Assignment 1: Labor Supply and Children

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Setup

For completeness I choose to include the baseline model specification below.

$$V_t(n_t, a_t, k_t) = \max_{c_t, h_t} \frac{c_t^{1+\eta}}{1+\eta} - \beta(n_t)\frac{h_t^{1+\gamma}}{1+\gamma} + \rho \mathbb{E}_t[V_{t+1}(n_{t+1}, a_{t+1}, k_{t+1})] \tag{1}$$

s.t.

$$a_{t+1} = (1+r)(a_t + (1-\tau)w_t h_t - c_t) \tag{2}$$

$$k_{t+1} = k_t + h_t \tag{3}$$

$$w_t = w\,(1+\alpha k_t) \tag{4}$$

$$\beta(n_t) = \beta_0 + \beta_1 \cdot n_t \tag{5}$$

$$n_{t+1} = \begin{cases} n_t + 1 & \text{with probability } p(n_t) \\ n_t & \text{with probability } 1 - p(n_t) \end{cases} \tag{6}$$

$$p(n_t) = \begin{cases} p_n & \text{if } n_t = 0 \\ 0 & \text{if } n_t = 1. \end{cases} \tag{7}$$

Terminal conditions are given by

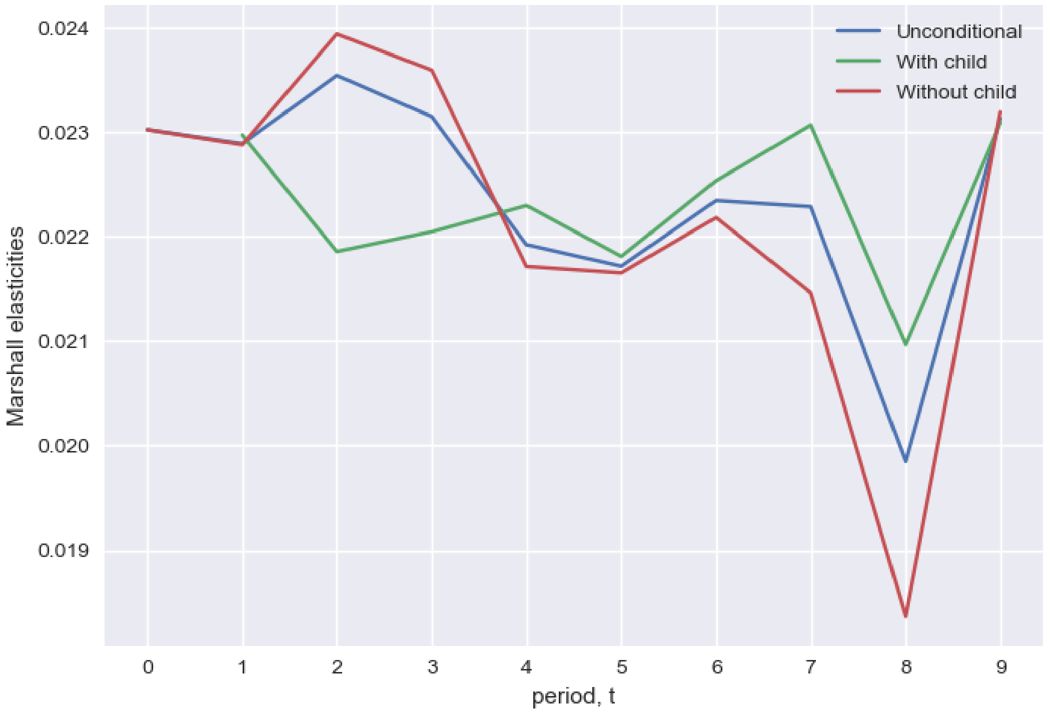
$$V_T(n_T, a_T, k_T) = \max_{h_T} \frac{c_T^{1+\eta}}{1+\eta} - \beta(n_T)\frac{h_T^{1+\gamma}}{1+\gamma} \tag{8}$$

s.t.

$$c_T = a_T + (1-\tau)w_T h_T. \tag{9}$$

I use the following baseline parameter values: $\eta = -2$, $\gamma = 2.5$, $\rho = 1/1.02$, $r = 0.02$, $w = 1$, $\tau = 0.1$, $\alpha = 0.3$, $p_n = 0.1$, $\beta_0 = 0.1$, and $\beta_1 = 0.053$ based on my calibration in section 1.

