1.) a) $\begin{pmatrix} 3 & 3 \\ -1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$3x + 3y = 0$
$-x - y = 0$
$3(x + y) = 0$
$x + y = 0$

$y = -x$
$-x - y = 0$
$-x + x = 0$

$\begin{pmatrix} x \\ -x \end{pmatrix} = x \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

To form a basis column vectors must be
linearly independent but the columns are equal
i.e $1 \times \begin{pmatrix} 3 \\ -1 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$

b) $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$x + z = 0$
$y + z = 0$
$x + y + 2z = 0$
$x = -z$
$y = -z$
$-z - z + 2z = 0$ True

$$\begin{pmatrix} -2 \\ -2 \\ 2 \end{pmatrix} = 2 \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}$$

2.) $\begin{pmatrix} 1 & -1 & 2 \\ 3 & -3 & 6 \end{pmatrix}$

$$\begin{pmatrix} 1 & -1 & 2 \\ 3 & -3 & 6 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 6 \end{pmatrix}$$

$x - y + 2z = 0$
$3x - 3y + 6z = 0 \Rightarrow x - y + 2z = 0$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} y - 2z \\ y \\ z \end{pmatrix} = y \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + z \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix}$$

3.) for M
Rank (M) + Nullity (M) = 3

Rank (M) = 2 - Nullity (M) $\leq$ 3 - 1 = 2

4.) Non-trivial null space means some non-zero
   vector mapped to zero, indicating loss of
dimension. for 3×3 matrix transformation it transforms
R3 to a lower dimension space as the range dimension
drops below 3

5-)

$\theta = 240°$

Matrix

$$\begin{pmatrix} -0.5 & \sqrt{3}/2 \\ -\sqrt{3}/2 & -0.5 \end{pmatrix}$$

Position $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$

New position.

$$\begin{pmatrix} -0.5 & \sqrt{3}/2 \\ -\sqrt{3}/2 & -0.5 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} -1 + \dfrac{3\sqrt{3}}{2} \\ -\sqrt{3} - 1.5 \end{pmatrix}$$