# MQE: Economic Inference from Data: Module 2: Fixed Effects

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6/9/2020

#### Module 2: Fixed Effects

- Data Structures
- Fixed Effects
- -A simulation
- -Fixed effects as demeaned data
- -Thinking about variation
- -Example: Crime and Unemployment

## Controlling for unobservables

We saw with AGG(2006) that even with many covariates, unobservables are a problem.

Certain types of data allow us to control for more of these unobservables by using fixed effects.

### Example:

$$Income_i = \beta_0 + \beta_1 Schooling_i + \epsilon$$

 $\beta_1$  cannot be interpreted as causal: big OVB problems, even with lots of control variables. Unlikely to have good measures of 'ability', 'enthusiasm', 'grit'...

What if I can control for unchanging individual characteristics?

## Data Structures: Cross-Section

Individual	Income	Schooling	Female
1 2	22000 57000	12 16	1 1
 N	15000	12	0

Each individual is observed once.

## Data Structures: Panel Data

Individual	Income	Schooling	Female	Year
1	22000	12	1	2001
1	23000	12	1	2002
2	57000	16	1	2001
2	63000	17	1	2002
N	15000	12	0	2001
N	13000	12	0	2002

Each individual is observed multiple times.

## Data Structures: Panel Data Subscripts

Unique observations must be identified by both the individual and time dimensions. . . notice the new subscripts:

$$Income_{it} = \beta_0 + \beta_1 Schooling_{it} + \epsilon.$$

#### Data Structures: Panel Data

Panel Data can be

- -balanced: same number of observations for each unit
- -unbalanced: some units are observed more often then others (probably good to look into why)

## Review: Indicator (Dummy) Variables

If I have multiple Female observation and multiple non-female observations I can control for the effect of being female on wages:

$$Income_{it} = \beta_0 + \beta_1 Schooling_{it} + \beta_2 Female_i + \epsilon.$$

#### Fixed Effects as Individual Indicator Variables

Indiv	Income	School	Female	Year	Indiv1	Indiv2		IndivN
1	22000	12	1	2007	1	0	0	0
1	23000	12	1	2008	1	0	0	0
2	57000	16	1	2007	0	1	0	0
2	63000	17	1	2008	0	1	0	0
 N N	15000 13000	12 12	0 0	2007 2008	0	0	0 0	1 1

#### Fixed Effects as Individual Indicator Variables

#### I can estimate:

$$\mathit{Inc}_{it} = \beta_0 + \beta_1 \mathit{School}_{it} + \beta_2 \mathit{Fem}_i + \beta_{a1} \mathit{Ind1}_i + \beta_{a2} \mathit{Ind2}_i + \ldots + \beta_{aN-1} \mathit{Ind}(N-1)_i + \epsilon.$$

What do the  $\beta_{ak}$  coefficients tell me?

#### Also:

- -Why do the *IndN* indicators only have an *i* subscript?
- -What is the implied assumption if *Fem* only has an *i* subscript?
- -Why are there only (N-1) individual dummies?

#### Fixed Effects as Individual Indicator Variables

#### What will these individual controls control for?

- $-\beta_{a1}$  will control for the effect of being individual 1 on income that is not explained by that person's gender or schooling.
- -Any **time invariant** characteristic that affects individual 1's income, such as ability, grit, enthusiasm. . . will be controlled for by adding this individual dummy variable.
- -These controls are known as individual **fixed effects**.

#### For notational convenience:

$$Income_{it} = \beta_0 + \beta_1 Schooling_{it} + \beta_2 Female_i + \gamma_i + \epsilon.$$

#### Fixed Effects

#### With my panel data, what else can I control for?

$$Income_{it} = \beta_0 + \beta_1 Schooling_{it} + \beta_2 Female_i + \gamma_i + \tau_t + \epsilon.$$

- -What is  $\tau_t$ ?
- -What is this estimation equivalent to?

You are a principle of a small school composed of four classrooms. You have just implemented a new option available to teachers for students to spend some small group reading time with a para-educator. You would like to know how this reading time is affecting reading scores.

#### You have data for ten students in each class that tells you:

- -the class the student is in
- -whether they participated in small group reading
- -their reading score.

