CAP5768_Assignment5_Corbin

November 3, 2019

1 CAP 5768 - Data Science - Adam Corbin- Fall 2019

1.1 Assignment 5: Hypothesis Testing

1.2 Starter code

1.2.1 Goals

- To learn how to test hypoteses using different test functions, compute p-values, and derive reasonable conclusions.
- To expand upon the prior experience of manipulating, summarizing, and visualizing small datasets.
- To increase our statistical analysis skills.

1.2.2 Instructions

- This assignment is structured in 3 parts, each one using their own dataset(s).
- As usual, there will be some Python code to be written and questions to be answered.
- At the end, you should export your notebook to PDF format; it will become your report.
- Submit the report (PDF), notebook (.ipynb file), and (optionally) link to the "live" version of your solution on Google Colaboratory via Canvas.
- The total number of points is 96 (sorry, no bonus points this time).

1.2.3 Important

- It is OK to attempt the bonus points, but please **do not overdo it!**

[2]:
```
#Imports
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
import seaborn as sns
import scipy.stats as ss
%matplotlib inline
```

1.2.4 Formulating and simulating a hypothesis

In this assignment we will look at how to test hypotheses in a more systematic way than we have done so far, i.e., going from "apparent effects" to "rigorous hypothesis testing".

The fundamental question we want to address is whether the effects we see in a sample are likely to appear in the larger population.

Or, put differently, you will learn how to use classical hypothesis testing to answer the question:

Given a sample and an apparent effect, what is the probability of seeing such an effect by chance?

Pipeline for hypothesis testing

- Clearly state the null hypothesis
- Define your test statistics
- Generate many simulated data assuming that your null hypothesis is true
- Compute the test statistic for each simulated data set
- The p-value is the fraction of your simulated data sets for which each test statistics is at least as extreme as for the real data

---

1.3 Part 1: Computing and visualizing permutation samples

We will use the Sheffield Weather Station data (see https://www.metoffice.gov.uk/pub/data/weather/uk/clima if you're interested in the entire dataset), paying particular attention to the monthly rainfall in June (a dry month) and November (a wet month). We expect these might be differently distributed, so we will take permutation samples to see how their ECDFs would look if they were identically distributed.

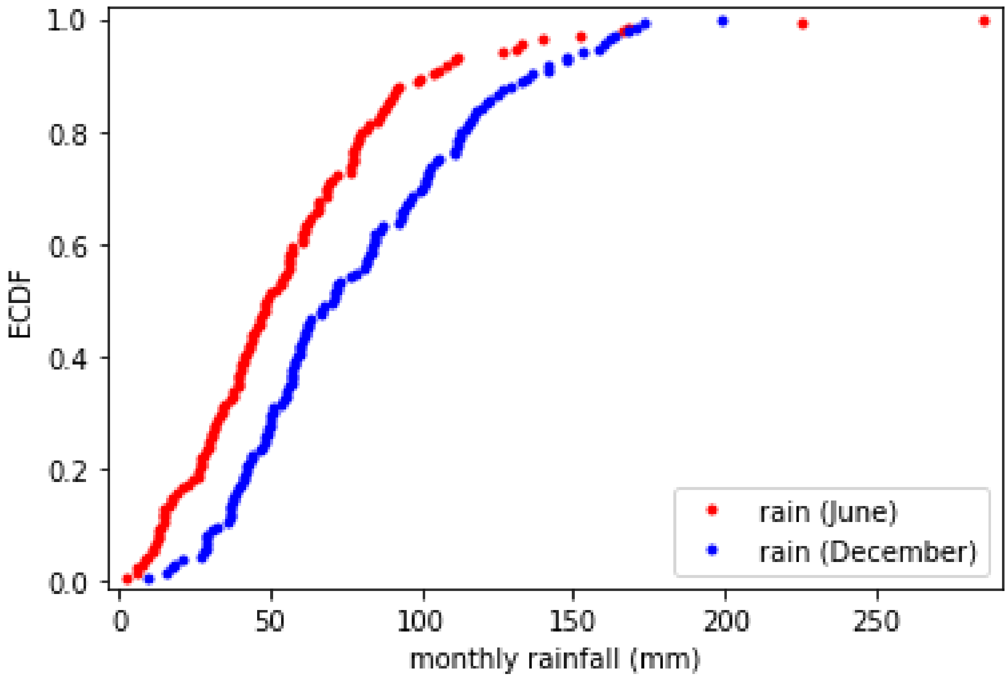
The data are stored in the Numpy arrays `rain_june` and `rain_november`, respectively.

