ECE 470 Prelab 4

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① Repulsion upward from Workspace plane parallel to x_0-y_0 plane, with $z_0 = 32\,\text{mm}$: $\quad o_i(q) = [o_{ix} \ o_{iy} \ o_{iz}]^T$

Let $b = [o_{ix} \ o_{iy} \ z_0]^T = [o_{ix} \ o_{iy} \ 32]^T$

$$o_i(q) - b = [0 \ 0 \ (o_{iz} - z_0)]^T = [0 \ 0 \ (o_{iz} - 32)]^T$$

$$\|o_i(q) - b\| = o_{iz} - z_0 = o_{iz} - 32$$

Now
$$F^0_{i,rep} = \begin{cases} \left(\dfrac{1}{\|o_i(q) - b\|} - \dfrac{1}{e_0}\right) \cdot \dfrac{o_i(q) - b}{\|o_i(q) - b\|^3}, & \text{if } \|o_i(q) - b\| \le e_0 \\ [0 \ 0 \ 0]^T, & \text{if } \|o_i(q) - b\| > e_0 \end{cases}$$

$$= \begin{cases} \left(\dfrac{1}{o_{iz} - z_0} - \dfrac{1}{e_0}\right) \cdot \dfrac{[0 \ 0 \ (o_{iz} - 32)]^T}{(o_{iz} - z_0)^3}, & \text{if } \|o_i(q) - b\| \le e_0 \\ [0 \ 0 \ 0]^T, & \text{if } \|o_i(q) - b\| > e_0 \end{cases}$$

② Repulsion from a finite length cylinder of height h and with bottom lying in x_0-y_0 plane: $o_i(q) = [o_{ix} \ o_{iy} \ o_{iz}]^T$

Radius $\equiv R$, centre $\equiv c = [c_x \ c_y]^T$

$$b = \begin{cases} \begin{bmatrix} c_x \\ c_y \\ o_{iz} \end{bmatrix} + \begin{bmatrix} \dfrac{R}{\|[o_{ix} \ o_{iy}]^T - c\|} \cdot ([o_{ix} \ o_{iy}]^T - c) \\ 0 \end{bmatrix}, & \text{if } \|[o_{ix} \ o_{iy}] - c\| \ge R, \ o_{iz} < h \\ \begin{bmatrix} c_x \\ c_y \\ h \end{bmatrix} + \begin{bmatrix} \dfrac{R}{\|[o_{ix} \ o_{iy}]^T - c\|} \cdot ([o_{ix} \ o_{iy}]^T - c) \\ 0 \end{bmatrix}, & \text{if } \|[o_{ix} \ o_{iy}] - c\| \ge R, \ o_{iz} \ge h \\ [o_{ix} \ o_{iy} \ h]^T, & \text{if } \|[o_{ix} \ o_{iy}] - c\| < R, \ o_{iz} \ge h \\ [0 \ 0 \ 0]^T, & \text{if } \|[o_{ix} \ o_{iy}] - c\| < R, \ o_{iz} < h \end{cases}$$

$$o_i(q) - b = \begin{cases} \begin{bmatrix} ([o_{ix} \ o_{iy}]^T - c)\left(1 - \dfrac{R}{\|[o_{ix} \ o_{iy}]^T - c\|}\right) \\ 0 \end{bmatrix}, & \text{if } \|[o_{ix} \ o_{iy}] - c\| \ge R, \ o_{iz} < h \\ \begin{bmatrix} ([o_{ix} \ o_{iy}]^T - c)\left(1 - \dfrac{R}{\|[o_{ix} \ o_{iy}] - c\|}\right) \\ o_{iz} - h \end{bmatrix}, & \text{if } \|[o_{ix} \ o_{iy}] - c\| \ge R, \ o_{iz} \ge h \\ [0 \ 0 \ (o_{iz} - h)]^T, & \text{if } \|[o_{ix} \ o_{iy}] - c\| < R, \ o_{iz} \ge h \\ o_i(q), & \text{if } \|[o_{ix} \ o_{iy}] - c\| < R, \ o_{iz} < h \end{cases}$$

$$\|o_i(q) - b\| = \begin{cases} \|[o_{ix} \ o_{iy}] - c\| - R, & \text{if } \|[o_{ix} \ o_{iy}] - c\| \ge R, \ o_{iz} < h \\ \sqrt{(\|[o_{ix} \ o_{iy}] - c\| - R)^2 + (o_{iz} - h)^2}, & \text{if } \|[o_{ix} \ o_{iy}] - c\| \ge R, \ o_{iz} \ge h \\ o_{iz} - h, & \text{if } \|[o_{ix} \ o_{iy}] - c\| < R, \ o_{iz} \ge h \\ \sqrt{o_{ix}^2 + o_{iy}^2 + o_{iz}^2}, & \text{if } \|[o_{ix} \ o_{iy}] - c\| < R, \ o_{iz} < h \end{cases}$$

Now
$$F^0_{i,rep} = \begin{cases} \left(\dfrac{1}{\|o_i(q) - b\|} - \dfrac{1}{e_0}\right) \cdot \dfrac{o_i(q) - b}{\|o_i(q) - b\|^3}, & \text{if } \|o_i(q) - b\| \le e_0 \\ [0 \ 0 \ 0]^T, & \text{if } \|o_i(q) - b\| > e_0 \end{cases}$$

(using expressions defined above)