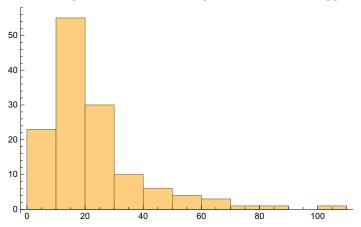
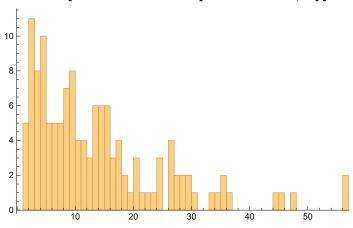
```
(* Adam Beck *)
      (* Problem 1*)
      (* hosp-heart.nb data *)
      (* {M,V} M = one year mortality rate,
      percentage of patiuents that died within one year of the
       transplant operation,
      V = average annual number of transplants at that center during the same 4 years *)
      heart = \{\{17.9, 27\}, \{23.1, 4\}, \{40, 3\}, \{6.5, 35\}, \{14.9, 17\}, \{12.5, 4\}, \{15.7, 45\},
          \{9.8, 28\}, \{24, 6\}, \{5.0, 10\}, \{15.4, 13\}, \{4.8, 7\}, \{0, 1\}, \{19.1, 47\}, \{4.5, 6\},
          \{15, 56\}, \{12.5, 4\}, \{33.9, 8\}, \{10.7, 9\}, \{13, 14\}, \{28.3, 12\}, \{57.1, 2\}, \{6.3, 4\},
          \{10, 3\}, \{8.3, 12\}, \{17.5, 10\}, \{20, 3\}, \{29.3, 10\}, \{21.4, 7\}, \{27.3, 8\}, \{13.6, 6\},
          \{21.8, 30\}, \{36.4, 3\}, \{18.2, 11\}, \{33.3, 2\}, \{20, 4\}, \{38.5, 7\}, \{20.8, 18\}, \{12.2, 19\},
          \{22.2, 18\}, \{29, 8\}, \{0, 9\}, \{5.7, 9\}, \{50, 2\}, \{21.7, 15\}, \{66.7, 4\}, \{29.4, 17\},
          \{12.1, 27\}, \{10.7, 14\}, \{6.3, 4\}, \{16.2, 9\}, \{21.1, 5\}, \{17.4, 33\}, \{23.9, 17\},
          \{42.9, 2\}, \{40, 2\}, \{6.7, 15\}, \{44.4, 3\}, \{18.7, 34\}, \{14.7, 24\}, \{7.4, 7\}, \{12.6, 24\},
          \{9.7, 26\}, \{44.4, 2\}, \{16.7, 6\}, \{15.8, 14\}, \{83.3, 2\}, \{10.9, 22\}, \{13.3, 5\},
          \{11.1, 5\}, \{75, 2\}, \{19, 20\}, \{14, 13\}, \{60, 1\}, \{21.2, 8\}, \{9.7, 8\}, \{50, 2\}, \{25, 14\},
          \{18.6, 15\}, \{0.0, 1\}, \{35.3, 9\}, \{23.5, 85\}, \{15.6, 11\}, \{37.5, 2\}, \{14.3, 28\},
          \{14.3, 4\}, \{16.7, 6\}, \{20.0, 15\}, \{13.0, 17\}, \{9.6, 26\}, \{66.7, 3\}, \{30.8, 3\},
          \{14.0, 13\}, \{27.5, 10\}, \{37.5, 8\}, \{18.9, 13\}, \{0.0, 4\}, \{12.2, 44\}, \{57.1, 4\},
          \{21.4, 35\}, \{23.4, 16\}, \{10.9, 12\}, \{15.6, 8\}, \{16.7, 2\}, \{13.9, 9\}, \{18.2, 11\},
          \{11.5, 26\}, \{18.4, 13\}, \{16.7, 3\}, \{20.4, 14\}, \{40.0, 5\}, \{20.7, 56\}, \{19.6, 13\},
          \{13.5, 9\}, \{29.9, 36\}, \{8.4, 21\}, \{28.4, 24\}, \{7.7, 23\}, \{19.3, 29\}, \{0.0, 1\},
          \{22.2, 20\}, \{30.0, 5\}, \{7.0, 11\}, \{23.8, 7\}, \{18.8, 29\}, \{14.5, 16\}, \{17.0, 16\},
          {20.0, 15}, {6.7, 15}, {11.4, 20}, {100.0, 1}, {31.4, 9}, {17.6, 26}, {19.6, 14}};
      (* Split this M and V data into separate
       lists via Transpose[] in order to parse through *)
      heartTranspose = Transpose[heart];
      MData = heartTranspose[[1]];
      VData = heartTranspose[[2]];
      (* Define mean, median, quantile, and variance functions *)
      mean[x_] := Sum[x[[i]], {i, 1, Length[x]}] / Length[x];
      (* Sum elements, divide by length *)
In[38]:= median[x_] := (s = Sort[x]; s[[IntegerPart[.5 * Length[s]]]]);
      (* Sort list, take element at index 1/2*length *)
ln[39]:= quantile[x_, alpha_] := (s = Sort[x]; s[[IntegerPart[alpha * Length[s]]]])
       (* Sort list, take element at index alpha*length *)
ln[40] = variance[x_] := (m = mean[x]; Sum[(x[[i]] - m)^2, {i, 1, Length[x]}]/Length[x]);
      (* difference of every element from mean, squared, times 1/length *)
ln[41]:= (* Find the mean, median, q1 and q3, and variance *)
      hospMeanM = mean[MData]
Out[41]= 21.9045
```

```
In[42]:= hospMeanV = N[mean[VData]]
Out[42]= 13.8657
In[43]:= hospMedianM = median[MData]
Out[43]= 18.2
In[44]:= hospMedianV = median[VData]
Out[44]= 10
In[45]:= hospQ1M = quantile[MData, .25]
Out[45]= 12.2
In[46]:= hospQ1V = quantile[VData, .25]
Out[46]= 4
In[47]:= hospQ3M = quantile[MData, .75]
Out[47]= 25
In[48]:= hospQ3V = quantile[VData, .75]
Out[48]= 17
In[49]:= hospVarianceM = variance[MData]
Out[49]= 268.634
In[50]:= hospVarianceV = N[variance[VData]]
Out[50]= 166.46
      (* Histograms using two difference bin sizes *)
      (* I will use a bin size Length/2 for a very large bin count,
      and Length/10 for a smaller bin count *)
      (* MData large bin count *)
     Histogram[MData, IntegerPart[Length[MData] / 2]]
      12
      10
      8
      6
      2
```

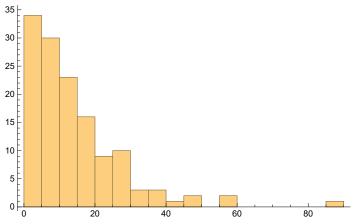
(* MData small bin count *) ${\tt Histogram}\big[{\tt MData,\ IntegerPart}\big[{\tt Length}\,[{\tt MData}]\,\big/\,{\tt 10}\big]\big]$



(* VData, large bin count *) Histogram[VData, IntegerPart[Length[VData] / 2]]



(* VData, small bin count*) ${\tt Histogram}\big[{\tt VData,\ IntegerPart}\big[{\tt Length}\big[{\tt VData}\big] \, \big/ \, {\tt 10}\big]\big]$



