MTH8408 : Méthodes d'optimisation et contrôle optimal

Laboratoire 4: Optimisation sans contraintes et méthodes itératives

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In [ ]:
```
#import Pkg
#Pkg.add("Krylov")
using LinearAlgebra, Krylov, NLPModels, Printf, Logging, SolverCore, Test, ADNLPMod
```

Exercice 0: Introduction aux NLSModels

On a vu dans les lab précédents l'utilisation des NLPModels pour représenter un problème d'optimisation. Dans le cas de l'optimisation de moindre carrées non-linéaires, il existe un type spécifique: **NLSModel**.

$$\min_x \frac{1}{2}\|F(x)\|^2$$

Comme un NLPModel classique on peut faire appels aux fonctions: obj, grad, hprod ...

Mais on peut aussi utiliser des fonctions relatives à F: https://juliasmoothoptimizers.github.io/NLPModels.jl/stable/#Nonlinear-Least-Squares

In [ ]:
```
#? NLPModels.residual
```

L'équivalent des ADNLPModel pour ce cas est la fonction: ADNLSModel. Lien vers le site: https://juliasmoothoptimizers.github.io/ADNLPModels.jl/stable/

In [ ]:
```
#? ADNLPModels.ADNLSModel
```

En utilisant les ADNLSModels écrire un modèle dont la fonction résidue est donné par FH ci-dessous.

In [ ]:
```
#Test problem:
FH(x) = [x[2]+x[1].^2-11, x[1]+x[2].^2-7]
x0H = [10., 20.]
###########################
#Utilise FH et x0H pour créer un ADNLSModel
himmelblau_nls = ADNLSModel(FH,x0H,2)
###########################
```

```
ADNLSModel - Nonlinear least-squares model with automatic differentiation backend A
DNLPModels.ForwardDiffAD{ForwardDiff.GradientConfig{ForwardDiff.Tag{ADNLPModels.va
r"#59#62"{typeof(FH)}, Float64}, Float64, 2, Vector{ForwardDiff.Dual{ForwardDiff.Ta
g{ADNLPModels.var"#59#62"{typeof(FH)}, Float64}, Float64, 2}}}}(3, 0, ForwardDiff.G
radientConfig{ForwardDiff.Tag{ADNLPModels.var"#59#62"{typeof(FH)}, Float64}, Float6
4, 2, Vector{ForwardDiff.Dual{ForwardDiff.Tag{ADNLPModels.var"#59#62"{typeof(FH)},
Float64}, Float64, 2}}}((Partials(1.0, 0.0), Partials(0.0, 1.0)), ForwardDiff.Dual
{ForwardDiff.Tag{ADNLPModels.var"#59#62"{typeof(FH)}, Float64}, Float64, 2}[Dual{Fo
rwardDiff.Tag{ADNLPModels.var"#59#62"{typeof(FH)}, Float64}}(1.27402949085e-311,1.2
73959003872e-311,1.273959004022e-311), Dual{ForwardDiff.Tag{ADNLPModels.var"#59#6
2"{typeof(FH)}, Float64}}(1.2739784245647e-311,1.2739784245964e-311,1.274351721895
e-311)]))
  Problem name: Generic
   All variables: ████████████████████ 2      All constraints: ⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅ 0
All residuals: ████████████████████ 2
            free: ████████████████████ 2                 free: ⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅ 0
linear: ⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅ 0
           lower: ⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅ 0                lower: ⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅ 0            n
onlinear: ████████████████████ 2
           upper: ⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅ 0                upper: ⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅ 0
nnzj: (  0.00% sparsity)   4
         low/upp: ⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅ 0              low/upp: ⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅ 0
nnzh: (  0.00% sparsity)   3
           fixed: ⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅ 0                fixed: ⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅ 0
          infeas: ⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅ 0               infeas: ⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅ 0
            nnzh: (  0.00% sparsity)   3              linear: ⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅ 0
                                                   nonlinear: ⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅ 0
                                                        nnzj: (------% sparsity)

  Counters:
             obj: ⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅ 0                 grad: ⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅ 0
cons: ⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅ 0
        cons_lin: ⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅ 0             cons_nln: ⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅ 0
jcon: ⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅ 0
           jgrad: ⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅ 0                  jac: ⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅ 0
jac_lin: ⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅ 0
         jac_nln: ⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅ 0                jprod: ⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅ 0            j
prod_lin: ⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅ 0
       jprod_nln: ⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅ 0               jtprod: ⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅ 0           jt
prod_lin: ⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅ 0
      jtprod_nln: ⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅ 0                 hess: ⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅ 0
hprod: ⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅ 0
           jhess: ⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅ 0               jhprod: ⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅ 0
residual: ⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅ 0
    jac_residual: ⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅ 0       jprod_residual: ⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅ 0      jtprod_
residual: ⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅ 0
   hess_residual: ⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅ 0       jhess_residual: ⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅ 0       hprod_
residual: ⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅⋅ 0
```

Exercice 1: Gauss-Newton

Dans cet exercice, on complète une implémentation de la méthode Gauss-Newton avec région de confiance (paramétrée par Δ) discutée en cours.

Il faut compléter les morceaux:

- utiliser les fonctions des NLSModels pour obtenir F et sa jacobienne (ici on utilise pas la jacobienne mais juste le produit jacobienne-vecteur). Parcourez la documentation de NLPModels pour déterminer la fonction adéquat, indice les fonctions pour les NLSModels indiquent des `nls` au lieu de `nlp` dans la documentation.
- Utiliser la fonction `lsmr` du package `Krylov.jl` pour résoudre le système linéaire avec une contrainte de `radius` . Lisez la [documentation de `lsmr`]() .

In [ ]:
```julia
function gauss_newton(nlp      :: AbstractNLSModel,
                      x        :: AbstractVector,
                      ϵ        :: AbstractFloat;
                      η₁       :: AbstractFloat = 1e-3,
                      η₂       :: AbstractFloat = 0.66,
                      σ₁       :: AbstractFloat = 0.25,
                      σ₂       :: AbstractFloat = 2.0,
                      max_eval :: Int = 1_000,
                      max_time :: AbstractFloat = 60.,
                      max_iter :: Int = typemax(Int64)
                      )
    ######################################################
    Fx = residual(nlp, x)# le résidu
    Jx = jac_residual(nlp, x)# operateur qui représente le jacobien du résidu
    ######################################################
    normFx = norm(Fx)

    Δ = 1.

    iter = 0

    el_time = 0.0
    tired   = neval_residual(nlp) > max_eval || el_time > max_time
    status  = :unknown

    start_time = time()
    too_small  = false
    normdual   = norm(Jx' * Fx)
    optimal    = min(normFx, normdual) ≤ ϵ

    @info log_header([:iter, :nf, :primal, :status, :nd, :Δ],
    [Int, Int, Float64, String, Float64, Float64],
    hdr_override=Dict(:nf => "#F", :primal => "‖F(x)‖", :nd => "‖d‖"))

    while !(optimal || tired || too_small)

        #################################
        #Compute a direction satisfying the trust-region constraint
        (d, stats)  = lsmr(Jx, -Fx, radius = Δ)
        #################################

        too_small = norm(d) < 1e-15
        if too_small #the direction is too small
            status = :too_small
        else
            xp      = x + d
            ###########################
            Fxp     = residual(nlp, xp) # évalue le résidu en xp
            ###########################
            normFxp = norm(Fxp)

            Pred = 0.5 * (normFx^2 - norm(Jx * d + Fx)^2)
            Ared = 0.5 * (normFx^2 - normFxp^2)

            if Ared/Pred < η₁
                Δ = max(1e-8, Δ * σ₁)
                status = :reduce_Δ
            else #success
```

```julia
                x  = xp
                ###########################
                Jx = jac_residual(nlp, x)# réevalue le jacobien en x
                ###########################
                Fx = Fxp
                normFx = normFxp
                status = :success
                if Ared/Pred > η₂ && norm(d) >= 0.99 * Δ
                    Δ *= σ₂
                end
            end
        end

        @info log_row(Any[iter, neval_residual(nlp), normFx, status, norm(d), Δ])

        el_time      = time() - start_time
        iter   += 1

        many_evals   = neval_residual(nlp) > max_eval
        iter_limit   = iter > max_iter
        tired        = many_evals || el_time > max_time || iter_limit
        normdual     = norm(Jx' * Fx)
        optimal      = min(normFx, normdual) ≤ ϵ
    end

    status = if optimal
        :first_order
    elseif tired
        if neval_residual(nlp) > max_eval
            :max_eval
        elseif el_time > max_time
            :max_time
        elseif iter > max_iter
            :max_iter
        else
            :unknown_tired
        end
    elseif too_small
        :stalled
    else
        :unknown
    end

    return GenericExecutionStats(nlp; status, solution = x,
                                 objective = normFx^2 / 2,
                                 dual_feas = normdual,
                                 iter = iter,
                                 elapsed_time = el_time)
end
```

```
gauss_newton (generic function with 1 method)
```

