2/21 Homework 5

1.) $x'(t) = 2\sin(t)$, $x(\pi) = 2$

$x'(t) = 2\sin(t)$

$x(\pi) = 2$

$dx/dt = 2\sin(t)$

$dx = 2\sin(t)\,dt$

$\int dx = \int 2\sin(t)\,dt$

$\int dx = 2\int \sin(t)\,dt$

$x(t) = 2 - \cos(t) + C$

$x(t) = -2\cos(t) + C$

$-2\cos(\pi) + C = 2$

$-2 \times -1 + C = 2$

$2 + C = 2$

$C = 0$

$x(t) = -2\cos(t)$

2.) $x'(t) = 3\cos(t)$, $x(0) = 1$

$dx/dt = 3\cos(t)$

$dx = 3\cos(t) \cdot dt$

$\int dx = 3\int \cos(t) \cdot dt$

$x(t) = 3\sin(t) + C$

$3\sin(0) + C = 1$

$C = 1$

$x(t) = 3\sin(t) + 1$

3.) $x'(t) = e^{-t}$, $x(1) = 0$

$$\frac{dx}{dt} = e^{-t}$$

$$\int dx = e^{-t}\,dt$$

$$x(t) = \int e^{-t}\,dt$$

$$x(t) = -e^{-t} + C$$

$$-e^{-1} + C = 0$$

$$C = e^{-1}$$

$$x(t) = -e^{-t} + e^{-1}$$

4.) $x'(t) = x$, $x(0) = 1$

$$\frac{dx}{dt} = x$$

$$dx = x \cdot dt$$

$$\int dx = \int x \cdot dt$$

$$x(t) = x^2/2 + C$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$\frac{0}{0} + C = 1$$

$$0 + C = 1$$

$$C = 1$$

$$x(t) = x^2/2 + 1 = \frac{x^2 + 2}{2}$$

5.) $x'(t) = x^2$, $x(0) = 1$

$$\int dx/dt = \int x^2$$

$$x(t) = \frac{x^3}{3} + C$$

$$\frac{0}{3} + C = 1$$

$$C = 1$$

$$x(t) = x^3/3 + 1$$

6.) $x'(t) = xt$, $x(0) = 0$

$$dx/dt = xt$$

$$\int dx/dt = \int xt$$

$$\int dx/dt = x \int t$$

$$x(t) = x \times t^2/2 + C$$

$$x(t) = \frac{xt^2}{2} + C$$

$$\frac{0}{2} + C = 0$$

$$C = 0$$

$$x(t) = \frac{xt^2}{2}$$

7.) $x'(t) = e^{-x}$, $x(1) = 1$

$$\frac{dx}{dt} = e^{-x}$$

$$\int dx = \int e^{-x}\, dt$$

$$x(t) = -e^{-x} + C$$

$$x(1) = -e^{-1} + C$$

$$1 = -e^{-1} + C$$

$$C = e^{-1} + 1$$

$$x(t) = -e^{-x} + e^{-1} + 1$$