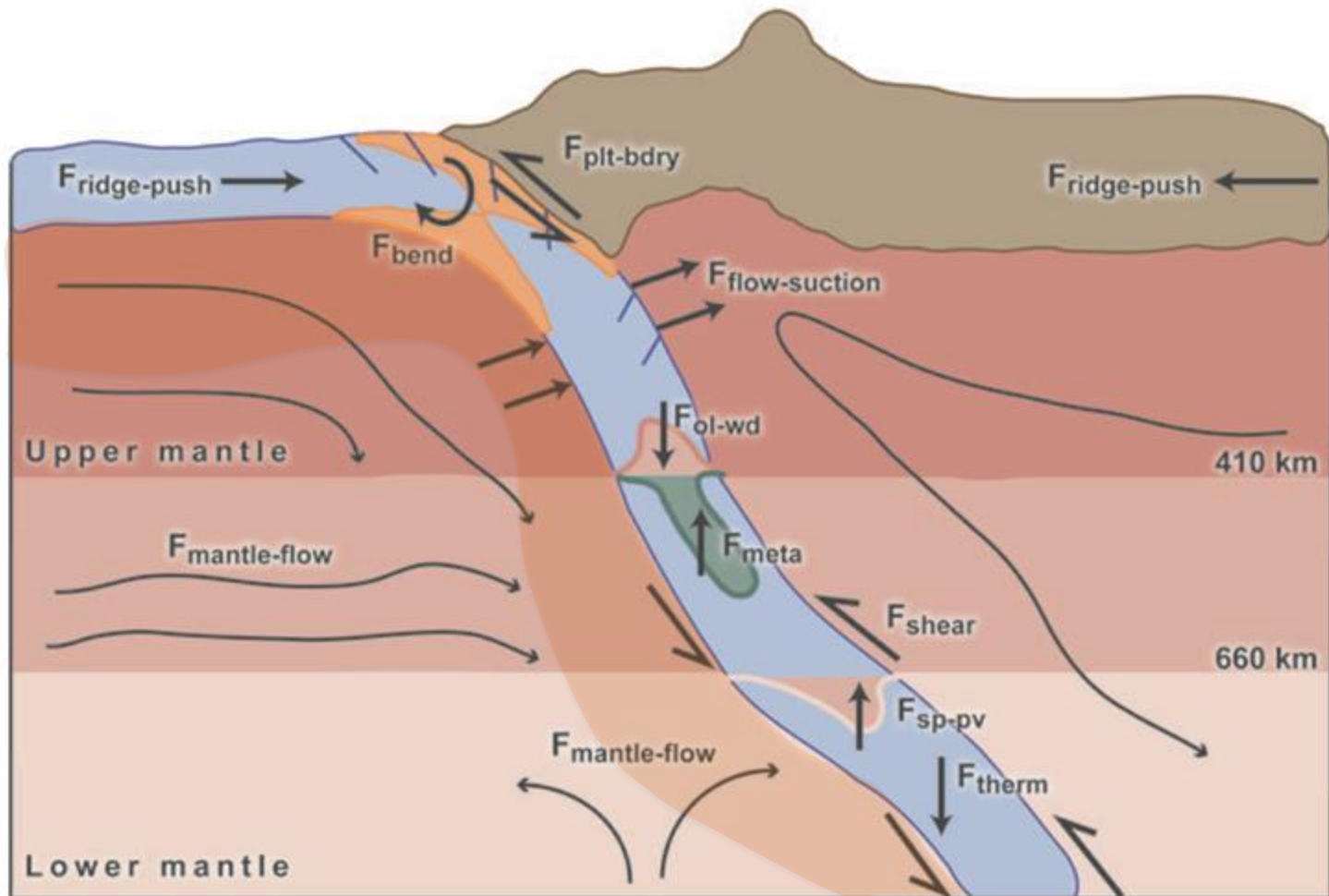
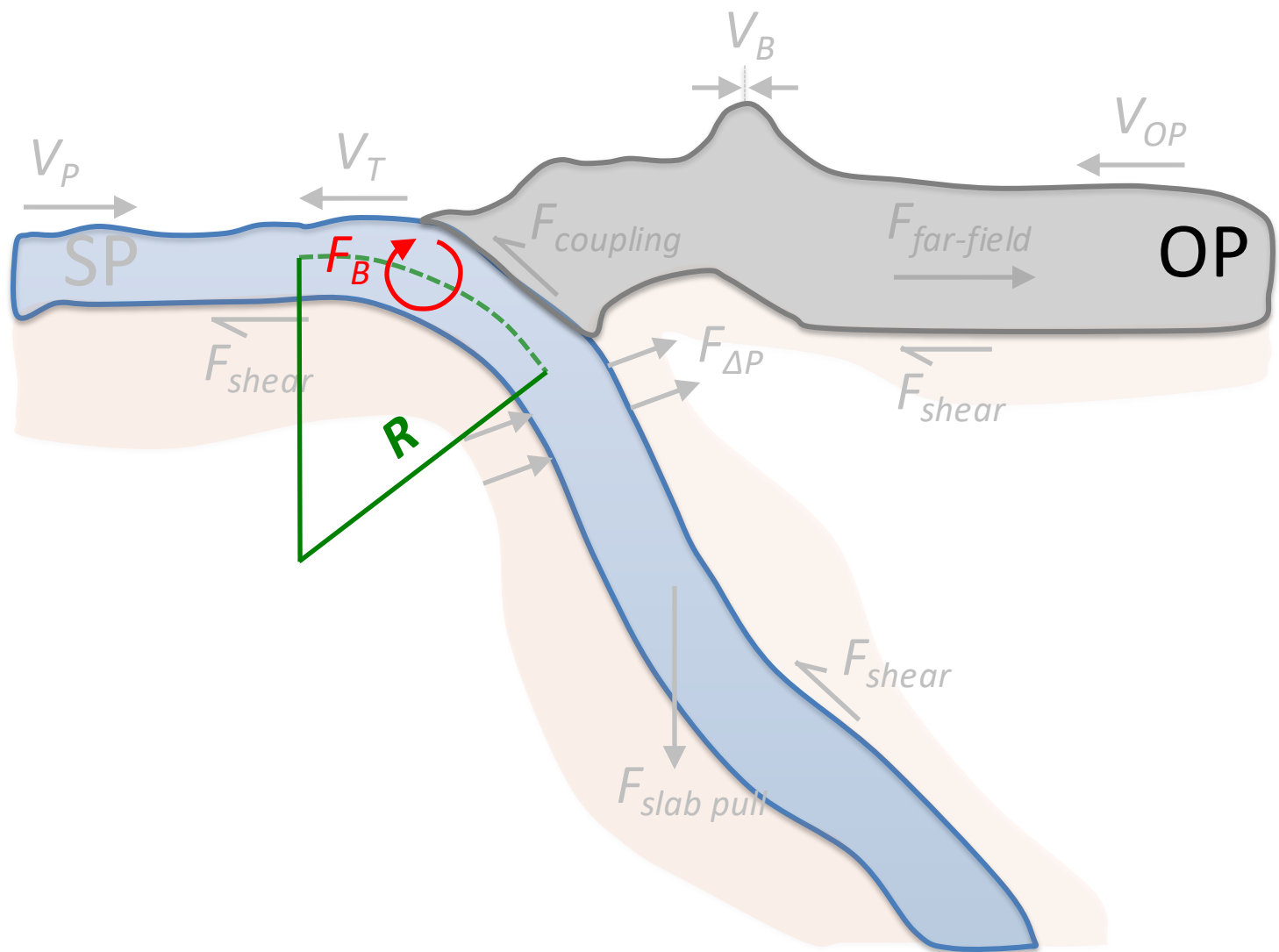


