Homework 4: Fluid Simulation - Semi-Lagrangian with projection.

Navier-Stookes equation: $\begin{cases} \frac{D\vec{v}}{Dt} = \vec{g} - \frac{1}{\rho}\nabla p + \nu \nabla^2 \vec{v} \\ \nabla \cdot v = 0 \end{cases}$

1. Advection: $\frac{Dq}{Dt} = \frac{\partial q}{\partial t} + v \cdot \nabla q = 0 \Rightarrow \frac{q_i^{n+1} - q_i^n}{\Delta t} + v^n \cdot \frac{q_{i+1}^n - q_{i-1}^n}{2\Delta x} = 0$

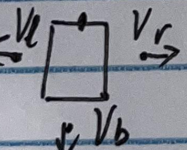
$\Rightarrow q_i^{n+1} = q_i^n - \Delta t\, v^n \cdot \frac{q_{i+1}^n - q_{i-1}^n}{2\Delta x}$

Use backtrace() to find q^n and bilerp to interpolate the value.
where q = ① the color "dyef"
② the velocity "vf"

2. Applying forces: $\Delta v = (g + \nu \nabla^2 v + f_{ext})\Delta t$
where "ν" viscosity is adjustable by pressing "↑" or "↓" key.
f_{ext} is applied by clicking and draging mouse.

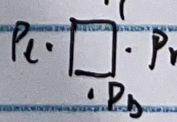
3. Projection: $\cancel{\frac{\Delta t}{\rho} \nabla \cdot \nabla p = -\nabla \cdot v^n}$

$\begin{cases} \frac{\partial v}{\partial t} = -\frac{1}{\rho}\nabla p \\ \nabla \cdot v = 0 \end{cases} \xrightarrow[\text{in time } \Delta t]{\text{discretize}} \begin{cases} v^{n+1} - v^n = -\frac{\Delta t}{\rho}\nabla p \\ \nabla \cdot v^{n+1} = 0 \end{cases} \Rightarrow -\nabla \cdot v^n = -\frac{\Delta t}{\rho} \nabla \cdot \nabla p$

$\nabla \cdot v^n = \sum$  $= \frac{1}{2}[(v_r - v_l).x + (v_t - v_b).y]$

& $-\nabla^2 p = \frac{1}{\Delta x^2}(4p_{i,j} - p_{i+1,j} - p_{i-1,j} - p_{i,j+1} - p_{i,j-1})$

Iterate over p[][] for 500 times by $p_{ij} = \frac{1}{4}[p_{i+1,j} + p_{i-1,j} + p_{i,j+1} + p_{i,j-1} - \nabla \cdot v_{ij}]$
to get the solution of p

$\frac{1}{2}[p_r - p_l, p_t - p_b]$ 

4. Update: $v^{n+1} = v^n - \frac{1}{\rho}\nabla p$

[Ref] Used Taichi example code: stable-fluid.py as starter code.