MTH8408 : Méthodes d'optimisation et contrôle optimal

Laboratoire 5: Optimisation avec contraintes et calcul variationnel

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Rédigé par Adam Osmani

In [ ]:
```
using Krylov, LinearAlgebra, Logging, NLPModels, NLPModelsIpopt, Printf, SolverCore
```

In [ ]:
```
import Pkg; Pkg.add("PDENLPModels")
import Pkg; Pkg.add("Gridap")
using PDENLPModels, Gridap
```

```
   Resolving package versions...
  No Changes to `C:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\P
roject.toml`
  No Changes to `C:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\M
anifest.toml`
Precompiling project...
  ? PDENLPModels
   Resolving package versions...
  No Changes to `C:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\P
roject.toml`
  No Changes to `C:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\M
anifest.toml`
Precompiling project...
  ? PDENLPModels
WARNING: Method definition testargs(Gridap.Arrays.PosNegReindex{A, B} where B where
A, Integer) in module Arrays at C:\Users\adamo\.julia\packages\Gridap\EZQEK\src\Arr
ays\PosNegReindex.jl:10 overwritten in module PDENLPModels at C:\Users\adamo\.julia
\packages\PDENLPModels\pW0Iv\src\PDENLPModels.jl:16.
ERROR: Method overwriting is not permitted during Module precompilation. Use `__pre
compile__(false)` to opt-out of precompilation.
WARNING: method definition for _compute_hess_structure at C:\Users\adamo\.julia\pac
kages\PDENLPModels\pW0Iv\src\hessian_struct_nnzh_functions.jl:70 declares type vari
able T but does not use it.
WARNING: method definition for _compute_hess_structure at C:\Users\adamo\.julia\pac
kages\PDENLPModels\pW0Iv\src\hessian_struct_nnzh_functions.jl:74 declares type vari
able T but does not use it.
```

Quelques commentaires en Julia

Les kwargs: choix optionnels

Dans le projet du dernier labo, une des questions demandait d'ajouter une option pour utiliser la fonction `lsmr` ou `lsqr`. C'est le cas typique d'arguments optionnels:

- On veut proposer un choix par défaut à l'utilisateur, par exemple `lsqr` ;
- On veut laisser la possibilité à l'utilisateur de changer;
- On voudrait aussi pouvoir ajouter d'autres par la suite (sans avoir à tout modifier).

In [ ]:
```
function dsol(A, b, ϵ; solver :: Function = lsqr)
    (d, stats) = solver(A, b, atol = ϵ)
    return d
end
```

```
dsol (generic function with 1 method)
```

A noter que l'on donne des valeurs par défaut aux arguments qui apparaissent après le `;` .

Exercice 1: Pénalité quadratique pour les ADNLPModels

Dans cet exercice, on va étudier une version simple d'une méthode de pénalité quadratique pour les problèmes d'optimisation avec contraintes d'égalité.

```
math
min f(x) s.à c(x) = 0.
```

Dans les labos précédents, on a déjà utilisé un NLPModel particulier, le ADNLPModel:

