Testing

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Chapter 1

VectorSpaces

1.1 ZeroScalarMultiplication

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Definition 1. A *vector space* is a space over a field K with an abelian group V. It has four main properties:

- **smul_add**: $\forall(a : K)(xy : V), a \bullet (x + y) = a \bullet x + a \bullet y$

- **add_smul**: $\forall(ab : K)(x : V), (a + b) \bullet x = a \bullet x + b \bullet x$

- **mul_smul**: $\forall(ab : K)(x : V), (a * b) \bullet x = a \bullet (b \bullet x)$

- **one_smul**: $\forall(x : V), (1 : K) \bullet x = x$

Theorem 2. *In any vector space V over K, the scalar 0 multiplied by any vector gives the zero vector: $(0 : K) \bullet w = (0 : V)$*

1.2 MultiplyingByTheZeroVector

Theorem 3. *In any vector space V over K, any scalar a multiplied by the zero vector gives the zero vector: $(a : K) : a \bullet (0 : V) = (0 : V)$*