MA1 - Advanced Empirical Finance

9 May 2021

Contents

Introduction

In this assignment we show that estimation uncertainty for plug-in estimates is likely to cause inefficient portfolio weight allocations. We do so by replicating the findings of Jobson and Korkie (1980) who investigate the finite-sample properties of the plug-in portfolio estimates. We base our analysis on a data set consisting of monthly returns of 10 industry-sorted portfolios from July 1926 to November 2020. In order to replicate their results we assume that the historical sample moments are the true ones. We then simulate sets of 250 hypothetical finite samples of various lengths and determine their implied plug-in estimates of the mean-variance frontier. We use the results to get a sense of the economic loss measured by average Sharpe-ratios for the sets of 250 simulations with different sample lengths. We find that the economic is larger when smaller sample sizes are used to obtain the plug-in estimates. Finally, we consider if alternative portfolio optimization strategies can minimize the estimation uncertainty and hereby lower the economic loss.

Exercise 1

Our analysis is based on a data set consisting of monthly returns of 10 industry-sorted portfolios from July 1926 to November 2020, available from Kenneth French's homepage. We assume a risk-free rate of $R_f = 0$ and determine the monthly average return for each of the $N = 10$ industry portfolios as:

$$\mu_i = \frac{1}{T}\sum_{t=1}^{T} r_{i,t}$$

where $r_{i,t}$ is the realized return at time t for industry i and $T = 1133$ is the total number of months in the sample. This yields a (10 $\times$ 1 mean vector, μ, of the means for the individual industry portfolios, which we take as the true parameters.

For each industry i we also calculate the standard deviation as:

$$\sigma_i = \sqrt{\frac{\sum_{t=1}^{T}(r_{i,t} - \mu_i)}{T}}$$

where again $r_{i,t}$ is the realized return at time t for industry i and $T = 1133$ is the total number of months in the sample. The monthly Sharpe-ratio for each industry i is then simply calculated as:

$$SR_i = \frac{E\left[R_i - R_f\right]}{\sigma_i} = \frac{E\left[R_i\right]}{\sigma_i} = \frac{\mu_i}{\sigma_i}$$

where it is used that the risk-free rate is assumed to be 0. Results are presented below.

Table 1: Summary statistics

	Durbl	Enrgy	HiTec	Hlth	Manuf	NoDur	Other	Shops	Telcm	Utils
mu	1.15	0.96	1.14	1.09	1.03	0.96	0.91	1.04	0.86	0.87
sd	7.87	6.33	7.21	5.55	6.24	4.59	6.41	5.81	4.61	5.49
sharpe	0.15	0.15	0.16	0.20	0.16	0.21	0.14	0.18	0.19	0.16

We find that the Non-durable goods portfolio (NoDur) exhibits the highest sharpe-ratio of the 10 individual industry portfolios, which is also clear from table 1.

We then compute the variance-covariance matrix as:

$$\Sigma = \frac{1}{T-1}\sum_{t=1}^{T}((r_t - \mu)(r_t - \mu)')$$

where r_t is a (10×1) vector of the return vectors from the individual industry portfolio. The variance-covariance matrix, Σ is presented below, and we will take these parameters as the true ones.

Table 2: Covariance matrix

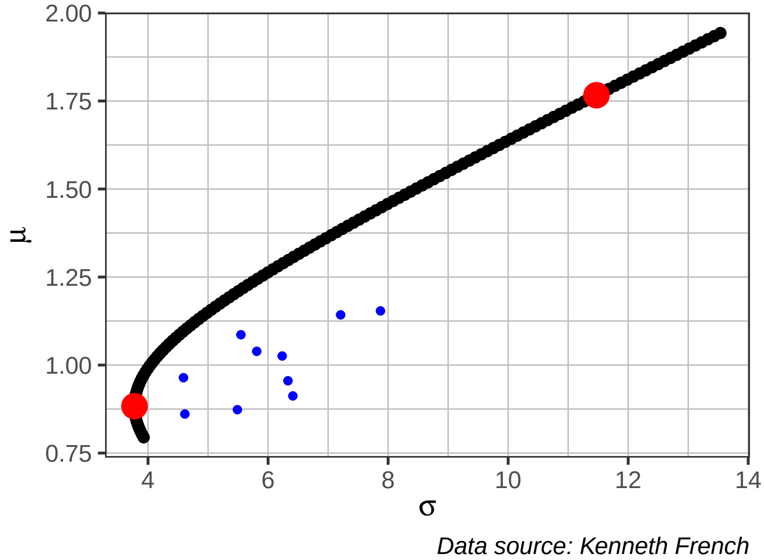
	NoDur	Durbl	Manuf	Enrgy	HiTec	Telcm	Shops	Hlth	Utils	Other
NoDur	21.07	26.53	24.27	18.04	24.37	14.54	23.04	20.26	17.77	24.76
Durbl	26.53	61.98	42.33	30.82	43.70	23.03	36.20	27.52	25.79	40.45
Manuf	24.27	42.33	38.88	28.85	38.75	19.88	30.73	26.26	23.65	36.33
Enrgy	18.04	30.82	28.85	40.09	27.74	15.46	21.65	19.73	20.78	28.16
HiTec	24.37	43.70	38.75	27.74	51.97	22.72	33.33	28.83	24.29	37.04
Telcm	14.54	23.03	19.88	15.46	22.72	21.28	18.27	15.56	15.86	21.02
Shops	23.04	36.20	30.73	21.65	33.33	18.27	33.77	24.03	20.62	30.82
Hlth	20.26	27.52	26.26	19.73	28.83	15.56	24.03	30.78	18.76	26.32
Utils	17.77	25.79	23.65	20.78	24.29	15.86	20.62	18.76	30.12	25.22
Other	24.76	40.45	36.33	28.16	37.04	21.02	30.82	26.32	25.22	41.12

