

# MECH 4110 - Pre-lab #1

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February 1, 2021

## 1 Equation of Motion Derivation

### 1.1 Configuration 1

Configuration 1 consists of the pump and tank 1. Based on this, we know that the input and output flowrates are given by,

$$q_{m,pump} = \rho K_p v_p \quad (1.1.1)$$

$$q_m = C_d A_o \sqrt{2\rho(P_1 - P_2)} \quad (1.1.2)$$

where,

$$P_1 = P_a + \rho gh \quad (1.1.3)$$

$$P_2 = P_a \quad (1.1.4)$$

Finally, we also know that,

$$\frac{dm}{dt} = \rho A_c \dot{h} \quad (1.1.5)$$

The change in mass with respect to time is given by conservation laws, we can develop the equation of motion by substituting Equations (1.1.1) - (1.1.5) and simplifying.

$$\frac{dm}{dt} = \sum q_{m,in} - \sum q_{m,out} \quad (1.1.6)$$

$$\rho A_c \dot{h} = \rho K_p v_p - C_d A_o \sqrt{2\rho(P_a + \rho gh - P_a)} \quad (1.1.7)$$

$$\rho A_c \dot{h} = \rho K_p v_p - C_d A_o \sqrt{2\rho^2 gh} \quad (1.1.8)$$

$$A_c \dot{h}_1 + C_d A_o \sqrt{2gh_1} = K_p v_p \quad (1.1.9)$$

Equation (1.1.9) is a non-linear ODE relating the height to the input voltage. We have added a subscript of 1 to  $h$  so that we can differentiate between the two heights further down the line.

## 1.2 Configuration 2

Configuration 2 consists of both tanks, with only 1 output from the pump. The flow into Tank 2 is the flow out of Tank 1.

$$q_{m,in} = \rho C_{d,1} A_{o,1} \sqrt{2gh_1} \quad (1.2.1)$$

$$q_{m,out} = \rho C_{d,2} A_{o,2} \sqrt{2gh_2} \quad (1.2.2)$$

Using the same conservation law, we can find the equation of motion for Tank 2.

$$\frac{dm}{dt} = \sum q_{m,in} - \sum q_{m,out} \quad (1.2.3)$$

$$\rho A_{c,2} \dot{h}_2 = \rho C_{d,1} A_{o,1} \sqrt{2gh_1} - \rho C_{d,2} A_{o,2} \sqrt{2gh_2} \quad (1.2.4)$$

$$A_{c,2} \dot{h}_2 + C_{d,2} A_{o,2} \sqrt{2gh_2} = C_{d,1} A_{o,1} \sqrt{2gh_1} \quad (1.2.5)$$

## 1.3 Configuration 3