Figure 1: Marshall elasticities



Note: The elasticities are based on the average hours worked of 1000 simulated individuals. 'With child' is the mean conditional on having a child, and 'Without child' is the mean conditional on not having a child. The number of observations in the calculations of the conditional means vary over time as children increase over time.

1 What value of β_1 corresponds to a decrease in hours of 10% upon receiving a child

I use a grid search approach over different values of β_1 and calculate the drop in hours for each value. I find that with my parameterization $\beta_1 = 0.053$ gives an initial drop in hours upon receiving a child of -10.04%.

2 Find the Marshall elasticity

The Marshall elasticities are shown in figure 1. I have calculated the Marshall elasticity as the relative deviation from hours worked in the baseline scenario as a result of a tax increase of 1%: $\frac{h_t^{shock} - h_t^{baseline}}{h_t^{baseline}}$, where h_t^{shock} is the hours worked in period t with a 1% higher tax rate, and $h_t^{baseline}$ is the hours worked in the baseline scenario. The shock is calculated as a regime change - hence I follow individuals over their lifetime in a regime with a 1% higher tax rate. This makes the interpretation of the Marshall elasticity a long run elasticity, as it is the effect for individuals who have only ever known the higher tax rate - in other words, the effect when the policy has been in place for a long time.

The elasticities are very noisy over time. I find that this is due to numerical error. The noise makes it difficult to interpret the elasticities as there is no clear trend over time and no clear positioning between the conditional Marshall elasticities. Therefore the interpretations presented below should be read with caution. I reflect on the issue of numerical imprecision in the end of this section.

The first observation I make is that the elasticities are small, around 0.022, but positive in all periods. This is a result of a dominating income effect. When people earn less, they choose to work more to keep up their income. The effect is kept down by the negative substitution effect - when the price of leisure decreases people substitute towards leisure. The positive elasticities show that the income effect dominates.

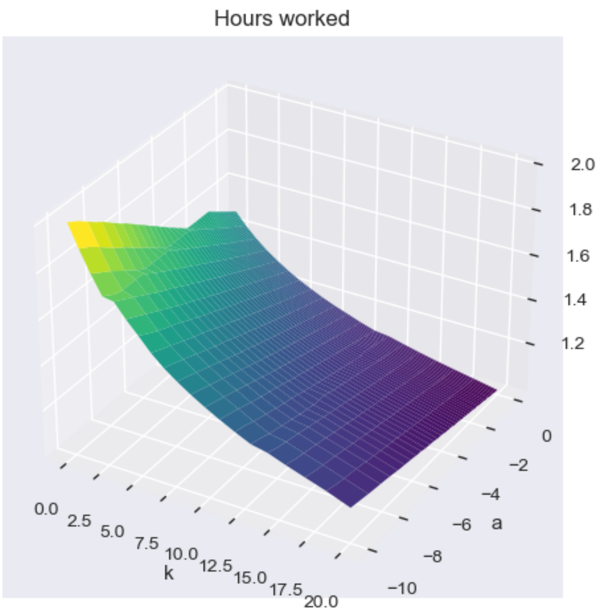
The second observation I make is that the elasticities seem to be declining over time. This is a result of human capital accumulation that makes income increase over time. The income effect is smaller for more rich people as their credit constraint is binding less, and hence their elasticities are lower. The jump in the last period is somewhat weird. I suspect that it is a result of the terminal condition where all debt must be paid back. If people have behaved suboptimally over their life time then they have to adjust for their mistakes in the last period. This makes the elasticity more volatile as the behavior in the last period in affected by this accumulated suboptimal behavior.

