1/21 Homework 1

1.) $f(x) = x^2 + 2x + 1$

given points $(\overset{x}{1}, \overset{y}{4})$ & $(\overset{x}{2}, \overset{y}{9})$

$$\frac{9-4}{2-1} = 5$$

derivative between $x=1$ & $x=2$ is 5

derivative at $x=2$ is 6.

2.) Given points

$(\overset{x}{\pi/6}, \sin(\overset{y}{\pi/6}))$ & $(\overset{x}{\pi/4}, \sin(\overset{y}{\pi/4}))$

$$\frac{\sin \pi/4 - \sin \pi/6}{\pi/4 - \pi/6} = \frac{1/\sqrt{2} - 1/2}{\pi/12} = \frac{\frac{2-\sqrt{2}}{2\sqrt{2}}}{\pi/12}$$

$$= \frac{12-6\sqrt{2}}{\pi\sqrt{2}} = 0.79$$

$$\frac{d}{dx} \sin(x) = \cos(x) = \cos(\pi/7) = 0.90$$

The approximation is 0.29 less than actual derivative

3.) $y(x) = \sqrt{x}$

Given $(0,0)$ & $(1,1)$

$$\frac{1-0}{1-0} = 1$$

$$\sqrt{x} = x^{1/2}$$

$$y'(x^{1/2}) = 1/2\, x^{-1/2}$$

$$1/2\,(0.5)^{-1/2} = \frac{1}{2\sqrt{0.5}} = 0.71$$

approximate = 1

actual = 0.71

4.)

x	1	1.5	2
$y(x)$	2	2.25	2.5

exhaustive

$(1,2)$ & $(1.5, 2.25)$ $\quad$ $(1.5, 2.25)$ & $(2, 2.5)$

$$\frac{2.25-2}{1.5-1} = \frac{0.25}{0.5} = 1/2 \qquad \frac{2.5-2.25}{2-1.5} = \frac{0.25}{0.5} = 1/2$$

$(1,2)$ & $(2, 2.5)$

$$\frac{2.5-2}{2-1} = 0.5 = 1/2$$

Actual = 1/2

Hence estimate $y'(1.5) = 1/2$