Mandatory Assignment 1 - Hand In

Jacob Wiberg Larsen - pxq563

2022-03-11

Ad 1

In this assignment we will be examining the *momentum* factor. The causes of this well-documented statistical phenomena are numerous, ranging from herding behavior among investors, to more firm specific dynamics. Whatever the cause, the momentum factor covers the empirical finding that in- or decreasing growth-rates in prices (or some related metric) for stocks in the nearby past, may be an powerful indicator of future short-term returns movements. Put simply, there's a tendency for winners to keep winning and losers to keep losing, within a reasonable time period.

We will primarily use monthly return data from the CRSP data set, as well as monthly market excess returns from the Fama-French data set. Key variables from the CRSP data are *permno*, which is an unique identifying key for each stock. *Mktcap*, which denotes the market capitalization (in millions of dollars) of the given stock at a given time, i.e the amount of stocks times the closing price at the given day. Finally, *ret_excess*, which denotes the monthly return of a stock in excess of the risk-free market return. The data initially contains 3.225.253 data entries from 1960-02-01 to 2020-12-01, but we will drop empty values throughout the exercises, to focus on the variables of interest and avoid backward extrapolating.

Table 1 provides initial summary statistics, which shows the time-series averages of the monthly cross-sectional moments for the market capitalization and excess return variables. These moments are calculated in accordance to Bali et al (2016), i.e. by first calculating them between all individual stocks for a given month t and then averaging across these.

Table 1: Average cross-sectional summary statistics

name	mean	sd	min	q05	q25	q50	q75	q95	max	n
mktcap	1814.47	8769.95	0.71	8.11	43.75	178.97	766.65	6846.56	242447.43	4412.11
ret_excess	0.01	0.15	-0.69	-0.19	-0.06	0.00	0.06	0.23	2.68	4412.11

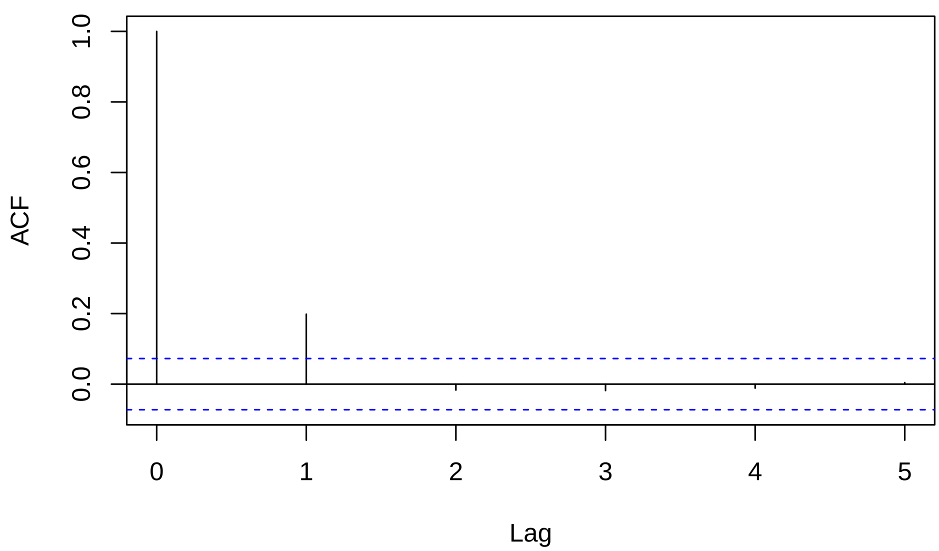
The average excess returns are fairly centered around 0, although mildly right-skewed as seen by the mean (1 pct. per month) being larger than the median (0 per cent), as well as comparing the 5th (-19 per cent) and 95th quantile (23 per cent). The market capitalization however is very much right-skewed by inspecting the same metrics, which indicates that a small amount of firms make up a large portion of the total market capitalization.

Ad 2

To examine the temporal dependence of the excess returns, we do two brief checks. First, we create an additional column with lagged excess returns r_{t-1}. This is done with the "add month" operator from the lubridate-package, i.e. (mutate(month = month %m+% months(1))). This is preferable compared to the lag() operator, as the latter method could cause problems if we had missing intermediate monthly observations for some stocks. As an example, consider a case where, say, the May observation was missing. The lag()-operator would then assign the r_{t-1} value for June as the April value, which **is not the previous months value** (rather, it is missing!). The lubridate procedure combined with a left-join on the original data set avoids such problems. We proceed by dropping missing values from our data set and then calculate a cross-sectional correlation between **ret_excess**$_t$ and **ret_excess**$_{t-1}$ at each month t across all available stocks i. Averaging across these cross-sectional correlations yields -0.042. That is, the one-month persistence between excess returns are very low.

