Error Analysis, Lab 1 - Concentric Heat Exchanger

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This script computes the results and uncertainty of Test #4, Counterflow.

1. Change in Temperature Uncertainty

```
clear variables; clc

syms t t1 t2 w_t w_t1 w_t2 t_bar

format shortG

delta_t = t2 - t1;

w_T = 0.01;

T_H_1 = 60.7;
T_H_2 = 58.1;

T_C_1 = 27.4;
T_C_2 = 41.2;

flow_cold = 0.5;
flow_hot = 3;
```

Calculate Nominal changes in temp

```
delta_T_H = T_H_2 - T_H_1
```

```
delta_T_H =
        -2.6
```

```
delta_T_C = T_C_2 - T_C_1
```

```
delta_T_C =
        13.8
```

Calculate Terms for Uncertainty

```
partial_t1 = diff(delta_t,t2);
partial_t2 = diff(delta_t,t1);
```

Calculate the Uncertainty in the Result

```
w_delta_t = sqrt((partial_t1*w_t1)^2+(partial_t2*w_t2)^2);

w_delta_t_Hot = subs(w_delta_t,{w_t1,w_t2},{w_T,w_T});
w_delta_t_Hot = vpa(w_delta_t_Hot,4)
```

w_delta_t_Hot = 0.01414

```
w_delta_t_Cold = subs(w_delta_t,{w_t1,w_t2},{w_T,w_T});
w_delta_t_Cold = vpa(w_delta_t_Cold,4)
```

w_delta_t_Cold = 0.01414

2. Average Temperature Uncertainty

```
avg_delta_t = (t1+t2)/2;
```

Calculate Nominal Average Temp

```
avg_hot = double(vpa(subs(avg_delta_t,{t1,t2},{T_H_1,T_H_2}),3))
```

```
avg_hot =
         59.4
```

```
avg_cold = double(vpa(subs(avg_delta_t,{t1,t2},{T_C_1,T_C_2}),3))
```

```
avg_cold =
         34.3
```

Calculate the Terms for Uncertainty

```
m_1 = diff(avg_delta_t,t1);
m_2 = diff(avg_delta_t,t2);
```

Calculate the Uncertainty in the Result

```
w_avg_delta_t = sqrt((m_1*w_t1)^2 + (m_2*w_t2)^2);

w_avg_delta_t_hot = subs(w_avg_delta_t,{w_t1,w_t2},{w_T,w_T});
w_avg_delta_t_hot = vpa(w_avg_delta_t_hot,4)
```

w_avg_delta_t_hot = 0.007071

```
w_avg_delta_t_cold = subs(w_avg_delta_t,{w_t1,w_t2},{w_T,w_T});
w_avg_delta_t_cold = vpa(w_avg_delta_t_cold,4)
```

w_avg_delta_t_cold = 0.007071

3. Density Uncertainty

```
p = vpa((999.839+16.952*t-7.990*10^-3*t^2-46.241*10^-6*t^3+105.84610*10^-9*t^4-281.030*10^-12*t
```

Calculate Nominal Density

```
p_hot = double(vpa(subs(p,t,avg_hot),6))
```

```
p_hot =
       983.49
```

```
p_cold = double(vpa(subs(p,t,avg_cold),6))
```

```
p_cold =
       994.26
```

Calculate the Terms for Uncertainty

```
m_1 = vpa(diff(p,t),6);
```

Calculate the Uncertainty in the Result

```
W_p = vpa(sqrt((m_1*w_t)^2),6);

W_p_hot = vpa(subs(W_p,{t,w_t},{avg_hot,w_T}),6)
```

W_p_hot = 0.00511017

```
W_p_cold = vpa(subs(W_p,{t,w_t},{avg_cold,w_T}),6)
```

W_p_cold = 0.003383

4. Volumetric Flow Rate Uncertainty

```
% cross sectional area * velocity

syms m_dot
w_m_dot = 0.01;

V_dot = vpa(m_dot / p,6);
```

Calculate Nominal Volumetric Flow Rate

```
V_hot = double(vpa(subs(V_dot,{m_dot,t},{flow_hot,avg_hot}),6))
```

```
V_hot =
    0.0030504
```

```
V_cold = double(vpa(subs(V_dot,{m_dot,t},{flow_cold,avg_hot}),6))
```

```
V_cold =
   0.00050839
```

Calculate the Terms for Uncertainty

```
m_1 = vpa(diff(V_dot,m_dot),6);
m_2 = vpa(diff(V_dot,t),6);
```

Calculate the Uncertainty in the Result

```
W_V_dot = vpa(sqrt((m_1*w_m_dot)^2 + (m_2*w_T)^2),6);

W_V_hot = vpa(subs(W_V_dot,{t m_dot},{avg_hot, flow_hot}),6)
```