Slab bending



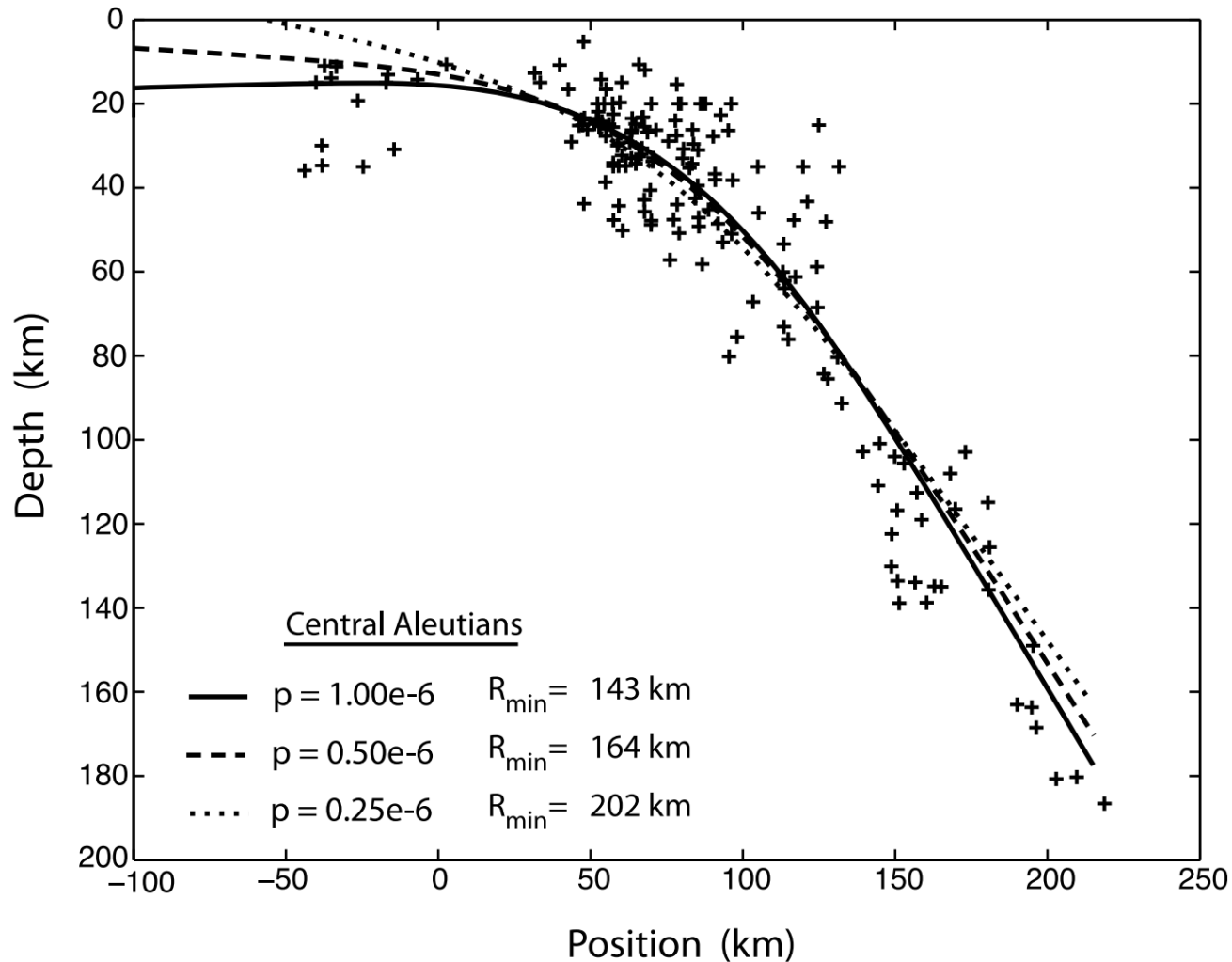
Billen, 2008



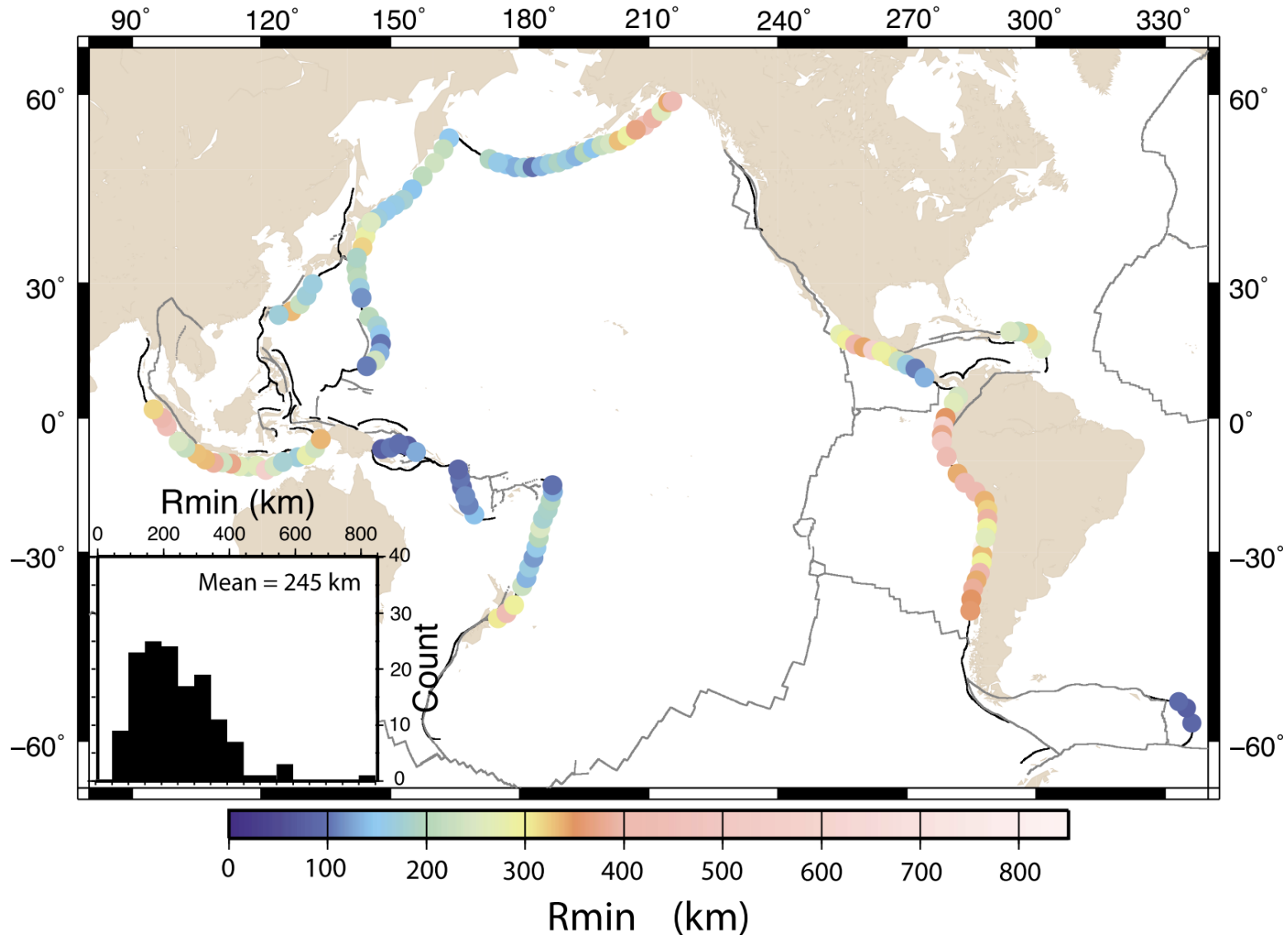
- Radius of curvature
- Viscous bending
Dissipation, force, plate motions, time-dependence.
- More complex viscous rheologies
Visco-plasticity; Elasticity
- Visco-elasto-plastic and comparing with seismicity.

Radius of curvature

Fitting splines (piecewise polynomials) through earthquake locations

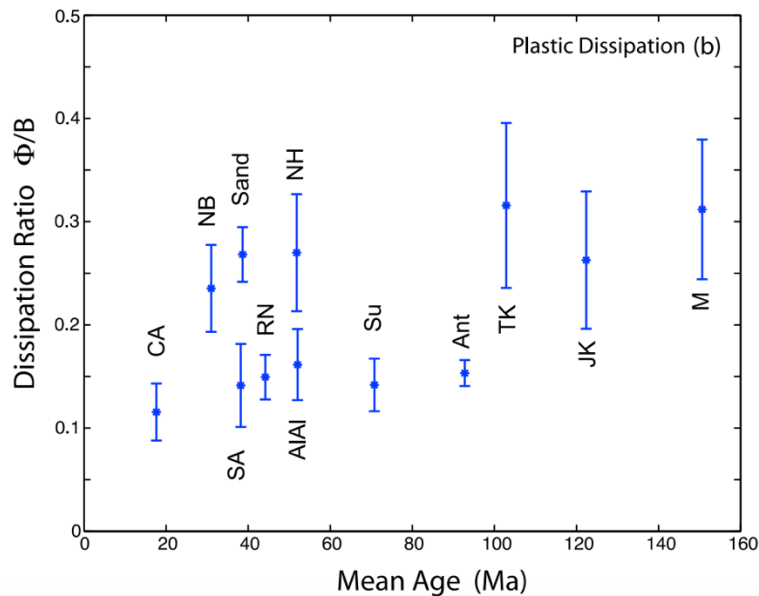
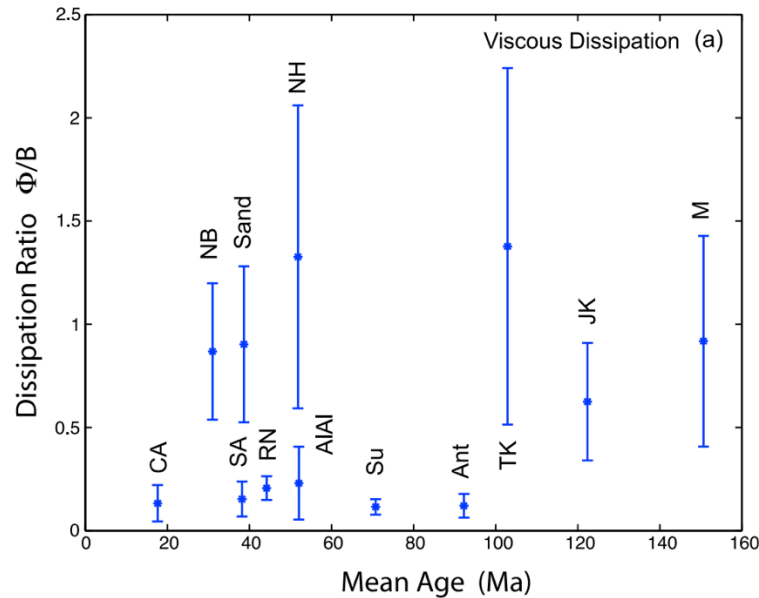


Radius of curvature



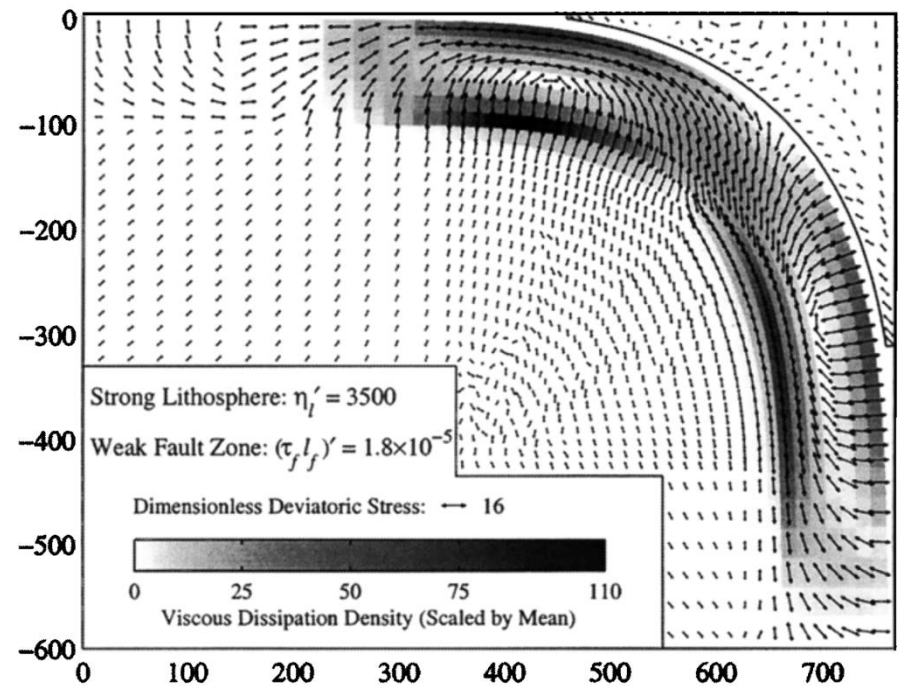
Radius of curvature

Radius of curvature – R – a key control on bending dissipation or forces, irrespective of assumed rheology (more to come...).



The baseline: Viscous bending

- Finite element models to probe the dependence of viscous dissipation (in the bending lithosphere) on subduction zone properties.

