Math 231 — Hw 24
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1. Explain how PCA transforms the data into a new coordinate system.

2. How can PCA be used for noise reduction in data?

3. Suppose you are working with a two-state system.

   - The element $A_{11} = 0.8$ represents the probability of staying in State 1 after one time step.
   - The element $A_{21} = 0.2$ represents the probability of transitioning from State 1 to State 2 after one time step.
   - The element $A_{12} = 0.3$ represents the probability of transitioning from State 2 to State 1 after one time step.
   - The element $A_{22} = 0.7$ represents the probability of staying in State 2 after one time step.

   Write down the transition matrix for A. Suppose you begin with an item in state 2. After n timesteps, what is the probability that this item is now in state 1.

4. Suppose you are analyzing customer retention for a subscription service with two states: *Subscribed* (State 1) and *Unsubscribed* (State 2).

   - The element $A_{11} = 0.9$ represents the probability of a customer remaining subscribed after one month.
   - The element $A_{21} = 0.1$ represents the probability of a customer transitioning from subscribed to unsubscribed after one month.
   - The element $A_{12} = 0.2$ represents the probability of a customer transitioning from unsubscribed to subscribed after one month.
   - The element $A_{22} = 0.8$ represents the probability of a customer remaining unsubscribed after one month.

   Suppose you begin with 450 subscribers and 50 who were previously subscribed but now have unsubscribed. After 12 months, what do you project to be the new proportion?

5. Suppose you are analyzing weather patterns in a region with two states: *Sunny* (State 1) and *Rainy* (State 2).

   - The element $A_{11} = 0.9$ represents the probability that a sunny day is followed by another sunny day.

- The element $A_{21} = 0.1$ represents the probability that a sunny day is followed by a rainy day.
- The element $A_{12} = 0.4$ represents the probability that a rainy day is followed by a sunny day.
- The element $A_{22} = 0.6$ represents the probability that a rainy day is followed by another rainy day.

Today is a rainy day. What is the probability that it will be sunny in 2 weeks?

6. Find the determinant of the following matrix:

$$\begin{pmatrix} 2 & 1 & 3 \\ 0 & 4 & 1 \\ 1 & 2 & 5 \end{pmatrix}$$

7. Suppose you have a vector $(1, 2)$. Write its representation in the bases $\{(1, 1), (1, -1)\}$. Write down a map that converts this basis to the standard normal basis.