

not-MIWAE: Deep Generative Modelling with Missing Not At Random Data

with sup-not-MIWAE: a Supervised Extension

Abstract

We study not-MIWAE (Ipsen et al., 2021)[1], a deep latent-variable model for learning and imputing when data are Missing Not At Random (MNAR). Unlike MAR/MCAR settings where the mask mechanism can be ignored, MNAR couples the generative model and the missingness mechanism, requiring joint modeling of data and mask. We (i) reproduce core results on UCI datasets, (ii) extend the MNAR clipping experiment to CelebA images, (iii) propose a transparent supervised MNAR baseline sup-not-MIWAE, and (iv) release unified PyTorch implementations (MIWAE / not-MIWAE / supMIWAE / sup-not-MIWAE).

Context

Data $\mathbf{x} \in \mathbb{R}^p$ is split into observed and missing parts: $\mathbf{x} = (\mathbf{x}^o, \mathbf{x}^m)$ with mask $\mathbf{s} \in \{0, 1\}^p$:

$$s_j = \mathbb{1}\{\mathbf{x}_j \text{ observed}\}.$$

Missingness regimes:

- MCAR: $p(\mathbf{s} | \mathbf{x}) = p(\mathbf{s})$
- MAR: $p(\mathbf{s} | \mathbf{x}) = p(\mathbf{s} | \mathbf{x}^o)$
- MNAR: $p(\mathbf{s} | \mathbf{x})$ depends on \mathbf{x}^m (non-ignorable)

Problem

Under MNAR, learning θ from only \mathbf{x}^o is biased because the observed likelihood must integrate missing values inside the mask model:

$$p_{\theta, \phi}(\mathbf{x}^o, \mathbf{s}, \mathbf{x}^m) = \int p_{\theta}(\mathbf{x}^o, \mathbf{x}^m) p_{\phi}(\mathbf{s} | \mathbf{x}^o, \mathbf{x}^m) d\mathbf{x}^m.$$

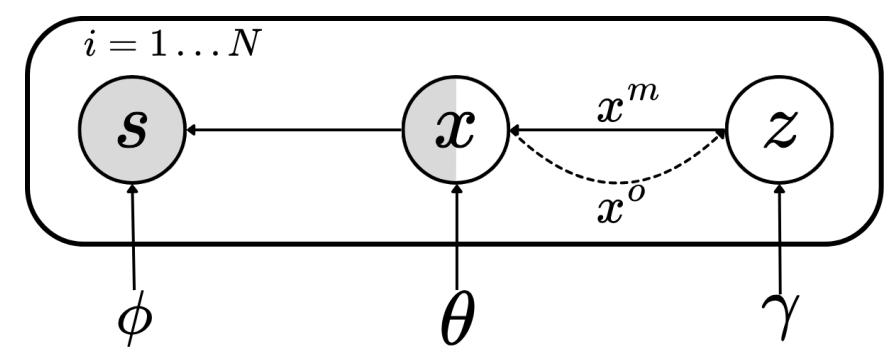
In MAR/MCAR, p_{ϕ} does not depend on \mathbf{x}^m and can be factored out; not so in MNAR. Hence we must model the missing mechanism jointly with the data distribution.

Model: not-MIWAE

Deep latent variable model with latent \mathbf{z} [1]:

$$p(\mathbf{x}^o, \mathbf{x}^m, \mathbf{s}, \mathbf{z}) = p(\mathbf{z}) p_{\theta}(\mathbf{x}^o, \mathbf{x}^m | \mathbf{z}) p_{\phi}(\mathbf{s} | \mathbf{x}^o, \mathbf{x}^m),$$

with conditional independence in the decoder: $p_{\theta}(\mathbf{x} | \mathbf{z}) = \prod_j p_{\theta}(x_j | \mathbf{z})$.



