Determination of the Velocity of Sound in Water with an Ultrasonic Grating

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Introduction

Underwater vibrations cause regions in the water to alternate between compressed and decompressed states, commonly known as waves. When waves propagate in a fixed, rectangular container, waves that reflect from the walls are superimposed with the directed waves to form standing waves with constant high and low density regions, or nodes and antinodes, of water molecules. These standing waves act as a diffraction grating, so that when light waves travel perpendicularly through them, interference pattern of the light can be observed on the other side, parallel to the standing waves. The relationship between the wavelength of a monochromatic light source λ and grating spacing d is given by equation (1):

$$m\lambda = d\sin\theta \quad (1)$$

where m and θ are the order and angle of diffraction, respectively.

The objective of this experiment is to determine the velocity of sound in water, which can be found by solving for d in equation (1) and substituting it for λ_s in equation (2):

$$v_s = f_s\lambda_s \quad (2)$$

where f_s is the frequency of the standing wave and λ_s is its wavelength. It is also important to note that the velocity of sound in liquids is dependent on their temperature; constant temperature of water is assumed for this experiment. The velocity of water can also be represented by equation (3):

$$v_s = \sqrt{B/\rho} \quad (3)$$

where B is the resistance to compression, also known as the bulk modulus of water, and ρ is the density of water. In this experiment, the bulk modulus for adiabatic compression is determined, in contrast to the isothermal bulk modulus, as no energy exchanges with the region next to the sound wave occur.

Apparatus

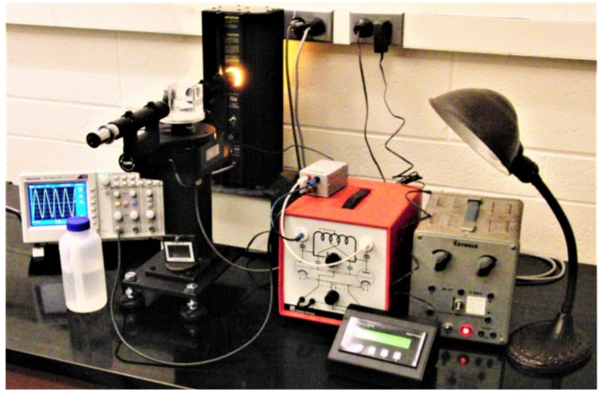


Figure 1 - The experimental setup.

- Spectrometer
- Sodium-vapour lamp
- Diffraction grating with 2500 lines per inch
- Ultrasound transducer
- Oscillator with a piezoelectric crystal
- Oscilloscope with a reading error of 0.005 MHz
- Frequency counter
- Rotating table
- Water inside a transparent, rectangular container
- Digital angle reader with a resolution of 0.00225°

Description of Experiment

The sodium-vapour lamp was turned on and was warmed up for 10 minutes until its colour reached a steady state of bright yellow from its initial state of bright red. In order to find the wavelength of the monochromatic yellow light, which is needed in solving for the velocity of sound in water by first solving for the wavelengths of the ultrasonic waves, the diffraction grating with 2500 lines per inch was placed on the rotating table in front of the spectroscope. The instrument slit located at the back of the spectroscope was adjusted as an attempt to bring the image and the crosshair into focus. The crosshair on the spectroscope was first aligned with the zero order, and the digital angle reader measuring the angle of the rotating table was zeroed by pressing the *Preset* button. The table was then rotated so that the crosshair was placed on the first and second orders, and measurements were taken from the digital angle reader.

Afterward, the standard diffraction grating was removed from the rotating table and it was replaced by a transparent, rectangular container filled with water. The ultrasound transducer was connected to the oscillator through the *Generator* box, and the oscilloscope was connected to the frequency counter. Next, the ultrasonic beam was switched on and its frequency was set as close to 2.10 MHz as possible by carefully turning the knob while reading the frequency value on the oscilloscope. Angles were measured for the different orders of diffraction, as it was previously explained for the standard diffraction grating, by first zeroing the digital angle reader when the crosshair was aligned with the zero order. This was repeated for frequencies between 2.10 MHz and 1.80 MHz, at intervals of 0.05 MHz. A total of eight trials were conducted in this part of the experiment.

Results

m	θ [°]
±1	3.4775 ± 0.0023
±2	6.9245 ± 0.0023

Table 1 - Direct measurements of angles of diffraction with diffraction grating.