The third observation I make is that people with children seem to have a higher elasticity especially in the late periods. Because people with children are working less their human capital is lower and so is their income. This makes the income effect stronger and hence the elasticity higher.

Mainly two things can explain the noise in the elasticities. First and foremost, the policy functions of the agents are not smooth, which I would expect it to be. This is likely a result of imprecision in the solver when solving 2-dimensional problems. Figure 2 highlights this issue. I use 'L-BFGS-B', and have tried other solvers such as 'Nelder-Mead', but not been able to achieve sufficient numerical precision. I have also increased the grid sizes substantially without any major improvements. Further work on this problem could be to use a nested EGM approach, where a solver only would have to solve in 1-dimension.

A second cause of the noisy Marshall elasticities is that the absolute change in the after-tax wage is very small. Hence the relative deviations in hours worked are vulnerable to small numerical errors. A different parameterization with a higher baseline tax rate, or a bigger change in the tax rate would accommodate this issue. I have tried both and get slightly less noisy results. However the level of the elasticities change drastically because of income effects - people's income are substantially lower. I have chosen to stick to the original parameterization.

Figure 2: Numerical error in policy function for hours worked



Note: The policy function is for an individual with a child in period 5

3 Extend the model with a spouse

An exogenous spouse only affects the agent through the budget constraint. The model can be extended with a spouse by replacing the budget constraint (2) and the corresponding terminal condition (9) with the following equations, and extending the model with (10).

$$a_{t+1} = (1+r)(a_t + (1-\tau)w_t h_t + y_t - c_t) \tag{2'}$$

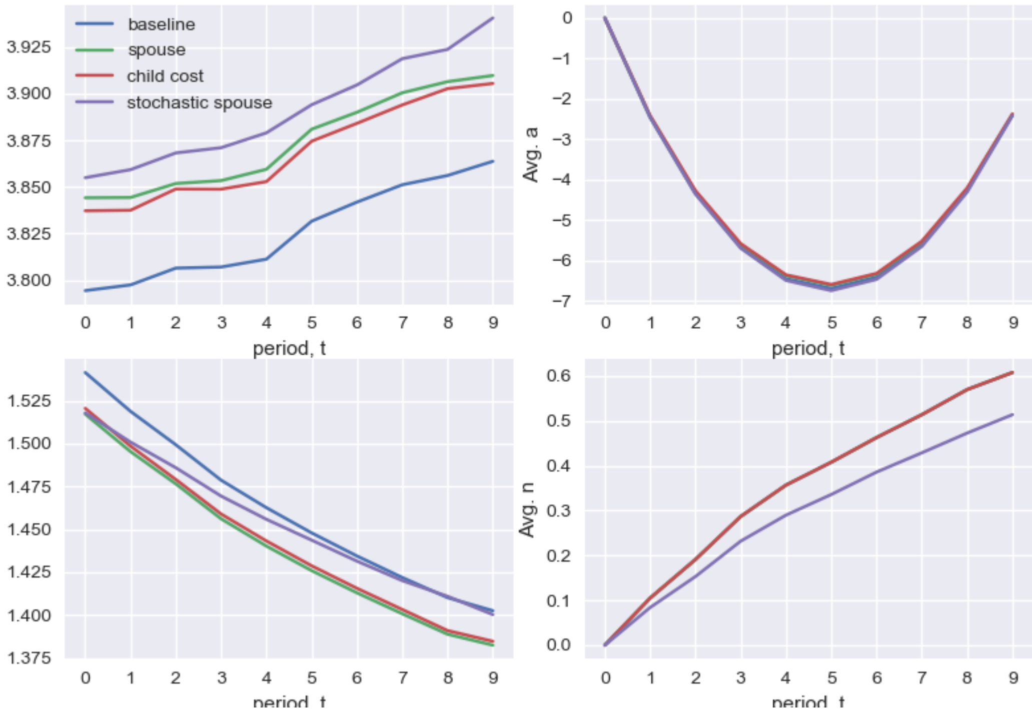
$$c_T = a_t + (1-\tau)w_T h_T + y_T \tag{9'}$$

$$y_t = 0.1 + 0.01t \tag{10}$$

The simulated behavior for all model extensions are shown in figure 3 and the Marshall elasticities are shown in figure 4. Comparing the 'spouse extension' to the baseline model the two main implications from spouse income are higher consumption and fewer hours worked. Both behaviors are due to consumption and leisure being normal goods, so when agents receive an additional income from spouses they choose to work less and consume more.

