AP Statistics Casino Game: Twins

Adam Omarali
Teacher: Mr. Stephens
Date: 9 February 2023

Stage 1: Game Design

Welcome to Twins, a game where you attempt to draw a number or face pair from a randomly determined amount of cards. This game uses a standard deck of 52 cards (no jokers) and a die.

Instructions & Rules:

1. The game begins with the player selecting how much they want to bet, $1, $2 or $3. The value will determine how much gets added to their dice roll explained in step 2.

2. The player then rolls a standard die. This value, in addition to the amount they bet, will determine the number of cards the player gets to draw according to the following table.

Bet Value	Cards Drawn (In addition to dice roll)
$1	+0
$2	+2
$3	+3

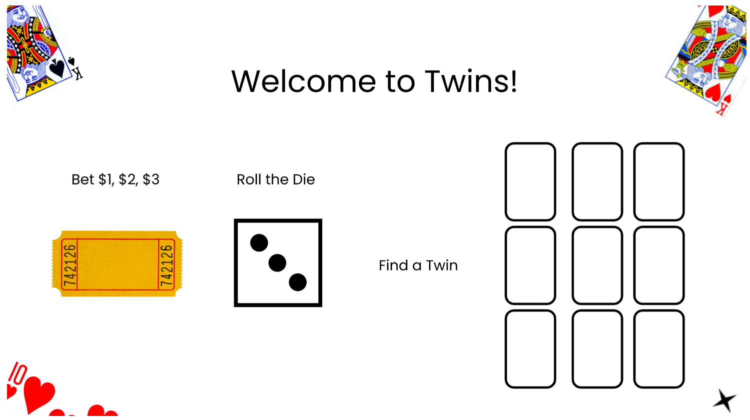
3. The dealer then draws the number of cards determined from the previous steps. These cards are drawn from the top of a standard, shuffled deck of cards, and displayed face-up one at a time.

4. Outcomes
   a. Player draws a face pair: Payout of $4
   b. Player draws a number pair: Payout of $2
   c. Player draws a face and number pair
      i. If the player bets $1: Payout $4
      ii. If the player bets more than $1: Payout $6
   d. Player does not draw a face or number pair: Payout of $0

   For these outcomes, only pairs matter. In other words, three and four-of-a-kind have no effect on the payout. The number of pairs also has no effect on the payout.

5. The game is reset by shuffling the cards and awaiting the next player.

Initial Mockup of Board Game Design



[All Random Events](#)
[10 Practice Games](#)

Stage 2: Theoretical Probabilities

This project utilizes two main chance processes, a singular dice roll and drawing X number of cards from a standard deck. We are interested in the probability of drawing a face pair, number pair, or both. This probability is dependent on the number of cards drawn, which is also dependent on the dice roll. Although the bet value will also affect the probability, it is not a chance process. As a result, separate probabilities and distributions will be formed for each possible bet amount ($1, $2, $3).

For each dice roll, there are four possible outcomes:

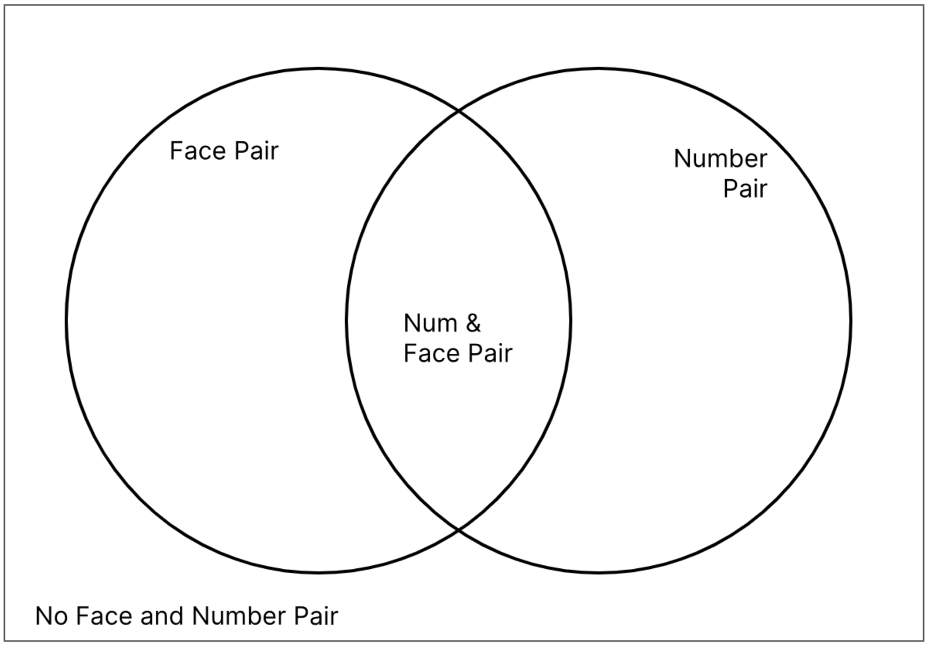
Face Pair	False	True	False	True
Number Pair	False	False	True	True