An alternative method would be to look at the **autocorrelation function** (ACF) for the monthly cross-sectional averages of excess returns, which can be seen in Figure 1. The confidence interval is technically not valid since it relies on quite strict moment conditions ($E(X^8) < \infty$), see Franq & Zakoian (2019), which are not usually fulfilled for financial time series. Nevertheless, we see that the ACF for the 1st lag is quite low and almost non-existent for higher lags, i.e. persistence is quite low.

Figure 1 − Autocorrelation function for average excess returns



Ad 3

We compute a stocks momentum-variable for a given time t as the relative change between the previous months market capitalization and the market capitalization 12 months prior.

$$Mom_{i,t} = 100(mc_{i,t-1} - mc_{i,t-12})/mc_{i,t-12}$$

Using *market capitalization* instead of the traditional measure of prices to calculate momentum of a given security, captures much of the same dynamics. Increasing prices will by definition (*Market capitalization = Stock close price x Number of stocks*) increase the market capitalization and vice versa for decreasing prices. Using $mktcap_t$ will however likely better capture scenarios, where a firm may be setting up for future growth by raising capital through issuing new stocks. All things equal, the stock dilution could put downward pressure on prices, but the $mktcap_t$ should also be positively affected by the increased number of stocks (which prices would not).

Table 2: Average cross-sectional summary statistics

name	mean	sd	min	q05	q25	q50	q75	q95	max
mom	19	95.77	-89.13	-48.23	-16.29	5.57	32.99	117.89	3200.82

Table 2 shows summary statistics for various average cross-sectional monthly moments for the momentum variable $mom_{i,t}$. The method follows the two-step procedure that was specified for the summary statistics Table 1. The mean momentum-value in an average month is 19 percent, while the median is 5.57 percent, which again indicates a right-skewed distribution. This is also backed up by the very large maximal value of 3200.82 percent. The standard deviation of 95.77 indicates quite a large dispersion in the average monthly values. Finally, the positive average mean value implies that on average, the total market capitalization has been increasing steadily throughout the sample period.

The momentum variable exhibits a positive, but quite low correlation with the *size* variable (log(mktcap)) of 0.09981. This is helpful in applications with multi-factor models, as the low correlation indicates that the variables captures different dynamics of excess stock returns. Standard inference ($\pm 1.96/\sqrt{(T)}$) for 719 valid monthly observations also yields $\pm$ 0.073, i.e. a non-significant correlation.

Ad 4

We proceed to examine the relationship between momentum and future excess stock returns, as outlined in the assignment description. In short, we use our momentum-variable $mom_{i,t}$ as a sorting variable, where for each period t, we sort stocks into 10 different portfolios, based on breakpoints (deciles) for the cross-sectional distribution of $mom_{i,t}$. Finally, we analyze certain characteristics for each of these portfolios, before moving on to evaluating a long-short strategy.

