Advanced Empirical Finance: Hand-In 3

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Exercise 1.

We obtain a balanced panel of monthly (excess) returns for the entire time series from January 1962 to November 2020. Below, we provide the minimum, maximum and mean excess return of each industry. Furthermore, we provide the industry count as of November 2020, as names can change industry in the dataset:

Descriptive summary of balanced panel of stocks

industry	count_as_of_2020	max_return	min_return	mean_return
Finance	1	0.5482	-0.3685	0.0090
Manufacturing	66	1.5134	-0.7295	0.0083
Mining	6	0.8602	-0.6039	0.0089
Missing	0	0.1366	-0.1696	-0.0030
Public	2	0.5049	-0.3880	0.0061
Retail	4	0.7702	-0.6135	0.0082
Services	7	1.3018	-0.5927	0.0084
Transportation	3	0.4883	-0.4555	0.0102
Utilities	29	1.1129	-0.6743	0.0056
Wholesale	1	0.4046	-0.2336	0.0081
Total	119	1.5134	-0.7295	0.0077

The total data-set consists of 119 stocks, each with 707 observations. The max/min returns, don't indicate any errors. Furthermore, we see that a large part of the dataset is made up of two industries, Manufacturing and Utilities. This may be more reflective of the market of 1962 (which we require the firms to be present in), however, may not reflect the market today, where Finance and Technology firms are a much larger part of the market. In fact, the SIC doesn't even consider computer, software and information technology sectors. As a result, strategies used on this dataset will not allow for investing in some of the emerging industries, which may limit their return.

Exercise 2.

We wish to find the following weights

$$
\begin{aligned}
\omega^*_{t+1} &:= \arg\max_{\omega\in\mathbb{R}^N, \iota'\omega=1} \omega'\mu - \frac{\lambda}{2}(\omega-\omega_{t^+})'\Sigma(\omega-\omega_{t^+}) - \frac{\gamma}{2}\omega'\Sigma\omega \\
&= \arg\max_{\omega\in\mathbb{R}^N} \omega'\mu - \frac{\lambda}{2}\omega'\Sigma(\omega-\omega_{t^+}) + \frac{\lambda}{2}\omega'_{t^+}\Sigma(\omega-\omega_{t^+}) - \frac{\gamma}{2}\omega'\Sigma\omega \\
&= \arg\max_{\omega\in\mathbb{R}^N} \omega'\mu - \frac{\lambda}{2}\omega'\Sigma\omega + \frac{\lambda}{2}\omega'\Sigma\omega_{t^+} + \frac{\lambda}{2}\underbrace{\omega'_{t^+}\Sigma\omega}_{=\omega'\Sigma\omega_{t^+}} - \frac{\lambda}{2}\underbrace{\omega'_{t^+}\Sigma\omega_{t^+}}_{\text{Constant}} - \frac{\gamma}{2}\omega'\Sigma\omega \\
&= \arg\max_{\omega\in\mathbb{R}^N} \omega'\underbrace{\left(\mu+\lambda\Sigma w_{t^+}\right)}_{=:\mu^*} - \frac{\gamma}{2}\omega'\underbrace{\Sigma\left(I+\frac{\lambda}{\gamma}\right)}_{=:\Sigma^*}\omega,
\end{aligned}
$$

and we know from the notes that this maximization problem has the solution:

$$
\omega^*_\gamma = \frac{1}{\gamma}\left(\left(\Sigma^*\right)^{-1} - \frac{1}{\iota'\left(\Sigma^*\right)^{-1}\iota}\left(\Sigma^*\right)^{-1}\iota\iota'\left(\Sigma^*\right)^{-1}\right)\mu^* + \frac{1}{\iota'\left(\Sigma^*\right)^{-1}\iota}\left(\Sigma^*\right)^{-1}\iota \tag{1}
$$

The assumption of transaction costs proportional to volatility can be argued as the transaction cost reflects the risk of the market-makers[1]; the more volatile the price, the higher change the price will exceed their selling price or fall short of the buy price. Furthermore, the bid-ask spread widens if there is excess demand or supply, and as a consequence, transaction cost increases. We estimate μ^* and Σ^*, and assume we hold the naive portfolio (with $1/N$) in each stock; we then find the optimal portfolio weights in the next period for different transaction costs using (1). Below we plot the L^1-norm distance between the new next period optimal weights and efficient portfolio (i.e. our portfolio choice for $\lambda = 0$). To no surprise, the higher the transaction cost, the greater the distance to the efficient portfolio, as we will be more reluctant to change our portfolio. For $\lambda \to \infty$ we won't make any trades, and the distance will converge to 13.58, i.e. the L^1-norm distance between the naive and efficient portfolio.