In [ ]:
```
using ADNLPModels, LinearAlgebra, Test
fH(x) = (x[2]+x[1].^2-11)^2 + (x[1]+x[2].^2-7)^2
x0H = [10., 20.]
cH(x) = [x[1]-1]
himmelblau = ADNLPModel(fH, x0H, cH, [0.], [0.])
```

```
ADNLPModel - Model with automatic differentiation backend ADNLPModels.ForwardDiffAD
{ForwardDiff.GradientConfig{ForwardDiff.Tag{typeof(fH), Float64}, Float64, 2, Vecto
r{ForwardDiff.Dual{ForwardDiff.Tag{typeof(fH), Float64}, Float64, 2}}}}(3, 2, Forwa
rdDiff.GradientConfig{ForwardDiff.Tag{typeof(fH), Float64}, Float64, 2, Vector{Forw
ardDiff.Dual{ForwardDiff.Tag{typeof(fH), Float64}, Float64, 2}}}((Partials(1.0, 0.
0), Partials(0.0, 1.0)), ForwardDiff.Dual{ForwardDiff.Tag{typeof(fH), Float64}, Flo
at64, 2}[Dual{ForwardDiff.Tag{typeof(fH), Float64}}(1.000782451328e-311,6.952696361
10307e-310,1.000782451336e-311), Dual{ForwardDiff.Tag{typeof(fH), Float64}}(6.95269
636110307e-310,1.000782451344e-311,6.95269636110307e-310)]))
  Problem name: Generic
   All variables: ████████████████████ 2     All constraints: ████████████████████
1
            free: ████████████████████ 2                free: ···················· 0
           lower: ···················· 0              lower: ···················· 0
           upper: ···················· 0              upper: ···················· 0
         low/upp: ···················· 0            low/upp: ···················· 0
           fixed: ···················· 0              fixed: ████████████████████ 1
          infeas: ···················· 0             infeas: ···················· 0
            nnzh: (  0.00% sparsity)   3                     linear: ···················· 0
                                                        nonlinear: ████████████████████
1
                                                             nnzj: (  0.00% sparsity)
2

  Counters:
             obj: ···················· 0                 grad: ···················· 0
cons: ···················· 0
        cons_lin: ···················· 0             cons_nln: ···················· 0
jcon: ···················· 0
           jgrad: ···················· 0                  jac: ···················· 0
jac_lin: ···················· 0
         jac_nln: ···················· 0                jprod: ···················· 0             j
prod_lin: ···················· 0
       jprod_nln: ···················· 0              jtprod: ···················· 0           jt
prod_lin: ···················· 0
      jtprod_nln: ···················· 0                 hess: ···················· 0
hprod: ···················· 0
            jhess: ···················· 0              jhprod: ···················· 0
```

Attention: dans toute la suite de l'exercice on suppose que les bornes sur les contraintes `nlp.meta.lcon` et `nlp.meta.ucon` sont 0 pour simplifier.

Question 1: Transformer un ADNLPModel en un problème pénalisé

Coder la fonction `quad_penalty_adnlp` qui prend en entrée un ADNLPModel, et un paramètre ρ et qui retourne un nouveau ADNLPModel qui correspond au problème sans contrainte:

$$\min_x f(x) + \frac{\rho}{2}\|c(x)\|^2.$$

Remarque: on peut accèder aux fonctions f et c par `NLPModels.obj()` et `NLPModels.cons()`.

In [ ]:
```
function quad_penalty_adnlp(nlp :: ADNLPModel, ρ :: Real)
    # TODO
    f(x) = obj(nlp,x) + ρ/2*norm(cons(nlp,x))^2
    x0 = nlp.meta.x0
    nlp_quad = ADNLPModel(f,x0)

    if((nlp.meta.lcon ≠ [0.0]) || (nlp.meta.ucon ≠ [0.0]))
        print("lcon or ucon not equal to 0.0\n")
        print("lcon: ",nlp.meta.lcon,"\n")
        print("ucon: ",nlp.meta.ucon,"\n")
    end

   return nlp_quad
end
```

```
quad_penalty_adnlp (generic function with 1 method)
```

In [ ]:
```
#Faire des tests pour vérifier que ça fonctionne.
#= fH(x) = (x[2]+x[1].^2-11).^2+(x[1]+x[2].^2-7).^2
x0H = [10., 20.]
himmelblau = ADNLPModel(fH, x0H) =#

himmelblau_quad = quad_penalty_adnlp(himmelblau, 1)
@test himmelblau_quad.meta.ncon == 0
@test obj(himmelblau_quad, zeros(2)) == 170.5
```

```
Test Passed
```

In [ ]:
```
#Ajouter au moins un autre test similaire avec des contraintes.
```

In [ ]:
```
# Ajouter un test au cas ou `nlp.meta.lcon` ou `nlp.meta.ucon` ont des composantes
```

Question 2: KKT

Coder une fonction `KKT_eq_constraint(nlp :: AbstractNLPModel, x, λ)` qui vérifie si le point `x` avec multiplicateur de Lagrange `λ` satisfait les conditions KKT d'un problème avec contraintes d'égalités.