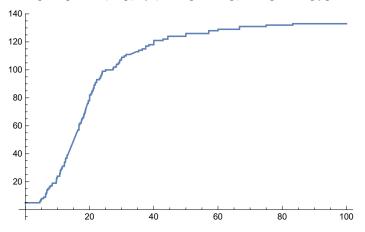
```
_{\text{ln}[5\,1]}= (* Produce plots of quantile functions, moment functions, and CDFs *)
     (* Define functions for moments and CDF *)
     (* Sum elements raised to the kth power, divide by length *)
     moment[x\_, k\_] := N[Sum[x[[i]]^k, \{i, 1, Length[x]\}] / Length[x]];
ln[52] = cdf[x_, xi_] := N[Sum[If[x[[i]]  <= xi, 1, 0], {i, 1, Length[x]}]];
     (* Count that an element is less than or equal to a given element *)
     (* Plot the quantile functions *)
     (* MData *)
     Plot[quantile[MData, i], {i, 0, 1}]
     60
     50
     40
     30
     20
     10
                0.2
                                                          1.0
                           0.4
                                     0.6
                                                8.0
     (* VData *)
     Plot[quantile[VData, i], {i, 0, 1}]
     50
     40
     30
     20
     10
                0.2
                                                          1.0
                                     0.6
                                                8.0
In[53]:= (* Plot moment functions *)
     (* Get the first 10 moments for MData and VData in a Table *)
     momMData = Table[moment[MData, i], {i, 1, 10}];
In[54]:= momVData = Table[moment[VData, i], {i, 1, 10}];
```

```
(* Plot the two moments *)
ListPlot[{momMData, momVData}, PlotStyle → PointSize[.03], PlotRange → {0, 9.9 * 10^17}]
(* Blue is MData, Orange is VData*)
8 \times 10^{17}
6 \times 10^{17}
4 \times 10^{17}
2\times10^{17}
(* Plot CDF functions *)
(* MData from range 0 to maximum element in the data set *)
Plot[cdf[MData, i], {i, 0, Max[MData]}]
140
120
100
 80
 60
 40
 20
                                                        100
(* VData *)
(* VData from range 0 to maximum element in the data set *)
Plot[cdf[VData, i], {i, 0, Max[VData]}]
140
120
100
 80
 60
 40
 20
```

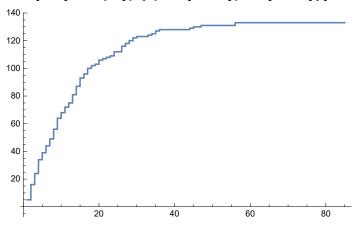
40

(∗ MData from range minimum element to maximum element in the data set ∗)

Plot[cdf[MData, i], {i, Min[MData], Max[MData]}]



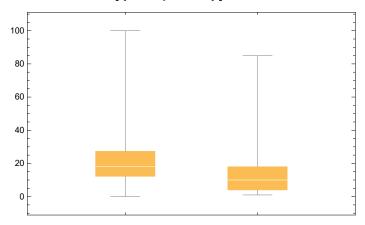
(* MData from range minimum element to maximum element in the data set *) Plot[cdf[VData, i], {i, Min[VData], Max[VData]}]



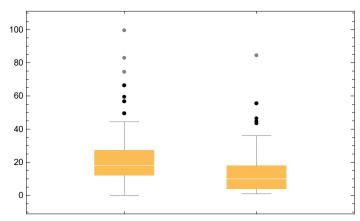
(* Although the instructions do not say to compare any box and whisker and QQ plots for this hospital data against other sets of data, I will product them anyways. QQ will be MData(x axis) against VDaya (y axis) *)

- (* Box an whisker plots *)
- (* A box and whisker plot takes a min, q1, q2 (median), q3, and max *)

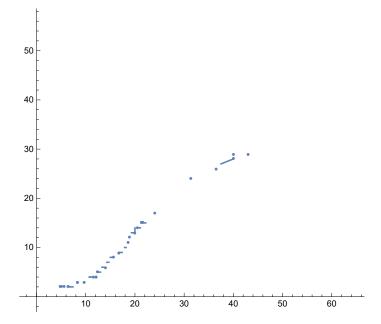
(∗ Box and whisker, MData and VData, outliers not shown ∗) BoxWhiskerChart[{MData, VData}]



(* Box and whisker, MData and VData, outliers shown *) BoxWhiskerChart[{MData, VData}, "Outliers"]



(* Parametric Plots (QQ Plots), MData on x axis, VData on y axis *)
ParametricPlot[{quantile[MData, i], quantile[VData, i]}, {i, 0, 1}]



```
_{	ext{In}[55]:=} (* If the two sets come from a population with the same distribution,
    the points should fall approximately
      along a 45 degree reference line. As we can see,
    the 2 batches do not appear to have come from populations with
      a common distribution, as they do not fit along a straight line. However,
     I would consider it closer to a straight line
      than, say, a quadratic. So it suggests that the data
      sets came from population of fairly equal distributions. *)
     (* resistor.nb data *)
     (* The data represents a listing of the resistances
      (in ohms) of 200 resistors which are all rated at 10 kiloohms. *)
     resistors = {9.97910927, 9.833997401, 10.48797923, 9.778286587, 10.4127049, 9.729651074,
        10.34005333, 9.894176108, 10.07983211, 9.933230947, 9.977783398, 10.13141411,
        10.1266421, 9.37852757, 10.26785423, 9.907086669, 9.744503691, 9.971603949,
        9.693939764, 9.620137112, 12.28072506, 10.0580338, 10.33764317, 9.757096213,
        9.593230848, 9.713741738, 9.432574293, 9.62099431, 9.802732952, 9.971484578,
        10.22548428, 10.3352728, 9.989841592, 10.29860424, 9.52298034, 10.08499861,
        9.394148142, 9.944944954, 10.21438162, 10.36193691, 10.02987499, 9.603449021,
        9.742946181, 9.875414084, 10.05078967, 10.12314509, 10.15281111, 5.870566193,
        9.484863417, 9.973958404, 9.94911044, 9.374762262, 9.788310356, 10.06500849,
        9.77439594, 10.03864565, 10.32397119, 9.916142963, 9.967350072, 10.09860352,
        9.987682395, 10.15563395, 9.537918791, 9.945042157, 10.02686399, 9.74540807,
        10.26915708, 9.696347652, 10.13930795, 9.51572712, 9.367227099, 9.831637831,
        10.1807235, 9.88921993, 9.923452458, 9.944225885, 9.779727284, 10.26538836, 10.2298635,
        10.2461264, 9.694717951, 9.771545526, 9.679096242, 10.15118993, 10.25894345,
        9.613968464, 10.14607857, 10.3809408, 10.00425765, 10.30422606, 9.938641588,
        10.14989447, 9.62901378, 6.613345698, 10.48706974, 10.10426569, 10.15476425,
        9.839152246, 9.74229305, 9.712882265, 10.09355753, 9.655283966, 10.01073951,
        10.23032052, 9.896222755, 9.646005983, 10.22741355, 9.916736976, 9.853518852,
        9.797304974, 9.542975581, 9.582644329, 10.06420074, 10.1110437, 9.09833499,
        9.694181349, 10.0837185, 9.990310834, 9.680224016, 9.544769559, 10.12220661,
        10.35625939, 9.68922915, 9.816272486, 9.838797828, 9.787675983, 10.01512384,
        9.672549018, 9.166747182, 9.839861368, 10.0490497, 9.9589975, 9.707653239, 9.642065029,
        10.14670044, 9.704657023, 9.851454583, 9.92931813, 10.05903936, 9.749898131,
        10.12904658, 9.776733909, 9.956306817, 10.10913774, 9.25291271, 9.823820724,
        9.581313056, 9.84027462, 9.738894951, 9.923279654, 9.815685862, 9.754906605,
        10.19531748, 9.718578829, 9.830784043, 9.860661512, 9.665515781, 9.956836598,
        10.06308718, 9.401201273, 10.10992616, 9.738494773, 9.991823154, 9.877411846,
        10.23755441, 10.04556889, 9.978626954, 10.06519891, 9.774786454, 10.26202664,
        10.10298671, 9.558598995, 9.352852535, 9.611078544, 9.807194024, 9.684415081,
        10.17326848, 9.683191811, 10.03918111, 9.891267714, 9.707087079, 9.68933829,
        10.10867702, 9.770431111, 9.697278747, 10.15024178, 10.17638293, 9.676198933,
        9.765484028, 9.952918381, 10.15444308, 10.03372073, 9.607316647, 9.856609145,
        9.805244863, 9.728007162, 9.951510938, 10.03217857, 10.19504918, 10.23059564};
```

```
In[56]:= (* Find the mean, median, q1 and q3, and variance *)
      (* functions were defined above for the hospital data *)
     resistorMean = mean[resistors]
Out[56]= 9.87989
In[57]:= resistorMedian = median[resistors]
Out[57] = 9.91674
In[58]:= resistorQ1 = quantile[resistors, .25]
Out[58]= 9.71288
In[59]:= resistorQ3 = quantile[resistors, .75]
Out[59]= 10.1043
In[60]:= resistorVariance = variance[resistors]
Out[60]= 0.229448
      (* Histograms using two difference bin sizes *)
      (* I will use a bin size Length/2 for a very large bin count,
     and Length/10 for a smaller bin count*)
      (* Large bin count*)
     Histogram[resistors, IntegerPart[Length[resistors] / 2]]
      15
      10
      5
```

