Distributed Command Filtered Robust Tracking Control of Wave-Adaptive Modular Vessel with Uncertainty

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***Abstract*— This paper considers formation control of multiple wave-adaptive-modular vessels (WAM-Vs) with the aid of neighbors' information when there are parametric uncertainty and non-parametric uncertainty in the dynamics of each WAM-V. With the aid of backstepping techniques, distributed robust tracking controllers are proposed. To avoid calculation of the derivative of signals, distributed command filtered controllers are also proposed. Simulation results show the effectiveness of the proposed controllers.**

I. INTRODUCTION

A Wave-Adaptive Modular Vessel (WAM-V) is an underactuated system. There are fewer control inputs than the numbers of the degree of freedom. Control of a WAM-V is challenging due to its underactuated nature. In oceans, there are always unknown currents applied to a WAM-V. Therefore, in the model of a WAM-V, there are always disturbances. Considering the complexity of tasks, robustness and flexibility of multiple WAM-Vs, coordination of multiple systems has been an important mission. In this paper, we considered formation control of multiple WAM-Vs with uncertainty. It is assumed that there is parametric uncertainty and non-parametric uncertainty in the model of each WAM-V and the information of the leader WAM-V is available only to a portion of the follower WAM-Vs.

Cooperative control of multiple agents has received considerable attention in recent years. Various methods have been proposed, which include behavior-based method [1], virtual structure based method [2], leader-follower method [3], artificial potentials based method [4], graph theory based method [5].

Formation control of surface vessels is particularly interesting and has many applications in practice. For example, several autonomous vessels trace a target and enclose it; several vessels search a target in a large water area cooperatively; and several vessels with some sensors on them, monitor a large area cooperatively for security, etc. One important type of formation control is to design cooperative control laws such that a group of surface vessels converges to a geometric pattern which is moving along a desired trajectory or path. This formation control problem is complicated due to the nonlinearity and underactuated features of the dynamics of each surface vehicle. In [6], [7], applications of the null-space-based behavioral control to a fleet of marine surface vessels were discussed. Cooperative controls which could achieve multiple tasks were proposed. In [8], a planar formation control problem of multiple agents was considered. A guided formation control scheme was developed by combining guidance laws with synchronization algorithms and collision avoidance techniques. In [9], robust formation control of marine craft was considered. Cooperative control laws were proposed with the aid of a set of inter-body constraint functions and Lagrangian multipliers. In [10], the formation control along a desired path was discussed. Decentralized controllers were proposed with the aid of Lyapunov-based techniques and graph theory results. In [11], cross-track formation control of underactuated 3-DOF surface vessels was considered. Decentralized controllers with two blocks were proposed such that a group of vessels asymptotically converged to a desired formation which followed a given straight-line path with a given forward speed profile. In papers [6], [7], [8], [9], the controlled systems are assumed to be fully actuated. In [12], dynamic positioning of multiple surface vessels subject to unknown time-varying environmental disturbances and input saturation was considered. Distributed controllers were proposed with the aid of disturbance observer and dynamic surface control technique.

In this paper, we consider formation control of multiple WAM-Vs with uncertainty and a leader vehicle. Distributed robust controllers are proposed with the aid of adaptive backstepping techniques. The proposed control laws ensure that the formation errors and the tracking errors exponentially converge to zero. To reduce the computation load in controller design, distributed command filtered controllers are proposed. To verify the effectiveness of the proposed cooperative control laws, simulation results are presented.

II. PROBLEM STATEMENT

A. WAM-V Modeling

It is considered a group of m wave-adaptive modular vessels (WAM-Vs). Each WAMV has two propellers in its pontoons. With the aid of the results in [13], the kinematics of the j-th WAM-V can be written as

$$\dot{x}_j = u_j \cos\psi_j - v_j \sin\psi_j \quad (1)$$

$$\dot{y}_j = u_j \sin\psi_j + v_j \cos\psi_j \quad (2)$$

$$\dot{\psi}_j = r_j \quad (3)$$

where (x_j, y_j) denotes the coordinates of the center of the WAM-V in the earth-fixed frame, ψ_j is the orientation of the WAM-V, and u_j, v_j and r_j are the velocities of the WAM-V

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in surge, sway and yaw, respectively. For simplicity of analysis, it is assumed that the jth WAM-V has port/starboard and fore/aft symmetry and the motion in heave, roll and pitch can be neglected. The dynamics of the jth WAM-V can be written as ([13])

$$m_{1j}\dot{u}_j - m_{2j}v_j r_j + d_{1j}u_j + D_{1j} = F_{Lj} + F_{Rj} \quad (4)$$
$$m_{2j}\dot{v}_j + m_{1j}u_j r_j + d_{2j}v_j + D_{2j} = 0 \quad (5)$$
$$m_{3j}\dot{r}_j - (m_{1j} - m_{2j})u_j v_j + d_{3j}r_j + D_{3j} = L_j(F_{Lj} - F_{Rj}) \quad (6)$$

where $m_{ij}(> 0)$ and $d_{ij}(> 0)$ for $i = 1, 2, 3$ are (effective) inertia and hydrodynamic damping of the WAM-V, respectively; L_j is the moment arm of the forces with respect to the center of geometry and mass of the WAM-V, which are assumed to coincide; D_{ij} is un-modeled dynamics and disturbance, and F_{Lj} and F_{Rj} are the force generated by the propellers.

