## Adam Frenkel

First one piece of notation: When referring to a budget that is smaller than the given budget, but must be solved in order to do tabulation, I'll use the notation budget<sub>k</sub>.

<u>Key Insights</u>: The first insight was that that you must take into account using every possible type of each class. The second insight was that you can take it one step at a time adding in another class of seforim each time, and store the best possible total spent for every budget<sub>k</sub> (bottom up tabulation). The main insight was that when adding another class, figuring out the best type to add, for a budget<sub>k</sub> is trivial given the previous classes. This is because one can simply find the max of all the types of the given class when adding: (the previous class's value for the (budget<sub>k</sub> -  $T_i$ )) +  $T_i$ . By taking the max of this you will find the best type to add for this budget<sub>k</sub>. Here is a diagram to explain these insights:

Budget: 8

Class: Mishna. Types: Zeraim – 1, Moed – 4, Nashim – 7

Class: Chumash. Types: Bereshis – 2, Shemos – 4, Vayikra – 6

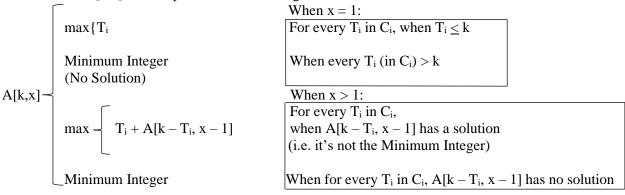
	Mishna	Mishna & Chumash	
Budget <sub>1</sub>	1	X	
Budget <sub>2</sub>	1	X	
Budget <sub>3</sub>	1	1+2=3 Ma	ax=3
Budget <sub>4</sub>	4	1+2=3 Ma	ax=3
Budget <sub>5</sub>	4	1+2=3, 1+4=5 Ma	ax=5
Budget <sub>6</sub>	4	4+2=6, 1+4=5 Ma	ax=6
Budget <sub>7</sub>	7	4+2=6, 1+4=5, 1+6=7 Ma	ax=7
Budget <sub>8</sub>	7	4+2=6, 4+4=8, 1+6=7 Ma	ax=8

Optimal Substructure: At each step, you can calculate the optimal solution for each C<sub>i</sub>, adding in one class at a time.

Overlapping Subproblems: At every step you can rely on the previous step's calculation of the optimal solution for the  $budget_k - each T_i$ .

## Recursion:

Notation: Let A[k,x] be the optimal value for budget<sub>k</sub> with x classes



## Runtime:

For maxAmountThatCanBeSpent(), the runtime is:

 $\textit{O}(C_j \cdot T_j \cdot budget)$ 

(Terminology:  $C_j$  is the number of classes,  $T_j$  is the largest number of Types in any of the given classes, and budget is the given budget for the problem.)

For solution(), the runtime is:  $O(C_i)$