Math 231 — Hw 17

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The null space is the space of all vectors that are sent to 0 by a matrix. For example, the null space of

$$\begin{pmatrix} 2 & 4 \\ -1 & -2 \end{pmatrix}$$

is the set of vectors of the form

$$\begin{pmatrix} 2x \\ -x \end{pmatrix}.$$

To demonstrate this, we see that

$$\begin{pmatrix} 2 & 4 \\ -1 & -2 \end{pmatrix} \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

1. Consider the following matrices. What is their null space? Based on their null space, do their column vectors form a basis?

   (a)
   $$\begin{pmatrix} 3 & 3 \\ -1 & -1 \end{pmatrix}$$

   (b)
   $$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 2 \end{pmatrix}$$

2. If a null space has more than just the 0 vector, we call it "nontrivial." Give the basis of the nontrivial null space of the following matrix
   $$\begin{pmatrix} 1 & -1 & 2 \\ 3 & -3 & 6 \end{pmatrix}$$

3. Suppose M is a 3×3 matrix. We said in class that M can be thought of as changing the basis of the matrix. For this reason, the columns of M represent a basis. If the null space is nontrivial, then the vectors don't form a basis of the 3 dimensional vector space. What does that mean about dimensionality of the range of M?

4. Based on your answer to the previous question, what does it mean geometrically if a matrix has a nontrivial null space?

5. To capture rotations in two dimensions, we can use the following matrix:
   $$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$
   Suppose a camera is pointed downward looking at a specimen located at $(2, 3)$. If the camera is rotated by 240 degrees in the positive direction, what is the vector that represents its location with this new orientation?