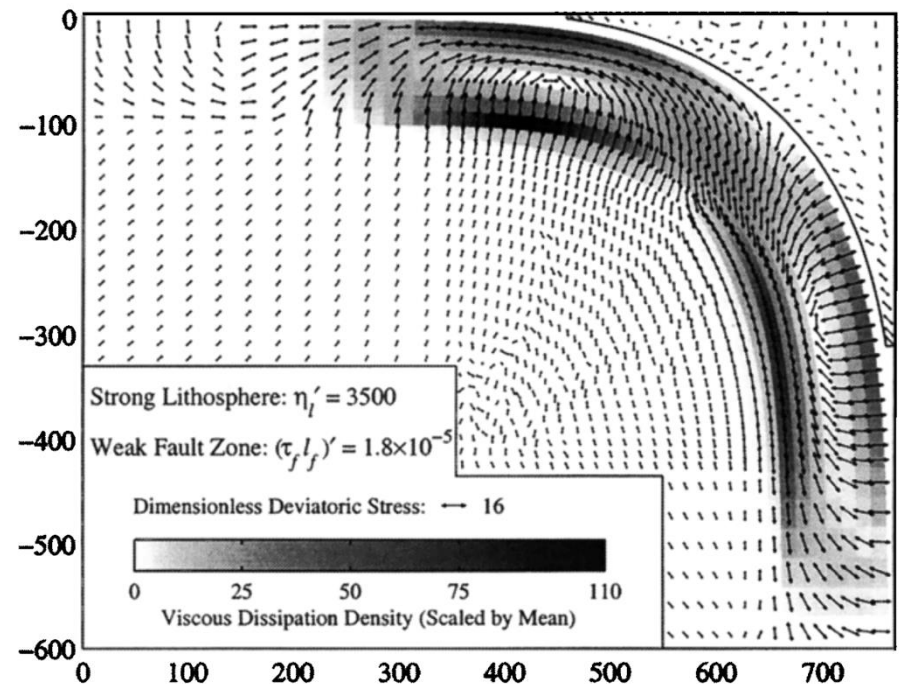


The baseline: Viscous bending

- Finite element models to probe the dependence of viscous dissipation (in the bending lithosphere) on subduction zone properties.
- Dissipation balance assuming a totally viscous system:

$$\Phi^{\text{pe}} = \Phi_m^{\text{vd}} + \Phi_f^{\text{vd}} + \Phi_l^{\text{vd}}$$

(The rate of energy supplied by mass anomalies moving through the gravity field is to equal the work done by stresses in deforming the material.)



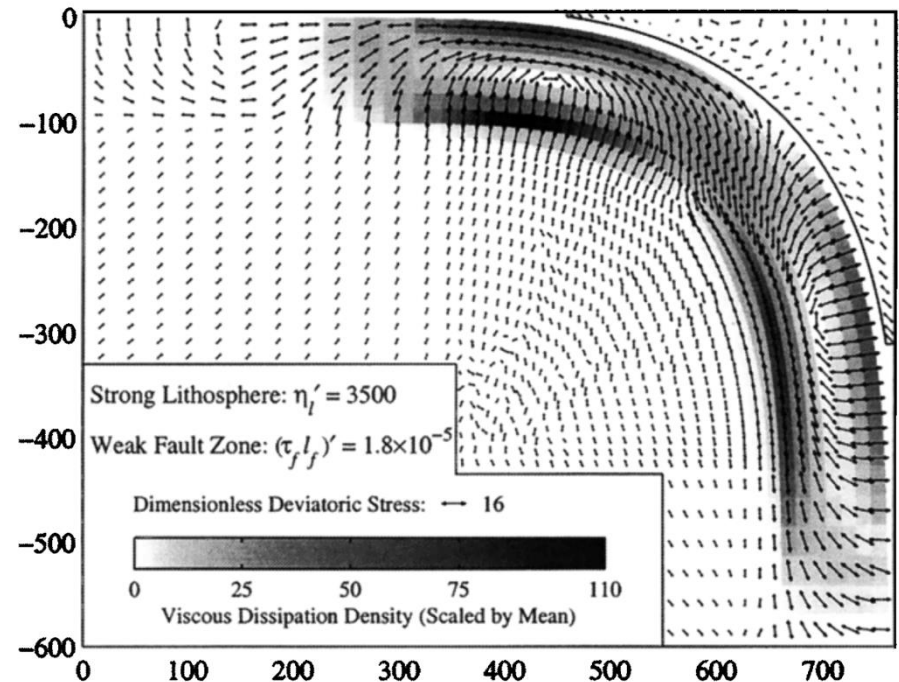
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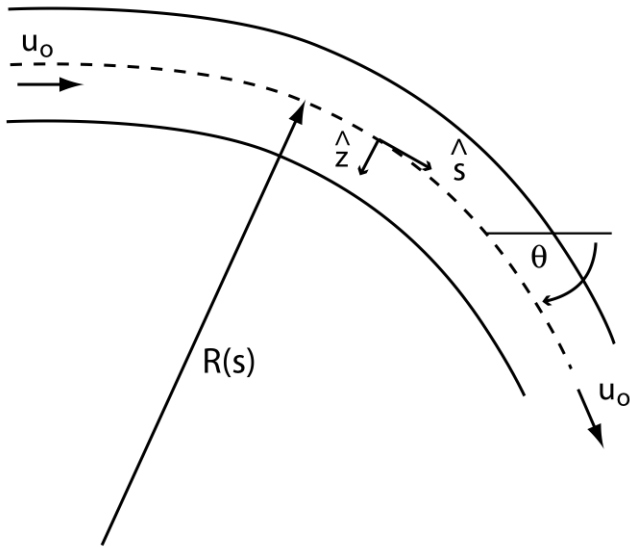
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- Where the ϕ_f^{vd} given by:

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$$\Phi_l^{vd} = C_l v_p^2 \eta_l \left(\frac{h_s}{R} \right)^3$$



The baseline: Viscous bending

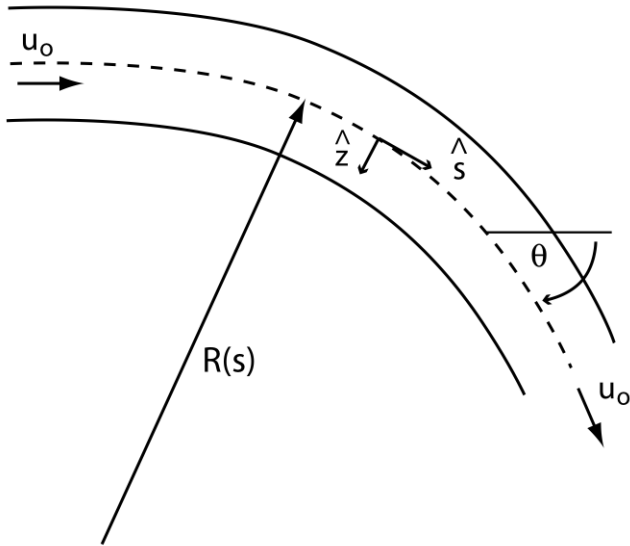


$$N = \int_{-H/2}^{H/2} \tau_{ss} dz \quad M = \int_{-H/2}^{H/2} \tau_{ss} z dz$$

$$M(s) = -\frac{1}{3} \eta u_0 H^3 \left(\frac{dK}{ds} \right) \quad \frac{dK}{ds} \approx \frac{1}{R_{\min}^2}$$

$$\Delta N(0) = -\frac{2}{3} \left(\frac{H}{R_{\min}} \right)^3 \eta u_0$$

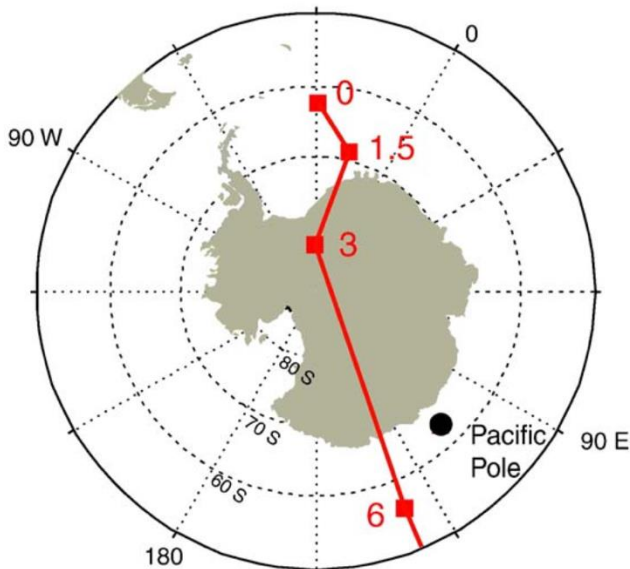
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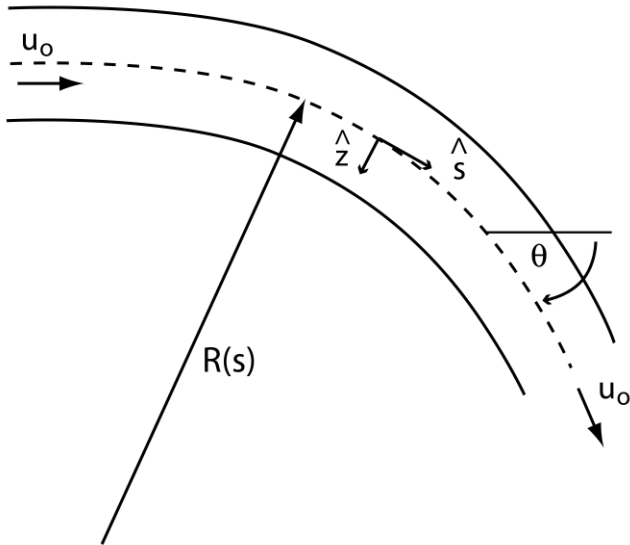
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Proposed to affect plate velocities. Plot shows predicted (red) and observed (black) Euler poles for different slab viscosities (units of 10^{22} Pas).

