How PID works from a mathematical perspective

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PID:

$$output = P(error) + I \cdot \Sigma error + D \cdot \frac{\delta error}{\delta t}$$

Explanation

Output

This is the desired output you want your control system to achieve.

Proportional gain

$$P = proportional\ gain$$

Proportional gain is a constant that determines the proportional contribution to the control output based on the current error. Typically it's a positive number.

$P \cdot error$
Responds to the current error value.

Error

Error is the difference between the desired position and the current position.

Integral gain

This is a constant that determines the integral affect on the control output. It accumulates the error over time and helps eliminate steady-state errors.
$I \cdot \Sigma error$

Responds to the accumulation of error over time. The accumulation is elaborated in the section below.

Summation of errors

Expressed in the equation as:
$\Sigma error$

This basically means to take the sum of all previous errors.

$error_1 = 2$
$error_2 = 3$
$error_3 = 4$
$\Sigma error = error_1 + error_2 + error_3 + ...$

Derivative gain

D is the derivative gain, which is a constant that determines the derivative affect on the output. It reacts to the rate of change of the error.

$$D \cdot \frac{\delta error}{\delta t}$$

Responds to the rate of change of the error.

$\frac{\delta error}{\delta t}$
Represents the rate of change of the error with respect to time. It's the derivative of the error signal.

Why is this useful in robotics?

PID (proportional, integral, derivative) controllers are useful in robotics because you can give them sensor data like an inertial sensor and they will calculate how to move your robot according to the desired position.

Citations:

Tools used:
Chat GPT 3.5

Websites and PDFs

https://georgegillard.com/resources/documents

George Gillard wrote an amazing paper on the subject of PID control systems.

FRC team 3749 wrote an amazing article detailing how each term contributes to the equation.
https://www.team3749.org/post/pid-control