Note on Elasticities
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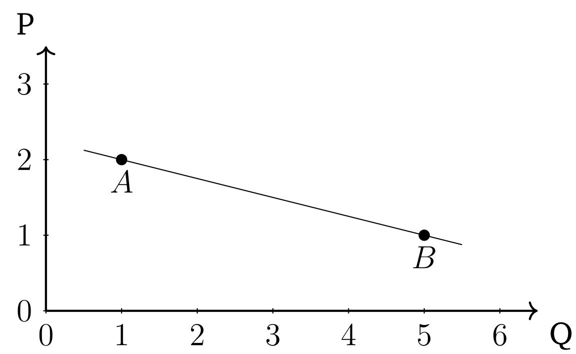
There is some confusion regarding the formula I use to calculate elasticities, its relation to the mid-point method, and why it is useful. This note should explain it.

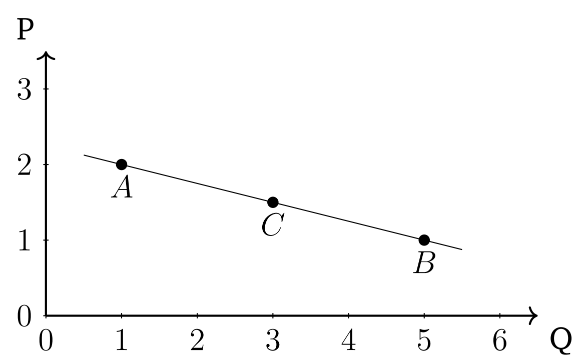
1. What elasticity does the mid-point method calculate? It calculates the elasticity of the point that is *between two given points*. Suppose you're given two points on the demand curve for popcorn. At a price of 2, people only want 1 bag of popcorn. At a price of 1, people want 5 bags of popcorn. Illustrated:



Now, suppose a problem asks you to find the elasticity between these two points. You throw the above numbers into the mid-point method formula:

$$\frac{\frac{Q_2 - Q_1}{(Q_2 + Q_1)/2}}{\frac{P_2 - P_1}{(P_2 + P_1)/2}} = \frac{\frac{5-1}{(5+1)/2}}{\frac{1-2}{(1+2)/2}} = \frac{\frac{4}{3}}{-\frac{1}{3/2}} = \frac{4}{3} \cdot -\frac{3}{2} = -\frac{12}{6} = -2$$

We get an elasticity of -2. Note that we did **not** calculate the elasticity of point A or point B. We instead calculated the elasticity for the point between them: the point is marked as C below.



That was simple enough. But what if we were presented with a different problem?

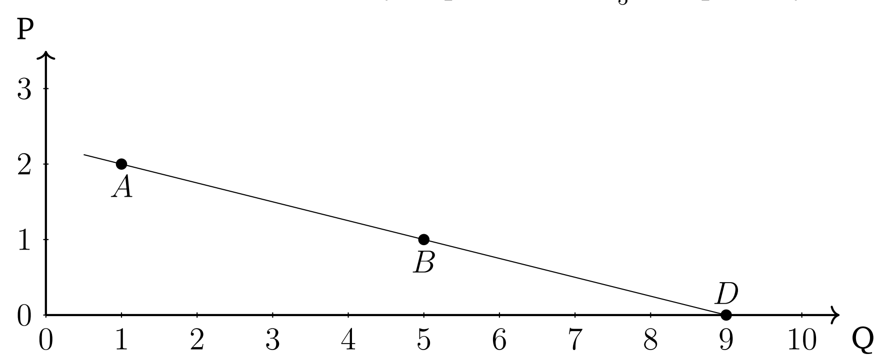
2. What do you do when you want the elasticity *at a particular point*, given two points? Suppose we have the same problem as above with the same numbers but we are instead asked to find the elasticity at point B (price = 1, quantity = 5).

Now we have to do extra work to use the mid-point method. First we need to find two points on the curve equidistant from point B. We can use point A which is to the left of B, but now we need to come up with a new point D that is to the right. Let's go through the steps of finding point D and then the elasticity of point B.

1. Calculate the differences in prices and quantities of A and B. Price difference = 1 - 2 = -1 and quantity difference = 5 - 1 = 4.
2. Use the distances to move from B in the opposite direction of A to find D. New price = 1 + (-1) = 0 and new quantity = 5 + 4 = 9.
3. Now use the two points, A and D, in the mid-point method. By construction, the mid-point is exactly the point we are interested in, B.

$$\frac{\frac{Q_2 - Q_1}{(Q_2 + Q_1)/2}}{\frac{P_2 - P_1}{(P_2 + P_1)/2}} = \frac{\frac{9-1}{(9+1)/2}}{\frac{0-2}{(0+2)/2}} = \frac{\frac{8}{5}}{\frac{-2}{1}} = \frac{8}{5} \cdot -\frac{1}{2} = -\frac{4}{5}$$

There we have it – the elasticity at point B is $-\frac{4}{5}$. Graphically, these are the points:



