$$\frac{\partial^2}{\partial s^2}\left[\frac{\mu_p L_p^3}{12\cos\theta}\frac{\partial^3(w_{next}-w)}{\partial s^2\partial t}\right) + \left[\frac{\partial^2(w_{next}-w)}{\partial s^2}+\frac{\partial^2 w}{\partial s^2}\right]\cdot L\left(\int_0^s q_x ds - \tan\theta\int_0^s(q_z+\rho_l g L)\right)$$

$$-(q_n^t+q_n^b)-\rho_l g l\cos\theta = 0 \qquad (1)$$

Let's assume $\frac{1}{\cos\theta}\cdot\frac{\partial^2(w_{next}-w)}{\partial s^2} = T$

$$\frac{\partial^2}{\partial s^2}\left[\frac{\mu_p l_p^3}{12\cos\theta}\frac{\partial^3(w_{next}-w)}{\partial s^2\partial t}\right] = \frac{\mu_p l_p^3}{12\Delta t}\left(\frac{T_{i+1}+T_{i-1}-2T_i}{\Delta s^2}\right) \qquad (2)$$

$$T_i = \left[\frac{\partial^2(w_{next}-w)}{\partial s^2}\right]_i\frac{1}{\cos\theta_i}$$

Then Let's say $(w_{next}-w)_i = U_i$

$$T_i = \frac{\partial^2(w_{next}-w)}{\partial s^2}\cdot\frac{1}{\cos\theta_i} = \frac{U_{i+1}+U_{i-1}-2U_i}{\Delta s^2}\cdot\frac{1}{\cos\theta_i}$$

So (2) could be rewritten as:

$$\frac{\mu_p l_p^3}{12\Delta t(\Delta s)^4}\left[\frac{1}{\cos\theta_{i+1}}U_{i+2}+\left(\frac{2}{\cos\theta_i}-\frac{2}{\cos\theta_{i+1}}\right)U_{i+1}+\left(\frac{1}{\cos\theta_{i-1}}+\frac{1}{\cos\theta_{i+1}}-\frac{4}{\cos\theta_i}\right)U_i\right.$$
$$\left.+\left(\frac{2}{\cos\theta_i}-\frac{2}{\cos\theta_{i-1}}\right)U_{i-1}+\frac{1}{\cos\theta_{i-1}}U_{i-2}\right] \qquad (3)$$

$$\frac{\partial^2(w_{next}-w)}{\partial s^2}+\frac{\partial^2 w}{\partial s^2} = \frac{1}{(\Delta s)^2}\left(U_{i+1}+U_{i-1}-2U_i+W_{i+1}+W_{i-1}-2W_i\right) \qquad (4)$$

assume

$$\alpha_i = \int_0^s q_x ds - \tan\theta_i\int_0^s(q_z-\rho_l g l)\,ds \qquad (5)$$

