

# Linear Algebra Game

Adam Kern

Justin Morrill

Letian Yang

Huiyu Chen

July 7, 2025

# Chapter 1

## VectorSpaces

### 1.1 ZeroScalarMultiplication

Add commentMore actions

**Definition 1.** A *vector space* is a space over a field  $K$  with an abelian group  $V$ . It has four main properties:

- **smul\_add:**  $\forall a \in K, x, y \in V, a \bullet (x + y) = a \bullet x + a \bullet y$
- **add\_smul:**  $\forall a, b \in K, x \in V, (a + b) \bullet x = a \bullet x + b \bullet x$
- **mul\_smul:**  $\forall a, b \in K, x \in V, (a * b) \bullet x = a \bullet (b \bullet x)$
- **one\_smul:**  $\forall x \in V, 1 \in K \bullet x = x$

**Theorem 2.** In any vector space  $V$  over  $K$ , the scalar  $0$  multiplied by any vector gives the zero vector:  $(0 : K) \bullet w = (0 : V)$

### 1.2 MultiplyingByTheZeroVector

**Theorem 3.** In any vector space  $V$  over  $K$ , any scalar  $a$  multiplied by the zero vector gives the zero vector:  $(a : K) \bullet (0 : V) = (0 : V)$

### 1.3 ScalingByNegativeOne

**Theorem 4.** In any vector space  $V$  over  $K$ , multiplying a vector by  $-1$  gives its additive inverse:  $(v : V) : (-1 : K) \bullet v = -v$

### 1.4 ZeroMustBelong

**Definition 5.** A *subspace* is a subset of a vector space that acts similarly to a vector space itself. It has three main properties:

- **Nonempty:**  $W.Nonempty$

- **Closure\_Under\_Addition:**  $\forall (x, y : V), x \in W \rightarrow y \in W \rightarrow x + y \in W$
- **Closure\_Under\_Scalar\_Multiplication:**  $\forall (a : K)(x : V), x \in W \rightarrow a \bullet x \in W$

**Theorem 6.** *This is a proof that any subspace contains the zero vector:  $(0 : V) \in W$*

## 1.5 NegativesInSubspace

**Theorem 7.** *This is a proof that if a subspace contains a vector 'x', it also contains '-x':  $\forall (x : V), x \in W \rightarrow (-x) \in W$*