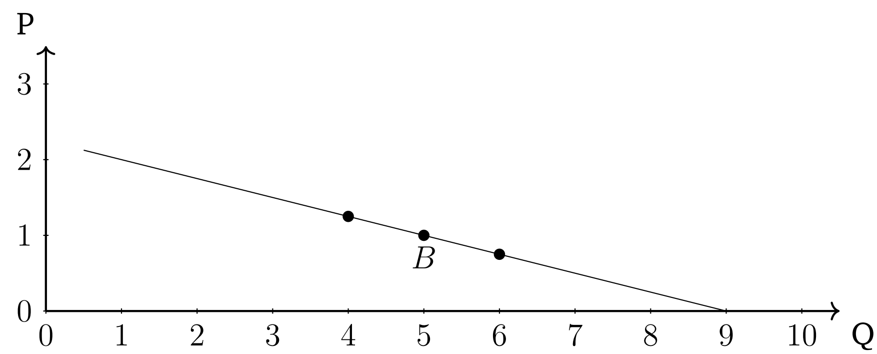
But what about another scenario?

3. What do you do when you want the elasticity *at a particular point*, given the demand equation? Suppose we are given the equation $P = -0.25Q + 2.25$ (which is conveniently the equation for the line in the example above) and want to find the elasticity at point B where price = 1 and quantity = 5. To find the elasticity, we need to choose two points equidistant from B and then use the mid-point method to find the elasticity. Let's go through the steps (I will pretend I don't already know the points A and D).

1. Move 1 to the left of B ($5 - 1 = 4$) and calculate the price: $P = -0.25 \cdot 4 + 2.25 = 1.25$. We have a point to the left: $(4, 1.25)$.
2. Move 1 to the right of B ($5 + 1 = 6$) and calculate the price: $P = -0.25 \cdot 6 + 2.25 = 0.75$. We have a point to the right: $(6, 0.75)$.
3. Now use the two points in the mid-point method. Again, by construction, the mid-point is exactly the point we are interested in, B.

$$\frac{\frac{Q_2 - Q_1}{(Q_2 + Q_1)/2}}{\frac{P_2 - P_1}{(P_2 + P_1)/2}} = \frac{\frac{6 - 4}{(6 + 4)/2}}{\frac{0.75 - 1.25}{(0.75 + 1.25)/2}} = \frac{\frac{2}{5}}{\frac{-0.5}{1}} = \frac{2}{5} \cdot -2 = -\frac{4}{5}$$

Here are the points we used and B:



Those are some examples of calculating elasticities using the mid-point method. But what if there was an easier way?

4. A Convenient Formula. I leave the derivation below (which you can ignore) and offer you a nice and easy formula to calculate the elasticity *at a point*. Make sure you have P on the y-axis and Q on the x-axis when finding the slope. In the case of price elasticity of demand, the elasticity at price p and quantity q is:

$$elasticity = \frac{1}{slope} \cdot \frac{p}{q}$$

Let's go back to part 2 and part 3 where we were finding the elasticity at point B (price = 1, quantity = 5) to verify that it works. All we need is the slope, which is $-\frac{1}{4}$, and plug it into the formula:

$$elasticity = -\frac{1}{1/4} \cdot \frac{1}{5} = -\frac{4}{5}$$

The elasticity is $-\frac{4}{5}$ which is exactly what we calculated from above! Pretty nice!

But does this formula help us in the situation in part 1, where we were finding the elasticity between two given points? It still works but it is the same amount of calculation. Remember that in that part, the mid-point method gives us the elasticity of point C. The mid-point (C) of A (p = 2, q = 1) and B (p = 1, q = 5) is p = $\frac{3}{2}$, q = 3. The slope = $-\frac{1}{4}$. Plugging into our formula:

$$elasticity = -\frac{1}{1/4} \cdot \frac{3/2}{3} = -4 \cdot \frac{3}{6} = -\frac{12}{6} = -2$$

So the elasticity is -2, which is what we calculated in part 1.

5. Formula Derivation – only for those interested.

We want the elasticity at a point on a demand curve with price p and quantity q. Note that the price elasticity of demand is defined as,

$$elasticity(p, q) = \frac{\text{\% change in quantity}}{\text{\% change in price}} = \frac{\frac{\triangle q}{q}}{\frac{\triangle p}{p}} = \frac{\triangle q}{\triangle p} \cdot \frac{p}{q}$$

Recognize that the demand curve defines a function $q(x)$ (the quantity $q(x)$ is demanded at price x), so what we really have is:

$$elasticity(p) = \frac{\triangle q(p)}{\triangle p} \cdot \frac{p}{q(p)}$$

Now take the limit as $\triangle p \to 0$:

$$\lim_{p' \to p} \frac{q(p') - q(p)}{p' - p} \cdot \frac{p}{q} = \frac{dq}{dp} \cdot \frac{p}{q(p)}$$

To apply this to the problems we usually see in class, we are given p, $q(p)$ (which we just call q), and the $slope = \frac{dp}{dq}$. So for point (q, p) and skipping some important details:

$$elasticity = \frac{dq}{dp} \cdot \frac{p}{q(p)} = \frac{1}{\frac{dp}{dq}} \cdot \frac{p}{q(p)} = \frac{1}{slope} \cdot \frac{p}{q}$$

But of course you do not need to know any of this.