For control purpose, each WAM-V knows its own state and the states of its neighbors by wireless communication or sensors. If each WAM-V is considered as a node, the communications between WAM-Vs can be described by a directed graph (i.e., digraph) $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, where $\mathcal{V} = \{1, 2, \ldots, m\}$ is a node set, $\mathcal{E}$ is an edge set with elements (i, j) which describes the communication from node i to node j. If the state of node i is available to node j, there is an edge (i, j) in $\mathcal{E}$, and vice versa. Node i is a *neighbor* of node j if the state of node i is available to node j. Since communication is directional, (i, j) is an ordered pair, which means that $(i, j) \in \mathcal{E}$ does not mean $(j, i) \in \mathcal{E}$. For node j, the indexes of its neighbors form a set which is denoted by $\mathcal{N}_j$. Therefore, the available states to node j are the state of node j and the state of node i for all $i \in \mathcal{N}_j$. A directed path in a digraph is an ordered sequence of vertices such that any ordered pair of vertices appearing consecutively in the sequence is an edge of the digraph. Node i is reachable to node j ($j \neq i$) if there exists a directed path from node i to node j. For more terminology on graph theory, interested readers may refer to [14], [15].

B. Problem Statement

In the dynamics (4)-(6), the inertia parameters m_{ij} and d_{ij} are not exactly known in practice.

It is given a leader WAM-V which moves along a smooth trajectory $(x_0(t), y_0(t))$. The leader WAM-V is labeled as node 0. The m WAM-Vs in (1)-(3) are called the follower WAM-Vs. The state of the leader WAM-V is available to some of the follower WAM-Vs. The communication between m follower WAM-Vs and the leader WAM-V is described by a digraph $\mathcal{G}^e = \{\mathcal{V}^e, \mathcal{E}^e\}$ with a node set $\mathcal{V}^e = \{0, 1, 2, \ldots, m\}$. For node j ($0 \le j \le m$), its neighbor set is denoted as $\mathcal{N}_j^e$. Node 0 is said to be globally reachable if node 0 is reachable to node j for $1 \le j \le m$. In this paper, the following assumption is made on the communication digraph $\mathcal{G}^e$.

Assumption 1: In the communication digraph $\mathcal{G}^e$, node 0 is globally reachable.

It is also given a desired geometric pattern $\mathcal{P}$ defined by constant vectors $[p_{jx}, p_{jy}]^\top (1 \le j \le m)$ which satisfy $\sum_{j=1}^m p_{jx} = 0$ and $\sum_{j=1}^m p_{jy} = 0$. We consider the following formation control problem.

Formation Control Problem: Design a controller (F_{Lj}, F_{Rj}) for j-th follower WAM-V based on its neighbors' state information such that

$$\lim_{t\to\infty} \begin{bmatrix} x_i - x_j \\ y_i - y_j \end{bmatrix} = \begin{bmatrix} p_{ix} - p_{jx} \\ p_{iy} - p_{jy} \end{bmatrix}, 1 \le i, j \le m \quad (7)$$

$$\lim_{t\to\infty} \sum_{j=1}^{m} \left(\frac{x_j}{m} - x_0 \right) = 0 \quad (8)$$

$$\lim_{t\to\infty} \sum_{j=1}^{m} \left(\frac{y_j}{m} - y_0 \right) = 0 \quad (9)$$

In the above problem, eqn. (7) means that m follower WAM-Vs come into the desired geometric pattern $\mathcal{P}$. Eqn. (8)-(9) mean that the centroid of m follower WAM-Vs follows the leader WAM-V.

III. Distributed Robust Controller Design

It is assumed that the nominal values of m_{ij} and d_{ij} are $\bar{m}_{ij}$ and $\bar{d}_{ij}$, respectively. The errors between the nominal values and the real values are bounded by known constants, i.e., $|m_{1j} - \bar{m}_{1j}| \le \rho_{1j}$, $|m_{2j} - \bar{m}_{2j}| \le \rho_{2j}$, $|m_{3j} - \bar{m}_{3j}| \le \rho_{3j}$, $|d_{1j} - \bar{d}_{1j}| \le \rho_{4j}$, $|d_{3j} - \bar{d}_{3j}| \le \rho_{5j}$, $|D_{1j}| \le \rho_{6j}$, and $|D_{3j}| \le \rho_{7j}$, where ρ_{ij} $(1 \le i \le 7)$ are known. The system in (1)-(6) has a cascade structure. We will take this advantage and design a distributed controller for the j-th WAM-V with the aid of backstepping techniques [16].