The Python code below shows how to concatenate the two arrays, permute the concatenated array, and split the permuted array into two. It also shows how to plot the ECDF for the original arrays as well as the ECDF for a particular permutation.

[3]:
```
# Relevant numpy arrays
rain_june = np.array([ 66.2,  39.7,  76.4,  26.5,  11.2,  61.8,   6.1,  48.4, ␣
↪89.2,
       104,  34,  60.6,  57.1,  79.1,  90.9,  32.3,  63.8,  78.2,
       27.5,  43.4,  30.1,  17.3,  77.5,  44.9,  92.2,  39.6,  79.4,
       66.1,  53.5,  98.5,  20.8,  55.5,  39.6,  56,  65.1,  14.8,
       13.2,  88.1,   8.4,  32.1,  19.6,  40.4,   2.2,  77.5, 105.4,
       77.2,  38,  27.1, 111.8,  17.2,  26.7,  23.3,  77.2,  87.2,
       27.7,  50.6,  60.3,  15.1,   6,  29.4,  39.3,  56.3,  80.4,
       85.3,  68.4,  72.5,  13.3,  28.4,  14.7,  37.4,  49.5,  57.2,
       85.9,  82.1,  31.8, 126.6,  30.7,  41.4,  33.9,  13.5,  99.1,
       70.2,  91.8,  61.3,  13.7,  54.9,  62.5,  24.2,  69.4,  83.1,
       44,  48.5,  11.9,  16.6,  66.4,  90,  34.9, 132.8,  33.4,
       225,   7.6,  40.9,  76.5,  48, 140,  55.9,  54.1,  46.4,
       68.6,  52.2, 108.3,  14.6,  11.3,  29.8, 130.9, 152.4,  61,
       46.6,  43.9,  30.9, 111.1,  68.5,  42.2,   9.8, 285.6,  56.7,
```

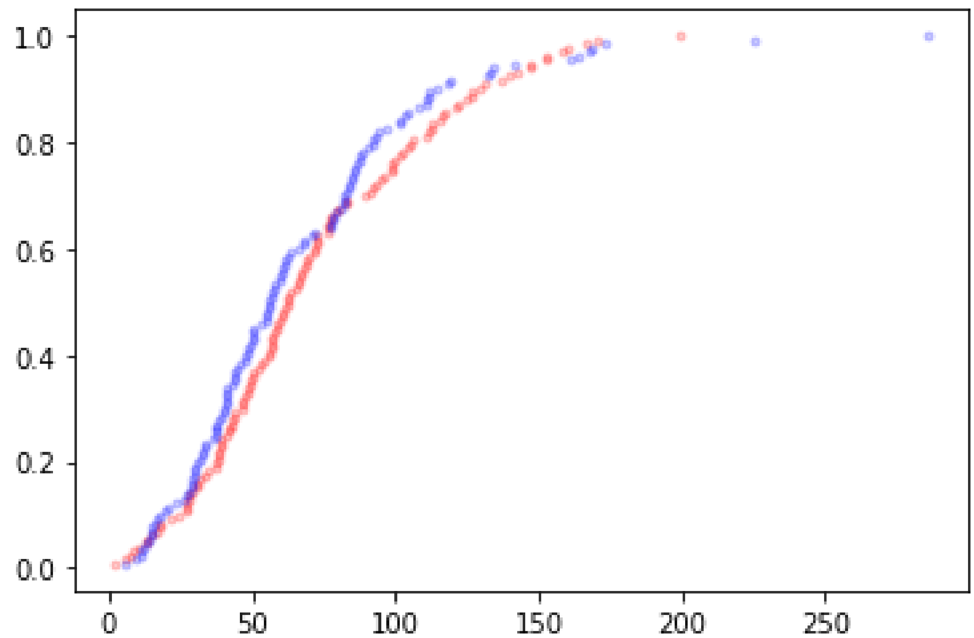
```
      168.2,  41.2,  47.8, 166.6,  37.8,  45.4,  43.2])

rain_november = np.array([ 83.6,  30.9,  62.2,  37,  41, 160.2,  18.2, 122.4, ␣
↪71.3,
        44.2,  49.1,  37.6, 114.5,  28.8,  82.5,  71.9,  50.7,  67.7,
       112,  63.6,  42.8,  57.2,  99.1,  86.4,  84.4,  38.1,  17.7,
       102.2, 101.3,  58,  82, 101.4,  81.4, 100.1,  54.6,  39.6,
        57.5,  29.2,  48.8,  37.3, 115.4,  55.6,  62,  95,  84.2,
       118.1, 153.2,  83.4, 104.7,  59,  46.4,  50, 147.6,  76.8,
        59.9, 101.8, 136.6, 173,  92.5,  37,  59.8, 142.1,   9.9,
       158.2,  72.6,  28, 112.9, 119.3, 199.2,  50.7,  44, 170.7,
        67.2,  21.4,  61.3,  15.6, 106, 116.2,  42.3,  38.5, 132.5,
        40.8, 147.5,  93.9,  71.4,  87.3, 163.7, 141.4,  62.6,  84.9,
        28.8, 121.1,  28.6,  32.4, 112,  50,  96.9,  81.8,  70.4,
       117.5,  41.2, 124.9,  78.2,  93,  53.5,  50.5,  42.6,  47.9,
        73.1, 129.1,  56.9, 103.3,  60.5, 134.3,  93.1,  49.5,  48.2,
       167.9,  27, 111.1,  55.4,  36.2,  57.4,  66.8,  58.3,  60,
       161.6, 112.7,  37.4, 110.6,  56.6,  95.8, 126.8])
```

