Two-Phase Flow Simulation with Level-Set Method

Objective

- Simulate incompressible flow of two immiscible fluids.
- Use **level-set method** to track the interface.
- Variable density and viscosity based on level-set function $l(\boldsymbol{x}, t)$.
- Solved with finite elements in **Firedrake**.

Governing Equations

Navier-Stokes with variable coefficients:

$$\rho(\ell)\left(\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v}\cdot\nabla)\boldsymbol{v}\right) = \nabla\cdot\mathbb{T} + \rho(\ell)\boldsymbol{g} + \boldsymbol{f_{sf}}$$

$$\mathbb{T} = -p\mathbb{I} + 2\mu(\ell)(\nabla\boldsymbol{v} + \nabla\boldsymbol{v}^T)$$

$$\rho(\ell) = \rho_1 H(\ell) + \rho_2(1 - H(\ell))$$

$$\mu(\ell) = \mu_1 H(\ell) + \mu_2(1 - H(\ell))$$

Level-set advection:

$$\frac{\partial \ell}{\partial t} + \boldsymbol{v}\cdot\nabla\ell = 0$$

$H(\ell)$ is a Heaviside function

Interface: $\ell = 0$

Finite Element Method and Solver

- **Velocity**
  - Continuous Galerkin, Polynomial degree 2
- **Pressure**
  - Continuous Galerkin, Polynomial degree 1
- **Level–set function**
  - Continuous Galerkin, Polynomial degree 2
- **Degrees of freedom:** 1 853 162
- **Solver for non–homogenous Navier–Stokes**
  - Non–linear system: Newton's method with the SNES solver from the PETSc library.
  - Linear system: Flexible GMRES method, preconditioned by a field split Schur complement approach with Hypre BoomerAMG on velocity block and SOR smoothing on the pressure block.