

## Math 231 — Hw 6

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1. Prove or disprove if this is a vector space using theorem 1.34 from the textbook:  
 $\{(x_1, x_2, x_3) \mid x_1 x_2 x_3 = 0, x_i \in \mathbb{R}\}$ .
2. Construct an example of a vector space  $W$  with two subspaces,  $W_1, W_2$  where  $W_1 + W_2 \neq W$ .
3. Let  $V = \mathbb{R}^3$ , and define two subspaces:
  - $V_1 = \{(x, y, 0) \mid x, y \in \mathbb{R}\}$
  - $V_2 = \{(0, y, z) \mid y, z \in \mathbb{R}\}$

Prove that  $V_1 + V_2$  forms a subspace of  $V$ .

4. Prove that  $V_1 + V_2 = V$  in the previous problem.

5. Let  $V = \mathbb{R}^3$ , and define two subspaces:
  - $V_1 = \{(x, y, 0) \mid x + y = 0, x, y \in \mathbb{R}\}$
  - $V_2 = \{(0, y, z) \mid y + z = 0, y, z \in \mathbb{R}\}$

Prove or provide a counter example to the statement:  $V_1 + V_2 = V$ .