Math 231 — Hw 18
Sara Jamshidi, Mar 31, 2025

1. In class, we discussed the differential operator, D, over the space P_3, the space of polynomials up to degree 3. The differential operator takes polynomials to their derivatives. Solve the following equation: $D(ax^3 + bx^2 + cx + d) =$

   The differential operator D takes a polynomial and returns its derivative. For a polynomial $ax^3 + bx^2 + cx + d$, the derivative is:

   $$D(ax^3 + bx^2 + cx + d) = 3ax^2 + 2bx + c$$

2. If $ax^3 + bx^2 + cx + d$ is represented as the column vector

   $$\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix},$$

   write out $M(D)$. (Hint: Use your previous answer.)

   The polynomial $ax^3 + bx^2 + cx + d$ can be represented as the column vector $\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$. The differential operator D can be represented as a matrix $M(D)$ that acts on this vector to produce the coefficients of the derivative. From the derivative $3ax^2 + 2bx + c$, we see that:

   $$M(D) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

3. In class, we stated that the null D is the space of constant functions. What is the representation of this null space? In other words, what is null $M(D)$?

   The null space of D consists of all polynomials that are mapped to zero by D. These are the constant functions, since the derivative of a constant is zero. Thus, the null space of D is:

   $$\text{null } D = \text{span}\left\{ \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right\}$$

4. Suppose S is a map that represents a shift in vectors over $\mathbb{R}^3$. $S(a, b, c) = (b, c, 0)$. Describe its null space and give a representation $M(S)$.

   The shift operator S maps (a, b, c) to $(b, c, 0)$. The matrix representation $M(S)$ is:

$$M(S) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

   The null space of S consists of all vectors that are mapped to zero by S. This is the space spanned by $(1, 0, 0)$:

$$\text{null } S = \text{span} \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right\}$$

5. Now suppose we define a function P that represents a permutation over the vector space $\mathbb{R}^3$. $P(a, b, c) = (b, c, a)$. Describe its null space and give a representation $M(P)$.

   The permutation operator P maps (a, b, c) to (b, c, a). The matrix representation $M(P)$ is:

$$M(P) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

   The null space of P is the zero vector, since P is a bijection and does not map any non-zero vector to zero:

$$\text{null } P = \{\mathbf{0}\}$$