Table 3: Average cross-sectional summary statistics

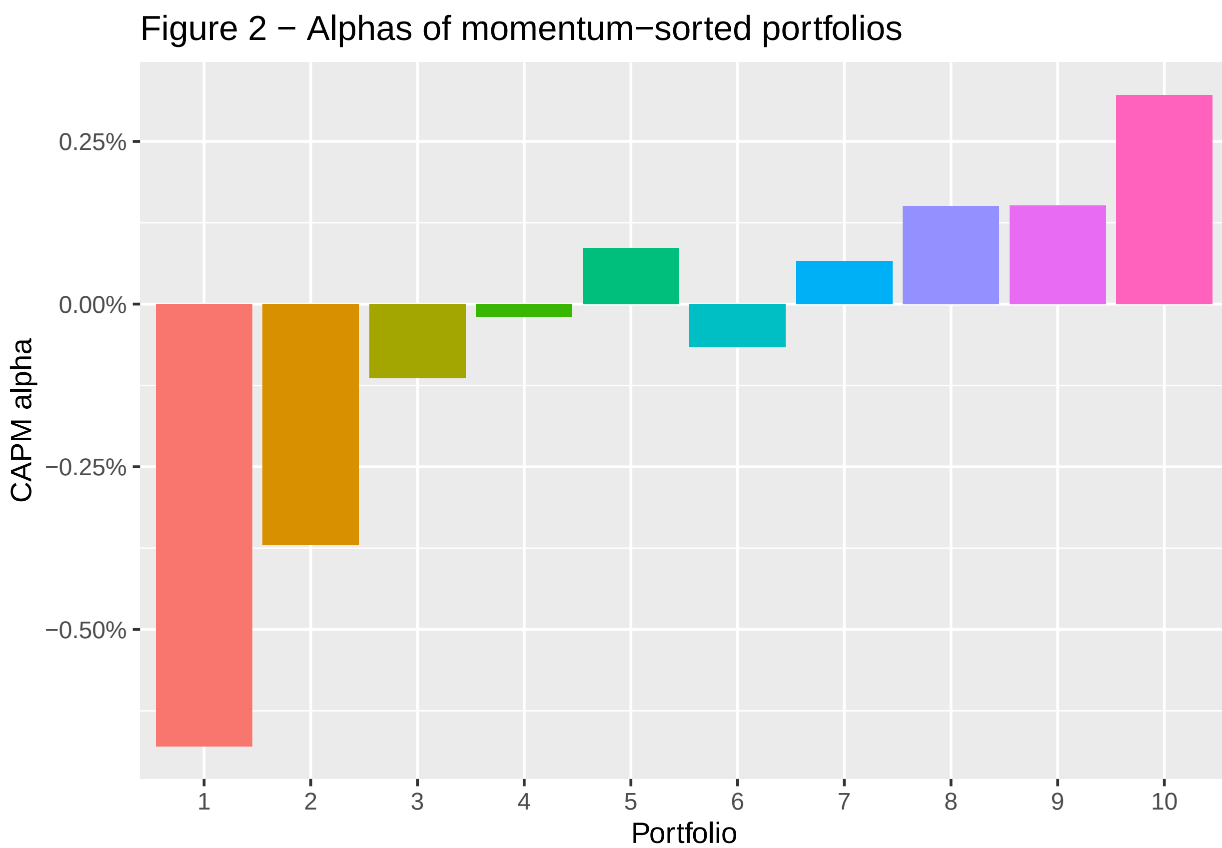
name	1	2	3	4	5	6	7	8	9	10
equal_mom	-50.88	-28.44	-16.39	-7.06	1.37	9.99	19.96	33.30	55.97	172.04
equal_mktcap	300.26	931.08	1627.81	2142.18	2509.69	2796.85	2867.25	2787.64	2261.98	1175.55

Table 3 contains information about the market capitalization and momentum for each of the 10 sorted portfolios. The increasing trend for the average values of momentum is not surprising, given that this is the variable that we sorted on. A more insightful observation however, is that there seems to be a concentration of larger firms for the middle-most portfolios (with a mildly right skew). These are the portfolios that does not have the most negative nor most positive momentum values. Intuitively, this makes fairly good sense. The smaller firms often exhibit more volatile stock price behavior as they operate on a lower absolute level (think penny-stocks). Larger firms usually require more price movement (in absolute value) to match the relative growth rates of these smaller firms, simply because they come from a larger absolute level.

Table 4: Average cross-sectional summary statistics

name	1	2	3	4	5	6	7	8	9	10
alpha	-0.0068	-0.0037	-0.0011	-0.0002	0.0009	-0.0007	0.0007	0.0015	0.0015	0.0032
beta	1.6658	1.3540	1.1696	1.0572	0.9770	0.9570	0.9414	0.9390	1.0228	1.2196
vw_ret	0.0026	0.0040	0.0055	0.0058	0.0064	0.0047	0.0060	0.0068	0.0073	0.0101

We proceed to look into each of the 10 portfolios. Table 4 shows the CAPM alpha, the market beta and the value-weighted average returns (across all months t) for each of the portfolios. The first row shows the CAPM alphas, which are also shown in Figure 2. In general, there seems to be a clear trend of higher alphas for portfolios sorted into higher *momentum-portfolios*, with the 10th decile momentum portfolio (with value-weighted portfolio weights) earning an alpha of 0.3 percent per month. Conversely, there seems to be an overall downward trend in the portfolios co-movement with the general market (i.e. the market beta), with low momentum-portfolios having a higher market beta. Finally, the value-weighted average excess return seems to be generally increasing for portfolios with higher momentum. This ties well into our momentum-introduction from our first exercise of *winners who keeps on winning and losers who keep on losing.*



We now consider a long-short strategy, that at each time t takes a long position in the portfolio with the highest decile of momentum values, while taking a short position in the portfolio with the lowest decile of momentum values (that is, *long-short = long(top decile) - short(bottom decile)*).