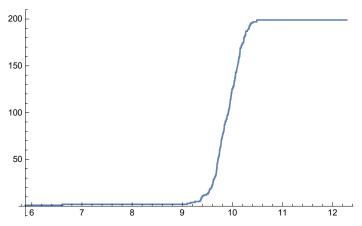
```
(* Small bin count *)
Histogram[resistors, IntegerPart[Length[resistors] / 10]]
50
40
30
20
10
                                                      11.0
                   9.5
                                           10.5
       9.0
                              10.0
```

```
(* Produce plots of quantile functions, moment functions, and CDFS \star)
(* functions were defined above for the hospital data *)
(* Plot the quantile functions *)
Plot[quantile[resistors, i], {i, 0, 1}]
10.5
10.0
9.5
                                                    1.0
            0.2
                      0.4
                                0.6
                                          0.8
```

```
In[61]:= (* Plot the moment functions *)
      (* Get the first 10 moments for the resistor data in a table *)
      momResistorData = Table[moment[resistors, i], {i, 1, 10}];
      (* Plot the moment *)
      ListPlot[momResistorData, PlotStyle \rightarrow PointSize[.03], PlotRange \rightarrow {0, 9.9 * 10^9}]
      8 \times 10^{9}
      6 \times 10^{9}
Out[62]=
      4 \times 10^{9}
      2 \times 10^{9}
      (* Plot the CDF functions *)
      (* CDF for range 0 to the maximum element in the data set *)
      Plot[cdf[resistors, i], {i, 0, Max[resistors]}]
      200
      150
      100
       50
                                                               12
                                                      10
```

(* CDF for range minimum element to maximum element in the data set *)

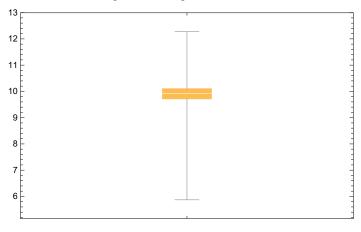
Plot[cdf[resistors, i], {i, Min[resistors], Max[resistors]}]



(* Box an whisker plots *)

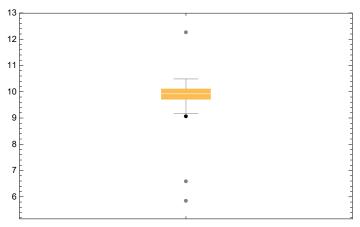
(* A box and whisker plot takes a min, q1, q2 (median), q3, and max *)

(* Box and whisker, resistor data, outliers not shown *) BoxWhiskerChart[resistors]



(* Box and whisker, resistor data, outliers shown *)

BoxWhiskerChart[resistors, "Outliers"]



0.12082240, 0.22755640, 0.14818550, 0.22244030, 0.17957990, 0.24217980, 0.20160190, 0.22291860, 0.14020770, 0.25848130, 0.13665170, 0.17852080, 0.11437730, 0.15921670, 0.13596970, 0.26868740, 0.10135420, 0.22488520, 0.20168690, 0.20958320, 0.22973200,

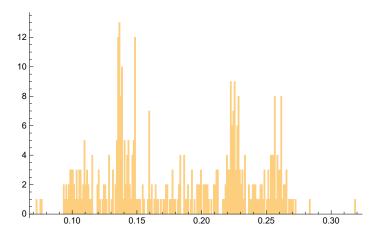
```
0.22528830, 0.10985200, 0.25246890, 0.14021050, 0.17207190, 0.14624990, 0.14421290,
0.13553570, 0.19721490, 0.11316860, 0.22365350, 0.19277550, 0.22177710, 0.22281810,
0.22673220, 0.18224740, 0.22948630, 0.13615970, 0.14343280, 0.13595380, 0.13332310,
0.13758030, 0.17235540, 0.13477980, 0.22750740, 0.27205230, 0.22723570, 0.22356740,
0.26976620, 0.16408940, 0.25041400, 0.13610470, 0.09729680, 0.10675350, 0.14689520,
0.10447090, 0.19264200, 0.11169570, 0.25101950, 0.24382670, 0.25675530, 0.23134120,
0.19872520, 0.15405980, 0.26169180, 0.12361070, 0.11594210, 0.13849870, 0.12892750,
0.15996210, 0.18149290, 0.10359830, 0.20306880, 0.22509590, 0.22510400, 0.22840200,
0.24895140, 0.19837270, 0.25859040, 0.18603360, 0.13769650, 0.13587430, 0.13107450,
0.12671680, 0.17744900, 0.10807120, 0.15458710, 0.22524160, 0.22834810, 0.26317470,
0.25623170, 0.14897020, 0.25305620, 0.19255750, 0.13608600, 0.13614730, 0.11293430,
0.13007420, 0.16590880, 0.10560190, 0.19604770, 0.22842020, 0.22890840, 0.19967150,
0.17581550, 0.21840620, 0.26584180, 0.18360270, 0.13641250, 0.15187060, 0.17108980};
```

```
In[64]:= (* Find the mean, median, q1, q3, and variance *)
      (* functions were defined above for the hospital data *)
      dripMean = mean[Drp]
Out[64]= 0.182304
In[65]:= dripMedian = median[Drp]
Out[65]= 0.181493
In[66]:= dripQ1 = quantile[Drp, .25]
Out[66]= 0.136332
In[67]:= dripQ3 = quantile[Drp, .75]
Out[67]= 0.227645
In[68]:= dripVariance = variance[Drp]
```

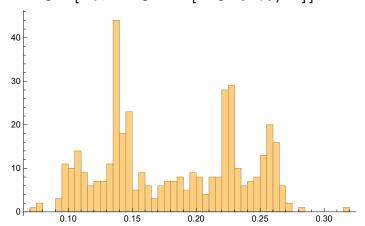
Out[68]= **0.00291283**

(* Histograms using two different bin sizes *) (* I will use a bin size of Length/2 for a very large bin count, and Length/10 for a smaller bin count *)

(* Large bin count *) Histogram[Drp, IntegerPart[Length[Drp] / 2]]

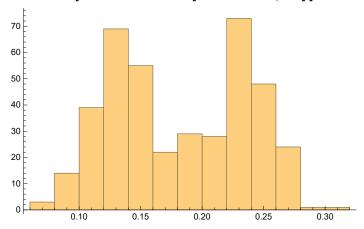


(* Small bin count *) Histogram[Drp, IntegerPart[Length[Drp] / 10]]



```
(* As there is 200 data elements,
Igit will use Length/30 for an even smaller bin count,
as it is appropriate for this data size, unlike the other data set sizes *)
```

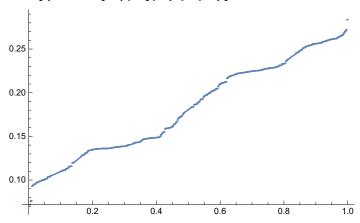
Histogram[Drp, IntegerPart[Length[Drp] / 30]]