As a result we are interested in the following four probabilities for each dice roll value:

$$P(\neg Face \cap \neg Number \mid Roll = x)$$
$$P(Face \cap \neg Number \mid Roll = x)$$
$$P(Number \cap \neg Face \mid Roll = x)$$
$$P(Number \cap Face \mid Roll = x)$$

Here, "Face" represents a face pair and "Number" represents a number pair.

Drawing face pairs and drawing numbers are not mutually exclusive since one can draw a face and number pair in the same hand, but drawing no face and number pairs is mutually exclusive of face, number and face and number pairs.



The number of number pairs drawn is also dependent on the number of face cards drawn and vice versa. This means we will have to consider the probability for each case of n face/number cards drawn. All pairs drawn are also dependent on the number of cards drawn, as stated before. Given these facts, probability statements can be formed for the four possible events for a given hand.

$$P(\neg Face \cap \neg Number \mid Cards\ Drawn = x) = \frac{\binom{13}{x} * 4^x}{\binom{52}{x}}$$

$$P(Face \cap \neg Number \mid Cards\ Drawn = x)$$

$$= \sum_{i=0}^{x} (1 - P(\neg Face \mid Face\ Cards = i)) * P(\neg Number \mid Number\ Cards = x - i) * P(\ Picking\ i\ Face\ Cards)$$

$$= \sum_{i=0}^{x} (1 - \frac{\binom{3}{i} * 4^i}{\binom{12}{i}}) * \frac{\binom{10}{x-i} * 4^{x-i}}{\binom{40}{x-i}} * \frac{\binom{12}{i} * \binom{40}{x-i}}{\binom{52}{x}}$$

$$P(\neg Face \cap Number \mid Cards\ Drawn = x)$$

$$= \sum_{i=0}^{x} P(\neg Face \mid Face\ Cards = x - i)) * (1 - P(\neg Number \mid Number\ Cards = i)) * P(\ Picking\ i\ Number\ Cards)$$

$$= \sum_{i=0}^{x} \frac{\binom{3}{x-i} * 4^{x-i}}{\binom{12}{x-i}}) * (1 - \frac{\binom{10}{i} * 4^{x-i}}{\binom{40}{i}}) * \frac{\binom{40}{i} * \binom{12}{x-i}}{\binom{52}{x}}$$

$$P(Face \cap Number \mid Cards\ Drawn = x)$$

$$= \sum_{i=0}^{x} (1 - P(\neg Face \mid Face\ Cards = i)) * (1 - P(\neg Number \mid Number\ Cards = x - i)) * P(\ Picking\ i\ Face\ Cards)$$

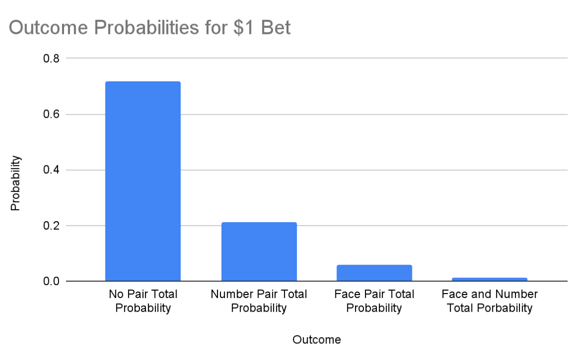
$$= \sum_{i=0}^{x} (1 - \frac{\binom{3}{i} * 4^{i}}{\binom{12}{i}}) * (1 - \frac{\binom{10}{x-i} * 4^{x-i}}{\binom{40}{x-i}}) * \frac{\binom{12}{i} * \binom{40}{x-i}}{\binom{52}{x}}$$

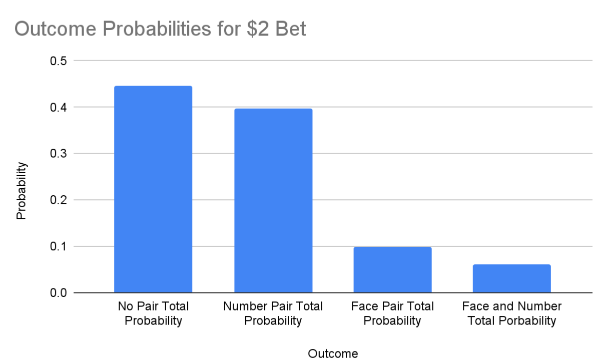
The probability statements take into account the number of cards drawn as dependent, and that drawing a varying number of face and number cards result in different probabilities (accounted by the sum via iterating through each case of 1,2 3... face/number cards drawn). Since we also care about *at least* one face/number pair appearing, we make use of the complement rule in a variety of areas. The calculations are performed for each bet value, which will influence “Cards Drawn”.

