3/31 Homework 18

1-) $D(ax^3+bx^2+cx+d) = 3ax^2+2bx+c$

2-)

$$\begin{pmatrix} 3a \\ 2b \\ c \end{pmatrix}$$

$D(x^3) = 3x^2 \rightarrow \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}$

$D(x^2) = 2x \rightarrow \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}$

$D(x) = 1 \rightarrow \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$$M(D) = \begin{pmatrix} 3 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} 3a \\ 2b \\ c \end{pmatrix}$$

$$M(D) \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} 3a \\ 2b \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$a = 0$
$b = 0$
$c = 0$

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ d \end{pmatrix} = d \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

So

null

$$M(D) = \text{Span}\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

4.)

$$S(a,b,c) = (b,c,0) = (0,0,0)$$

eg

$a = 0$

$b = 0$

a is free

$$(a,b,c) = (a,0,0) \quad \text{null } S = \text{spans}\{(1,0,0)\}$$

$M(S)$: Map $\mathbb{R}^3 \to \mathbb{R}^3$. basis $\{(1,0,0),(0,1,0),(0,0,1)\}$

$$S(1,0,0) = (0,0,0)$$

$$S(0,1,0) = (1,0,0)$$

$$S(0,0,1) = (0,1,0)$$

$$M(S) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

5.)

Null space: $P(a,b,c) = (b,c,a) = (0,0,0)$

If

$b = 0$

$c = 0$

$a = 0 \Rightarrow (a,b,c) = (0,0,0)$: Null $P = (0,0,0)$

Matrix $M(P)$

$P(1,0,0) = (0,0,1)$

$P(0,1,0) = (1,0,0)$

$\rightarrow P(0,0,1) = (0,1,0)$

$$M(P) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$