Let $v_1, v_2 \in null(T)$ Notice $T(v_1, v_2) = Tv_1 + Tv_2 = 0 + 0 = 0$

T is a linear Operator

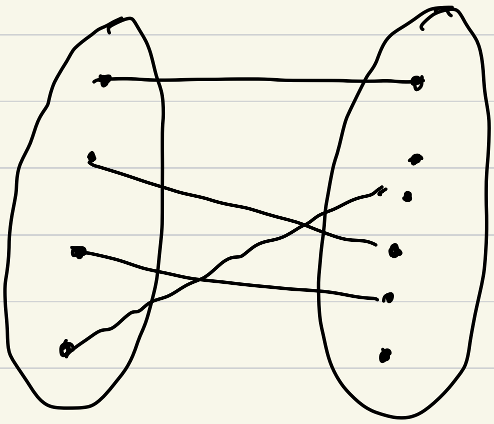
$v_1\ v_2$ are in null space

Then $v_1 + v_2 \in null(T)$. Thus $null(T)$ is closed under addition

Let $v \in null(T)$ & $\lambda \in \mathbb{F}$ of V $T(\lambda v) = \lambda Tv = \lambda 0 = 0$
Thus $null(T)$ is closed under scalar multiplication

By theorem 1.34, $null(T)$ is a subspace of V $\blacksquare$

What is injectivity? 1 to 1
A function is injective if
$x_1 \neq x_2$
$f(x_1) \neq f(x_2)$



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Homework 19

1.) $\dim V = \dim \operatorname{null} T + \dim \operatorname{range} T$

for example
Define $T: \mathbb{R}^5 \to \mathbb{R}^2$

$$T(x_1, x_2, x_3, x_4, x_5) = (x_1, x_2)$$

Null space of T

$T(x_1, x_2 \cdots x_5) = (0,0)$

$x_1 = 0, \ x_2 = 0$

$(0, 0, x_3, x_4, x_5) \to \dim \operatorname{null} T = 3$

$\dim \operatorname{Range} T = \dim \mathbb{R}^2 = 2$

$\dim V = 5$ $\dim \operatorname{null} T = 3$ & $\dim \operatorname{Range} T = 2$