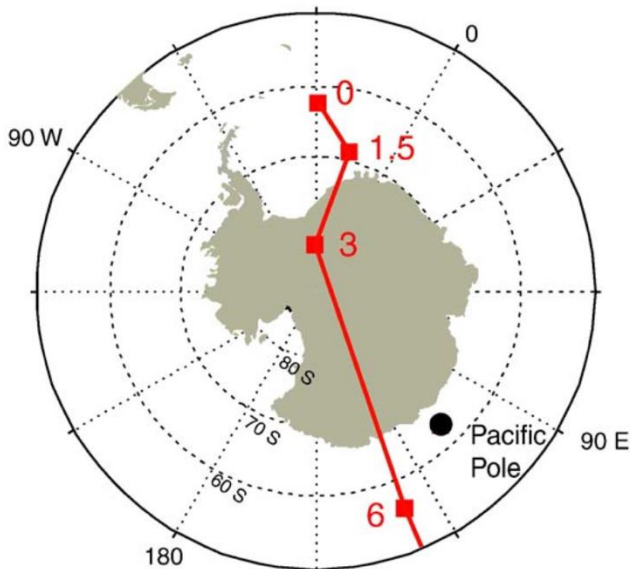
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Proposed to affect plate velocities. Plot shows predicted (red) and observed (black) Euler poles for different slab viscosities (units of 10^{22} Pas). Assumes $R = 200$ km and proposes that bending dissipates $\sim 40\%$ of slab's potential energy.

The baseline: Viscous bending

But not so quick, e.g.:

Geochemistry
Geophysics
Geosystems

G³

AN ELECTRONIC JOURNAL OF THE EARTH SCIENCES

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Article

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Dynamics of plate bending at the trench and slab-plate coupling

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Evolution of the slab bending radius and the bending dissipation in three-dimensional subduction models with a variable slab to upper mantle viscosity ratio

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The baseline: Viscous bending

But not so quick, e.g.:

(Friday's discussion!)

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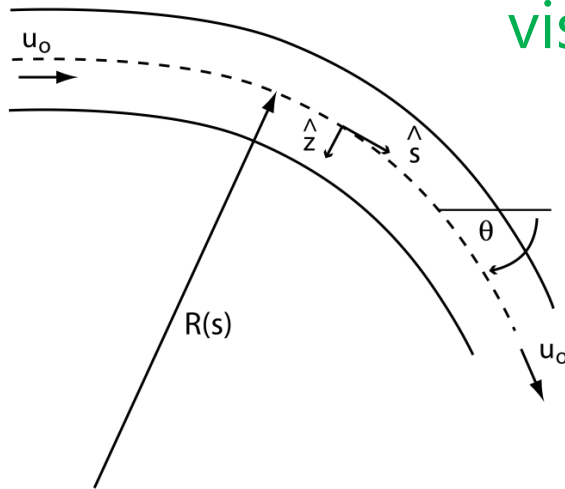
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Adding complexity: Plastic bending



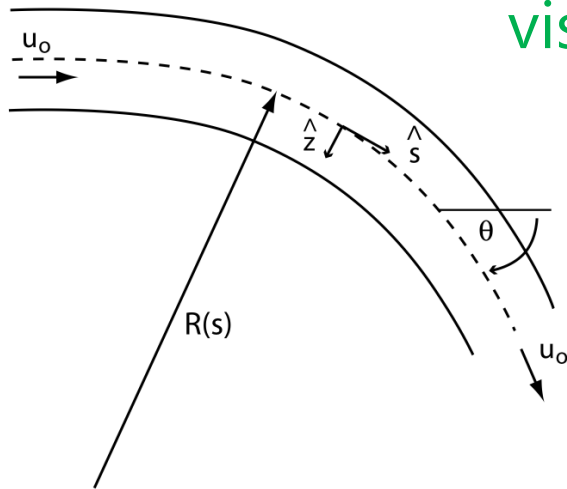
viscous:

$$N = \int_{-H/2}^{H/2} \tau_{ss} dz \quad M = \int_{-H/2}^{H/2} \tau_{ss} z dz$$

$$\Delta N(0) = -\frac{2}{3} \left(\frac{H}{R_{\min}} \right)^3 \eta u_0$$

$$\Phi = \frac{2}{3} \left(\frac{H^3}{R_{\min}^3} \right) \eta u_0^2$$

Adding complexity: Plastic bending



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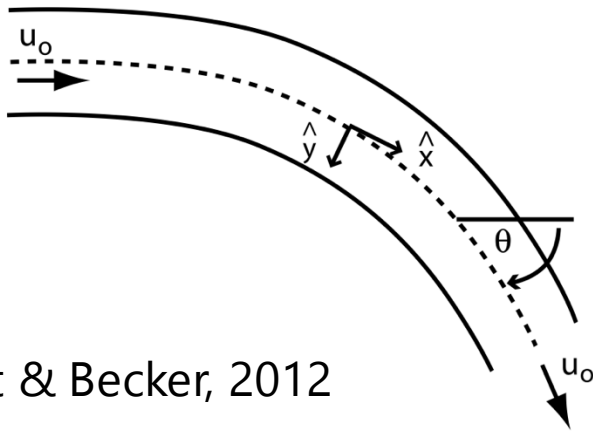
plastic:

$$z < 0: \quad \tau_y(z) = \tau_y(0)(1 + 2z/H)$$

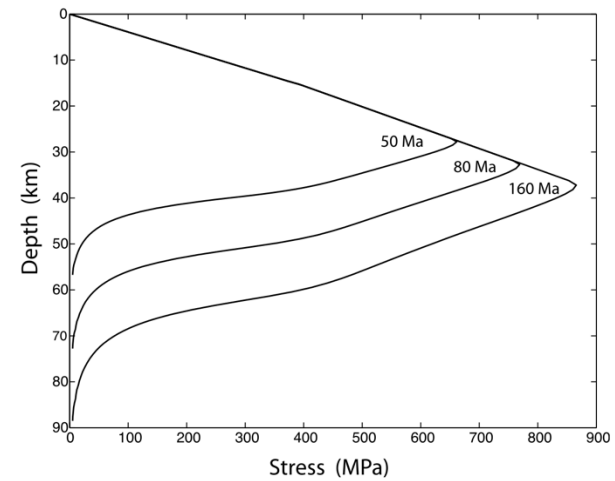
$$z > 0: \quad \tau_y(z) = \tau_y(0)(1 - 2z/H)$$

$$\Phi = \frac{1}{6} \left(\frac{H^2}{R_{\min}} \right) \tau_y(0) u_0 \quad \Delta N(0) = -\frac{1}{6} \left(\frac{H^2}{R_{\min}} \right) \tau_y(0)$$

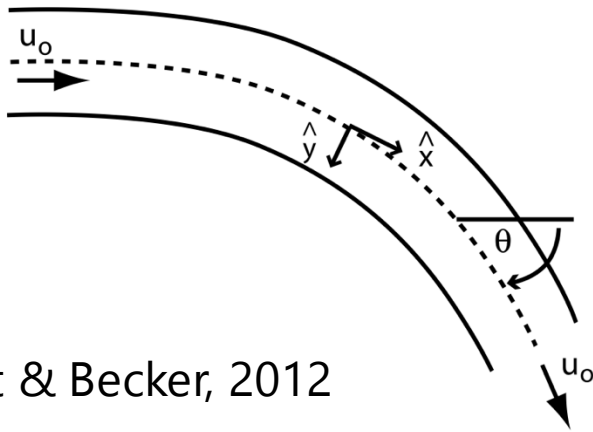
More sophisticated visco-plastic rheologies



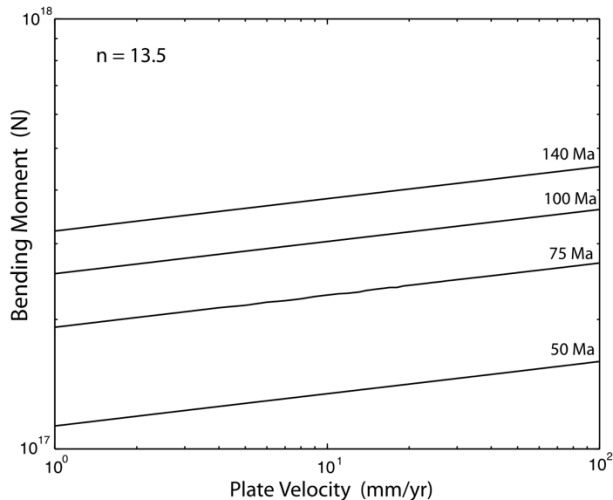
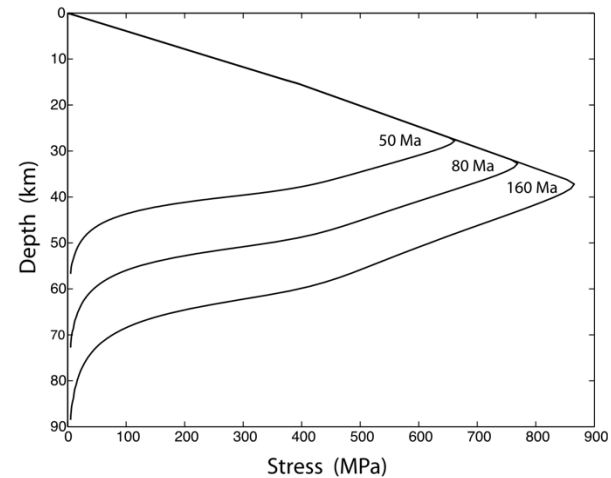
Buffett & Becker, 2012