## Generating Simulated Data

I will work with a simulated dataset to show how the use of fixed effects can help us recover the true treatment effect.

I start by loading the dplyr package and "setting the seed":

```
#install.packages("dplyr")
#install.packages("lfe")
#install.packages("stargazer")
library(dplyr)
## Warning: package 'dplyr' was built under R version 3.6.3
##
## Attaching package: 'dplyr'
## The following objects are masked from 'package:stats':
##
##
       filter, lag
## The following objects are masked from 'package:base':
##
##
       intersect, setdiff, setequal, union
library(lfe)
## Loading required package: Matrix
library(stargazer)
## Please cite as:
```

#### I generate a vector of class identifiers and a random error term.

```
class<-c(1,2,3,4)
scores<-as.data.frame(class)
scores<-rbind(scores,scores,scores,scores,scores,scores,scores,scores,scores,scores)
scores*error<-rnorm(40, mean=0, sd=5)
#note: if you are not working in markdown you would just write head(scores)
knitr::kable(head(scores))</pre>
```

class	error
1	3.6633624
2	-0.1891486
3	6.0150457
4	7.3490101
1	0.6684515
2	2.5991362

I simulate some selection into treatment. The probability of getting treated is

-0.8 for students in classrooms 3 and 4

#### -0.2 in classrooms 1 and 2.

```
scores$treat1<-rbinom(40,1,0.2)
scores$treat2<-rbinom(40,1,0.8)
scores$treat[scores$class%in%c(1,2)]<-scores$treat1[scores$class%in%c(1,2)]
scores$treat[scores$class%in%c(3,4)]<-scores$treat2[scores$class%in%c(3,4)]</pre>
knitr::kable(head(scores))
```

class	error	treat1	treat2	treat
1	3.6633624	0	1	0
2	-0.1891486	0	1	0
3	6.0150457	0	1	1
4	7.3490101	1	0	0
1	0.6684515	0	1	0
2	2.5991362	0	1	0

## I drop unneeded variables and generate a dummy variable for each classroom

```
scores<->%dplyr::select(class,error,treat)
scores <- fastDummies::dummy_cols(scores, select_columns = "class")
knitr::kable(head(scores))</pre>
```

class	error	treat	class_1	class_2	class_3	class_4
1	3.6633624	0	1	0	0	0
2	-0.1891486	0	0	1	0	0
3	6.0150457	1	0	0	1	0
4	7.3490101	0	0	0	0	1
1	0.6684515	0	1	0	0	0
2	2.5991362	0	0	1	0	0

#### Finally! I simulate the DGP (Data Generating Process):

- -The true treatment effect = 15
- -students in classrooms 1 and 2 have higher reading scores
- -students in classrooms 3 and 4 have lower reading scores.

```
scores$score<-80+15*scores$treat+10*scores$class_2+-30*scores$class_3+
-35*scores$class_4+scores$error</pre>
```

knitr::kable(head(scores))

class	error	treat	class_1	class_2	class_3	class_4	score
1	3.6633624	0	1	0	0	0	83.66336
2	-0.1891486	0	0	1	0	0	89.81085
3	6.0150457	1	0	0	1	0	71.01505
4	7.3490101	0	0	0	0	1	52.34901
1	0.6684515	0	1	0	0	0	80.66845
2	2.5991362	0	0	1	0	0	92.59914

I estimate three specifications. The first:

$$\mathit{Score}_{\mathit{ci}} = \beta_0 + \beta_1 \mathit{Treat}_{\mathit{ci}} + \epsilon$$

nofe<-felm(score~treat,scores)</pre>

The second:

$$\textit{Score}_{ci} = \beta_0 + \beta_1 \textit{Treat}_{ci} + \beta_2 \textit{Class}2_c + \beta_3 \textit{Class}3_c + \beta_4 \textit{Class}4_c + \epsilon$$

dummies<-felm(score~treat+class\_2+class\_3+class\_4, scores)</pre>

The third:

$$Score_{ci} = \beta_0 + \beta_1 Treat_{ci} + \kappa_c + \epsilon$$

where  $\kappa_i$  is a classroom fixed effect.

