A STUDY OF CHAOTIC PENDULUM DYNAMICS

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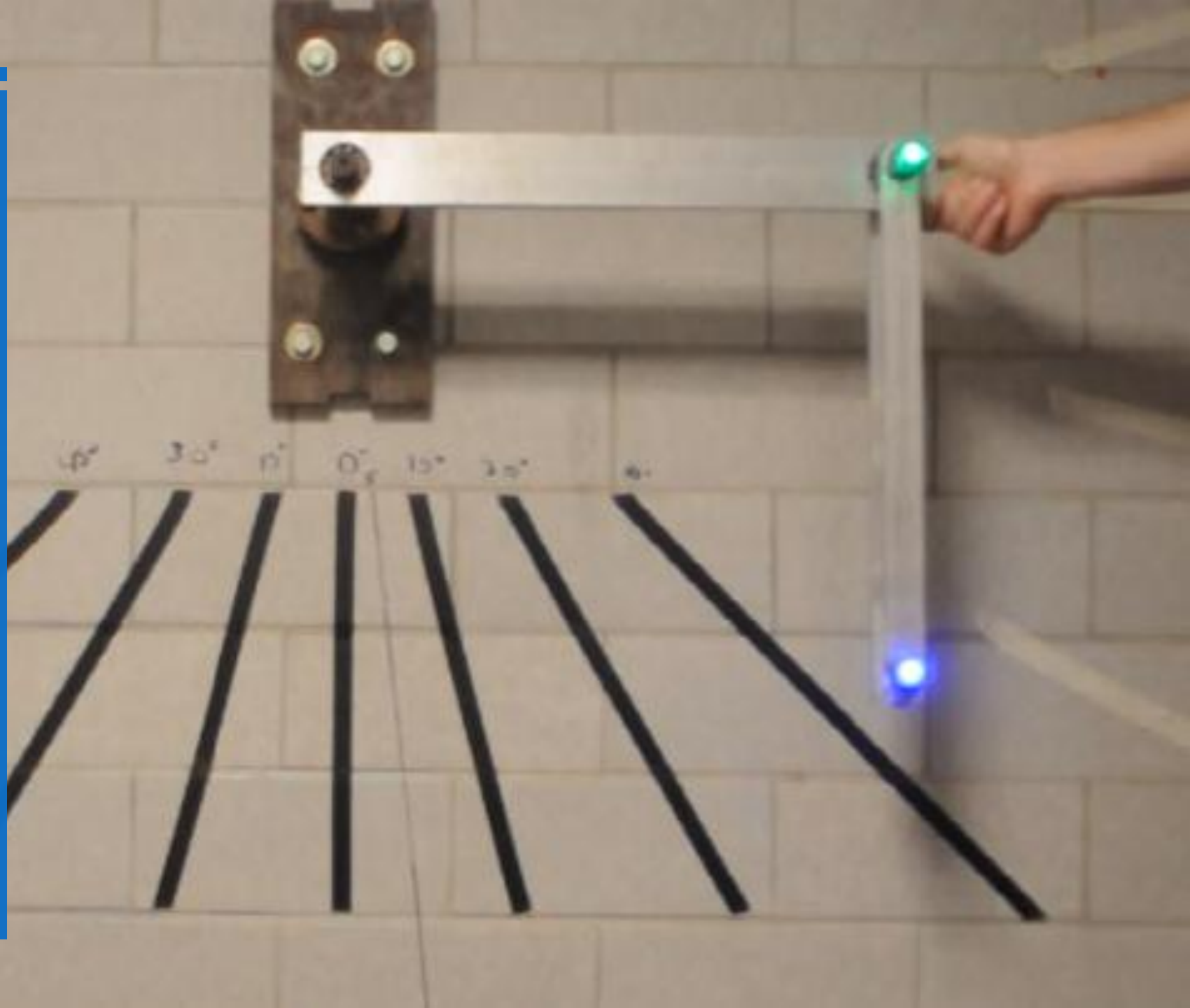
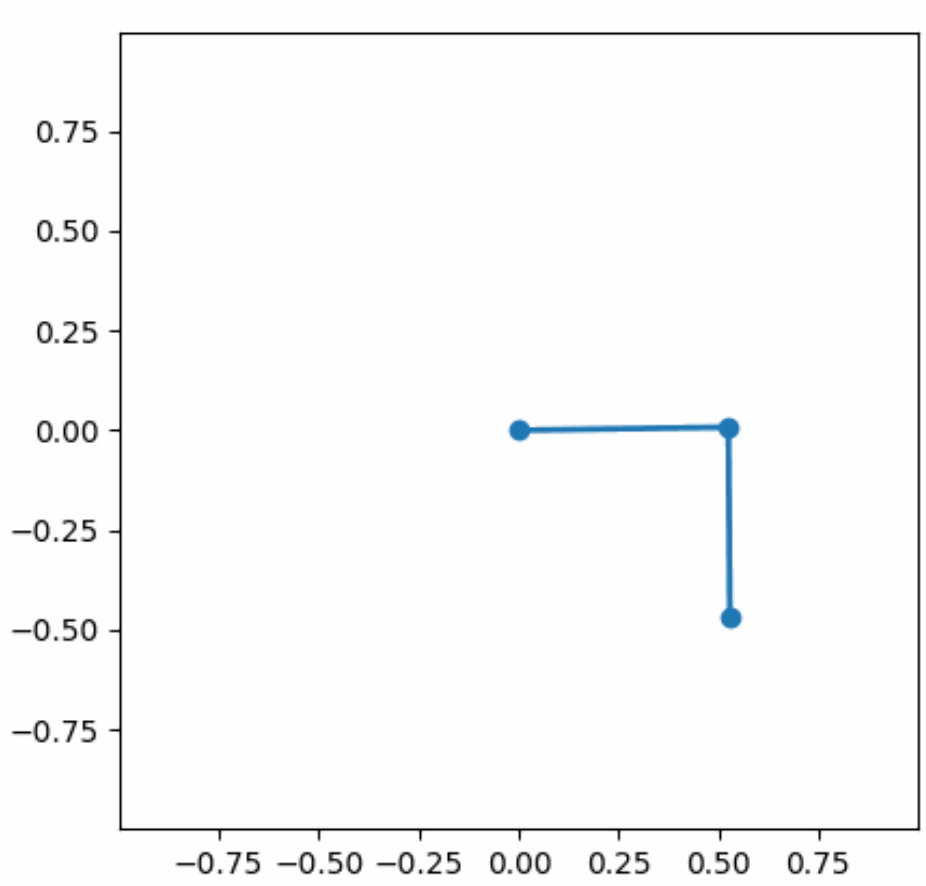


