Math 231 — Hw 7

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1. Let V be a vector space and $0 \in V$ the additive identity. Prove that $0 + 0 = 0$. Then prove that $0 + \ldots + 0 = 0$ for any finite number of sums.

2. Let $V = \mathbb{R}^3$ and consider the subspaces:

$$W_1 = \{(x, y, 0) \mid x, y \in \mathbb{R}\}, \quad W_2 = \{(0, 0, z) \mid z \in \mathbb{R}\}.$$

   Prove that $V = W_1 \oplus W_2$ using the last theorem from class.

3. Let $V = \mathbb{R}^3$. Consider the subspace $U = \{(x, y, 0) \mid x + y = 0\}$. Find a space W such that $V = U \oplus W$.