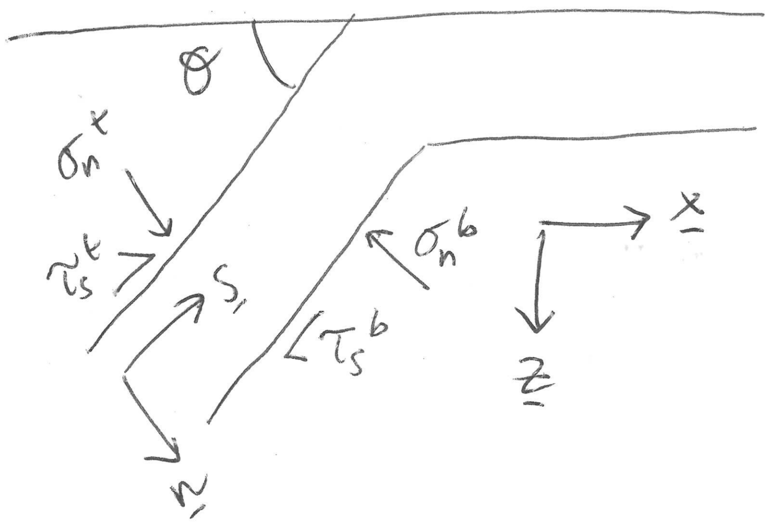
force balance

4/23/24



In a typical subduction [redacted] scenario, signs as follows:

$\sigma_n^t = -ve$ (i.e. towards wedge)

$\sigma_n^b = +ve$ (i.e towards wedge)

$\tau_s^t = +ve$ (i.e. up-dip)

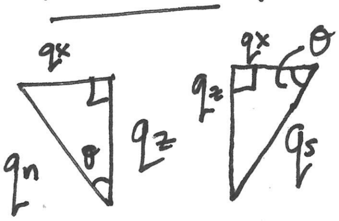
$\tau_s^b = -ve$ (i.e. up-dip)

Totals:

$$q_s = \tau_s^t - \tau_s^b$$ (i.e. +ve, up-dip)

$$q_n = \sigma_n^t - \sigma_n^b$$ (i.e. -ve, towards wedge)

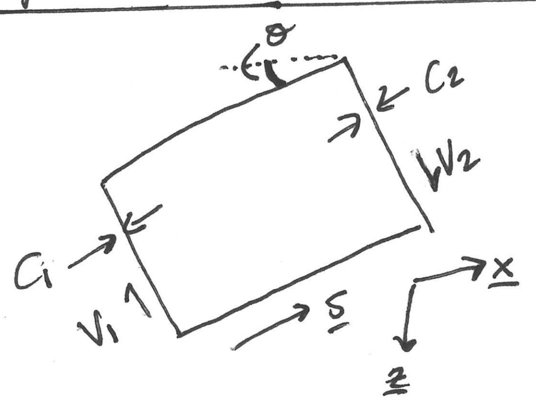
x and z:



$$q_z = q_n \cos\theta - q_s \sin\theta$$ (i.e. -ve, up)

$$q_x = q_n \sin\theta + q_s \cos\theta$$ (+ve or -ve, depends whether shear or normal greater)

force balance on a segment



x-direction

$$C_1 \cos\theta - C_2 \cos\theta - V_1 \sin\theta + V_2 \sin\theta + q_x ds = 0$$

$$-\frac{\partial}{\partial s} C \cos\theta + \frac{\partial}{\partial s} V \sin\theta + q_x = 0$$

z-direction

$$-C_1 \sin\theta + C_2 \sin\theta - V_1 \cos\theta + V_2 \cos\theta + q_z ds + \rho g L \, ds = 0$$

$$\frac{\partial}{\partial s} C \sin\theta + \frac{\partial}{\partial s} V \cos\theta + q_z + \rho g L = 0$$

Integrate x: $-C\cos\theta + V\sin\theta + \int_0^s q_x \, ds = 0$ — (1)

Int. z: $C\sin\theta + V\cos\theta + \int_0^s (q_z + \rho g L) \, ds$ — (2)