More sophisticated visco-plastic rheologies



Buffett & Becker, 2012



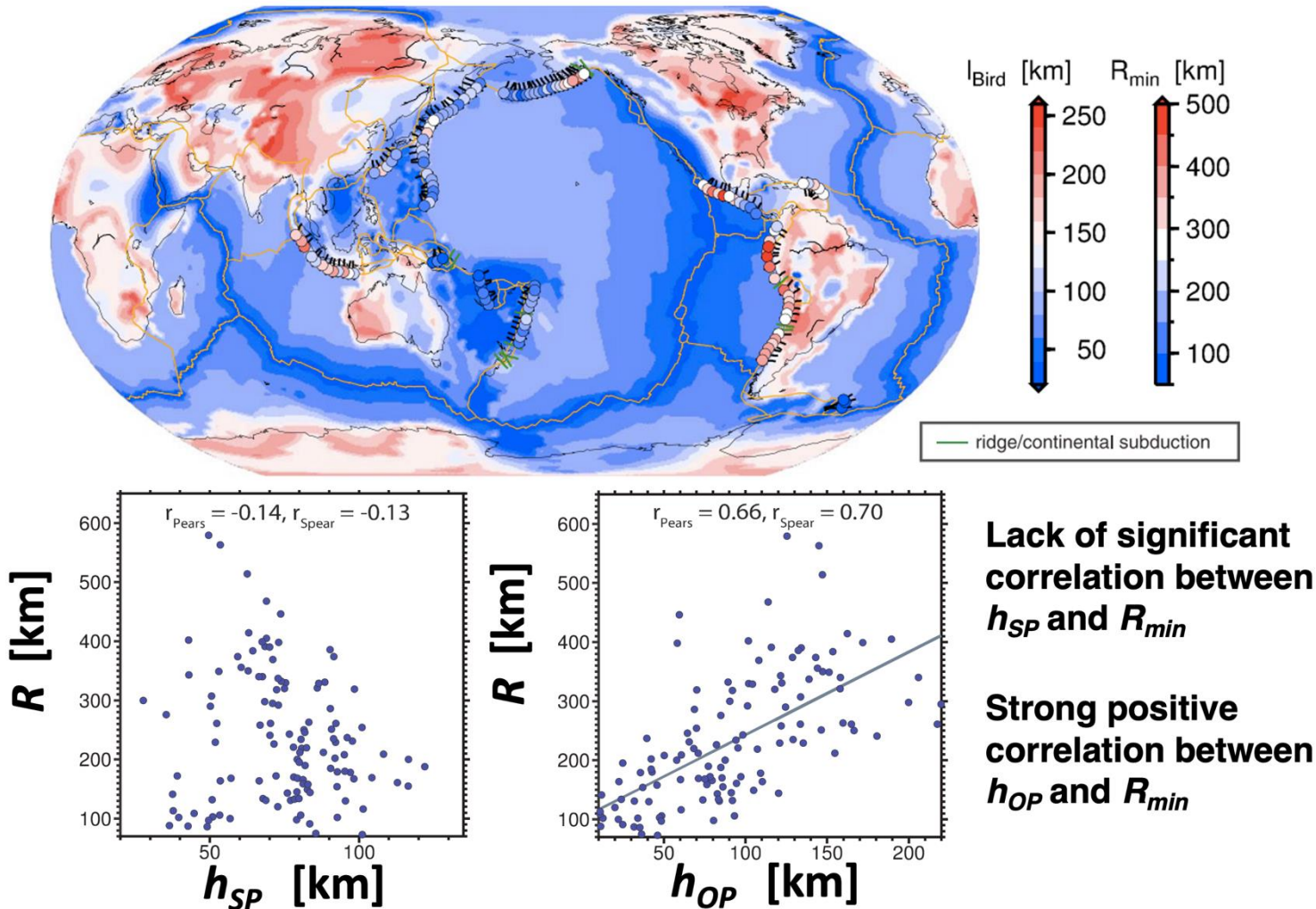
- Both bending moment (M) and bending force (N) relatively insensitive to subduction velocity.
- Moment and force can be approximated by a power-law fluid with $n = 13.5$ (i.e., almost perfectly plastic).

"Stresses associated with bending in a plastic plate cannot balance the torque associated with buoyancy forces. Instead, surface forces on the plate must play an important role: the conditions in the OP should contribute, and possibly control, to the evolution of curvature."

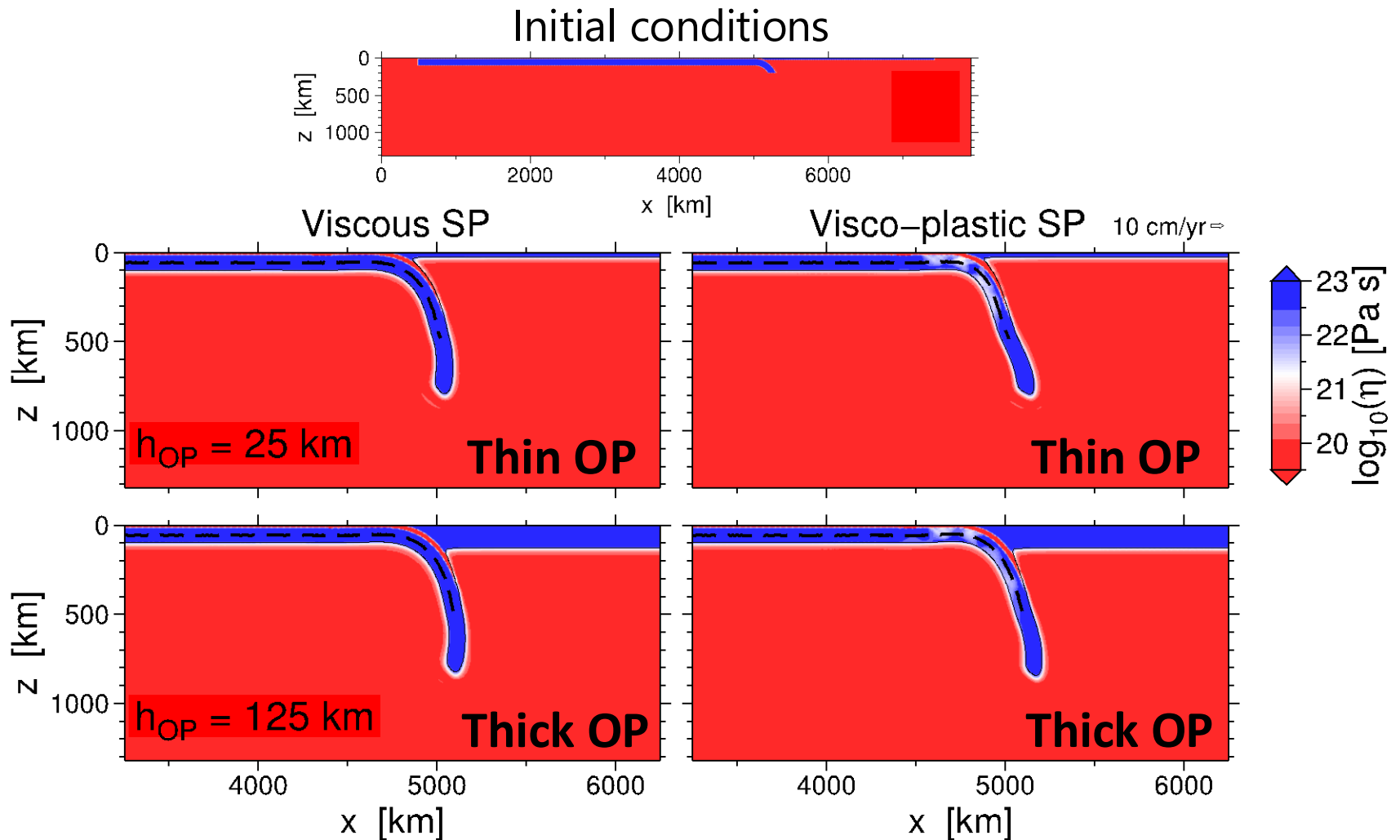
More sophisticated visco-plastic rheologies

R from Benioff zone spline fits (Buffett & Heuret, 2011)

Lithospheric thickness estimated from seismic tomography (Bird et al., 2008)

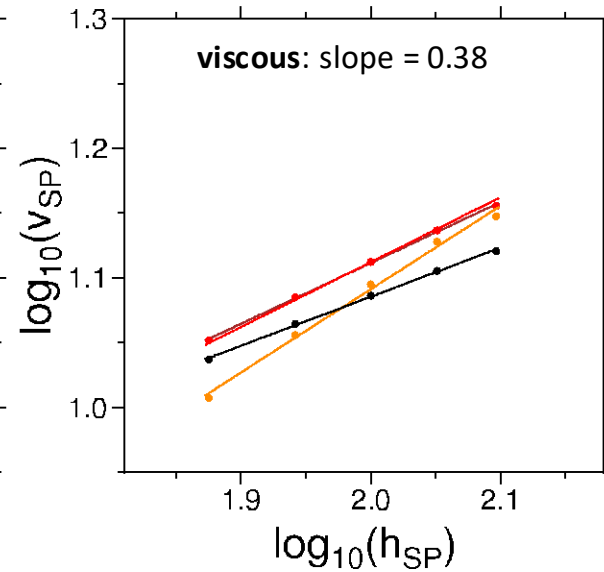
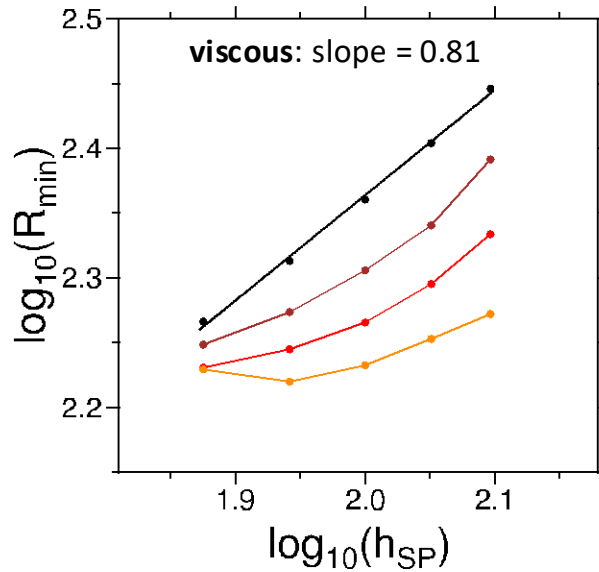


More sophisticated visco-plastic rheologies



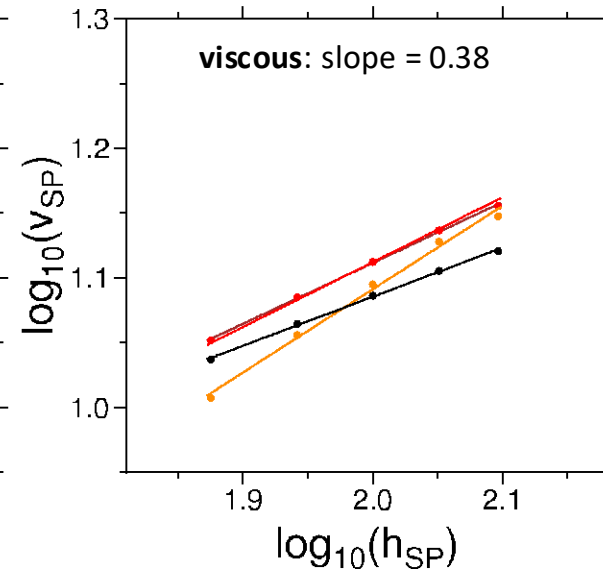
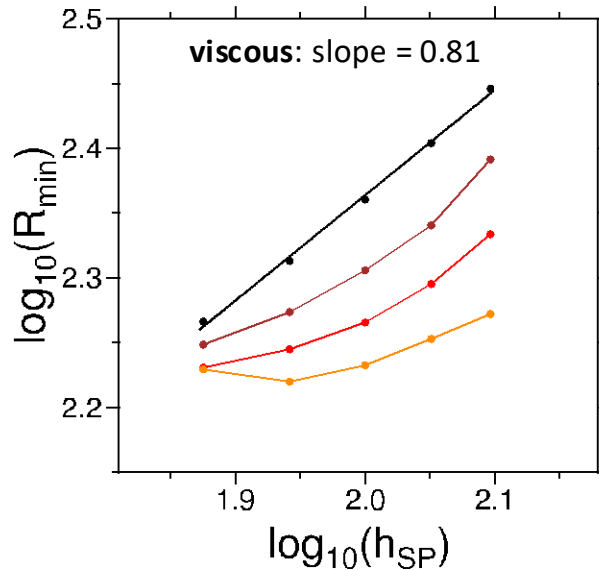
More sophisticated visco-plastic rheologies