(* Product plots of quantile functions, moment functions, and CDFS *)

(* functions were defined above for the hospital data *)

(*Plot the quantile functions *) Plot[quantile[Drp, i], {i, 0, 1}]

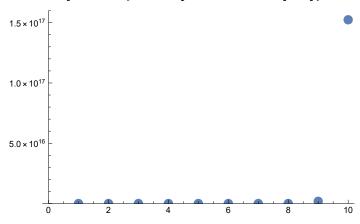


In[69]:= (* Plot the moment functions *)

(* Get the first 10 moments for the resistor data in a table *) momDripData = Table[moment[Drp, i], {i, 1, 10}];

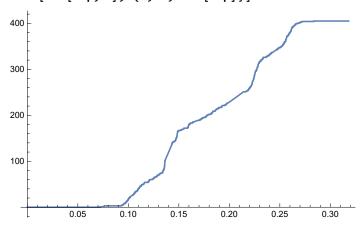
```
(* Note that since the drip data's elements are all less than 1, raising these values
  to a power 1-10 be decreasing this value,
as opposed to the other data sets where raising their
  values to a power 1-10 will increase the value *)
ListPlot[momDripData, PlotStyle → PointSize[.03], PlotRange → {0, .19}]
0.15
0.10
0.05
(* In comparison, here is what the MData and VData moments looked like \star)
(* MData moment *)
ListPlot[momMData, PlotStyle → PointSize[.03], PlotRange → {0, 9.6 * 10^17}]
8 \times 10^{17}
6 \times 10^{17}
4 \times 10^{17}
2 \times 10^{17}
(* VData moment *)
```

ListPlot[momVData, PlotStyle \rightarrow PointSize[.03], PlotRange \rightarrow {0, 1.6 * 10^17}]

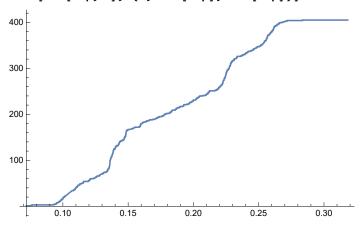


(* Plot the CDF functions *)

(* CDf for range 0 to the maximum element in the data set *) Plot[cdf[Drp, i], {i, 0, Max[Drp]}]



(* CDf for range minimum element to maximum element in the data set *) Plot[cdf[Drp, i], {i, Min[Drp], Max[Drp]}]

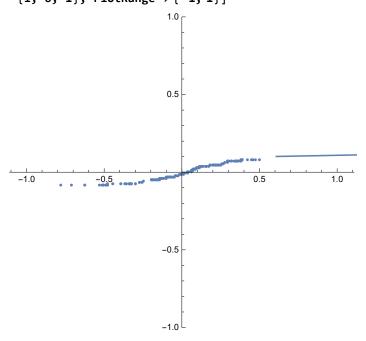


```
(* Box and Whisker plots *)
     (* A box and whisker plot takes a min, q1, q2 (median), q3, and max *)
     (* Box and whisker, drip, data, outliers not shown *)
     BoxWhiskerChart[Drp]
     0.30
     0.25
     0.20
     0.15
     0.10
     0.05
     (* Box and whisker, drip, data, outliers shown *)
     BoxWhiskerChart[Drp, "Outliers"]
     0.30
     0.25
     0.20
     0.15
     0.10
     0.05
In[70]:= (* QQ plot compairson for drip and resistor data *)
     (* Data must be centered (subtract the means) for both data sets *)
     (* "Centering simply means subtracting a constant from every value of a variable.
       What it does is redefine the 0 point for that predictor to be whatever
       value you subtracted. It shifts the scale over, but retains the units." *)
     (* Centering function *)
     (* Subtract the mean off of every value in the data set *)
     Centering[x_, meanValue_] :=
       (s = x; For[i = 1, i <= Length[s], i++, s[[i]] = s[[i]] - meanValue]; Return[s]);</pre>
In[71]:= (* Center the drip and resistor data *)
```

DripCentered = Centering[Drp, dripMean];

In[72]:= ResistorCentered = Centering[resistors, resistorMean];

(* Parametric Plots (QQ Plots), resistor data on x axis, drip data on y axis *) ParametricPlot[{quantile[ResistorCentered, i], quantile[DripCentered, i]}, {i, 0, 1}, PlotRange \rightarrow {-1, 1}]



(* If the two sets come from a population with the same distribution, the points should fall approximately along a 45 degree reference line. As we can see, the 2 batches do not appear to have come from populations with a common distribution, as they do not fit along a straight line. *) (* The QQ plot looks logarithmic, meaning that it dos not look like a straight line. This suggests that the two data sets came from populations with different distributions. *)

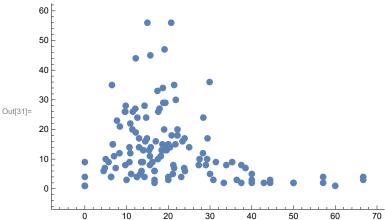
```
(* Problem 2 *)
      (* Calculate the correlation coefficient between the volume V and mortality M
       of the heart transplants based on the data in hospheart.nb *)
      (* hospheart.np data *)
      (* {M,V} M = one year mortality rate,
      percentage of patients that died within one year of the
       transplant operation,
      V = average annual number of transplants at that center during the same 4 years *)
      heart = \{\{17.9, 27\}, \{23.1, 4\}, \{40, 3\}, \{6.5, 35\}, \{14.9, 17\}, \{12.5, 4\}, \{15.7, 45\},
          \{9.8, 28\}, \{24, 6\}, \{5.0, 10\}, \{15.4, 13\}, \{4.8, 7\}, \{0, 1\}, \{19.1, 47\}, \{4.5, 6\},
          \{15, 56\}, \{12.5, 4\}, \{33.9, 8\}, \{10.7, 9\}, \{13, 14\}, \{28.3, 12\}, \{57.1, 2\}, \{6.3, 4\},
          \{10, 3\}, \{8.3, 12\}, \{17.5, 10\}, \{20, 3\}, \{29.3, 10\}, \{21.4, 7\}, \{27.3, 8\}, \{13.6, 6\},
          \{21.8, 30\}, \{36.4, 3\}, \{18.2, 11\}, \{33.3, 2\}, \{20, 4\}, \{38.5, 7\}, \{20.8, 18\}, \{12.2, 19\},
          \{22.2, 18\}, \{29, 8\}, \{0, 9\}, \{5.7, 9\}, \{50, 2\}, \{21.7, 15\}, \{66.7, 4\}, \{29.4, 17\},
          \{12.1, 27\}, \{10.7, 14\}, \{6.3, 4\}, \{16.2, 9\}, \{21.1, 5\}, \{17.4, 33\}, \{23.9, 17\},
          \{42.9, 2\}, \{40, 2\}, \{6.7, 15\}, \{44.4, 3\}, \{18.7, 34\}, \{14.7, 24\}, \{7.4, 7\}, \{12.6, 24\},
          \{9.7, 26\}, \{44.4, 2\}, \{16.7, 6\}, \{15.8, 14\}, \{83.3, 2\}, \{10.9, 22\}, \{13.3, 5\},
          \{11.1, 5\}, \{75, 2\}, \{19, 20\}, \{14, 13\}, \{60, 1\}, \{21.2, 8\}, \{9.7, 8\}, \{50, 2\}, \{25, 14\},
          \{18.6, 15\}, \{0.0, 1\}, \{35.3, 9\}, \{23.5, 85\}, \{15.6, 11\}, \{37.5, 2\}, \{14.3, 28\},
          \{14.3, 4\}, \{16.7, 6\}, \{20.0, 15\}, \{13.0, 17\}, \{9.6, 26\}, \{66.7, 3\}, \{30.8, 3\},
          {14.0, 13}, {27.5, 10}, {37.5, 8}, {18.9, 13}, {0.0, 4}, {12.2, 44}, {57.1, 4},
          \{21.4, 35\}, \{23.4, 16\}, \{10.9, 12\}, \{15.6, 8\}, \{16.7, 2\}, \{13.9, 9\}, \{18.2, 11\},
          \{11.5, 26\}, \{18.4, 13\}, \{16.7, 3\}, \{20.4, 14\}, \{40.0, 5\}, \{20.7, 56\}, \{19.6, 13\},
          \{13.5, 9\}, \{29.9, 36\}, \{8.4, 21\}, \{28.4, 24\}, \{7.7, 23\}, \{19.3, 29\}, \{0.0, 1\},
          \{22.2, 20\}, \{30.0, 5\}, \{7.0, 11\}, \{23.8, 7\}, \{18.8, 29\}, \{14.5, 16\}, \{17.0, 16\},
          \{20.0, 15\}, \{6.7, 15\}, \{11.4, 20\}, \{100.0, 1\}, \{31.4, 9\}, \{17.6, 26\}, \{19.6, 14\}\};
      (* Split this M and V data into separate
       lists via Transpose[] in order to parse through *)
      heartTranspose = Transpose[heart];
      MData = heartTranspose[[1]];
      VData = heartTranspose[[2]];
      (* To calculate the correlation coefficient,
      we need to define functions to find the means of a data set *)
      mean[x_] := Sum[x[[i]], {i, 1, Length[x]}] / Length[x];
      (* Sum elements, divide by length *)
      meanM = mean[MData]
Out[15]= 21.9045
In[16]:= meanV = N[mean[VData]]
Out[16]= 13.8657
```

```
In[17]:= (* Create a function to sum (m-meanM) * (v-meanV),
     where m and v are elemnts of M and V respectively. *)
     differenceMeanSum[m_, mBAR_, v_, vBAR_] :=
      Sum[(m[[i]] - mBAR) * (v[[i]] - vBAR)), \{i, 1, Length[m]\}]
     (* Sum the product of (elementInM - meanOfM) (elementInV - meanOfV) *)
In[18]= (* find the sum of the mean difference as noted above,
     this is the numerator of our correlation coefficient equation *)
In[19]= MVdifferenceMeanSum = differenceMeanSum[MData, meanM, VData, meanV]
Out[19]= -7238.42
ln[20]:= (* The denominator of the correlation coefficient equation is
      the square root of: sum of (x-xBAR)^2 times sum of (y-yBAR)^2 *
     (* Create a function to Sum a data sets elemnts, by taking an element,
     subtracting the mean from it, and squating the value *)
ln[21] = squaredSum[a_, aMean_] := Sum[(a[[i]] - aMean)^2, {i, 1, Length[a]}];
_{\ln[22]:=} (* Now find the the squared difference sum for the M and V data *)
     squareDifferenceM = squaredSum[MData, meanM]
Out[22]= 35 996.9
In[23]:= squareDifferenceV = squaredSum[VData, meanV]
Out[23]= 22305.6
In[24]:= (* Take the root of the product of these sums,
     and that is the denominator of the correlation coefficient equation *)
     rootOfSums = Sqrt[squareDifferenceM * squareDifferenceV]
Out[24]= 28 336.1
In[25]:= (* Now take the numerator and denominator,
     find the decimal of that fraction and we have the correlation coefficient *)
     coefficient = MVdifferenceMeanSum / rootOfSums
```

