Math 231 — Hw 16
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In class, we continued to discuss how we to use matrix representations of linear maps between vector spaces. Suppose we define the horrible map you came up with in class, $T : \mathbb{R}^4 \to \mathbb{R}^3$

- $T(1,2,3,4) = (5,6,7)$
- $T(11,10,9,8) = (2,3,1)$
- $T(1,5,7,2) = (7,8,6)$
- $T(0,0,0,1) = (9,1,1)$

Our strategy for handling this is to define a map $U : \mathbb{R}^4 \to \mathbb{R}^4$ that translates from a nice basis $\{(1,0,0,0),(0,1,0,0),...\}$ to the basis upon which T is defined: $\{(1,2,3,4),(11,10,9,8),...\}$. We did this in class and defined the matrix representation for U to be

$$M(U) = \begin{pmatrix} \frac{25}{24} & \frac{-17}{6} & \frac{15}{8} & 0 \\ \frac{1}{24} & \frac{1}{6} & \frac{-1}{8} & 0 \\ \frac{-1}{2} & 1 & \frac{-1}{2} & 0 \\ \frac{-7}{2} & 8 & \frac{-11}{2} & 1 \end{pmatrix}$$

If we define the set of mapped elements in $\mathbb{R}^3$ to be $\{(5,6,7),(2,3,1),(7,8,6),(9,1,1)\}$, then the matrix representation of T is

$$M(T) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

But this is not enough to get the full answer. Why? Suppose there is a vector v for which we apply $M(U)$ and then $M(T)$ and we get the result $(1,0,1,1)$. This vector represents: $1 \cdot (5,6,7) + 0 \cdot (2,3,1) + 1 \cdot (7,8,6) + 1 \cdot (9,1,1)$. Ideally we'd like the result to actually be our answer directly and not have to do this extra step. So we can define a map S that goes from $\{(5,6,7),(2,3,1),(7,8,6),(9,1,1)\}$ to $\{(1,0,0),(0,1,0),(0,0,1)\}$.
Define $M(S)$.