HOMEWORK 6
DIVIDE AND CONQUER ALGORITHM

1. WRITTEN QUESTIONS

Problem 1. Let A_0, A_1, A_2, A_3 be a sequence of matrices of sizes 3×2, 2×4, 4×6 and 6×3.

    (1) What is the dimension of the product $A_1 A_2 A_3 A_4$?

    (2) What is the minimum number of multiplications needed to calculate this product?

Problem 2. Describe how mergesort works with the following list

```
a_list =[1,2, -1, -3, 4, 5, 8, 7, -1, 10]
```

2. CODING QUESTIONS

Problem 3 (The coin change problem). Given an list of coin values `coin_list` and a target value N. During the lecture, we wrote a program to find the minimum number of coins required to achive the total value of N. In this problem, your task is to return a dictionary that outlines the specific counts of each coin value used to reach this optimal solution. If multiple combinations are possible, any of those solutions is acceptable. For example, for

```
coin_list = [1, 2, 4]
N = 5
```

the answer should be $\{1 : 1, 4 : 1\}$ (there is one coin of value 1 and one coin of value 4). On the other hand, for

```
coin_list = [2, 4, 5]
N = 10
```

the answer should be $\{5 : 2\}$.

Problem 4. Implement the solution for the matrix chain problem. Specifically, suppose we want to optimize the number of multiplications required to calcualte the product $A_0 A_1 \ldots A_n$. Each matrix A_i has dimensions represented by the list $d = [d_0, d_1, \ldots, d_n, d_{n+1}]$, where A_i is of size $d_i \times d_{i+1}$ for $0 \leq i \leq n$.

    Write a function that takes d as an input and return the minimal number of multiplications required to calculate the product $A_0 A_1 \ldots A_n$.

Problem 5. Given two sorted list `list_1` and `list_2`, write a program to merge them into a single sorted list.