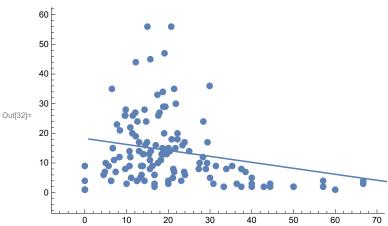
Out[25]= -0.255449

```
In[26]:= (* This is negative, so as x increases,
     y decreases. As is is close to zero, it is not a strong correlation *)
     (* Now, we need to make a scatter plot of this data *)
     (* We also need the line of best fit,
     which will be calculated manually. Assuming the V data is y axis, M data is x axis*)
     (* The numerator for the slope of our
      best fit line is the variable MVdifferenceMeanSum *)
     (* The denominator for the slope of our best fit line is the sum of x -
      xMean squared *)
     (* This denominator is as follows *)
     denom = Sum[(MData[[i]] - meanM)^2, {i, 1, Length[MData]}];
     slope = MVdifferenceMeanSum / denom
Out[27]= -0.201084
ln[28]:= (* The b value is calculated as the y mean, minus slope * x mean *)
     b = meanV - slope * meanM
Out[28]= 18.2703
In[29]:= (* The best fit equation is as follows *)
     bestFit[x_] := slope * x + b;
In[30]:= (* Now, superimpose this regression line on the scatter plot *)
     bestFitGraph = Plot[bestFit[x], {x, 1, 100}]
     15
     10
Out[30]=
      5
                20
                          40
                                    60
                                                        100
```

```
In[31]:= scatterPlot = ListPlot[heart, PlotStyle → PointSize[.02],
       PlotRangePadding → Scaled[0.1], Axes → False, Frame → {True, True, False, False}]
     (* Extra paramets on this scatter plot to best show the data *)
```



In[32]:= superimpose = Show[scatterPlot, bestFitGraph]



In[33]:= (* To double check my calculations are correct, I will let Mathematica generate a scatter plot and best fit line *)

In[34]:= Fit[heart, {1, x}, {x}] Out[34]= 18.2703 - 0.201084 x

```
ln[35]:= bestLine[x] := 18.270320917115342 - 0.20108442453442926 x;
     Show[ListPlot[heart, PlotStyle \rightarrow PointSize[.02], PlotRangePadding \rightarrow Scaled[0.1],
        Axes \rightarrow False, Frame \rightarrow {True, True, False, False}], Plot[bestLine[x], {x, 1, 100}]]
     60
     50
      40
     30
Out[36]=
     20
      10
      0
                   10
                          20
                                 30
                                        40
                                              50
                                                     60
      (* Conclusions *)
      (* This is negative, so as x increases,
     y decreases. As is is close to zero, it is not a strong correlation *)
      (* The one year mortality rate,
      and average annual number of transplants at that center during the same 4 years, are
       not correlated strongly. The correlation is very weak *)
```

```
(* Problem 3 *)
(* Select long texts in three different languages. Calculate the entropy
for each and compare them, draw your conclusions *)
(* I will first select an English text *)
```

In[73]:= (* Sample text from The Adventures of Sherklock Holmes *) text =

> "To Sherlock Holmes she is always the woman. I have seldom heard him mention her under any other name. In his eyes she eclipses and predominates the whole of her sex. It was not that he felt any emotion akin to love for Irene Adler. All emotions, and that one particularly, were abhorrent to his cold, precise but admirably balanced mind. He was, I take it, the most perfect reasoning and observing machine that the world has seen, but as a lover he would have placed himself in a false position. He never spoke of the softer passions, save with a gibe and a sneer. They were admirable things for the observer--excellent for drawing the veil from men's motives and actions. But for the trained reasoner to admit such intrusions into his own delicate and finely adjusted temperament was to introduce a distracting factor which might throw a doubt upon all his mental results. Grit in a sensitive instrument, or a crack in one of his own high-power lenses, would not be more disturbing than a strong emotion in a nature such as his. And yet there was but one woman to him, and that woman was the late Irene Adler, of dubious and questionable memory.

I had seen little of Holmes lately. My marriage had drifted us away from each other. My own complete happiness, and the home-centred interests which rise up around the man who first finds himself master of his own establishment, were sufficient to absorb all my attention, while Holmes, who loathed every form of society with his whole Bohemian soul, remained in our lodgings in Baker Street, buried among his old books, and alternating from week to week between cocaine and ambition, the drowsiness of the drug, and the fierce energy of his own keen nature. He was still, as ever, deeply attracted by the study of crime, and occupied his immense faculties and extraordinary powers of observation in following out those clues, and clearing up those mysteries which had been abandoned as hopeless by the official police. From time to time I heard some vague account of his doings: of his summons to Odessa in the case of the Trepoff murder, of his clearing up of the singular tragedy of the Atkinson brothers at Trincomalee, and finally of the mission which he had accomplished so delicately and successfully for the reigning family of Holland. Beyond these signs of his activity, however, which I merely shared with all the readers of the daily press, I knew little of my former friend and companion.

