Hw 20

1.) $A = \frac{1}{2} \pi r^2 = \frac{1}{2} \pi (1)^2 = \frac{\pi}{2}$

$AS = 4 \times \frac{\pi}{2} = 2\pi$

2.)

$\begin{vmatrix} 1 & -1 \\ 2 & -1 \end{vmatrix} = (1 \times -1) - (-1 \times 2) = 1$

$\begin{vmatrix} 2 & -1 \\ 1 & -1 \end{vmatrix} = (2 \times -1) - (-1 \times 1) = -1$

$\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} = (2 \times 2) - (1 \times 1) = 3$

$\det (A) = (1 \times 1) - (1 \times -1) - (2 \times 3) =$

$\qquad 1 + 1 - 6 = 4$

$\det (A) \neq 0)$

$Null (A) = \{\vec{0}\}$

3.)

$Tv_1 = \frac{1}{3} e_1 - \frac{1}{2} e_2$

$Tv_2 = \frac{2}{3} e_1 + \frac{1}{2} e_2$

$T = \begin{pmatrix} \frac{1}{3} & \frac{2}{3} \\ -\frac{1}{2} & \frac{1}{2} \end{pmatrix}$

$$\det(T) = \left(\frac{1}{3} \cdot \frac{1}{2} \right) - \left(\frac{2}{3} \cdot \frac{-1}{2} \right) = \frac{1}{2}$$

$$A = \frac{\pi}{\frac{1}{2}} = 2\pi$$

4) $T = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$

$$\det(T) = \frac{1}{2}$$

$$A^{-1} = \frac{1}{\det(A)} = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$T^{-1} = \begin{bmatrix} 1 & -\frac{4}{3} \\ 1 & \frac{2}{3} \end{bmatrix}$$

5.) $T \cdot T^{-1} = I$

$T = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$ $T^{-1} = \begin{bmatrix} 1 & -\frac{4}{3} \\ 1 & \frac{2}{3} \end{bmatrix}$

$$\begin{bmatrix} \frac{1}{3} \cdot 1 + \frac{2}{3} \cdot 1 & \frac{1}{3} \cdot (-\frac{4}{3}) + \frac{2}{3} \cdot \frac{2}{3} \\ -\frac{1}{2} \cdot 1 + \frac{1}{2} \cdot 1 & -\frac{1}{2} \cdot (-\frac{4}{3}) + \frac{1}{2} \cdot \frac{2}{3} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$