THE UNIVERSITY OF ADELAIDE

SCHOOL OF MECHANICAL ENGINEERING

Advanced Digital Control

Assignment 1

Due: Friday, 15th September

By
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Introduction:

This assignment focuses on designing a filter, using MATLAB, to remove an electromagnetic disturbance from an old Jimi Hendrix recording. The electromagnetic disturbance resonates throughout the whole recording and can be described as a low frequency hum. The filter must be designed using only the provided recording and its known sample rate of 8kHz. Therefore, to design an appropriate filter we must first:

- Characterise the frequency of the electromagnetic disturbance
- Design a filter to attenuate the disturbance by selecting specific zeros and poles
- Test that the filter works as intended

Characterisation of Disturbance Signal:

Before an appropriate filter can be designed the frequency/s at which the electromagnetic disturbance resonates at must be discovered. By listening to the recording, using MATLAB's sound function, the electromagnetic disturbance can be heard as a low frequency hum that exists throughout the entire recording.

Further observation of the recording in the time-domain, as seen below in figure 1, shows us how the recording looks as a WAV file. Considering the recording only consists of a single guitar riff and a simple drum beat it is obvious to lovers of music that this WAV file is too noisy. There should be some gaps between the waves as there are pauses in the recording, and the fact that there is not is clear proof that the electromagnetic disturbance exists.

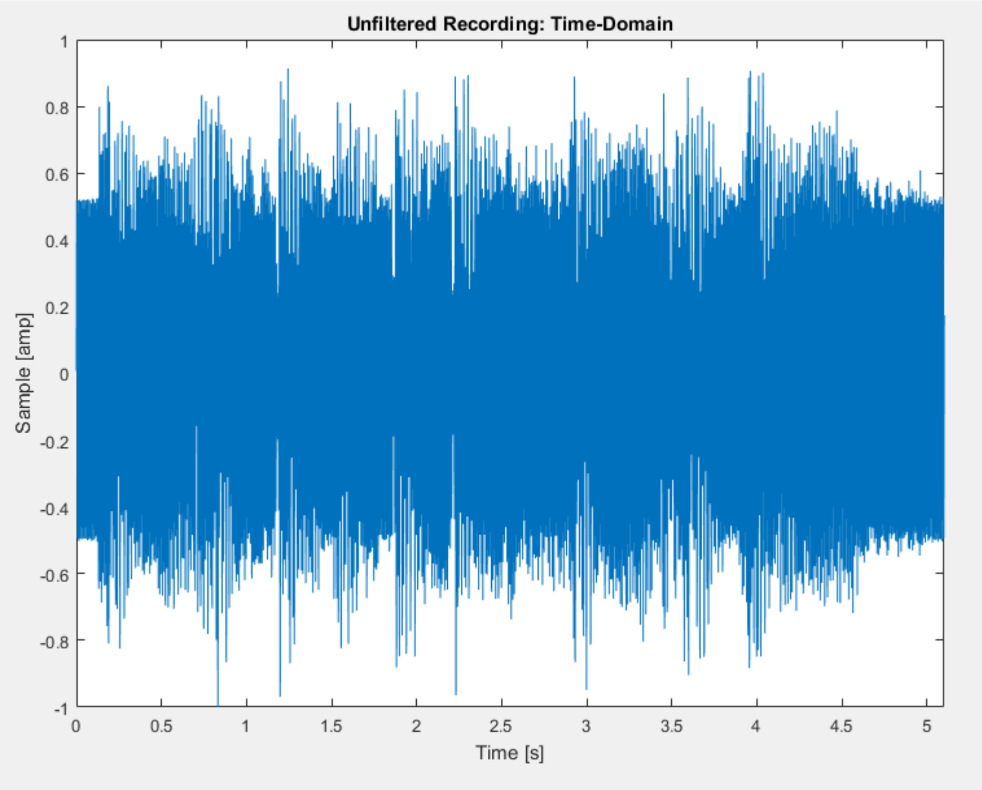


Figure 1: Original recording in the time-domain (WAV File)

Although the above time-domain analysis may prove useful for visualising the recording there is no way to determine the frequency of which the electromagnetic disturbance resonates at. For that we must venture into the frequency-domain by using MATLAB's "Fast Fourier Transform" function to look at the range of frequencies which exist within the recording (figure 2).

