GAM_BETA_SD_Prior_Simulation

Adam C. Smith

08/03/2022

Prior Simulation of the SD of GAM parameters

Running a prior simulation for alternative prior distributions on the SD of the GAM BETA parameters, which are the hyperparameters that control the shape and wiggliness of the survey-wide mean smoothed population trajectory. This simulation compares the effects of alternative priors on derived estimates of the long-term (53-year), and short-term (10-year) population trend estimates that would result from a population trajectory estimated with the same spline basis function used in this paper. This SD parameter controls the scale and variation of the parameters that link the spline basis function to the estimated smooth population trajectory. The posterior estimate of this SD parameter is what determines the complexity (wiggliness) and magnitude of the population change ((Crainiceanu, Ruppert, and Wand 2005; Bürkner 2017; Goodrich et al. 2020)).

The semi-parametric nature of smooths and the variation among alternative basis functions complicates the use of default or informative priors ((Lemoine 2019; Banner, Irvine, and Rodhouse 2020)). We've used a prior simulation to translate these alternative priors into intuitive values of population trends that are directly interpretable as biological parameters. We then compare the prior distribution of trends that would result from these alternative priors to:

1. the collection of realised long-term trend estimates from a different statistical model applied to the North American Breeding Bird Survey ((Link, Sauer, and Niven 2020)).
2. our own prior knowledge about probable rates of change in wild bird populations at continental scales.

We compared half normal and half t-distributions for the priors on the Standard Deviation on the GAM parameters.

1. normal
2. t-distribution with 3 degrees of freedom

And, for each of the half-normal and half-t-distributions, we compared 5 different values to set the prior-scale: (0.5, 1, 2, 3, and 4). Given the log-link in the trend model and the scaling of the low-rank thin-plate regression spline with identifiability constraints ((Wood 2020)), these 5-values of prior-scales should cover the range of plausible parameter values.

Selected prior

We suggest a half t-distribution, with a scale parameter = 2, fits the realised distributions of both long and short-term trends for most bird species surveyed by the BBS (i.e., most bird species with the best information

on population trends at continental scales and for ~50 years). This half-t prior results in prior distributions of long-term and short-term trends that fit the realized distributions and includes long tails that cover the range of plausible trend estimates without including large amounts of prior probability mass at implausibly extreme values.

