2/4 Homework 7

Thm 1.34 A subset $\mathcal{U}$ of V is a subspace of V if $\mathcal{U}$ satisfies
1.) additive identify in $\mathcal{U}$
2.) Closed under addition
3.) Closed under scalar multiplication

$$W = \{(x, y, z) \mid x - (y+1) + 2(z+1) = 1 \quad x, y, z \in R$$

is zero vector in the vector space?

$$0 - (0+1) + 2(0+1)$$
$$0 - 1 + 2 = 1$$

hence zero vector is in the vector space
additive identity in $\mathcal{U}$ is met

Closed under addition
$\mathcal{U} = x, y, z$
$\mathcal{U}_1 = x_1, y_1, z_1$

$$\mathcal{U} + \mathcal{U}_1 = (x + x_1), (y + y_1), (z + z_1)$$

$$(x - (y+1) + 2(z+1)) + (x_2 - (y_2+1) + 2(z_2+1)) = 2$$

$$(x + x_1) - ((y + y_1) + 1 + 2((\cancel{z} + 1 + (z_2 + 1)) = 2$$

$$(x + x_1) - ((y + y_1) + 1 + 2((z + z_1) + 1 = 1$$

Substract 1 from both sides The new equation follows same structure

Closure under scalar multiplication

such that

$c \cdot u = cu$

$c(x, y, z) = cx, cy, cz$

$x - y - 1 + 2z + 2 = 1$

$x - y + 2z + 1 = 1$

$x - y + 2z = 0$

$cx - cy + 2cz = 0$

$c(x - y + 2z) = 0$

$c \cdot 0 = 0$

Closed under scalar multiplication

2.) Vector space W such that $W = (x, y, z)$

& $x, y, z \in \mathbb{R}$

Subspace

$W_1 = (x_1, 0, z_1)$

$W_2 = (x_2, y_2, 0)$

$x = x_1 + x_2 \rightarrow x_2 = x - x_1$ x can be anything

$y = 0 + y_2 = y_2$ so any $y_2 = y$ in vector space

$z = z_1 + 0 = z_1$

so let

$x_2 = 0$

$(x_1, 0, z) + (0, y_2, 0) \rightarrow (x, y, z)$

3.) $u_1 = (x_1, y_1, 0, 0) \in V_1$, $u_2 = (0, 0, z_1, \omega_1) \in V_2$

Closure under addition

$u_1 + u_2 = (x_1, y_1, 0, 0) + (0, 0, z_1, \omega_1) = (x_1, y_1, z_1, \omega_1)$

$u_1 + u_2 \in V_1 + V_2$ as the sum of elements in V_1 and V_2
remain in $V_1 + V_2$. Closed

Closure under scalar multiplication

$u = (x, y, 0, 0) + (0, 0, z, \omega) = (x, y, z, \omega) \in V_1 + V_2$ &
let $\alpha \in \mathbb{R}$

$\alpha u = \alpha (x, y, z, \omega) = (\alpha x, \alpha y, \alpha z, \alpha \omega)$

$\alpha u \in V_1 + V_2$

Zero vector / additive identity

$(0, 0, 0, 0) + u_1 = u_1$

$(0, 0, 0, 0) + u_2 = u_2$

4.) $v = (x, y, z, \omega)$

$v = (x, y, 0, 0) + (0, 0, z, \omega)$, $(x, y, 0, 0) \in V_1$ & $(0, 0, z, \omega) \in V_2$

Since

$\mathbb{R}^4 = V_1 + V_2 = V$

5.) $V_1 = \{(\omega_1, \omega_2) \in \mathbb{R}^2 \mid \omega_1 + \omega_2 = 0\}$.

$\omega_1 + 2\omega_2 = 0$

$\omega_1 = -2\omega_2 \longrightarrow (-2\omega_2, \omega_2) = \omega_2(-2, 1)$

$V_2 = \{(v_1, v_2) \in \mathbb{R}^2 \mid v_1 + v_2 = 0\}$

$v_1 + v_2 = 0$

$v_1 = -v_2 \rightarrow (-v_2, v_2) = v_2(-1, 1)$

$V_1 + V_2 = \{(x_1, x_2) \mid (x_1, x_2) = \alpha(-2, 1) + \beta(-1, 1), \alpha, \beta \in \mathbb{R}$

$(x_1, x_2) = \alpha(-2, 1) + \beta(-1, 1)$

Combine Components

$x_1 = -2\alpha - \beta$

$x_2 = \alpha + \beta$