

## Math 231 — Hw 11

Sara Jamshidi, Feb 14, 2025

1. In [this video](#), the speaker explains that historically, rainbows in Western art were depicted with only three colors until Isaac Newton expanded the concept to seven. This shift highlights how different cultural and scientific perspectives influence our understanding of color.

In modern computing, colors are represented mathematically using vector spaces such as RGB (Red-Green-Blue) and CMYK (Cyan-Magenta-Yellow-Black). We can approximate these models using the vector spaces  $\mathbb{R}^3$  and  $\mathbb{R}^4$ , respectively, where each coordinate represents a component's intensity. For example, in RGB, the vector  $(1, 0, 0)$  represents pure red, while in CMYK, the vector  $(1, 0, 0, 0)$  represents pure cyan.

The RGB and CMYK models describe color in fundamentally different ways:

- (a) RGB is an additive color model, where colors are created by combining light.
- (b) CMYK is a subtractive model, used in printing, where colors result from absorbing certain wavelengths.
- (c) What do you think it means in this context if a given set spans a set? Mathematically, could three distinct shades of a color we recognize, say three shades of red, be enough to span the entire space of colors?

Pick three vectors in the RGB space with values between 0 and 1 (e.g.,  $(.33, .25, 0)$ ). Prove that your three vectors span  $\mathbb{R}^3$ . With the provided code, convert them to CMYK. Prove that the result does not span  $\mathbb{R}^4$ .

2. In music theory, chords can be represented as vectors in a musical space, where each coordinate represents the presence or absence of a particular note in a chord. Let's look at the [5 chords within the B-minor scale famously used in the song "15 Step"](#) from *In Rainbows*, Radiohead's 7th album.<sup>1</sup>

Chord	<i>B</i>	<i>C#</i>	<i>D</i>	<i>E</i>	<i>F#</i>	<i>G</i>	<i>A</i>
<b>Bm</b>	1	0	1	0	1	0	0
<b>Cdim5</b>	0	1	0	1	0	1	0
<b>A6</b>	0	0	1	0	1	0	1
<b>Gmaj7</b>	0	0	1	0	1	1	1

- (a) Are the given chord vectors linearly independent in  $\mathbb{R}^7$ ? Justify your answer.

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<sup>1</sup>Fun fact: They released this album themselves (without a record label) and allowed customers to pay what they wanted for it. The band made a much bigger profit on this album than their previous ones.

- (b) Do these chords span the space of all possible chords in the B minor scale (i.e., all possible vectors in  $\mathbb{R}^7$  with only 0s and 1s)? If not, add additional chords to the set in the provided code. Alternatively, feel free to create an entirely new set of chords from the basis that span.
- (c) What do you think it means in this context if a given set is linearly dependent?