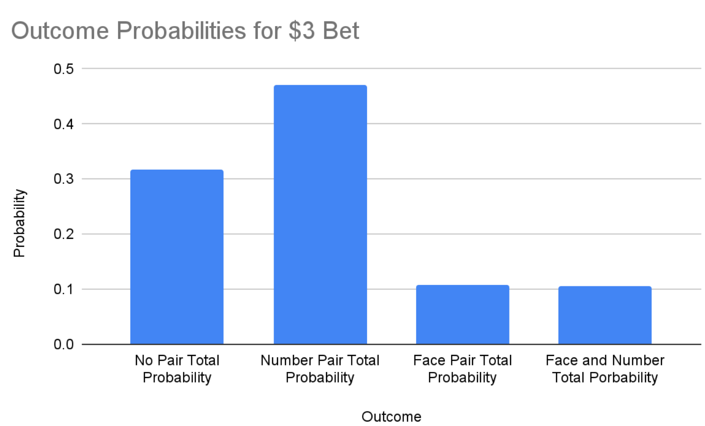
[Probability Distribution Tables]

Factoring in the dice roll just requires each probability to be multiplied by ⅙, since there is a ⅙ chance of that event occurring and a dice roll of the specified value. This is shown in the probability table as ‘Total Probability’. Notice the sum row highlights the total probability sums to 1 for each bet value, indicating accurate calculations.

For probability distributions, we can display the probability for each outcome for each bet type influenced by the roll of a die.

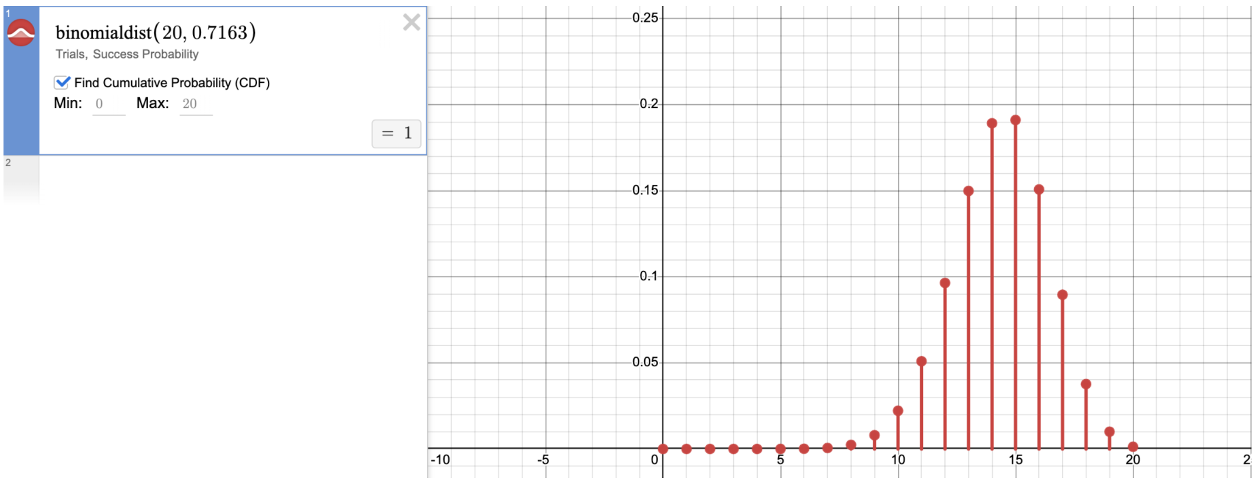






We can also consider a binomial distribution for a discrete binomial random variable Y = # of trials with no pairs for $1 bets. This meets the requirements, most importantly binary, since each trial either has no pairs or has pairs. Given the probability from the probability tables, we can also form a binomial distribution assuming n=20 trials.

$$P(\neg Face \cap \neg Number \mid Bet = 1) = 0.7163$$



The distribution is skewed left since n is low, and p is high.