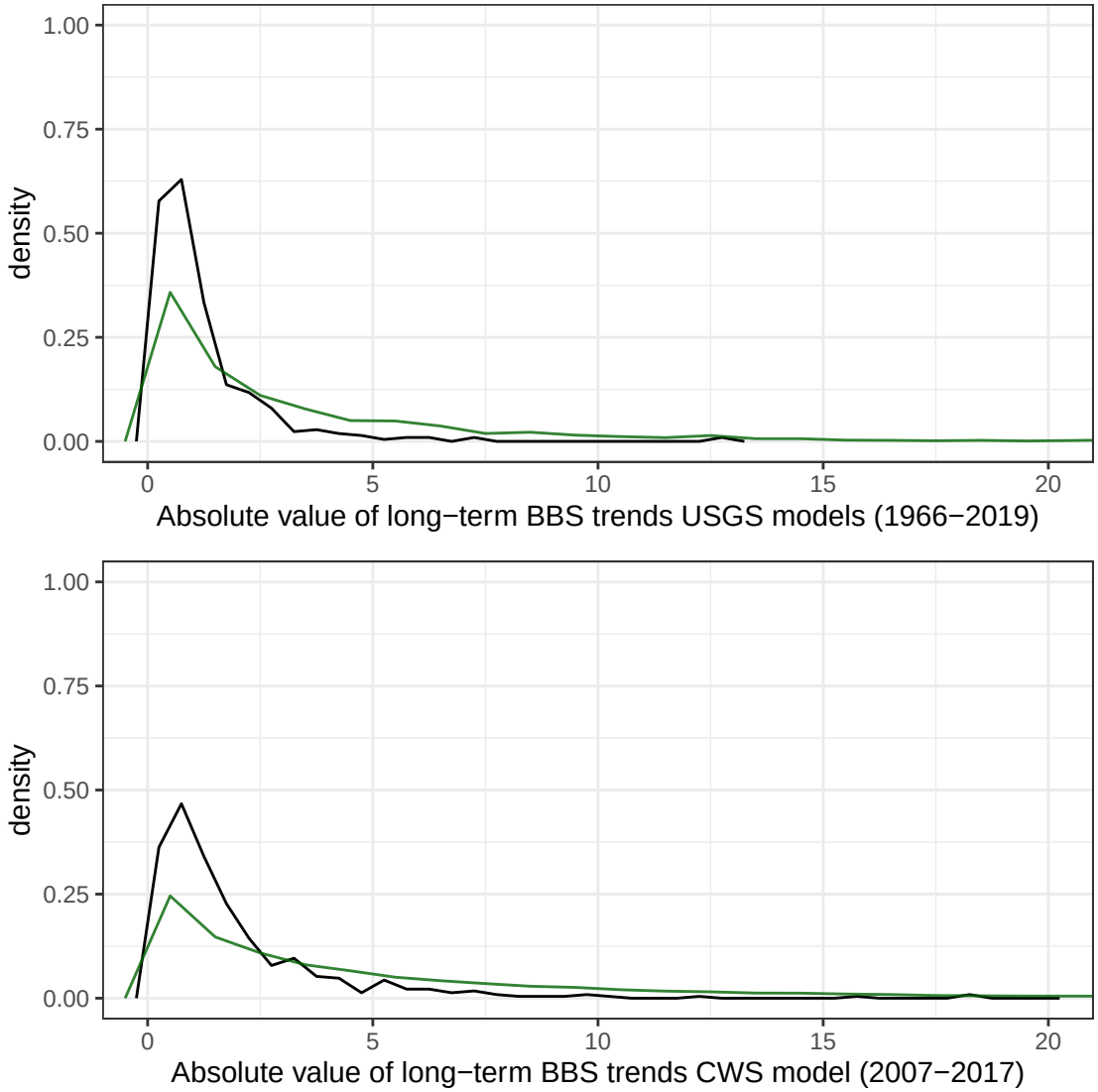


Figure 1: Observed distributions of the absolute values of long-term (top) and short-term (bottom) population trends from the BBS data using non-GAM models (in black), and the simulated prior distribution (in green) of long- and short-term trends from the spline smooth basis used in this paper, with a half-t (df = 3 and scale parameter = 2) prior distribution on the standard deviation of the spline parameters

Details on the prior simulations

Here's the code that runs the simulations in Stan.

```
# this is all set-up
library(tidyverse)
library(cmdstanr)
library(mgcv)
library(patchwork)


for(pp in c("norm","t")){
  for(prior_scale in c(0.5,1,2,3,4)){

  tp = paste0("GAM_",pp,prior_scale,"_rate")

  #STRATA_True <- log(2)
  output_dir <- "output/"
  out_base <- tp
  csv_files <- paste0(out_base,"-",1:3,".csv")


  if(pp == "norm"){
    pnorm <- 1
  }
  if(pp == "t"){
    pnorm <- 2
  }

  if(!file.exists(paste0(output_dir,csv_files[1]))){

    nyears = 54 #to match the time-scales of BBS and CBC analyses
  dat = data.frame(year = 1:nyears)
  nknots = 13
  nknots_realised = nknots-1

  M = mgcv::smoothCon(s(year,k = nknots, bs = "tp"),data = dat,
                      absorb.cons=TRUE,#this drops the constant
                      diagonal.penalty=TRUE) ## If TRUE then the smooth is reparameterized to turn the


    year_basis = M[[1]]$X




    stan_data = list(#scalar indicators
      nyears = nyears,
```

```
      #GAM structure
      nknots_year = nknots_realised,
      year_basis = year_basis,

      prior_scale = prior_scale,
      pnorm = pnorm
      )




    # Fit model ----------------------------------------------------------------

    print(paste("beginning",tp,Sys.time()))

    mod.file = "models/GAM_prior_sim.stan"

    ## compile model
    model <- cmdstan_model(mod.file)


    # Initial Values -----------------------------------------------------------


    init_def <- function(){
      list(sdbeta = runif(1,0.01,0.1),
           BETA_raw = rnorm(nknots_realised,0,0.01))}

    stanfit <- model$sample(
      data=stan_data,
      refresh=100,
      chains=2, iter_sampling=1000,
      iter_warmup=500,
      parallel_chains = 2,
      #pars = parms,
      adapt_delta = 0.8,
      max_treedepth = 14,
      seed = 123,
      init = init_def,
      output_dir = output_dir,
      output_basename = out_base)


   save(list = c("stanfit","stan_data","csv_files",
                 "out_base"),
        file = paste0(output_dir,"/",out_base,"_gamye_iCAR.RData"))



  }

}#end prior_scale loop
}#end pp loop
```

The simulation model

The Stan simulation model is very simple. We implemented it in Stan to match the implementation in the full model (although this simulation doesn't require MCMC sampling).

```
// simple GAM prior simulation

data {
  int<lower=1> nyears;
  //scale of the prior distribution
  real<lower=0>  prior_scale;
  // indicator for the prior distribution 0 = t, 1 = normal
  int<lower=0,upper=1> pnorm;
// data for spline s(year)
  // number of knots in the basis function for year
  int<lower=1> nknots_year;
  // basis function matrix
  matrix[nyears, nknots_year] year_basis;
}

parameters {
  real<lower=0> sdbeta;    // sd of spline coefficients
  // unscaled spline coefficients
  vector[nknots_year] BETA_raw;
}

transformed parameters {
  vector[nknots_year] BETA;  //scaled spline parameters
  vector[nyears] smooth_pred;

    BETA = sdbeta * BETA_raw; //scaling the spline parameters
     smooth_pred = year_basis * BETA; //log-scale smooth trajectory
  }

model {

//Conditional statements to select the prior distribution
if(pnorm == 1){
 sdbeta ~ normal(0,prior_scale); //prior on sd of GAM parameter
}
if(pnorm == 0){
  sdbeta ~ student_t(3,0,prior_scale); //prior on sd of GAM parameter
}

   BETA_raw ~ normal(0,1); //non-centered parameterisation

}

 generated quantities {
  //estimated smooth on a count-scale
   vector[nyears] nsmooth = exp(smooth_pred);

  }
```