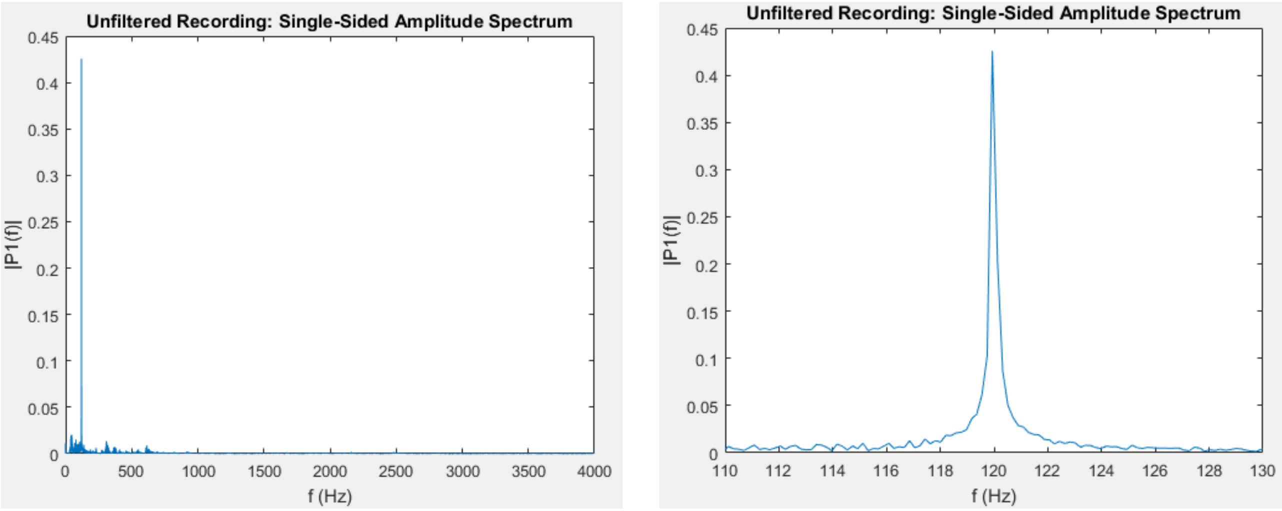


Figure 2: Original recording in the frequency-domain

The large spike seen figure 2 indicates that the electromagnetic disturbance exists at a frequency of 120Hz and has an amplitude of around 0.425. Therefore, in order to restore the recording a filter must be designed to attenuate the frequency at 120Hz to lower its amplitude.

Filter Design:

The first step taken towards the design of a filter involves selecting its type. A low or high pass filter could be used to attenuate the 120Hz frequency but using either one of these filters would destroy the sound quality of the recording. This is because we are interested in frequencies both above and below 120Hz as the recording has sounds in the range of 0 – 4000Hz (highest frequency in recording should be around half the sampling frequency, Fs = 8kHz). Therefore, a notch filter should be designed to attenuate the 120Hz frequency over a small bandwidth of say 10Hz. A bandwidth of this size should ensure that the desired frequency is attenuated whilst at the same time the rest of the recording is left unaffected.

Calculations for the design of the notch filter are located in the Appendices Section A. Results from these calculations include:

Notch Filter Transfer Function:

$$H(z) = \frac{z^2 - 1.991z + 1}{z^2 - 1.983z + 0.9922}$$

Resulting Difference Equation:

$$y(n) = x(n) - 1.991x(n-1) + x(n-2) + 1.983y(n-1) - 0.9922y(n-2)$$

Analysis towards the behaviour of this transfer function at different frequencies can be observed by looking at the bode plot shown below in figure 3.