• **Step 1:** For system j, the weighted average of its neighbors information is defined as

$$\zeta_{1j} = \frac{\sum_{i\in\mathcal{N}_j^e} a_{ji}(x_i - p_{ix})}{\sum_{i\in\mathcal{N}_j^e} a_{ji}}, \quad \zeta_{2j} = \frac{\sum_{i\in\mathcal{N}_j^e} a_{ji}(y_i - p_{iy})}{\sum_{i\in\mathcal{N}_j^e} a_{ji}} \quad (10)$$

where $\mathcal{N}_j^e$ is the neighbor set of node j in the digraph $\mathcal{G}^e$, a_{ji} is a positive constant for $j = 1, 2, \ldots, m$. In the system (1)-(2), we consider $u_j \cos\psi_j$ and $u_j \sin\psi_j$ as virtual control inputs and design tracking controllers such that

$$\lim_{t\to\infty} (x_j(t) - p_{jx} - \zeta_{1j}(t)) = 0 \quad (11)$$
$$\lim_{t\to\infty} (y_j(t) - p_{jy} - \zeta_{2j}(t)) = 0. \quad (12)$$

For convenience, we let

$$u_j \cos\psi_j = \eta_{1j}, \quad u_j \sin\psi_j = \eta_{2j}. \quad (13)$$

where η_{1j} and η_{2j} will be chosen later. Let the tracking error

$$e_{*j} = \begin{bmatrix} e_{1j} \\ e_{2j} \end{bmatrix} = \begin{bmatrix} x_j - p_{jx} - \zeta_{1j} \\ y_j - p_{jy} - \zeta_{2j} \end{bmatrix} \quad (14)$$

then

$$\dot{e}_{*j} = \begin{bmatrix} \eta_{1j} \\ \eta_{2j} \end{bmatrix} + \begin{bmatrix} -\sin\psi_j \\ \cos\psi_j \end{bmatrix} v_j - \begin{bmatrix} \dot{\zeta}_{1j} \\ \dot{\zeta}_{2j} \end{bmatrix} \quad (15)$$

(15) can be considered as a linear system with perturbations. Stabilizing controllers can be designed as

$$\eta_{1j} = -a_{jj}e_{1j} + v_j \sin\psi_j - \frac{\rho_x e_{1j}}{\sqrt{e_{1j}^2 + h(t)}} \quad (16)$$

$$\eta_{2j} = -a_{jj}e_{2j} - v_j \cos\psi_j - \frac{\rho_y e_{2j}}{\sqrt{e_{2j}^2 + h(t)}} \quad (17)$$

where $a_{jj} = \sum_{i\in\mathcal{N}_j^e} a_{ji}$, ρ_x and ρ_y are sufficiently large constants, $h(t) > 0$, and $h(t)$ exponentially converges to zero.

To verify that the proposed virtual controller (16)-(17) ensures that (11)-(12) hold, we substitute the virtual controller (16)-(17) to (1)-(2) and have

$$\dot{x}_j = -\sum_{i\in\mathcal{N}_j^e} a_{ji}(x_j - p_{jx} - x_i + p_{ix}) - \frac{\rho_x e_{1j}}{\sqrt{e_{1j}^2 + h}} \quad (18)$$

$$\dot{y}_j = -\sum_{i\in\mathcal{N}_j^e} a_{ji}(y_j - p_{jy} - y_i + p_{iy}) - \frac{\rho_y e_{2j}}{\sqrt{e_{2j}^2 + h}}. \quad (19)$$

Define $\tilde{x}_j = x_j - p_{jx} - x_0$ and $\tilde{y}_j = y_j - p_{jy} - y_0$ where $p_{0x} = p_{0y} = 0$, we have

$$\dot{\tilde{x}}_j = -\sum_{i\in\mathcal{N}_j^e} a_{ji}(\tilde{x}_j - \tilde{x}_i) - \dot{x}_0 - \frac{\rho_x e_{1j}}{\sqrt{e_{1j}^2 + h}} \quad (20)$$

$$\dot{\tilde{y}}_j = -\sum_{i\in\mathcal{N}_j^e} a_{ji}(\tilde{y}_j - \tilde{y}_i) - \dot{y}_0 - \frac{\rho_y e_{2j}}{\sqrt{e_{2j}^2 + h}}. \quad (21)$$

For the communication between vehicles we make the following assumption.

Assumption 2: The leader vehicle is globally reachable.

If

$$\rho_x \geq \max\{|\dot{x}_0(t)|\}, \quad \rho_y \geq \max\{|\dot{y}_0(t)|\}, \quad (22)$$

it can be proved that $\tilde{x}_j$ and $\tilde{y}_j$ exponentially converge to zero if Assumption 2 is satisfied with the aid of the results in [17].

