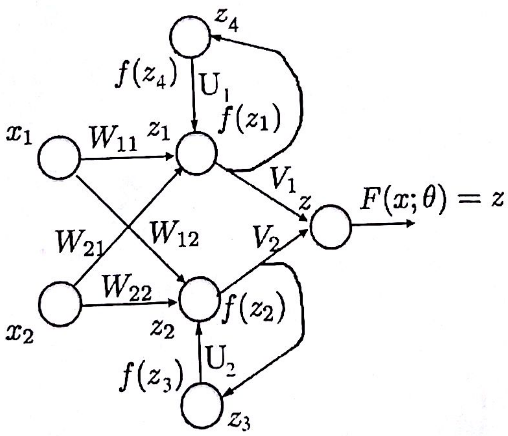
four regions of the x−space where

$$\begin{aligned}
&\text{region 1} \quad && f(z_1) = 0, f(z_2) = 0 && (8)\\
&\text{region 2} \quad && f(z_1) > 0, f(z_2) = 0 && (9)\\
&\text{region 3} \quad && f(z_1) = 0, f(z_2) > 0 && (10)\\
&\text{region 4} \quad && f(z_1) > 0, f(z_2) > 0 && (11)
\end{aligned}$$

Note that some of the regions may not contain the boundary at all.

Recursive Neural Networks

Suppose that a simple recursive layer is now added to the neural network above.



In this recursive network the outputs from the hidden layer are fed into a new layer of neurons, and the output from these neurons are fed back into the hidden layer during the next time step. There is no offset included for this problem. Therefore, you can assume that the offset is equal to 0 for all of the nodes.

3. If we take a look when $x_1 = 2$, $x_2 = 0$

in the graph $f_1(z_1) = +$

$f_2(z_2) = 0$

$\Rightarrow$ vector $[+, \phi]$

$\approx [1, \phi]$

if we plug it in

$z_1 = (2)(1) + 0(1) - 1 = 1$

$z_2 = (2)(1) - 1(0) - 1 = 1$

$[1, 1]$

4. ~~x~~

$z_1 = x_1 + x_2 - 1$

$z_2 = x_1 - x_2 - 1$

$z = (x_1 + x_2 - 1)V_1 + (x_1 - x_2 - 1)V_2 + U_0$

$= (x_1 + x_2 - 1)(1) + (x_1 - x_2 - 1)(1) + (-1)$

R1) $z = -1$

R2) $x_1 + x_2 - 1 - 1 = x_1 + x_2 - 2$

R3) $x_1 - x_2 - 2$

R4) $2x_1 - 3$

Recursive

1.) ~~f(...)~~ $z_3^{(t)} = f(z_3^{(t-1)}) \cdot U_2 + Z_2$

$$f(\, f(z_3^{(t-1)}) \cdot U_2 + z_2)$$

$$z_3^{(t)} \Rightarrow f(\, f(z_3^{(t-1)}) \cdot U_2 + (x_1 w_{12} + x_2 w_{22}))\, V_2$$

$$z_4^{(t)} = f(f(z_4^{(t-1)})\, U_1 + (x_1 w_{11} + x_2 w_{21}))\, V_1$$

2) $z^{(0)} = V_1 z_3^{(0)} + V_2 z_4^{(0)} = 0$

$$z_3^{(t)} = f(\, f(z_3^{(t-1)}) \cdot 2 + (2x_1 - 3x_2))\, 2$$

$-1, 5, 1$

$4,$

$$z_4^{(t)} = f(f(z_4^{(t-1)}) \cdot (-3) + (2x_1 + 2x_2))\, 2$$

$$z^{(1)} = (2)(z_3^{(1)} + z_4^{(1)}) = 2(2+2) = 8$$

$$z^{(2)} = 2(z_3^{(2)} + z_4^{(2)}) = 2(2+0) = 4$$

$$z^{(3)} = 2(z_3^{(3)} + z_4^{(3)}) = 2(2+0) = 4$$

3) $U_2 = -1, 5, 1$

$U_1 = 4, 0, -4$

$\Rightarrow f(z_n^{t-1}) = 1$

$\Rightarrow U_2 = -5$

$U_1 = -4$

$\Rightarrow z_4^{(1)}, z_4^{(2)}, z_4^{(3)} = 0$

z_4

Back.

1) $\delta_{3j} = \frac{\partial}{\partial z_{3j}} Loss(y(Fx;\theta))$

$= \frac{\partial}{\partial z_{3j}} Loss(y(\sum_{j=1}^{m} f(z_{3j}) V_j + V_0))$

if Loss = 0 derivative = 0.

else $\frac{\partial}{\partial z_{3j}} (1 - y(\sum_{j=1}^{m} f(z_{3j}) V_j + V_0))$

$= \frac{\partial}{\partial z_{3j}} (y(\sum_{j=1}^{m} f(z_{3j}) V_j + V_0))$

$\delta_{3j} = f(z_{3j})(1 - f(z_{3j}))[V_j][-y]$

2) $\frac{\partial}{\partial w_{3ij}} = \delta_{w_{3ij}} = \boxed{(\delta_{3j})(\sum_{i} V_j)}$

3) $\delta_{2j} = \delta_{3j}$

$\boxed{\delta_{2j} = f(z_{2j})(1 - f(z_{2j}))(\sum_{i=1}^{m} w_{2j \to 3i} \delta_{3i})}$

4) $\delta_{1j} = f(z_{1j})(1 - f(z_{1j}))(\sum_{i=1}^{m} w_{1j \to 2i} \delta_{2i})$

5

$$\frac{\partial}{\partial w_{ij}} Loss(yF(x;\theta))$$

$$= [x_i] \; \delta_{ij}$$

6. $w_{\ell,ij} \leftarrow w_{ij} + \eta_k f(z_{\ell j}) \delta_{\ell j}$