[1] Agents constantly offering to buy/sell assets at a lower/higher price respectively.

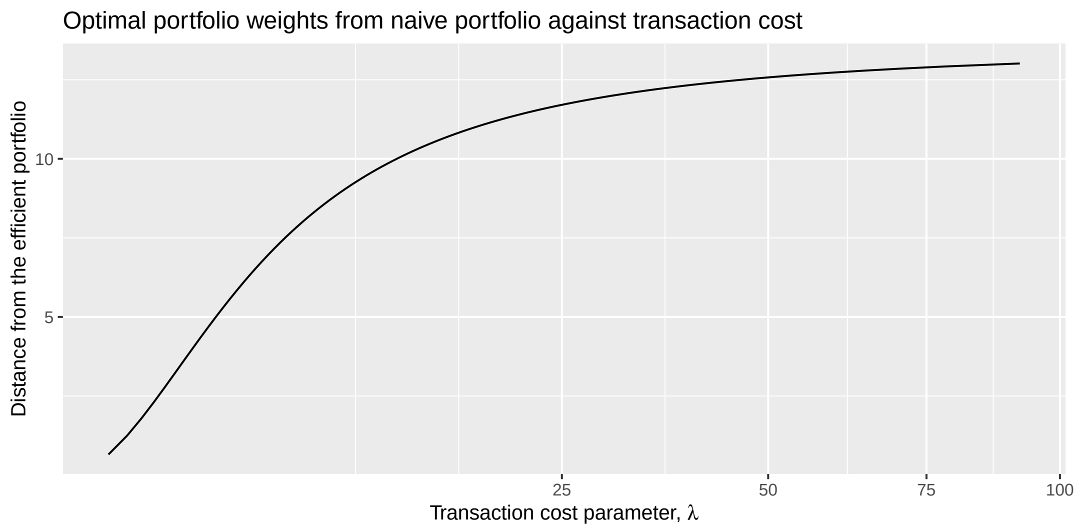

Figure 1: L^1-norm distance between the new next period optimal weights and efficient portfolio for different transaction costs

Exercise 3.

We want to backtest the methods, providing a rolling-window estimate of μ and Σ. This is problematic when having 119 stocks in our dataset, and as a consequence we need a large rolling-window. For that reason, we choose a window of 432 time points, equivalent of 432 months/36 years. This is a rather large estimation period, but given our rich data-source, it should be in order. Nevertheless, we recognize that we still may run into issues of $\hat{\Sigma}$ being invertible, which we combat through linear shrinkage to the equicorrelation matrix, estimating $\hat{\rho}$ as the average of the correlations in the rolling window. To estimate our α, we let us be inspired by a single data-fold from machine-learning, by using the following method

1. Estimate $\hat{\mu}$ as standard.
2. Split our rolling-window into two data sets consisting of the first 18 years used for training and the remaining 18 years for validating.
3. Estimate $\hat{\Sigma}_{\text{train}}$ and $\hat{\rho}$ from the first 18 years of data.
4. Estimate $\hat{\Sigma}_{\text{validating}}$ on the remaining 18 years; pick the α that minimizes the error:

$$\left\| \alpha F + (1-\alpha)\hat{\Sigma}_{\text{train}} - \hat{\Sigma}_{\text{validating}} \right\|_2$$