By (13), we solve u_j and ψ_j and obtain $u_j = \eta_{3j}$ and $\psi_j = \eta_{4j}$ where

$$\eta_{3j} = \sqrt{\eta_{1j}^2 + \eta_{2j}^2}, \quad \eta_{4j} = \mathrm{atan2}(\eta_{2j}, \eta_{1j}) \quad (23)$$

In (23), atan2 is not defined if $\eta_{2j} = 0$ and $\eta_{1j} = 0$. To avoid this, we make the following assumption.

Assumption 3: $0 < \epsilon < \dot{x}_0^2(t) + \dot{y}_0^2(t) < \infty$ for any time t, where ϵ is a small positive constant.

• **Step 2:** u_j and ψ_j are not the real control inputs and $[u_j, \psi_j]^\top \neq [\eta_{3j}, \eta_{4j}]^\top$. Let

$$z_{*j} = [z_{1j}, z_{2j}]^\top = [u_j - \eta_{3j}, \psi_j - \eta_{4j}]^\top$$

then

$$\begin{aligned} \dot{\tilde{x}}_j &= -\sum_{i\in\mathcal{N}_j^e} a_{ji}(\tilde{x}_j - \tilde{x}_i) - \dot{x}_0 - \frac{\rho_x e_{1j}}{\sqrt{e_{1j}^2 + h}} \\ &\quad +\Lambda_j \quad (24) \\ \dot{\tilde{y}}_j &= -\sum_{i\in\mathcal{N}_j^e} a_{ji}(\tilde{y}_j - \tilde{y}_i) - \dot{y}_0 - \frac{\rho_y e_{2j}}{\sqrt{e_{2j}^2 + h}} \\ &\quad +\Omega_j \quad (25) \\ m_{1j}\dot{z}_{1j} &= m_{2j}v_j r_j - d_{1j}u_j + F_{Lj} + F_{Rj} \\ &\quad -m_{1j}\dot{\eta}_{3j} - D_{1j} \quad (26) \\ \dot{z}_{2j} &= r_j - \dot{\eta}_{4j} \quad (27) \end{aligned}$$

where

$$\begin{aligned} \Lambda_j &= z_{1j}\cos\eta_{4j} + u_j z_{2j}\cos\eta_{4j}\frac{(\cos z_{2j} - 1)}{z_{2j}} \\ &\quad -u_j z_{2j}\sin\eta_{4j}\frac{\sin z_{2j}}{z_{2j}} \\ \Omega_j &= z_{1j}\sin\eta_{4j} + u_j z_{2j}\sin\eta_{4j}\frac{(\cos z_{2j} - 1)}{z_{2j}} \\ &\quad +u_j z_{2j}\cos\eta_{4j}\frac{\sin z_{2j}}{z_{2j}} \end{aligned}$$

for $1 \leq j \leq m$.

For the system in (24)-(25), the following input-to-state property can be shown.

Lemma 1: For the systems in (24)-(25), Under Assumption 2,

1) if z_{1j} and z_{2j} are bounded and converge to a small neighborhood of the origin with radius r, $\tilde{x}_j$ and $\tilde{y}_j$ are bounded and converge to a neighborhood of the origin whose radius can be made as small as possible by choosing r very small.
2) if z_{1j} and z_{2j} are bounded and converge to zero, $\tilde{x}_j$ and $\tilde{y}_j$ are bounded and converge to zero.
3) if z_{1j} and z_{2j} exponentially converge to zero, $\tilde{x}_j$ and $\tilde{y}_j$ exponentially converge to zero.

The proof of Lemma 1 is omitted due to space limit.

Thanks to Lemma 1, we design controllers such that z_{1j} and z_{2j} are bounded and converge to zero. Choose a Lyapunov function candidate

$$V_{2j} = \frac{1}{2}\sum_{j=1}^{m}(m_{1j}z_{1j}^2 + z_{2j}^2)$$

and differentiate V_{2j} along the solution of the systems in (26)-(27), we have

$$\begin{aligned} \dot{V}_{2j} &= \sum_{j=1}^{m} z_{1j}[m_{2j}v_j r_j - d_{1j}u_j + F_{Lj} + F_{Rj} \\ &\quad -m_{1j}\dot{\eta}_{3j} - D_{1j}] + \sum_{j=1}^{m} z_{2j}(r_j - \dot{\eta}_{4j}) \end{aligned}$$

We choose

$$
\begin{aligned}
F_{Lj}+F_{Rj} &= -K_3 z_{1j}+\bar{m}_{1j}\dot{\eta}_{3j}-\bar{m}_{2j}v_j r_j+\bar{d}_{1j}u_j \\
&\quad -(\rho_{1j}|\dot{\eta}_{3j}|+\rho_{2j}|v_j r_j| \\
&\quad +\rho_{4j}|u_j|+\rho_{6j})\mathrm{sign}(z_{1j}) =: \delta_{1j} \qquad (28)\\
r_j &= \eta_{5j} \qquad (29)\\
\eta_{5j} &= -K_4 z_{2j}+\dot{\eta}_{4j} \qquad (30)
\end{aligned}
$$

where K_3 and K_4 are positive definite matrices. Then

$$
\dot{V}_{2j} \le -\sum_{j=1}^{m}(K_3 z_{1j}^2+K_4 z_{2j}^2)
$$

It can be shown that z_{1j} and z_{2j} $(1\le j\le m)$ are bounded and exponentially converge to zero.