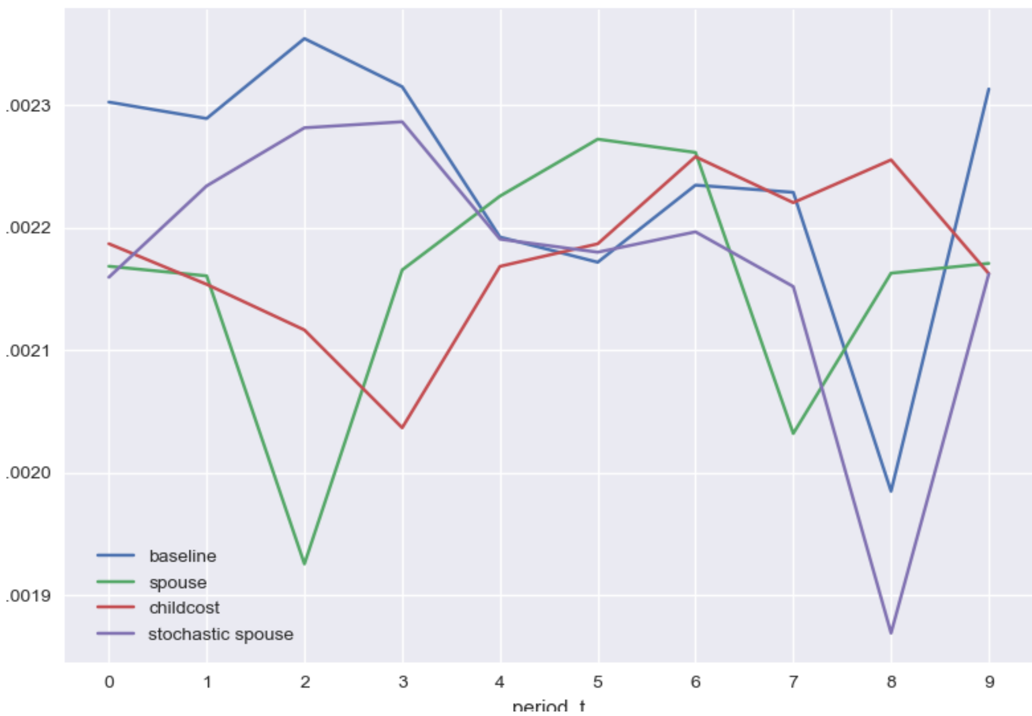
The Marshall elasticity in the 'spouse extension' seems to be a little lower. This is because people are richer and so the income effect is smaller, which makes the elasticity lower.

Figure 3: Simulated behavior for all models



Note: The behavior is based on the unconditional average hours worked of 1000 simulated individuals.

Figure 4: Marshall elasticities for all models



Note: The elasticities are based on the unconditional average hours worked of 1000 simulated individuals.

4 Extend the model with child costs

Child costs also only affects the agent through its budget constraint. The model can be extended with child costs by replacing the budget constraint (2') and terminal condition (9') with the following equations:

$$a_{t+1} = (1+r)(a_t + (1-\tau)w_t h_t + y_t - c_t - \theta n_t) \tag{2''}$$

$$c_T = a_T + (1-\tau)w_T h_T + y_T - \theta n_T) \tag{9''}$$

The consumption is a little lower and the hours worked a little higher in the 'child cost extension' compared to the 'spouse extension'. Again, this can be explained by income effects - people have more expenses so they consume less consumption goods and leisure. It is interesting that people work more and consume less already in period 0 before anyone has children. People anticipate that they might get children in the future and account for the expenses in the present by consuming less, working more, and borrowing less.