In [ ]:

```julia
stats = gauss_newton(himmelblau_nls, himmelblau_nls.meta.x0, 1e-6)
@test stats.status == :first_order
include("test1.jl")
```

```
┌ Info:   iter      #F    ‖F(x)‖           status       ‖d‖         Δ
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:31
┌ Info:      0       2   3.8e+02          success   1.0e+00   2.0e+00
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:72
┌ Info:      1       3   3.1e+02          success   2.0e+00   4.0e+00
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:72
┌ Info:      2       4   1.9e+02          success   4.0e+00   8.0e+00
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:72
┌ Info:      3       5   4.5e+01          success   7.7e+00   8.0e+00
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:72
┌ Info:      4       6   9.5e+00          success   3.4e+00   8.0e+00
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:72
┌ Info:      5       7   1.6e+00          success   1.3e+00   8.0e+00
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:72
┌ Info:      6       8   1.2e-01          success   3.5e-01   8.0e+00
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:72
┌ Info:      7       9   8.8e-04          success   3.0e-02   8.0e+00
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:72
┌ Info:      8      10   5.3e-08          success   2.3e-04   8.0e+00
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:72
  0.270666 seconds (805.59 k allocations: 55.594 MiB, 99.40% compilation time)
┌ Info:   iter      #F    ‖F(x)‖           status       ‖d‖         Δ
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:31
┌ Info:      0       2   4.9e+01          success   1.0e+00   2.0e+00
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:72
┌ Info:      1       3   1.2e+01          success   2.0e+00   4.0e+00
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:72
┌ Info:      2       4   8.3e+00          success   2.5e+00   4.0e+00
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:72
┌ Info:      3       5   2.2e-12          success   8.3e-01   4.0e+00
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:72
  0.324860 seconds (818.98 k allocations: 56.484 MiB, 17.13% gc time, 99.67% compil
ation time)
```

```
┌ Info:   iter      #F    ‖F(x)‖           status       ‖d‖         Δ
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:31
┌ Info:      0       2   3.0e+00          success   1.0e+00   2.0e+00
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:72
┌ Info:      1       3   1.0e+00          success   2.0e+00   4.0e+00
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:72
┌ Info:      2       4   0.0e+00          success   1.0e+00   4.0e+00
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:72
  0.279884 seconds (823.31 k allocations: 57.104 MiB, 98.95% compilation time)
┌ Info:   iter      #F    ‖F(x)‖           status       ‖d‖         Δ
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:31
┌ Info:      0       2   7.6e+00          success   1.0e+00   2.0e+00
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:72
┌ Info:      1       3   9.6e-01          success   1.1e+00   2.0e+00
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:72
┌ Info:      2       4   4.6e-02          success   2.1e-01   2.0e+00
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:72
┌ Info:      3       5   1.4e-04          success   1.2e-02   2.0e+00
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:72
┌ Info:      4       6   1.4e-09          success   3.8e-05   2.0e+00
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:72
  0.432691 seconds (903.36 k allocations: 62.217 MiB, 99.66% compilation time)
Test Summary:            | Pass  Total  Time
Test set for Gauss-Newton |    4      4  1.6s
```

```
┌ Info:   iter      #F    ‖F(x)‖           status       ‖d‖          Δ
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:31
┌ Info:      0       2   2.9e+02          success   1.0e+00   2.0e+00
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:72
┌ Info:      1       3   1.8e+02          success   2.0e+00   4.0e+00
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:72
┌ Info:      2       4   4.2e+01          success   4.0e+00   8.0e+00
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:72
┌ Info:      3       5   1.3e+01          success   3.1e+00   8.0e+00
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:72
┌ Info:      4       6   1.1e+00          success   1.2e+00   8.0e+00
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:72
┌ Info:      5       7   3.4e-02          success   9.8e-02   8.0e+00
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:72
┌ Info:      6       8   3.9e-05          success   3.0e-03   8.0e+00
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:72
┌ Info:      7       9   5.2e-11          success   3.2e-06   8.0e+00
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:72
Test.DefaultTestSet("Test set for Gauss-Newton", Any[], 4, false, false, true, 1.70
9500006756e9, 1.709500008384e9, false, "c:\\Users\\adamo\\OneDrive\\Documents\\POLY
\\H24\\MTH8408\\MTH8408-Hiv24\\lab4\\test1.jl")
```

Exercice 2: Méthode Levenberg-Marquard inexacte

Dans cet exercice, on complète une implémentation de la méthode Levenberg-Marquardt. Pour compléter le code `lm_param` on va utiliser les fonctions suivantes:

- `dsol` qui calcul la solution du système $\min_x \frac{1}{2}\|J(x)d + F(x)\| + \lambda\|x\|^2$ avec la fonction `lsqr` du package `Krylov.jl` .
- `multi_sol` qui pour un entier nl donné et un μ va résoudre le problème de dsol pour nl valeurs de λ (autour de la valeur μ). Par exemple, pour $\mu = 10^{-6}$ et $nl = 3$, on prendra $\lambda = 10^{-7}, 10^{-6}, 10^{-5}$. Parmis les `nl` directions calculées, on retourne celle qui donne la plus petite valeur de $\|F(x+d)\|^2$.

In [ ]:
```julia
function dsol(Fx, Jx, λ, τ)
    # TODO.

    (d, stats) = lsmr(Jx, -Fx, λ = λ, timemax = τ)

    return d
end
```

```
dsol (generic function with 1 method)
```

In [ ]:
```
function multi_sol(nlp, x, Fx, Jx, λ, τ; nl = 3)
    # TODO
    μ = λ
    N = Any[]
    dk = Any[]
    for i = 1:nl
        λ = μ*exp10(floor(-nl/2)+i)
        push!(dk,dsol(Fx, Jx, λ, τ))
        push!(N,residual(nlp, x + dk[i]))
    end
    d = dk[argmin(norm(N))]
    return d
end
```

```
multi_sol (generic function with 1 method)
```

