HOMEWORK 2
CS 417

Problem 1. In class, we discussed the modified Euclidean algorithm. Namely, for $a, b > 0$ we can write

$$b = aq + r$$

where $|r| \leq \frac{a}{2}$. Furthermore, we know that

$$\gcd(b, a) = \gcd(a, |r|).$$

This is called the Euclidean algorithm with the least absolute remainder (see `https://en.wikipedia.org/wiki/Euclidean_algorithm` for some furhter disucssion).

(1) Write a function that take a, b as input and return $|r|$.
(2) Write a new function using the previous function to calculate the gcd of a and b.

Problem 2. Write a function named numeric_values(a_list) that takes a list as input and returns a new list with only the numeric elements. Numeric values include both integers and floating-point numbers. For example

```
numeric_values("1", "apple", 1, 1.2, -4])
```

should return $[1, 1.2, -4]$.

Problem 3. Write a function named remove_element(a_list, element) that takes a list and an element as input and returns a new list with all occurrences of that element removed. For example

```
removed_element([0, "test", 1, "apple", 0, 1.1], 0)
```

should return

```
["test", 1, "apple", 1.1]
```

Problem 4. Write a function named higher_than_average(d) that takes a dictionary d as input. In this dictionary d, the keys represent students enrolled in Chem 101, and the values represent their midterm scores. The function should return a list of students who scored above the average midterm score. For example, for

```
d = {
    "Alice": 85,
    "Bob": 78,
    "Charlie": 92,
    "Daisy": 88,
    "Ethan": 76}
```

```
higher_than_average(d)
```

the average score is 83.8. As a result, the function should return the list

```
["Alice", "Charlie", "Daisy"]
```

Problem 5. Write a function update_inventory(inventory, new_shipment) that takes two dictionaries as input.

- inventory: A dictionary representing current stock (e.g., {`"apple": 10, "banana": 5, "orange": 7`}).
- new_shipment: A dictionary representing new items arriving (e.g., {`"banana": 10, "orange": 5, "mango": 3`}).

The function should update the inventory with the quantities from new_shipment. If an item in the shipment is not in the inventory, add it. The function should return the updated inventory. For example

```
inventory = {"apple": 10, "banana": 5, "orange": 7}
new_shipment = {"banana": 10, "orange": 5, "mango": 3}
update_inventory(inventory, new_shipment)
```

should return

```
{"apple": 10, "banana": 15, "orange": 12, "mango": 3}
```

Problem 6. In class, we talked about creating a class for Complex numbers. Recall that a complex number is a number of the form $a + bi$ where a, b are real numbers and i is an imaginary number such that $i^2 = -1$. Addition and multiplication are done algebraically. For example

$$(a + bi)(c + di) = ac + adi + bci + bdi^2 = (ac - bd) + (bc + ad)i.$$

Write a class named `ComplexNumber` that accepts parameters a and b (the real and imaginary parts, respectively). Implement the following methods within the class:

- `__add__(self, other)`: add two complex numbers.
- `__mul__(self, other)`: multiply two complex numbers.
- `is_zero(self)`: Determine if the complex number is equal to zero.
- `is_real(self)`: Check if the complex number is a real number (i.e., $b = 0$).
- `is_pure_imaginary(self)`: Check if the complex number is a pure imaginary number (i.e., $a = 0$).
- `modulus(self)`: Calculate the modulus (magnitude) of the complex number, which is given by the formula $\sqrt{a^2 + b^2}$.