[4]:
```
def ecdf(data):
    """Compute ECDF for a one-dimensional array of measurements."""

    # Number of data points: n
    n = len(data)

    # x-data for the ECDF: x
    x = np.sort(data)

    # y-data for the ECDF: y
    y = np.arange(1, n + 1) / n

    return x, y

# Create and plot ECDFs from original data
x_1, y_1 = ecdf(rain_june)
x_2, y_2 = ecdf(rain_november)
_ = plt.plot(x_1, y_1, marker='.', linestyle='none', color='red', label='rain␣
↪(June)')
_ = plt.plot(x_2, y_2, marker='.', linestyle='none', color='blue', label='rain␣
↪(December)')

# Label axes, set margin, and show plot
plt.margins(0.02)
_ = plt.xlabel('monthly rainfall (mm)')
_ = plt.ylabel('ECDF')
plt.legend();
plt.show()
```



[5]:

```
data1 = rain_june
data2 = rain_november

# Concatenate the data sets: data
data = np.concatenate((data1, data2))

# Permute the concatenated array: permuted_data
permuted_data = np.random.permutation(data)

# Split the permuted array into two: perm_sample_1, perm_sample_2
perm_sample_1 = permuted_data[:len(data1)]
perm_sample_2 = permuted_data[len(data1):]

# Compute ECDFs
x_1, y_1 = ecdf(perm_sample_1)
x_2, y_2 = ecdf(perm_sample_2)

# Plot ECDFs of permutation sample
_ = plt.plot(x_1, y_1, marker='.', linestyle='none', color='red', alpha=0.2)
_ = plt.plot(x_2, y_2, marker='.', linestyle='none', color='blue', alpha=0.2)
```



1.4 Your turn! (24 points, i.e., 12 pts each)

1. Create an auxiliary function `permutation_sample()` to encapsulate the functionality of lines 4-12 in the example above.
2. Write a `for` loop to generate 50 permutation samples, compute their ECDFs, and plot them.