INTRODUCTION – WHAT DOES IT MEAN TO BE CHAOTIC?



Small changes to the initial state will lead to significant differences in the system's outcome.

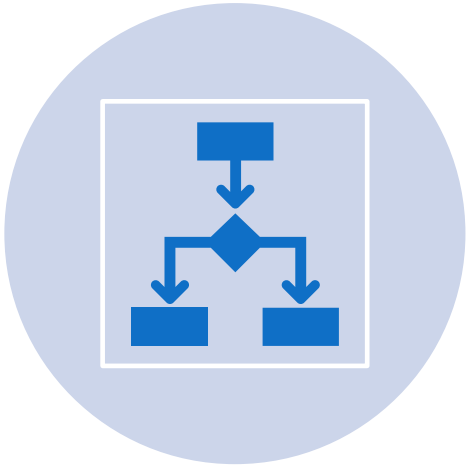
THEORY – THE LAGRANGIAN

- Determined position equations for each arm
- Develop the equations of Kinetic and Potential Energy
- Derive the equations of motion below using the Euler-Lagrange formalism

$$\ddot{\theta}_1 = \frac{-6(m_1 + 2m_2)g\sin\theta_1 - 2m_2 l_1 \dot{\theta}_1^2 \sin(2\theta_1 - 2\theta_2) + 12m_2 g \sin\theta_2 \cos(\theta_1 - \theta_2) - 2m_2 l_2 \dot{\theta}_2^2 \sin(\theta_1 - \theta_2)}{l_1(m_1 + 4m_2) - 4m_2 l_1 \cos^2(\theta_1 - \theta_2)}$$

$$\ddot{\theta}_2 = -2\frac{l_1}{l_2}\ddot{\theta}_1 \cos(\theta_1 - \theta_2) + 2\frac{l_1}{l_2}\dot{\theta}_1^2 \sin(\theta_1 - \theta_2) - 6\frac{g}{l_2}\sin\theta_2$$