We then summarized the estimated trajectories as well as all possible 1-year, 10-year, and long-term trends from the alternative priors.

```
source("Functions/posterior_summary_functions.R") ## loads the saved simulation results

nsmooth_out <- NULL
trends_out <- NULL
summ_out <- NULL

for(pp in c("norm","t")){
  for(prior_scale in c(0.5,1,2,3,4)){

    tp = paste0("GAM_",pp,prior_scale,"_rate")

    #STRATA_True <- log(2)
    output_dir <- "output/"
    out_base <- tp


load(paste0(output_dir,"/",out_base,"_gamye_iCAR.RData"))

summ = stanfit$summary()

summ <- summ %>%
  mutate(prior_scale = prior_scale,
         distribution = pp)

nsmooth_samples <- posterior_samples(stanfit,
                                     parm = "nsmooth",
                                     dims = c("Year_Index"))



BETA_samples <- posterior_samples(stanfit,
                                      parm = "BETA",
                                      dims = c("k"))
BETA_wide <- BETA_samples %>%
  pivot_wider(.,id_cols = .draw,
              names_from = k,
              names_prefix = "BETA",
              values_from = .value)


nsmooth_samples <- nsmooth_samples %>%
  left_join(., BETA_wide,by = ".draw") %>%
  mutate(prior_scale = prior_scale,
         distribution = pp)



nyears = max(nsmooth_samples$Year_Index)
# function to calculate a %/year trend from a count-scale trajectory
trs <- function(y1,y2,ny){
  tt <- (((y2/y1)^(1/ny))-1)*100
```

```
}

for(tl in c(2,11,nyears)){ #estimating all possible 1-year, 10-year, and full trends
  ny = tl-1
  yrs1 <- seq(1,(nyears-ny),by = ny)
  yrs2 <- yrs1+ny
  for(j in 1:length(yrs1)){
    y2 <- yrs2[j]
    y1 <- yrs1[j]

nyh2 <- paste0("Y",y2)
nyh1 <- paste0("Y",y1)
trends <- nsmooth_samples %>%
  filter(Year_Index %in% c(y1,y2)) %>%
  select(.draw,.value,Year_Index) %>%
  pivot_wider(.,names_from = Year_Index,
              values_from = .value,
              names_prefix = "Y") %>%
  rename_with(.,~gsub(pattern = nyh2,replacement = "YE", .x)) %>%
  rename_with(.,~gsub(pattern = nyh1,replacement = "YS", .x)) %>%
  group_by(.draw) %>%
  summarise(trend = trs(YS,YE,ny))%>%
  mutate(prior_scale = prior_scale,
         distribution = pp,
         first_year = y1,
         last_year = y2,
         nyears = ny)
trends_out <- bind_rows(trends_out,trends)
}
}
nsmooth_out <- bind_rows(nsmooth_out,nsmooth_samples)
summ_out <- bind_rows(summ_out,summ)
print(paste(pp,prior_scale))

  }#prior_scale
}# pp

save(file = "output/GAM_prior_sim_summary.RData",
     list = c("nsmooth_out",
              "trends_out",
              "summ_out"))
```

Comparing simulation priors to realised data

Realised trend estimates

First, here is the distribution of long-term trends from a different model for the BBS data from 1966-2019, for 426 species.

```
bbs_trends_usgs <- read.csv("data/BBS_1966-2019_core_best_trend.csv")
```