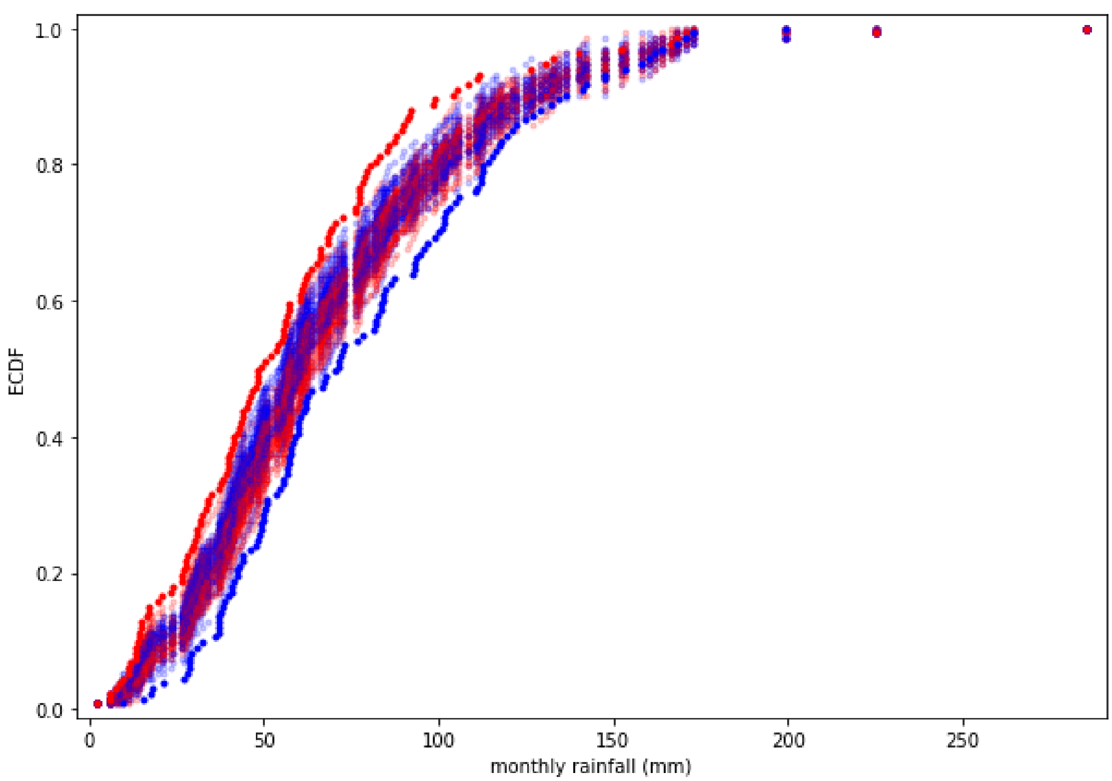
The plot should look like this:

1.5 Solution

[6]:
```
def permutation_dample(data1,data2):
    # Concatenate the data sets: data
    data = np.concatenate((data1, data2))

    # Permute the concatenated array: permuted_data
    permuted_data = np.random.permutation(data)

    # Split the permuted array into two: perm_sample_1, perm_sample_2
    perm_sample_1 = permuted_data[:len(data1)]
    perm_sample_2 = permuted_data[len(data1):]
    return perm_sample_1, perm_sample_2
```

[22]:
```
plt.figure(figsize=(10,7))
for i in range(50):
    perm_sample_1, perm_sample_2 = permutation_dample(data1,data2)
    # Compute ECDFs
    x_1, y_1 = ecdf(perm_sample_1)
```

```
    x_2, y_2 = ecdf(perm_sample_2)

    # Plot ECDFs of permutation sample
    _ = plt.plot(x_1, y_1, marker='.', linestyle='none', color='red', alpha=0.2)
    _ = plt.plot(x_2, y_2, marker='.', linestyle='none', color='blue', alpha=0.
 ↪2)

# Adding origional ECDF graphs
x_1, y_1 = ecdf(rain_june)
x_2, y_2 = ecdf(rain_november)
_ = plt.plot(x_1, y_1, marker='.', linestyle='none', color='red', label='rain␣
 ↪(June)')
_ = plt.plot(x_2, y_2, marker='.', linestyle='none', color='blue', label='rain␣
 ↪(December)')

plt.margins(0.02)
_ = plt.xlabel('monthly rainfall (mm)')
_ = plt.ylabel('ECDF')
plt.show()
```