One night--it was on the twentieth of March, 1888--I was returning from a journey to a patient (for I had now returned to civil practice), when my way led me through Baker Street. As I passed the well-remembered door, which must always be associated in my mind with my wooing, and with the dark incidents of the Study in Scarlet, I was seized with a keen desire to see Holmes again, and to know how he was employing his extraordinary powers. His rooms were brilliantly lit, and, even as I looked up, I saw his tall, spare figure pass twice in a dark silhouette against the blind. He was pacing the room swiftly, eagerly, with his head sunk upon his chest and his hands clasped behind him. To me, who knew his every mood and habit, his attitude and manner told their own story. He was at work again. He had risen out of his drug-created dreams and was hot upon the scent of some new problem. I rang the bell and was shown up to the chamber which had formerly been in part my own.";

```
In[74]:= lowerEnglishText = ToLowerCase[text]; (* Convert text to lowercase *)
          In[75]:= charcode = ToCharacterCode[lowerEnglishText];
                                           (* Get a numerical character code for each letter in the string *)
          Im[76]: charentp = Table[{Length[Position[charcode, i]], FromCharacterCode[i]}, {i, 96, 122}]
                                                  (* make a table with the counts for each character *)
       Out[76] = \{\{0, `\}, \{218, a\}, \{43, b\}, \{69, c\}, \{118, d\}, \{335, e\}, \{66, f\}, \{41, g\}, \{170, h\}, \{18, d\}, \{18, b\}, \{19, 
                                                  \{211, i\}, \{2, j\}, \{21, k\}, \{110, 1\}, \{93, m\}, \{196, n\}, \{208, o\}, \{42, p\}, \{1, q\},
                                                  \{161, r\}, \{191, s\}, \{214, t\}, \{61, u\}, \{20, v\}, \{81, w\}, \{4, x\}, \{50, y\}, \{1, z\}\}
          In[77]:= charentp[[1]] = {Length[Position[charcode, 32]], "space"}
                                                  (* Also, get the character count for spaces *)
       Out[77]= { 611, space }
          IN[78]= TableForm[charentp] (* Put the counts and characters in a table to visualize easier *)
Out[78]//TableForm=
                                         611
                                                                                      space
                                         218
                                                                                     а
                                         43
                                                                                     b
                                          69
                                                                                      C
                                          118
                                                                                      d
                                          335
                                                                                      e
                                          66
                                                                                      f
                                         41
                                                                                      g
                                          170
                                                                                     h
                                          211
                                                                                      i
                                          2
                                                                                      j
                                          21
                                                                                      k
                                         110
                                                                                      1
                                        93
                                                                                     m
                                          196
                                                                                     n
                                          208
                                                                                     0
                                         42
                                                                                      р
                                          1
                                                                                      q
                                         161
                                         191
                                                                                      s
                                          214
                                                                                     t
                                          61
                                                                                     u
                                          20
                                                                                     ٧
                                          81
                                                                                     W
                                                                                     Х
                                          50
                                                                                      У
           In[79]:= sortcharentp = Reverse[Sort[charentp]]
                                                  (* Sort by most common to least common frequencies *)
       Out[79] = \{ \{611, space\}, \{335, e\}, \{218, a\}, \{214, t\}, \{211, i\}, \{208, o\}, \{196, n\}, \{191, s\}, \{211, i\}, \{211, i\}
                                                  \{170, h\}, \{161, r\}, \{118, d\}, \{110, 1\}, \{93, m\}, \{81, w\}, \{69, c\}, \{66, f\}, \{61, u\}, \{61, u
                                                  \{50, y\}, \{43, b\}, \{42, p\}, \{41, g\}, \{21, k\}, \{20, v\}, \{4, x\}, \{2, j\}, \{1, z\}, \{1, q\}\}
           In[80]:= sumchars = Sum[sortcharentp[[i, 1]], {i, Length[sortcharentp]}]
                                                  (* Get the total character count in the text *)
       Out[80]= 3338
```

```
ln[81]= charfreq = N[Table[{sortcharentp[[i, 1]]/sumchars, sortcharentp[[i, 2]]},
          {i, 1, Length[sortcharentp]}] (* Get the frequency for each character in the text *)
 Out[81] = \{\{0.183044, space\}, \{0.100359, e\}, \{0.0653086, a\}, \{0.0641102, t\}, \}
        {0.0632115, i}, {0.0623128, o}, {0.0587178, n}, {0.0572199, s}, {0.0509287, h},
        \{0.0482325, r\}, \{0.0353505, d\}, \{0.0329539, 1\}, \{0.027861, m\}, \{0.024266, w\},
        {0.0206711, c}, {0.0197723, f}, {0.0182744, u}, {0.014979, y}, {0.012882, b},
        \{0.0125824, p\}, \{0.0122828, g\}, \{0.00629119, k\}, \{0.00599161, v\},
        \{0.00119832, x\}, \{0.000599161, j\}, \{0.000299581, z\}, \{0.000299581, q\}\}
 Out[82]//TableForm=
      0.183044
                     space
      0.100359
                     е
      0.0653086
                     а
      0.0641102
                     t
      0.0632115
                     i
      0.0623128
      0.0587178
      0.0572199
      0.0509287
      0.0482325
      0.0353505
                     d
      0.0329539
                     1
      0.027861
                     m
      0.024266
                     W
      0.0206711
                      c
      0.0197723
                     f
      0.0182744
                     u
      0.014979
                     У
      0.012882
      0.0125824
                     р
      0.0122828
                     g
      0.00629119
                     k
      0.00599161
                     ٧
      0.00119832
                     Х
      0.000599161
                     j
      0.000299581
                     Z
      0.000299581
 In[83]:= entplang = 0;
       (* Loop through the entire table, using the entropy formula,
      calculate the entropy of the text given our frequencies *)
      For [i = 1, i <= 27, i++,
                   If[charfreq[[i, 1]] == 0, entplang,
                       entplang = entplang - charfreq[[i, 1]] * Log[charfreq[[i, 1]]]
                   ]
               ]
 _{\text{ln[85]:=}} (* This represents the average minimum number of bits needed to
       encode a string of symbols, based on the frequency of the symbols. *)
      entplang
 Out[85]= 2.83089
```

In[86]:= (* Take this entropy, multiply it by the size of our text, and that gives the amount of bits required to optimally encode the string *) optimalEnglishEncoding = entplang * sumchars

Out[86]= 9449.52

- In[87]:= (* Next, I will analyze a French text *)
 - (* Le tour du monde en quatre-vingts jours by Jules Verne is the French version of Around the World in Eighty Days *)

In[88]:=

frenchText = "

Mais si le rétablissement de la jeune Indienne ne fit pas question dans l'esprit du brigadier général, celui-ci se montrait moins rassuré pour l'avenir. Il n'hésita pas à dire à Phileas Fogg que si Mrs. Aouda restait dans l'Inde, elle retomberait inévitablement entre les mains de ses bourreaux. Ces énergumènes se tenaient dans toute la péninsule, et certainement, malgré la police anglaise, ils sauraient reprendre leur victime, fût-ce à Madras, à Bombay, à Calcutta. Et Sir Francis Cromarty citait, à l'appui de ce dire, un fait de même nature qui s'était passé récemment. A son avis, la jeune femme ne serait véritablement en sûreté qu'après avoir quitté l'Inde.

Phileas Fogg répondit qu'il tiendrait compte de ces observations et qu'il aviserait.

Vers dix heures, le guide annonçait la station d'Allahabad. Là reprenait la voie interrompue du chemin de fer, dont les trains franchissent, en moins d'un jour et d'une nuit, la distance qui sépare Allahabad de Calcutta.

