MECH 3340 - Assignment #3

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Exercise 1

Resolve feedback loop between blocks $1/s$ and 8.

$$\frac{1/s}{1+8/s} = \frac{1/s}{\frac{s+8}{s}} = \frac{1}{s+8}.$$

Blocks $4, 1/s$ are in series with the sum of $G(s), 1/(s+8)$.

$$(4/s + G(s))\frac{1}{s+8}.$$

Which is in a feedback loop with 6.

$$\frac{(4/s+G(s))\frac{1}{s+8}}{1+\frac{6}{s+8}(4/s+G(s))}.$$

$$= \frac{\frac{4}{s(s+8)} + \frac{G(s)}{s+8}}{\frac{s(s+8)+24+sG(s)}{s(s+8)}}.$$

$$\frac{X(s)}{F(s)} = \frac{4+sG(s)}{s^2+(G(s)+8)s+24} \tag{1.1}$$

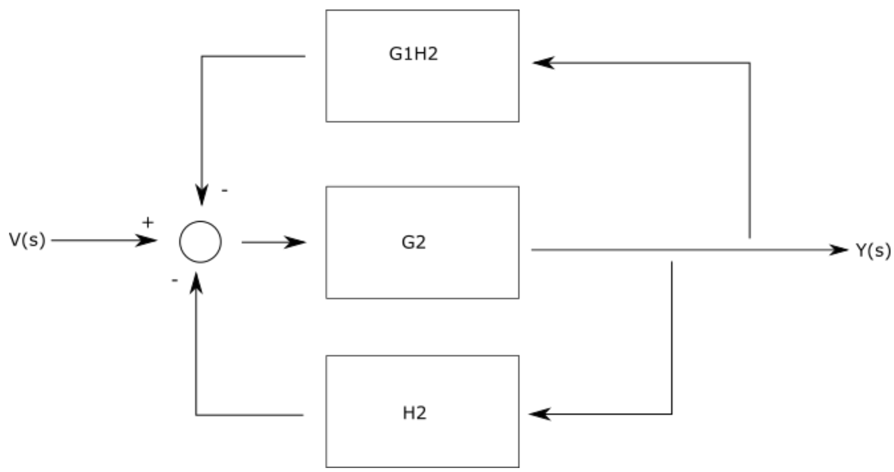

Figure 1: Equivalent block diagram derived from $U(s) = 0$. (Exercise 2)

Exercise 2

Note that G_n, H_n are functions of s.

2.1 $V(s) = 0$

Combine G_2, H_2 using feedback. This is in series with G_1.

$$\frac{G_1 G_2}{1 + G_2 H_2}.$$

Finally, combine the above with H_1 using feedback.

$$\frac{\frac{G_1 G_2}{1 + G_2 H_2}}{1 + \frac{G_1 G_2 H_1}{1 + G_2 H_2}}.$$

$$\frac{Y(s)}{U(s)} = \frac{G_1(s) G_2(s)}{1 + G_2(s) H_2(s) + G_1(s) G_2(s) H_1(s)} \tag{2.1}$$

2.2 $U(s) = 0$.

With $U(s) = 0$, we can draw an equivalent block diagram (see Figure 1). Combine G_2, H_2 using feedback.

$$\frac{G_2}{1+G_2H_2}.$$

Finally, combine the above result with G_1H_2 using feedback.

$$\frac{\frac{G_2}{1+G_2H_2}}{1+\frac{G_1G_2H_1}{1+G_2H_2}}.$$

$$\frac{Y(s)}{V(s)} = \frac{G_2(s)}{1+G_2(s)H_2(s)+G_1(s)G_2(s)H_1(s)} \tag{2.2}$$

Exercise 3

Given,

$$5\ddot{x}+3\dot{x}+7x = 10f(t)-4g(t).$$

Take the laplace transform of both sides,

$$\left(5s^2+3s+7\right)X(s) = 10F(s)-4G(s).$$

$$X(s) = \frac{10}{(5s^2+3s+7)}F(s) - \frac{4}{(5s^2+3s+7)}G(s) \tag{3.1}$$

Let,

$$H_1 = \frac{10}{(5s^2+3s+7)}, \quad H_2 = \frac{4}{(5s^2+3s+7)}.$$

Equation (3.1) can be represented in block diagram form. See Figure 2.

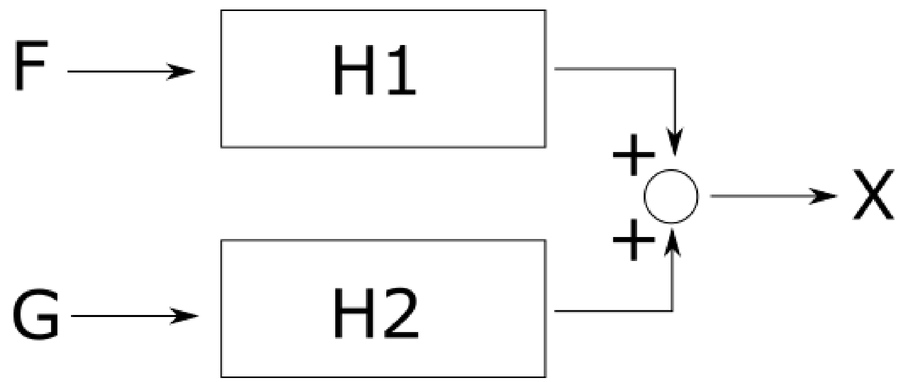

Figure 2: Block diagram solution for Exercise 3

Exercise 4

Given the system,

$$\begin{aligned} 2\ddot{y} + \dot{y} + y &= z \\ \dot{z} + 3z + y &= f(t) \end{aligned}$$

Let,

$$x = \begin{bmatrix} y \\ \dot{y} \\ z \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

$$\Rightarrow \dot{x} = \begin{bmatrix} x_2 \\ x_3 - x_2 - x_1 \\ -x_1 - 3x_3 + f(t) \end{bmatrix}.$$

Write $\dot{x}$ in the form of $\dot{x} = Ax + Bu$.