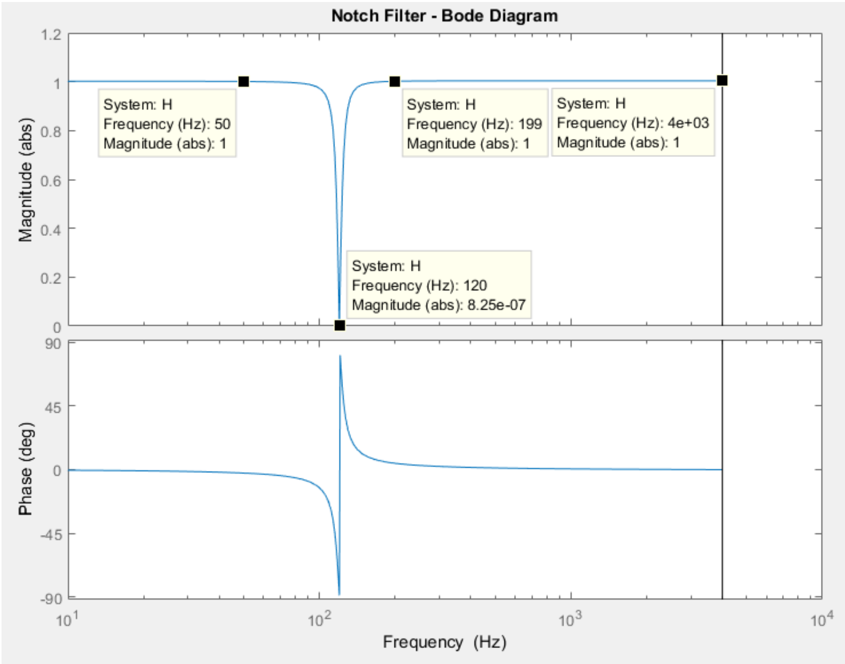


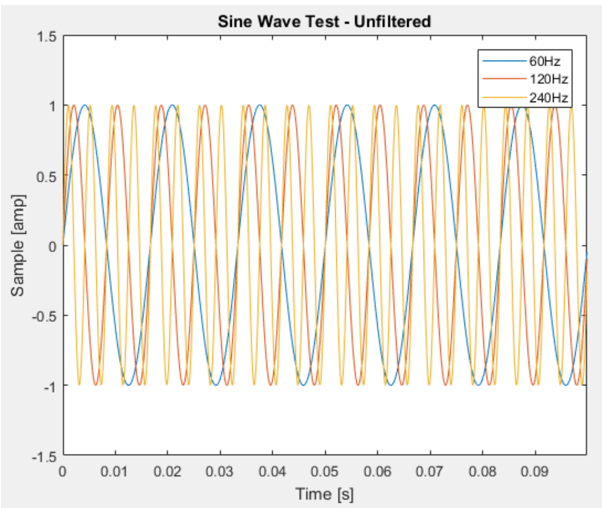
Figure 3: Bode plot of the designed notch filter

This bode plots shows us that the notch filter is successful at attenuating the electromagnetic disturbance (120Hz) while at the same time not having an effect of the rest of the recording (frequencies that lie above or below 120Hz).

Testing:

To make sure that the notch filter works as intended a test was performed. This test involved passing three sine waves, of differing frequencies, through the notch filter to see how they were effected. The frequencies of the sine waves were selected as 60Hz, 120Hz and 240Hz. This way the effects of the notch filter can be visualised before, at and after the desired attenuation frequency of 120Hz. If the notch filter works only the sine wave at 120Hz should be effected.

The results of the test can be seen below in figure 4:



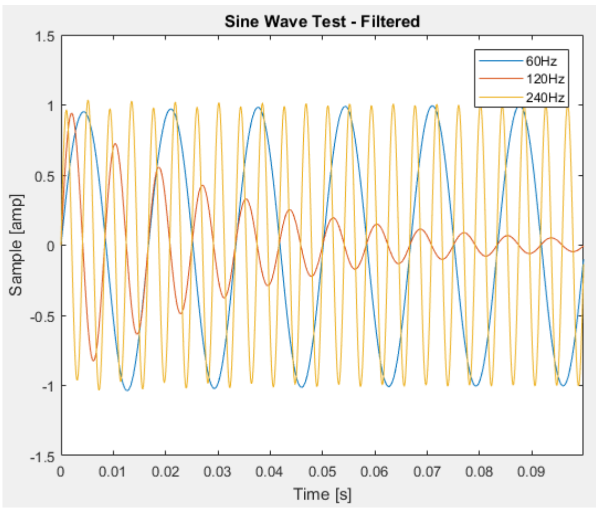


Figure 4: Original sine waves on the left, filtered sine waves on the right

As hypothesised the notch filter works as intended as only the 120Hz sine wave was attenuated. Therefore, filtering the recording using this notch filter should remove the electromagnetic disturbance and improve the overall sound quality.

