**MGS 723 – Problem Set #2 – Viscous creep**

Assigned: 9/15/21

Due: 9/29/21

Background:

From the classic Hirth and Kohlstedt (2004) paper, the generalized diffusion/dislocation creep flow law for olivine at upper mantle conditions is presented as follows:

(1)

Here is uniaxial strain rate, a constant pre-factor, *s* (= ) the differential stress, *n* the power-law exponent, *d* grain size, *m* the grain size exponent, *CH2O*the water fugacity (~ content) and its exponent *r,* the melt fraction and a melt constant . Finally, the Arrhenius (exponential) temperature dependence is specified by an activation energy (*E*), activation volume (*V*), pressure (*p*), temperature (*T*), and gas constant (*R* = 8.314 J/molK)

To simplify this expression let’s assume constant water content and hence fold in to a new prefactor, *A’*:

(2)

For typical “dry” conditions = 1 H/106 Si, and for typical “wet” conditions, = 1000 H/106 Si (note the units: water fugacity is specified in number of hydrogen ions per million Silicate atoms in the olivine).

Typically, when we compute effective viscosity values (which will be the focus of the exercise), we use second invariants of the deviatoric stress and strain rate (and , respectively). To convert (1) to be in terms of and , we need to again modify the pre-factor.

(3)

Lastly, Hirth and Kohlstedt use some non-SI units. Namely, m for *d* and MPa for stress. To use SI units, we need to modify the pre-factor by one final factor to convert to SI units:

(3)

This leaves us with the following constitutive law (for stress/strain rate second invariants, SI units, and no melt)

(4)

**Table

Description automatically generated**Using the ***A*** constants from Hirth and Kohlstedt for dry and wet olivine (and = 1 or 1000 H/106 Si), the table below shows the new prefactors for Equation 4. Note also that temperatures in Equation 4 are in Kelvin (= T[°C] + 273.15)

?

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**We will now use this constitutive law (Equation 4) and these mainly experimentally derived parameters (table) to understand viscous creep in the upper mantle.**

Questions:

1. Use Equation 3 to calculate the new pre-factors (A’’’) for dry and wet disclocation creep. Assume *r* (the fugacity exponent for dislocation creep) = 1.2
2. Why are the *n* and *m* parameters so different for diffusion vs. dislocation creep?
3. To calculate effective viscosity (), we need to use the following relationship between stress and strain rate: . Re-write Equation 4 as an effective viscosity [i.e., f()].

*Hint: Plug in into Eq. 4, solve for .*

1. Inspecting the effective viscosity expression from ***Part 3***, is the temperature-dependence of viscosity stronger or weaker for diffusion creep (relative to dislocation creep)?
2. From Equation 4 (or the rearranged version): What are the units of A’’’ for dislocation and diffusion creep, respectively?
3. Vary parameters such as *T* and *d* and describe the behavior of diff and disl for dry rheologies. In particular, plot against temperature (273 to 2073 K) for variable grain sizes (1 mm and 10 mm) and strain rates (10-14, 10-15, 10-16 s-1) and a constant (upper mantle) pressure.
   1. How much do viscosities change for temperature changes of 100 K, say, at asthenospheric temperatures of 1600 K?
   2. What are the predicted viscosities for surface temperatures (T ~ 273 K)? Do you expect cold slabs to deform purely viscously?
   3. Describe the role that grain-size and strain-rate variations play.

*Note: Use a log y-(viscosity)-axes as changes over many orders of magnitude.*

1. Plot diff and disl for wet and dry rheologies as a function of depth (100 km < z < 600 km), if d = 4 mm, strain rate = 10-15 s-1, and temperature and pressure vary with depth following:

where *z* is inserted into the above expressions in units of km to get *T* [K] and *p* [Pa].

1. First plot temperature and pressure versus depth (in one plot with two different axes, or two plots). Which physical processes lead to the behavior approximated by the two functions for *p* and *T*?
2. Plot four curves for dislocation and diffusion creep viscosities, and wet and dry conditions, against depth.
3. Comment on the overall trend of viscosity with depth. Would you expect this trend to continue to the lower mantle?
4. By assuming that the two strain rates ( and ) sum together to give a total strain rate, we can compute a joint (or composite) viscosity as follows:

Plot as a function of depth for the dry and wet rheologies. Over which depth range does each deformation mechanism (diffusion vs. dislocation) dominate? How does this change if you modify the strain rate?

Note: These problems are modified from Exercise #4, Becker and Faccenna, Tectonic Geodynamics (draft)

References

Hirth, G., and D. L. Kohlstedt (2004), Rheology of the upper mantle and the mantle wedge: A view from the experimentalists, in Inside the Subduction Factory, Geophys. Monograph, vol. 138, edited by J. Eiler, pp. 83–105, American Geophysical Union,Washington DC..