Homework 4.

1.) $V = \mathbb{Z}_5^2 \to \{2, 3, 4, 5$

a) $(a,b) + (c,d) = (a+c) \bmod 5 + (b+d) \bmod 5$

$((a+c) \bmod 5 + (b+d) \bmod 5 \in \mathbb{Z}_5^2$

Closed under addition.

b $\lambda(a,b) = (\lambda \cdot a \bmod 5, \lambda \cdot b \bmod 5)$

Product of an $\mathbb{Z}_5^2 \bmod 5 \in \mathbb{Z}_5^2$ Hence closed under scalar

c.) Commutability

$(a,b) + (c,d) = ((a+c) \bmod 5, (b+d) \bmod 5$

$= ((c+a) \bmod 5, (d+b) \bmod 5$

d.) Additive identity

$(a,b) + (0,0) \to (a,b)$

$(a,b) + (0,0) \to ((a+0) \bmod 5, (b+0) \bmod 5)$

$= (a,b)$

e.) Additive inverse

$(a,b) + (-(a,b)) = 0$

$(a \bmod 5 + (-a \bmod 5)), (b \bmod 5 + (-b \bmod 5)) = 0$

2.) $P^3 = \{ax^3+bx^2+cx+d \mid a,b,c,d \in \mathbb{R}\}$

Addition:

$(ax^3+bx^2+cx+d)+(\tilde{a}x^3+\tilde{b}x^2+\tilde{c}x+\tilde{d})$

$$\underbrace{(a+\tilde{a})}_{\in \mathbb{R}}x^3+\underbrace{(b+\tilde{b})}_{\in \mathbb{R}}x^2+\underbrace{(c+\tilde{c})}_{\in \mathbb{R}}x+\underbrace{(d+\tilde{d})}_{\in \mathbb{R}}$$

Closed under addition

Scalar multiplication

$\lambda(ax^3+bx^2+cx+d)$

$$\underbrace{\lambda ax^3}_{\in \mathbb{R}}+\underbrace{\lambda bx^2}_{\in \mathbb{R}}+\underbrace{\lambda cx}_{\in \mathbb{R}}+\underbrace{\lambda d}_{\in \mathbb{R}}$$

Closed under scalar multiplication

Axioms

Commutability

$(a+\tilde{a})x^3 = (\tilde{a}+a)x^3 \in \mathbb{R}$

$(b+\tilde{b})x^2 = (\tilde{b}+b)x^2 \in \mathbb{R}$

$(c+\tilde{c})x = (\tilde{c}+c)x \in \mathbb{R}$

$(d+\tilde{d}) = (\tilde{d}+d) \in \mathbb{R}$

Additive identity

$(ax^3+bx^2+cx+d)+(0x^3+0x^2+0x+0)$

$(a+0)x^3 = ax^3 \in \mathbb{R}$ | $d+0=d \in \mathbb{R}$

$(b+0)x^2 = bx^2 \in \mathbb{R}$

$(c+0)x = cx \in \mathbb{R}$

Additive Inverse

$(ax^3 + bx^2 + cx + d) + (-ax^3 + (-bx^2) + (-cx) + (-d))$

$$\left.\begin{array}{l} (a - a)x^3 = 0 \\ (b - b)x^2 = 0 \\ (c - c)x = 0 \\ d - d = 0 \end{array}\right\} \in \mathbb{R}$$

Multiplicative identity

$1(ax^3 + bx^2 + cx + d)$

$1 \cdot ax^3 + 1 \cdot bx^2 + 1 \cdot cx + 1 \cdot d$

$ax^3 + bx^2 + cx + d \in \mathbb{R}$