DESIGN



CREATED A SIMULATION BASED ON OUR EQUATIONS OF MOTION

DEVELOPED A COMPUTER VISION ALGORITHM TO TRACK LEDS

SIMULATION DESIGN

We implemented an ODE solver to evaluate the previously shown equations

We iterated through steps by solving one equation in terms of the other, and then substituting that new value into the second equation

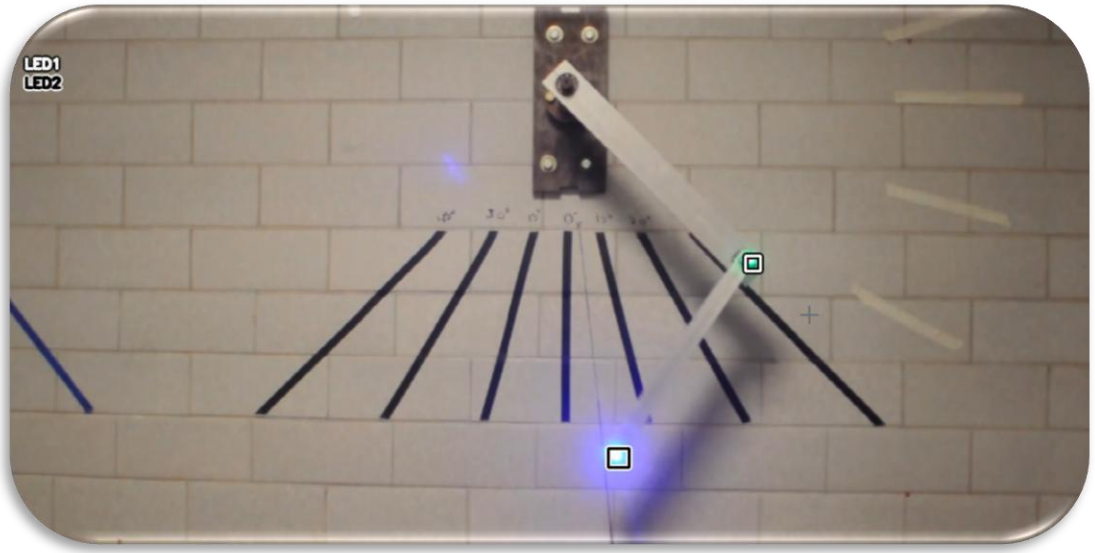
Constructed our values into a graph of angular displacement