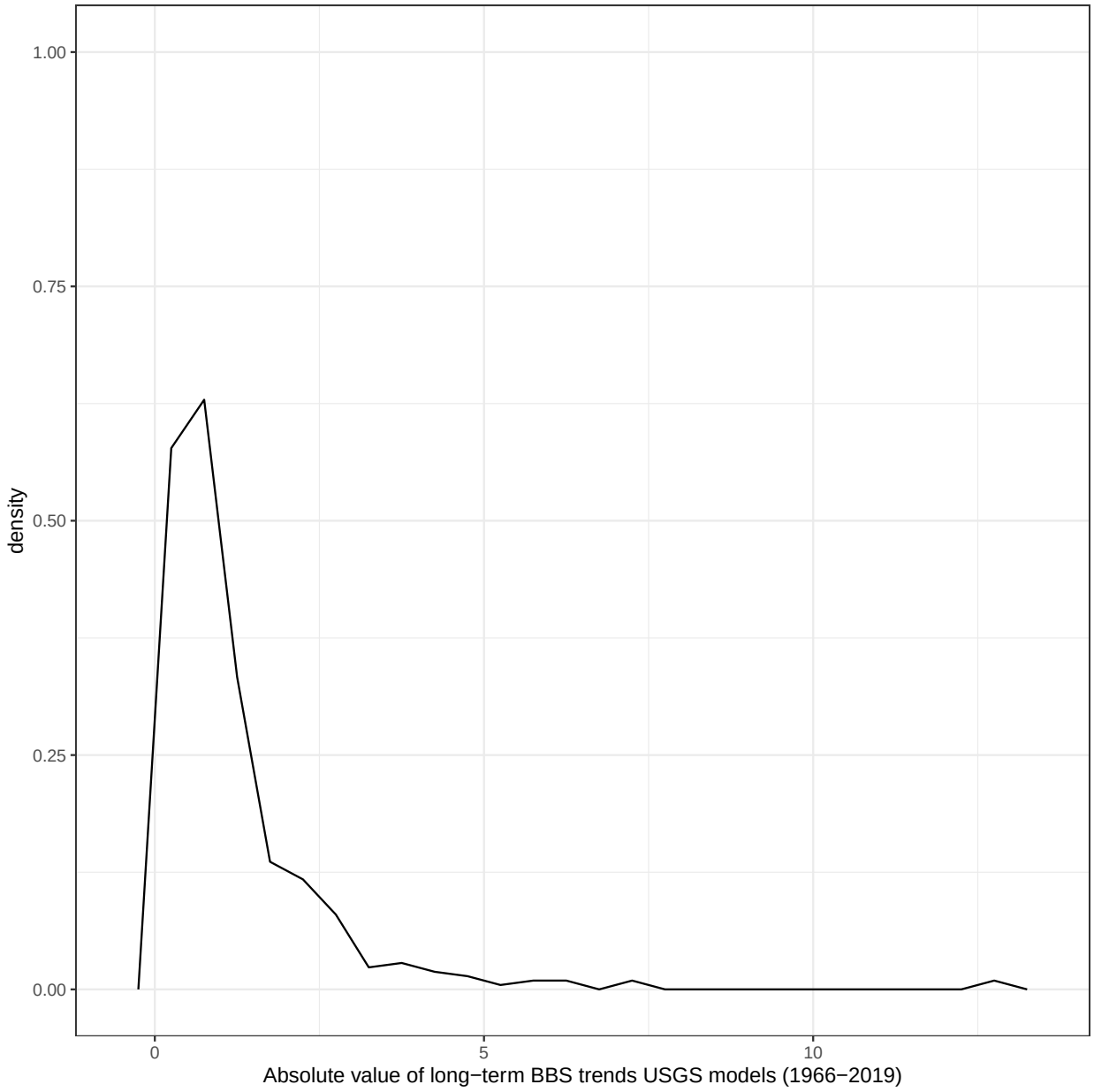
```r
## selecting survey-wide trend estimates
bbs_trends_usgs_long <-bbs_trends_usgs %>%
  filter(Region == "SU1")%>%
  select(Trend,Species.Name) %>%
  mutate(abs_trend = abs(Trend)) %>% #calculating absolute values of the trends
  arrange(-abs_trend)

G_long_usgs <- max(bbs_trends_usgs_long$abs_trend)

realised_long_bbs_hist <- ggplot(data = bbs_trends_usgs_long,
                           aes(abs_trend,after_stat(density)))+
  geom_freqpoly(breaks = seq(0,13,0.5),center = 0)+
  xlab("Absolute value of long-term BBS trends USGS models (1966-2019)")+
  theme_bw()+
  scale_y_continuous(limits = c(0,1))
print(realised_long_bbs_hist)
```

Table 1: Most extreme long-term BBS trends

Trend	Species.Name	abs_trend
12.92	Cave Swallow	12.92
12.91	Eurasian Collared-Dove	12.91
7.27	Canada Goose	7.27



The maximum absolute value of an observed long-term trends are for Cave Swallow and Eurasian Collared Dove, which have increased at an annual rate of 12.9 %/year. This annual rate of change implies an approximate 63000 % overall increase in the populations since 1966. As such, we feel this represents example of an "extreme" long-term trend that is unlikely to be observed in most of the BBS dataset. For example, the next largest values of trend in the data is a 7.3 %/year increase in Canada Goose populations.

And the largest absolute value of trends for a declining species is < 4%/year for King Rail, Bank Swallow,

Table 2: Most extreme long-term BBS declines

Trend	Species.Name	abs_trend
-3.82	King Rail	3.82
-3.72	Bank Swallow	3.72
-3.66	Lark Bunting	3.66

and Lark Bunting.

Summarize the long-term trends from the prior simulations

```
load("output/GAM_prior_sim_summary.RData")

trends_out <- trends_out %>%
  mutate(abs_trend = abs(trend),# absolute values of trends
         scale_factor = factor(prior_scale,ordered = TRUE)) #just for plotting

trends_long <- trends_out %>%  #select long-term trends only
  filter(nyears == 53 )

trends_normal <- trends_long %>%
  filter(distribution == "norm")
```

