Math 240 — Hw 2
Sara Jamshidi, Jan 31, 2025

In today's class, we introduced Lagrange polynomials, which provide us with a way of finding the polynomial of degree $n-1$ that goes through n points. The example we did today was three points to find a degree 2 polynomial (a quadratic). For a reminder, the Lagrange polynomial is constructed from basis polynomials, denoted as $\ell_i(x)$. It is

$$L(x) = y_1\ell_1(x) + y_2\ell_2(x) + \ldots + y_n\ell_n(x).$$

These basis elements are constructed from only the x-values of the points. They are defined as multiplying the following terms:

$$\frac{x - x_m}{x_i - x_m}$$

where m never equals i. The notation used in math is

$$\ell_i(x) = \prod_{m, m\neq i} \frac{x - x_m}{x_i - x_m}.$$

That big pi-looking symbol means multiply all the terms.
The example we did in class was with the points $\{(1,9),(2,4),(3,3)\}$. So we will use the index i for the first point, 2 for the second, and 3 for the third. This means:

i	x_i	y_i
1	1	9
2	2	4
3	3	3

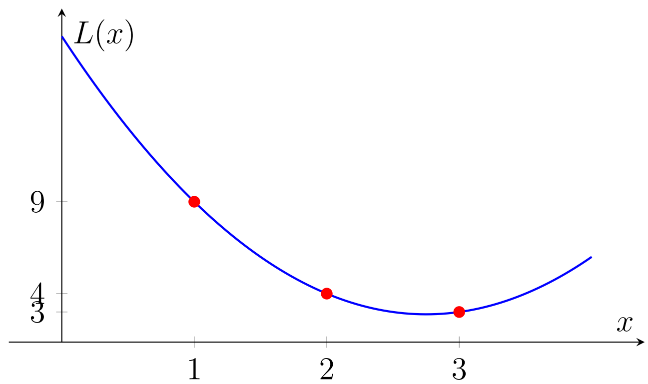
The fact that i is equal to x in this example is purely coincidental, as you will see in the ones you attempt. When we compute the basis elements, we compute the following:

- $\ell_1(x) = \left(\frac{x-2}{1-2}\right)\left(\frac{x-3}{1-3}\right) = \frac{1}{2}(x^2 - 5x + 6)$
- $\ell_2(x) = \left(\frac{x-1}{2-1}\right)\left(\frac{x-3}{2-3}\right) = (-1)(x^2 - 4x + 3)$
- $\ell_3(x) = \left(\frac{x-1}{3-1}\right)\left(\frac{x-2}{3-2}\right) = \frac{1}{2}(x^2 - 3x + 2)$

To construct the Lagrange polynomial, we now multiply each basis polynomial by its corresponding y_i value:

$$\begin{aligned} L(x) &= 9\ell_1(x) + 4\ell_2(x) + 3\ell_3(x) \\ &= \frac{9}{2}(x^2 - 5x + 6) - 4(x^2 - 4x + 3) + \frac{3}{2}(x^2 - 3x + 2) \\ &= 2x^2 - 11x + 18 \end{aligned}$$

We see that we get a degree 2 polynomial—yay! Let's check that it works with our points and even graph our points with the function we found.



Let's also verify by plugging in:

- $2(1)^2 - 11(1) + 18 = 2 - 11 + 18 = 9$
- $2(2)^2 - 11(2) + 18 = 8 - 22 + 18 = 4$
- $2(3)^2 - 11(3) + 18 = 18 - 33 + 18 = 3$

Over the weekend, I would like you to do a similar example on your own–following the same procedure.

1. Use the method of Lagrange polynomials illustrated in the example above and from class to show that the quadratic that passes through $\{(-1, 2), (0.4), (1, 8)\}$ is $y = x^2 + 3x + 4$.