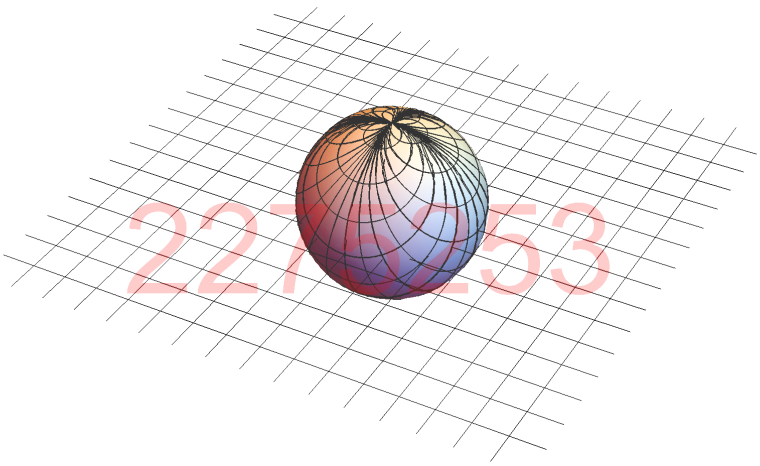
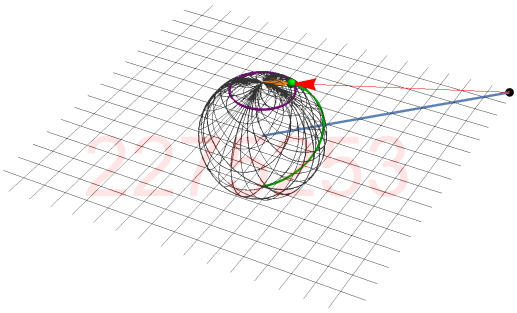
Adam Gordon	Assignment 02	APPM2007A
2275253	Parameteric equations	2021-09-14

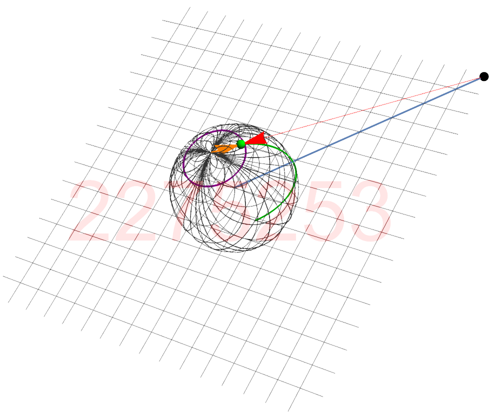
Question 1

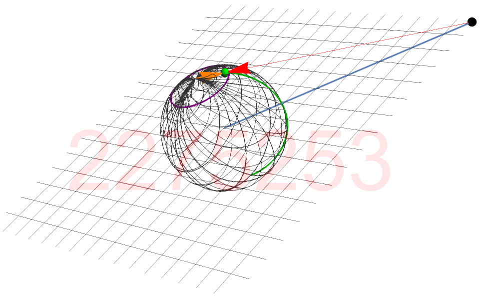
We assign coordinates to surface of the sphere in S^2 by mapping points in R^2 in a continues way. The point z on the plane and a point a on the sphere is now represented by a pair of numbers. We can now fill in all points on the sphere from the projection map of R to S. The coordinate grid is pinched at north pole as it vanishes into a point.



Question2 (More than one angle as been provided in order for the marker to better see the image)







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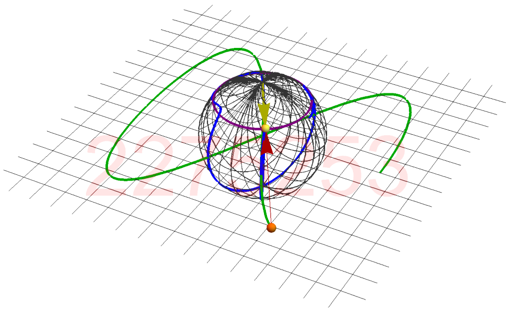
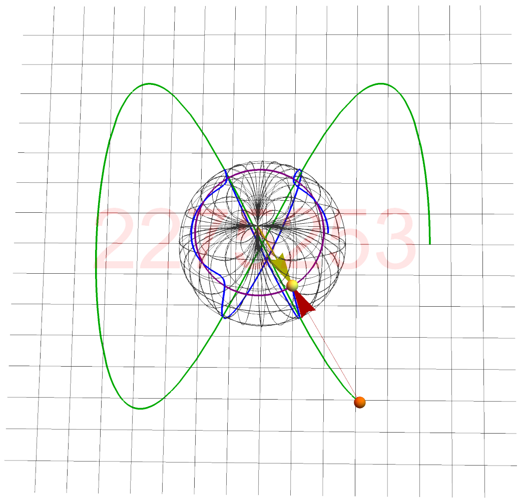
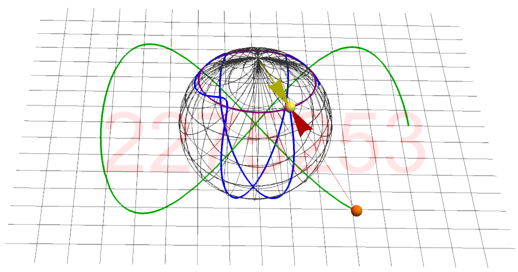
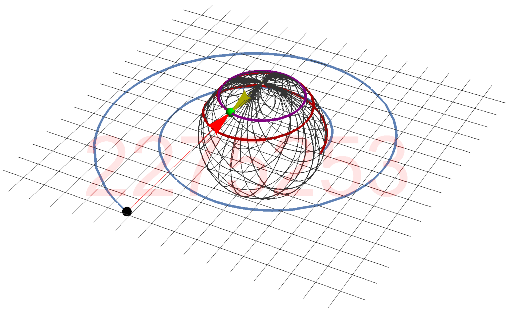
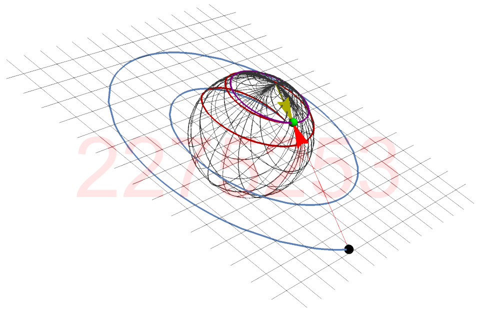