In [ ]:
```julia
function lm_param(nlp      :: AbstractNLSModel,
                  x        :: AbstractVector,
                  ϵ        :: AbstractFloat;
                  η₁       :: AbstractFloat = 1e-3,
                  η₂       :: AbstractFloat = 0.66,
                  σ₁       :: AbstractFloat = 10.0,
                  σ₂       :: AbstractFloat = 0.5,
                  max_eval :: Int = 10_000,
                  max_time :: AbstractFloat = 60.,
                  max_iter :: Int = typemax(Int64)
                  )
    ######################################################
    Fx = residual(nlp, x)# le résidu
    Jx = jac_residual(nlp, x)# operateur qui représente le jacobien du résidu
    ######################################################
    normFx   = norm(Fx)
    normdual = norm(Jx' * Fx)

    iter = 0
    λ = 0.0
    λ₀ = 1e-6
    η = 0.5
    τ = η * normdual

    el_time = 0.0
    tired   = neval_residual(nlp) > max_eval || el_time > max_time
    status  = :unknown

    start_time = time()
    too_small  = false
    optimal    = min(normFx, normdual) ≤ ϵ

    @info log_header([:iter, :nf, :primal, :status, :nd, :λ],
    [Int, Int, Float64, String, Float64, Float64],
    hdr_override=Dict(:nf => "#F", :primal => "‖F(x)‖", :nd => "‖d‖"))

    while !(optimal || tired || too_small)

        ###########################
        # (d, stats)  = lsqr(Jx, -Fx, λ = λ, atol = τ)
        d = multi_sol(nlp, x, Fx, Jx, λ, τ)
        ###########################

        too_small = norm(d) < 1e-16
        if too_small #the direction is too small
            status = :too_small
        else
            xp      = x + d
            ###########################
            Fxp     = residual(nlp, xp)# évalue le résidu en xp
            ###########################
            normFxp = norm(Fxp)

            Pred = 0.5 * (normFx^2 - norm(Jx * d + Fx)^2 - λ*norm(d)^2)
            Ared = 0.5 * (normFx^2 - normFxp^2)

            if Ared/Pred < η₁
                λ = max(λ₀, σ₁ * λ)
                status = :increase_λ
```

```
                status = :increase_λ
            else #success
                x  = xp
                ###########################
                Jx = jac_residual(nlp,x)# réevalue le jacobien en x
                ###########################
                Fx = Fxp
                normFx = normFxp
                status = :success
                if Ared/Pred > η₂
                    λ = max(λ * σ₂, λ₀)
                end
            end
        end

        @info log_row(Any[iter, neval_residual(nlp), normFx, status, norm(d), λ])

        el_time      = time() - start_time
        iter        += 1
        many_evals   = neval_residual(nlp) > max_eval
        iter_limit   = iter > max_iter
        tired        = many_evals || el_time > max_time || iter_limit
        normdual     = norm(Jx' * Fx)
        optimal      = min(normFx, normdual) ≤ ϵ

        η = λ == 0.0 ? min(0.5, 1/iter, normdual) : min(0.5, 1/iter)
        τ = η * normdual
    end

    status = if optimal
        :first_order
    elseif tired
        if neval_residual(nlp) > max_eval
            :max_eval
        elseif el_time > max_time
            :max_time
        elseif iter > max_iter
            :max_iter
        else
            :unknown_tired
        end
    elseif too_small
        :stalled
    else
        :unknown
    end

    return GenericExecutionStats(nlp; status, solution = x,
                                objective = normFx^2 / 2,
                                dual_feas = normdual,
                                iter = iter,
                                elapsed_time = el_time)

end
```

```
lm_param (generic function with 1 method)
```

In [ ]:
```
stats = lm_param(himmelblau_nls, himmelblau_nls.meta.x0, 1e-6)
@test stats.status == :first_order
include("test2.jl")
```

```
┌ Info:   iter      #F    ‖F(x)‖           status       ‖d‖         λ
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:33
┌ Info:      0      15   1.0e+02          success   1.1e+01   1.0e-06
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:74
┌ Info:      1      19   2.4e+01          success   5.2e+00   1.0e-06
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:74
┌ Info:      2      23   4.9e+00          success   2.2e+00   1.0e-06
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:74
┌ Info:      3      27   7.0e-01          success   8.5e-01   1.0e-06
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:74
┌ Info:      4      31   2.8e-02          success   1.7e-01   1.0e-06
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:74
┌ Info:      5      35   5.1e-05          success   7.2e-03   1.0e-06
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:74
  0.312587 seconds (805.37 k allocations: 55.588 MiB, 18.17% gc time, 99.66% compil
ation time)
┌ Info:      6      39   1.8e-10          success   1.3e-05   1.0e-06
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:74
┌ Info:   iter      #F    ‖F(x)‖           status       ‖d‖         λ
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:33
┌ Info:      0       5   9.0e+01          success   1.2e+01   0.0e+00
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:74
┌ Info:      1       9   1.5e-11          success   9.0e+00   1.0e-06
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:74
┌ Info:   iter      #F    ‖F(x)‖           status       ‖d‖         λ
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:33
┌ Info:      0       5   0.0e+00          success   4.0e+00   1.0e-06
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:74
  0.248143 seconds (818.67 k allocations: 56.453 MiB, 99.63% compilation time)
```

```
┌ Info:   iter      #F    ‖F(x)‖           status       ‖d‖          λ
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:33
┌ Info:      0       5   2.4e+00          success   1.8e+00   1.0e-06
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:74
┌ Info:      1       9   2.0e-01          success   4.5e-01   1.0e-06
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:74
┌ Info:      2      13   2.4e-03          success   5.0e-02   1.0e-06
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:74
┌ Info:      3      17   4.0e-07          success   6.4e-04   1.0e-06
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:74
  0.265526 seconds (823.36 k allocations: 56.772 MiB, 99.13% compilation time)
  0.302985 seconds (903.42 k allocations: 62.212 MiB, 99.24% compilation time)
Test Summary:    | Pass  Total  Time
Test set for LM |    4      4  1.4s
┌ Info:   iter      #F    ‖F(x)‖           status       ‖d‖          λ
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:33
┌ Info:      0       5   1.3e+02          success   1.5e+01   1.0e-06
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:74
┌ Info:      1       9   1.9e+01          success   1.0e+01   1.0e-06
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:74
┌ Info:      2      13   3.3e+00          success   9.8e-01   1.0e-06
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:74
┌ Info:      3      17   2.4e-01          success   2.5e-01   1.0e-06
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:74
┌ Info:      4      21   1.8e-03          success   2.1e-02   1.0e-06
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:74
┌ Info:      5      25   1.1e-07          success   1.5e-04   1.0e-06
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:74
Test.DefaultTestSet("Test set for LM", Any[], 4, false, false, true, 1.709500008704
e9, 1.709500010133e9, false, "c:\\Users\\adamo\\OneDrive\\Documents\\POLY\\H24\\MTH
8408\\MTH8408-Hiv24\\lab4\\test2.jl")
```

Exercice 3: Rocket Control

Dans les cellules ci-dessous nous introduisons un modèle de contrôle optimal (cf. https://en.wikipedia.org/wiki/Optimal_control) pour le contrôle d'une fusée dont une version discrétisée a été modélisé avec JuMP:

Le lien vers le tutoriel: https://nbviewer.jupyter.org/github/jump-dev/JuMPTutorials.jl/blob/master/notebook/modelling/rocket_control.ipynb

In [ ]:
```julia
using JuMP, Ipopt

# Create JuMP model, using Ipopt as the solver
rocket = Model(optimizer_with_attributes(Ipopt.Optimizer, "print_level" => 0))

# Constants
# Note that all parameters in the model have been normalized
# to be dimensionless. See the COPS3 paper for more info.
h_0 = 1    # Initial height
v_0 = 0    # Initial velocity
m_0 = 1    # Initial mass
g_0 = 1    # Gravity at the surface

T_c = 3.5  # Used for thrust
h_c = 500  # Used for drag
v_c = 620  # Used for drag
m_c = 0.6  # Fraction of initial mass left at end

c     = 0.5 * sqrt(g_0 * h_0)  # Thrust-to-fuel mass
m_f   = m_c * m_0             # Final mass
D_c   = 0.5 * v_c * m_0 / g_0    # Drag scaling
T_max = T_c * g_0 * m_0        # Maximum thrust

n = 800   # Time steps

@variables(rocket, begin
    Δt ≥ 0, (start = 1/n) # Time step
    # State variables
    v[1:n] ≥ 0            # Velocity
    h[1:n] ≥ h_0          # Height
    m_f ≤ m[1:n] ≤ m_0    # Mass
    # Control
    0 ≤ T[1:n] ≤ T_max    # Thrust
end)

# Objective: maximize altitude at end of time of flight
@objective(rocket, Max, h[n])

# Initial conditions
@constraints(rocket, begin
    v[1] == v_0
    h[1] == h_0
    m[1] == m_0
    m[n] == m_f
end)

# Forces
# Drag(h,v) = Dc v^2 exp( -hc * (h - h0) / h0 )
@NLexpression(rocket, drag[j = 1:n], D_c * (v[j]^2) * exp(-h_c * (h[j] - h_0) / h_0
# Grav(h)   = go * (h0 / h)^2
@NLexpression(rocket, grav[j = 1:n], g_0 * (h_0 / h[j])^2)
# Time of flight
@NLexpression(rocket, t_f, Δt * n)

# Dynamics
for j in 2:n
    # h' = v
```

```
    # Rectangular integration
    # @NLconstraint(rocket, h[j] == h[j - 1] + Δt * v[j - 1])

    # Trapezoidal integration
    @NLconstraint(rocket,
        h[j] == h[j - 1] + 0.5 * Δt * (v[j] + v[j - 1]))

    # v' = (T-D(h,v))/m - g(h)

    # Rectangular integration
    # @NLconstraint(rocket, v[j] == v[j - 1] + Δt *(
    #                 (T[j - 1] - drag[j - 1]) / m[j - 1] - grav[j - 1]))

    # Trapezoidal integration
    @NLconstraint(rocket,
        v[j] == v[j-1] + 0.5 * Δt * (
            (T[j] - drag[j] - m[j] * grav[j]) / m[j] +
            (T[j - 1] - drag[j - 1] - m[j - 1] * grav[j - 1]) / m[j - 1]))

    # m' = -T/c

    # Rectangular integration
    # @NLconstraint(rocket, m[j] == m[j - 1] - Δt * T[j - 1] / c)

    # Trapezoidal integration
    @NLconstraint(rocket,
        m[j] == m[j - 1] - 0.5 * Δt * (T[j] + T[j-1]) / c)
end
```