fe<-felm(score~treat|class,scores)</pre>

stargazer(nofe, dummies, fe,header=FALSE, type='latex')

Table 8

		Dependent variable:				
		score				
	(1)	(2)	(3)			
treat	2.280 (5.822)	15.998*** (1.771)	15.998*** (1.771)			
class_2		11.678*** (2.275)				
class_3		-27.916*** (2.435)				
class_4		-32.332*** (2.376)				
Constant	72.557*** (3.905)	78.527*** (1.643)				
Observations R <sup>2</sup> Adjusted R <sup>2</sup> Residual Std. Error	40 0.004 -0.022 18.318 (df = 38)	40 0.930 0.922 5.072 (df = 35)	40 0.930 0.922 5.072 (df = 35)			
Note:		*p<0.1; **p	<0.05; ***p<0.01			

Recall:  $\beta_1=15$  (the true treatment effect)  $\Rightarrow \hat{\beta}_1^{nofe}$  is very biased!

#### Why?

-The classes are an important omitted variable: cor(Score, Class3/4) < 0 and cor(Treat, Class3/4) > 0 creating substantial downward bias.

We can correct for this in two (equivalent) ways:

- -adding the dummy variables for the class to the regression,
- -adding a class fixed effect.

Either approach returns an identical unbiased estimate such that  $E[\hat{\beta}_1] = \beta_1$ .

Fixed effect estimates are also known as the **within estimator**, because it identifies  $\beta$  using within-unit variation.

 $\Rightarrow$  we only using the variation that exists **within the classroom** to estimate the treatment effect.

This is the equivalent of "correcting" our data by demeaning each observation using it's classroom mean, so that the corrected data represents deviations from the classroom mean.

Our fixed effect estimation is

$$y_{ci} = \beta_1 x_{ci} + \kappa_c + \epsilon_{ci}$$

For each class, the average across the students is

$$\bar{y}_i = \beta_1 \bar{x}_i + \kappa_c + \bar{\epsilon}_i$$

Subtracting this from the fixed effect model gives

$$y_{ic} - \bar{y}_i = \beta_1(x_{ic} - \bar{x}_i) + (\epsilon_{ic} - \bar{\epsilon}_i)$$

1. Calculate the mean score, and the mean treatment, in each classroom

```
#getting the mean score in each classroom
cl_mean<-scores %>%
group_by(class) %>%
dplyr::summarize(Classmean = mean(score, na.rm=TRUE), treatmean=mean(treat, na.rm=TRUE))

## `summarise()` ungrouping output (override with `.groups` argument)
knitr::kable(head(cl mean))
```

class	Classmean	treatmean		
1	81.72638	0.2		
2	95.00442	0.3		
3	61.80916	0.7		
4	55.79295	0.6		

#### 2. Merging the means into full data

```
scores<-left_join(scores, cl_mean, by = "class")</pre>
```

knitr::kable(head(scores))

class	error	treat	class_1	class_2	class_3	class_4	score	Classmean	treatmean
1	3.6633624	0	1	0	0	0	83.66336	81.72638	0.2
2	-0.1891486	0	0	1	0	0	89.81085	95.00442	0.3
3	6.0150457	1	0	0	1	0	71.01505	61.80916	0.7
4	7.3490101	0	0	0	0	1	52.34901	55.79295	0.6
1	0.6684515	0	1	0	0	0	80.66845	81.72638	0.2
2	2.5991362	0	0	1	0	0	92.59914	95.00442	0.3

#### 3. Calculating the demeaned score

```
scores$demeansc<-scores$score-scores$Classmean
scores$demeantrt<-scores$treat-scores$treatmean
```

knitr::kable(head(scores[,c("class","treat","score","Classmean","treatmean","demeansc","demeantrt")]))

class	treat	score	Classmean	treatmean	demeansc	demeantrt
1	0	83.66336	81.72638	0.2	1.936980	-0.2
2	0	89.81085	95.00442	0.3	-5.193566	-0.3
3	1	71.01505	61.80916	0.7	9.205885	0.3
4	0	52.34901	55.79295	0.6	-3.443942	-0.6
1	0	80.66845	81.72638	0.2	-1.057931	-0.2
2	0	92.59914	95.00442	0.3	-2.405281	-0.3