Modeled
dependence
on SP
thickness:



More sophisticated visco-plastic rheologies

Modeled
dependence
on SP
thickness:



For viscous
SP, follows
from a simple
scaling:

buoyancy flux \propto bending dissipation

$$\Delta \rho g h_{\text{SP}} D v_{\text{SP}} \propto \left(\frac{h_{\text{SP}}}{R_{\min}} \right)^3 \eta' v_{\text{SP}}^2,$$

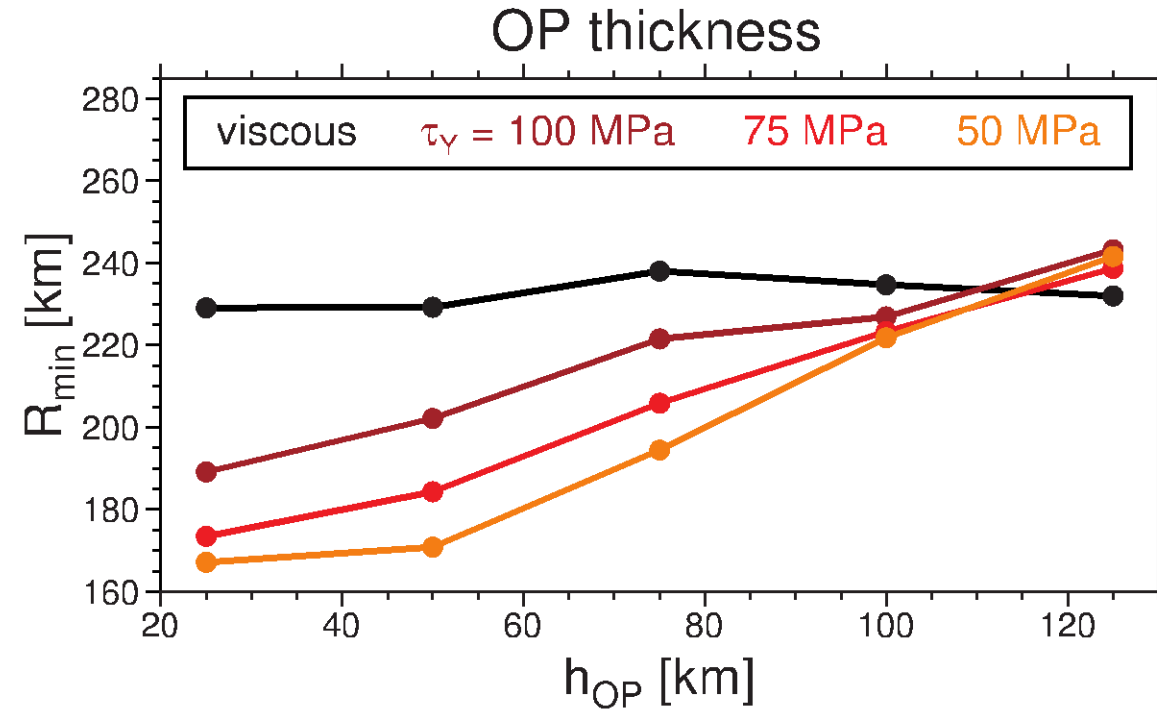
$$R_{\min} \propto h_{\text{SP}}^{2/3} v_{\text{SP}}^{1/3}$$

$$R_{\min} \propto h_{\text{SP}}^{0.79}$$

But no positive correlation on Earth: **Plasticity** reduces this!

More sophisticated visco-plastic rheologies

Now, OP thickness, h_{OP} :



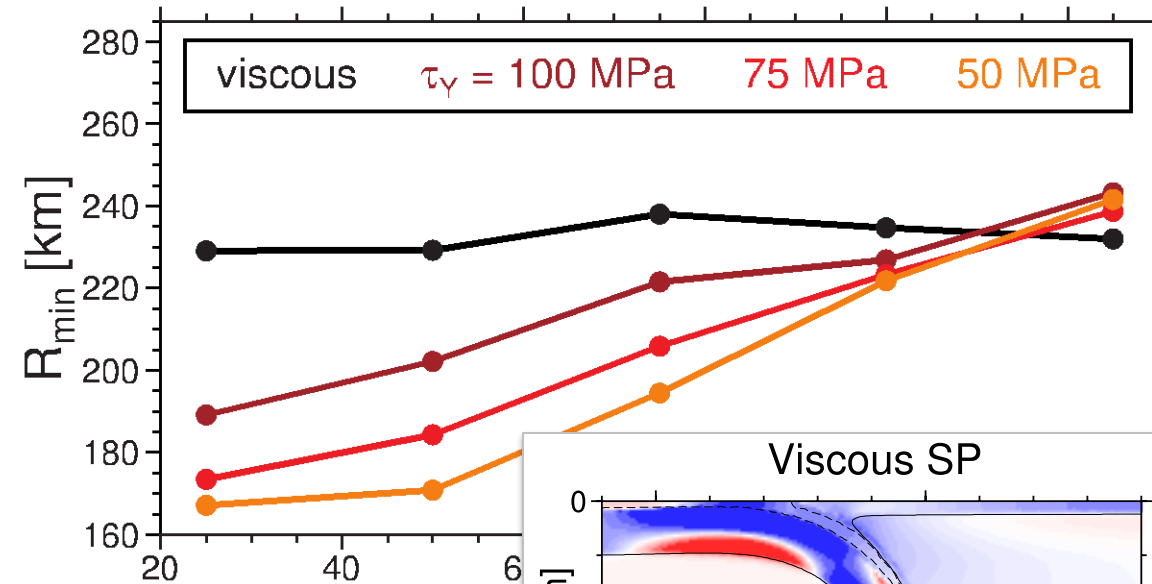
Increasing the degree of SP plasticity, i.e. reducing yield stress:

- Reduces the strength of R dependence on h_{SP} .
- Introduces dependence of R on h_{OP} .

More sophisticated visco-plastic rheologies

Now, OP thickness, h_{OP} :

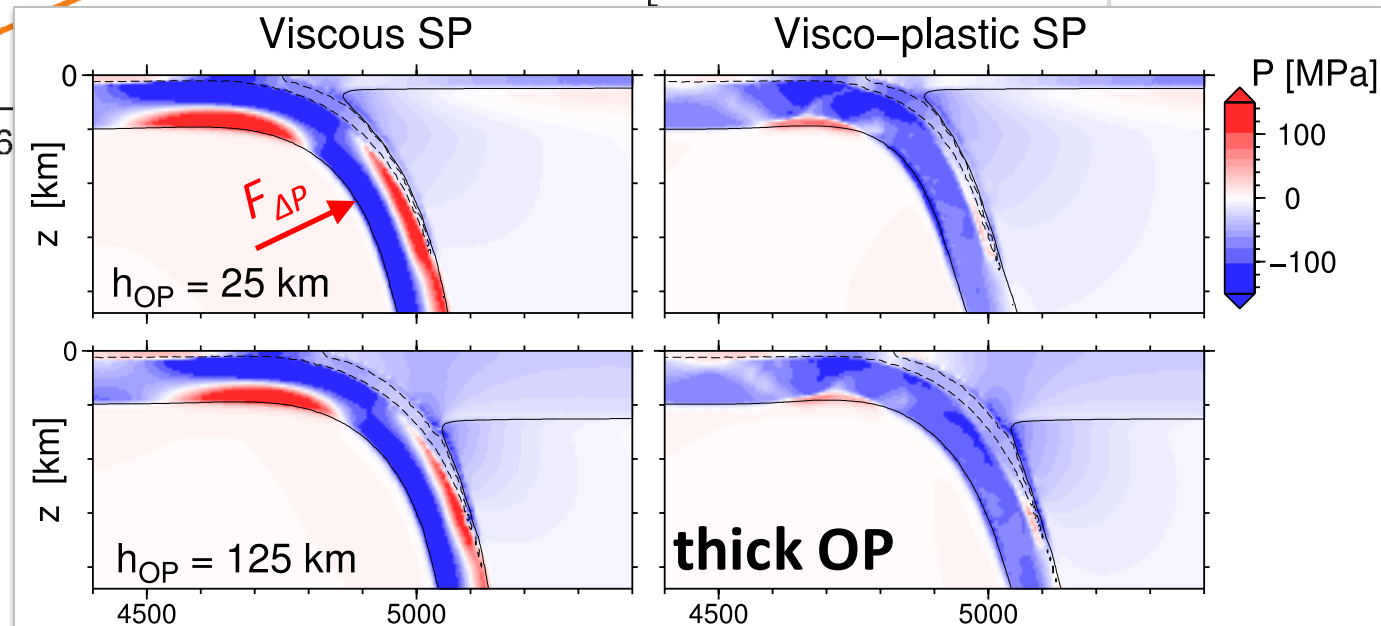
OP thickness



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- Introduces dependence

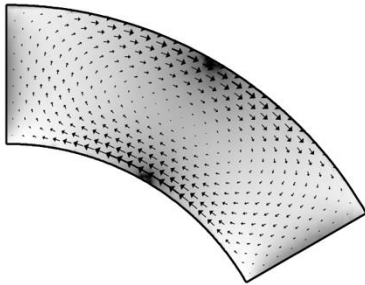
Weaker plastic slab more strongly affected by lifting force associated with OP?



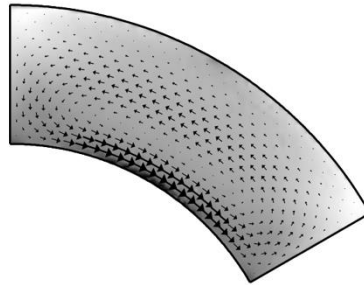
Testing variable viscous and visco-plastic rheologies

Like Conrad & Hager 1999 but with more sophisticated rheologies
(including lab-derived olivine flow laws)

(a) isoviscous

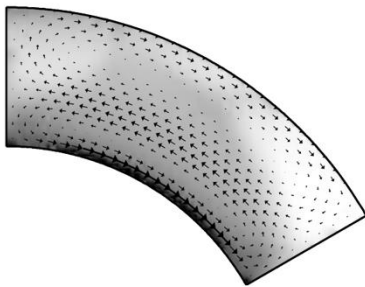


(b) T-dependent

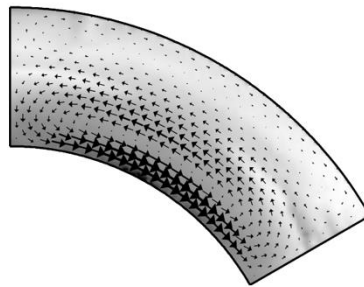


Rose & Korenaga, 2011

(c) pseudoplastic (L.E.)