In [ ]:
```
function KKT_eq_constraint(nlp :: AbstractNLPModel, x, λ)
   # TODO
   cx = cons(nlp,x)
   gfx = grad(nlp,x)
   gcx = grad(cx,x)
   if(gfx == dot(λ,gcx))
      return true
   else
      return false
   end
end
```

```
KKT_eq_constraint (generic function with 1 method)
```

In [ ]:
```
#test
using NLPModelsIpopt
stats = ipopt(himmelblau)
KKT_eq_constraint(himmelblau,stats.solution,himmelblau.meta.y0)
```

```
******************************************************************************
This program contains Ipopt, a library for large-scale nonlinear optimization.
 Ipopt is released as open source code under the Eclipse Public License (EPL).
         For more information visit https://github.com/coin-or/Ipopt
******************************************************************************

This is Ipopt version 3.14.14, running with linear solver MUMPS 5.6.2.

Number of nonzeros in equality constraint Jacobian...:        2
Number of nonzeros in inequality constraint Jacobian.:        0
Number of nonzeros in Lagrangian Hessian.............:        3

Total number of variables............................:        2
                     variables with only lower bounds:        0
                variables with lower and upper bounds:        0
                     variables with only upper bounds:        0
Total number of equality constraints.................:        1
Total number of inequality constraints...............:        0
        inequality constraints with only lower bounds:        0
   inequality constraints with lower and upper bounds:        0
        inequality constraints with only upper bounds:        0

iter    objective    inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du alpha_pr  ls
   0  1.7429000e+05 9.00e+00 1.00e+02  -1.0 0.00e+00    -  0.00e+00 0.00e+00   0
   1  3.0904536e+04 0.00e+00 2.92e+01  -1.0 9.00e+00    -  1.00e+00 1.00e+00f  1
   2  5.8693387e+03 0.00e+00 8.57e+00  -1.0 4.39e+00    -  1.00e+00 1.00e+00f  1
   3  1.0806931e+03 0.00e+00 2.48e+00  -1.0 2.87e+00    -  1.00e+00 1.00e+00f  1
   4  2.0907560e+02 0.00e+00 6.88e-01  -1.0 1.82e+00    -  1.00e+00 1.00e+00f  1
   5  7.0572121e+01 0.00e+00 1.69e-01  -1.7 1.06e+00    -  1.00e+00 1.00e+00f  1
   6  5.5541596e+01 0.00e+00 2.84e-02  -2.5 4.92e-01    -  1.00e+00 1.00e+00f  1
   7  5.4952659e+01 0.00e+00 1.57e-03  -2.5 1.23e-01    -  1.00e+00 1.00e+00f  1
   8  5.4950704e+01 0.00e+00 5.88e-06  -3.8 7.65e-03    -  1.00e+00 1.00e+00f  1
   9  5.4950704e+01 0.00e+00 8.33e-11  -8.6 2.88e-05    -  1.00e+00 1.00e+00f  1

Number of Iterations....: 9

                                   (scaled)                 (unscaled)
Objective...............:   1.6929787436250029e-01    5.4950704060580350e+01
Dual infeasibility......:   8.3309050636453264e-11    2.7040451655580000e-08
Constraint violation....:   0.0000000000000000e+00    0.0000000000000000e+00
Variable bound violation:   0.0000000000000000e+00    0.0000000000000000e+00
Complementarity.........:   0.0000000000000000e+00    0.0000000000000000e+00
Overall NLP error.......:   8.3309050636453264e-11    2.7040451655580000e-08


Number of objective function evaluations             = 10
Number of objective gradient evaluations             = 10
Number of equality constraint evaluations            = 10
Number of inequality constraint evaluations          = 0
Number of equality constraint Jacobian evaluations   = 10
Number of inequality constraint Jacobian evaluations = 0
Number of Lagrangian Hessian evaluations             = 9
Total seconds in IPOPT                               = 2.876

EXIT: Optimal Solution Found.
```

```
MethodError: no method matching grad(::Vector{Float64}, ::Vector{Float64})


Closest candidates are:

  grad(!Matched::AbstractNLPModel, ::AbstractVector)

   @ NLPModels C:\Users\adamo\.julia\packages\NLPModels\XBcWL\src\nlp\api.jl:28




Stacktrace:

 [1] KKT_eq_constraint(nlp::ADNLPModel{Float64, Vector{Float64}}, x::Vector{Float6
4}, λ::Vector{Float64})

   @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab5\Lab
5-notebook.ipynb:5

 [2] top-level scope

   @ c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab5\Lab5-not
ebook.ipynb:4
```