COMPUTER VISION DESIGN

Developed Python code to identify and track the position of two LEDs

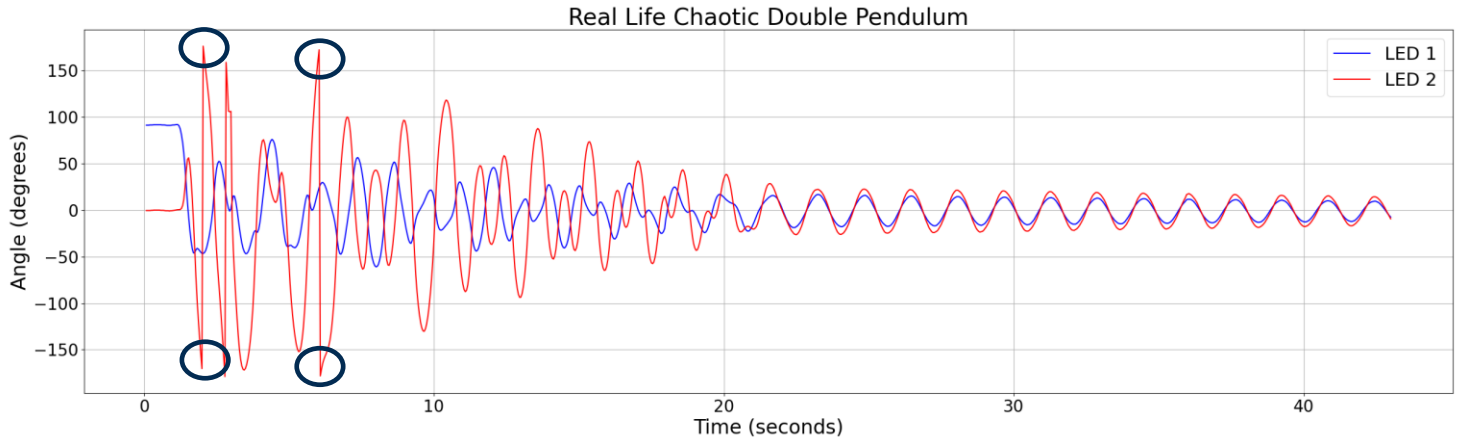
Graphed the angular positions of the LEDs

