Determination of the Charge-to-Mass Ratio for the Electron

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Introduction

The mass of an elementary particle cannot be measured directly by weighing them on a traditional scale, due to the limited precision of the instrument and the impracticality of isolating the particle from its surroundings. Alternatively, the mass can be derived from its relationship with the charge of the particle.

Newton's second law from classical mechanics states that the acceleration of a particle is equal to the force applied on it divided by its mass. The relationship between the force *F* and the charge of the particle *q* in a magnetic field *B* is given by equation (1):

$$F = qv \times B \quad (1)$$

where *v* is the velocity of the particle.

The objective of this experiment is to determine the charge-to-mass ratio for the electron. A uniform *B* and a *v* perpendicular to *B* will cause the electron to experience centripetal motion. In this experiment, a uniform B_c is created by applying current through a pair of Helmholtz coils, given by equation (2):

$$B_c = \left(\frac{4}{5}\right)^{3/2} \frac{\mu_0 n I}{R} \quad (2)$$

where μ_0 is the vacuum permeability equal to $4\pi \times 10^{-7}$ H/m, *R* is the radius of coils, and n is the number of turns in each coil. However, the magnetic field $B = B_c + B_e$, where B_e is the external magnetic field generated from Earth, nearby ferromagnetic materials, and other sources. Combining the centripetal version of Newton's second law and equation (1) for the electron above gives us equation (3):

$$evB = m\frac{v^2}{r} \quad (3)$$

where *r* is the radius of the orbit.

In order for the electron to reach speed *v*, it is accelerated through an electric potential *V*. After the electron has been accelerated, it will have gained kinetic energy equal to the amount of work done on it, according to conservation of energy. This is represented by equation (4):

$$eV = \frac{1}{2}mv^2 \quad (4)$$

which is an approximation that does not take special relativity into account.

Thus, from equations (2) and (3), the curvature of the electron orbit is given by equation (5):

$$\frac{1}{r} = \sqrt{\frac{e}{2m}}\frac{B}{\sqrt{V}} \quad (5)$$

which can be used to solve for *e/m*, the charge-to-mass ratio for the electron. Equation (5) can be rewritten as equation (6):

$$\frac{1}{r} = \sqrt{\frac{e}{2m}}\frac{1}{\sqrt{V}}\left[\left(\frac{4}{5}\right)^{3/2}\frac{\mu_0 n I}{R}\right] \quad (6)$$

which can also be rewritten as equation (7):

$$\frac{1}{r} = \sqrt{\frac{e}{2m}}k\frac{(I - I_0)}{\sqrt{V}} \quad (7)$$

where k is the characteristic of the coil dimensions equal to $\frac{1}{\sqrt{2}}(\frac{4}{5})^{3/2}\frac{\mu_0 n}{R}$, and I_0 is a constant equal to B_e/k.

Apparatus

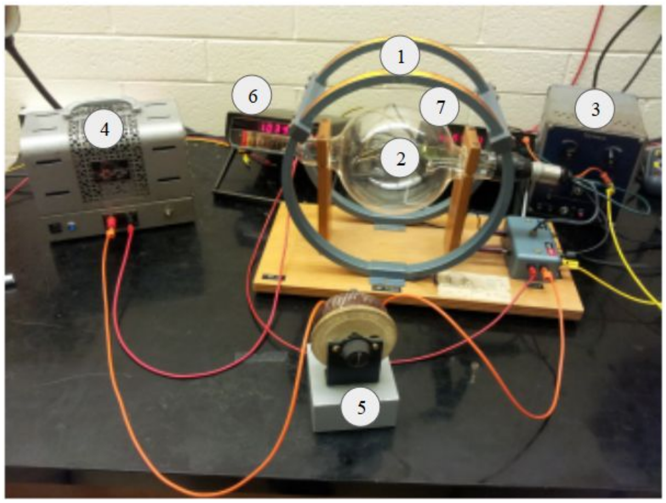

Figure 1 - The experimental setup with numbered labels corresponding to the equipment listed below.

1. Helmholtz coils with 15.4 ± 0.1 cm radius and 130 turns in each coil
2. Glass bulb containing low pressure hydrogen gas and electron gun
3. 0 to 300 V power supply
4. 8 V DC power supply (around 10 V under load)
5. Rheostat
6. Ammeter with a reading error of 0.001 mA
7. Voltmeter with a reading error of 0.001 mA

In addition, a self-illuminated 12 cm scale with a 2 cm gap in the middle and a plastic reflector was placed 11.5 cm away from the front face of the glass bulb. It produced an image of the scale that appeared to be superimposed on the circular trajectory of the electron beam, in order to eliminate problems of parallax.

Description of Experiment

With the apparatus positioned and connected appropriately as in Figure 1 above, the filament of the electron gun inside the glass bulb was turned on. In order to prevent the tube from becoming damaged, the anode voltage was only turned on after waiting at least 30 seconds after the filament of the electron gun had been turned on, and the anode voltage was turned off before the filament of the electron gun had been turned off. The 8 V DC power supply was turned on to provide current to the Helmholtz coils. In order to effectively make out the curved trajectories of the electron beams, the lights in the lab were turned off.

