This Course
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Models
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Programming in Python
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Dynamic Programming
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References

Introduction

Thomas H. Jørgensen

2023

Outline

1. This Course

2. Models

3. Programming in Python

4. Dynamic Programming

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  (Associate Professor, University of Copenhagen)

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  Dynamic programming in Python
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  Introduction to Programming and Numerical Analysis

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- Plan for today:
    1. Course description
    2. Programming in Python
    3. Introduction to Dynamic Programming

Teaching Methods

- **Lectures:** Wednesday 12-15
  - ~2 hours lecture
  - ~1 hour of problem-solving (e.g. programming)

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  I will provide code.
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- **Material:**
  Web: https://sites.google.com/view/householdbehavior
  GitHub:
  https://github.com/ThomasHJorgensen/HouseholdBehaviorCourse

Course Plan (preliminary)

Introduction

1. Introduction
2. Dynamic Programming and Structural Estimation

Part 1: Labor Supply

3. Static and Dynamic labor supply
4. Dynamic labor supply and human capital
5. Career costs of children
.. *No lecture: Work on assignment*
6. Household Labor Supply and Taxation
7. Household Labor Supply and Child-Related Transfers

Part 2: Family Formation and Dissolution

8. Models of Household Behavior
9. Divorce Law and Intra-Household Bargaining
10. Marriage and Divorce
11. Fertility and labor supply
12. Taxes, Transfers and Intra-Household Inequality

Outroduction

13. Buffer

Assignments and Exam

- **Exam** (Portfolio):
    1. 3 individual assignments.
       + peer feedback.
    2. 48 hour individual take-home exam.
       Model formulation, code modification, simulations, economic
       interpretations.

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- 3 individual **assignments** (hand-in on Absalon)
    1. Based on our dynamic **labor supply** model
       Deadline: March 17 (no lecture that week)
    2. Based on our dynamic **bargaining** model
       Deadline: April 28
    3. **Free**: Formulate a research question + model + data.
       Deadline: May 12
       Deadline for peer feedback: May 19

- All feedback can be used to improve assignments before exam date

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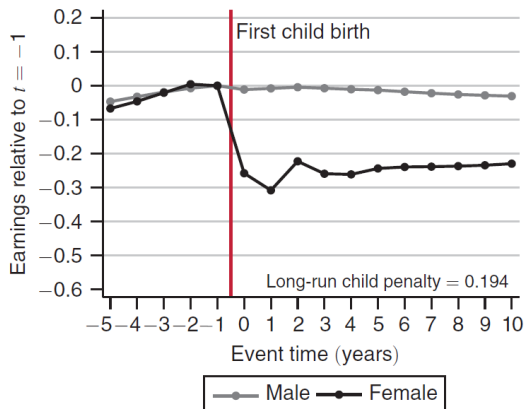
Models! What are they good for!?

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    Example: "Child penalty" (Kleven, Landais and Søgaard, 2019)

Panel A. Earnings

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  Example: "Career costs of children" (Adda, Dustmann and Stevens, 2017)
  "We estimate a dynamic life cycle model of labor supply, fertility, and savings,
  incorporating occupational choices, with specific wage paths and skill atrophy
  that vary over the career. This allows us to understand the trade-off between
  occupational choice and desired fertility, as well as sorting both into the labor
  market and across occupations. We **quantify** the life cycle career costs
  associated with children, how they **decompose** into loss of skills during
  interruptions, lost earnings opportunities, and selection into more child-friendly
  occupations. We analyze the **long-run effects of policies** that encourage
  fertility and show that they are considerably smaller than short-run effects."

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  "All models are wrong, but some are useful"

  George E.P. Box

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- Why ever use a model then?

- **The Lucas critique:** *Behavioral rules change with policy*
  $\Rightarrow$ policy advice can not rely on estimated behavioral rules
  (reduced-form estimates using existing variation)
  $\Rightarrow$ we need to estimate *structural (deep) parameters*

  > *"Invariance of parameters in an economic model is not, of course, a property which can be assured in advance, but it seems reasonable to hope that neither tastes nor technology vary systematically with variations in counter-cyclical policies." (Lucas, 1977)*

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  Implications of not-yet implemented policy reforms (e.g. child subsidies)
  Responses to changes in the economic environment (e.g. wages)
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$\rightarrow$ Tight link between **research question** and model formulation.

Learning Objectives

See all at https://kurser.ku.dk/course/a%c3%98kk08427u/2022-2023

- **Knowledge (two):**
  Define, formulate and interpret *models* of household behaviour
  Account for backwards induction and how to *solve* dynamic
  programming models

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- **Competences (two):**
  Discuss and evaluate *research* on household behavior over the life cycle
  Modify computer *code* to analyze small changes to simple models

How to read a research paper (in this course)

- Each lecture will be based on 1 mandatory (*) **research paper**
  What is the main *research question*?
  What is the *(empirical) motivation*?
  What are the central *mechanisms in the model*?
  What is the *simplest model* in which we could capture these?
  Challenging: Research frontier.