Phileas Fogg devait donc arriver à temps pour prendre un paquebot qui ne partait que le lendemain seulement, 25 octobre, à midi, pour Hong-Kong.

La jeune femme fut déposée dans une chambre de la gare. Passepartout fut chargé d'aller acheter pour elle divers objets de toilette, robe, châle, fourrures, etc., ce qu'il trouverait. Son maître lui ouvrait un crédit illimité.

Passepartout partit aussitôt et courut les rues de la ville. Allahabad, c'est la cité de Dieu, l'une des plus vénérées de l'Inde, en raison de ce qu'elle est bâtie au confluent de deux fleuves sacrés, le Gange et la Jumna, dont les eaux attirent les pèlerins de toute la péninsule. On sait d'ailleurs que, suivant les légendes du Ramayana, le Gange prend sa source dans le ciel, d'où, grâce à Brahma, il descend sur la terre.";

```
lowerFrenchText = ToLowerCase[frenchText]; (* Convert text to lowercase *)
```

```
In[90]:= charcode = ToCharacterCode[lowerFrenchText];
     (* Get a numerical character code for each letter in the string *)
```

```
In[01]:= charentp = Table[{Length[Position[charcode, i]], FromCharacterCode[i]}, {i, 96, 122}];
                                 (* make a table with the counts for each character *)
  In[92]:= charentp[[1]] = {Length[Position[charcode, 32]], "space"}
                                        (* Also, get the character count for spaces *)
Out[92]= { 262, space}
  In[93]= TableForm[charentp] (* Put the counts and characters in a table to visualize easier *)
                                262
                                                                         space
                                129
                                                                         а
                                19
                                                                         b
                                40
                                                                         C
                                                                         d
                                64
                                198
                                                                         e
                                 16
                                                                         f
                                 22
                                                                         g
                                                                         h
                               16
                               107
                                                                         i
                                6
                                                                         j
                               1
                                                                         k
                               87
                                                                         1
                                38
                                                                         m
                                97
                                                                         n
                                54
                                                                         0
                                36
                                                                         р
                                14
                                                                         q
                                98
                               97
                                                                         s
                                108
                                                                         t
                                78
                                                                         u
                                19
                                                                         ٧
                               0
                                                                         W
                                4
                                                                         Х
                               3
                                                                         У
                               0
  In[94]:= sortcharentp = Reverse[Sort[charentp]]
                                        (* Sort by most common to least common frequencies *)
\texttt{Out} \texttt{[4262, space], \{198, e\}, \{129, a\}, \{108, t\}, \{107, i\}, \{98, r\}, \{97, s\}, \{97, n\}, 
                                        \{87, 1\}, \{78, u\}, \{64, d\}, \{54, o\}, \{40, c\}, \{38, m\}, \{36, p\}, \{22, g\}, \{19, v\}, \{40, c\}, \{
                                       \{19, b\}, \{16, h\}, \{16, f\}, \{14, q\}, \{6, j\}, \{4, x\}, \{3, y\}, \{1, k\}, \{0, z\}, \{0, w\}\}
  In[95]:= sumchars = Sum[sortcharentp[[i, 1]], {i, Length[sortcharentp]}]
                                        (* Get the total character count in the text *)
Out[95]= 1613
  In[97]:=
                                 (* Get the frequency for each character in the text *)
```

```
In[98]:= charfreq = N[Table[{sortcharentp[[i, 1]]/sumchars, sortcharentp[[i, 2]]},
          {i, 1, Length[sortcharentp]}]]
 Out[98]= \{\{0.16243, \text{space}\}, \{0.122753, e\}, \{0.0799752, a\}, \{0.066956, t\},
        \{0.066336, i\}, \{0.0607564, r\}, \{0.0601364, s\}, \{0.0601364, n\}, \{0.0539368, l\},
        {0.0483571, u}, {0.0396776, d}, {0.033478, o}, {0.0247985, c}, {0.0235586, m},
        \{0.0223187, p\}, \{0.0136392, g\}, \{0.0117793, v\}, \{0.0117793, b\},
        {0.0099194, h}, {0.0099194, f}, {0.00867948, q}, {0.00371978, j},
        \{0.00247985, x\}, \{0.00185989, y\}, \{0.000619963, k\}, \{0., z\}, \{0., w\}\}
 Out[99]//TableForm=
      0.16243
                      space
      0.122753
                      е
      0.0799752
                      а
      0.066956
                     t
      0.066336
                      i
      0.0607564
      0.0601364
      0.0601364
                      n
      0.0539368
      0.0483571
                      u
      0.0396776
                      d
      0.033478
                      0
      0.0247985
                      C
      0.0235586
                      m
      0.0223187
                      р
      0.0136392
                      g
      0.0117793
      0.0117793
                      b
      0.0099194
                      h
      0.0099194
      0.00867948
                      q
      0.00371978
                      j
      0.00247985
                     Х
      0.00185989
                      У
      0.000619963
                      k
      0.
                      Z
      0.
 In[100]:= entplang = 0;
       (* Loop through the entire table, using the entropy formula,
      calculate the entropy of the text given our frequencies *)
      For [i = 1, i <= 27, i++,
                   If[charfreq[[i, 1]] == 0, entplang,
                       entplang = entplang - charfreq[[i, 1]] * Log[charfreq[[i, 1]]]
                   ]
               ]
 _{
m In[102]:=} (* This represents the average minimum number of bits needed to encode a string
       of symbols, based on the frequency of the symbols. *)
      entplang
Out[102]= 2.78247
```

```
In[103]:= (* Take this entropy, multiply it by the size of our text,
      and that gives the amount of bits required to optimally encode the string *)
     optimalFrenchEncoding = entplang * sumchars
Out[103]= 4488.12
In[104]:= (* Next, I will analyze an Italian text *)
      (* Orlando Furioso, and Italian epic poem by Ludovico Ariosto *)
In[105]:= italianText = "Le donne, i cavallier, l'arme, gli amori,
        le cortesie, l'audaci imprese io canto,
        che furo al tempo che passaro i Mori
        d'Africa il mare, e in Francia nocquer tanto,
        seguendo l'ire e i giovenil furori
        d'Agramante lor re, che si diè vanto
        di vendicar la morte di Troiano
        sopra re Carlo imperator romano. Dirò d'Orlando in un medesmo tratto
        cosa non detta in prosa mai, né in rima:
        che per amor venne in furore e matto,
        d'uom che sì saggio era stimato prima;
        se da colei che tal quasi m'ha fatto,
        che 'l poco ingegno ad or ad or mi lima,
        me ne sarà però tanto concesso,
        che mi basti a finir quanto ho promesso. Piacciavi, generosa Erculea prole,
        ornamento e splendor del secol nostro,
        Ippolito, aggradir questo che vuole
        e darvi sol può l'umil servo vostro.
        Quel ch'io vi debbo, posso di parole
        pagare in parte e d'opera d'inchiostro;
        né che poco io vi dia da imputar sono,
        che quanto io posso dar, tutto vi dono.Voi sentirete fra i più degni eroi,
        che nominar con laude m'apparecchio,
        ricordar quel Ruggier, che fu di voi
        e de' vostri avi illustri il ceppo vecchio.
        L'alto valore e' chiari gesti suoi
        vi farò udir, se voi mi date orecchio,
        e vostri alti pensier cedino un poco,
        sì che tra lor miei versi abbiano loco. Orlando, che gran tempo innamorato
        fu de la bella Angelica, e per lei
        in India, in Media, in Tartaria lasciato
        avea infiniti ed immortal trofei,
        in Ponente con essa era tornato,
        dove sotto i gran monti Pirenei
        con la gente di Francia e de Lamagna
        re Carlo era attendato alla campagna, per far al re Marsilio e al re Agramante
        battersi ancor del folle ardir la guancia,
        d'aver condotto, l'un, d'Africa quante
        genti erano atte a portar spada e lancia;
        l'altro, d'aver spinta la Spagna inante
        a destruzion del bel regno di Francia.
        E così Orlando arrivò quivi a punto:
        ma tosto si pentì d'esservi giunto:
```

```
che vi fu tolta la sua donna poi:
                                ecco il giudicio uman come spesso erra!