Question 3: méthode de pénalité quadratique

In [ ]:
```
using NLPModelsIpopt
```

In [ ]:
```
function quad_penalty(nlp      :: AbstractNLPModel,
                      x        :: AbstractVector;
                      ϵ        :: AbstractFloat = 1e-3,
                      η        :: AbstractFloat = 1e6,
                      σ        :: AbstractFloat = 2.0,
                      max_eval :: Int = 1_000,
                      max_time :: AbstractFloat = 60.,
                      max_iter :: Int = typemax(Int64)
                      )
    ##### Initialiser cx et gx au point x;
    cx = cons(nlp,x)# Initialiser la violation des contraintes
    gx = grad(nlp,x)# Initialiser le gradient
    ########################################################
    normcx = normcx_old = norm(cx)

    ρ = 1.

    iter = 0

    el_time = 0.0
    tired   = neval_cons(nlp) > max_eval || el_time > max_time
    status  = :unknown

    start_time = time()
    too_small  = false
    normdual   = norm(gx) #exceptionnellement on ne va pas vérifier toute l'optimal
    optimal    = max(normcx, normdual) ≤ ϵ

    nlp_quad   = quad_penalty_adnlp(nlp, ρ)

    @info log_header([:iter, :nf, :primal, :status, :nd, :Δ],
    [Int, Int, Float64, String, Float64, Float64],
    hdr_override=Dict(:nf => "#F", :primal => "‖F(x)‖", :nd => "‖d‖"))

    while !(optimal || tired || too_small)

        #Appeler Ipopt pour résoudre le problème pénalisé en partant du point x0 =
        #utiliser l'option print_level = 0 pour enlever les affichages d'ipopt.
        stats = ipopt(nlp, x0 = x, print_level = 0)#...
        ################################################

        if stats.status == :first_order
            ###### Mettre à jour cx avec la solution renvoyé par Ipopt
            x = stats.solution#...
            cx = cons(nlp,x)#...
            ############################################################
            normcx_old = normcx
            normcx = norm(cx)
        end

        if normcx_old > 0.95 * normcx
            ρ *= σ
        end

        @info log_row(Any[iter, neval_cons(nlp), normcx, stats.status])

        nlp_quad   = quad_penalty_adnlp(nlp, ρ)

        el_time = time() - start_time
```

```
        el_time      = time() - start_time
        iter   += 1
        many_evals   = neval_cons(nlp) > max_eval
        iter_limit   = iter > max_iter
        tired        = many_evals || el_time > max_time || iter_limit || ρ ≥ η
        ##### Utiliser la réalisabilité dual renvoyé par Ipopt pour `normdual`
        normdual     = stats.dual_feas#....
        ########################################################################
        optimal      = max(normcx, normdual) ≤ ϵ
    end

    status = if optimal
        :first_order
    elseif tired
        if neval_cons(nlp) > max_eval
            :max_eval
        elseif el_time > max_time
            :max_time
        elseif iter > max_iter
            :max_iter
        else
            :unknown_tired
        end
    elseif too_small
        :stalled
    else
        :unknown
    end

    return GenericExecutionStats(nlp, status = status, solution = x,
                                 objective = obj(nlp, x),
                                 primal_feas = normcx,
                                 dual_feas = normdual,
                                 iter = iter,
                                 elapsed_time = el_time,
                                 solver_specific = Dict(:penalty => ρ))
end
```

```
quad_penalty (generic function with 1 method)
```

In [ ]:

```
#Faire des tests pour vérifier que ça fonctionne.
stats = quad_penalty(himmelblau, x0H)
@test stats.status == :first_order
@test stats.solution ≈ [1.0008083416169895, 2.709969135758311] atol=1e-2
@test norm(cons(himmelblau, stats.solution)) ≈ 0. atol=1e-3
```

```
┌ Info:   iter      #F    ‖F(x)‖           status       ‖d‖         Δ
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab5\Lab
5-notebook.ipynb:31
┌ Info:      0      24   0.0e+00      first_order
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab5\Lab
5-notebook.ipynb:55
Test Passed
```

Vérifier que la solution rendue vérifie les conditions KKT avec la fonction de la question précédente.

