Introduction

Thomas H. Jørgensen

2023

Outline

1. This Course
2. Models
3. Programming in Python
4. Dynamic Programming

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  (Associate Professor, University of Copenhagen)

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  Dynamic programming in Python
- **Prerequisite:**
  Introduction to Programming and Numerical Analysis

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- Plan for today:
  1. Course description
  2. Programming in Python
  3. Introduction to Dynamic Programming

Teaching Methods

- **Lectures:** Wednesday 12-15
  2 hours lecture
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  I will provide code.
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- **Material:**
  Web: `https://sites.google.com/view/householdbehavior`
  GitHub:
  `https://github.com/ThomasHJorgensen/HouseholdBehaviorCourse`

Course Plan (preliminary)

Introduction

1. Introduction
2. Dynamic Programming and Structural Estimation

Part 1: Labor Supply

3. Static and Dynamic labor supply
4. Dynamic labor supply and learning by doing
5. Career costs of children
6. *No lecture: Work on assignment*
7. Individual vs. Joint taxation
8. Labor supply and Retirement of couples

Part 2: Family Formation and Dissolution

9. Marriage and Divorce.
10. Household Bargaining: Limited Commitment
11. Divorce Law and Labor Supply.
12. Taxes, Transfers and Intra-Household Inequality.
13. Fertility.

Outroduction

14. Buffer

Assignments and Exam

- **Exam** (Portfolio):
  1. 3 individual assignments.
     $+$ peer feedback.
  2. 48 hour individual take-home exam.
     Model formulation, code modification, simulations, economic interpretations.

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- 3 individual **assignments** (hand-in on Absalon)
  1. Based on our dynamic **labor supply** model
     Deadline: March 17 (no lecture that week)
  2. Based on our dynamic **bargaining** model
     Deadline: April 28
  3. **Free**: Formulate a research question + model + data.
     Deadline: May 12
     Deadline for peer feedback: May 19
- All feedback can be used to improve assignments before exam date

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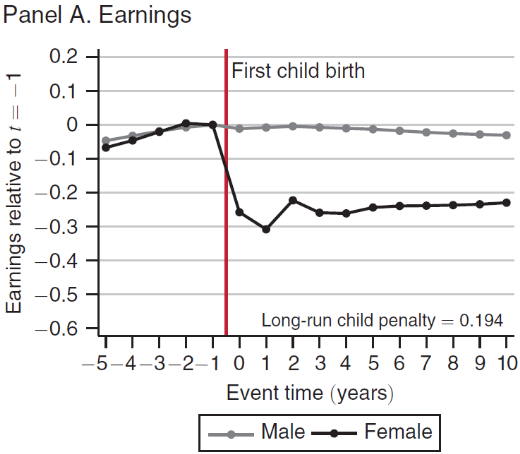
Example: "Child penalty" (Kleven, Landais and Søgaard, 2019)



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  Example: "Career costs of children" (Adda, Dustmann and Stevens, 2017)
  "We estimate a dynamic life cycle model of labor supply, fertility, and savings, incorporating occupational choices, with specific wage paths and skill atrophy that vary over the career. This allows us to understand the trade-off between occupational choice and desired fertility, as well as sorting both into the labor market and across occupations. We **quantify** the life cycle career costs associated with children, how they **decompose** into loss of skills during interruptions, lost earnings opportunities, and selection into more child-friendly occupations. We analyze the **long-run effects of policies** that encourage fertility and show that they are considerably smaller than short-run effects."

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> *"All models are wrong, but some are useful"*
>
> *George E.P. Box*

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- **The Lucas critique:** *Behavioral rules change with policy*
  $\Rightarrow$ policy advice can not rely on estimated behavioral rules (reduced-form estimates using existing variation)
  $\Rightarrow$ we need to estimate *structural (deep) parameters*

  > *"Invariance of parameters in an economic model is not, of course, a property which can be assured in advance, but it seems reasonable to hope that neither tastes nor technology vary systematically with variations in counter-cyclical policies." (Lucas, 1977)*

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→ Tight link between **research question** and model formulation.

