HOMEWORK 3
CS 417

Please read the remaining of Chapter 3 of the online version of the textbook `https://runestone.academy/ns/books/published/pythonds/index.html?mode=browsing`

Submit a picture/pdf file for the following proof questions.

Problem 1. Use Gauss's method to show that

$$1 + 3 + 5 + \ldots + (2n - 1) = n^2.$$

For example

$$1 + 3 = 4 = 2^2, 1 + 3 + 5 = 9 = 3^2.$$

Problem 2. (10 points) Do 5 questions in Section 3.10 (Discussion questions).

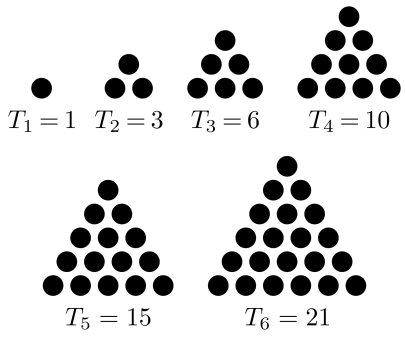
For the remaining problems, please submit a python file (jupyter notebook is also fine). For questions that ask for an explanation, please provide your answers as comments.

Problem 3. (10 points) Do Questions 4 and 5 in Section 3.11 (Programming Exercises).

Problem 4. A triangular number is a number that can be arranged in the shape of an equilateral triangle. Mathematically, n is a triangular number if we can find a positive integer k such that

$$n = \frac{k(k+1)}{2}.$$

The first few triangular numbers are described in the picture below.



Write a function to check whether a given number n is triangualr or not. What is the big O-performance of your program?

Problem 5. Let a_m be a sequence given by the following recursive formula

$$a_0 = 2, \quad a_1 = 5, \quad a_m = 5a_{m-1} - 6a_{m-2} \text{ for } m \geq 2.$$

The following questions are considered to be independent from each other (though, if you want to use one to solve the others, that is fine).

(1) Write a function to calculate the kth term of this sequence. You function should take k as the argument and return a_k.
(2) Given a number n. Write a function to check whether n belongs to this sequence (namely, there exists k such that $a_k = n$). For example, 13 belongs to this sequence because $a_2 = 13$. On the other hand, 20 is not a member of this sequence. Use a count variable inside your function to estimate the number of assignments in your function for $n = 10^2, 10^3, 10^4, 10^5, 10^6, 10^7$. What do you think is the big O-performance of your algorithm?
(3) What if I tell you that the general formula for a_m is $a_m = 2^m + 3^m$.