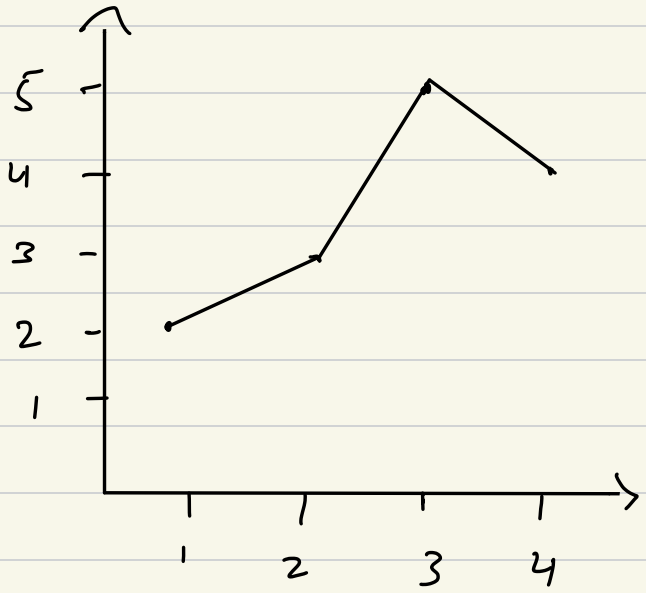
3/23

Homework 7

1.)

x	1	2	3	4
y	2	3	5	4



1.) Identify a good set of initial parameters (1,2) & (4,4) -> so it accounts for the entire class

$$m = \frac{4-2}{4-1} = \frac{2}{3}$$

$$\frac{y-2}{x-1} = 2/3$$

$$3y - 6 = 2x - 2$$

$$3y = 2x + 4$$

$$y = 2/3\,x + 4/3$$

$$b = 4/3$$

$$E(m,b) = \frac{1}{n}\sum_{i=1}^{n} | y_i - (mx_i + b) |$$

$y = \frac{2}{3}x + \frac{4}{3}$ | $y_{actual} - y_{pred} = y_i - (mx_i + b)$

$x = 1$, $y_{actual} = 2$

$y = \frac{2}{3} + \frac{4}{3} = \frac{6}{3} = 2$

$y_{pred} = 2$

$AE = 2 - 2 = 0$

$x = 2$ $y_{actual} = 3$

$y = \frac{2}{3}(2) + \frac{4}{3}$

$\frac{4}{3} + \frac{4}{3} = \frac{8}{3}$

$y_{pred} = \frac{8}{3}$

$AE = \frac{9}{3} - \frac{8}{3} = \frac{1}{3}$

$x = 3$ $y_{actual} = 5$

$y = \frac{2}{3}(3) + \frac{4}{3}$

$2 + \frac{4}{3} = \frac{6}{3} + \frac{4}{3} = \frac{10}{3}$

$y_{pred} = \frac{10}{3}$

$AE = 5 - \frac{10}{3} = \frac{15}{3} = \frac{10}{3} = \frac{5}{3}$

$x = 4 \quad y_{actual} = 4$

$$y = \frac{2}{3}x + \frac{4}{3} = \frac{2}{3}(4) + \frac{4}{3} = \frac{12}{3} = 4$$

$y_{pred} = 4$

$AE = 4 - 4 = 0$

ΣMAE

$$0 + \frac{1}{3} + \frac{5}{3} + 0 = \frac{6}{3} = 2$$

$$E\left(\frac{2}{3}, \frac{4}{3}\right) = \frac{1}{4} \times 2 = 0.5 \rightarrow MAE = 0.5$$

3.) $$\frac{dE}{dm} = -\frac{1}{n} \sum_{i=1}^{n} sign(y_i - (mx_i + b)) \cdot x_i$$

$$\frac{dE}{db} = -\frac{1}{n} \sum_{i=1}^{n} sign(y_i - (mx_i + b))$$

$x = 1 : 2 - 2 = 0 \quad sign(0) = 0$

$x = 2 : 3 - 6/3 = 1/3 \quad sign(1/3) = 1$

$x = 3 : 5 - 10/3 = 5/3, \quad sign(5/3) = 1$

$x = 4 : 4 - 4 = 0, \quad sign(0) = 0$

$$\frac{dE}{dm} = -\frac{1}{4}(0 \cdot 1 + 1 \cdot 2 + 1 \cdot 3 + 0 \cdot 4)$$

$$\frac{dE}{dm} = -\frac{5}{4}$$

$$\frac{dE}{db} = -1/4 (0+1+1+0)$$

$$-1/4 \times 2 = -1/2$$

$L = 0.07$

$$m - L\frac{dE}{dm} = 2/3 - 0.07 \times \left(-5/4\right) = 0.75$$

$$b - L\frac{dE}{db} = 4/3 - 0.07\left(-1/2\right) = 1.37$$

Adjusted model:

$$y = 0.75x + 1.37$$

$x = 1$ $y_{actual} = 2$:

$y = 0.75 + 1.37$

$y_{pred_1} = 2.12$

$AE = |2 - 2.12| = 0.12$

$x = 2$ $y_{actual} = 3$

$y = 0.75(2) + 1.37$

$y_{pred_2} = 2.87$

$3 - 2.87 = 0.13$

$x = 3$ $y_{actual} = 5$

$y = 0.75(3) + 1.37 = 3.62$

$5 - 3.62 = AE = 1.38$

$x = 4 \quad y_{actual} = 4$

$y_{pred_4} = 0.75(4) + 1.37$

$y_{pred_4} = 4.37$

$|4 - 4.37| = 0.37$

$0.12 + 0.13 + 1.36 + 0.37 = 2$

$\frac{1}{4} \times 2 = 0.5$

The MAE remains 0.5 which is the same as initial error. Suggesting that the learning rate might be too small to make a significant change