

Introduction

Thomas H. Jørgensen

2023

Outline

- 1 This Course
- 2 Models
- 3 Programming in Python
- 4 Dynamic Programming

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- **Teacher:** Thomas Jørgensen
(Associate Professor, University of Copenhagen)

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 - Dynamic programming in Python
- **Prerequisite:**
 - Introduction to Programming and Numerical Analysis

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- Plan for today:
 1. Course description
 2. Programming in Python
 3. Introduction to Dynamic Programming

Teaching Methods

- **Lectures:** Wednesday 12-15
 - 2 hours lecture
 - 1 hour of problem-solving (e.g. programming)

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1. Discussions of research papers on different *topics* related to *household behavior over the life cycle*
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- I will provide code.

- You will be asked to modify it to answer certain questions.

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- **Material:**

- Web: <https://sites.google.com/view/householdbehavior>

- GitHub:

- <https://github.com/ThomasHJorgensen/HouseholdBehaviorCourse>

Course Plan (preliminary)

Introduction

1. Introduction
2. Dynamic Programming and Structural Estimation

Part 1: Labor Supply

3. Static and Dynamic labor supply
4. Dynamic labor supply and learning by doing
5. Career costs of children
6. *No lecture: Work on assignment*
7. Individual vs. Joint taxation
8. Labor supply and Retirement of couples

Part 2: Family Formation and Dissolution

9. Marriage and Divorce.
10. Household Bargaining: Limited Commitment
11. Divorce Law and Labor Supply.
12. Taxes, Transfers and Intra-Household Inequality.
13. Fertility.

Outroduction

14. Buffer

Assignments and Exam

- **Exam** (Portfolio):

1. 3 individual assignments.
+ peer feedback.
2. 48 hour individual take-home exam.
Model formulation, code modification, simulations, economic interpretations.

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Model formulation, code modification, simulations, economic interpretations.
- 3 individual **assignments** (hand-in on Absalon)
 1. Based on our dynamic **labor supply** model
Deadline: March 17 (no lecture that week)
 2. Based on our dynamic **bargaining** model
Deadline: April 28
 3. **Free:** Formulate a research question + model + data.
Deadline: May 12
Deadline for peer feedback: May 19
- All feedback can be used to improve assignments before exam date

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Models! What are they good for!?

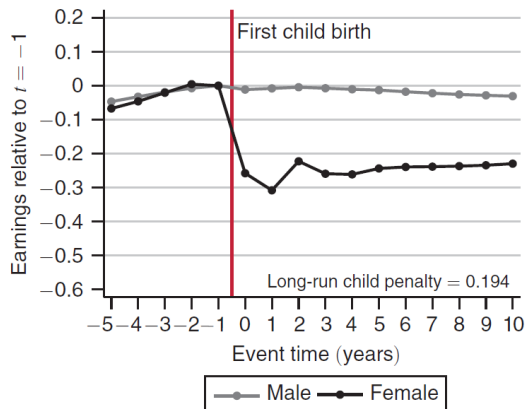
- We will use **empirical regularities as motivation**

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Example: “Child penalty” (Kleven, Landais and Sørensen, 2019)

Panel A. Earnings



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Example: “Career costs of children” (Adda, Dustmann and Stevens, 2017)

“We estimate a dynamic life cycle model of labor supply, fertility, and savings, incorporating occupational choices, with specific wage paths and skill atrophy that vary over the career. This allows us to understand the trade-off between occupational choice and desired fertility, as well as sorting both into the labor market and across occupations. We **quantify** the life cycle career costs associated with children, how they **decompose** into loss of skills during interruptions, lost earnings opportunities, and selection into more child-friendly occupations. We analyze the **long-run effects of policies** that encourage fertility and show that they are considerably smaller than short-run effects.”

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“All models are wrong, but some are useful”

George E.P. Box

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- Why ever use a model then?
- **The Lucas critique:** *Behavioral rules change with policy*
 - ⇒ policy advice can not rely on estimated behavioral rules (reduced-form estimates using existing variation)
 - ⇒ we need to estimate *structural (deep) parameters*
 - “Invariance of parameters in an economic model is not, of course, a property which can be assured in advance, but it seems reasonable to hope that neither tastes nor technology vary systematically with variations in counter-cyclical policies.” (Lucas, 1977)*

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Implications of not-yet implemented policy reforms (e.g. child subsidies)

Responses to changes in the economic environment (e.g. wages)

Quantifying the importance of mechanisms (e.g. risk)

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→ Tight link between **research question** and model formulation.