Filtered Recording:

The original recording was filtered using the notch filter and the results can be seen below in figure 5:

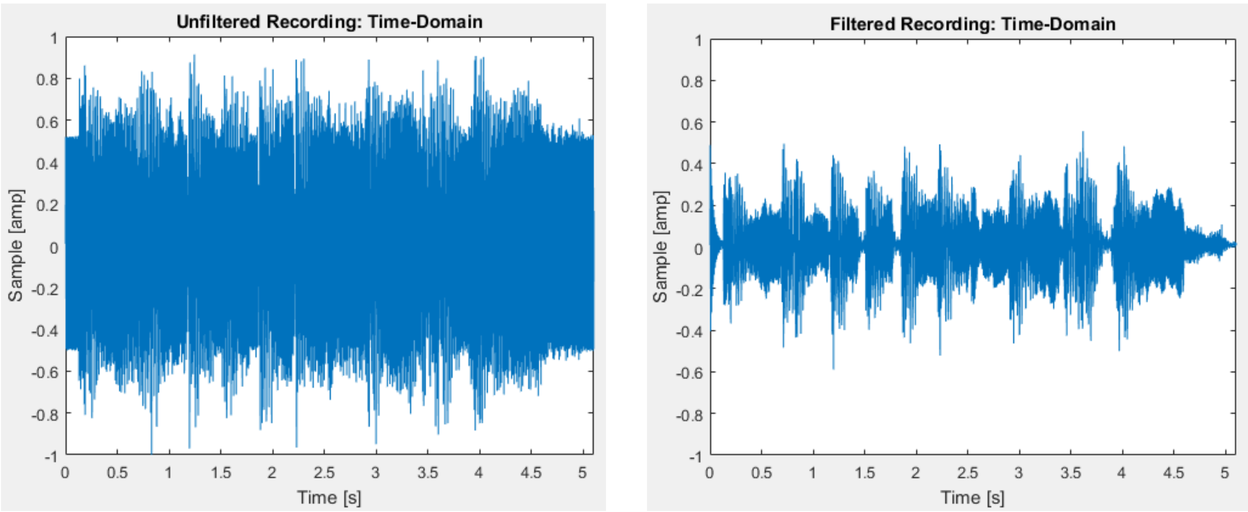


Figure 5: Unfiltered Recording Vs. Filtered Recording

By comparing the unfiltered and filtered recordings it is clear that significant changes have occurred. More specifically the overall size and magnitude of the filtered recording has decreased and there are now visible gaps between the waveforms where pauses in the song take place. These changes can be contributed to the notch filter attenuating the electromagnetic disturbance from the recording.

Physical proof that this disturbance has in fact been removed from the recording can be found by simply listening to the "Filtered_Recording.wav" file supplied with this assignment.

Conclusion:

In conclusion, during the creation of this assignment a filter was designed to remove an electromagnetic disturbance from an old Jimi Hendrix recording. Through the use of MATLAB, the frequency of the electromagnetic disturbance was determined to resonate at 120Hz. A notch filter was then designed and tested to remove this and only this frequency from the recording. In doing so the recording has been restored to its former glory.

Appendices:

Section A: Notch Filter Calculations

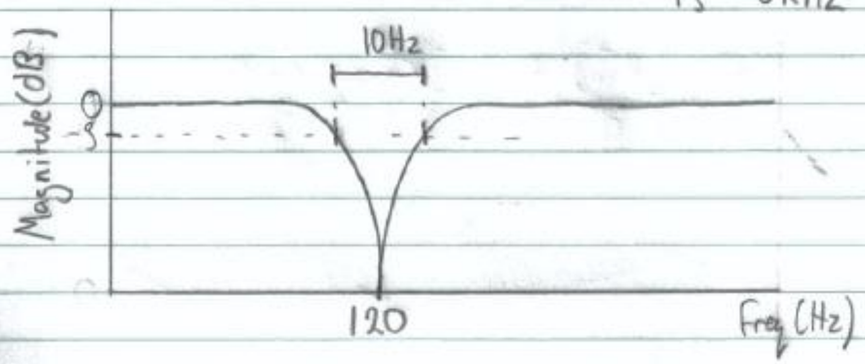
Pole - Zero Design

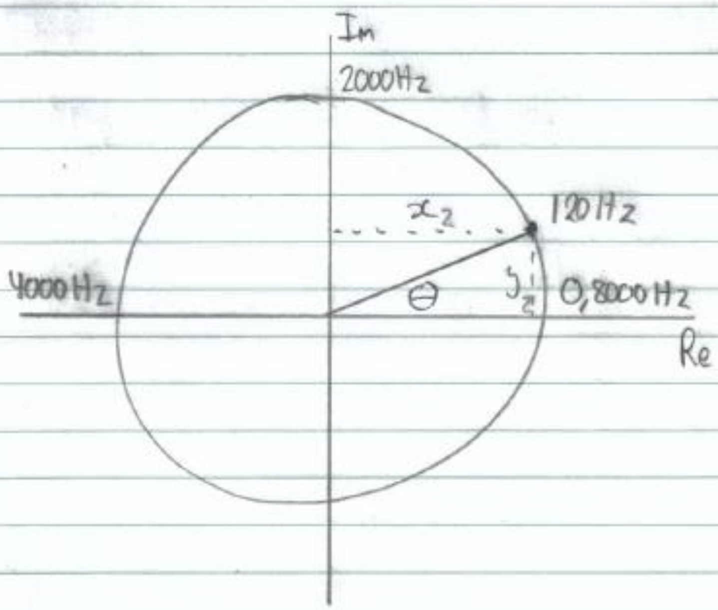
Notch Filter Design: Pole - Zero

Removal target frequency: 120 Hz

Desired Bandwidth: 10 Hz

$F_s = 8KHz$





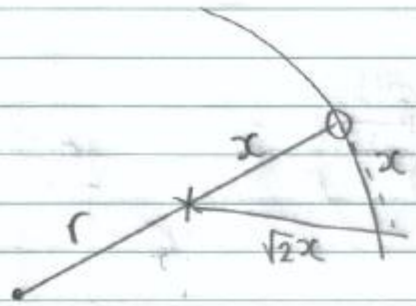
to find θ assume linear relationship between frequencies on the unit circle:

$$\therefore \quad \theta = \frac{120Hz}{4000Hz} \times 180^\circ = 5.4^\circ$$

$$\therefore \ x_z = \cos 5.4 = 0.9956$$
$$y_z = \sin 5.4 = 0.09411$$

∴ Zeros located @ $\left[z_z = 0.9956 \pm 0.09411j \right]$

Next Find the location of the notch filter poles.



$x = 1 - r$

$BW = 2x = 2(1-r)$

We know $F_s = 8\,KHz = 2\pi$

$\therefore \quad \pi = 4\,KHz$

$\therefore \quad 10\,Hz = \frac{\pi}{400} = BW$

∴ Find r:

$1 - r = \frac{1}{2} BW$

$\therefore \quad r = 1 - \frac{1}{2} BW = 1 - \frac{1}{2} \frac{\pi}{400} \approx 0.9961$

∴ Poles are located at:

$x_p = 0.9961 \cos 5.4 = 0.9917$

$y_p = 0.9961 \sin 5.4 = 0.0937$

$\left[\therefore \; z_p = 0.9917 \pm 0.0937j \right]$

∴ Notch filter is:

$$H(z) = \frac{(z - 0.9956 - 0.09411j)(z - 0.9956 + 0.09411j)}{(z - 0.9917 - 0.0937j)(z - 0.9917 + 0.0937j)}$$

$$\therefore H(z) = \frac{z^2 - 1.991z + 1}{z^2 - 1.983z + 0.9922}$$

$$\text{also } H(z) = \frac{1 - 1.991z^{-1} + z^{-2}}{1 - 1.983z^{-1} + 0.9922z^{-2}}$$

$\therefore$ Difference equation is:

$$H(z) = \frac{Y(n)}{X(n)} = \frac{1 - 1.991z^{-1} + z^{-2}}{1 - 1.983z^{-1} + 0.9922z^{-2}}$$

$$\boxed{\therefore y(n) = x(n) - 1.991x(n-1) + x(n-2) + 1.983y(n-1) - 0.9922y(n-2)}$$