Finally, we can also form discrete random variables for the house's profit given each bet value. A, is the discrete random variable describing the chance process for $1 bets. B and C are for $2, $3 bets respectively.

Event	Payout	Probability	(Bet - Payout) * Probability
No Pair	$0	0.7163088214	0.7163088214
Face Pair	$4	0.0592969037	-0.1778907111
Number Pair	$2	0.2135000645	-0.2135000645
Face and Number	$4	0.01240247326	-0.03720741979

$$\mu_A = E(A) = \sum A_i * p_i = 0.2877$$

$$\mu_B = E(B) = \sum B_i * p_i = 0.4547$$

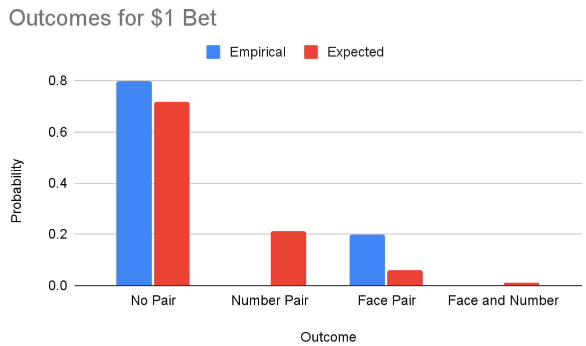
$$\mu_C = E(C) = \sum C_i * p_i = 0.9986$$

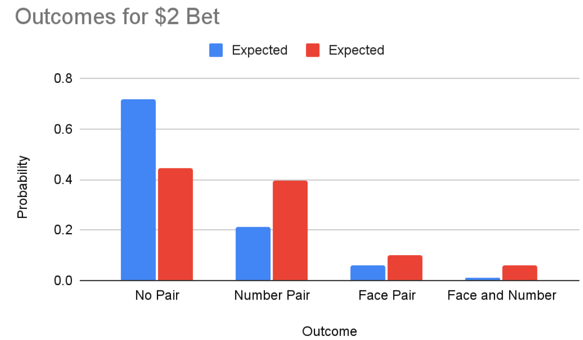
An event would be the roll of dice and the X drawn cards, while the outcome is the presence of a number pair, face pair, number and face pair, and no pair.

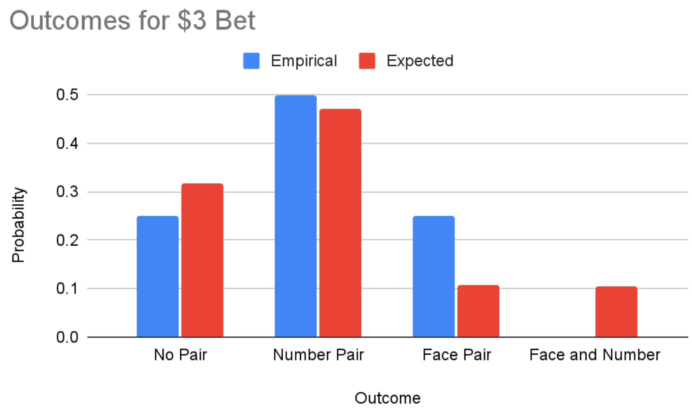
Stage 3: Casino Game Day

Casino Day Data

$1 Bet	Outcome Occurances	Probability
No Pair	8	0.8
Number Pair	0	0
Face Pair	2	0.2
Face and Number	0	0
Expected Value	0.2	
Expected Value Error	-0.0877	
$2 Bet		
No Pair	12	0.5
Number Pair	7	0.2916666667
Face Pair	2	0.08333333333
Face and Number	3	0.125
Expected Value	0.3333333333	
Expected Value Error	-0.1214	
$3 Bet		
No Pair	1	0.25
Number Pair	2	0.2
Face Pair	1	0.1
Face and Number	0	0
Expected Value	0.85	
Expected Value Error	0.0014	







The expected value and probabilities all have quite small margins of error. Any margin of error is due to the random chance process at hand and the limited number of trials. Only as the number of trials increases infinitely can we be certain theoretical and empirical probabilities align. It's also important to note the expected probability has some value for all outcomes, even very unlikely outcomes such as drawing a face and number pair. Since the number of trials is so low, these low-probability events never occurred empirically.

Reflection:

The most difficult aspect of this project was the probability calculations. Performing and ensuring these were accurate before settling on the game may have eased the difficulty of the project. In addition, the game takes a while to explain and run, resulting in a longer game time and therefore fewer opportunities to play and for the casino to make money. Having multiple tables run the game or selecting a game that can be played quicker would help increase casino profit.