#### 4. Running the basic regression on the demeaned scores

```
regdemean<-felm(demeansc-demeantrt, scores)
stargazer( fe, regdemean,header=FALSE, type='latex', omit.stat=c("all" ))</pre>
```

Table 12

	Depende	Dependent variable:			
	score	demeansc			
	(1)	(2)			
treat	15.998*** (1.771)				
demeantrt		15.998*** (1.700)			
Constant		0.000 (0.770)			
Note:	*p<0.1; **p<0.05; ***p<0.01				

Careful: the standard errors on the demeaned regression are incorrect because the cases are not independent of each other.

#### Variation

What would happen if none of the students in classes 1 and 2 went to the small reading group and all of the students in class 3 and 4 did?

#### Variation

#### Creating a new treatment variable to reflect this:

```
scores$treat2[scores$class%in%c(1,2)]<-0
scores$treat2[scores$class%in%c(3,4)]<-1
scores$score2<-80+15*scores$treat2+10*scores$class_2+ -30*scores$class_3+ -35*scores$class_4+scores$error
nofe2<-felm(score2-treat2, scores)
dummies2<-felm(score2-treat2+class_2+class_3+class_4, scores)

## Warning in chol.default(mat, pivot = TRUE, tol = tol): the matrix is either
## rank-deficient or indefinite
## rank-deficient or indefinite
## fe2<-felm(score2-treat2)class, scores)</pre>
```

#### Variation

stargazer(nofe2, dummies2, header=FALSE, type='latex')

Table 13

	Dependent variable: score2	
	(1)	(2)
treat2	-20.564*** (2.118)	-16.933*** (2.247)
class_2		11.778*** (2.247)
class_3		4.516* (2.247)
class_4		
Constant	84.615*** (1.497)	78.726*** (1.589)
Observations	40	40
$R^2$	0.713	0.847
Adjusted R <sup>2</sup>	0.705	0.834
Residual Std. Error	6.697 (df = 38)	5.024 (df = 36)
Note:	*p<0.1; **p<0.05; ***p<0.01	

## Example: Crime and Unemployment

You are interested in the relationship between unemployment and crime.

You have data on the crime and unemployment rates for 46 cities for 1982 and 1987.

I start by using the data from the 1987 cross section and run the following simple regression of the crime rate on unemployment,

$$crimerate_i = \beta_0 + \beta_1 unemployment_i + \epsilon$$

## Example: Crime and Unemployment

```
#install.packages("wooldridge")
library(wooldridge)
```

## Warning: package 'wooldridge' was built under R version 3.6.3

#note: this dataset comes from the wooldridge textbook. Conveniently there is an R package that #includes all the wooldridge datasets.

```
crime<-data('crime2')
crime<-crime2</pre>
```

## Example: Crime and Unemployment

```
regcrime<-felm(crmrte-unem, crime[crime$year=="87",])
summary(regcrime)</pre>
```

```
##
## Call:
     felm(formula = crmrte ~ unem, data = crime[crime$year == "87",
##
## Residuals:
     Min 1Q Median
                          30
                                Max
## -57 55 -27 01 -10 56 18 01 79 75
##
## Coefficients:
              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 128.378
                          20.757 6.185 1.8e-07 ***
              -4.161 3.416 -1.218
## unem
                                            0.23
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 34.6 on 44 degrees of freedom
## Multiple R-squared(full model): 0.03262 Adjusted R-squared: 0.01063
## Multiple R-squared(proj model): 0.03262 Adjusted R-squared: 0.01063
## F-statistic(full model):1.483 on 1 and 44 DF, p-value: 0.2297
## F-statistic(proj model): 1.483 on 1 and 44 DF, p-value: 0.2297
```

Wierd.

The culprit? Probably omitted variables.

Reflex: lets add controls for more observable city characteristics: the area of the city, if the city is in the west, police officers per square mile, expenditure on law enforcement, per capita income...