		Trial #1	Trial #2	Trial #3	Trial #4	Trial #5	Trial #6	Trial #7	Trial #8
f_s [MHz]	m	θ [°]	θ [°]	θ [°]	θ [°]	θ [°]	θ [°]	θ [°]	θ [°]
2.10	±1	0.0400	0.0475	0.0475	0.0400	0.0400	0.0400	0.0550	0.0525
2.05	±1	0.0450	0.0450	0.0350	0.0450	0.0475	0.0475	0.0450	0.0375
2.00	±1	0.0575	0.0425	0.0450	0.0500	0.0500	0.0400	0.0525	0.0500
1.95	±1	0.0250	0.0400	0.0425	0.0450	0.0400	0.0400	0.0450	0.0400
	±2	0.0575	0.0925	0.0775	0.0850	0.0800	0.0900	0.0800	0.0850
1.90	±1	0.0375	0.0325	0.0300	0.0500	0.0450	0.0450	0.0400	0.0425
	±2	0.0750	0.0825	0.0575	0.0825	0.0900	0.0850	0.0925	0.0825
1.85	±1	0.0425	0.0475	0.0375	0.0425	0.0425	0.0425	0.0350	0.0450
	±2	0.0775	0.0975	0.0825	0.0725	0.0750	0.0850	0.0775	0.0875
1.80	±1	0.0250	0.0450	0.0375	0.0300	0.0375	0.0500	0.0400	0.0375
	±2	0.0625	0.0925	0.0750	0.0725	0.0825	0.0825	0.0825	0.0775

Table 2 - Direct measurements of angles of diffraction with ultrasonic waves. Uncertainty is ± 0.0023° for all angles measured above. Second orders of diffraction were indistinguishable for frequencies of 2.00, 2.05, and 2.10 MHz.

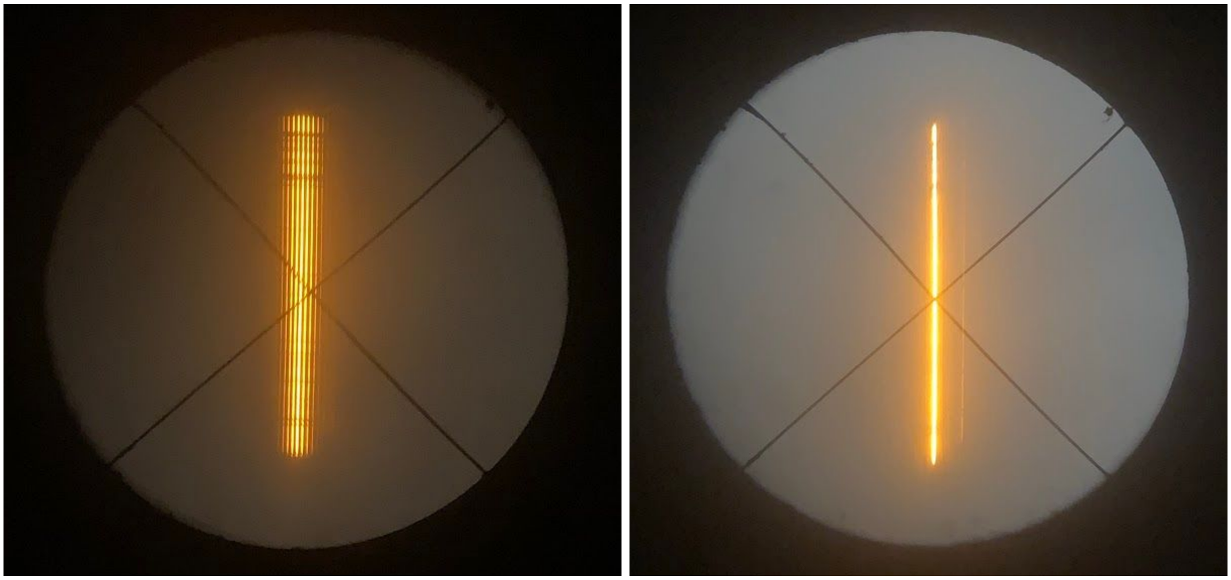


Figure 2 - Diffraction of yellow light due to the ultrasound with frequency of 1.80 MHz (left) and 2.10 MHz (right). Photos were taken on an iPhone XR with a single, optically stabilised, wide-angle, 12-megapixel f/1.8 camera.

Obtaining sharp images of the interference pattern and the crosshair was proven difficult to do as a concrete agreement was not reached on the clearest state of the spectroscope in focus. This, with poor eyesight, as the usage of the spectroscope required the removal of corrective lenses, led to the observations of blurred spectra of light. Consequently, the crosshair was imprecisely placed on the orders of diffraction, which resulted in the inconsistent data of angles of diffraction. The inconsistency was more significant for higher frequencies as lines in the spectra for the different orders of diffraction were closer together. Therefore, in order to improve the results of the measurements, the spectroscope must be in focus, and the spectra should be observed with better vision. An external monitor displaying an enlarged image of the spectroscope may help with this.