Inference: IW variational lower bound

Use variational posterior $q_{\gamma}(\mathbf{z} | \mathbf{x}^o)$ and draw K importance samples $\mathbf{z}_{ki} \sim q_{\gamma}(\mathbf{z} | \mathbf{x}_i^o)$, $\mathbf{x}_{ki}^m \sim p_{\theta}(\mathbf{x}^m | \mathbf{z}_{ki})$. Optimize [2] :

$$\mathcal{L}_K(\theta, \phi, \gamma) = \sum_{i=1}^n \mathbb{E} \left[\log \frac{1}{K} \sum_{k=1}^K w_{ki} \right], \quad w_{ki} = \frac{p_{\phi}(\mathbf{s}_i | \mathbf{x}_i^o, \mathbf{x}_{ki}^m) p_{\theta}(\mathbf{x}_i^o | \mathbf{z}_{ki}) p(\mathbf{z}_{ki})}{q_{\gamma}(\mathbf{z}_{ki} | \mathbf{x}_i^o)}.$$

\mathcal{L}_K is a lower bound and tightens monotonically as $K \uparrow$ [3].

Imputation & OT interpretation

Imputation is a Bayesian decision problem under $p_{\theta, \phi}(\mathbf{x}^m | \mathbf{x}^o, \mathbf{s})$:

$$\hat{\mathbf{x}}^m = \arg \min_{\hat{\mathbf{x}}^m} \mathbb{E}[L(\mathbf{x}^m, \hat{\mathbf{x}}^m) | \mathbf{x}^o, \mathbf{s}].$$

Squared loss \Rightarrow conditional mean (via SNIS), absolute loss \Rightarrow conditional median. [1]. Which is equivalent, by the Optimal transport point of view as imputations using the distance between distributions \mathbb{W}^p restricted to the variational family of Dirac measures $q = \delta_{\hat{\mathbf{x}}^m}$, studied in details in [4], then collapsing posterior uncertainty (motivating a distributional imputation).

Using prior knowledge on the MNAR mechanism

The missing model typically factorizes as Bernoulli:

$$p_{\phi}(\mathbf{s} | \mathbf{x}) = \prod_{j=1}^p \pi_{\phi, j}(\mathbf{x})^{s_j} (1 - \pi_{\phi, j}(\mathbf{x}))^{1-s_j}.$$

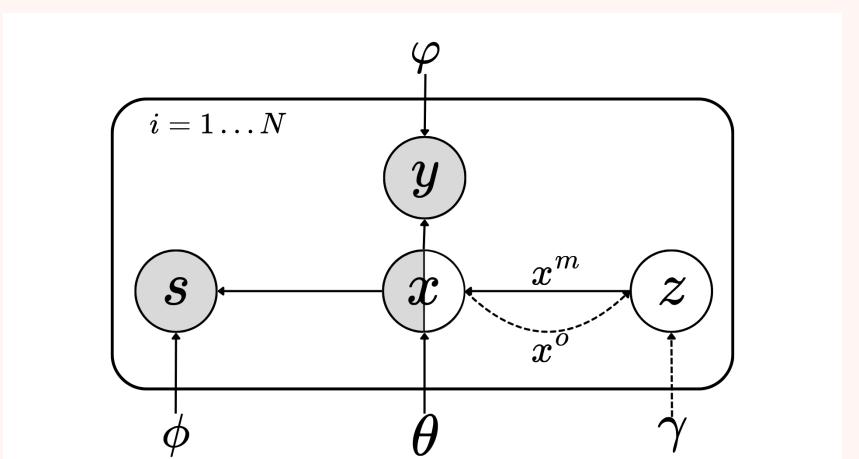
When we know a self-masking structure, we can constrain $\pi_{\phi, j}(\mathbf{x}) = \sigma(ax_j + b)$ (logistic censoring), improving identifiability and performance. [1]

sup-not-MIWAE

Goal: predict y from partially observed covariates by marginalizing missing features:

$$p(y | \mathbf{x}^o, \mathbf{s}) = \int p_{\varphi}(y | \mathbf{x}^o, \mathbf{x}^m) p_{\theta, \phi}(\mathbf{x}^m | \mathbf{x}^o, \mathbf{s}) d\mathbf{x}^m.$$

Following existing supervised deep generative approaches [4], we augment not-MIWAE [1] with a predictor head $p_{\varphi}(y | \mathbf{x})$ (labels depend on complete covariates).