In [ ]:

```
# TODO
```

In [ ]:
```
#Fichier de tests à demander.
KKT_eq_constraint(himmelblau,stats.solution,himmelblau.meta.y0)
```

```
MethodError: no method matching grad(::Vector{Float64}, ::Vector{Float64})



Closest candidates are:

  grad(!Matched::AbstractNLPModel, ::AbstractVector)

   @ NLPModels C:\Users\adamo\.julia\packages\NLPModels\XBcWL\src\nlp\api.jl:28




Stacktrace:

 [1] KKT_eq_constraint(nlp::ADNLPModel{Float64, Vector{Float64}}, x::Vector{Float6
4}, λ::Vector{Float64})

   @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab5\Lab
5-notebook.ipynb:5

 [2] top-level scope

   @ c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab5\Lab5-not
ebook.ipynb:2
```

Exercice 2: Calcul Variationnel

Dans cet exercice, on considère le problème de calcul variationnel suivant:

$$\min \int_0^1 (\dot{x}(t)^2 + 2x(t)^2)e^t dt, \quad x(0) = 0, x(1) = e - e^{-2}$$

modélisé avec `PDENLPModels` .

In [ ]:
```
function cv_model(n :: Int)

  domain = (0,1) # set the domain
  partition = n
  model = CartesianDiscreteModel(domain,partition) # set discretization

  labels = get_face_labeling(model)
  add_tag_from_tags!(labels,"diri1",[2])
  add_tag_from_tags!(labels,"diri0",[1]) # boundary conditions

  order=1
  valuetype=Float64
  reffe = ReferenceFE(lagrangian, valuetype, order)
  V0 = TestFESpace(model, reffe; conformity=:H1, dirichlet_tags=["diri0","diri1"])
  U = TrialFESpace(V0,[0., exp(1)-exp(-2)])

  trian = Triangulation(model)
  degree = 2
  dΩ = Measure(trian,degree) # integration machinery

  # Our objective function
  w(x) = exp(x[1])
  function f(y)
    ∫((∇(y)⊙∇(y) + 2 * y * y) * w) * dΩ
  end

  xin = zeros(Gridap.FESpaces.num_free_dofs(U))
  nlp = GridapPDENLPModel(xin, f, trian, U, V0)
  return nlp
end
```

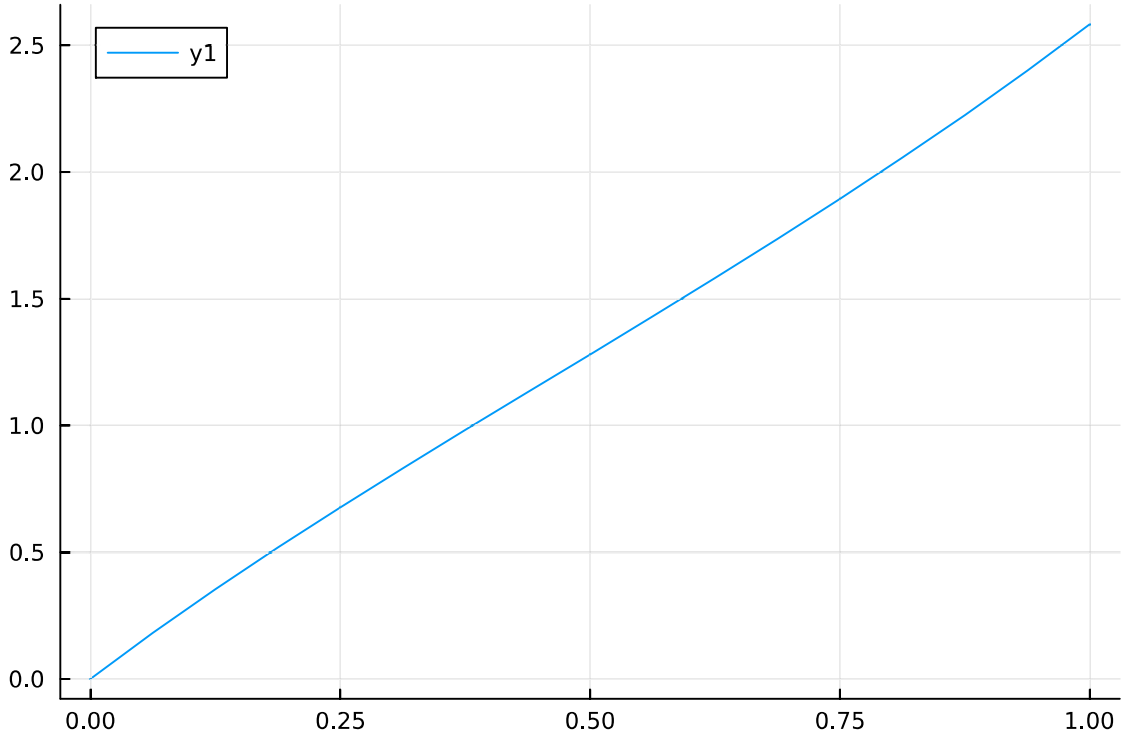
```
cv_model (generic function with 1 method)
```

Question 1: Résoudre

Résoudre le NLPModel généré par la fonction `cv_model` pour `n = 16` avec `ipopt` et afficher la solution (attention la solution rendue ne contient pas les valeurs aux bords qu'il faut rajouter).