(d) pseudoplastic (KK08)



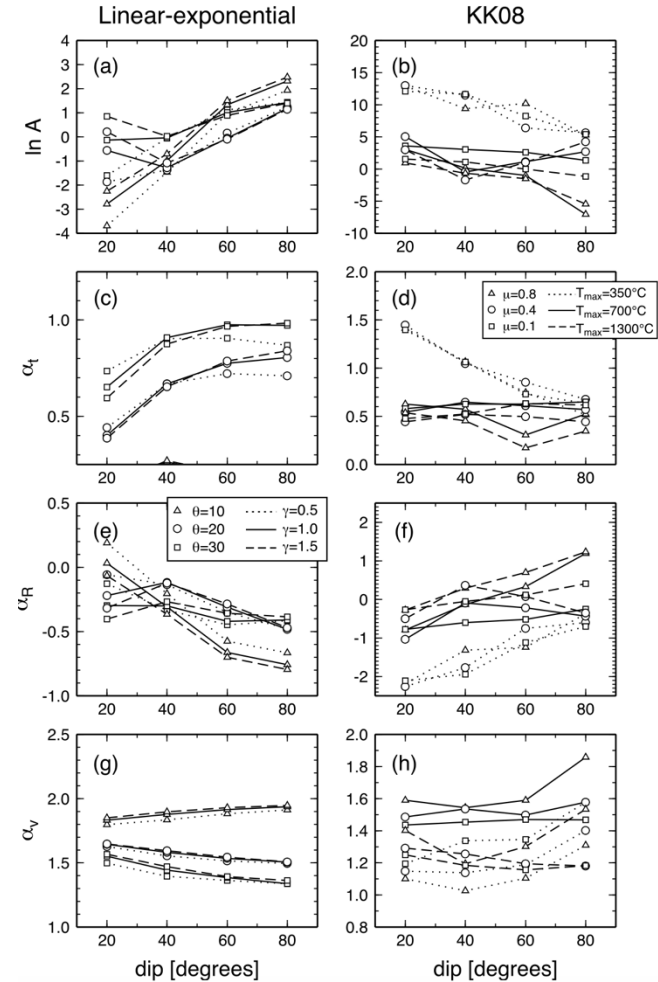
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Like Conrad & Hager 1999 but with more sophisticated rheologies (including lab-derived olivine flow laws)



Rose & Korenaga, 2011

Linear regression for dissipation scaling factors points to values that are highly variable relative to isoviscous case.



$$\Phi_{BD}(t, R, v, \phi) = A(\phi) t^{\alpha_t(\phi)} R^{\alpha_R(\phi)} v^{\alpha_v(\phi)},$$

How about elasticity?

$$D_{ij} = D_{ij}^v + D_{ij}^e = \frac{\tau_{ij}}{2\eta} + \frac{\dot{\tau}_{ij}}{2\mu}$$

$$\dot{\tau}_{ij} = \lim_{\delta t \rightarrow 0} \frac{\tau(t, \mathbf{x}(t)) - \tau(t - \delta t, \mathbf{x}(t - \delta t))}{\delta t}$$

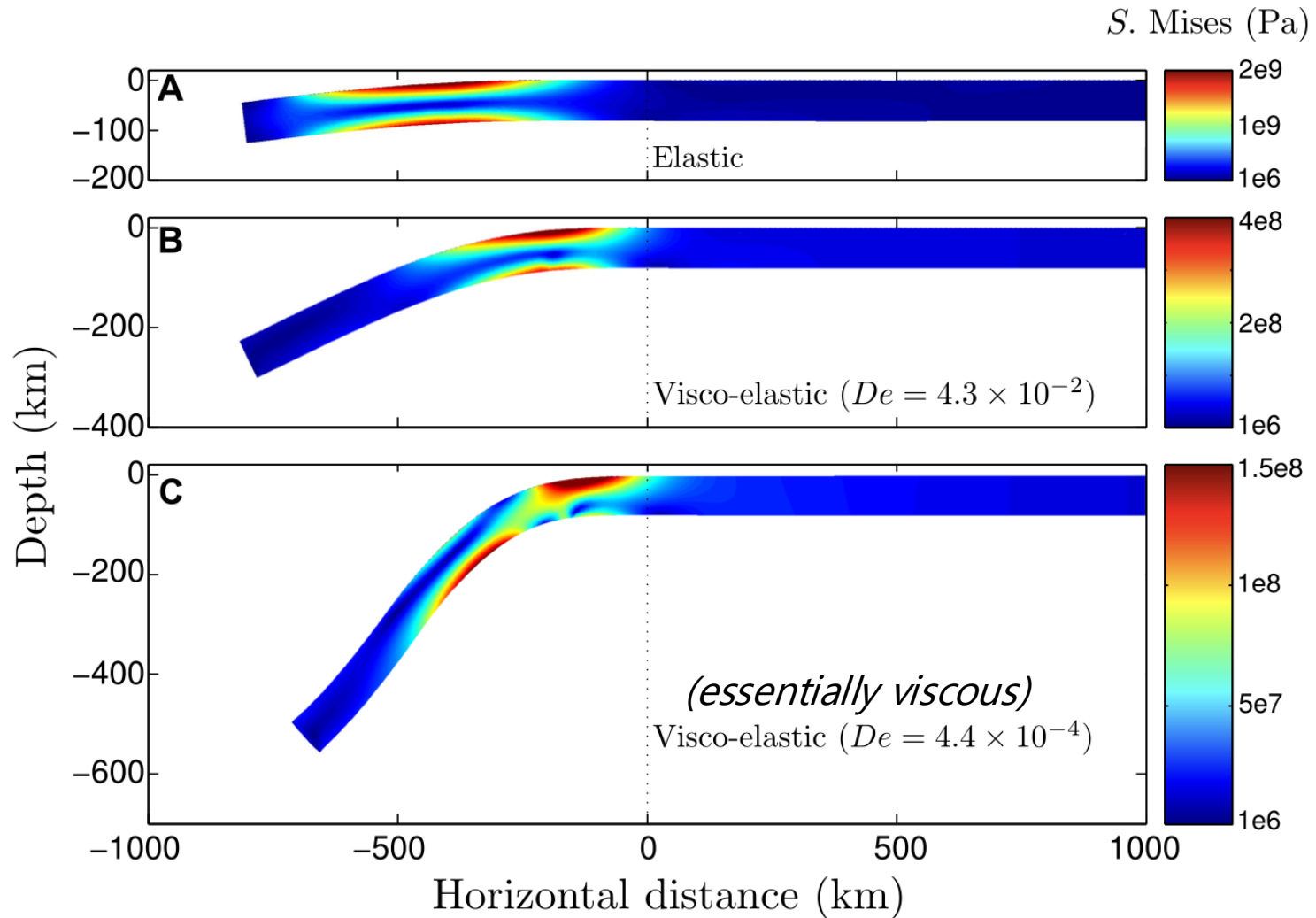
$$\tau_{ij}(t, \mathbf{x}(t)) = 2\eta_{\text{eff}} D_{ij}(t, \mathbf{x}(t)) + \frac{\eta_{\text{eff}}}{\mu \Delta t} \tau_{ij}^v(t - \Delta t, \mathbf{x}(t))$$

$$\sigma_{ij,j} = \tau_{ij,j} - p_i = f_i$$

$$(2\eta_{\text{eff}} D_{ij})_j - p_{,i} = f_i - \frac{\eta_{\text{eff}}}{\mu \Delta t} \tau_{ij,j}^v$$

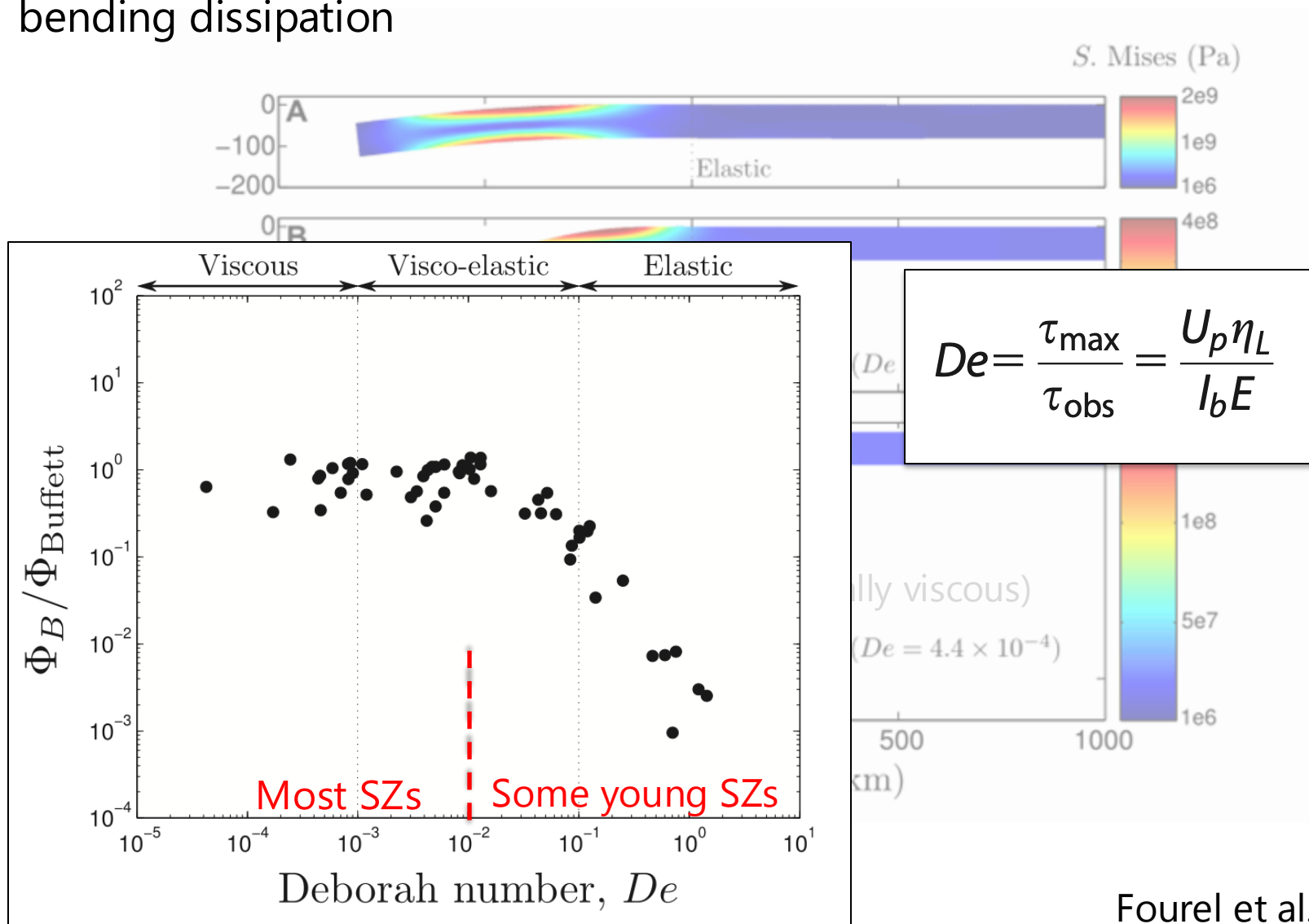
Example implementation from Moresi et al., 2003 (and Farrington et al., 2014)

How about elasticity?



