Math 231 — Hw 2

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1. Today we looked at an example of a finite field, a field with finitely many objects: $\mathbb{Z}_p$. Such structures are always fields when p is a prime number. For $\mathbb{Z}_5$, find all the additive and multiplicative inverse of the elements in the field: $\{0, 1, 2, 3, 4\}$. (Note that 0 will have no multiplicative inverse.)

   A table would illustrate the elements well, but here, I will just list them:

   0: The additive inverse is 0. There is no multiplicative inverse for the additive identity.

   1: The additive inverse is 4: $1 + 4 \mod 5 \equiv 0$. The multaplicative inverse is itself.

   2: The additive inverse is 3: $2 + 3 \mod 5 \equiv 0$. The multaplicative inverse is 3: $2 \times 3 = 6 \equiv_5 1$

   3: The additive inverse is 2: $3 + 2 \mod 5 \equiv 0$. The multaplicative inverse is 2: $3 \times 2 = 6 \equiv_5 1$

   4: The additive inverse is 1: $4 + 1 \mod 5 \equiv 0$. The multaplicative inverse is itself: $4 \times 4 = 16 \equiv_5 1$

2. For finite fields, p must be a prime number. To illustrate why $\mathbb{Z}_4$ is not a field, construct its multiplication table.

   Recall that a multiplication table is a table where the header row and first column list the elements of the set, and each cell contains the product of the corresponding row and column elements.

   In our solution below, we see that both 2 and 3 lack multaplicative inverses, so $\mathbb{Z}_4$ could not form a field.

$\times_p$	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	0	2
3	0	3	2	3