In [ ]:
```
# TODO
using Plots
cv16 = cv_model(16)
stats = ipopt(cv16)

x = range(0,1,length = 17)
y = [0;stats.solution;exp(1)-exp(-2)]
plot(x,y)
```

```
This is Ipopt version 3.14.14, running with linear solver MUMPS 5.6.2.

Number of nonzeros in equality constraint Jacobian...:        0
Number of nonzeros in inequality constraint Jacobian.:        0
Number of nonzeros in Lagrangian Hessian.............:       44

Total number of variables............................:       15
                     variables with only lower bounds:        0
                variables with lower and upper bounds:        0
                     variables with only upper bounds:        0
Total number of equality constraints.................:        0
Total number of inequality constraints...............:        0
        inequality constraints with only lower bounds:        0
   inequality constraints with lower and upper bounds:        0
        inequality constraints with only upper bounds:        0

iter    objective    inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du alpha_pr  ls
   0  2.8202747e+02 0.00e+00 1.00e+02  -1.0 0.00e+00    -  0.00e+00 0.00e+00   0
   1  2.0987074e+01 0.00e+00 3.02e-14  -1.0 2.40e+00    -  1.00e+00 1.00e+00f  1

Number of Iterations....: 1

                                   (scaled)                 (unscaled)
Objective...............:   9.6484800135597375e+00    2.0987074475792042e+01
Dual infeasibility......:   3.0216136547373433e-14    6.5725203057809267e-14
Constraint violation....:   0.0000000000000000e+00    0.0000000000000000e+00
Variable bound violation:   0.0000000000000000e+00    0.0000000000000000e+00
Complementarity.........:   0.0000000000000000e+00    0.0000000000000000e+00
Overall NLP error.......:   3.0216136547373433e-14    6.5725203057809267e-14


Number of objective function evaluations             = 2
Number of objective gradient evaluations             = 2
Number of equality constraint evaluations            = 0
Number of inequality constraint evaluations          = 0
Number of equality constraint Jacobian evaluations   = 0
Number of inequality constraint Jacobian evaluations = 0
Number of Lagrangian Hessian evaluations             = 1
Total seconds in IPOPT                               = 5.260

EXIT: Optimal Solution Found.
```



Question 2: Convergence en n

Afficher sur un même graphique la solution obtenue par `ipopt` pour plusieurs valeurs de `n`.

