

4.17

## Homework 22

1.) Find the eigenvalues of  $T$ . For a  $2 \times 2$  matrix

$$\det(T - \lambda I) = 0$$

Here,  $T - \lambda I = \begin{pmatrix} 0.7 - \lambda & 0.4 \\ 0.3 & 0.6 - \lambda \end{pmatrix}$ , so the determinant is

$$\begin{aligned} (0.7 - \lambda)(0.6 - \lambda) - (0.4)(0.3) &= 0 \\ &= \lambda^2 - 1.3\lambda + 0.42 \end{aligned}$$

Subtract the 2<sup>nd</sup> term

$$0.4 \times 0.3 = 0.12$$

X-static equation

$$\lambda^2 - 1.3\lambda + 0.42 - 0.12 = \lambda^2 - 1.3\lambda + 0.3 = 0$$

Solve

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \quad a=1, \quad b=-1.3+0.3=0$$

$$\begin{aligned} &= \frac{(-1.3)^2 - 4 \cdot 1 \cdot 0.3}{2\sqrt{0.49}} \\ &= 0.7 \end{aligned}$$

$$\lambda = \frac{1.3 \pm 0.7}{2}$$

$$\lambda_1 = 1 \quad \lambda_2 = 0.3$$

$$2.) x_0 = \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix}$$

$$x_n = T^n x_0$$

So

$$x_{60} = T^{60} x_0$$

$$T - I = \begin{pmatrix} 0.7-1 & 0.4 \\ 0.3 & 0.6-1 \end{pmatrix} = \begin{pmatrix} -0.3 & 0.4 \\ 0.3 & -0.4 \end{pmatrix}$$

$$(T - I)v = 0$$

$$\begin{pmatrix} -0.3 & 0.4 \\ 0.3 & -0.4 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$-0.3 v_1 + 0.4 v_2 = 0$$

$$0.4 v_2 = 0.3 v_1$$

$$v_2 = 3/4 v_1$$

$$v = \begin{pmatrix} 1 \\ 3/4 \end{pmatrix}$$

$$v_1 + 3/4 v_1 = v_1 \left(1 + 3/4\right) = v_1 \cdot 7/4 = 1$$

$$v_1 = 4/7$$

$$v_2 = 3/4 \cdot 4/7 = 3/7$$

$$v = \begin{pmatrix} 4/7 \\ 3/7 \end{pmatrix}$$

$$x_{60} = \begin{pmatrix} 4/7 \\ 3/7 \end{pmatrix}$$