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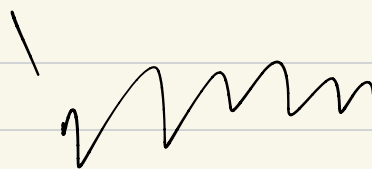
1.)  $0 + V = V$

Additive identity

Such that  $0 + v \in V = V$

as  $0 \in V$

$0 + 0 = 0$



For additive identity of a Vector Space

$(v_1 + v_2 + v_3 + \dots + v_m) + (0 + \dots + 0) = (v_1 + \dots + v_m)$

if  $(v_1 + v_2 + v_3 + \dots + v_m) = (0 + 0 + 0 + \dots + 0)$

then

$(0 + 0 + 0 + \dots + 0) + (0 + 0 + 0 + \dots + 0)$

$= (0 + 0 + 0 + \dots + 0) = 0$

2.)  $V = \mathbb{R}^3$

$\omega_1 = \{ (x, y, 0) \mid x, y \in \mathbb{R} \}$

$\omega_2 = \{ (0, 0, z) \mid z \in \mathbb{R} \}$

Prove  $V = \omega_1 \oplus \omega_2$

let

$(x, y, z) \in V$

$(x, y, 0) + (0, 0, z) = (x, y, z) \in V$   
 $\in \omega_1 \quad \in \omega_2$

Show  $\omega_1 \cap \omega_2$

Suppose  $(a, b, c) \in \omega_1 \cap \omega_2$

Since  $(a, b, c) \in \omega_1 \rightarrow (a, b, 0)$

Since  $(a, b, c) \in \omega_2 \rightarrow (0, 0, c)$

holds true if  $(a, b, c) = (0, 0, 0)$  i.e.  $\omega_1 \cap \omega_2 = \{(0, 0, 0)\}$

proving the direct sum

$$3.) \quad V = \mathbb{R}^3 \rightarrow (x, y, z) \\ U = \{(x, y, 0) \mid x+y=0\}$$

$$\begin{array}{ccccc} (x, y, z) & = & (x, y, 0) & + & (0, 0, z) \\ \in V & & \in U & & \in W \end{array}$$

$$U \cap W \rightarrow (0, 0, 0) \\ \text{hence}$$

$V$  is the direct sum of  $U \oplus W$  because  
 $U \cap W \rightarrow (0, 0, 0)$