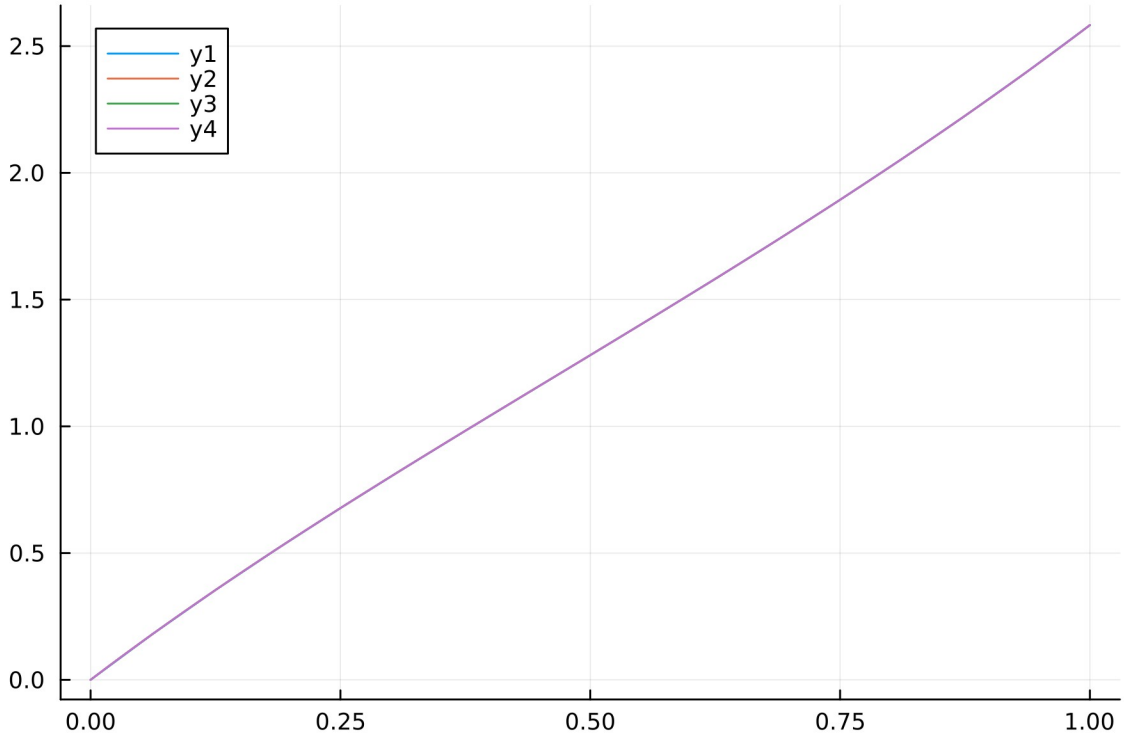
In [ ]:

```
n = 16
cv = cv_model(n)
stats = ipopt(cv)

x = range(0,1,length=(n+1))
y = [0;stats.solution;exp(1)-exp(-2)]
plot(x,y)

n = 64
cv = cv_model(n)
stats = ipopt(cv)

x = range(0,1,length=(n+1))
y = [0;stats.solution;exp(1)-exp(-2)]
plot!(x,y)

n = 128
cv = cv_model(n)
stats = ipopt(cv)

x = range(0,1,length=(n+1))
y = [0;stats.solution;exp(1)-exp(-2)]
plot!(x,y)

x = range(0,1,length=128)
y = exp.(x) - exp.(-2*x)
plot!(x,y)
```

```
This is Ipopt version 3.14.14, running with linear solver MUMPS 5.6.2.

Number of nonzeros in equality constraint Jacobian...:        0
Number of nonzeros in inequality constraint Jacobian.:        0
Number of nonzeros in Lagrangian Hessian.............:       44

Total number of variables............................:       15
                     variables with only lower bounds:        0
                variables with lower and upper bounds:        0
                     variables with only upper bounds:        0
Total number of equality constraints.................:        0
Total number of inequality constraints...............:        0
        inequality constraints with only lower bounds:        0
   inequality constraints with lower and upper bounds:        0
        inequality constraints with only upper bounds:        0

iter    objective    inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du alpha_pr  ls
   0  2.8202747e+02 0.00e+00 1.00e+02  -1.0 0.00e+00    -  0.00e+00 0.00e+00   0
   1  2.0987074e+01 0.00e+00 3.02e-14  -1.0 2.40e+00    -  1.00e+00 1.00e+00f  1

Number of Iterations....: 1

                                   (scaled)                 (unscaled)
Objective...............:   9.6484800135597375e+00    2.0987074475792042e+01
Dual infeasibility......:   3.0216136547373433e-14    6.5725203057809267e-14
Constraint violation....:   0.0000000000000000e+00    0.0000000000000000e+00
Variable bound violation:   0.0000000000000000e+00    0.0000000000000000e+00
Complementarity.........:   0.0000000000000000e+00    0.0000000000000000e+00
Overall NLP error.......:   3.0216136547373433e-14    6.5725203057809267e-14


Number of objective function evaluations             = 2
Number of objective gradient evaluations             = 2
Number of equality constraint evaluations            = 0
Number of inequality constraint evaluations          = 0
Number of equality constraint Jacobian evaluations   = 0
Number of inequality constraint Jacobian evaluations = 0
Number of Lagrangian Hessian evaluations             = 1
Total seconds in IPOPT                               = 0.072

EXIT: Optimal Solution Found.
This is Ipopt version 3.14.14, running with linear solver MUMPS 5.6.2.

Number of nonzeros in equality constraint Jacobian...:        0
Number of nonzeros in inequality constraint Jacobian.:        0
Number of nonzeros in Lagrangian Hessian.............:      188

Total number of variables............................:       63
                     variables with only lower bounds:        0
                variables with lower and upper bounds:        0
                     variables with only upper bounds:        0
Total number of equality constraints.................:        0
Total number of inequality constraints...............:        0
        inequality constraints with only lower bounds:        0
   inequality constraints with lower and upper bounds:        0
        inequality constraints with only upper bounds:        0

iter    objective    inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du alpha_pr  ls
   0  1.1518283e+03 0.00e+00 1.00e+02  -1.0 0.00e+00    -  0.00e+00 0.00e+00   0
```

```
   1  2.0986033e+01 0.00e+00 4.46e-14  -1.0 2.54e+00    -  1.00e+00 1.00e+00f  1

