

Testing

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Chapter 1

VectorSpaces

1.1 ZeroScalarMultiplication

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Definition 1. A *vector space* is a space over a field K with an abelian group V . It has four main properties:

- **smul_add:** $\forall(a : K)(xy : V), a \bullet (x + y) = a \bullet x + a \bullet y$
- **add_smul:** $\forall(ab : K)(x : V), (a + b) \bullet x = a \bullet x + b \bullet x$
- **mul_smul:** $\forall(ab : K)(x : V), (a * b) \bullet x = a \bullet (b \bullet x)$
- **one_smul:** $\forall(x : V), (1 : K) \bullet x = x$

Theorem 2. In any vector space V over K , the scalar 0 multiplied by any vector gives the zero vector: $(0 : K) \bullet w = (0 : V)$

1.2 MultiplyingByTheZeroVector

Theorem 3. In any vector space V over K , any scalar a multiplied by the zero vector gives the zero vector: $(a : K) \bullet (0 : V) = (0 : V)$

1.3 ScalingByNegativeOne

Theorem 4. In any vector space V over K , multiplying a vector by -1 gives its additive inverse: $(v : V) : (-1 : K) \bullet v = -v$

1.4 ZeroMustBelong

Definition 5. A *subspace* is a subset of a vector space that acts similarly to a vector space itself. It has three main properties:

- **Nonempty:** $W.Nonempty$

- **Closure_Under_Addition:** $\forall (x, y : V), x \in W \rightarrow y \in W \rightarrow x + y \in W$
- **Closure_Under_Scalar_Multiplication:** $\forall (a : K)(x : V), x \in W \rightarrow a \bullet x \in W$