Table 5: Test for statistical significant excess returns

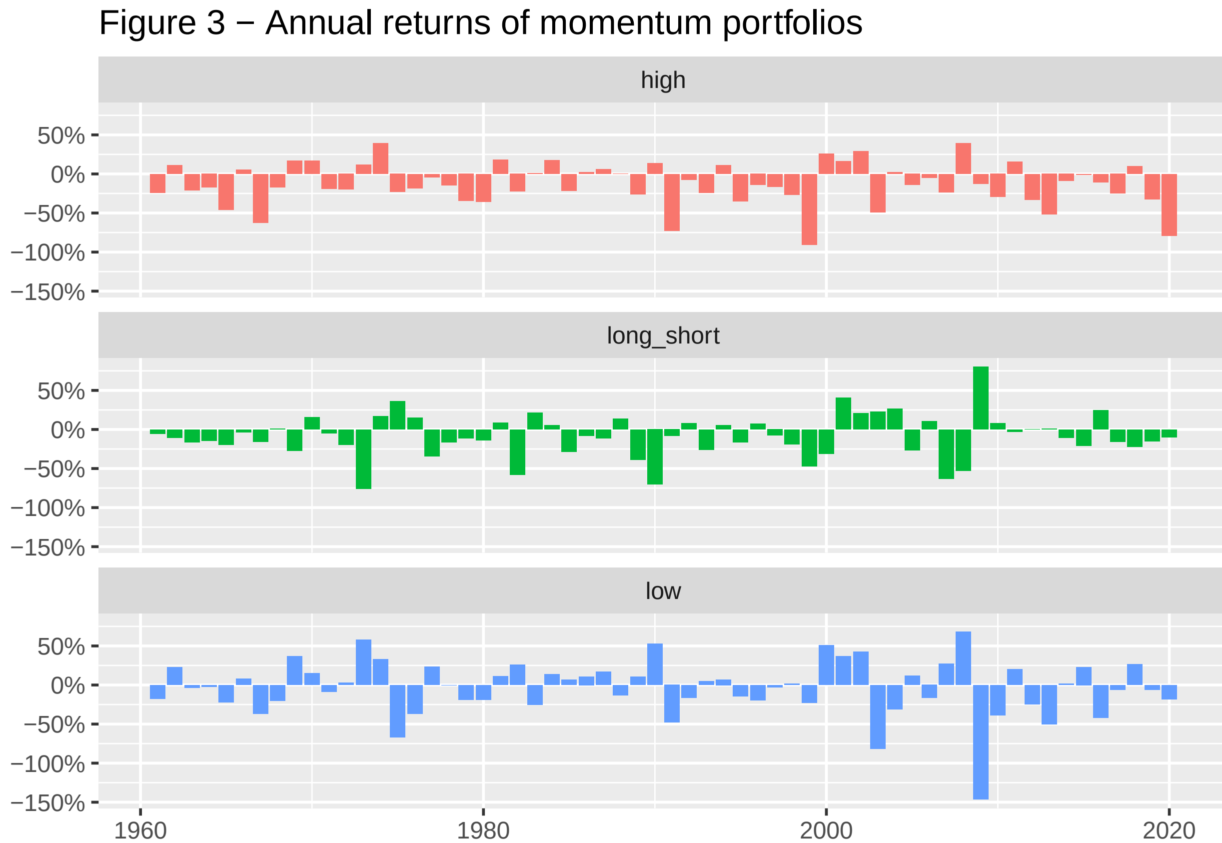
term	estimate	std.error	statistic	p.value
(Intercept)	0.0075	0.0032	2.3293	0.0201

To check for whether the long-short strategy provides statistical significant excess returns, we first regress the long-short portfolio on a constant with Newey-West robust std. errors. This is basically equivalent to checking whether the intercept is different from 0, or in our case, if the excess returns are different from 0. As seen in Table 5, the returns seems to be significant at 0.75 percent per month, at a 2 percent significance level.