Learning Objectives

See all at <https://kurser.ku.dk/course/a%c3%98kk08427u/2022-2023>

- **Knowledge (two):**

Define, formulate and interpret *models* of household behaviour
Account for backwards induction and how to *solve* dynamic programming models

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- **Competences (two):**

Discuss and evaluate *research* on household behavior over the life cycle
Modify computer *code* to analyze small changes to simple models

How to read a research paper (in this course)

- Each lecture will be based on 1 mandatory (*) **research paper**
What is the main *research question*?
What is the (*empirical*) *motivation*?
What are the central *mechanisms in the model*?
What is the *simplest model* in which we could capture these?
Challenging: Research frontier.

How to read a research paper (in this course)

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What is the (*empirical*) *motivation*?

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Challenging: Research frontier.

- **How to read** a research paper in this course?

- **Focus** on the questions above

How do the questions interact and inform each other?

- Try not to **get stuck** in too many details!
(we can discuss some in class if you want)
 - Research papers often include many **“bells and whistles”**
 - Read **~30 min before** each lecture.
See reading-guide for each lecture

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→ We need numerical methods

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- **Purpose** of you programming:
Learn best by implementing!
Appreciate bottlenecks and challenges
Better understanding of frontier research
Set you free...!

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→ We need numerical methods
- **Purpose** of you programming:
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Better understanding of frontier research
Set you free...!
- **Goal** is to keep things simple!
No fancy numerical tricks (at least in beginning)
Code should be “intuitive”
→ slow...
(might have to do more efficient code)

Programming in Python

- **Setup** as

Introduction to Programming and Numerical Analysis

<https://sites.google.com/view/numeconcph-introprog/>

- Jupyter Lab (execution of code)
- Visual Studio Code (editor, Python modules)

- **Installation** guide:

<https://sites.google.com/view/numeconcph-introprog/guides/installation>

- **Packages** (all by Jeppe Druedahl):

EconModel

consav

See “01. Introduction to EconModel and consav.ipynb”

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- **Agents maximize** expected discounted sum of utility throughout life
Maximize wrt. $\{C_t\}_{t=1}^T$
Forward looking \rightarrow dynamic
Assume *optimal* behavior in all periods

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$$V_t(\mathcal{S}_t) = \max_{\mathcal{C}_t} U(\mathcal{C}_t, \mathcal{S}_t) + \beta \mathbf{E}[V_{t+1}(\mathcal{S}_{t+1}) | \mathcal{C}_t, \mathcal{S}_t]$$

s.t.

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- $U(C_t, \mathcal{S}_t)$: flow-utility

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- $\beta \mathbb{E}[V_{t+1}(\mathcal{S}_{t+1}) | \mathcal{C}_t, \mathcal{S}_t]$: expected discounted value of next-period
- $\mathcal{S}_{t+1} \sim F(\mathcal{C}_t, \mathcal{S}_t)$: transition density of states (fcn of \mathcal{C}_t !)
(there might be other constraints)

Backwards Induction

- Solved by **backwards induction**

1. Start with last/terminal period, T (no future)

$$V_T(\mathcal{S}_T) = \max_{\mathcal{C}_T} U(\mathcal{C}_T, \mathcal{S}_T)$$

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3. Continue backwards...

Introduction to Dynamic Programming (DP)

- **On a computer**, everything is discrete \rightarrow arrays + loops

Introduction to Dynamic Programming (DP)

- **On a computer**, everything is discrete \rightarrow arrays + loops
- ***Numerical*** Dynamic Programming
 - Backwards induction on arrays
 - Grids
 - Interpolation
- See “02. Consumption-Saving Model.ipynb”

Next Time

- **Next time:**

Dynamic programming with *uncertainty*
Structural *estimation*

- **Literature:**

Blundell, Dias, Meghir and Shaw (2016)

“Female Labour Supply, Human Capital and Welfare Reform”

- **Skim** before lecture

- **Reading guide:**

Section 1: Introduction – *Key*

Section 2: UK Policy reforms. *Get the idea* – important for estimation.

Section 3: Empirical results. *Get the idea. Introduction probably enough.*

Section 4: *Key to get the idea.* Lot of technicalities (less important).

Section 5: Estimation: short.

References I

ADDA, J., C. DUSTMANN AND K. STEVENS (2017): “The Career Costs of Children,” *Journal of Political Economy*, 125(2), 293–337.

BLUNDELL, R., M. C. DIAS, C. MEGHIR AND J. M. SHAW (2016): “Female Labour Supply, Human Capital and Welfare Reform,” *Econometrica*, 84(5), 1705–1753.

KLEVEN, H. J., C. LANDAIS AND J. E. SØGAARD (2019): “Children and gender inequality: Evidence from Denmark,” *American Economic Journal: Applied Economics*, 11, 181–209.