Math 231 — Hw 10
Sara Jamshidi, Feb 13, 2025

1. Is the following set of vectors from $\mathbb{R}^3$ a linearly independent set?

$$\{(1,2,3),(4,5,6),(7,8,9)\}$$

Prove or disprove.

To determine if the set of vectors $\{(1,2,3),(4,5,6),(7,8,9)\}$ is linearly independent, we need to check that there is only one linear combination for $(0,0,0)$ with coefficients $c_1 = c_2 = c_3 = 0$. Consider

$$c_1(1,2,3) + c_2(4,5,6) + c_3(7,8,9) = (0,0,0).$$

Solving the system of equations, we can see that when $c_1 = c_3$ and $c_2 = -2c_3$, we can get $(0,0,0)$. This means that there are non-trivial solutions (i.e., not all c_i are zero), indicating that the vectors are NOT linearly independent.

2. Remember that vectors are just elements of a vector space. Since P_2, the space of polynomials up to degree 2, is a vector space, then below is a set of vectors from that space.

$$S = \{2, x-1, x^2 - x\}$$

Is it true that $\mathbf{span}(S) = P_2$. Prove your answer.

To determine if $\mathbf{span}(S) = P_2$, where $S = \{2, x-1, x^2 - x\}$, we need to check if any polynomial of the form $ax^2 + bx + c$ can be written as a linear combination of the vectors in S. We want to express $ax^2 + bx + c$ as:

$$k_1 \cdot 2 + k_2 \cdot (x-1) + k_3 \cdot (x^2 - x)$$

Expanding and combining like terms, we get:

$$k_3 x^2 + (k_2 - k_3)x + (2k_1 - k_2)$$

We need this to equal $ax^2 + bx + c$. Thus, we equate the coefficients:

$$k_3 = a$$

$$k_2 - k_3 = b$$

$$2k_1 - k_2 = c$$

Solving this system of equations, we find:

$$k_1 = \frac{a}{2} + \frac{b}{2} + \frac{c}{2}$$

$$k_2 = a + b$$

$$k_3 = a$$

Since we can find such k_1, k_2, k_3 for any a, b, c. So $\mathbf{span}(S) = P_2$.