1.6 Part 2: Test statistics and p-values

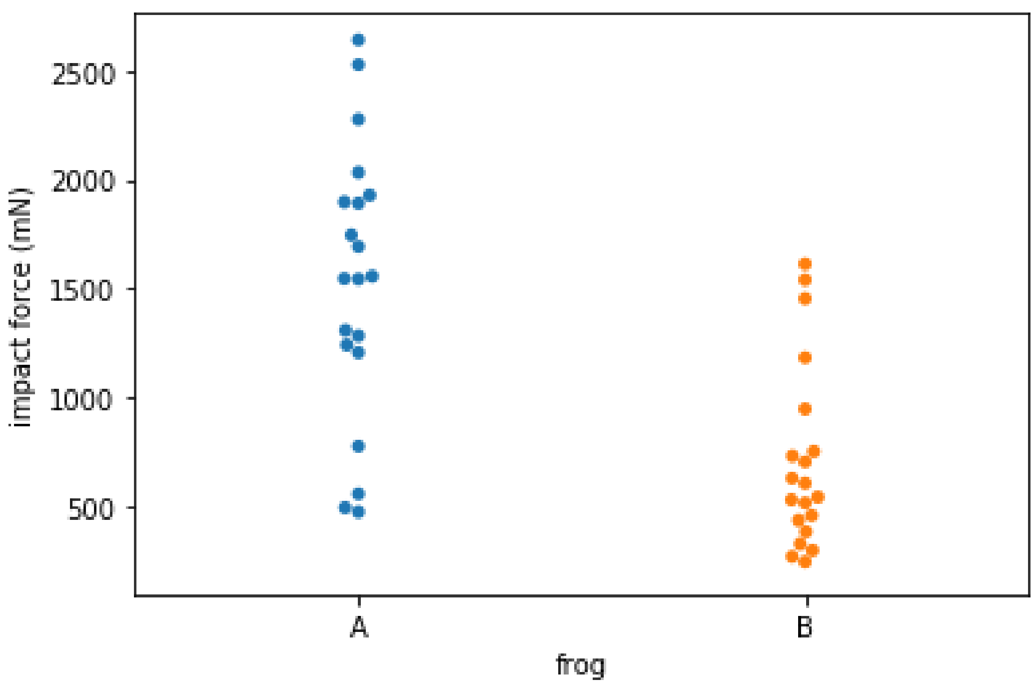
Kleinteich and Gorb (Sci. Rep., 4, 5225, 2014) performed an interesting experiment with South American horned frogs. They held a plate connected to a force transducer, along with a bait fly, in front of them. They then measured the impact force and adhesive force of the frog's tongue when it struck the target. (See https://www.nature.com/articles/srep05225 for full paper, if interested.)

Frog A is an adult and Frog B is a juvenile. The researchers measured the impact force of 20 strikes for each frog.

In this part, we will test the hypothesis that the two frogs have the same distribution of impact forces.

The Python code below reads the data from a CSV file, creates a Pandas data frame, `df`, and makes a bee swarm plot for the data.

[23]:
```
# Read CSV and build dataframe
df = pd.read_csv('data/horned_frog_tongue.csv')

# Make bee swarm plot
_ = sns.swarmplot(df['ID'], df['impact force (mN)'])

# Label axes
_ = plt.xlabel('frog')
_ = plt.ylabel('impact force (mN)')

# Show the plot
plt.show()
```



A *permutation replicate* is a single value of a statistic computed from a permutation sample.

The Python code below shows the `draw_perm_reps()`, which is useful to generate permutation replicates.

The function has call signature `draw_perm_reps(data_1, data_2, func, size=1)`. Importantly, `func` must be a function that takes two arrays as arguments. In most circumstances, `func` will be a function you write yourself.

[24]:
```
# Auxiliary functions
def permutation_sample(data1, data2):
    """Generate a permutation sample from two data sets."""

    # Concatenate the data sets: data
    data = np.concatenate((data1, data2))

    # Permute the concatenated array: permuted_data
    permuted_data = np.random.permutation(data)

    # Split the permuted array into two: perm_sample_1, perm_sample_2
    perm_sample_1 = permuted_data[:len(data1)]
    perm_sample_2 = permuted_data[len(data1):]

    return perm_sample_1, perm_sample_2

def draw_perm_reps(data_1, data_2, func, size=1):
    """Generate multiple permutation replicates."""

    # Initialize array of replicates: perm_replicates
    perm_replicates = np.empty(size)

    for i in range(size):
        # Generate permutation sample
        perm_sample_1, perm_sample_2 = permutation_sample(data_1, data_2)

        # Compute the test statistic
        perm_replicates[i] = func(perm_sample_1, perm_sample_2)

    return perm_replicates
```

1.6.1 Testing a difference in means

The code below computes the average strike force of Frog A (1.53 N), and that of Frog B (0.71 N) for a difference of 0.82 N. It is possible the frogs strike with the same force and this observed difference was by chance.