In [ ]:
```
# Solve for the control and state
println("Solving...")
status = optimize!(rocket)

# Display results
# println("Solver status: ", status)
println("Max height: ", objective_value(rocket))
```

```
Solving...
Max height: 1.0128340648308067
```

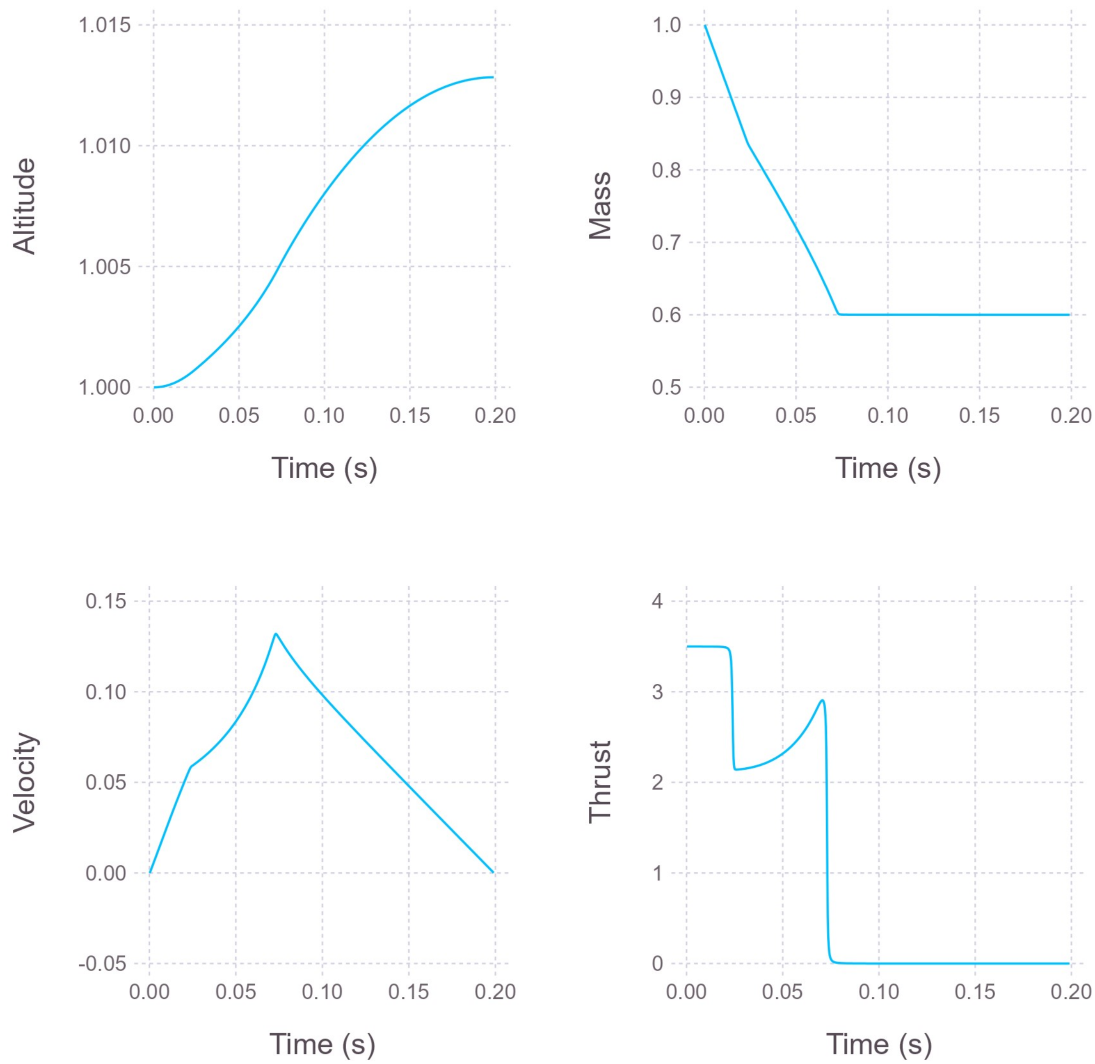
In [ ]:
```
value.(h)[n]
```

```
1.0128340648308067
```

In [ ]:
```
# Can visualize the state and control variables
import Pkg
Pkg.add("Gadfly")
using Gadfly
```

```
   Resolving package versions...
  No Changes to `C:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\Project.toml`
  No Changes to `C:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\Manifest.toml`
```

In [ ]:
```
h_plot = plot(x = (1:n) * value.(Δt), y = value.(h)[:], Geom.line,
                Guide.xlabel("Time (s)"), Guide.ylabel("Altitude"))
m_plot = plot(x = (1:n) * value.(Δt), y = value.(m)[:], Geom.line,
                Guide.xlabel("Time (s)"), Guide.ylabel("Mass"))
v_plot = plot(x = (1:n) * value.(Δt), y = value.(v)[:], Geom.line,
                Guide.xlabel("Time (s)"), Guide.ylabel("Velocity"))
T_plot = plot(x = (1:n) * value.(Δt), y = value.(T)[:], Geom.line,
                Guide.xlabel("Time (s)"), Guide.ylabel("Thrust"))
draw(SVG(6inch, 6inch), vstack(hstack(h_plot, m_plot), hstack(v_plot, T_plot)))
```



Questions:

```
- i) Transformer le modèle JuMP utilisé ci-dessus en un NLPModel en
utilisant le package `NLPModelsJuMP`.
- ii) Résoudre ce nouveau modèle avec `Ipopt` en utilisant
`NLPModelsIpopt`.
- iii) Calcul séparément la différence entre les h,v,m,T, Δt
calculés.
- iv) Est-ce que le contrôle T atteint ses bornes ?
- v) Reproduire les graphiques ci-dessous avec la solution calculée
via `NLPModelsIpopt`.
```