W_V_hot = 0.0000101679

```
W_V_cold = vpa(subs(W_V_dot,{t m_dot},{avg_cold, flow_cold}),6)
```

W_V_cold = 0.0000100577

5. Specific Heat Uncertainty

```
c_p_15 = 4815.5;

c_p = vpa((0.996185+0.0002874*((t+100)/100)^(5.26)+0.011160*10^(-0.036*t))*c_p_15,6);
```

Calculate Nominal Specific Heat Capacity

```
c_p_cold = double(vpa(subs(c_p,t,avg_cold),6))
```

```
c_p_cold =
      4806.8
```

```
c_p_hot = double(vpa(subs(c_p,t,avg_hot),6))
```

```
c_p_hot =
      4813.6
```

Calculate the Terms for Uncertainty

```
m_1 = vpa(diff(c_p,t),6);
```

Calculate the Uncertainty in the Result

```
w_c_p_cold = vpa(subs(sqrt((m_1*w_avg_delta_t_cold)^2),t,avg_cold),6)
```

w_c_p_cold = 0.0000264444

```
w_c_p_hot = vpa(subs(sqrt((m_1*w_avg_delta_t_hot)^2),t,avg_hot),6)
```

w_c_p_hot = 0.00352241

6. Heat Transfer Rate Uncertainty

```
Q = vpa((V_dot*p*c_p*delta_t),6);
```

Calculate Nominal Heat Transfer Rate

```
nominal_Q_e = double(vpa((V_hot*p_hot*c_p_hot*delta_T_H),6))
```

```
nominal_Q_e =
      -37546
```

```
nominal_Q_a = double(vpa((V_cold*p_cold*c_p_cold*delta_T_C),6))
```

```
nominal_Q_a =
       33530
```

Calculate the Terms for Uncertainty

```
m_1_cold = vpa(V_cold*p_cold*c_p_cold,6); % wrt delta t
m_2_cold = vpa(V_cold*p_cold*delta_T_C,6); % wrt c_p
m_3_cold = vpa(V_cold*c_p_cold*delta_T_C,6); % wrt p
m_4_cold = vpa(p_cold*c_p_cold*delta_T_C,6); % wrt V

m_1_hot = vpa(V_hot*p_hot*c_p_hot,6);
m_2_hot = vpa(V_hot*p_hot*delta_T_H,6);
m_3_hot = vpa(V_hot*c_p_hot*delta_T_H,6);
m_4_hot = vpa(p_hot*c_p_hot*delta_T_H,6);
```

Calculate the Uncertainty in the Result

```
W_Q_a = vpa(sqrt((m_1_cold*w_delta_t_Cold)^2 + (m_2_cold*w_c_p_cold)^2 + (m_3_cold*W_p_cold)^2+
```

W_Q_a = 664.226

```
W_Q_e = vpa(sqrt((m_1_hot*w_delta_t_Hot)^2 + (m_2_hot*w_c_p_hot)^2 + (m_3_hot*W_p_hot)^2+(m_4_h
```

W_Q_e = 239.522

7. Thermal Energy Efficiency Uncertainty

a. η_h and η_c

Calculate Nominal Thermal Energy Efficiency

```
syms h1 h2 c1 c2

n_h = symfun((h1 - h2)/(h1 - c1),[h1 h2 c1]);
n_c = symfun((c2 - c1)/(h1 - c1),[c1 c2 h1]);

nominal_n_h = double(vpa(n_h(T_H_1,T_H_2,T_C_1),6))*100
```

```
nominal_n_h =
       7.8078
```

```
nominal_n_c = double(vpa(n_c(T_C_1,T_C_2,T_H_1),6))*100
```

```
nominal_n_c =
       41.441
```

Calculate the Terms for Uncertainty

```
m_1 = diff(n_h,h1);
m_1 = vpa(m_1(T_H_1,T_H_2,T_C_1),6);

m_2 = diff(n_h,h2);
m_2 = vpa(m_2(T_H_1,T_H_2,T_C_1),6);

m_3 = diff(n_h,c1);
m_3 = vpa(m_3(T_H_1,T_H_2,T_C_1),6);
```

```
n_1 = diff(n_c,c1);
n_1 = vpa(n_1(T_C_1,T_C_2,T_H_1),6);

n_2 = diff(n_c,c2);
n_2 = vpa(n_2(T_C_1,T_C_2,T_H_1),6);

n_3 = diff(n_c,h1);
n_3 = vpa(n_3(T_C_1,T_C_2,T_H_1),6);
```

