Homework 24

1.) PCA transforms data to a new coordinate system by identifying the directions along which the variance of data is maximized. The first PCA is the direction of the greatest variance. The second is the greatest variance. The data is then projected onto these components to reduce dimensionality while preserving as much variability

2.) Projects data to a lower-dimensional space that captures most of the variance (signal) while discarding components with low variance, which are likely to represent noise.

3.)

$$A = \begin{bmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{bmatrix}$$

$$v_0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$v_n = A^n v_0$$

4.)

$$A = \begin{bmatrix} 0.9 & 0.2 \\ 0.1 & 0.8 \end{bmatrix}$$

$$V_0 = \begin{bmatrix} 0.9 \\ 0.1 \end{bmatrix}$$

$$v_{12} = A^{12} v_0$$

5.) $A = \begin{bmatrix} 0.9 & 0.4 \\ 0.1 & 0.6 \end{bmatrix}$

$V_0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$V_{14} = A^{14} V_0$

6.) Matrix = $\begin{bmatrix} 2 & 1 & 3 \\ 0 & 4 & 1 \\ 1 & 2 & 5 \end{bmatrix}$

$\det = 2[4\cdot5 - 1\cdot2] - 1[0\cdot5 - 1\cdot1] + 3[0\cdot2 - 4\cdot1]$
$= 2(20-2) - 1(0-1) + 3(0-4) = 36 + 1 - 12 = 25$

7.)
Given vector $v = (1,2)$, basis vectors:
$b_1 = (1,1)$
$b_2 = (1,-1)$
we want $v = a\cdot b_1 + b\cdot b_2$

$a(1,1) + b(1,-1) = (1,2) \rightarrow (a+b, a-b) = (1,2)$

Solving
$a+b=1$
$a-b=2$
Add: $2a = 3 \rightarrow a = 1.5$
$b = -0.5$
Conversion Map