Training uses an IW bound with weights

$$w_{ki}^{\text{sup}} = \frac{p_{\phi}(\mathbf{s}_i | \mathbf{x}_i^o, \mathbf{x}_{ki}^m) p_{\varphi}(y_i | \mathbf{x}_i^o, \mathbf{x}_{ki}^m) p_{\theta}(\mathbf{x}_i^o | \mathbf{z}_{ki}) p(\mathbf{z}_{ki})}{q_{\gamma}(\mathbf{z}_{ki} | \mathbf{x}_i^o)}$$

Imputation under MNAR

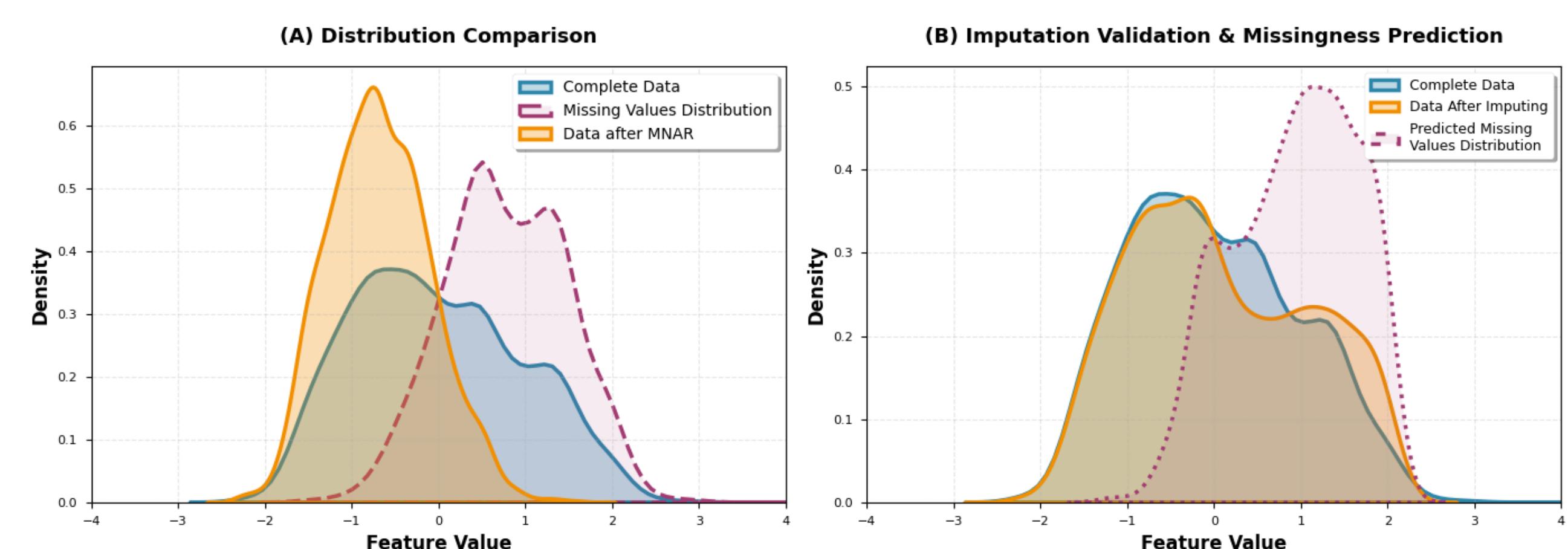
Experimental Setup:

- Scenario: Synthetic MNAR injected into UCI datasets and CelebA (images).
- Baselines: Compared against Mean, MICE, and standard MIWAE (MAR assumption).
- Metric: RMSE calculated against the ground truth of missing values.

Table 1. Imputation RMSE on UCI datasets under MNAR self-masking. Lower is better.

Model	Banknote (61.8%)	Concrete (50.3%)	Breast (43.6%)	White (44.6%)	CelebA (Images)
Mean	1.546	1.329	1.548	1.460	0.466
KNN	1.543	1.366	1.290	1.410	0.350
MICE	1.540	1.247	1.130	1.290	-
MIWAE	1.243	1.148	1.113	1.310	-
not-MIWAE variants					
Self-masking (Unknown parameters)	0.770	1.099	0.836	0.950	0.093
Linear	1.023	1.149	1.018	1.040	-
Non-linear	1.133	1.128	0.938	1.080	-

Quantitative Results: Table 1 shows that the proposed not-MIWAE (self-masking) consistently achieves the lowest RMSE, significantly outperforming standard baselines (Mean, KNN, MICE) and the MAR-based MIWAE.