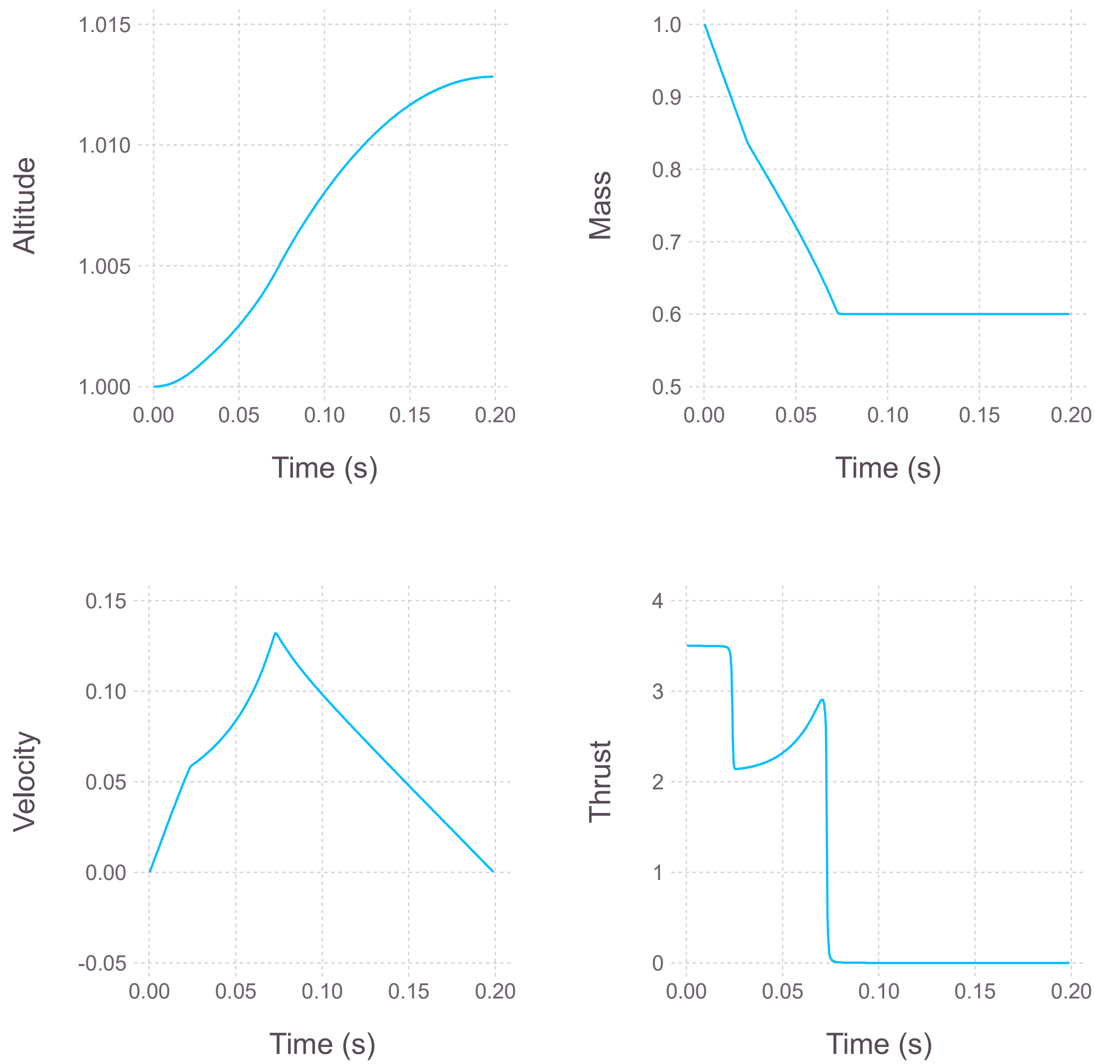
In [ ]:

```
#import Pkg
#Pkg.add("NLPModelsJuMP")
#Pkg.add("NLPModelsIpopt")
using NLPModels, LinearAlgebra, NLPModelsJuMP, NLPModelsIpopt

nlp = NLPModelsJuMP.MathOptNLPModel(rocket)
stats = ipopt(nlp)
vecteur_solution = stats.solution
Δt2 = vecteur_solution[1]
v2 = vecteur_solution[2:n+1]
h2 = vecteur_solution[n+2:2*n+1]
m2 = vecteur_solution[2*n+2:3*n+1]
T2 = vecteur_solution[3*n+2:4*n+1]

println("T2 valeur initiale:",T2[1])
println("T2 valeur finale:",T2[800])
println("On voit que les conditions aux bornes sont respectées pour T")

h_plot = plot(x = (1:n) * value.(Δt2), y = value.(h2)[:], Geom.line,
                Guide.xlabel("Time (s)"), Guide.ylabel("Altitude"))
m_plot = plot(x = (1:n) * value.(Δt2), y = value.(m2)[:], Geom.line,
                Guide.xlabel("Time (s)"), Guide.ylabel("Mass"))
v_plot = plot(x = (1:n) * value.(Δt2), y = value.(v2)[:], Geom.line,
                Guide.xlabel("Time (s)"), Guide.ylabel("Velocity"))
T_plot = plot(x = (1:n) * value.(Δt2), y = value.(T2)[:], Geom.line,
                Guide.xlabel("Time (s)"), Guide.ylabel("Thrust"))
draw(SVG(6inch, 6inch), vstack(hstack(h_plot, m_plot), hstack(v_plot, T_plot)))
```


Altitude
1.015
1.010
1.005
1.000
0.00
0.05
0.10
0.15
0.20
Time (s)
Mass
1.0
0.9
0.8
0.7
0.6
0.5
Velocity
0.15
0.10
0.05
0.00
-0.05
Thrust
4
3
2
1
0

```
This is Ipopt version 3.14.14, running with linear solver MUMPS 5.6.2.

Number of nonzeros in equality constraint Jacobian...:    15185
Number of nonzeros in inequality constraint Jacobian.:        0
Number of nonzeros in Lagrangian Hessian.............:    45543

Total number of variables............................:     3201
                     variables with only lower bounds:     1601
                variables with lower and upper bounds:     1600
                     variables with only upper bounds:        0
Total number of equality constraints.................:     2401
Total number of inequality constraints...............:        0
        inequality constraints with only lower bounds:        0
   inequality constraints with lower and upper bounds:        0
        inequality constraints with only upper bounds:        0

iter    objective    inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du alpha_pr  ls
   0  1.0100000e+00 3.96e-01 2.13e+00  -1.0 0.00e+00    -  0.00e+00 0.00e+00   0
   1  1.2110479e+00 7.40e-03 6.00e+03  -1.0 4.97e-01    -  1.32e-02 9.84e-01f  1
   2  1.2048591e+00 5.86e-03 1.11e+04  -1.0 3.15e+00    -  1.44e-01 1.57e-01f  1
   3  1.2629237e+00 5.19e-03 1.46e+04  -1.0 1.82e+00    -  7.26e-02 1.13e-01f  1
   4  1.4170550e+00 5.07e-03 3.15e+03  -1.0 1.67e+01   0.0 1.17e-02 2.18e-02f  1
   5  1.1124928e+00 2.20e-03 5.06e+05  -1.0 5.28e-01   1.3 2.12e-01 5.77e-01h  1
   6  1.1282562e+00 1.79e-03 1.44e+06  -1.0 1.83e+01    -  1.25e-02 1.82e-01f  1
   7  1.0529956e+00 3.50e-04 3.37e+05  -1.0 4.64e-01   0.9 9.47e-01 7.93e-01h  1
   8  1.0386949e+00 4.62e-04 2.32e+05  -1.0 9.23e+00    -  4.08e-02 3.63e-01f  1
   9  1.0298777e+00 3.71e-04 1.76e+05  -1.0 7.48e+00    -  2.05e-01 3.07e-01h  1
iter    objective    inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du alpha_pr  ls
  10  1.0232606e+00 2.51e-04 1.34e+05  -1.0 4.79e+00    -  3.50e-01 3.18e-01h  1
  11  1.0175352e+00 1.63e-04 9.98e+04  -1.0 5.79e+00    -  4.28e-01 3.88e-01h  1
  12  1.0143410e+00 1.05e-04 1.03e+05  -1.0 4.65e+00    -  1.00e+00 3.36e-01h  1
  13  1.0080148e+00 2.85e-05 5.04e+04  -1.0 1.45e+00    -  1.00e+00 9.90e-01h  1
  14  1.0078402e+00 4.17e-06 2.33e+03  -1.0 4.80e-01    -  1.00e+00 1.00e+00h  1
  15  1.0078153e+00 1.55e-08 1.82e+01  -1.0 3.60e-02    -  1.00e+00 1.00e+00f  1
  16  1.0078153e+00 4.53e-13 1.31e+01  -2.5 8.59e-05    -  1.00e+00 1.00e+00h  1
  17  1.0078190e+00 3.01e-10 1.01e-03  -2.5 2.80e-03    -  1.00e+00 1.00e+00h  1
  18  1.0078229e+00 3.09e-10 3.15e+02  -5.7 2.98e-03    -  9.99e-01 1.00e+00h  1
  19  1.0094243e+00 8.64e-05 3.87e+00  -5.7 2.27e+00    -  9.88e-01 9.72e-01f  1
iter    objective    inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du alpha_pr  ls
  20  1.0111368e+00 6.10e-05 2.94e+00  -5.7 2.05e+00    -  1.00e+00 9.00e-01f  1
  21  1.0111174e+00 4.64e-07 2.00e-02  -5.7 7.19e-01    -  1.00e+00 1.00e+00f  1
  22  1.0111207e+00 3.30e-09 6.81e-06  -5.7 6.70e-02    -  1.00e+00 1.00e+00h  1
  23  1.0122700e+00 2.31e-05 3.00e+01  -8.6 8.96e-01    -  7.15e-01 8.77e-01f  1
  24  1.0127033e+00 1.51e-05 8.16e+00  -8.6 8.55e-01    -  7.50e-01 7.80e-01h  1
  25  1.0128033e+00 1.01e-05 3.13e+00  -8.6 1.26e+00    -  6.60e-01 7.31e-01h  1
  26  1.0128269e+00 5.34e-06 1.23e+00  -8.6 1.39e+00    -  6.46e-01 7.34e-01h  1
  27  1.0128326e+00 2.56e-06 3.91e-01  -8.6 1.36e+00    -  7.03e-01 7.73e-01h  1
  28  1.0128339e+00 1.11e-06 5.08e-03  -8.6 1.18e+00    -  9.71e-01 8.96e-01f  1
  29  1.0128341e+00 2.45e-07 4.92e-05  -8.6 9.30e-01    -  1.00e+00 1.00e+00f  1
iter    objective    inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du alpha_pr  ls
  30  1.0128341e+00 3.51e-09 1.19e-06  -8.6 3.15e-01    -  1.00e+00 1.00e+00h  1
  31  1.0128341e+00 8.93e-11 5.13e-09  -8.6 3.70e-02    -  1.00e+00 1.00e+00h  1

Number of Iterations....: 31

                                   (scaled)                 (unscaled)
Objective...............:  -1.0128340648308067e+00    1.0128340648308067e+00
Dual infeasibility......:   5.1308433105302535e-09    5.1308433105302535e-09
Constraint violation....:   8.9276780412816947e-11    8.9276780412816947e-11
```

```
Variable bound violation:   3.0377522475760316e-41    3.0377522475760316e-41
Complementarity.........:   2.5098720261142263e-09    2.5098720261142263e-09
Overall NLP error.......:   5.1308433105302535e-09    5.1308433105302535e-09


Number of objective function evaluations             = 32
Number of objective gradient evaluations             = 32
Number of equality constraint evaluations            = 32
Number of inequality constraint evaluations          = 0
Number of equality constraint Jacobian evaluations   = 32
Number of inequality constraint Jacobian evaluations = 0
Number of Lagrangian Hessian evaluations             = 31
Total seconds in IPOPT                               = 0.649

EXIT: Optimal Solution Found.
T2 valeur initiale:3.4995900203628953
T2 valeur finale:0.0023318862190573626
On voit que les conditions aux bornes sont respectées pour T
```