Learning Objectives

See all at `https://kurser.ku.dk/course/a%c3%98kk08427u/2022-2023`

- **Knowledge (two):**
  Define, formulate and interpret *models* of household behaviour
  Account for backwards induction and how to *solve* dynamic programming models

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- **Skills (one):**
  Analyze *counterfactual* policy reform simulations from simple and more complex models of household behavior
- **Competences (two):**
  Discuss and evaluate *research* on household behavior over the life cycle
  Modify computer *code* to analyze small changes to simple models

How to read a research paper (in this course)

- Each lecture will be based on 1 mandatory (*) **research paper**
  What is the main *research question*?
  What is the *(empirical) motivation*?
  What are the central *mechanisms in the model*?
  What is the *simplest model* in which we could capture these?
  Challenging: Research frontier.

How to read a research paper (in this course)

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  What is the main *research question*?
  What is the *(empirical) motivation*?
  What are the central *mechanisms in the model*?
  What is the *simplest model* in which we could capture these?
  Challenging: Research frontier.

- **How to read** a research paper in this course?
  - **Focus** on the questions above
    How do the questions interact and inform each other?
  - Try not to **get stuck** in too many details!
    (we can discuss some in class if you want)
  - Research papers often include many **"bells and whistles"**
  - Read ~**30 min before** each lecture.
    See reading-guide for each lecture

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  $\rightarrow$ We need numerical methods

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  Learn best by implementing!
  Appreciate bottlenecks and challenges
  Better understanding of frontier research
  Set you free...!

Programming in Python

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- **Purpose** of you programming:
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  Set you free...!
- **Goal** is to keep things simple!
  No fancy numerical tricks (at least in beginning)
  Code should be "intuitive"
  $\rightarrow$ slow...
  (might have to do more efficient code)

Programming in Python

- **Setup** as
  Introduction to Programming and Numerical Analysis
  `https://sites.google.com/view/numeconcph-introprog/`
  - Jupyter Lab (execution of code)
  - Visual Studio Code (editor, Python modules)
- **Installation** guide:
  `https://sites.google.com/view/numeconcph-introprog/guides/installation`
- **Packages** (all by Jeppe Druedahl):
  EconModel
  consav
  See "`01. Introduction to EconModel and consav.ipynb`"

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  Maximize wrt. $\{\mathcal{C}_t\}_{t=1}^{T}$
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s.t.

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  - $\beta \mathbb{E}[V_{t+1}(\mathcal{S}_{t+1}) | \mathcal{C}_t, \mathcal{S}_t]$: expected discounted value of next-period
  - $\mathcal{S}_{t+1} \sim F(\mathcal{C}_t, \mathcal{S}_t)$: transition density of states (fcn of $\mathcal{C}_t$!)
    (there might be other constraints)

Backwards Induction

- Solved by **backwards induction**

1. Start with last/terminal period, T (no future)

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3. Continue backwards...

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- **On a computer**, everything is discrete $\rightarrow$ arrays $+$ loops

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- **On a computer**, everything is discrete $\rightarrow$ arrays + loops
- ***Numerical*** Dynamic Programming
  - Backwards induction on arrays
  - Grids
  - Interpolation
- See "`02. Consumption-Saving Model.ipynb`"

Next Time

- **Next time:**
  Dynamic programming with *uncertainty*
  Structural *estimation*

- **Literature:**
  Gourinchas and Parker (2002): "Consumption Over the Life Cycle"
  - **Read** before lecture
  - **Reading guide:**
    Section 1: Introduction – *Key* (page 50 is not that important)
    Section 2: Model – *Key*, we will discuss. Do not get stuck.
    Section 3: Estimation method (SMM). *Key*, we will discuss.
    Section 4: First stage calibrations. Skim fast.
    Section 5: Data. Skim fast.
    Section 6: Results. Focus on 6.1. Figures 5 and 7 are main results.

References I

ADDA, J., C. DUSTMANN AND K. STEVENS (2017): "The Career Costs of Children," *Journal of Political Economy*, 125(2), 293–337.

GOURINCHAS, P.-O. AND J. A. PARKER (2002): "Consumption Over the Life Cycle," *Econometrica*, 70(1), 47–89.

KLEVEN, H. J., C. LANDAIS AND J. E. SØGAARD (2019): "Children and gender inequality: Evidence from Denmark," *American Economic Journal: Applied Economics*, 11, 181–209.