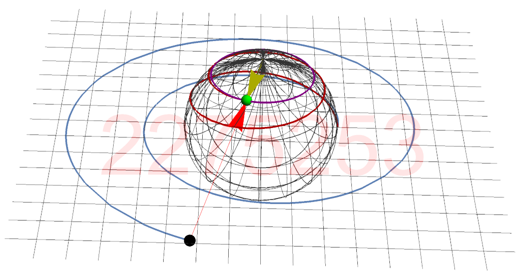




Question 4 (More than one angle as been provided in order for the marker to better see the image)







Question 5

1) The path length of $p_1(t) = 3.14159$
2) The path length of $p_2(t) = 12.5301$
3) The path length of $p_3(t) = 10.159$

Question 6 (More than one angle as been provided in order for the marker to better see the image)

parametric equation p_1 on S:

$$p1S[t] = S[Tan[\pi/2 * t] * Cos[\pi/4], Tan[\pi/2 * t] * Sin[\pi/4]];$$

1.1) Construction of Unit tangent vector $\vec{T_1}(t)$(Red)

$$T1[t] = Simplify[p1S'[t]/Norm[p1S'[t]], t > 1];$$

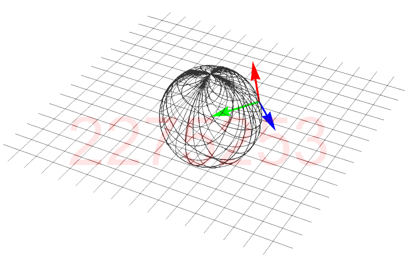
1.2)Construction of Unit acceleration $\vec{N_1}(t)$(Green)

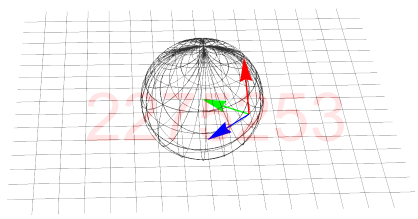
$$N1[t] = Simplify[T1'[t]/Norm[T1'[t]], t > 1];$$

1.3) Construction of Unit bi-normal $\vec{B_1}(t) = \vec{T_1}(t) x \vec{N_1}(t)$(Blue)

$$B1[t] = Simplify[Cross[T1[t], N1[t]]];$$

The curve moves radially outward from origin in the plane. The velocity vector, acceleration vector and bi-normal vector's are attached to the surface of the sphere. Acceleration vector always points toward the centre of the sphere, the velocity vector points in the direction in which curve is moving(parameter increases). The Bi-normal vector is normal to curve, Velocity vector and Acceleration vector and is tangent to the sphere at all times while the Velocity and Bi-normal vectors remain normal to curve.
Image of Construction with t=0.55:





Question 7 (More than one angle as been provided in order for the marker to better see the image)

parametric equation p_2 on S:

$$p2S[t] = S[2Cos[(2\pi) * t], 2Sin[2 * (2\pi) * t]];$$

1.1) Construction of Unit tangent vector $\vec{T_2}(t)$(Red)

$$T2[t] = Simplify[p2S'[t]/Norm[p2S'[t]], t > 1];$$

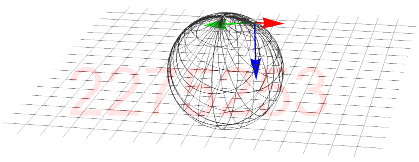
1.2)Construction of Unit acceleration $\vec{N_2}(t)$(Green)