                                Quella che dagli esperi ai liti eoi
                                avea difesa con sì lunga guerra,
                                or tolta gli è fra tanti amici suoi,
                                senza spada adoprar, ne la sua terra.
                                Il savio imperator, ch'estinguer volse
                               un grave incendio, fu che gli la tolse.Nata pochi dì inanzi era una gara
                                tra il conte Orlando e il suo cugin Rinaldo,
                                che entrambi avean per la bellezza rara
                                d'amoroso disio l'animo caldo.
                                Carlo, che non avea tal lite cara,
                                che gli rendea l'aiuto lor men saldo,
                                questa donzella, che la causa n'era,
                                tolse, e diè in mano al duca di Bavera; in premio promettendola a quel d'essi,
                                ch'in quel conflitto, in quella gran giornata,
                                degl'infideli più copia uccidessi,
                                e di sua man prestasse opra più grata.
                               Contrari ai voti poi furo i successi;
                                ch'in fuga andò la gente battezzata,
                                e con molti altri fu 'l duca prigione,
                                e restò abbandonato il padiglione. ";
  In[106]:= lowerItalianText = ToLowerCase[italianText]; (* Convert text to lowercase *)
  in[107]:= charcode = ToCharacterCode[lowerItalianText];
                         (* Get a numerical character code for each letter in the string *)
  In[108]= charentp = Table[{Length[Position[charcode, i]], FromCharacterCode[i]}, {i, 96, 122}]
                              (* make a table with the counts for each character *)
Out[108] = \{ \{0, `\}, \{260, a\}, \{14, b\}, \{97, c\}, \{88, d\}, \{220, e\}, \{27, f\}, \{47, g\}, \{34, h\}, \{280, e\}, \{14, b\}, \{14,
                            \{215, i\}, \{0, j\}, \{0, k\}, \{121, 1\}, \{56, m\}, \{146, n\}, \{204, o\}, \{62, p\}, \{14, q\}, \{140, n\}, \{140, o\}, \{
                             \{168, r\}, \{94, s\}, \{122, t\}, \{64, u\}, \{39, v\}, \{0, w\}, \{0, x\}, \{0, y\}, \{8, z\}\}
 In[109]:= charentp[[1]] = {Length[Position[charcode, 32]], "space"}
                              (* Also, get the character count for spaces *)
Out[109]= { 550, space }
```

```
In[110]:= TableForm[charentp] (* Put this frequency in a table to visualize easier *)
Out[110]//TableForm=
                                                                     space
                                  550
                                  260
                                                                     a
                                  14
                                                                     b
                                 97
                                                                     C
                                  88
                                                                     d
                                   220
                                                                      e
                                                                     f
                                   27
                                  47
                                                                     g
                                   34
                                                                     h
                                   215
                                                                      i
                                                                      j
                                 0
                                                                     k
                                 121
                                                                     1
                                  56
                                                                     m
                                 146
                                                                     n
                                  204
                                                                     0
                                 62
                                                                     р
                                 14
                                                                      q
                                 168
                                                                      r
                                 94
                                                                      S
                                 122
                                                                      t
                                  64
                                                                     u
                                  39
                                                                      ٧
                                 0
                                                                     W
                                  0
                                                                     Х
                                 0
                                                                     У
                                  8
     in[111]:= sortcharentp = Reverse[Sort[charentp]]
                                        (* Sort by most common to least common frequencies *)
   \label{eq:output} \text{Output} = \{\{550, \, space\}, \, \{260, \, a\}, \, \{220, \, e\}, \, \{215, \, i\}, \, \{204, \, o\}, \, \{168, \, r\}, \, \{146, \, n\}, \, \{122, \, t\}, \, \{146, \, n\}, \, \{146, \,
                                         \{121, 1\}, \{97, c\}, \{94, s\}, \{88, d\}, \{64, u\}, \{62, p\}, \{56, m\}, \{47, g\}, \{39, v\},
                                        \{34, h\}, \{27, f\}, \{14, q\}, \{14, b\}, \{8, z\}, \{0, y\}, \{0, x\}, \{0, w\}, \{0, k\}, \{0, j\}\}
     In[112]:= sumchars = Sum[sortcharentp[[i, 1]], {i, Length[sortcharentp]}]
                                         (* Get the total character count in the text *)
  Out[112]= 2650
     In[113]:= (* Get the frequency for each character in the text *)
                                   charfreq = N[Table[{sortcharentp[[i, 1]] / sumchars, sortcharentp[[i, 2]]},
                                                   {i, 1, Length[sortcharentp]}]]
  \label{eq:output} \textit{Out[113]=} \ \{ \{0.207547, \, space\}, \, \{0.0981132, \, a\}, \, \{0.0830189, \, e\}, \, \{0.0811321, \, i\}, \, \{0.0769811, \, o\}, \, \{0.0811321, \, i\}, \, \{0.081
                                        {0.0633962, r}, {0.0550943, n}, {0.0460377, t}, {0.0456604, 1}, {0.0366038, c},
                                        \{0.0354717, s\}, \{0.0332075, d\}, \{0.0241509, u\}, \{0.0233962, p\}, \{0.0211321, m\},
                                        {0.0177358, g}, {0.014717, v}, {0.0128302, h}, {0.0101887, f}, {0.00528302, q},
                                        \{0.00528302, b\}, \{0.00301887, z\}, \{0., y\}, \{0., x\}, \{0., w\}, \{0., k\}, \{0., j\}\}
```

```
In[114]:= TableForm[charfreq] (* Put this frequency in a table to visualize easier *)
Out[114]//TableForm=
                     space
      0.207547
      0.0981132
                     а
      0.0830189
                     e
      0.0811321
                     i
      0.0769811
                     0
      0.0633962
      0.0550943
      0.0460377
                     t
      0.0456604
                     1
      0.0366038
                     C
      0.0354717
                     s
      0.0332075
      0.0241509
                     u
      0.0233962
                     р
      0.0211321
                     m
      0.0177358
      0.014717
                     ٧
      0.0128302
                     h
      0.0101887
      0.00528302
                     q
      0.00528302
                     b
      0.00301887
                     Z
      0.
                     У
      0.
                     Х
      0.
                     W
      0.
                     k
      0.
                     j
 In[115]:= entplang = 0;
       (* Loop through the entire table, using the entropy formula,
       calculate the entropy of the text given our frequencies *)
      For [i = 1, i <= 27, i++,
                    If[charfreq[[i, 1]] == 0, entplang,
                        entplang = entplang - charfreq[[i, 1]] * Log[charfreq[[i, 1]]]
                    ]
               ]
 In[117]:= (* This represents the average minimum number of bits needed to
        encode a string of symbols, based on the frequency of the symbols .*)
      entplang
Out[117]= 2.70014
 IN[118]:= (* Take this entropy, multiply it by the size of our text, and that gives the amount
        of bits required to optimally encode the string *)
      optimalItalianEncoding = entplang * sumchars
Out[118]= 7155.37
```

```
In[119]:= (* Analysis on languages *)
     (* English: 2.83089, French: 2.78246, Italian: 2.70014 *)
     (* In general, all three of these languages have a very similar entropy. Analyzing even
      longer texts can yeild more accurate results due to the law of large numbers *)
     (* Since English is a Germanic language, and Italian and French are Romance languages,
     I expected English' entropy to be very different from French and Italian *)
     (* The space, letter e, and a,
     were all top 3 for these languages for relative frequency,
     and were similar frequencies among
      the three languages. Since the top 3 were similar across all three languages,
     it's not suprising
      that the entropies of the languages are relatively the same *)
     (* From this test, I am concluding that there isn't a significant difference between the
      language's entropies. *)
```