

MECH 3340 - Assignment #2

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February 1, 2021

Exercise 1

Given,

$$x(t) = e^{-2t} \cos 3t.$$

The laplace transform of $x(t)$ can be found via lookup table.

$$\mathcal{L}\{x(t)\} = \frac{s+2}{(s+2)^2+9} \quad (1.1)$$

Exercise 2

Given,

$$\ddot{x} + \omega^2 x = 0; \quad x(0^-) = 1, \dot{x}(0^-) = 0.$$

We can solve the IVP with laplace transforms.

$$s^2 X(s) - sx(0) + \omega^2 X(s) = 0 \quad (2.1)$$

$$(s^2 + \omega^2) X(s) = s \quad (2.2)$$

$$X(s) = \frac{s}{s^2 + \omega^2} \Rightarrow x(t) = \cos \omega t \quad (2.3)$$

We can verify our solution with the homogeneous solution. The characteristic equation is,

$$\lambda^2 e^{\lambda t} + \omega^2 e^{\lambda t} = 0 \quad (2.4)$$

$$\lambda^2 + \omega^2 = 0 \quad (2.5)$$

Which gives us,

$$\lambda = j\omega, \quad (\text{Re}(\lambda) = 0) \quad (2.6)$$

We can write $x_h(t)$ as,

$$x_h(t) = C_i e^{j\omega} + C_{i+1} e^0 \quad (2.7)$$

$$x_h(t) = C'_i \cos \omega t + C'_{i+1} \sin 0 \quad (2.8)$$

Clearly, x_h and $x(t)$ have the same form, which suggests our solution is correct.

Exercise 3

Given,

$$F(s) = \frac{1}{s^2(s^2 + 3s + 2)}.$$

Using the final value theorem, we can find that

$$\lim_{s \rightarrow \infty} sF(s) = \frac{1}{\infty} = 0 \quad (3.1)$$

For the initial value theorem, we can find that

$$\lim_{s \rightarrow 0^-} sF(s) = -\infty \quad (3.2)$$

$$\lim_{s \rightarrow 0^+} sF(s) = \infty \quad (3.3)$$

This means that the initial value of $F(s)$ does not exist.

Exercise 4

Given,

$$F(s) = \frac{2s+1}{s^2+4s+3} + \frac{1}{s^2}.$$

We can find $\mathcal{L}^{-1}\{F(s)\}$ with a partial fraction expansion.

$$\frac{2s+1}{(s+3)(s+1)} = \frac{A}{s+3} + \frac{B}{s+1} \quad (4.1)$$

Using the cover up method, we get that $A = B = 5/2$. Therefore,

$$\mathcal{L}^{-1}\{F(s)\} = \frac{5}{2} (e^{3t} + e^{-t}) + t \quad (4.2)$$

Exercise 5

Given,

$$F(s) = \frac{1}{s(s^2 + 2s + 2)}.$$

Do a partial fraction expansion of $F(s)$.

$$\frac{A}{s} + \frac{Bs + C}{s^2 + 2s + 2} = \frac{A(s^2 + 2s + 2) + s(Bs + C)}{s(s^2 + 2s + 2)} \quad (5.1)$$

$$\frac{(A + B)s^2 + (2A + C)s + 2A}{s(s^2 + 2s + 2)} = \frac{1}{s(s^2 + 2s + 2)} \quad (5.2)$$

$$2A = 1$$

$$A + B = 0$$

$$2A + C = 0$$

This gives us $A = 1/2, B = -1/2, C = -1$.

Finally,

$$F(s) = \frac{1}{2} \cdot \frac{1}{s} + \frac{1/2(s + 1)}{(s + 1)^2 + 1} - \frac{1}{(s + 1)^2 + 1} \quad (5.3)$$

$$\mathcal{L}^{-1}\{F(s)\} = \frac{1}{2} + \frac{1}{2}e^{-t} \cos t - e^{-t} \sin t \quad (5.4)$$

Exercise 6

6.1 $5\dot{x} + 7x = f(t)$

Take the laplace transform of both sides,

$$(5s + 7)X(s) = F(s) \quad (6.1)$$

$$\frac{X(s)}{F(s)} = \frac{1}{5s + 7} \Rightarrow p = -7/5 \quad (6.2)$$

6.2 $\ddot{x} + 10\dot{x} + 21x = 4f(t)$

Take the laplace transform of both sides,

$$(s^2 + 10s + 21)X(s) = 4F(s) \quad (6.3)$$

$$\frac{X(s)}{F(s)} = \frac{4}{(s+3)(s+7)} \Rightarrow p = \{-3, -7\} \quad (6.4)$$

6.3 $\ddot{x} + 14\dot{x} + 58x = 6\dot{f}(t) + 4f(t)$

Take the laplace transform of both sides,

$$(s^2 + 14s + 58)X(s) = (6s + 4)F(s) \quad (6.5)$$

$$\frac{X(s)}{F(s)} = \frac{6s + 4}{s^2 + 14s + 58} \quad (6.6)$$

Compute p with the quadratic formula.

$$\frac{-14 \pm \sqrt{196 - 232}}{2} = \frac{-14 \pm j6}{2} \quad (6.7)$$

$$p = -7 \pm j3 \quad (6.8)$$

Exercise 7

Given,

$$3\dot{x} = y.$$

$$\dot{y} = f(t) - 3y - 15x.$$

Take the laplace transform of both sides of each equation,

$$\begin{aligned} 3sX(s) &= Y(s) \\ (s+3)Y(s) &= F(s) - 15X(s) \end{aligned} \tag{7.1}$$

Substitute $3sX(s)$ for $Y(s)$ on the bottom equation

$$(s+3)3sX(s) = F(s) - 15X(s) \tag{7.2}$$

$$(3s^2 + 9s + 15)X(s) = F(s) \tag{7.3}$$

$$\frac{X(s)}{F(s)} = \frac{1}{3s^2 + 9s + 15} \tag{7.4}$$

We can compute $Y(s)/F(s)$ by substituting $1/3sX(s)$ for $Y(s)$.

$$\frac{Y(s)}{F(s)} = \frac{1}{3s(3s^2 + 9s + 15)} \tag{7.5}$$

Exercise 8

Given,

$$F(s) = \frac{5(s+2)}{s^2(s+1)(s+3)}.$$

Compute the partial fraction expansion.

$$F(s) = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+1} + \frac{D}{s+3} \quad (8.1)$$

using the cover up method, we can find that

$$B = 10/3 \quad (8.2)$$

$$C = 5/2 \quad (8.3)$$

$$D = 5/18 \quad (8.4)$$

The value of A can be found by equating the top two equations.

$$5s+2 = As(s+1)(s+3) + \frac{10}{3}(s+1)(s+3) + \frac{5}{2}s^2(s+3) + \frac{5}{18}s^2(s+1) \quad (8.5)$$

Group the powers of s together and equate their corresponding powers

$$5s+2 = s^3(\dots) + s^2(\dots) + s(3A + \frac{40}{3}) + 10 \quad (8.6)$$

$$A = -25/9 \quad (8.7)$$

Therefore,

$$F(s) = \frac{-25}{9s} + \frac{10}{3s^2} + \frac{5}{2(s+1)} + \frac{5}{18(s+3)} \quad (8.8)$$

Exercise 9

Given,

$$F(s) = \frac{3s + 1}{s^4 + 3s^3 + 2s^2}.$$

Factor the denominator to get the poles and zeroes,

$$F(s) = \frac{3s + 1}{s^2(s + 2)(s + 1)} \tag{9.1}$$

$$p = 0, 0, -2, -1$$

$$z = -1/3$$