In [ ]:

```
## Exercice 1
#
#import Pkg
#Pkg.add("NLSProblems")
#Pkg.add("SolverBenchmark")
#Pkg.add("Plots")
using NLPModels, NLPModelsJuMP, NLSProblems, SolverBenchmark, Plots

nls_mgh30 = mgh30()

stats = gauss_newton(nls_mgh30, nls_mgh30.meta.x0, 1e-6)
@test stats.status == :first_order
```

```
┌ Info:   iter      #F     ‖F(x)‖           status       ‖d‖          Δ
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:31
┌ Info:      0       2   6.6e-01          success    9.6e-01   1.0e+00
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:72
┌ Info:      1       3   3.0e-02          success    1.9e-01   1.0e+00
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:72
┌ Info:      2       4   9.2e-05          success    9.8e-03   1.0e+00
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:72
┌ Info:      3       5   1.1e-09          success    3.1e-05   1.0e+00
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:72
Test Passed
```

In [ ]:

```
## Exercice 2
#

stats = lm_param(nls_mgh30, nls_mgh30.meta.x0, 1e-6)
@test stats.status == :first_order
```

```
┌ Info:   iter      #F    ‖F(x)‖           status       ‖d‖         λ
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:33
┌ Info:      0      10   6.6e-01          success   9.6e-01   1.0e-06
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:74
┌ Info:      1      14   3.0e-02          success   1.9e-01   1.0e-06
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:74
┌ Info:      2      18   9.2e-05          success   9.8e-03   1.0e-06
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:74
┌ Info:      3      22   1.1e-09          success   3.1e-05   1.0e-06
└ @ Main c:\Users\adamo\OneDrive\Documents\POLY\H24\MTH8408\MTH8408-Hiv24\lab4\Lab
4-notebook.ipynb:74
Test Passed
```

