LES Simulation around a Torus

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Objective and Simulation Setup

- Simulate 3D airflow around a torus using Large Eddy Simulation (LES)
- Subgrid-scale turbulence modeled by WALE (Wall-Adapting Local Eddy-viscosity)
- Domain: Torus in cylindrical tunnel
- Inflow velocity: 5 m/s
- Diameter of torus: 1 m
- Reynolds number: $\sim 10^6$
- FEM framework: Firedrake (mixed space, $\sim 10^6$ DOF)

Mathematical Model

Incompressible Navier–Stokes equations:

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right) - \nabla\cdot\mathbb{T}(\mathbf{v},p) = 0$$

$$\nabla\cdot\mathbf{v} = 0$$

Cauchy stress tensor with WALE model:

$$\mathbb{T}(\mathbf{v},p) = -pI + 2(\mu+\mu_t)\mathbb{D}(\mathbf{v})$$

Discretized using Crank-Nicolson scheme ($\theta = 0.5$), adaptive time-stepping, nonlinear variational formulation in Firedrake.

WALE Subgrid-Scale Model

- WALE model for turbulent viscosity:

$$\mu_t = \rho \left(C \frac{\Delta^{1/3}}{K} \right)^2 \cdot \frac{||W_d||^{3/2}}{||S||^{5/2} + ||W_d||^{5/4} + \varepsilon}$$

- **Key idea:** based on local velocity gradient tensor
- μ_t updated in each timestep (operator-splitting iteration)
- Parallel solver configuration (PETSc + HYPRE + GMRES)
- Adaptive timestep based on convergence efficiency