Math 240 — Hw 3
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In today's class, we continued Lagrange polynomials, which provide us with a way of finding the polynomial of degree $n-1$ that goes through n points. The example we did today was three points to find a degree 3 polynomial (a quadratic). For a reminder, the Lagrange polynomial is constructed from basis polynomials, denoted as $\ell_i(x)$. It is

$$L(x) = y_1\ell_1(x) + y_2\ell_2(x) + \ldots + y_n\ell_n(x).$$

These basis elements are constructed from only the x-values of the points. They are defined as multiplying the following terms:

$$\frac{x - x_m}{x_i - x_m}$$

where m never equals i. The notation used in math is

$$\ell_i(x) = \prod_{m, m\neq i} \frac{x - x_m}{x_i - x_m}.$$

That big pi-looking symbol means multiply all the terms.
The example we did in class was with the points $\{(-2,-8),(-1,-1),(1,1),(2,8)\}$. So we will use the index i for the first point, 2 for the second, and 3 for the third. This means:

i	x_i	y_i
1	-2	-8
2	-1	-1
3	1	1
4	2	8

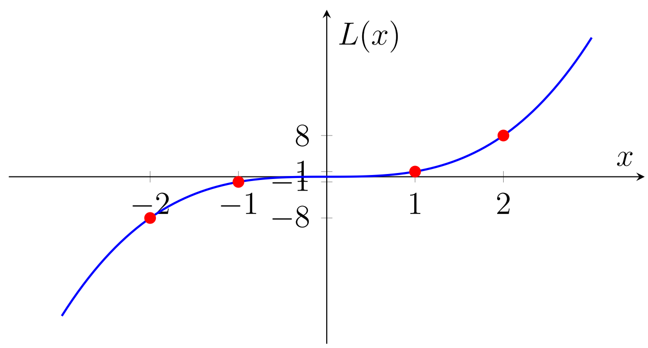
When we compute the basis elements, we compute the following:

- $\ell_1(x) = \left(\frac{x+1}{-2+1}\right)\left(\frac{x-1}{-2+1}\right)\left(\frac{x-2}{-2-2}\right) = -\frac{1}{12}(x^3 - 2x^2 - x + 2)$
- $\ell_2(x) = \left(\frac{x+2}{-1+2}\right)\left(\frac{x-1}{-1-1}\right)\left(\frac{x-8}{-1-2}\right) = \frac{1}{6}(x^3 - x^2 - 4x + 4)$
- $\ell_3(x) = \left(\frac{x+2}{1+2}\right)\left(\frac{x+1}{1+1}\right)\left(\frac{x-2}{1-2}\right) = -\frac{1}{6}(x^3 + x^2 - 4x - 4)$
- $\ell_3(x) = \left(\frac{x+2}{2+2}\right)\left(\frac{x+1}{2+1}\right)\left(\frac{x-1}{2-1}\right) = \frac{1}{12}(x^3 + 2x^2 - x - 2)$

To construct the Lagrange polynomial, we now multiply each basis polynomial by its corresponding y_i value:

$$\begin{aligned} L(x) &= -8\ell_1(x) - 1\ell_2(x) + 1\ell_3(x) + 2\ell_4(x) \\ &= x^3 \end{aligned}$$

We see that we get a degree 3 polynomial. Let's check that it works with our points and even graph our points with the function we found.



Now, I would like you to repeat the same procedure with the following points.

1. Find the cubic polynomial that passes through $(0, 3), (-1, 2), (2, 11), (1, 4)$ using the method of Lagrange polynomials illustrated in the example above.