Note on Elasticities
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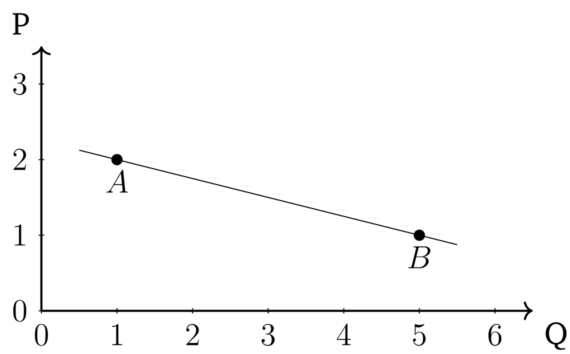
There is some confusion regarding the formula I use to calculate elasticities, its relation to the mid-point method, and why it is useful. This note should explain it.

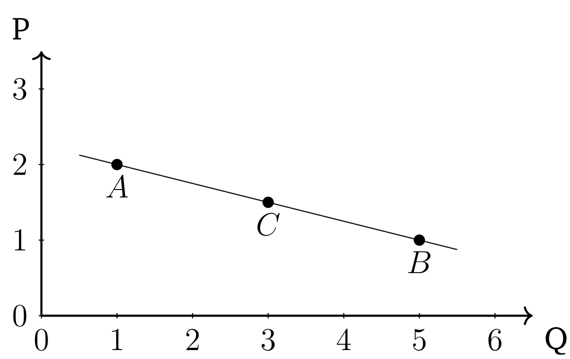
1. What elasticity does the mid-point method calculate? It calculates the elasticity of the point that is *between two given points*. Suppose you're given two points on the demand curve for popcorn. At a price of 2, people only want 1 bag of popcorn. At a price of 1, people want 5 bags of popcorn. Illustrated:



Now, suppose a problem asks you to find the elasticity between these two points. You throw the above numbers into the mid-point method formula:

$$\frac{\frac{Q_2 - Q_1}{(Q_2 + Q_1)/2}}{\frac{P_2 - P_1}{(P_2 + P_1)/2}} = \frac{\frac{5-1}{(5+1)/2}}{\frac{2-1}{(2+1)/2}} = \frac{\frac{4}{3}}{\frac{1}{3/2}} = \frac{4}{3} \cdot \frac{3}{2} = \frac{12}{6} = 2$$

We get an elasticity of 2. Note that we did **not** calculate the elasticity of point A or point B. We instead calculated the elasticity for the point between them: the point is marked as C below.



That was simple enough. But what if we were presented with a different problem?

2. What do you do when a question asks you for the elasticity *at a particular point*? Suppose we have the same problem as above with the same numbers but we are instead asked to find the elasticity at point B (price = 1, quantity = 5).

Now we have to do a lot of extra work to use the mid-point method. First we need to find two points on the curve equidistant from point B. We can use point A which is to the left of B, but now we need to come up with a new point D that is to the right. Let's go through the steps of finding point D and then the elasticity of point B.

1. Derive the equation for the demand curve. Use points A and B to find the slope: $\frac{1-2}{5-1} = -\frac{1}{4} = -0.25$.

2. Now use either point to find the intercept (I use point A):

$$y = mx + b$$
$$2 = -0.25 \cdot 1 + b$$
$$b = 2.25$$

So the demand curve equation is $P = -0.25Q + 2.25$.

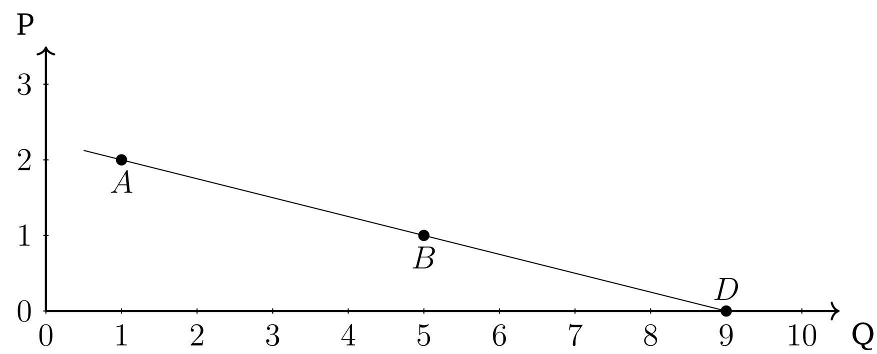
3. Find the quantity at point D. It will be the same distance as point A is to B, just in the opposite direction. The quantity at A (1) is 4 away from the quantity at B (5). So the quantity at point D is 9.

