**MGS 723 – Problem Set #1 – Stress and Strain**

Assigned: 9/01/21

Due: 9/15/21

PART A: Stresses in the mantle

**A few straightforward problems can guide your understanding of the orders of magnitude and typical states of strain and stress for the Earth.**

1. The Pacific Plate is moving at 44 mm/yr with respect to North America. Assume that deformation is accommodated in a 150 km wide viscous zone. Estimate i) the strain-rate in the shear zone ii) the shear stress in the shear zone. State your assumptions.
2. Assume the whole mantle is convecting and 44 mm/yr is a reasonable average plate velocity. How might you estimate mantle strain-rates? Again, state assumptions, and provide an estimate.
3. A picture containing graphical user interface

   Description automatically generatedThe lithostatic pressure due to an overburden is given by the integral of the product of gravitational acceleration, density, and thickness:
   1. At a depth of 500 km, estimate the pressure below a column that holds 15 km thick oceanic crust, 85 km thick oceanic mantle lithosphere, and the underlying mantle. Using g = 10 m/s2 and appropriate constant density values for each of the three layers.
   2. Repeat this calculation but now estimate the pressure deeper, at the core-mantle boundary (CMB). Compare your calculated CMB estimate with pressure estimates from PREM (*Figure 1*, next page). What do you think causes the discrepancy with PREM?
4. Linear viscous deformation predicts that shear stress in a fluid, *τ*, is given by 2*η* where *η* is the viscosity (~ 1021 Pas for the mantle). Using your strain rate estimate (from 2), i) predict the shear stress in the mantle, ii) compare with your lithostatic estimate at 500 km depth, iii), comment on the magnitude differences between the deviatoric () and full stress () values deep within Earth’s mantle.

Diagram

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**The preliminary reference Earth model (PREM)**. LEFT: Density and mass, in terms of total Earth mass (ME = 5.97 · 1024 kg), below a given depth, as a function of depth. Note the density jumps at phase transitions. RIGHT: gravitational acceleration relative to the surface gE = 9.807 m/s2, and pressure. All based on integration of density as a function of depth. Original paper: Dziewonski and Anderson (1981). Figure from [Becker and Faccenna (draft).](http://www-udc.ig.utexas.edu/external/becker/preprints/tectonic_geodynamics_draft.pdf)

PART B: Stresses in the lithosphere: *Uniaxial strain and sedimentation*.

**This should give you a sense for simple estimates of stress within the lithosphere.**

1. Under uniaxial strain, only one major strain axis, *ε*1, is non-zero, *e.g.,* aligned with the vertical axis as *εzz* = *ε*1; and all other strain tensor components are zero, *εxx* = *εyy* = *εxy* . . . = 0. For this state to hold, forces have to act in the horizontal plane to keep the compressed body from deforming.

In general, linear elasticity for an isotropic solid with shear modulus *μ* and Lame ́ parameter *λ* can be written as:

Where is stress, is the Kronecker delta (1 for i = j, 0 for i j), and = .

* 1. Write out the only components of stress that are non-zero for uniaxial strain: and :
  2. Express as a function of the other normal stresses:
  3. Text, whiteboard

     Description automatically generatedA picture containing diagram

     Description automatically generatedWrite the expression in terms of the Poisson’s ratio () using the following relationships between the elastic properties:
  4. Comment on the physical meaning of the Poisson’s ratio. Using an appropriate value for , comment on the size of horizontal stresses relative to vertical stresses for rock in uniaxial strain

1. Chart, line chart

   Description automatically generatedConsider a point *Q* which has been covered by a sediment layer of thickness *h*.

Q is at the boundary between sediments and underlying rock and so before sedimentation, = = 0, and now .

* 1. Using 1) write out the horizontal normal stresses ( and ) assuming a uniaxial strain state after sedimentation. What are the relative magnitudes and sense (compressive vs. extensional) of the horizontal normal stresses?
  2. Now consider the deviatoric stresses () that lead to shear deformation, where is pressure defined as ). Write as a function of and . What are the magnitudes and sense (compressive vs. extensional) of the horizontal deviatoric normal stresses (for a typical rock value of )?
  3. Maximum shear stress is given by which, in our coordinate system, is equivalent to . Now write as a function of and . What is the maximum shear stress for a sediment layer that is 1.5 km thick and has density of 2600 kg/m3. Do you expect these shear stresses to be large enough to produce earthquakes? *(Hint: see Byerlee’s Law)*

Note: These problems are modified from Becker and Faccenna, Tectonic Geodynamics (draft)

References

Dziewonski, A. M., and D. L. Anderson. 1981. "Preliminary reference Earth model." Phys. Earth Plan. Int. 25:297-356.