① $\times \sin\theta$:

$$-C\cos\theta\sin\theta + V\sin^2\theta + \sin\theta\int_0^s q_x\,ds = 0$$

② $\times \cos\theta$:

$$C\sin\theta\cos\theta + V\cos^2\theta + \cos\theta\int_0^s (q_z + \rho g L)\,ds = 0$$

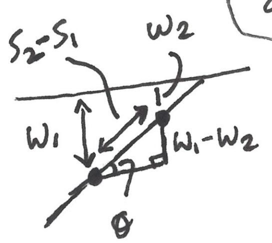
SUM

$$V + \sin\theta\int_0^s q_x\,ds + \cos\theta\int_0^s (q_z + \rho g L)\,ds = 0$$

$\partial/\partial s$

$$\frac{\partial V}{\partial s} + \frac{\partial \sin\theta}{\partial s}\int_0^s q_x\,ds + \sin\theta\cdot q_x + \frac{\partial\cos\theta}{\partial s}\int_0^s q_z\,ds + \cos\theta\cdot q_z$$
$$+ \frac{\partial\cos\theta}{\partial s}\int_0^s \rho g L\,ds + \cos\theta\cdot\rho g L = 0$$

writing trig functions as f(w)



$$\sin\theta = \frac{w_1 - w_2}{s_2 - s_1} = \frac{-\Delta w}{\Delta s} \approx \frac{-\partial w}{\partial s} \Rightarrow \frac{\partial \sin\theta}{\partial s} = -\frac{\partial^2 w}{\partial s^2}$$

$$\cos\theta = (1 - \sin^2\theta)^{\frac{1}{2}} = \left(1 - \left(\frac{\partial w}{\partial s}\right)^2\right)^{\frac{1}{2}}$$

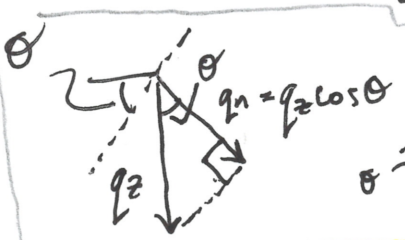
$$\frac{\partial\cos\theta}{\partial s} = \frac{\partial}{\partial s}\left[1 - \left(\frac{\partial w}{\partial s}\right)^2\right]^{\frac{1}{2}} = \frac{\partial u^{\frac{1}{2}}}{\partial u}\frac{\partial u}{\partial s} \quad \text{where } u = 1 - \frac{\partial w}{\partial s}\frac{\partial w}{\partial s}$$

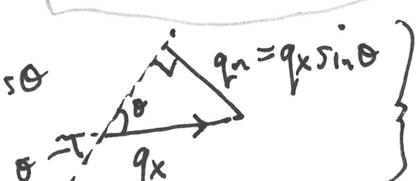
$$= \frac{1}{2}u^{-\frac{1}{2}}\cdot\left(-\frac{\partial^2 w}{\partial s^2}\frac{\partial w}{\partial s} - \frac{\partial w}{\partial s}\frac{\partial^2 w}{\partial s^2}\right) = \frac{1}{2}\left(1 - \left(\frac{\partial w}{\partial s}\right)^2\right)^{-\frac{1}{2}}\cdot -2\cdot\frac{\partial w}{\partial s}\frac{\partial^2 w}{\partial s^2}$$

$$= -\left(1 - \left[\frac{\partial w}{\partial s}\right]^2\right)^{-\frac{1}{2}}\frac{\partial w}{\partial s}\frac{\partial^2 w}{\partial s^2} = \frac{-1}{\cos\theta}\cdot -\sin\theta\cdot\frac{\partial^2 w}{\partial s^2} = \tan\theta\frac{\partial^2 w}{\partial s^2}$$

plugging in:

$$\frac{\partial V}{\partial s} - \frac{\partial^2 w}{\partial s^2}\int_0^s q_x\,ds + \tan\theta\frac{\partial^2 w}{\partial s^2}\int_0^s (q_z + \rho g L)\,ds + (q_x\sin\theta + q_z\cos\theta)$$
$$+ \rho g L\cos\theta = 0$$





$$q_n = q_x\sin\theta + q_z\cos\theta$$

$$\boxed{\frac{\partial V}{\partial s} - \frac{\partial^2 w}{\partial s^2}\int_0^s q_x\,ds + \tan\theta\frac{\partial^2 w}{\partial s^2}\int_0^s (q_z + \rho g L)\,ds + q_n + \rho g L\cos\theta}$$

