Math 231 — Hw 7

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1. Determine whether the following set is a vector space. Justify your answer using theorem 1.34 from the textbook:

$$W = \{(x, y, z) \mid x - (y + 1) + 2(z + 1) = 1,\ x, y, z \in \mathbb{R}\}.$$

2. Construct an example of a vector space W with two subspaces, W_1, W_2, where you know $W_1 + W_2 = W$. Attempt to prove this.

3. Let $V = \mathbb{R}^4$, and define two subspaces:

   - $V_1 = \{(x, y, 0, 0) \mid x, y \in \mathbb{R}\}$
   - $V_2 = \{(0, 0, z, w) \mid z, w \in \mathbb{R}\}$

   Prove that $V_1 + V_2$ forms a subspace of V.

4. Prove that $V_1 + V_2 = V$ in the previous problem.

5. Let $V = \mathbb{R}^2$, and define two subspaces:

   - $V_1 = \{(w_1, w_2) \mid w_1 + 2w_2 = 0, w_1, w_2 \in \mathbb{R}\}$
   - $V_2 = \{(v_1, v_2) \mid v_1 + v_2 = 0, v_1, v_2 \in \mathbb{R}\}$

   Prove or provide a counterexample to the statement: $V_1 + V_2 = V$.