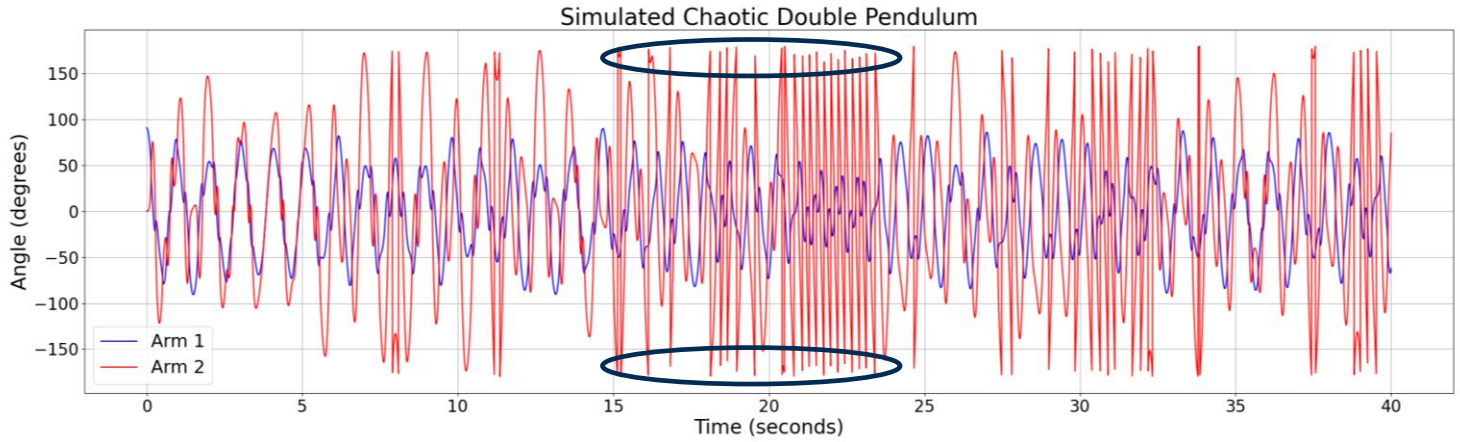
Used to compare with our simulation



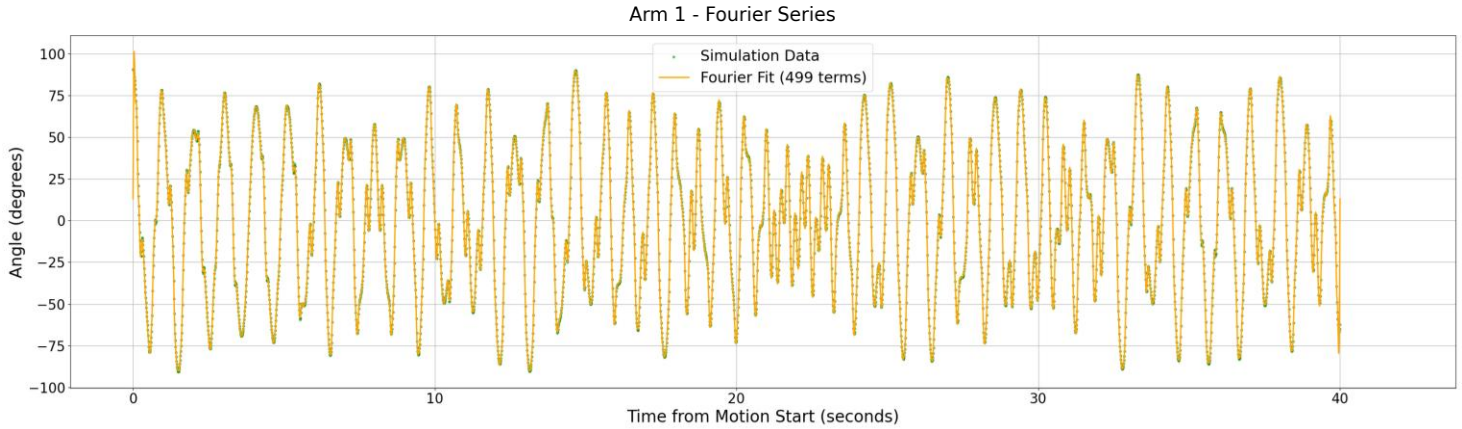
DATA AND GRAPHS

- Real life approaches steady state
- Simulation lacks dampening
  - Never loses energy or approach a steady state
- LED 2/Arm 2 make "jumps" when completing over-the-top swings





ANALYSIS

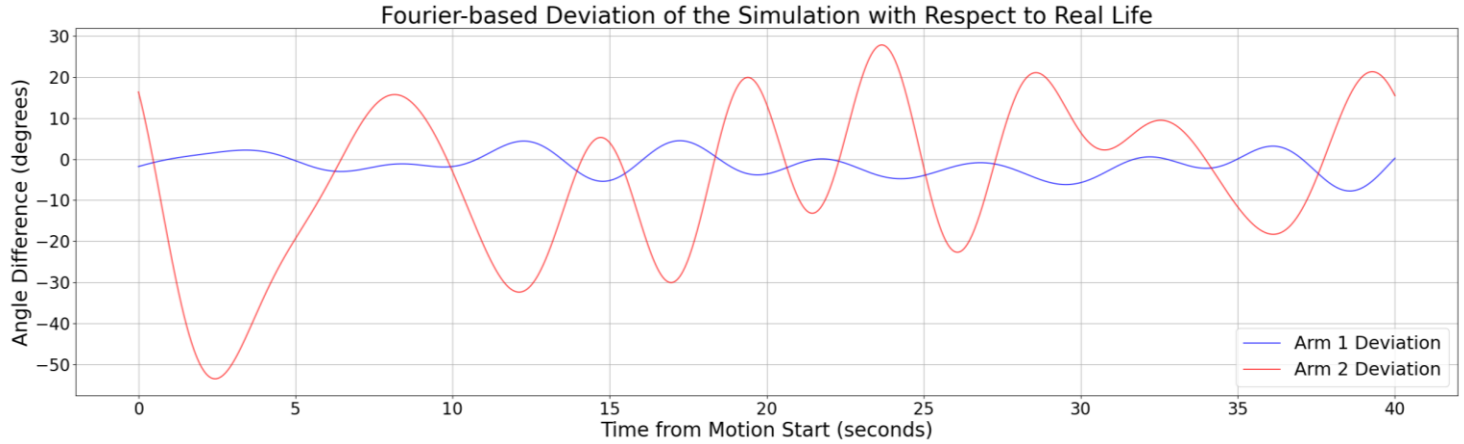


Fourier Series (left):

- Difficult to compare discrete datasets
- A continuous function representative of the data solves this problem
- Continuous function overlays scatterplot of data

Comparison of Graphs (right):

- Subtracted our simulated angular position from our experimental data
- Allows us to assess the validity of our simulation



THANKS FOR WATCHING

Always remember! If one pendulum arm won't confuse you, TOUCAN!!!!!!