[25]:
```
frog_A = df['ID'] == 'A'
frog_B = df['ID'] == 'B'
force_a = np.array(df['impact force (mN)'][frog_A])/1000
force_b = np.array(df['impact force (mN)'][frog_B])/1000
```

```
print('The average strike force of Frog A is {:.2f} N'.format(force_a.mean()))
print('The average strike force of Frog B is {:.2f} N'.format(force_b.mean()))
```

```
The average strike force of Frog A is 1.53 N
The average strike force of Frog B is 0.71 N
```

1.7 Your turn! (25 points)

Write code to compute the probability of getting at least a 0.82 N difference in mean strike force under the hypothesis that the distributions of strike forces for the two frogs are identical. You will use a permutation test with a test statistic of the *difference of means* to test this hypothesis.

Hints: 1. Create a function `diff_of_means()` that takes two arrays as parameters, computes their mean values and returns the difference between the two mean values. 2. Use that function to compute the difference of mean impact force, i.e., call `diff_of_means(force_a, force_b)`. 3. Use function `draw_perm_reps()` to draw 100,000 permutation replicates. 4. Compute and print the p-value.

Important: Don't panic if your p-value is extremely low.

1.8 Solution

[31]:
```
def diff_of_means(array1, array2):
    return np.mean(array1) - np.mean(array2)
```

[32]:
```
perm_replicates = draw_perm_reps(force_a,force_b,
                                 diff_of_means, size=100_000)
# Computing p-value
empirical_diff_means = diff_of_means(force_a,force_b)
p = np.sum(perm_replicates >= empirical_diff_means) / len(perm_replicates)
print("p-value:", p)
```

```
p-value: 1e-05
```

1.9 Question 1 (6 points)

1. How do you describe the result of your hypothesis testing based on the p-value calculated above?

1.10 Solution

When the p-value is small, it means the they are statistically significantly different. Meaning based on the simulation its highly likely that the 2 different sets are different and support the null hypothesis

---

1.11 Part 3: Test of correlation

In this part, we will look at the correlation between female literacy and fertility (defined as the average number of children born per woman) throughout the world. For ease of analysis and interpretation, we will work with the *illiteracy* rate.

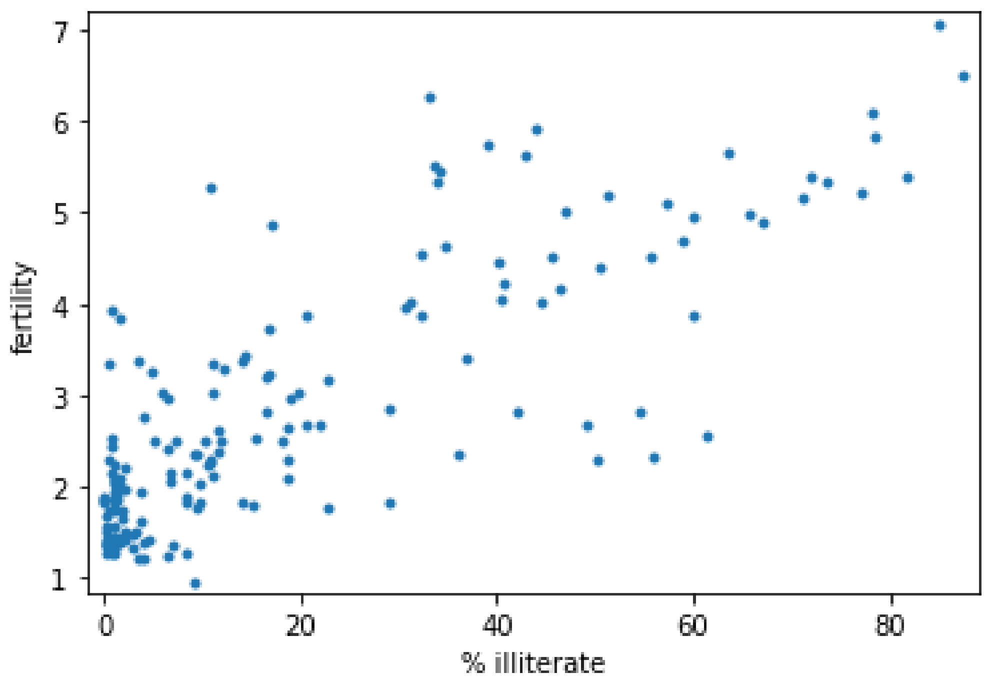
The Python code below plots the fertility versus illiteracy and computes the Pearson correlation coefficient. The Numpy array `illiteracy` has the illiteracy rate among females for most of the world's nations. The array `fertility` has the corresponding fertility data.