We note that all industry portfolios display positive co-variances, which is not surprising from an empirical viewpoint, as market portfolios tend to move in the same direction.

Exercise 2

Exercise 3

Figure 1: Efficient frontier based on true moments



Exercise 4

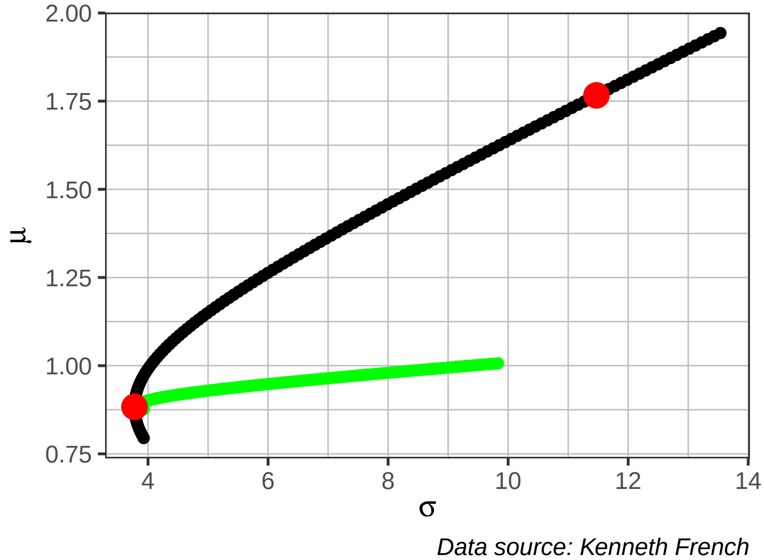
Table 3: Tangency portfolio weights

	NoDur	Durbl	Manuf	Enrgy	HiTec	Telcm	Shops	Hlth	Utils	Other
weights	0.75	0.05	-0.14	0.18	0.04	0.37	0.01	0.26	0.05	-0.58

Exercise 5

Exercise 6

Figure 2: Simulated Efficient Frontier



Exercise 7

Figure 3: Simulated efficient frontier, 100 observations

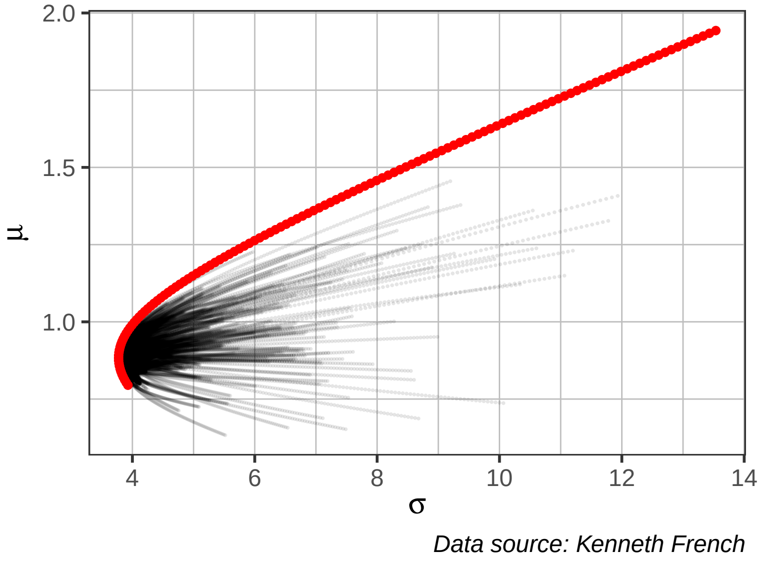
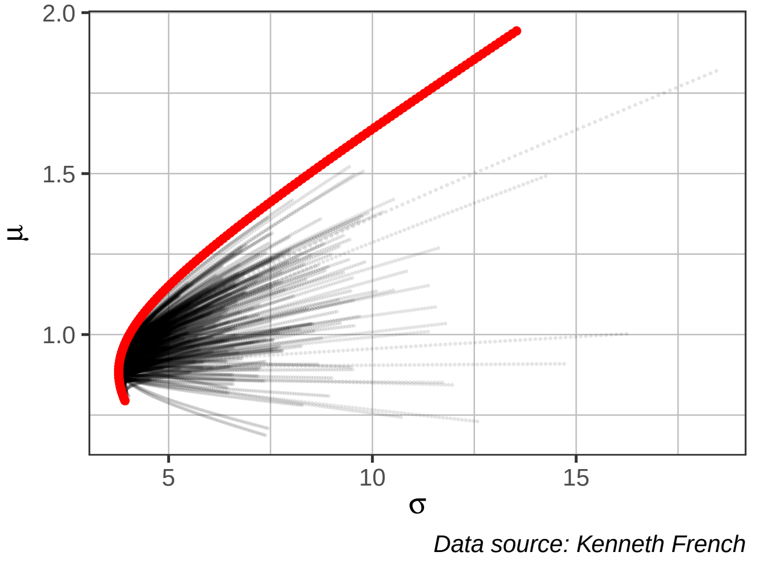