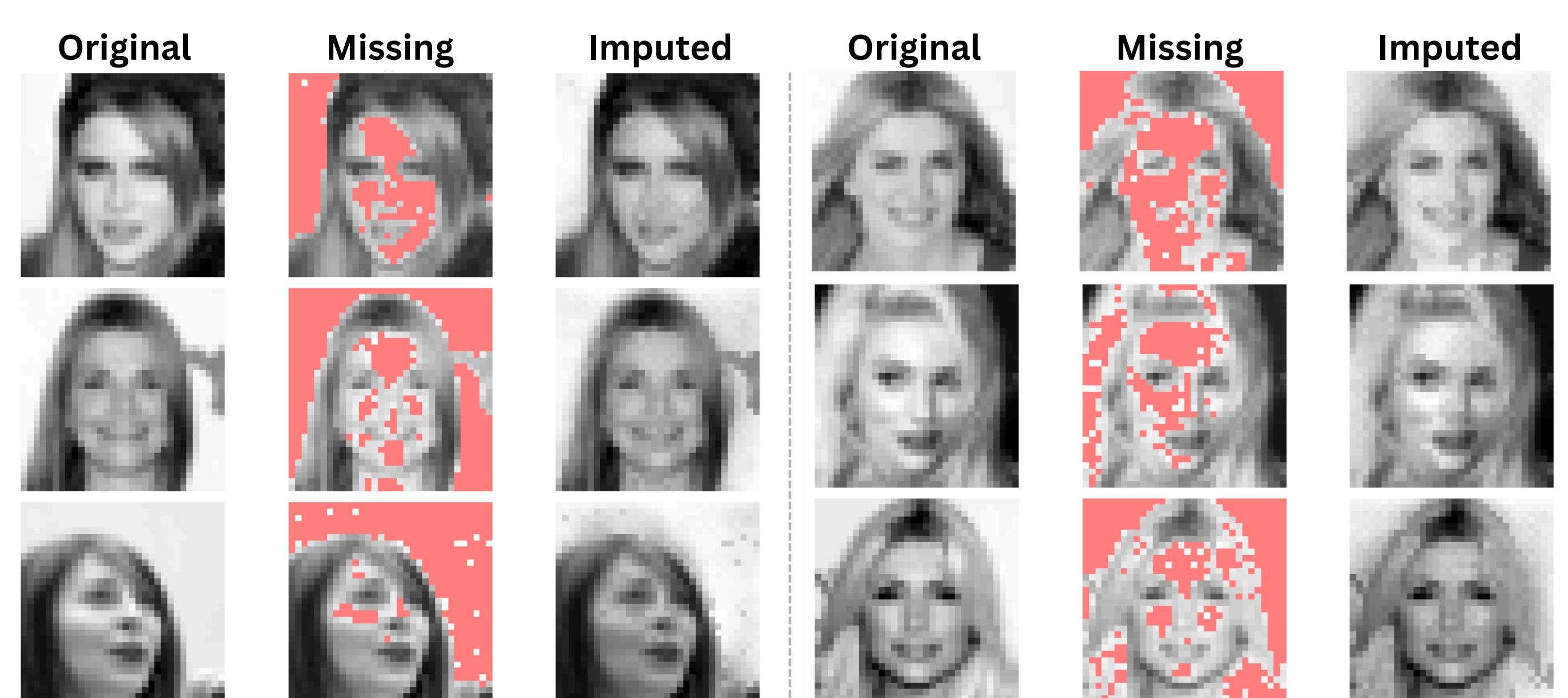
Distribution Recovery: Figure 1 illustrates this efficacy qualitatively. The model accurately predicts missing values and pushes the imputed distribution (orange) back to align with the original complete data (blue), effectively correcting the MNAR bias.

CelebA MNAR clipping

Self-masking censoring for pixels:

$$P(s_{ij} = 1 | x_{ij}) = \sigma(W(x_{ij} - b)), \quad W = -50, b = 0.75.$$

The model recovers missingness parameters and imputes clipped regions.



Supervised learning under MNAR

Model	Accuracy	Test RMSE
Mean + Ridge	0.5405	0.6966
MICE + Ridge	0.5425	0.6713
Two-Stage (not-MIWAE + LR)	0.5445	0.6503
Sup-not-MIWAE	0.5690	0.6718
Oracle (complete data)	0.7123	0.6173

Takeaway: For classification (Covertype), marginalization improves over naive baselines; for regression (Wine Quality), strong point imputations paired with ridge regression can be very competitive.

References

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