[33]:
```
df = pd.read_csv('data/female_literacy_fertility.csv')
df.head()
```

[33]:

	Country	Continent	female literacy	fertility	population
0	Chine	ASI	90.5	1.769	1,324,655,000
1	Inde	ASI	50.8	2.682	1,139,964,932
2	USA	NAM	99.0	2.077	304,060,000
3	Indonésie	ASI	88.8	2.132	227,345,082
4	Brésil	LAT	90.2	1.827	191,971,506

[34]:
```
illiteracy = 100 - df['female literacy']

fertility = df['fertility']

def pearson_r(x, y):
    """Compute Pearson correlation coefficient between two arrays."""
    # Compute correlation matrix: corr_mat
    corr_mat = np.corrcoef(x, y)

    # Return entry [0,1]
    return corr_mat[0,1]

# Plot the illiteracy rate versus fertility
_ = plt.plot(illiteracy, fertility, marker='.', linestyle='none')

# Set the margins and label axes
plt.margins(0.02)
_ = plt.xlabel('% illiterate')
_ = plt.ylabel('fertility')

# Show the plot
plt.show()

# Show the Pearson correlation coefficient
print('Pearson correlation coefficient between illiteracy and fertility: {:.
 ↪5f}'.format(pearson_r(illiteracy, fertility)))
```




```
Pearson correlation coefficient between illiteracy and fertility: 0.80413
```

1.12 Your turn! (25 points)

The observed correlation between female illiteracy and fertility may just be by chance; the fertility of a given country may actually be totally independent of its illiteracy.

You will test this hypothesis.

To do so, permute the illiteracy values but leave the fertility values fixed. This simulates the hypothesis that they are totally independent of each other. For each permutation, compute the Pearson correlation coefficient (using the `pearson_r()` function above) and assess how many of your permutation replicates have a Pearson correlation coefficient greater than the observed one.

Hint: use a `for` loop to draw 100,000 permutation replicates and compute the Pearson correlation coefficient for each of them.

Important: Don't panic if your p-value is extremely low.

1.13 Solution

[38]:
```python
r_obs = pearson_r(illiteracy, fertility)

perm_replicates = np.empty(100_000)
for _ in range(100_000):
    illiteracy_permuted = np.random.permutation(illiteracy)

    # Compute Pearson correlation
    perm_replicates[i] = pearson_r(illiteracy_permuted, fertility)
```

```
    # Compute p-value: p
empirical_pearson = p = np.sum(perm_replicates > r_obs) / len(perm_replicates)
print('p-val =', p)
```

```
p-val = 4e-05
```

1.14 Question 2 (6 points)

2. How do you describe the result of your hypothesis testing based on the p-value calculated above?

1.15 Solution

Similar to before since the p-value is so small its statistically significantly different.

1.16 Conclusions (10 points)

Write your conclusions and make sure to address the issues below: 1. What have you learned from this assignment? 2. Which parts were the most fun, time-consuming, enlightening, tedious? 3. What would you do if you had an additional week to work on this?

1.17 Solution

1. How to compare 2 different datasets to see if they are at all statistically significant or correlated. This helps measure if the results are due to chance or thats really a trend. This also helps with showing if the comparison is noteworthy or not.
2. Even though I could follow the steps to recreate what appears to be the solutions I find that I have to go over the material many times to catch exactly what I am trying to do. It could be that its this simple and that I am overthinking it but I am not sure. I also think if you told me to do this from scratch on my own I probably would have no idea where to start. I feel like I know when I should use this tool but I when it comes to using it in the filed. I think i might forget to use it or might not know when I should use it. Probably just not confident on this topic yet.

3. I seem to really enjoying the visual graphs when trying to explain things. Might be why just computing the p-value seeing the results was not as exciting. Maybe look for otherways to represent the statistically significant

[ ]:

```
```