How about elasticity?

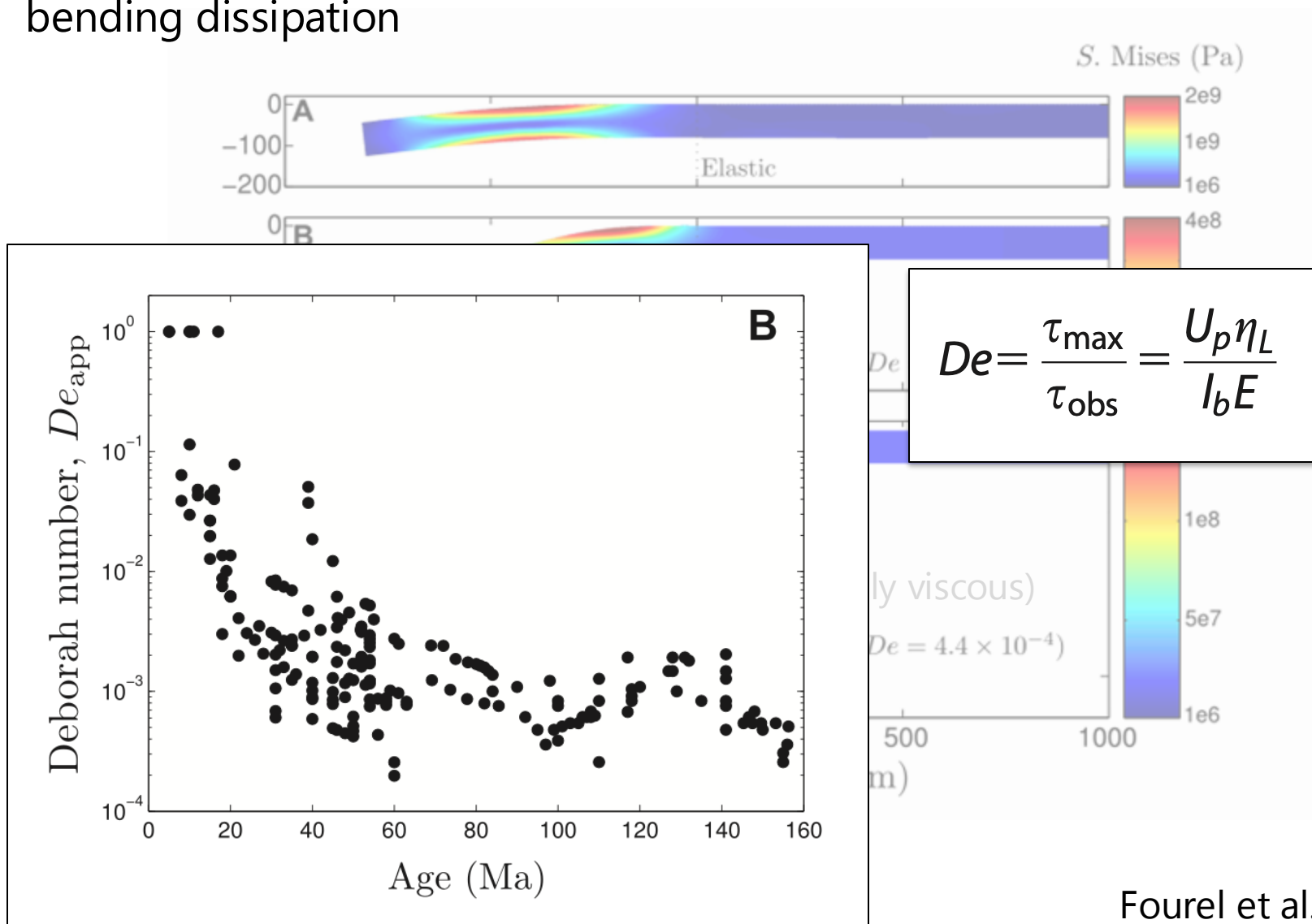
For high Deborah numbers, viscous scaling significantly overestimate bending dissipation



Fourel et al., 2014
cf. Farrington et al., 2014

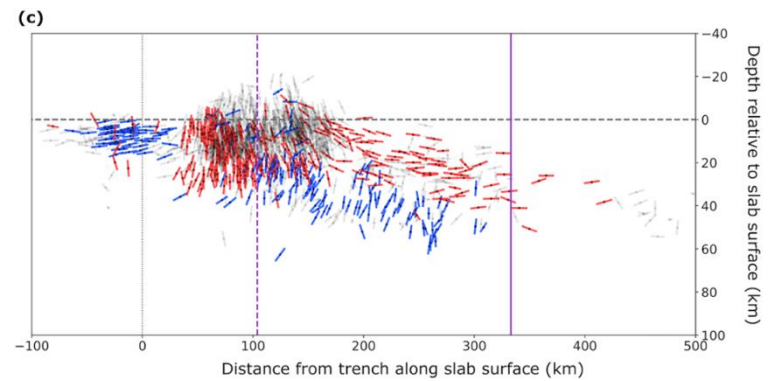
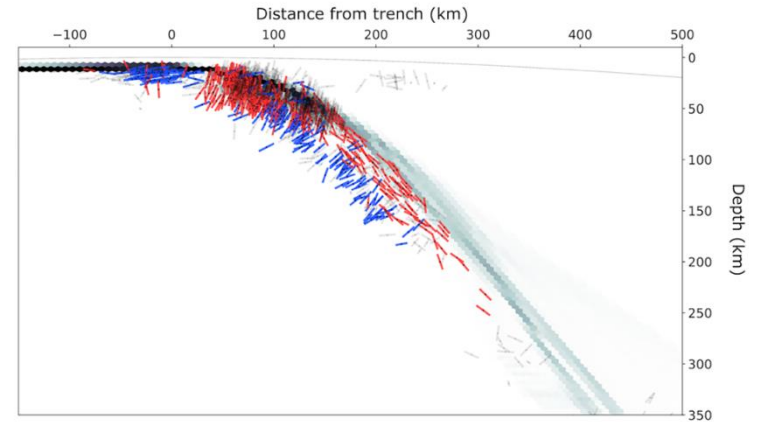
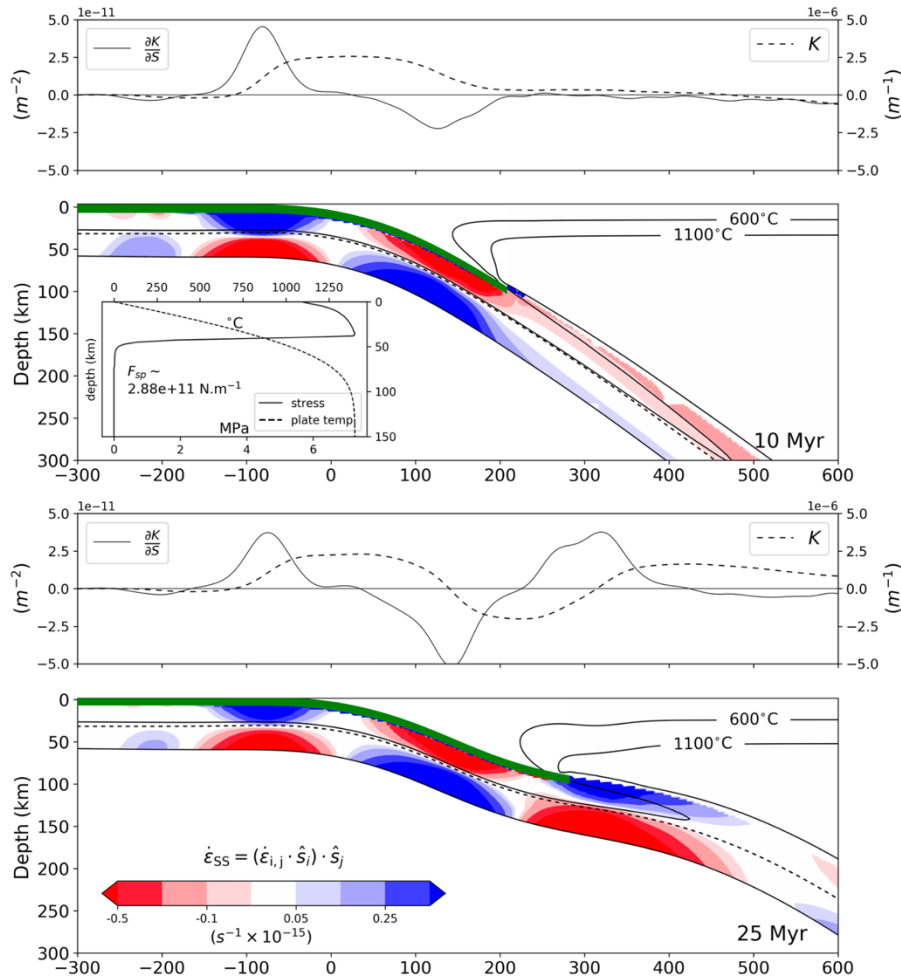
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Fourel et al., 2014
cf. Farrington et al., 2014

Visco-elasto-plastic



Sandiford et al., 2020

cf. Sandiford & Craig, 2023; Craig et al., 2023