Calculate the Uncertainty in the Result

```
W_n_h = vpa(sqrt((m_1 * w_T)^2 + (m_2*w_T)^2 + (m_3*w_T)^2),6)*100
```

W_n_h = $0.040911837870405181223633173391102$

```
W_n_c = vpa(sqrt((n_1 * w_T)^2 + (n_2*w_T)^2 + (n_3*w_T)^2),6)*100
```

W_n_c = $0.036958290965017420382962623112575$

b. $\overline{\eta}$

Calculate Nominal Thermal Efficiency

```
syms avg_n(n_h,n_c)

avg_n(n_h,n_c) = (n_h + n_c)/2;
nominal_avg_n = double(vpa(avg_n(nominal_n_h, nominal_n_c),6))
```

```
nominal_avg_n =
       24.625
```

Calculate the Terms for Uncertainty

```
m_1 = diff(avg_n,n_h);
m_1 = vpa(m_1(nominal_n_h,nominal_n_c),6);

m_2 = diff(avg_n,n_c);
m_2 = vpa(m_2(nominal_n_h,nominal_n_c),6);
```

Calculate the Uncertainty in the Result

```
w_n_avg = vpa(sqrt((m_1 * W_n_h)^2 + (m_2 * W_n_c)^2),6)
```

w_n_avg = 0.0275667

8. Energy Balance Coefficient Uncertainty

```
syms Q_dot_a Q_dot_e

C_EB = symfun(Q_dot_a/Q_dot_e,[Q_dot_a Q_dot_e]);
```

Calculate the Nominal Energy Balance Coefficient

```
Nominal_C_EB = double(vpa((nominal_Q_a/nominal_Q_e),6))
```

```
Nominal_C_EB =
    -0.89304
```

Calculate the Terms for Uncertainty

```
m_1 = diff(C_EB,Q_dot_a);
m_1 = vpa(m_1(nominal_Q_a, nominal_Q_e),6);

m_2 = diff(C_EB,Q_dot_e);
m_2 = vpa(m_2(nominal_Q_a,nominal_Q_e),6);
```

Calculate the Uncertainty in the Result

```
W_C_EB = vpa(sqrt((m_1*W_Q_a)^2 + (m_1*W_Q_e)^2),6)
```

W_C_EB = 0.018806

9. LMTD Uncertainty

```
LMTD = symfun(((h2 - c2) - (h1 - c1))/(log((h2 - c2)/(h1 - c1))),[h1 h2 c1 c2]);
```

Calculate the Nominal LMTD

```
Nominal_LMTD = double(vpa(LMTD(T_H_1,T_H_2,T_C_1,T_C_2),6))
```

```
Nominal_LMTD =
       24.18
```

Calculate the Terms for Uncertainty

```
m_1 = diff(LMTD,h1);
m_1 = vpa(m_1(T_H_1,T_H_2,T_C_1,T_C_2),6);

m_2 = diff(LMTD,h2);
m_2 = vpa(m_2(T_H_1,T_H_2,T_C_1,T_C_2),6);

m_3 = diff(LMTD,c1);
m_3 = vpa(m_3(T_H_1,T_H_2,T_C_1,T_C_2),6);

m_4 = diff(LMTD,c2);
m_4 = vpa(m_4(T_H_1,T_H_2,T_C_1,T_C_2),6);
```

Calculate the Uncertainty in the Result

```
W_LMTD = vpa(sqrt((m_1*w_T)^2 + (m_2*w_T)^2 + (m_3*w_T)^2 + (m_4*w_T)^2),6)
```

W_LMTD = 0.0106437

10. Heat Transfer Coefficient Uncertainty

```
syms A LMTD
w_a = 0.01;
area = 0.02;

U = symfun(Q_dot_e/(A*LMTD), [Q_dot_e A LMTD]);
```

Calculate Nominal Heat Transfer Coefficient

```
nominal_U = double(vpa(U(nominal_Q_e, area, Nominal_LMTD),3))
```

```
nominal_U =
      -77638
```

Calculate the Terms for Uncertainty

```
m_1 = diff(U,Q_dot_e);
m_1 = vpa(m_1(nominal_Q_e, area, Nominal_LMTD),6);

m_2 = diff(U,A);
m_2 = vpa(m_2(nominal_Q_e, area, Nominal_LMTD),6);

m_3 = diff(U,LMTD);
m_3 = vpa(m_3(nominal_Q_e, area, Nominal_LMTD),6);
```

Calculate the Uncertainty in the Result

```
W_U = vpa(sqrt((m_1*W_Q_e)^2 + (m_2*w_a)^2 + (m_3*W_LMTD)^2),3)
```

W_U = $3.88e+4$