Homework 3

1.) Given points: $(0,3), (-1,2), (2,11), (1,4)$
         1   2   3   4

$$L(x) = \sum_{j=1}^{k} y_j L_j(x)$$

$$L_j(x) = \prod_{\substack{0 < m < k \\ m \neq j}} \frac{x - x_m}{x_j - x_m}$$

$$x_1 = 0, \quad x_2 = -1, \quad x_3 = 2, \quad x_4 = 1$$

$$L_1(x) = \left(\frac{x+1}{0+1}\right)\left(\frac{x-2}{0-2}\right)\left(\frac{x-1}{0-1}\right)$$
$j = 0$

$$L_2(x) = \left(\frac{x-0}{-1-0}\right)\left(\frac{x-2}{-1-2}\right)\left(\frac{x-1}{0-1}\right)$$
$j = -1$

$$L_3(x) = \left(\frac{x-0}{2-0}\right)\left(\frac{x+1}{2+1}\right)\left(\frac{x-1}{2-1}\right)$$
$j = 2$

$$L_4(x) = \left(\frac{x-0}{1-0}\right)\left(\frac{x+1}{2+1}\right)\left(\frac{x-2}{1-2}\right)$$
$j = 1$

$$\frac{x^3 - 2x^2 - x + 2}{2}, \frac{x^3 - 3x^2 + 2x}{-3}, \frac{x^3 - x}{6}, \frac{x^3 - x^2 - 2x}{-3}$$

$$\frac{3}{2}\cdot\left(x^3 - 2x^2 - x + 2\right) - \frac{2}{3}\left(x^3 - 3x^2 + 2x\right) + \frac{11}{6}\left(x^3 - x\right) - \frac{4}{3}\left(x^3 - x^2 - 2x\right)$$

$$\frac{3}{2}x^3 - 3x^2 - \frac{3x}{2} + 3 - \frac{2}{3}x^3 + 2x^2 - \frac{4x}{3} + \frac{11x^3}{6} - \frac{11}{6}x - \frac{4x^3}{3} + \frac{4x^2}{3} + \frac{8x}{3}$$

$$\left(\frac{3}{2} - \frac{2}{3} + \frac{11}{6} - \frac{4}{3}\right)x^3 = \frac{4}{3}x^3$$

$$\left(-3 + 2 + \frac{4}{3}\right)x^2 = \frac{1}{3}x^2$$

$$\left(-\frac{3}{2} - \frac{4}{3} - \frac{11}{6} + \frac{8}{3}\right)x = -2x$$

$$(3) = 3$$

$$f(x) = \frac{4}{3}x^3 + \frac{1}{3}x^2 - 2x^2 + 3$$