Math 231 — Hw 12

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1. Consider the vector space $\mathbb{F}_2^2$. What are all the possible bases for this space? (You do not need to prove this).

2. Consider the linearly independent set in $\mathbb{R}^3$ and construct a basis by adding one element to it. Then prove that it is a basis.

$$\{(1, 0, 1), (-1, 0, 1)\}$$

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4. In the previous two examples, you wrote two distinct bases of $\mathbb{R}^3$. Given a vector $(x, y, z) \in \mathbb{R}^3$, write a set of functions $f_i(a_1, a_2, a_3) = b_i$ where each function takes in the coefficients of the first basis and produces the i^{th} coefficient of the second basis. What kind of functions are these?

5. Prove or give a counterexample: If v_1, v_2, v_3, v_4 is a basis of V and U is a subspace of V such that $v_1, v_2 \in U$ and $v_3 \notin U$ and $v_4 \notin U$, then v_1, v_2 is a basis of U.