Necessary Derivations

Finding λ_s

We know that $d' = \lambda_s$

$$m'\lambda_L = d' sin'\theta'$$

$$d' = \frac{m'\lambda_L}{sin'\theta'}$$

$$\lambda_s = \frac{m'\lambda_L}{sin'\theta'}$$

Equation Error

Taking result $\lambda_s = \frac{m'\lambda_L}{sin'\theta'}$

$$\sigma_\lambda = \lambda_s \sqrt{(\frac{\sigma_{m'}}{m'})^2 + (\frac{\sigma_{\lambda_L}}{\lambda_L})^2 + (\frac{\sigma_{\theta'}}{\theta'})^2}$$

Finding *B*

$$v_s = \sqrt{\frac{B}{\rho}}$$

$$v_s^2 = \frac{B}{\rho}$$

$$v_s^2 \rho = B$$

$$B = v_s^2 \rho$$

Equation Error

Taking result $B = v_s^2 \rho$

$$\sigma_B = B \sqrt{2(\frac{\sigma_{v_s}}{v_s})^2 + (\frac{\sigma_\rho}{\rho})^2}$$

Finding Error on λ_L

$$\sigma_{\lambda_L} = \lambda_L \sqrt{(\frac{\sigma_m}{m})^2 + (\frac{\sigma_d}{d})^2 + (\frac{\sigma_\theta}{\theta})^2}$$

Discussion/Analysis

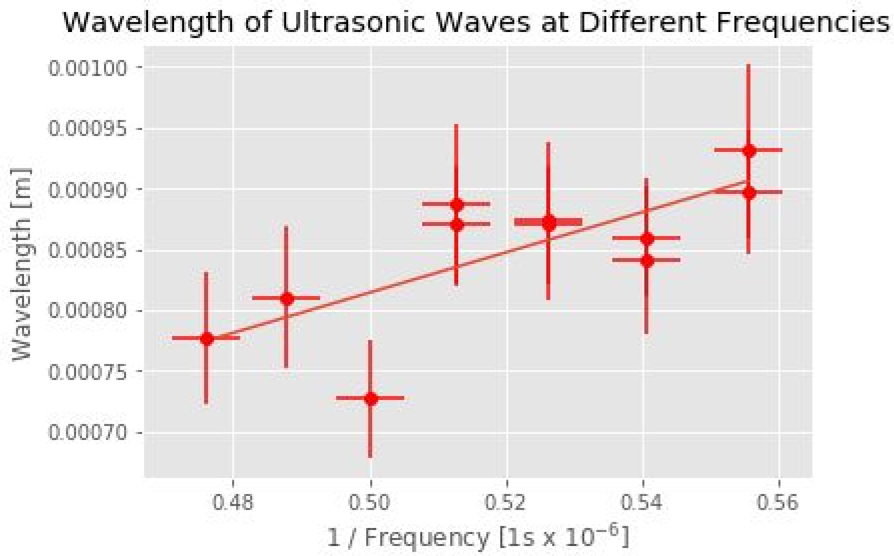
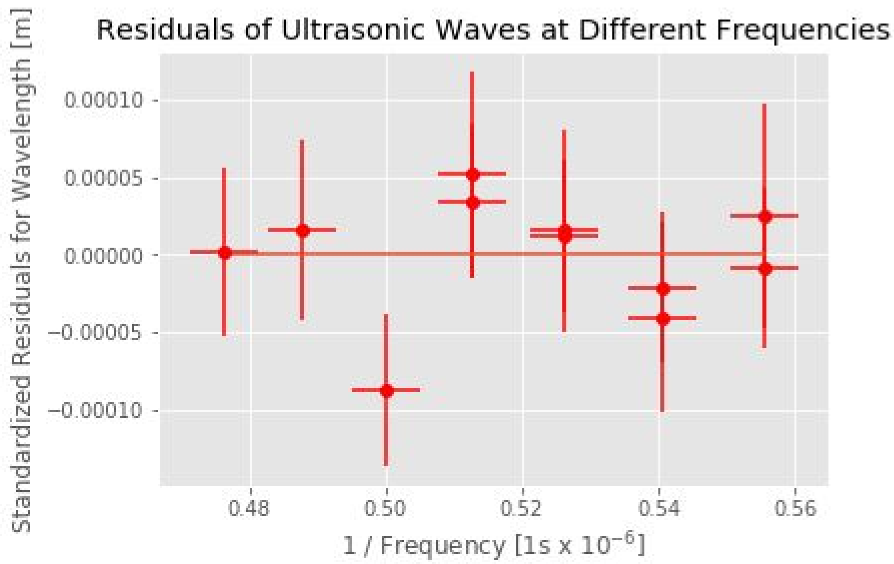


Figure 3 - The Wavelengths of Ultrasonic Waves plotted against the period of oscillation of the water

The figure above displays our result. Using the slope of the graph, it is shown that the velocity of the waves in water is approximately 1653 ± 491m/s. Using this value and the density of water (997kg/m^3), the bulk modulus can be calculated using the equations derived above to yield 2.72 ± 0.42 GN/m^2. All of the values have been plotted and calculated using the averages of our data points across all trials.