Number of Iterations....: 1

                                   (scaled)                 (unscaled)
Objective...............:   2.3536090441322592e+00    2.0986032544330911e+01
Dual infeasibility......:   4.4625409358967054e-14    3.9790393202565605e-13
Constraint violation....:   0.0000000000000000e+00    0.0000000000000000e+00
Variable bound violation:   0.0000000000000000e+00    0.0000000000000000e+00
Complementarity.........:   0.0000000000000000e+00    0.0000000000000000e+00
Overall NLP error.......:   4.4625409358967054e-14    3.9790393202565605e-13


Number of objective function evaluations             = 2
Number of objective gradient evaluations             = 2
Number of equality constraint evaluations            = 0
Number of inequality constraint evaluations          = 0
Number of equality constraint Jacobian evaluations   = 0
Number of inequality constraint Jacobian evaluations = 0
Number of Lagrangian Hessian evaluations             = 1
Total seconds in IPOPT                               = 0.048

EXIT: Optimal Solution Found.
This is Ipopt version 3.14.14, running with linear solver MUMPS 5.6.2.

Number of nonzeros in equality constraint Jacobian...:        0
Number of nonzeros in inequality constraint Jacobian.:        0
Number of nonzeros in Lagrangian Hessian.............:      380

Total number of variables............................:      127
                     variables with only lower bounds:        0
                variables with lower and upper bounds:        0
                     variables with only upper bounds:        0
Total number of equality constraints.................:        0
Total number of inequality constraints...............:        0
        inequality constraints with only lower bounds:        0
   inequality constraints with lower and upper bounds:        0
        inequality constraints with only upper bounds:        0

iter    objective    inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du alpha_pr  ls
   0  2.3123716e+03 0.00e+00 1.00e+02  -1.0 0.00e+00    -  0.00e+00 0.00e+00   0
   1  2.0985980e+01 0.00e+00 5.04e-14  -1.0 2.56e+00    -  1.00e+00 1.00e+00f  1

Number of Iterations....: 1

                                   (scaled)                 (unscaled)
Objective...............:   1.1721510689040417e+00    2.0985980229387991e+01
Dual infeasibility......:   5.0402056087532821e-14    9.0238927441532734e-13
Constraint violation....:   0.0000000000000000e+00    0.0000000000000000e+00
Variable bound violation:   0.0000000000000000e+00    0.0000000000000000e+00
Complementarity.........:   0.0000000000000000e+00    0.0000000000000000e+00
Overall NLP error.......:   5.0402056087532821e-14    9.0238927441532734e-13


Number of objective function evaluations             = 2
Number of objective gradient evaluations             = 2
Number of equality constraint evaluations            = 0
Number of inequality constraint evaluations          = 0
Number of equality constraint Jacobian evaluations   = 0
```

```
Number of inequality constraint Jacobian evaluations = 0
Number of Lagrangian Hessian evaluations             = 1
Total seconds in IPOPT                               = 0.071

EXIT: Optimal Solution Found.
```



Question 3: Comparer à la solution exacte

La solution exacte est $x(t) = e^t - e^{-2t}$ et la valeur optimale est $e^3 - 2e^{-3} + 1$.

In [ ]:

In [ ]:
```julia
## Exercice 1
#
using ADNLPModels, OptimizationProblems, JuMP
nlp = BOX2()
@test nlp.meta.ncon != 0

nlp_quad = quad_penalty_adnlp(nlp, 1)
@test nlp_quad.meta.ncon == 0
```

```
UndefVarError: `BOX2` not defined



Stacktrace:

 [1] top-level scope

   @ c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab5\Lab5-not
ebook.ipynb:4
```