• **Step 3:** r_j is not the real control inputs and $r_j \neq \eta_{5j}$. Let $z_{3j}=r_j-\eta_{5j}$, then

$$
\begin{aligned}
\dot{z}_{2j} &= -K_4 z_{2j}+z_{3j} \qquad (31)\\
m_{3j}\dot{z}_{3j} &= (m_{1j}-m_{2j})u_j v_j-d_{3j}r_j-m_{3j}\dot{\eta}_{5j} \\
&\quad -D_{3j}+L_j(F_{Rj}-F_{Lj}). \qquad (32)
\end{aligned}
$$

Choose a Lyapunov function candidate

$$
V_{3j}=V_{2j}+\frac{1}{2}\sum_{j=1}^{m}m_{3j}z_{3j}^2
$$

and differentiate it along the solutions of the system, we have

$$
\begin{aligned}
\dot{V}_{3j} &\le -\sum_{j=1}^{m}(K_3 z_{1j}^2+K_4 z_{2j}^2)+\sum_{j=1}^{m}z_{2j}z_{3j} \\
&\quad +\sum_{j=1}^{m}z_{3j}((m_{1j}-m_{2j})u_j v_j-d_{3j}r_j-m_{3j}\dot{\eta}_{5j} \\
&\quad -D_{3j}+L_j(F_{Rj}-F_{Lj}))
\end{aligned}
$$

We choose

$$
\begin{aligned}
L_j(F_{Rj}-F_{Lj}) &= -K_5 z_{3j}-z_{2j}-(\bar{m}_{1j}-\bar{m}_{2j})u_j v_j \\
&\quad +\bar{d}_{3j}r+\bar{m}_{3j}\dot{\eta}_{5j}-((\rho_{1j} \\
&\quad +\rho_{2j})|u_j v_j|+\rho_{5j}|r_j|+\rho_{3j}|\dot{\eta}_{5j}| \\
&\quad +\rho_{7j})\mathrm{sign}(z_{3j}) =: \delta_{2j} \qquad (33)
\end{aligned}
$$

where K_5 is a positive constant. Then

$$
\dot{V}_{3j} \le -\sum_{j=1}^{m}(K_3 z_{1j}^2+K_4 z_{2j}^2+K_5 z_{3j}^2)
$$

It can be shown that z_{1j}, z_{2j}, z_{3j} $(1\le j\le m)$ are bounded and exponentially converge to zero.

Solve the equations in (28) and (33), we obtain the control inputs as follows:

$$
F_{Rj}=\frac{1}{2}\left(\delta_{1j}+\frac{\delta_{2j}}{L_j}\right),\quad F_{Lj}=\frac{1}{2}\left(\delta_{1j}-\frac{\delta_{2j}}{L_j}\right) \qquad (34)
$$

The above results are summarized in the following theorem.

Theorem 1: For m vehicles in (1)-(6) and a leader vehicle with the desired trajectory (x_0,y_0), under Assumptions 1-3, the control laws in (34) ensure that (7)-(9) are satisfied.

Proof: By the above controller design procedure, z_{1j}, z_{2j}, and z_{3j} exponentially converge to zero. With the aid of Lemma 1, $\tilde{x}_j$ and $\tilde{y}_j$ are bounded and exponentially converge to zero, which means that (7)-(9) are satisfied. ■

IV. Distributed Command Filtered Tracking Control Laws

In the proposed controller in the last section, the derivatives of signals are needed. To avoid this, a command filtered controller can be designed with the aid of the ideas in [18], [19], [20]. To this end, we introduce the following command filters:

$$
\begin{aligned}
\dot{\chi}_{1j} &= -r_{1j}|\chi_{1j}-\eta_{3j}|^{\frac{1}{2}}\mathrm{sign}(\chi_{1j}-\eta_{3j})+\chi_{2j} \qquad (35)\\
\dot{\chi}_{2j} &= -r_{2j}\mathrm{sign}(\chi_{2j}-\dot{\chi}_{1j}) \qquad (36)\\
\dot{\chi}_{3j} &= -r_{3j}|\chi_{3j}-\eta_{4j}|^{\frac{1}{2}}\mathrm{sign}(\chi_{3j}-\eta_{4j})+\chi_{4j} \qquad (37)\\
\dot{\chi}_{4j} &= -r_{4j}\mathrm{sign}(\chi_{4j}-\dot{\chi}_{3j}) \qquad (38)\\
\dot{\chi}_{5j} &= -r_{5j}|\chi_{5j}-\eta_{5j}|^{\frac{1}{2}}\mathrm{sign}(\chi_{5j}-\eta_{5j})+\chi_{6j} \qquad (39)\\
\dot{\chi}_{6j} &= -r_{6j}\mathrm{sign}(\chi_{6j}-\dot{\chi}_{5j}) \qquad (40)
\end{aligned}
$$

where r_{ij} for $1\le i\le 6$ and $1\le j\le m$ are appropriate constants [21]. The compensated signals are defined by

$$
\begin{aligned}
\dot{\xi}_{1j} &= -\frac{K_3}{\bar{m}_{1j}}\xi_{1j} \qquad (41)\\
\dot{\xi}_{2j} &= -K_4\xi_{2j}+\xi_{3j}+(\chi_{5j}-\eta_{5j}) \qquad (42)\\
\dot{\xi}_{3j} &= -\frac{K_5}{\bar{m}_{3j}}\xi_{3j} \qquad (43)
\end{aligned}
$$

Let

$$
\begin{aligned}
w_{1j} &= u_j-\chi_{1j}-\xi_{1j}, \qquad (44)\\
w_{2j} &= \psi_j-\chi_{3j}-\xi_{2j} \qquad (45)\\
z_{3j} &= r_j-\chi_{5j}-\xi_{3j} \qquad (46)
\end{aligned}
$$

then

$$
\begin{aligned}
m_{1j}\dot{w}_{1j} &= m_{2j}v_j r_j-d_{1j}u_j+F_{Lj}+F_{Rj} \\
&\quad -m_{1j}\dot{\chi}_{1j}-D_{1j}-m_{1j}\dot{\xi}_{1j} \qquad (47)\\
\dot{w}_{2j} &= r_j-\dot{\chi}_{3j}-\dot{\xi}_{2j} \\
&= z_{3j}+\xi_{3j}+(\chi_{5j}-\eta_{5j})+\eta_{5j} \\
&\quad -\dot{\chi}_{3j}-\dot{\xi}_{2j} \qquad (48)\\
m_{3j}\dot{z}_{3j} &= (m_{1j}-m_{2j})u_j v_j-d_{3j}r_j-m_{3j}\dot{\chi}_{5j} \\
&\quad -D_{3j}+L_j(F_{Rj}-F_{Lj})-m_{3j}\dot{\xi}_{3j} \qquad (49)
\end{aligned}
$$

We choose

$$
\begin{aligned}
F_{Lj}+F_{Rj} &= -K_3(u_j-\chi_{1j})+\bar{m}_{1j}\dot{\chi}_{1j} \\
&\quad -\bar{m}_{2j}v_j r_j+\bar{d}_{1j}u_j-(\rho_{1j}|\dot{\chi}_{1j}| \\
&\quad +\rho_{2j}|v_j r_j|+\rho_{4j}|u_j| \\
&\quad +\rho_{6j})\mathrm{sign}(w_{1j}) =: \delta_{1j} \qquad (50)\\
\eta_5 &= -K_4\psi_j+\dot{\chi}_{3j} \qquad (51)\\
L_j(F_{Rj}-F_{Lj}) &= -K_5(r_j-\chi_{5j})-w_{2j}-(\bar{m}_{1j} \\
&\quad -\bar{m}_{2j})u_j v_j+\bar{d}_{3j}r+\bar{m}_{3j}\dot{\chi}_{5j} \\
&\quad -((\rho_{1j}+\rho_{2j})|u_j v_j|+\rho_{5j}|r_j| \\
&\quad +\rho_{3j}|\dot{\chi}_{5j}|+\rho_{7j})\mathrm{sign}(z_{3j}) \\
&=: \delta_{2j} \qquad (52)
\end{aligned}
$$

then (47)-(49) are in the following forms.

$$\begin{aligned} m_{1j}\dot{w}_{1j} &= -K_3 w_{1j} + \tilde{m}_{2j} v_j r_j - \tilde{d}_{1j} u_j \\ &\quad -\tilde{m}_{1j}\dot{\chi}_{1j} - D_{1j} + \frac{m_{1j}-\bar{m}_{1j}}{\bar{m}_{1j}} K_3 \xi_{1j} \\ &\quad -(\rho_{1j}|\dot{\chi}_{1j}| + \rho_{2j}|v_j r_j| \\ &\quad +\rho_{4j}|u_j| + \rho_{6j})\mathrm{sign}(w_{1j}) \qquad (53) \\ \dot{w}_{2j} &= -K_4 w_{2j} + z_{3j} \qquad (54) \\ m_{3j}\dot{z}_{3j} &= (\tilde{m}_{1j} - \tilde{m}_{2j}) u_j v_j - \tilde{d}_{3j} r_j - \tilde{m}_{3j}\dot{\chi}_{5j} - D_{3j} \\ &\quad -K_5 z_{3j} - w_{2j} + \frac{m_{3j}-\bar{m}_{3j}}{\bar{m}_{3j}} K_5 \xi_{3j} \\ &\quad -((\rho_{1j}+\rho_{2j})|u_j v_j| + \rho_{5j}|r_j| + \rho_{3j}|\dot{\chi}_{5j}| \\ &\quad +\rho_{7j})\mathrm{sign}(z_{3j}) \qquad (55) \end{aligned}$$