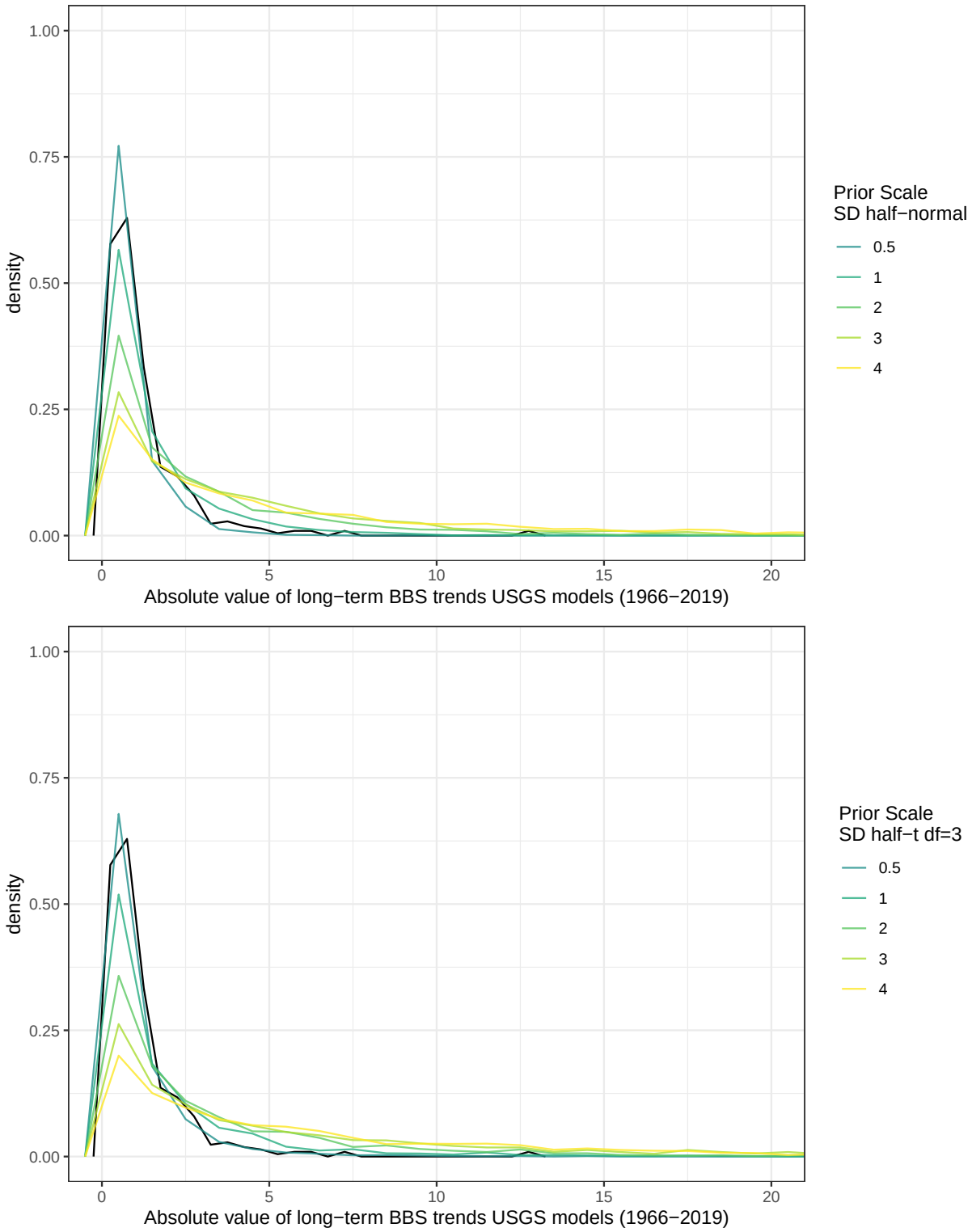
The distributions for the normal priors do a reasonable job of covering the range of possible long-term trend values, but the heavier-tailed t-distributions seem to better fit the shape of the distributions.

```
  overp_normal <- realised_long_bbs_hist +
    geom_freqpoly(data = trends_normal,
                  aes(abs_trend,after_stat(density),
                      colour = scale_factor),
                  breaks = seq(0,100,1),center = 0,
                  alpha = 0.8)+
    scale_colour_viridis_d(begin = 0.5,alpha = 0.8,
                           "Prior Scale\nSD half-normal")+
  coord_cartesian(xlim = c(0,20))


  trends_t <- trends_long %>%
  filter(distribution == "t")


  overp_t <- realised_long_bbs_hist +
    geom_freqpoly(data = trends_t,
                  aes(abs_trend,after_stat(density),
                      colour = scale_factor),
                  breaks = seq(0,100,1),center = 0,
                  alpha = 0.8)+
    scale_colour_viridis_d(begin = 0.5,alpha = 0.8,
                           "Prior Scale\nSD half-t df=3")+
  coord_cartesian(xlim = c(0,20))
```

The half-t-distributions with a scale value of 2 or 3 fit the shape of the realised trend distribution reasonably well, and the long-tail includes significant prior mass at and beyond the observed maximum absolute values of trends.



In addition, each of the prior distributions include some prior mass at trend values beyond the realised

Table 3: Prior simulated distribution quantiles for the long-term trends and proportion of the distributions greater than the realised maximum values from a separate analysis of the BBS data for > 400 species. Prior simulations for two prior distributions with 5 different scales (normal and t)

Distribution	Prior Scale	Median Prior Predicted Distribution	80th percentile	90th percentile	99th percentile	Proportion of prior distribution > realised maximum value
norm	0.5	0.403	1.123	1.777	3.686	0.000
norm	1.0	0.790	2.201	3.584	8.115	0.002
norm	2.0	1.465	4.383	6.800	16.521	0.021
norm	3.0	2.570	6.712	10.186	27.547	0.067
norm	4.0	3.056	8.876	13.785	32.492	0.112
t	0.5	0.524	1.590	2.439	7.597	0.002
t	1.0	0.927	2.863	4.746	15.990	0.016
t	2.0	1.743	5.549	8.890	30.854	0.049
t	3.0	2.947	9.169	14.859	39.754	0.122
t	4.0	4.087	11.833	18.989	52.542	0.174

