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1.) $0 + V = V$

Additive identity

Such ~~that~~ $0 + u \in V = u$

as $0 \in V$

$$0 + 0 = 0$$

For additive identity of a Vector Space

$$(V_1 + V_2 + V_3 - - - - V_m) + (0 + - - - - - - - - + 0) = (V_1 + \dots V_m)$$

if

$$(V_1 + V_2 + V_3 \dots V_m) = (0 + 0 + 0 - - - - 0)$$

then

$$(0 + 0 + 0 \dots\dots 0) + (0 + 0 + 0 - - - - 0)$$

$$= (0 + 0 + 0 - - - - - - 0) = 0$$

2.) $V = \mathbb{R}^3$

$W_1 = \{(x, y, 0) \mid x, y \in \mathbb{R}\}$.

$W_2 = \{(0, 0, z) \mid z \in \mathbb{R}$

Prove $V = W_1 \oplus W_2$

let

$(x, y, z) \in V$

$$\underbrace{(x, y, 0)}_{\in W} + \underbrace{(0, 0, z)}_{\in W_2} = (x, y, z) \in V$$

Show $W_1 \cap W_2$

Suppose $(a, b, c) \in W_1 \cap W_2$

Since $(a, b, c) \in W_1 \to (a, b, 0)$

Since $(a, b, c) \in W_2 \to (0, 0, c)$

holds true if $(a, b, c) = (0, 0, 0)$ i.e. $W_1 \cap W_2 = (0, 0, 0)$

proving the direct sum

3.) $V = \mathbb{R}^3 \to (x, y, z)$

$U = \{(x, y, 0) \mid x + y = 0\}$

$$\underset{\in V}{(x, y, z)} = \underset{\in U}{(x, y, 0)} + \underset{\in W}{(0, 0, z)}$$

$U \cap W \to (0, 0, 0)$

hence

V is the direct sum of $U \oplus W$ because

$\therefore U \cap W \to (0, 0, 0)$