In [ ]:

```julia
## Exercice 3
#
#Pkg.add("CaNNOLeS")
#Pkg.add("JSOSolvers")
using CaNNOLeS, JSOSolvers, NLPModels, NLPModelsJuMP, NLSProblems, SolverBenchmark

problems = (eval(problem)() for problem ∈ filter(x -> x != :NLSProblems, names(NLSP

solvers = Dict(
  :gauss => model -> gauss_newton(model,model.meta.x0,1e-6),
  :lm => model -> lm_param(model,model.meta.x0,1e-6),
)

stats = bmark_solvers(
  solvers, problems,
  skipif=prob -> (!unconstrained(prob) || get_nvar(prob) > 100 || get_nvar(prob) <
)
```

```
┌ Info:           Name    nvar    ncon          status      Time      f(x)      D
ual    Primal
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:127
┌ Info:           NZF1      13       0     first_order   0.0e+00   3.4e-20   1.0
e-09   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:          mgh17       5       0     first_order   1.0e-03   2.7e-05   1.5
e-08   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:          mgh18       6       0         stalled   0.0e+00   4.7e+00   1.5
e-02   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:          mgh19      11       0         stalled   4.0e-03   9.3e+14   5.8e
+41   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:          mgh20       6       0     first_order   3.9e-02   1.1e-03   1.0
e-06   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:          mgh21      20       0     first_order   0.0e+00   1.8e-13   4.0
e-06   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:          mgh22      20       0     first_order   0.0e+00   3.0e-10   1.9
e-07   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:          mgh25      10       0     first_order   0.0e+00   8.0e-16   7.8
e-07   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:          mgh26      10       0     first_order   0.0e+00   1.3e-13   5.7
e-07   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:          mgh27      10       0     first_order   1.0e-03   5.0e-01   1.6
e-10   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:          mgh28      10       0     first_order   0.0e+00   4.8e-16   6.2
e-09   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:          mgh29      10       0     first_order   0.0e+00   6.9e-14   4.8
e-07   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:          mgh30      10       0     first_order   0.0e+00   5.6e-19   3.3
e-09   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:          mgh31      10       0     first_order   0.0e+00   1.2e-16   9.2
e-08   0.0e+00
```

```
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:           mgh32      10       0      first_order   0.0e+00   5.0e+00   1.4
e-15   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:           mgh33      10       0      first_order   0.0e+00   2.3e+00   2.1
e-10   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:           mgh34      10       0      first_order   0.0e+00   3.1e+00   4.2
e-10   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:           tp266       5       0      first_order   3.1e-02   5.0e-01   9.6
e-07   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:           tp267       5       0         max_eval   5.1e-01       NaN
NaN   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:           tp271       6       0      first_order   0.0e+00   4.1e-29   1.1
e-13   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:           tp272       6       0          stalled   0.0e+00   4.7e+00   1.5
e-01   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:           tp273       6       0      first_order   0.0e+00   2.1e-17   7.3
e-08   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:           tp282      10       0         max_eval   4.7e-02   4.6e-01   2.0
e-02   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:           tp286      20       0      first_order   0.0e+00   1.8e-13   4.0
e-06   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:           tp288      20       0         max_eval   4.9e-02   4.2e-09   2.5
e-06   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:           tp291      10       0      first_order   0.0e+00   5.6e-10   5.0
e-07   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:           tp292      30       0         max_eval   5.2e-02   9.4e-05   3.3
e-03   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:           tp293      50       0         max_eval   9.0e-02   9.5e-05   3.3
e-03   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:           tp294       6       0         max_eval   4.3e-02   2.4e-05   3.5
```

```
e-03   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:           tp295      10       0         max_eval   2.4e-01   5.6e-05   5.3
e-03   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:           tp296      16       0         max_eval   1.0e-01   1.6e-04   9.0
e-03   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:           tp297      30       0         max_eval   1.6e-01   7.0e-04   1.9
e-02   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:           tp298      50       0         max_eval   1.3e-01   3.7e-02   1.7
e-01   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:           tp299     100       0         max_eval   9.2e-01   2.2e+01   1.2e
+00   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:           tp303      20       0      first_order   0.0e+00   6.2e-14   9.5
e-06   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:           tp304      50       0      first_order   2.0e-03   7.6e-21   1.3
e-08   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:           tp305     100       0      first_order   1.0e-03   3.7e-21   2.5
e-08   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:           tp370       6       0      first_order   3.8e-02   1.1e-03   1.0
e-06   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:           tp371       9       0      first_order   1.0e-03   7.0e-07   7.1
e-08   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:           tp379      11       0          stalled   2.0e-03   2.8e+00   5.6e
+01   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:            Name    nvar    ncon           status      Time      f(x)      D
ual    Primal
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:127
┌ Info:            NZF1      13       0      first_order   0.0e+00   9.2e-19   5.2
e-09   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:           mgh17       5       0      first_order   0.0e+00   2.7e-05   1.0
e-07   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
```

```
┌ Info:           mgh18       6       0      first_order   0.0e+00   2.8e-03   3.9
e-07   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:           mgh19      11       0      first_order   1.0e-03   2.0e-02   3.9
e-07   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:           mgh20       6       0      first_order   1.0e-03   1.1e-03   8.2
e-07   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:           mgh21      20       0      first_order   1.0e-03   7.1e-27   2.7
e-12   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:           mgh22      20       0      first_order   1.0e-03   1.2e-13   2.6
e-07   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:           mgh25      10       0      first_order   0.0e+00   8.0e-16   7.8
e-07   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:           mgh26      10       0      first_order   2.0e-03   1.4e-05   3.5
e-07   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:           mgh27      10       0      first_order   0.0e+00   1.1e-13   1.5
e-06   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:           mgh28      10       0      first_order   0.0e+00   4.8e-16   6.2
e-09   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:           mgh29      10       0      first_order   0.0e+00   6.9e-14   4.8
e-07   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:           mgh30      10       0      first_order   0.0e+00   5.6e-19   3.3
e-09   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:           mgh31      10       0      first_order   1.0e-03   1.2e-16   9.2
e-08   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:           mgh32      10       0      first_order   0.0e+00   5.0e+00   2.2
e-15   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:           mgh33      10       0      first_order   0.0e+00   2.3e+00   1.3
e-09   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:           mgh34      10       0      first_order   0.0e+00   3.1e+00   5.2
e-10   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
```

```
lver.jl:175
┌ Info:           tp266       5       0      first_order   0.0e+00   5.0e-01   3.2
e-07   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:           tp267       5       0      first_order   0.0e+00   8.6e-15   2.8
e-07   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:           tp271       6       0      first_order   0.0e+00   7.9e-30   4.7
e-14   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:           tp272       6       0      first_order   0.0e+00   2.8e-03   3.9
e-07   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:           tp273       6       0      first_order   0.0e+00   2.1e-13   7.2
e-06   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:           tp282      10       0      first_order   0.0e+00   1.2e-13   3.0
e-06   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:           tp286      20       0      first_order   0.0e+00   3.5e-26   5.9
e-12   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:           tp288      20       0      first_order   0.0e+00   2.0e-10   2.5
e-07   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:           tp291      10       0      first_order   0.0e+00   2.1e-10   2.6
e-07   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:           tp292      30       0      first_order   0.0e+00   5.1e-10   9.3
e-07   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:           tp293      50       0      first_order   1.0e-03   4.3e-10   9.8
e-07   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:           tp294       6       0      first_order   0.0e+00   3.2e-18   3.6
e-08   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:           tp295      10       0      first_order   0.0e+00   1.9e-26   2.8
e-12   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:           tp296      16       0      first_order   1.0e-03   1.4e-23   7.7
e-11   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:           tp297      30       0      first_order   2.0e-03   7.1e-22   5.5
e-10   0.0e+00
```

```
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:          tp298      50       0      first_order   1.0e-03   1.2e-16   2.3
e-07   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:          tp299     100       0      first_order   1.4e-02   3.0e-26   3.3
e-12   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:          tp303      20       0      first_order   0.0e+00   6.2e-14   9.5
e-06   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:          tp304      50       0      first_order   0.0e+00   7.6e-21   1.3
e-08   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:          tp305     100       0      first_order   1.0e-03   3.7e-21   2.5
e-08   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:          tp370       6       0      first_order   2.0e-03   1.1e-03   8.2
e-07   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:          tp371       9       0      first_order   0.0e+00   7.0e-07   8.1
e-07   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
┌ Info:          tp379      11       0      first_order   2.0e-03   2.0e-02   7.7
e-07   0.0e+00
└ @ SolverBenchmark C:\Users\adamo\.julia\packages\SolverBenchmark\YM13z\src\run_so
lver.jl:175
Dict{Symbol, DataFrames.DataFrame} with 2 entries:
  :lm    => 40×39 DataFrame…
  :gauss => 40×39 DataFrame…
```

In [ ]:

```
cols = [:id, :name, :nvar, :objective, :dual_feas, :neval_obj, :neval_grad, :neval_
header = Dict(
  :nvar => "n",
  :objective => "f(x)",
  :dual_feas => "‖∇f(x)‖",
  :neval_obj => "# f",
  :neval_grad => "# ∇f",
  :neval_hess => "# ∇²f",
  :elapsed_time => "t",
)

for solver ∈ keys(solvers)
  pretty_stats(stats[solver][!, cols], hdr_override=header)
end
```