maximum values. And the broadest priors (scales of 3 or 4), include prior mass at trend values that are extremely unlikely for a wild population to sustain over >50 years. For example, more than 10% of the two widest distributions are at values larger than the largest observed value of long-term trends, and the 99th percentiles extend to values that are truly extreme (40 - 50%/year).

```
quant_long_tends <- trends_long %>%
  group_by(distribution,prior_scale) %>%
  summarise(median_abs_t = median(abs_trend),
            U80 = quantile(abs_trend,0.80),
            U90 = quantile(abs_trend,0.90),
            U99 = quantile(abs_trend,0.99),
            pGTmax = length(which(abs_trend > G_long_usgs))/length(abs_trend))
```

```
## `summarise()` has grouped output by 'distribution'. You can override using the `.groups` argument.
```

```
kable(quant_long_tends, booktabs = TRUE,
      digits = 3,
      format.args = list(width = 7),
      col.names = c("Distribution",
                    "Prior Scale",
                    "Median Prior Predicted Distribution",
                    "80th percentile",
                    "90th percentile",
                    "99th percentile",
                    "Proportion of prior distribution > realised maximum value"),
      caption = "Prior simulated distribution quantiles for the long-term trends  and proportion of the
  kable_styling(font_size = 8)%>%
column_spec(column = 1:7,width = "2cm")
```

Short-term trends

The maximum observed short-term trend from the same USGS analysis of BBS trends is for Eurasian Collared-dove, which have a ten-year trend rate of 23.5%/year. Although the full set of short-term trends is not available for download, this extreme rate of short-term increase provides a useful benchmark against which we can compare the prior distributions of short-term trends. This ten-year trend rate implies a 730 % overall increase in the population over this time. This is an extreme increase over a short-period of time.

```
G_short_usgs <- 23.5 #Eurasian Collared Dove trend for short-term 2009-2019 analysis USGS
    # short-term trends not included in Science Base, but visible here:
    # https://www.mbr-pwrc.usgs.gov/bbs/reglist19v3.shtml
```

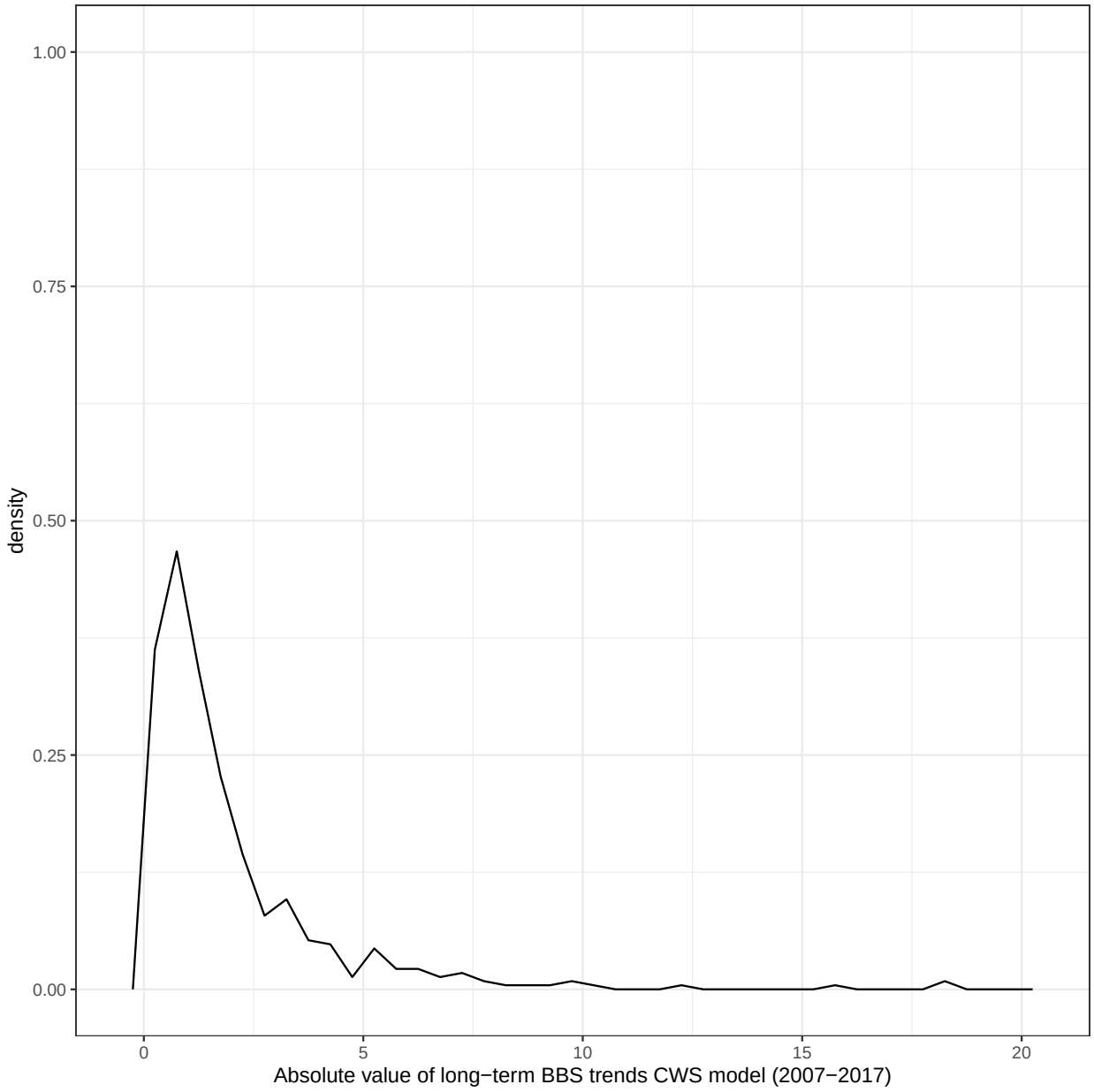
We can also use archived estimates from the Canadian Wildlife Service for 1970 - 2017, which were derived from a different model than the GAM used here.

```
bbs2017_Canada <- read.csv("data_basic/All BBS trends 2017 w reliab.csv")

bbs_short_survey_wide <- bbs2017_Canada %>%
  filter(region.type == "Continental",
         trendtype == "short-term",
         startyear == 2007,
         trendtime == "full") %>%
  select(species,trend) %>%
  mutate(abs_trend = abs(trend)) %>%
  arrange(-abs_trend)
```