Theorem 2: For m vehicles in (1)-(6) and a leader vehicle with the desired trajectory (x_0, y_0), under Assumptions 1-3, the control law in (34) with δ_{1j} in (50) and δ_{2j} in (52) ensure that (7)-(9) are satisfied.

Proof: Choose a Lyapunov function

$$V_4 = \frac{1}{2}\sum_{j=1}^{m}(m_{1j}w_{1j}^2 + w_{2j}^2 + m_{3j}z_{3j}^2)$$

its derivative along the solution of the closed loop system is

$$\begin{aligned} \dot{V}_4 \leq & -K_3 w_{1j}^2 - K_4 w_{2j}^2 - K_5 z_{3j}^2 \\ & + \frac{m_{1j}-\bar{m}_{1j}}{\bar{m}_{1j}} K_3 \xi_{1j} w_{1j} + \frac{m_{3j}-\bar{m}_{3j}}{\bar{m}_{3j}} K_5 \xi_{3j} z_{3j} \end{aligned}$$

Noting that ξ_{1j} and ξ_{3j} exponentially converge to zero, it can be shown that w_{1j}, w_{2j}, and z_{3j} exponentially converge to zero, respectively.

By the command filters, $\chi_{1j} - \eta_{3j}$, $\chi_{3j} - \eta_{4j}$, and $\chi_{5j} - \eta_{5j}$ are bounded and converge to zero in a finite time. By the definitions of the compensated signals, ξ_{1j}, ξ_{3j}, and ξ_{2j} exponentially converge to zero. So, z_{1j} and z_{2j} converge to zero. By Lemma 1, $\tilde{x}_j$ and $\tilde{y}_j$ converge to zero, which means that (7)-(9) are satisfied. ■

V. Simulation

Consider three identical WAM-Vs with the model parameters: $m_{1j} = 200$, $m_{2j} = 250$, $m_{3j} = 80$, $d_{1j} = 70$, $d_{2j} = 100$, $d_{3j} = 50$ for $j = 1, 2, 3$. The un-modeled dynamics and uncertainty is as follows: $D_{1j} = 2\cos 0.3t$, $D_{2j} = 2\cos 0.2t$, and $D_{3j} = 2\cos 0.1t$. Assume the desired geometric pattern $\mathcal{P}$ is a triangle defined by $(p_{1x}, p_{1y}) = (0, 20)$, $(p_{2x}, p_{2y}) = (-20, -10)$, and $(p_{3x}, p_{3y}) = (20, -10)$. (see Fig. 1). The desired trajectory $(x_0, y_0) = (20\cos 0.1t, 20\sin 0.1t)$.

Assume the communication digraph $\mathcal{G}$ among the WAM-Vs is fixed and is shown in Fig. 2. The communication graph is directed. If the estimates of inertia parameters are known as $\bar{m}_{1j} = 210$, $\bar{m}_{2j} = 240$, $\bar{m}_{3j} = 75$, $\bar{d}_{1j} = 75$, $\bar{d}_{2j} = 105$, $\bar{d}_{3j} = 55$ for $j = 1, 2, 3$. The bounds between the estimations can be chosen as $\rho_{1j} = 10$, $\rho_{2j} = 10$, $\rho_{3j} = 10$, $\rho_{4j} = 10$, $\rho_{5j} = 10$, $\rho_{6j} = 3$, and $\rho_{7j} = 4$. The cooperative control laws can be obtained by Theorem 2. Figs. 3-4 show the time response of $\tilde{x}_j$ and $\tilde{y}_j$ for $1 \leq j \leq 3$. Figs. 5-7 show the time response of z_{1j}, z_{2j}, and z_{3j} for $1 \leq j \leq 3$. The simulation results verify Theorem 1.