5. The resulting $\hat{\Sigma}$ we will use is $\hat{\alpha} F + (1-\hat{\alpha})\hat{\Sigma}_{\text{train}}$

Having defined the rolling-window period and the estimation methods of the covariance matrix and the mean return, we can now implement the 3 specified strategies to back-test, evaluating their performance based upon the net returns (adjusted for transaction costs) and Sharpe-Ratio (average return over standard deviation) and average Turnover (average of $|w_{t+1} - w_{t^+}|$ indicating how much the weights change on average. Note that turnover is given in percentage. The results are given below:

Descriptive summary of balanced panel of stocks

strategy	Mean	SD	Sharpe ratio	Turnover
a. Naive	9.75	16.12	0.61	5.44
b. MV with TC	-71.03	192.60	NA	4338.48
c. MV with TC, no short selling	10.79	21.23	0.51	11.57

We see that the mean-variance efficient portfolio with a no short-selling constraint in the spirit of Jagannathan and Ma obtains the highest mean net return, but the Naive portfolio performs better when risk-adjusting, obtaining a lower standard deviation and Sharpe ratio. The portfolio that minimizes transaction costs but allows for short-selling is performing very poorly. It is clear from the high turn over that we sometime go "very short" in some assets. It can be hypothesized that this is caused by a changing $\hat{\mu}$ and $\hat{\Sigma}$ estimates, and allowing the portfolio to be "too flexible" (as in it allows for shorting) results in extreme results. To demonstrate this effect, we plot the histogram of the difference between the maximum and minimum (equivalent to the L^∞-norm) estimated return of each stock:

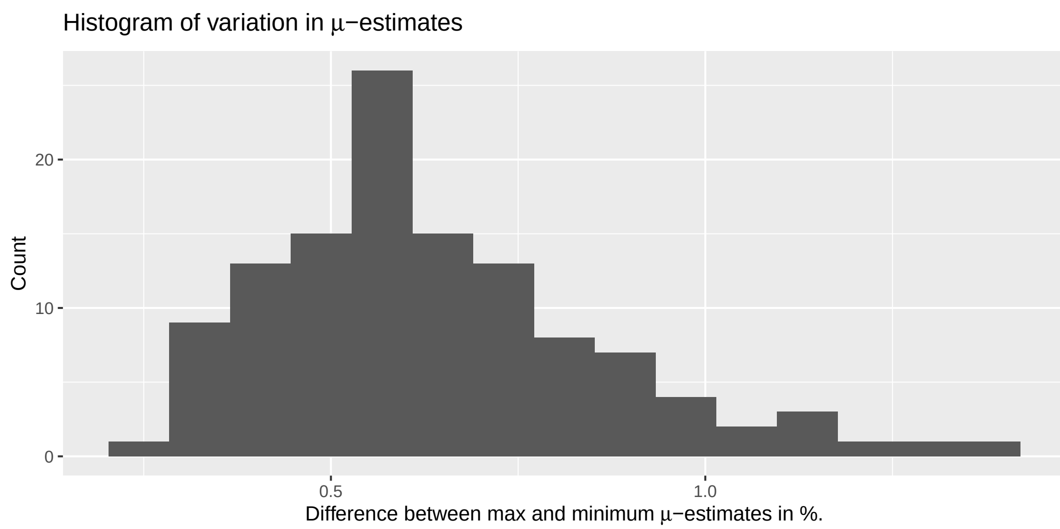

Figure 2: Difference between the maximum and minimum estimated return of each stock:

The estimate of monthly return can vary as much as 0.015 equivalent to 1.5%. Hence one testing period a stock can look rather attractive, providing an estimated mean monthly return 1.5% and hence we will go long that asset, but the next period it can have 0% return, meaning the portfolio changes dramatically. The Σ-estimates are most likely worse, since there are more parameters to fit. Hence, the problem may not be inherited in the strategy, but in the actual estimates being too volatile over time. The problem can be reduced by rethinking the estimation or changing the strategy to a multi-period maximization problem where we consider that the estimates may change over time.

There are more limitations of this out-of-sample experiment; our transaction costs dependent on Σ, which is not observable and we have just argued is fitted rather poorly. As a result, our transaction costs are not properly calculated. This alone will not help the 'b. MV with transaction costs' strategy alone, as it performs rather poorly even when using the raw-returns. However, it is influenced by the transaction costs in the maximization problem itself.