Initially, the output voltage of the 0 to 300 V power supply was set to approximately 150 V. The rheostat was adjusted so that the diameter of the circular path was 12.0 cm, measured with the image of the self-illuminating scale. Afterward, the current readings on the ammeter were recorded, along with the corresponding voltage and diameter measurements. This was repeated for decreasing diameters at 1.0 cm intervals up to 5.0 cm, when the rheostat could not be turned any farther to produce a lower resistance. Three more trials of the steps above were conducted at anode voltages of approximately 200 V, 250 V, and 300 V.

Results

	Trial #1	Trial #2	Trial #3	Trial #4
Voltage [V]	150.3 ± 0.1	200.2 ± 0.1	249.5 ± 0.1	301.2 ± 0.1
Diameter [cm]	Current [mA]	Current [mA]	Current [mA]	Current [mA]
12.0 ± 0.1	1.755 ± 0.001	2.000 ± 0.001	2.316 ± 0.001	2.530 ± 0.001
11.0 ± 0.1	1.807 ± 0.001	2.255 ± 0.001	2.539 ± 0.001	2.792 ± 0.001
10.0 ± 0.1	2.120 ± 0.001	2.448 ± 0.001	2.809 ± 0.001	3.085 ± 0.001
9.0 ± 0.1	2.329 ± 0.001	2.703 ± 0.001	3.058 ± 0.001	3.371 ± 0.001
8.0 ± 0.1	2.515 ± 0.001	2.981 ± 0.001	3.407 ± 0.001	3.804 ± 0.001
7.0 ± 0.1	2.720 ± 0.001	3.370 ± 0.001	3.866 ± 0.001	4.312 ± 0.001
6.0 ± 0.1	3.175 ± 0.001	3.728 ± 0.001	4.465 ± 0.001	N/A
5.0 ± 0.1	3.716 ± 0.001	4.392 ± 0.001	N/A	N/A

Table 1 - Directed measurements of current with varying voltages and electron orbit diameters.

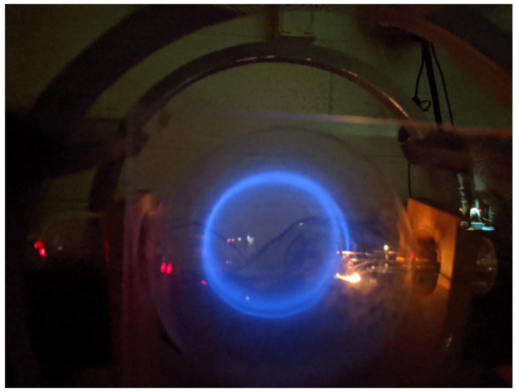

Figure 2 - Circular trajectory of the electron beam with 12.0 cm diameter.

Accurately positioning the plastic reflector away from the glass bulb was difficult, as there was no way to determine whether the image of the self-illuminating scale was perfectly superimposed on the circular trajectory of the electron, or whether it had been offset by a small, but not negligible, distance. The trajectory was often difficult to observe and measure due to its blur and lack of brightness, which worsened with the superimposed image of the scale. These results could be improved by eliminating all ambient light and observing the trajectory through a camera with good low light image processing, such as the one used to obtain Figure 2. Results for current and voltage readings could be improved by accounting taking their internal resistances into account, which was not considered for this experiment.