SLAB RHEOLOGY INDEPENDENT

for an elastic thin beam

flex. rigidity

term 1 $\quad V = -\dfrac{\partial M}{\partial s} = -\dfrac{\partial}{\partial s}\left(\dfrac{D}{\cos\theta}\dfrac{\partial^2 w}{\partial s^2}\right)$

term 2

plugging in (and multiplying by −1)

$$\frac{\partial^2}{\partial s^2}\left(\frac{D}{\cos\theta}\frac{\partial^2 w}{\partial s^2}\right) + \frac{\partial^2 w}{\partial s^2}\left(\int_0^s q_x\,ds - \tan\theta\int_0^s (q_z+\rho g l)\,ds\right) - q_n - \rho g l\cos\theta = 0$$

Need to solve for w (write derivatives using finite diff.)

firstly, term 1:

$$D\frac{\partial^2}{\partial s^2}\left(\frac{1}{\cos\theta}\frac{\partial^2 w}{\partial s^2}\right) \approx D\left[\frac{\frac{1}{\cos\theta_{i-1}}\frac{\partial^2 w}{\partial s^2}(i-1) - \frac{2}{\cos\theta_i}\frac{\partial^2 w}{\partial s^2}(i) + \frac{1}{\cos\theta_{i+1}}\frac{\partial^2 w}{\partial s^2}(i+1)}{\Delta s^2}\right]$$

$$= D\left[\frac{\frac{1}{\cos\theta_{i-1}}\left[\frac{w(i-2)-2w(i-1)+w(i)}{\Delta s^2}\right] - \frac{2}{\cos\theta_i}\left[\frac{w(i-1)-2w(i)+w(i+1)}{\Delta s^2}\right] + \frac{1}{\cos\theta_{i+1}}\left[\frac{w(i)-2w(i+1)+w(i+2)}{\Delta s^2}\right]}{\Delta s^2}\right]$$

Let's say $\grave{D}_i = \dfrac{D}{\Delta s^4\cos\theta_i}$

$$= \grave{D}_{i-1}(w_{i-2}-2w_{i-1}+w_i) - 2\grave{D}_i(w_{i-1}-2w_i+w_{i+1}) + \grave{D}_{i+1}(w_i-2w_{i+1}+w_{i+2})$$

$$= w_{i-2}(\grave{D}_{i-1}) + w_{i-1}(-2\grave{D}_{i-1}-2\grave{D}_i) + w_i(\grave{D}_{i-1}+4\grave{D}_i+\grave{D}_{i+1}) + w_{i+1}(-2\grave{D}_i-2\grave{D}_{i+1}) + w_{i+2}(\grave{D}_{i+1})$$

Now, term 2: $= \left[\dfrac{w_{i-1}-2w_i+w_{i+1}}{\Delta s^2}\right] q_{int}$ $\quad$ (where $q_{int} = \int_0^s q_x\,ds - \tan\theta\int_0^s q_z+\rho g l\,ds$)

$$= w_{i-1}\left(\frac{q_{int}}{\Delta s^2}\right) + w_i\left(\frac{-2q_{int}}{\Delta s^2}\right) + w_{i+1}\left(\frac{q_{int}}{\Delta s^2}\right)$$

plugging it all back in:

$$\boxed{w_{i-2}(\grave{D}_{i-1}) + w_{i-1}\left(\frac{q_{int}}{\Delta s^2}-2\grave{D}_{i-1}-2\grave{D}_i\right) + w_i\left(\frac{-2q_{int}}{\Delta s^2}+\grave{D}_{i-1}+4\grave{D}_i+\grave{D}_{i+1}\right) + w_{i+1}\left(\frac{q_{int}}{\Delta s^2}-2\grave{D}_i-2\grave{D}_{i+1}\right) + w_{i+2}(\grave{D}_{i+1}) = q_n + \rho g l\cos\theta}$$