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & -1 & 1 \\ -1 & 0 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} f(t) \tag{4.1}$$

Exercise 5

Listing 1 and Figure 3 show the matlab solution and resulting plot, respectively.

Listing 1: Script for Problem 5, a system and input are defined and simulated

```
set(0, 'defaulttextinterpreter', 'Latex')

% Define system
num = 6;
den = [3 21 30];
sys = tf(num, den);

% Define input
t = linspace(0, 6, 500);
f = cos(3*t);

% Plot simulation
[y, t] = lsim(sys, f, t);

figure;plot(t, f, t, y)
title("Linear Simulation")
xlabel('Time, $t$')
ylabel('$x(t)$ and $f(t)$')
xlim([0 6])
legend('$6\cos 3t$', '$x(t)$', 'Interpreter','latex'
    )
```

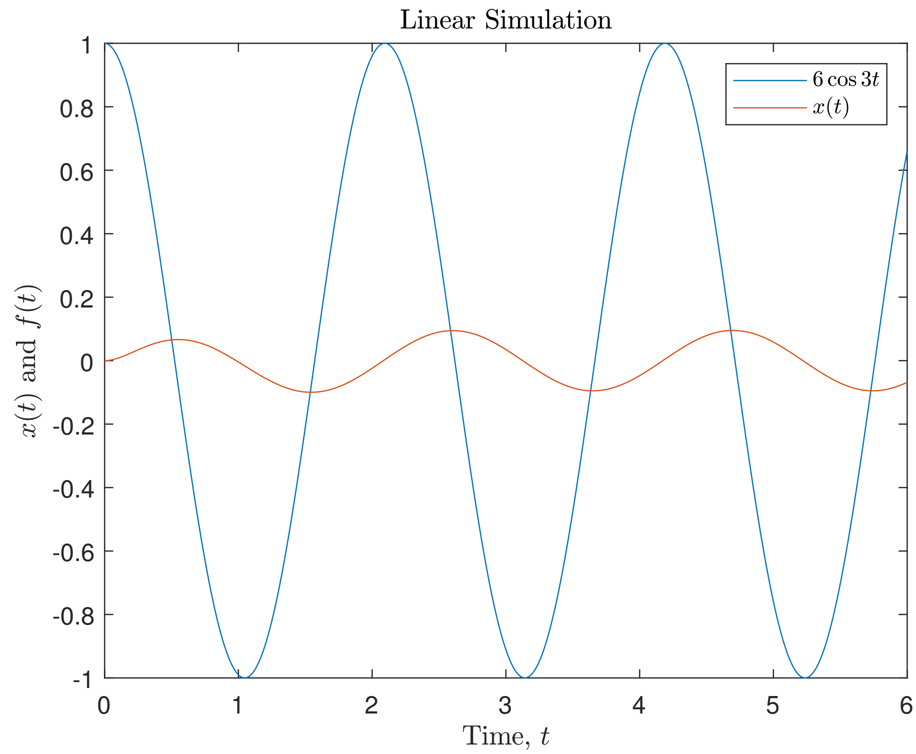

Figure 3: System Response versus Input Force

Exercise 6

Assume that $D(s) = 0$. The result from listing 2 is,

$$\frac{C(s)}{R(s)} = \frac{6}{12s^3 + 11s^2 + 12s + 6} \tag{6.1}$$

```
>> problem6

s4 =

                 6
  ----------------------------
  12 s^3 + 11 s^2 + 12 s + 6

Continuous-time transfer function.
```

Figure 4: Output of Listing 2

Listing 2: Script for Problem 6, Transfer function blocks are defined and block diagram algebra is preformed

```
% Define blocks
E = tf(6, [1 0]);
M = tf(1, [4 1]);
G = tf(1, [3 2]);


% Without D
s1 = series(M, G);
s2 = feedback(s1, 10);
s3 = series(s2, E);
s4 = feedback(s3, 1)
```

Exercise 7

Calculate $C\,(sI - A)^{-1}\,B + D$ by hand,

$$
\begin{aligned}
(sI - A)^{-1} &= \begin{bmatrix} s & -1 \\ 1/2 & s+1 \end{bmatrix}^{-1} \\
&= \frac{1}{s(s+1)+1/2}\begin{bmatrix} s+1 & 1 \\ -1/2 & s \end{bmatrix}.
\end{aligned}
$$

Multiply the above result by $\begin{bmatrix} 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$.

$$
\frac{1}{s(s+1)+1/2}\begin{bmatrix} -1/2 & s \end{bmatrix}\begin{bmatrix} 0 \\ 2 \end{bmatrix}.
$$

Which gives us,

$$
C\,(sI - A)^{-1}\,B + D = \frac{2s}{s^2 + s + 1/2} \tag{7.1}
$$

We can check our answer in matlab with listing 3.

```
>> problem7

sys_tf =

       2 s
  -------------
  s^2 + s + 0.5

Continuous-time transfer function.
```

Figure 5: Output of Listing 3

Listing 3: Script for problem 7, conversion from state space to transfer function

```
A = [0 1; -1/2 -1];
B = [0; 2];
C = [0 1];
D = 0;

sys = ss(A, B, C, D);

sys_tf = tf(sys)
```