Topic: Intermediate value theorem with an interval

Question: Which statement is true?

Answer choices:

A The IVT only applies to discontinuous functions.

B The IVT only applies when there's no interval.

C The IVT only applies to open intervals.

D The IVT only applies to closed intervals.

Solution: D

The Intermediate Value Theorem states that for a function on a closed interval $[a, b]$ where the function is continuous on the interval, a point c exists on the interval where $f(c) = k$.

$f(a) < k < f(b)$ and $a < c < b$

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Question: Use the Intermediate Value Theorem to choose an interval over which $f(x) = x^2 + 2x - 35$ is guaranteed to have a root.

Answer choices:

A $[0,2]$

B $[0,10]$

C $[8,10]$

D $[-2,0]$

Solution: B

This function is quadratic function, so we know that it's continuous. Evaluate the function at both endpoints of the interval $[0,10]$.

$$f(0) = 0^2 + 2(0) - 35$$

$$f(0) = -35$$

and

$$f(10) = 10^2 + 2(10) - 35$$

$$f(10) = 85$$

Because the function is below the x-axis at the left edge of the interval, and above the x-axis at the right edge of the interval, we can say $f(a) < f(c) < f(b)$, or more specifically, $-35 < f(c) < 85$, where $f(c) = 0$.

Therefore, by the intermediate value theorem, it must be true that the function has a root on the interval $[0,10]$.

Topic: Intermediate value theorem with an interval

Question: Is there a root for the function $f(x) = x^2 - 4$ on the interval $[1,6]$?

Answer choices:

A Yes, there's a root at $(0,4)$.

B Yes, there's a root at $(0, -4)$.

C Yes, there's a root at $(2,0)$.

D Yes, there's a root at $(-2,0)$.

Solution: C

This function is quadratic function, so we know that it's continuous. Evaluate the function at both endpoints of the interval $[1,6]$.

$$f(1) = 1^2 - 4$$

$$f(1) = 1 - 4$$

$$f(1) = -3$$

and

$$f(6) = 6^2 - 4$$

$$f(6) = 36 - 4$$

$$f(6) = 32$$

The IVT confirms that the function has a root on the interval, because the function's value crosses from below the x-axis to above the x-axis at some point within that interval.

To find the root, which is the point where the graph of the function crosses the x-axis, we'll set the function equal to 0.

$$x^2 - 4 = 0$$

$$x^2 = 4$$

$$x = \pm 2$$

Therefore, the root in the interval $[1,6]$ is at $x = 2$, and that coordinate point is $(2,0)$.