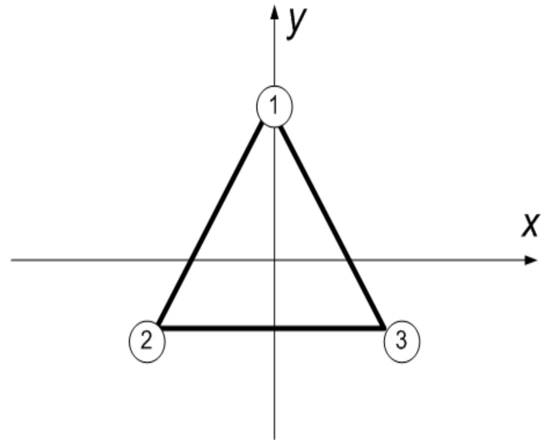

Fig. 1. Desired formation of three WAM-Vs

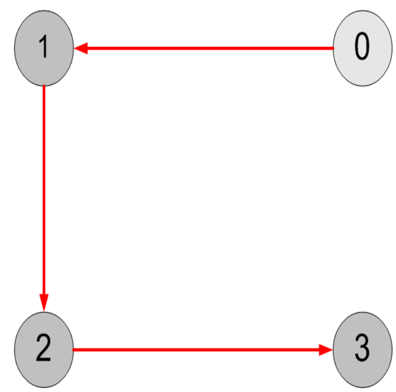

Fig. 2. Communication digraph

VI. Conclusion

In this paper, we considered formation control of multiple uncertain WAM-Vs with a leader WAM-V. If inertia parameters are not exactly known, distributed robust tracking laws were proposed with the aid of neighbors' information. To reduce the computation load in controller design, distributed command filtered controllers were proposed. Simulation results show the effectiveness of the proposed results.

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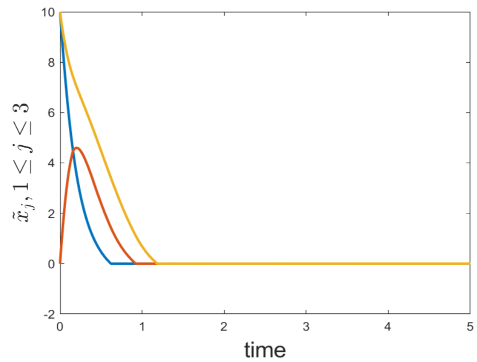

Fig. 3. Time response of $\tilde{x}_j$ for $1 \leq j \leq 3$

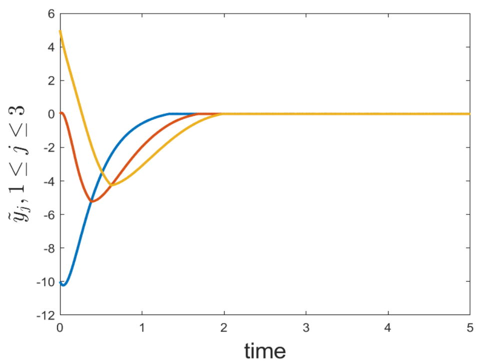

Fig. 4. Time response of $\tilde{y}_j$ for $1 \leq j \leq 3$

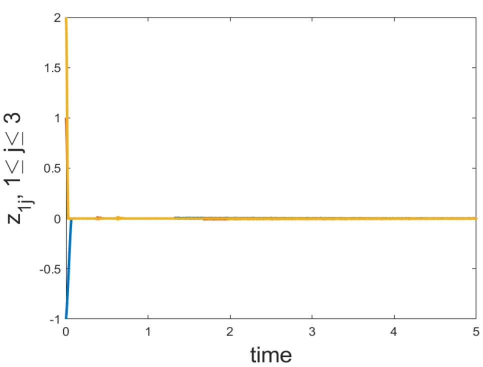

Fig. 5. Time response of z_{1j} for $1 \leq j \leq 3$

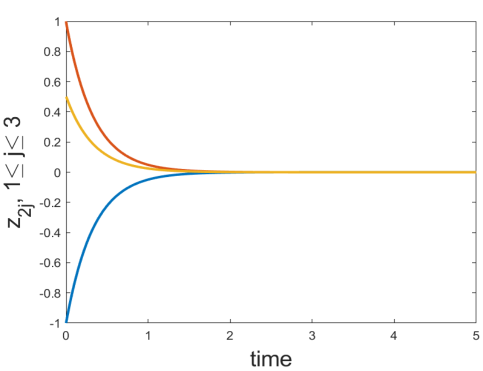

Fig. 6. Time response of z_{2j} for $1 \leq j \leq 3$

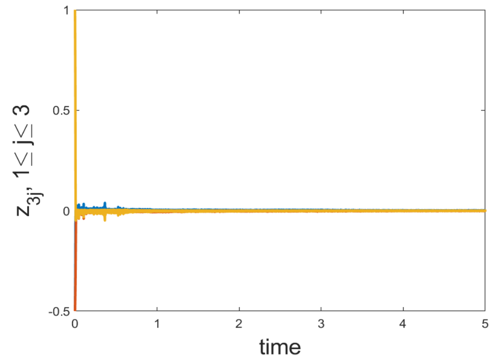

Fig. 7. Time response of z_{3j} for $1 \leq j \leq 3$

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