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	1	2	3	4	5
A	**1**,3	**1**	1,**2**	**2**	**2**
B	**2**	2,**3**	**3**	3,**1**	**1**
C					**3**

<u>Proof 1</u>

Theorem: In any execution of the algorithm, if a woman receives a proposal on day_i, then she receives some proposal on every subsequent day until the algorithm terminates.

Proof by induction:

Base case:

- On day_1 a $woman_1$ receives a proposal from man_A.

Inductive Step:

- Given a $woman_1$ who receives at least one proposal on day_i from a man.
- By part 1 of the algorithm, each man that proposed to $woman_1$ must have $woman_1$ in the front of his list.
- The second part of this algorithm states that $women_1$ must accept one of those offers, on day_i let's assume that's man_A.
- According to the third part of the algorithm if a man is rejected, he removes the woman from his list. Since Man_A's offers was accepted, he will not remove $women_1$ from his list, thus he will propose to her the next day as well. Hence on day_{i+1} she will receive a proposal.
- This cycle will continue every day. QED

<u>Proof 2</u>

Theorem: In any execution of the algorithm, if a woman receives no proposal on day_i, then she receives no proposal on any previous day_j , $1 \le j < i$

Proof by Contradiction:

Assume: In any execution of the algorithm, if a woman receives no proposal on day_i, then she receives a proposal on any previous day_j , $1 \le j < i$

The assumption stated another way: In any execution of the algorithm, if a woman receives a proposal on day_j, then she will not receive a proposal on any further day_i , $1 \le j < i$

- Given a $woman_1$ who receives a proposal on day_j from a man. (1)
- By proof 1, she will receive a proposal on all later days including day_i.
- This contradicts the assumption. QED

<u>Proof 3</u>

Theorem: In any execution of the algorithm, there is at least one woman who only receives a single proposal.

Direct Proof:

◆ The last part of the algorithm states that the process ends when all men and women are paired, that means that there are an equal number of men and women.

◆ It follows that if all women receive a proposal, then they will all receive only one, and at that point the process will end.

◆ The last part of the algorithm therefore shows us that **only on the last day** all women will get a proposal from exactly one man, otherwise the algorithm would have ended on an earlier day.

◆ Following this, it comes out that on the day before the last day, (at least) one of the women must not have received an offer.

-This is because if she would have received a proposal then that would mean that every woman already received a proposal making this the last day, and that can't be considering it's the second to last day.

◆ That means that on the second to last day there is a woman who didn't receive an offer but on the next day she did (because all women have an offer on the last day).

◆By Proof 2, she must not have received an offer any day prior to the final day.
-Thus, this woman will only receive an offer on one day (the last day).
-Therefore, in any execution of the algorithm, there is at least one woman who only receives a single proposal. QED