$$C_i = (q_n^t+q_n^b)+\rho_l g l\cos\theta_i \qquad (6)$$

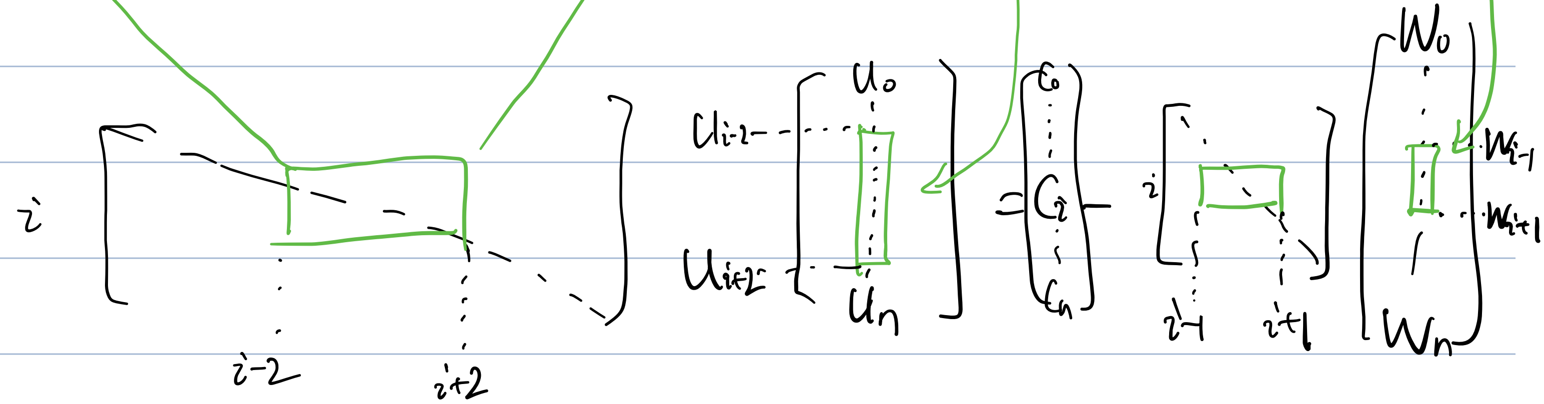
So, (1) could be rewritten by (3) − (6) as:

$$\frac{\mu_p l_p^3}{12\Delta t(\Delta s)^4}\left[\frac{1}{\cos\theta_{i+1}}U_{i+2}+\left(\frac{2}{\cos\theta_i}-\frac{2}{\cos\theta_{i+1}}\right)U_{i+1}+\left(\frac{1}{\cos\theta_{i-1}}+\frac{1}{\cos\theta_{i+1}}-\frac{4}{\cos\theta_i}\right)U_i\right.$$
$$\left.+\left(\frac{2}{\cos\theta_i}-\frac{2}{\cos\theta_{i-1}}\right)U_{i-1}+\frac{1}{\cos\theta_{i-1}}U_{i-2}\right]$$
$$+\frac{1}{(\Delta s)^2}\left(U_{i+1}+U_{i-1}-2U_i\right)\alpha_i = C_i - \frac{1}{(\Delta s)^2}\left(W_{i+1}+W_{i-1}-2W_i\right) \qquad (7)$$

assume $\frac{M p l p^3}{12\Delta t(\Delta s)^4} = C_f$

The matrix should be like:

$$\begin{bmatrix} C_f\left(\frac{1}{\cos\theta_{i-1}}\right) \\ C_f\left(\frac{2}{\cos\theta_i}-\frac{2}{\cos\theta_{i+1}}\right)+\frac{1}{(\Delta s)^2}\theta_i \\ C_f\left(\frac{1}{\cos\theta_{i-1}}+\frac{1}{\cos\theta_{i+1}}-\frac{4}{\cos\theta_i}\right)-\frac{2\theta_i}{(\Delta s)^2} \\ C_f\left(\frac{2}{\cos\theta_i}-\frac{2}{\cos\theta_{i+1}}\right)+\frac{1}{(\Delta s)^2}\theta_i \\ C_f\left(\frac{1}{\cos\theta_{i+1}}\right) \end{bmatrix}^T \cdot \begin{bmatrix} U_{i-2} \\ U_{i-1} \\ U_i \\ U_{i+1} \\ U_{i+2} \end{bmatrix} = G_i - \begin{pmatrix} \frac{1}{(\Delta s)^2} \\ -\frac{2}{(\Delta s)^2} \\ \frac{1}{(\Delta s)^2} \end{pmatrix}^T \begin{pmatrix} W_{i-1} \\ W_i \\ W_{i+1} \end{pmatrix}$$



In this part,

θ_i, G_i, W_i and C_f could be calculated from slab shape, toroidal flow, poloidal flow.

Once we get $U = W_{next} - W$, new slab shape W_{next} could be used to in the next time step calculation in time loops.