2/1 H.W 6

1.) Let $V = (x_1, x_2, x_3)$

$V + (0,0,0) = V$

$(x_1, x_2, x_3) + (0,0,0) = (x_1, x_2, x_3)$

Closure under Addition

Let $V_1 = (x_1, x_2, x_3) \in W$

$V_2 = (y_1, y_2, y_3) \in W$

$V_1 + V_2 = (x_1 + y_1, x_2 + y_2, + x_3 + y_3)$

$(x_1 + y_1)(x_2 + y_2)(x_3 + y_3) = 0$

But Not true for all elements in the subspace as $x_i \in \mathbb{R}$

Closure under Scalar multiplication

$V = (x_1, x_2, x_3)$ & $x_1 \cdot x_2 \cdot x_3 = 0$ & $C \in \mathbb{R}$

$C \cdot V = C(x_1, x_2, x_3)$

$x_1 x_2 x_3 = 0$

$C(0) = 0$

True

2.) let $W = \mathbb{R}^3$

$W = (x, y, z) \mid x, y, z \in \mathbb{R}$

$W_1 \to z = 0 \to (x_1, y_1, 0) \quad x_1, y \in$

$W_2 \to y = 0 \to (x_2, 0, z_2) \quad x, z \in R$

$W_1 + W_2 = (x_1 + x_2, y_1, z_2)$

$W_1 W_2 \notin W$ as

$x_1 + x_2 \in W$ ✓

$y_1 \in W_1$ ×

$z_2 \in W_2$ ×

3.) Additive identity

let $v \in V_1 + V_2$

$v = v_1 + v_2$ such that $v_1 \in V_1$ & $v_2 \in V_2$

$v_1 = (x, y_1, 0) \quad V_2 = (0, y_2, z)$

so:

$V = (x_1, y_1 + y_2, z_2)$

$V + 0 = (x_1, y_1 + y_2, z_2) = V$ holds for $v \in V_1 + V_2$

Closure under addition

let $u = (x_1, y_1 + y_2, z_2)$

$v = (x_3, y_3 + y_4, z_4)$

$x_1 + x_3 \in R$ ✓

$y_1 + y_2 + y_3 + y_4 \in R$ ✓

$z_2 + z_4 \in R$ ✓

Scalar Multiplication

$(x, y, z) \in V_1 + V_2$ & $c \in R$

$c \cdot v = c\,(x, y, z) = (cx, cy, cz)$

$cx \in R$

$cy \in R$

$cz \in R$

4.) $v_1 \in V_1$ & $v_1 = (x, y, 0)$

$v_2 \in V_2$ & $v_2 = (0, y_2, z)$

$V_1 + V_2 = (x_1, y_1 + y_2, z_2)$

$x \in R$

$y = y_1 + y_2 \in R$

$z \in R$ $V_1 + V_2$ includes all vectors in form $(x, y, z) \in V = R^3$

Arbitrary

$v_1 = (x, y, 0)$

$v_2 = (0, 0, z)$

$V_1 + V_2 = (x, y, 0) + (0, 0, z) = (x, y, z)$

any vector $v \in V \subseteq V_1 + V_2$

5.) $v_1 \in V_1 : v_1 = (x_1, y_1, 0)$

$v_2 \in V_2 : v_2 = (0, y_2, z_2)$

$V_1 + V_2 = (x_1, y_1 + y_2, z_2)$

following must hold for $v_1 + v_2 = v$

$x_1 = x$

$y_1 + y_2 = y$

$z_2 = z$

$V_1: x_1 + y_1 = 0$

$y_1 = -x$

$V_2: y_2 + z_2 = 0$

$y_2 = -z$

So.

$x_1 = x$

$y_1 = -x_1$

$y_2 = -z_2$

$y_1 + y_2 = -x - z$

for $v \in V_1 + V_2$; $y = -x - z$

So, if $y \neq -x - z$ then $v \notin V_1 + V_2$

let $v = (1, 1, 1) \in \mathbb{R}^3$:

$y = -1 - 1 = -2 \neq 1$

hence $v = (1, 1, 1) \notin V_1 + V_2$