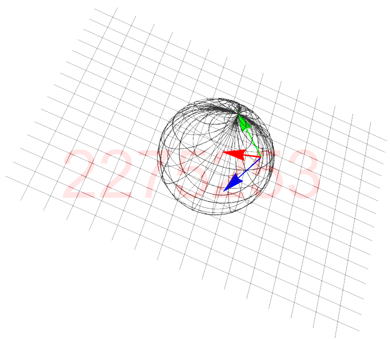
$$N2[t] = Simplify[T2'[t]/Norm[T2'[t]], t > 1];$$

1.2) Construction of Unit bi-normal $\vec{B_2}(t) = \vec{T_2}(t)x\vec{N_2}(t)$(Blue)

$$B2[t] = Simplify[Cross[T2[t], N2[t]]];$$

The Velocity vector moves in the direction of motion(increase). The Acceleration vector starts by pointing to the north pole of the sphere and as the time increases it switches from facing the north pole to the south pole and back to north pole. The Bi-normal vector starts facing out the sphere and rotates to face in the sphere and then out the sphere once again as t increases
Image of Construction with t=0.55:





Question 8 (More than one angle as been provided in order for the marker to better see the image)

parametric equation p_3 on S:

$$p3S[t] = S[(1 + (2 * t)) * Cos[2 * 2\pi * t], (1 + (2 * t)) * Sin[2 * 2\pi * t]];$$

1.1) Construction of Unit tangent vector $\vec{T_3}(t)$(Red)

$$T3[t] = Simplify[p3S'[t]/Norm[p3S'[t]], t > 1];$$

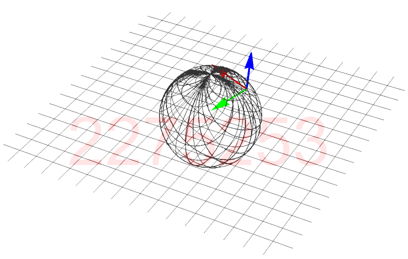
1.2)Construction of Unit acceleration $\vec{N_3}(t)$(Grene)

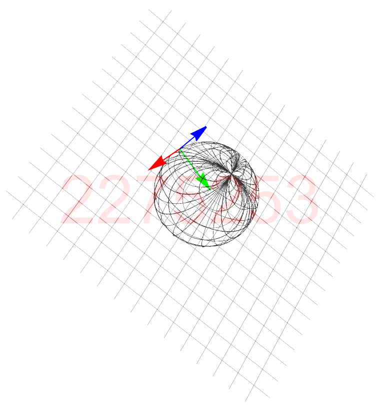
$$N3[t] = Simplify[T3'[t]/Norm[T3'[t]], t > 1];$$

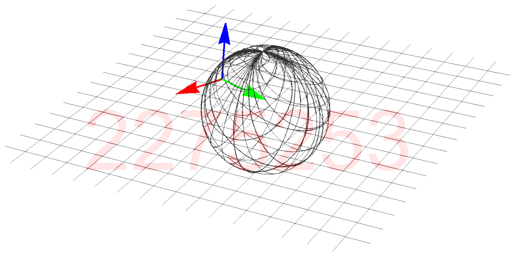
1.2) Construction of Unit bi-normal $\vec{B_3}(t) = \vec{T_3}(t)x\vec{N_3}(t)$(Blue)

$$B3[t] = Simplify[Cross[T3[t], N3[t]]];$$

The curve is spiral on the plane which maps to spiral on the sphere. The Velocity vector points in direction in which the curve is moving. The Bi-normal vector points vertically upwards and is no longer tangent to the sphere. The Acceleration vector is pointing inwards towards the internal area of sphere but not towards the geometric centre of sphere. As such the Bi-normal vector rotates off the surface of the sphere. The Velocity vector still points in direction of increase.Initially the Acceleration vector points towards the geometric centre and therefore the Bi-normal vector is tangent to sphere at the start. but as t increases Acceleration vector no longer points towards the geometric centre and therefore the Bi-normal vector points away from surface of sphere.
Images of Construction with t=0.1:







Question 9 (More than one angle as been provided in order for the marker to better see the image)

The plot of $\vec{T_1}(t)$ (blue) is in the shape of a parabola meaning that as $t \to 1$, $\vec{T_1}(t)$ will move and point in the direction of increase (movement) of the curve. It also points radially outward from origin to plane

The plot of $\vec{T_2}(t)$ (green) is in an almost zig-zaggy shape $t \to 1$, $\vec{T_2}(t)$ will move along this path. The Path deviates from what we would expect due to the nature of the parametrisation and the sphere itself

The plot of $\vec{T_3}(t)$ (orange) is following the shape of a spiral. meaning that as $t \to 1$, $\vec{T_3}(t)$ The tangent vector will move up and along this spiral. The Spiral on the plane maps to the spiral on sphere. They have the same starting point however they separate and do not follow the same curve. The tangent vector curves along sphere on spiral and points in direction of movement of curve(increase)