How to read a research paper (in this course)

- Each lecture will be based on 1 mandatory (*) **research paper**
  What is the main *research question*?
  What is the *(empirical) motivation*?
  What are the central *mechanisms in the model*?
  What is the *simplest model* in which we could capture these?
  Challenging: Research frontier.

- **How to read** a research paper in this course?
  - **Focus** on the questions above
    How do the questions interact and inform each other?
  - Try **not to get stuck** in too many details!
    (we can discuss some in class if you want)
  - Research papers often include many **"bells and whistles"**
  - Read ~**30 min before** each lecture.
    See reading-guide for each lecture

Outline

1. This Course

2. Models

3. **Programming in Python**

4. Dynamic Programming

Programming in Python

- **Only very simple**/unrealistic models can be solved with pen/paper
  $\rightarrow$ We need numerical methods

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- **Purpose** of having you programming:
  Learn best by implementing!
  Appreciate bottlenecks and challenges
  Better understanding of frontier research
  Set you free...!

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  $\rightarrow$ We need numerical methods

- **Purpose** of having you programming:
  Learn best by implementing!
  Appreciate bottlenecks and challenges
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- **Goal** is to keep things simple!
  No fancy numerical tricks (at least in beginning)
  Code should be "intuitive"
  $\rightarrow$ slow...
  (might have to do more efficient code)

Programming in Python

- **Setup** as
  Introduction to Programming and Numerical Analysis
  https://sites.google.com/view/numeconcph-introprog/
    - Visual Studio Code

- **Installation** guide:
  https://sites.google.com/view/numeconcph-introprog/guides/installation

- **Packages** (all by Jeppe Druedahl):
  EconModel
  consav
  See "01. Introduction to EconModel and consav.ipynb"

| This Course | Models | Programming in Python | Dynamic Programming | References |
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Introduction to Dynamic Programming (DP)

- **Agents maximize** expected discounted sum of utility throughout life
  Maximize wrt. $\{\mathcal{C}_t\}_{t=1}^{T}$
  Forward looking $\rightarrow$ dynamic
  Assume *optimal* behavior in all periods

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$$V_t(\mathcal{S}_t) = \max_{\mathcal{C}_t} U(\mathcal{C}_t, \mathcal{S}_t) + \beta \mathbb{E}[V_{t+1}(\mathcal{S}_{t+1}) | \mathcal{C}_t, \mathcal{S}_t]$$

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- $\beta \mathbb{E}[V_{t+1}(\mathcal{S}_{t+1})|\mathcal{C}_t, \mathcal{S}_t]$: expected discounted value of next-period
- $\mathcal{S}_{t+1} \sim F(\mathcal{C}_t, \mathcal{S}_t)$: transition density of states (fcn of $\mathcal{C}_t$!)
  (there might be other constraints)

Backwards Induction

- Solved by **backwards induction**

  1. Start with last/terminal period, T (no future)

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  2. Go to second-to-last (we now know period-T optimal behavior for all $\mathcal{S}_T$)

  $$V_{T-1}(\mathcal{S}_{T-1}) = \max_{\mathcal{C}_{T-1}} U(\mathcal{C}_{T-1}, \mathcal{S}_{T-1}) + \beta \mathbb{E}[V_T(\mathcal{S}_T)|\mathcal{C}_{T-1}, \mathcal{S}_{T-1}]$$
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  3. Continue backwards...

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- **Numerical** Dynamic Programming
    - Backwards induction on arrays
    - Grids
    - Interpolation
    - Integration
- See "02. Consumption-Saving Model.ipynb"

Next Time

- **Next time:**
  Dynamic programming with *uncertainty*
  Structural *estimation*

- **Literature:**
  Gourinchas and Parker (2002): "Consumption Over the Life Cycle"
    - **Read** before lecture
    - **Reading guide:**
      Section 1: Introduction – *Key* (page 50 is not that important)
      Section 2: Model – *Key*, we will discuss. Do not get stuck.
      Section 3: Estimation method (SMM). *Key*, we will discuss.
      Section 4: First stage calibrations. Skim fast.
      Section 5: Data. Skim fast.
      Section 6: Results. Focus on 6.1. Figures 5 and 7 are main results.

References I

ADDA, J., C. DUSTMANN AND K. STEVENS (2017): "The Career Costs of Children," *Journal of Political Economy*, 125(2), 293–337.

GOURINCHAS, P.-O. AND J. A. PARKER (2002): "Consumption Over the Life Cycle," *Econometrica*, 70(1), 47–89.

KLEVEN, H. J., C. LANDAIS AND J. E. SØGAARD (2019): "Children and gender inequality: Evidence from Denmark," *American Economic Journal: Applied Economics*, 11, 181–209.