Table 6: CAPM alpha and market beta for long-short strategy

term	estimate	std.error	statistic	p.value
(Intercept)	0.0100	0.0028	3.5500	4e-04
mkt_excess	-0.4462	0.1335	-3.3434	9e-04

Table 6 contains information about a simple CAPM regression on the long-short portfolio. We see that we get a highly significant abnormal excess return (alpha) of 0.01. The strategy is not market neutral, which is seen by the negative market beta of -0.4462. The strategy could be made market neutral by scaling either the long or short position up or down, until a market beta of 0 is achieved. By downscaling this strategy could still be kept *"self-financing"* in the sense that no additional financing is required, while upscaling would potentially require additional financing.



Finally, Figure 3 depicts the annual returns of the long-short strategy. At first glance, there is no clear tendency. We see from the middle row that the strategy has been quite underperforming within the last few years, while the strategy performed well in the early 2000's (e.g. during hype leading up to the dot-com bubble). Taking a short position in the low-momentum-portfolio during 2009 also appears to have been quite helpful for the overall return of the strategy.

Ad 5

We proceed with our analysis to examine whether momentum has predictive power for returns further ahead than next month. More specifically, we will now be using *k-months-ahead* future excess returns, for $k = \{1, 3, 6, 12\}$, as the outcome variable for our portfolio sorting analysis.

A little foreword on *timing* is in place. So far, we have been doing our portfolio sorting on $mom_{t,i}$ and analyzing the effect on ret_excess_t. Since $mom_{t,i}$ is $t-1$-measurable, we argue that the analysis so far has already been examining the 1-month-ahead returns (which materialize from period $t-1$ to t). Therefore, to set up our tibble, we add 3 additional columns with *(k-1)-ahead-excess-returns* (i.e. 2, 5 and 11 months ahead). This is done similarly to the join-procedure in exercise 2 with the lubridate package, only this time subtracting months. Entries containing missing values are removed, so we have effectively 2.647.474 data entries. We repeat some of the major steps from exercise 4, but now with the different k-month-ahead excess returns as our outcome variable.

Table 7: Average cross-sectional summary statistics

portfolio	a_k1	b_k1	r_k1	a_k3	b_k3	r_k3	a_k6	b_k6	r_k6	a_k12	b_k12	r_k12
1	-0.005	1.593	0.003	0.001	-0.173	0.000	0.002	0.093	0.002	0.011	-0.043	0.011
2	-0.003	1.321	0.005	0.002	-0.119	0.002	0.003	0.071	0.003	0.006	0.002	0.006
3	-0.001	1.151	0.005	0.004	-0.091	0.004	0.004	0.054	0.004	0.008	0.022	0.008
4	-0.001	1.020	0.005	0.005	-0.094	0.004	0.004	0.090	0.004	0.006	-0.014	0.006
5	0.001	0.977	0.006	0.005	-0.092	0.005	0.005	0.054	0.005	0.006	-0.020	0.006
6	0.000	0.947	0.005	0.005	-0.073	0.005	0.005	0.067	0.006	0.006	0.017	0.006
7	0.000	0.948	0.005	0.006	-0.079	0.006	0.005	0.086	0.006	0.006	0.006	0.006
8	0.001	0.948	0.006	0.007	-0.052	0.006	0.005	0.070	0.006	0.006	0.001	0.006
9	0.001	1.049	0.006	0.007	-0.009	0.007	0.007	0.083	0.008	0.006	-0.025	0.006
10	0.002	1.232	0.009	0.009	-0.001	0.009	0.006	0.029	0.006	0.003	-0.025	0.003

Table 7 contains an overview of all the CAPM alphas, the market betas and the value-weighted average excess returns for each of the 10 portfolios, for different values of k. An anomaly worth noting is that the relationship we saw before between alphas and momentum-portfolio-deciles, breaks down when using 12-month-ahead excess returns as the outcome variable. This could be due to the strategy not having enough predictive power 12 months ahead, such that we are essentially just viewing noise.

Table 8: Test on alphas for long-short strategies for different k's

Name	k___1	k___3	k___6	k_12
Alpha	0.008	0.009	0.004	-0.008
t-stat	2.722	2.928	1.707	-3.025

Again, we construct the long-short portfolios similarly to before. The values are shown in Table 8 which also contains t-statistics for these alphas, while Figure 4 shows a visual representation of them. It seems that the highest risk-adjusted performance is for the 3-months-ahead excess returns. Also, notice the large negative alpha for 12-months-ahead excess returns. This ties to the alphas described in Table 7, where the long-short portfolio is essentially constructed as $0.03 - 0.011 = -0.08$.

Figure 4 − Alphas of long−short portfolios for different values of k