4. Find the price at point D. Just plug in the quantity ($Q = 9$) into the demand equation. $P = -0.25 \cdot 9 + 2.25 = 0$. So point D is price = 0, quantity = 9.

5. Now use the two points, A and D, in the mid-point method. By construction, the mid-point is exactly the point we are interested in, B.

$$\frac{\frac{Q_2 - Q_1}{(Q_2 + Q_1)/2}}{\frac{P_2 - P_1}{(P_2 + P_1)/2}} = \frac{\frac{9-1}{(9+1)/2}}{\frac{2-0}{(2+0)/2}} = \frac{\frac{8}{5}}{\frac{2}{2/2}} = \frac{8}{5} \cdot \frac{1}{2} = \frac{4}{5}$$

There we have it – the elasticity at point B is $\frac{4}{5}$. Graphically, these are the points:



But what if I told you that you only have to do the first step?

3. A Convenient Formula. I leave the derivation below (which you can ignore) and offer you a nice and easy formula to calculate the elasticity *at a point*. Make sure you have P on the y-axis and Q on the x-axis when finding the slope. In the case of price elasticity of demand, the elasticity at price p and quantity q is:

$$elasticity = \frac{1}{slope} \cdot \frac{p}{q}$$

Let's go back to part 2 where we were finding the elasticity at point B (price = 1, quantity = 5) to verify that it works. We did step 1, which gave us slope = $-\frac{1}{4}$. Let's plug it into the formula:

$$elasticity = -\frac{1}{1/4} \cdot \frac{1}{5} = -\frac{4}{5}$$

Now we take the absolute value because Mankiw prefers that. So the elasticity is $\frac{4}{5}$ which is exactly what we calculated from above! Pretty nice!

But does this formula help us in the situation in part 1, where we were finding the elasticity between two given points? It still works but it is the same amount of calculation. Remember that in that part, the mid-point method gives us the elasticity of point C. The mid-point (C) of A (p = 2, q = 1) and B (p = 1, q = 5) is p = $\frac{3}{2}$, q = 3. The slope = $-\frac{1}{4}$. Plugging into our formula:

$$elasticity = -\frac{1}{1/4} \cdot \frac{3/2}{3} = -4 \cdot \frac{3}{6} = -\frac{12}{6} = -2$$

So the elasticity is 2, which is what we calculated in part 1.

4. Formula Derivation – only for those interested.

We want the elasticity at a point on a demand curve with price p and quantity q. Note that the price elasticity of demand is defined as,

$$elasticity(p,q) = \frac{\text{\% change in quantity}}{\text{\% change in price}} = \frac{\frac{\triangle q}{q}}{\frac{\triangle p}{p}} = \frac{\triangle q}{\triangle p} \cdot \frac{p}{q}$$

Recognize that the demand curve defines a function $q(x)$ (the quantity $q(x)$ is demanded at price x), so what we really have is:

$$elasticity(p) = \frac{\triangle q(p)}{\triangle p} \cdot \frac{p}{q(p)}$$

Now take the limit as $\triangle p \to 0$:

$$\lim_{p' \to p} \frac{q(p') - q(p)}{p' - p} \cdot \frac{p}{q} = \frac{dq}{dp} \cdot \frac{p}{q(p)}$$

To apply this to the problems we usually see in class, we are given p, $q(p)$ (which we just call q), and the $slope = \frac{dp}{dq}$. So for point (q, p) and skipping some important details:

$$elasticity = \frac{dq}{dp} \cdot \frac{p}{q(p)} = \frac{1}{\frac{dp}{dq}} \cdot \frac{p}{q(p)} = \frac{1}{slope} \cdot \frac{p}{q}$$

But of course you do not need to know any of this.