Math 231 — Hw 3

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1. Let $S = \{(x, y) \in \mathbb{R}^2 \mid x + y = 1\}$ be a space defined over the field $\mathbb{R}$ with addition defined as
$$(a, b) + (c, d) = (a + c, b + d)$$
and scalar multiplication as $x(a, b) = (xa, xb)$ where $x \in \mathbb{R}$ and $(a, b) \in S$. Show why this is **not** a vector space.

2. Let $U = \{(x, y) \in \mathbb{R}^2 \mid x \geq 0, y \geq 0\}$, with vector addition and scalar multiplication defined as the previous case. Show why this is **not** a vector space.

3. Define a set $W = \mathbb{R}^2$ with addition defined as $(x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$ and scalar multiplication defined as $c \cdot (x, y) = (cx, y)$. Show why this is **not** a vector space.

4. Let $X = \{(x, y, z) \in \mathbb{R}^3 \mid x + y + z = 0\}$, with vector addition and scalar multiplication defined as usual. Show why this is **not** a vector space.