 $\textit{crmrte}_i = \beta_0 + \beta_1 \textit{unemp}_i + \beta_2 \textit{area}_i + \beta_3 \textit{west}_i + \beta_4 \textit{offarea}_i + \beta_5 \textit{lawexp}_i + \beta_6 \textit{pcinc}_i + \epsilon$ 

```
regcrime2<-felm(crmrte-unem+area+west+offarea+lawexpc+pcinc, crime[crime$year=="87",])
summary(regcrime2)</pre>
```

```
##
## Call:
##
     felm(formula = crmrte ~ unem + area + west + offarea + lawexpc + pcinc, data = crime[crime$vea
##
## Residuals:
##
      Min
              10 Median
                              30
                                    Max
## -50.847 -21.511 -6.829 18.940 75.114
##
## Coefficients:
##
              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 140.06017 51.16000 2.738 0.00927 **
## unem
            -6.70024 3.71634 -1.803 0.07913 .
## area
            0.05867 0.04757 1.233 0.22491
## west -21.96336 12.27535 -1.789 0.08135 .
## offarea -0.11442 0.66876 -0.171 0.86504
## lawexpc 0.02137 0.01859 1.149 0.25736
## pcinc
              -0.00185 0.00352 -0.526 0.60215
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 34.27 on 39 degrees of freedom
## Multiple R-squared(full model): 0.1587 Adjusted R-squared: 0.02932
## Multiple R-squared(proj model): 0.1587 Adjusted R-squared: 0.02932
## F-statistic(full model):1.227 on 6 and 39 DF, p-value: 0.3138
## F-statistic(proj model): 1.227 on 6 and 39 DF, p-value: 0.3138
```

Even wierder.

So many potential omitted variables. . .

What if we can **capture all unobserved, time invariant factors** about a city that might affect crime rates?

Use data for 1987 and 1982, and add city **fixed effects**,( $\alpha_i$ ).

$$\begin{split} \textit{crmrte}_{\textit{it}} = & \beta_0 + \beta_1 \textit{unemp}_{\textit{it}} \\ & + \beta_2 \textit{area}_{\textit{i}} + \beta_3 \textit{west}_{\textit{i}} + \beta_4 \textit{offarea}_{\textit{it}} + \beta_5 \textit{lawexp}_{\textit{it}} + \beta_6 \textit{pcinc}_{\textit{it}} \\ & + \alpha_{\textit{i}} + \epsilon \end{split}$$

```
#note: the data does not have a unique city identifier. I am assuming the area of the city is
#1) time-invariant and
#2) uniquely identifies the 46 cities.
#The line of code below generates a unique identifier
crime <- transform(crime,city=as.numeric(factor(area)))
#I check that my assumptions were correct by seeing if I have 2 observations for 46 cities.
table(crime$city)</pre>
```

regcrime3<-felm(crmrte~unem+area+west+offarea+lawexpc+pcinc|city, crime)

```
## Warning in chol.default(mat. pivot = TRUE, tol = tol): the matrix is either
## rank-deficient or indefinite
summary(regcrime3)
## Warning in chol.default(mat, pivot = TRUE, tol = tol): the matrix is either
## rank-deficient or indefinite
##
## Call:
     felm(formula = crmrte ~ unem + area + west + offarea + lawexpc + pcinc | city, data = crime)
##
##
## Residuals:
     Min
             10 Median 30
                               Max
## -27.36 -6.85 0.00 6.85 27.36
##
## Coefficients:
         Estimate Std. Error t value Pr(>|t|)
##
## unem 1.491297 0.795972 1.874 0.0680 .
## area
                NΑ
                           NΑ
                                  NΑ
                                           NΑ
## west
                NΑ
                           NA
                                  NA
                                           NΑ
## offarea 1.348882 1.805672 0.747 0.4592
## lawexpc -0.005076 0.013915 -0.365 0.7171
## pcinc 0.003821 0.001644 2.324 0.0251 *
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 14.66 on 42 degrees of freedom
## Multiple R-squared(full model): 0.8887 Adjusted R-squared: 0.7588
## Multiple R-squared(proj model): 0.18 Adjusted R-squared: -0.7767
## F-statistic(full model):6.843 on 49 and 42 DF, p-value: 1.888e-09
## F-statistic(proj model): 1.537 on 6 and 42 DF, p-value: 0.19
```

Interpret the coefficient on unemployment.

