Math 231 — Hw 19
Sara Jamshidi, Apr 2, 2025

1. Give an example of a linear map T with dim null $T = 3$ and dim range $T = 2$.

   Let's define $T : \mathbb{R}^5 \to W$, where $\dim W \geq 2$. Define the basis of $\mathbb{R}^5$ to be

   $$\{(1,0,0,0,0), (0,1,0,0,0), (0,0,1,0,0), (0,0,0,1,0), (0,0,0,0,1)\}.$$

   We define T to work as follows:

   - $T(1,0,0,0,0) = 0_W$
   - $T(0,1,0,0,0) = 0_W$
   - $T(0,0,1,0,0) = 0_W$
   - $T(0,0,0,1,0) = w_1$
   - $T(0,0,0,0,1) = w_2$

   where w_1 and w_2 are linearly independent in W. Then T has a 3 dimensional null space and a 2 dimensional image. Note: there are *many* possible answers this question.