Generally, our result agrees very well with a linear relation between *d* and $1/f_s$. This is visible both in the line of best fit's accuracy in the plot as well as our analyzed χ^2 value. See the attached code for specifics of the implementation. The value of our fit quality is 5.23, producing a reduced value of 2.615. This value proves that our fit is fairly well done, especially considering the number of sources of error that the lab had, as well as the outlier point produced in our trials. This fit value would be greatly improved by including a technique that removes outliers from our data. Our general accuracy can also be plotted in a residual plot as:



Both the residual plot and chi-squared results indicate that the level of dispersive waves[1] was fairly low. This is a result of the depth of water used, the frequencies chosen for the experiment and the method by which the waves were produced. In all, our results were able to be made fairly accurate by our ability to mitigate reading errors while aligning the crosshair during distance readings, as well as the number of trials made during the experiment.

Conclusion

In conclusion, Our result was quite close to the expected bulk modulus of 2.2 GN/m^2 [3]. With a fair margin of error, the result is within error of the expected, making the experiment an accurate one. Moreover, the data presented was fairly precise, proved by the small variance in the data from trial to trial for the same frequency. In future iterations of the experiment, improvements could be made primarily to the perception of the bands of light under the crosshair by taking photos and measuring the distances digitally taking into account the lens. This would remove the error of human perception, increasing the accuracy further.

References

Dispersive Waves. (n.d.). Retrieved from http://physics.usask.ca/~hirose/ep225/animation/dispersion/anim-dispersion.html.

Serbanescu, R. M. (2019). The Velocity of Ultrasonic Waves in Water. Retrieved 2019, from https://www.physics.utoronto.ca/~phy224_324/experiments/velocity-in-water/The-Velocity-of-Ultrasonic-Waves-in-Water.pdf.

Akay, O. (n.d.). Fluid Properties. Retrieved from https://www.okan.edu.tr/uploads/pages/ce204-fluid-mechanics/Class2_25_02_2010.pdf.

Fitting Code

```
def get_fit_quality_chi_sq(y, fit_y, y_acc):
    """This returns the chi-squared value for the data & fit line"""
    y_diff = [ypt - fitpt for ypt, fitpt in zip(y, fit_y)]
    return sum([(ypt - fitpt)**2/acc**2 for ypt, fitpt, acc in zip(y,
fit_y, y_acc)])
# Get uncertainties
lambda_L = 0.000000614361
theta_L = 0.120855
d_sodium = 0.0254/2500
m_err = 0
theta_err = 0.000039269
d_err = 0.0000005
lambda_L_err = lambda_L * np.sqrt((d_err/d_sodium)**2 +
(theta_err/theta_L)**2)
# Get averages
avg_theta = np.array([sum([plot_data[v][trial] for trial in range(2,
len(plot_data[0]))])/(len(plot_data[0]) - 2) for v in range(0,
len(plot_data))])
avg_lambda_s = [compute_lambda_s(plot_data[row][1], avg_theta[row]) for
row in range(0, len(avg_theta))]
avg_lambda_err = [lambda_s*np.sqrt((lambda_L_err/lambda_L)**2 +
(theta_err/theta)**2) for lambda_s, theta in zip(avg_lambda_s, avg_theta)]
# Fit Quality
q = get_fit_quality_chi_sq(avg_lambda_s, ls_fit, avg_lambda_err)
N = 2 # Always 2 for linear fit, really DOF
print("Reduced chi-squared:{};Chi-squared:{};DOF:{};".format(q/N, q, N))
# Get uncertainty
pts_len = len(plot_data[:,0])
delta = pts_len*sum([(1/x)**2 for x in plot_data[:,0]]) - sum([1/x for x
in plot_data[:,0]])**2
s_yxsq = (1/(pts_len - 2))*sum([(ypt - yest)**2 for ypt, yest in
zip(avg_lambda_s, ls_fit)])
s_m = np.sqrt(pts_len*(s_yxsq/delta))
s_b = np.sqrt((s_yxsq*sum([(1/x)**2 for x in plot_data[:,0]]))/delta)
print("slope: {}; intercept: {};".format(m, b))
print("slope error: {}; intercept error: {};".format(s_m, s_b))
```