Why were we not able to estimate a coefficient for area; and west;?

Suppose I am concerned about how national factors could be affecting all cities simultaneously.

Add a year fixed effect,  $(\tau_t)$ .

$$\begin{split} \textit{crmrte}_{\textit{it}} = & \beta_0 + \beta_1 \textit{unemp}_{\textit{it}} \\ & + \beta_2 \textit{area}_{\textit{i}} + \beta_3 \textit{west}_{\textit{i}} + \beta_4 \textit{offarea}_{\textit{it}} + \beta_5 \textit{lawexp}_{\textit{it}} + \beta_6 \textit{pcinc}_{\textit{it}} \\ & + \alpha_{\textit{i}} + \tau_t + \epsilon \end{split}$$

regcrime4<-felm(crmrte-unem+area+west+offarea+lawexpc+pcinc|city+year, crime)

```
## Warning in chol.default(mat. pivot = TRUE, tol = tol): the matrix is either
## rank-deficient or indefinite
summary(regcrime4)
## Warning in chol.default(mat, pivot = TRUE, tol = tol): the matrix is either
## rank-deficient or indefinite
##
## Call:
     felm(formula = crmrte ~ unem + area + west + offarea + lawexpc + pcinc | city + year, data = c
##
##
## Residuals:
##
      Min
             10 Median
                              30
                                     Max
## -23.641 -7.441 0.000 7.441 23.641
##
## Coefficients:
         Estimate Std. Error t value Pr(>|t|)
##
## unem 2.931904 1.133562 2.586 0.0133 *
## area
                 NΑ
                           NΑ
                                   NΑ
                                           NΑ
## west
                 NΑ
                           NΑ
                                   NA
                                           NΑ
## offarea 1.838022 1.785312 1.030 0.3093
## lawexpc -0.006982 0.013632 -0.512 0.6113
## pcinc -0.005697 0.005683 -1.002 0.3220
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 14.31 on 41 degrees of freedom
## Multiple R-squared(full model): 0.8964 Adjusted R-squared: 0.77
## Multiple R-squared(proj model): 0.1709 Adjusted R-squared: -0.8402
## F-statistic(full model):7.094 on 50 and 41 DF, p-value: 1.405e-09
## F-statistic(proj model): 1.408 on 6 and 41 DF, p-value: 0.2347
```

Is this estimate causal?

What we have controlled for?

- -The city fixed effects: controls for any time invariant factors that always affect the crime rates in a city in a similar way. Such as...
- -The time fixed effects: controls for any patterns that are common to all cities in a given year. Such as...
- -In addition to this we are also controlling for some observable time varying variables: officers in an area, law enforcement expenditures per capita and income per capita.

So are these estimates causal?

What kind of omitted variables should we still be concerned about?

Any variable that changes within a city across years and that is correlated with both unemployment and crime rates could still be biasing our results.

This could be things like school funding, decriminalization of marijuana, housing costs. . . to name just a few.

# Example: Crime and Unemployment Variation

Suppose I get ambitious and want to control for all these factors as well.

I decide I am going to generate a city-by-year fixed effect,  $(\gamma_{it})$ , to control for these time variant omitted variables. I estimate,

$$\begin{aligned} \textit{crmrte}_{\textit{it}} = & \beta_0 + \beta_1 \textit{unemp}_{\textit{it}} \\ & + \beta_2 \textit{area}_{\textit{i}} + \beta_3 \textit{west}_{\textit{i}} + \beta_4 \textit{offarea}_{\textit{it}} + \beta_5 \textit{lawexp}_{\textit{it}} + \beta_6 \textit{pcinc}_{\textit{it}} \\ & + \gamma_{\textit{it}} + \epsilon \end{aligned}$$

## Example: Crime and Unemployment Variation

```
crime$city_year<-paste(crime$city, crime$year, sep="_")
#Note: the following regression will not run!
#regcrime5<-felm(crmrte-unem+area+west+offarea+lawexpc+pcinc/city_year, crime)
#summary(regcrime5)</pre>
```

### Why?!?

What kind of data would I need to be able to estimate this?