Lab 1: Numerical differentiation and Quadrature

Due: Friday 23 April 2021, 20:00

Instructions:

i) All your solutions must be in ONE file.

ii) The file name should be studentnumber.py, for example 87654321.py

Question 1. Write a function to perform Trapezoidal rule to approximate $\int_a^b f(x)dx$.

```
def function_name(f, n, a, b):
    ......................
    return integral_approx
```

Here f is function, n is the number of subintervals and a, b are the lower limit and upper limits of the integral respectively. Test the function for fs of your choice. [5]

Question 2. Write a function to perform Simposon rule to approximate $\int_a^b f(x)dx$.

```
def function_name(f, n, a, b):
    ......................
    return integral_approx
```

Here f is function, n is the number of subintervals and a, b are the lower limit and upper limits of the integral respectively. Test the function for fs of your choice. [5]

Question 3. Using your answers to 1 and 2 above and taking

$$f(x) = \frac{\cos(x)\ln(\sin(x))}{\sin^2(x)+1}, a = 0.1, \quad b = \pi/2,$$

write code to produce the following table:

```
n             Trapezoidal          Simpson        Absolute Difference
------------------------------------------------------------------------
2             -0.9298943           -0.6833796              0.2465147
4             -0.6871394           -0.6062211              0.0809183
8             -0.6145815           -0.5903955              0.0241860
16            -0.5941871           -0.5873889              0.0067981
```

32	-0.5887964	-0.5869996	0.0017969
64	-0.5874217	-0.5869635	0.0004582
128	-0.5870761	-0.5869609	0.0001152
256	-0.5869895	-0.5869607	0.0000288
512	-0.5869679	-0.5869607	0.0000072
1024	-0.5869625	-0.5869607	0.0000018
2048	-0.5869611	-0.5869607	0.0000005
4096	-0.5869608	-0.5869607	0.0000001
8192	-0.5869607	-0.5869607	0.0000000
16384	-0.5869607	-0.5869607	0.0000000
32768	-0.5869607	-0.5869607	0.0000000
65536	-0.5869607	-0.5869607	0.0000000
131072	-0.5869607	-0.5869607	0.0000000
262144	-0.5869607	-0.5869607	0.0000000
524288	-0.5869607	-0.5869607	0.0000000

Marks: [5]

Question 4. Write some code to determine the smallest n such that

$$|T_n - S_n| < 10^{-5}$$

where T_n is the value for Trapezoidal rule and S_n is the value of Simpson rule to approximate $\int_a^b f(x)dx$ where

$$f(x) = \frac{1}{\sqrt{1 + \exp(x) - x}}, \quad a = 0, \quad b = 3$$

using n subintervals. [5]

Question 5. The error function is defineed as

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt.$$

Write some code to approximate $\text{erf}(x)$ for some given x. You code should approximate using the midpoint rule. [5]

Question 6. Your friend, a postgraduate student in mathematics at Wits thinks that the quadrature rules she learnt in second year are not appropriate to approximate the error function. She has calculated an approximate value for $\text{erf}(x)$ at $x = 0.5$ and found that

$$\text{erf}(0.5) \approx 0.520500,$$

using a method called Gaussian quadrature with $n = 4$ which she learnt while studying heat transfer for her masters thesis. She is a very brilliant and confident student and you think that her approximation can be taken as exact.

Using your solution to Question 5, write code to determine the interval length h so that the absolute error (when you compare your value with your postgraduate friend's) is less that 10^{-6}. [5]

(Hint: Do not use equations 2.9 or 2.16 in your notes or similar expressions)