id	name	n	f(x)	‖∇f(x)‖	# f	# ∇f	# ∇²f	i
15	NZF1	13	3.42e-20	1.00e-09	0	0	0	
67	mgh17	5	2.73e-05	1.53e-08	0	0	0	
68	mgh18	6	4.70e+00	1.49e-02	0	0	0	
69	mgh19	11	9.34e+14	5.78e+41	0	0	0	
70	mgh20	6	1.14e-03	1.00e-06	0	0	0	3
71	mgh21	20	1.81e-13	4.00e-06	0	0	0	
72	mgh22	20	3.00e-10	1.86e-07	0	0	0	
75	mgh25	10	7.96e-16	7.84e-07	0	0	0	
76	mgh26	10	1.33e-13	5.71e-07	0	0	0	
77	mgh27	10	5.00e-01	1.58e-10	0	0	0	
78	mgh28	10	4.84e-16	6.19e-09	0	0	0	
79	mgh29	10	6.94e-14	4.79e-07	0	0	0	
80	mgh30	10	5.64e-19	3.31e-09	0	0	0	
81	mgh31	10	1.20e-16	9.20e-08	0	0	0	
82	mgh32	10	5.00e+00	1.37e-15	0	0	0	
83	mgh33	10	2.32e+00	2.08e-10	0	0	0	
84	mgh34	10	3.07e+00	4.16e-10	0	0	0	
120	tp266	5	5.00e-01	9.55e-07	0	0	0	6
121	tp267	5	NaN	NaN	0	0	0	25
124	tp271	6	4.10e-29	1.06e-13	0	0	0	
125	tp272	6	4.70e+00	1.52e-01	0	0	0	
126	tp273	6	2.09e-17	7.30e-08	0	0	0	
127	tp282	10	4.61e-01	2.02e-02	0	0	0	25
128	tp286	20	1.81e-13	4.00e-06	0	0	0	
129	tp288	20	4.17e-09	2.48e-06	0	0	0	25
131	tp291	10	5.61e-10	4.99e-07	0	0	0	
132	tp292	30	9.43e-05	3.26e-03	0	0	0	25
133	tp293	50	9.46e-05	3.27e-03	0	0	0	25
134	tp294	6	2.39e-05	3.47e-03	0	0	0	25
135	tp295	10	5.63e-05	5.34e-03	0	0	0	25
136	tp296	16	1.57e-04	8.95e-03	0	0	0	25
137	tp297	30	7.04e-04	1.93e-02	0	0	0	25
138	tp298	50	3.71e-02	1.68e-01	0	0	0	25
139	tp299	100	2.17e+01	1.21e+00	0	0	0	25
140	tp303	20	6.22e-14	9.46e-06	0	0	0	
141	tp304	50	7.56e-21	1.27e-08	0	0	0	
142	tp305	100	3.73e-21	2.51e-08	0	0	0	
169	tp370	6	1.14e-03	9.99e-07	0	0	0	3
170	tp371	9	7.00e-07	7.07e-08	0	0	0	
173	tp379	11	2.76e+00	5.61e+01	0	0	0	

id	name	n	f(x)	‖∇f(x)‖	# f	# ∇f	# ∇²f	i
15	NZF1	13	9.22e-19	5.21e-09	0	0	0	
67	mgh17	5	2.73e-05	1.02e-07	0	0	0	
68	mgh18	6	2.83e-03	3.87e-07	0	0	0	
69	mgh19	11	2.01e-02	3.91e-07	0	0	0	
70	mgh20	6	1.14e-03	8.16e-07	0	0	0	
71	mgh21	20	7.14e-27	2.67e-12	0	0	0	
72	mgh22	20	1.20e-13	2.62e-07	0	0	0	
75	mgh25	10	7.96e-16	7.84e-07	0	0	0	
76	mgh26	10	1.40e-05	3.47e-07	0	0	0	
77	mgh27	10	1.06e-13	1.46e-06	0	0	0	

78	mgh28	10	4.84e-16	6.19e-09	0	0	0	
79	mgh29	10	6.94e-14	4.79e-07	0	0	0	
80	mgh30	10	5.64e-19	3.31e-09	0	0	0	
81	mgh31	10	1.20e-16	9.20e-08	0	0	0	
82	mgh32	10	5.00e+00	2.23e-15	0	0	0	
83	mgh33	10	2.32e+00	1.34e-09	0	0	0	
84	mgh34	10	3.07e+00	5.22e-10	0	0	0	
120	tp266	5	5.00e-01	3.18e-07	0	0	0	
121	tp267	5	8.64e-15	2.80e-07	0	0	0	
124	tp271	6	7.89e-30	4.73e-14	0	0	0	
125	tp272	6	2.83e-03	3.87e-07	0	0	0	
126	tp273	6	2.10e-13	7.22e-06	0	0	0	
127	tp282	10	1.19e-13	3.01e-06	0	0	0	
128	tp286	20	3.47e-26	5.90e-12	0	0	0	
129	tp288	20	2.03e-10	2.51e-07	0	0	0	
131	tp291	10	2.07e-10	2.61e-07	0	0	0	
132	tp292	30	5.12e-10	9.33e-07	0	0	0	
133	tp293	50	4.28e-10	9.76e-07	0	0	0	1
134	tp294	6	3.20e-18	3.64e-08	0	0	0	
135	tp295	10	1.88e-26	2.82e-12	0	0	0	
136	tp296	16	1.39e-23	7.73e-11	0	0	0	
137	tp297	30	7.10e-22	5.54e-10	0	0	0	1
138	tp298	50	1.20e-16	2.29e-07	0	0	0	
139	tp299	100	2.98e-26	3.32e-12	0	0	0	3
140	tp303	20	6.22e-14	9.46e-06	0	0	0	
141	tp304	50	7.56e-21	1.27e-08	0	0	0	
142	tp305	100	3.73e-21	2.51e-08	0	0	0	
169	tp370	6	1.14e-03	8.16e-07	0	0	0	
170	tp371	9	7.00e-07	8.14e-07	0	0	0	
173	tp379	11	2.01e-02	7.68e-07	0	0	0	

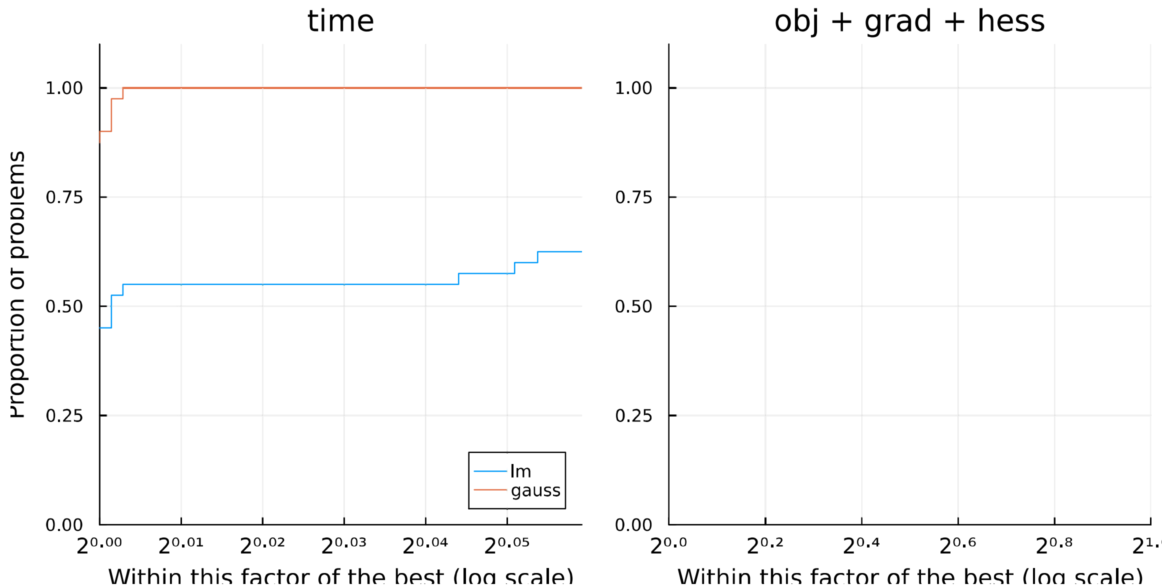
In [ ]:

```julia
first_order(df) = df.status .== :first_order
unbounded(df) = df.status .== :unbounded
solved(df) = first_order(df) .| unbounded(df)
costnames = ["time", "obj + grad + hess"]
costs = [
  df -> .!solved(df) .* Inf .+ df.elapsed_time,
  df -> .!solved(df) .* Inf .+ df.neval_obj .+ df.neval_grad .+ df.neval_hess,
]

gr()

profile_solvers(stats, costs, costnames)
```

```
┌ Warning: some measures are zero; shifting all by one
└ @ BenchmarkProfiles C:\Users\adamo\.julia\packages\BenchmarkProfiles\KnjdY\src\pe
rformance_profiles.jl:16
┌ Warning: some measures are zero; shifting all by one
└ @ BenchmarkProfiles C:\Users\adamo\.julia\packages\BenchmarkProfiles\KnjdY\src\pe
rformance_profiles.jl:16
```



On observe que 40 problèmes on été utilisés pour le benchmark précédent. On voit que le problème mgh19 (ID69) diverge fortement avec la méthode de Gauss-Newton. Cependant, la méthode de Levenberg-Marquard est capable de résoudre ce problème dans difficulté, trouvant les conditions d'optimalité de premier ordre.