The realised distribution of short-term trends from this archived analysis shows that short-term trends are generally more extreme than long-term trends.

```
realised_short_bbs_hist <- ggplot(data = bbs_short_survey_wide,
                            aes(abs_trend,after_stat(density)))+
  geom_freqpoly(breaks = seq(0,20,0.5),center = 0)+
  xlab("Absolute value of long-term BBS trends CWS model (2007-2017)")+
  theme_bw()+
  scale_y_continuous(limits = c(0,1))
print(realised_short_bbs_hist)
```



The short-term trends from the prior distributions represent all possible ten-year trends, not just the final ten-years of the trajectory.

```
trends_short <- trends_out %>%  #select long-term trends only
  filter(nyears == 10 )

trends_short_normal <- trends_short %>%
  filter(distribution == "norm")

trends_short_t <- trends_short %>%
  filter(distribution == "t")
```

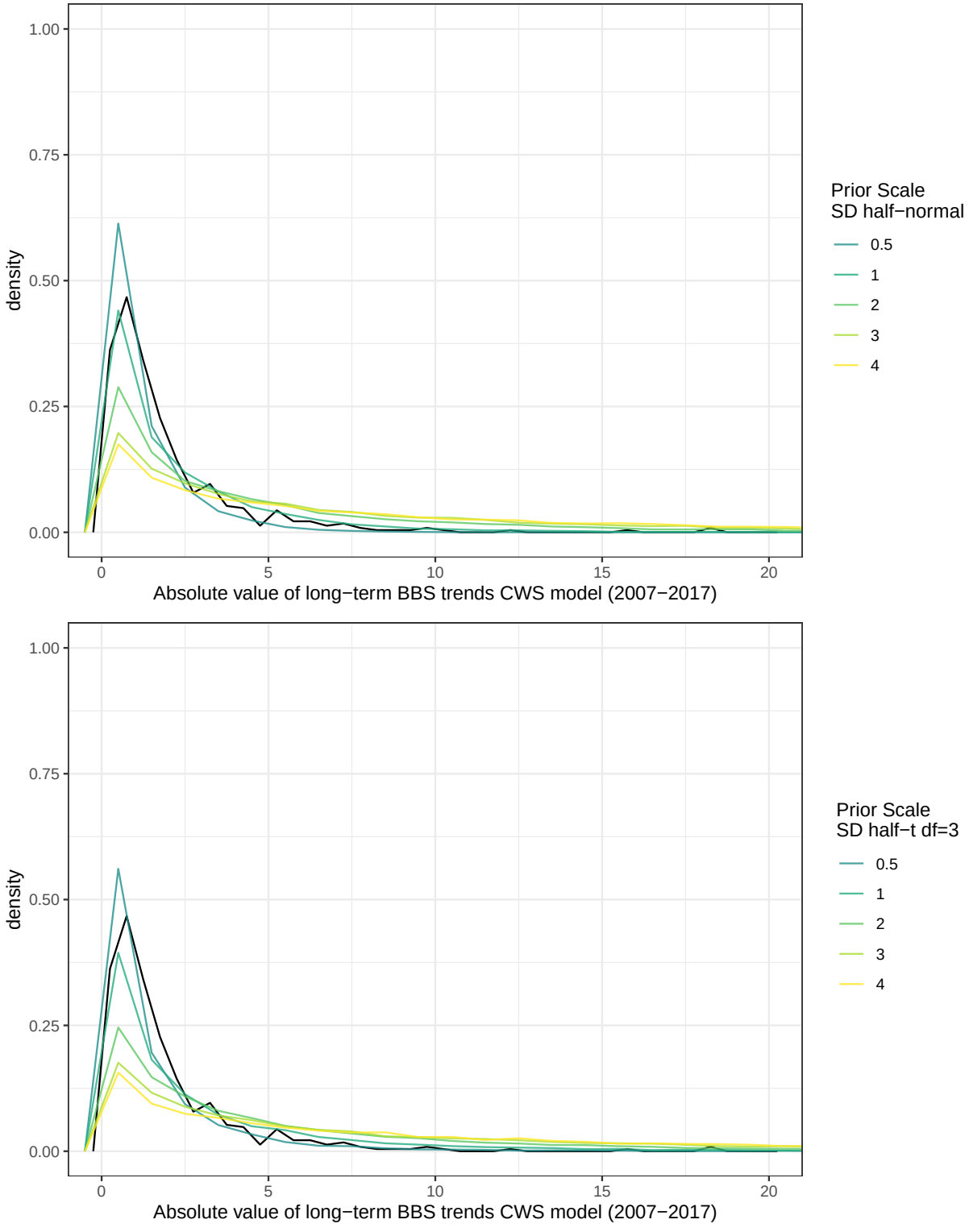
Overplotting the simulation prior distributions of short-term trends with the realised collection of short-term trends.

```
overp_normal_short <- realised_short_bbs_hist +
  geom_freqpoly(data = trends_short_normal,
                aes(abs_trend,after_stat(density),
                    colour = scale_factor),
                breaks = seq(0,100,1),center = 0,
                alpha = 0.8)+
  scale_colour_viridis_d(begin = 0.5,alpha = 0.8,
                         "Prior Scale\nSD half-normal")+
coord_cartesian(xlim = c(0,20))



overp_t_short <- realised_short_bbs_hist +
  geom_freqpoly(data = trends_short_t,
                aes(abs_trend,after_stat(density),
                    colour = scale_factor),
                breaks = seq(0,100,1),center = 0,
                alpha = 0.8)+
  scale_colour_viridis_d(begin = 0.5,alpha = 0.8,
                         "Prior Scale\nSD half-t df=3")+
coord_cartesian(xlim = c(0,20))
```