The Marshall elasticities are too noisy to comment on based on figure 4, but I would expect Marshall elasticities to increase as a result of people's effective income having decreased.

5 Discuss how endogenous fertility would affect the results

If people could choose whether to have children or not (or exert effort) then the number of children would be affected by child subsidies (or costs). As the model stands, there is no incentive to having children, but the model could be extended with utility and preferences for children, which would make some people choose to have children. In that case, the incentive to have children would be lower if it became more expensive to have children. Then people would have fewer children. I don't expect the effect to be very big as the direct costs of children is only part of the costs. The human capital missed out on has a huge impact on future earnings, which also is an important cost.

Another interesting aspect is that if preferences for children differ between agents, then people will be hit differently by the the reform. People with preferences for children will consume less, work more, and borrow less as seen in the model from section 4, while the people with preferences for no children won't be affected as much or not at all depending on the specifications of the model.

6 Extend the model with a stochastic spouse

A stochastic spouse changes multiple equations, because it affects states and expectations. The state of having a spouse would need to be introduced in the value function affecting, the budget constraint needs to address whether there is a spouse, and state transitions have to be specified. It is important to include the state of having a spouse, s_t, in the value function to be able to solve the model with backward induction, because people need to form expectations about all future states. The new equations are presented below, where a star marks that an old equation is being replaced. Note that I have modelled arrival of a child in the next period such that it can only happen if you have a spouse in the present period.

$$V_t(n_t, s_t, a_t, k_t) = \max_{c_t, h_t} \frac{c_t^{1+\eta}}{1+\eta} - \beta(n_t)\frac{h_t^{1+\gamma}}{1+\gamma} + \rho\mathbb{E}_t[V_{t+1}(n_{t+1}, s_{t+1}, a_{t+1}, k_{t+1})] \tag{1*}$$

$$a_{t+1} = (1+r)(a_t + (1-\tau)w_t h_t + y_t s_t - c_t) \tag{2*}$$

$$n_{t+1} = \begin{cases} n_t + 1 \text{ with probability } p(n_t, s_t), \\ n_t \text{ with probability } 1 - p(n_t, s_t) \end{cases} \tag{6*}$$

$$p(n_t, s_t) = \begin{cases} p_n \text{ if } n_t = 0, s_t = 1, \\ 0 \text{ else} \end{cases} \tag{7*}$$

$$s_{t+1} = \begin{cases} 1 \text{ with probability } p_s, \\ 0 \text{ with probability } 1 - p_s \end{cases} \tag{11}$$

$$V_T(n_T, s_T, a_T, k_T) = \max_{h_T} \frac{c_T^{1+\eta}}{1+\eta} - \beta(n_T)\frac{h_T^{1+\gamma}}{1+\gamma} \tag{8*}$$

$$c_T = a_T + (1-\tau)w_T h_T + y_T s_T - \theta n_T) \tag{9*}$$

When simulating this model, I need to consider the initial state of spouses. I assume that 80% start with a spouse and 20% doesn't corresponding to the probabilities of having a spouse in the next period.

With this extension lifetime income is affected in two ways: Lifetime income decreases, because it now is uncertain whether you will receive income from your spouse in the next period; but lifetime income also increases, because your chance of having children is lower. The overall effect seems to be an increase in lifetime income as consumption throughout life is significantly higher compared to the model from 4. Hours worked starts out at the same level, but increases relative to behavior from the model in section 4. In the beginning of life when no one has children, all share the same disutility of work. But as time passes and relatively fewer children are born, people have in average a lower disutility of work and hence they in average choose to work relatively more.

The Marshall elasticities are too noisy to comment on based on figure 4, and because there are effects going in both directions it is difficult to say how the Marshall elasticities are expected to behave with this extension.