Figure 4: Simulated efficient frontier, 250 observations



Exercise 8

Exercise 9

Figure 5: Histogram of simulated sharpe ratios with T=100

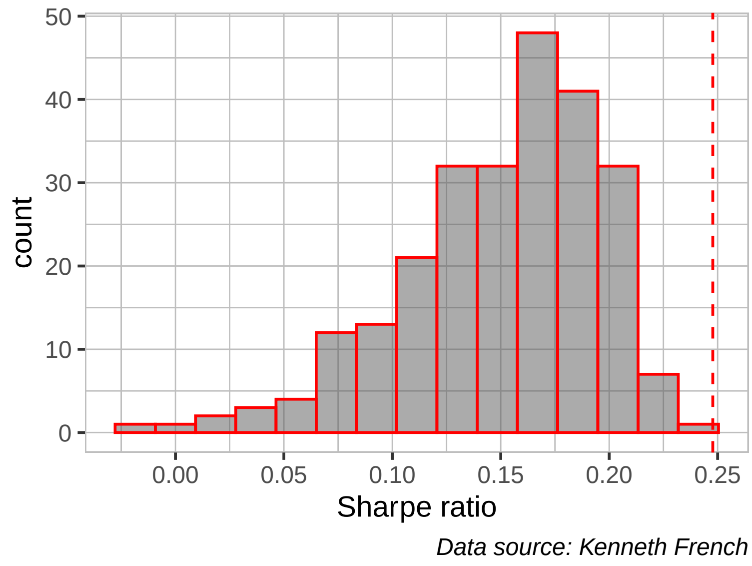


Figure 6: Histogram of simulated sharpe ratios with T=250

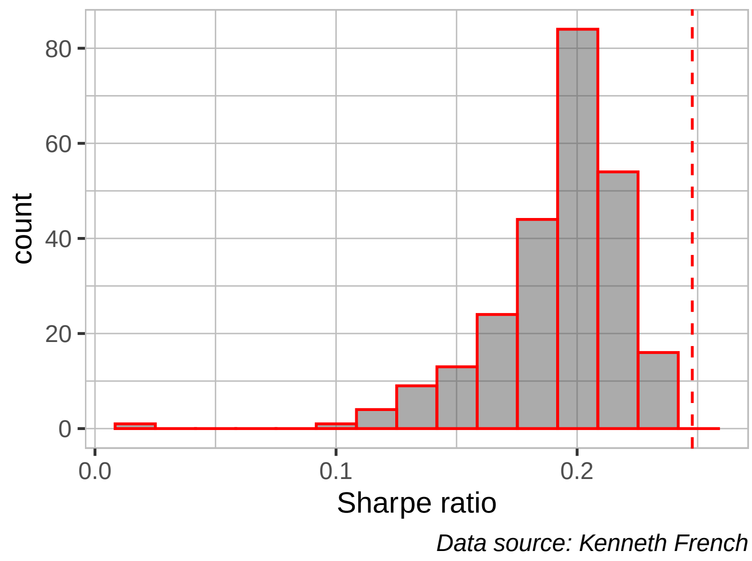


Table 4: Sharpe Ratios for Different strategies

	Unconstrained	Naive	Constrained	True
T=100	0.15	0.19	0.21	0.25
T=250	0.19	0.19	0.22	0.25