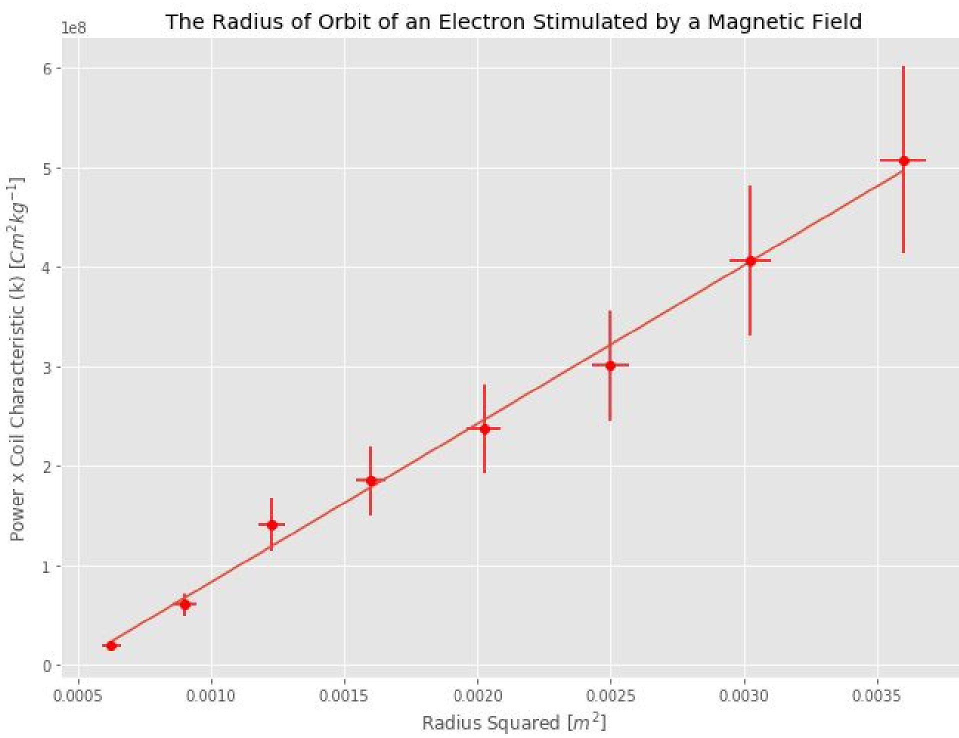

Figure 3 - Plot of radius of orbit squared relative to magnetic field strength

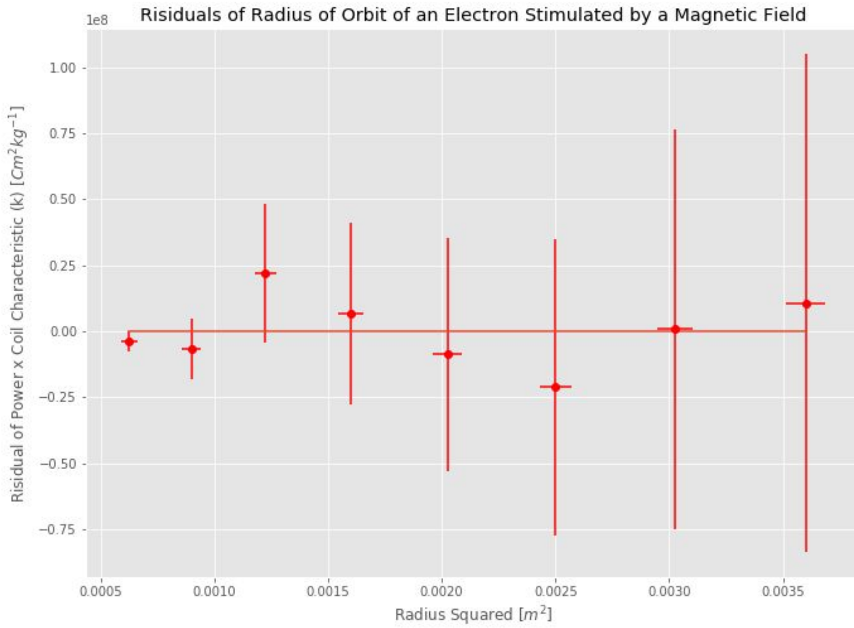

Figure 4 - Residual plot for the radius of orbit squared relative to magnetic field strength. Fit strength of 1.66 reduced chi squared method

Discussion & Analysis

<u>Uncertainty of Calculation</u>

Using the equation for the y-axis of the plot,

$$y = \frac{\sqrt{V}}{k(I - I_0)},$$

we can calculate uncertainty as follows

$$\sigma_y = y\sqrt{(\frac{\sigma_V}{\sqrt{V}})^2 + (\frac{\sigma_I}{I})^2 + (\frac{\sigma_k}{k})^2}$$

The plotted data yielded a result for the charge to mass ratio of electrons being -1.59 x 10^{11} ± 1.1 x 10^{10} C kg^{-1}. The determination of this result & the plot made use of our approximation of ambient magnetic field strength being around 3.116 x 10^{-6} ± 2.83 x 10^{-7} T which yields from equation the lab documentation [1] re-arranged (see code for more details)

$$B - B_c = B_e$$

This magnetic field was mostly simple and constant forces in the area, such as the field of the earth, technology in the building, and similar devices. The exception is the impact that can be seen in bringing a device with a strong electromagnetic field near the test chamber, in which case a deviation can be seen, and the orbit ceases.

Conclusion

In conclusion, we found a value for the ratio of charge to mass of an electron to be approximately $-1.59\text{x}10^{11}$ C kg^{-1} with account for the ambient magnetic fields and measurement errors which were partially negated using a light based measurement tool. The fit of our data was fairly strong, with a reduced chi squared value of only 1.166 [2] (see code for more details on calculation).

References

[1] “Charge-to-mass ratio for the electron - U of T Physics.” [Online]. Available: https://www.physics.utoronto.ca/~phy224_324/experiments/em-electrons/em-electron.pdf. [Accessed: 29-Oct-2019].

[2] ”Physics Labs Code Repository - PHY293” [Online]. Available: https://github.com/Thefaceofbo/physics_labs_PHY293/tree/master/Lab_2~Electron. [Accessed: 29-Oct-2019].