The t-distribution scale parameter = 2, does a reasonably good job of covering the range of realistic short-term trends in the BBS data, and includes a long tail that includes much more extreme values when supported by the data. The distributions with larger scale parameters place significant prior density at rates of short-term change that are highly improbable at continental scales. For example, approximately 10% of the two widest distributions are at values larger than the largest observed value, and the 99th percentiles extend to values that are truly extreme (64 - 92%/year).



Below, you can see that the half-t-distribution with scale parameter = 2 has a long-tail that will allow for extreme trends when supported by the data (e.g., it has approximately 5% of the prior distribution covering values larger than the maximum observed value and a 99th percentile far beyond the most extreme observed value).

Table 4: Prior simulated distribution quantiles for the short-term trends and proportion of the distributions greater than the realised maximum values from a separate analysis of the BBS data for > 400 species. Prior simulations for two prior distributions with 5 different scales (normal and t)

Distribution	Prior Scale	Median Prior Predicted Distribution	80th percentile	90th percentile	99th percentile	Proportion of prior distribution > realised maximum value
norm	0.5	0.687	1.823	2.818	6.123	0.000
norm	1.0	1.257	3.589	5.465	13.168	0.000
norm	2.0	2.480	7.281	11.498	26.876	0.015
norm	3.0	4.052	11.359	17.401	41.275	0.052
norm	4.0	5.178	14.872	22.987	57.107	0.097
t	0.5	0.796	2.395	3.953	11.813	0.001
t	1.0	1.526	4.738	7.834	26.698	0.012
t	2.0	2.995	8.872	14.497	48.119	0.041
t	3.0	4.832	14.696	23.364	64.604	0.099
t	4.0	6.237	18.549	29.864	92.292	0.147

```
quant_short_tends <- trends_short %>%
  group_by(distribution,prior_scale) %>%
  summarise(median_abs_t = median(abs_trend),
            U80 = quantile(abs_trend,0.80),
            U90 = quantile(abs_trend,0.90),
            U99 = quantile(abs_trend,0.99),
            pGTmax = length(which(abs_trend > G_short_usgs))/length(abs_trend))
```

```
## `summarise()` has grouped output by 'distribution'. You can override using the `.groups` argument.
```

```
kable(quant_short_tends, booktabs = TRUE,
      digits = 3,
      format.args = list(width = 7),
      col.names = c("Distribution",
                    "Prior Scale",
                    "Median Prior Predicted Distribution",
                    "80th percentile",
                    "90th percentile",
                    "99th percentile",
                    "Proportion of prior distribution > realised maximum value"), caption = "Prior simul
  kable_styling(font_size = 8)%>%
column_spec(column = 1:7,width = "2cm")
```

References

Banner, Katharine M., Kathryn M. Irvine, and Thomas J. Rodhouse. 2020. "The Use of Bayesian Priors in Ecology: The Good, the Bad and the Not Great." *Methods in Ecology and Evolution* 11 (8): 882–89. https://doi.org/https://doi.org/10.1111/2041-210X.13407.

Bürkner, Paul-Christian. 2017. "Brms: An R Package for Bayesian Multilevel Models Using Stan." *Journal of Statistical Software* 80 (1). https://doi.org/10.18637/jss.v080.i01.

Crainiceanu, Ciprian M., David Ruppert, and Matthew P. Wand. 2005. "Bayesian Analysis for Penalized Spline Regression Using WinBUGS." *Journal of Statistical Software* 14 (1): 1–24. https://doi.org/10.18637/jss.v014.i14.

Goodrich, Ben, Jonah Gabry, Imad Ali, and Sam Brilleman. 2020. *Rstanarm: Bayesian Applied Regression Modeling via Stan.* https://mc-stan.org/rstanarm.

Lemoine, Nathan P. 2019. "Moving Beyond Noninformative Priors: Why and How to Choose Weakly Informative Priors in Bayesian Analyses." *Oikos* 128 (7): 912–28. https://doi.org/10.1111/oik.05985.

Link, William A., John R. Sauer, and Daniel K. Niven. 2020. "Model Selection for the North American Breeding Bird Survey." *Ecological Applications* 30 (6): e02137. https://doi.org/10.1002/eap.2137.

Wood, Simon N. 2020. "Inference and Computation with Generalized Additive Models and Their Extensions." *TEST* 29 (2): 307–39. https://doi.org/10.1007/s11749-020-00711-5.