Calculus 2 Workbook Solutions

Physics

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MATH

MOMENTS OF THE SYSTEM

■ 1. Calculate the moments of the system.

$$m_1 = 3; \ P_1(2,5)$$

$$m_2 = 4; \ P_2(-2,6)$$

$$m_3 = 6; \ P_3(4, -5)$$

Solution:

If we plug the given points and masses into the formulas for the moments of a system, we get

$$M_y = m_1(x_1) + m_2(x_2) + m_3(x_3)$$

$$M_y = 3(2) + 4(-2) + 6(4) = 22$$

and

$$M_y = m_1(x_1) + m_2(x_2) + m_3(x_3)$$

$$M_x = 3(5) + 4(6) + 6(-5) = 9$$

■ 2. Calculate the moments of the system.

$$m_1 = 7; \ P_1(5,2)$$

$$m_2 = 3; \ P_2(-4,3)$$

$$m_3 = 5; \ P_3(-3,4)$$

Solution:

If we plug the given points and masses into the formulas for the moments of a system, we get

$$M_y = m_1(x_1) + m_2(x_2) + m_3(x_3)$$

$$M_y = 7(5) + 3(-4) + 5(-3) = 8$$

and

$$M_x = m_1(y_1) + m_2(y_2) + m_3(y_3)$$

$$M_x = 7(2) + 3(3) + 5(4) = 43$$

■ 3. Calculate the moments of the system.

$$m_1 = 9; \ P_1(7,5)$$

$$m_2 = -5; \ P_2(3,8)$$

$$m_3 = 4; \ P_3(5,4)$$

Solution:

If we plug the given points and masses into the formulas for the moments of a system, we get

$$M_y = m_1(x_1) + m_2(x_2) + m_3(x_3)$$

$$M_y = 9(7) + (-5)(3) + 4(5) = 68$$

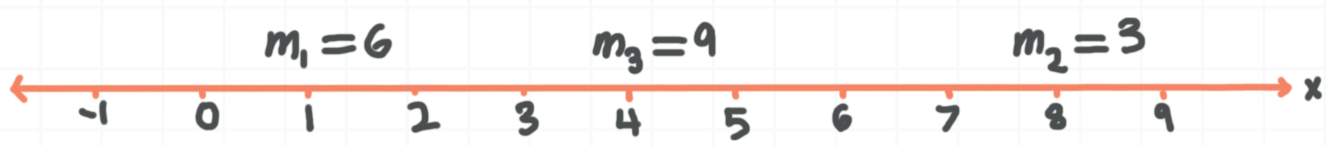
and

$$M_x = m_1(y_1) + m_2(y_2) + m_3(y_3)$$

$$M_x = 9(5) + (-5)(8) + 4(4) = 21$$

MOMENTS OF THE SYSTEM, X-AXIS

■ 1. Calculate the moments of the system.



Solution:

The moments of the system are

$$M_y = m_1(x_1) + m_2(x_2) + m_3(x_3)$$

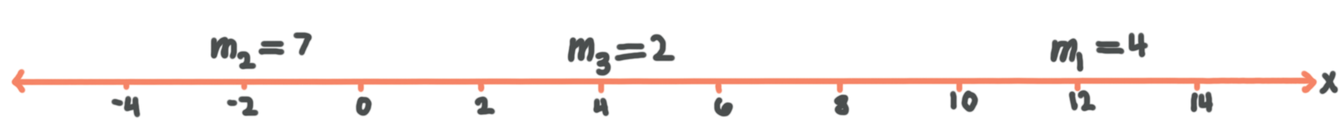
$$M_y = 6(1) + 3(8) + 9(4) = 66$$

and

$$M_x = m_1(y_1) + m_2(y_2) + m_3(y_3)$$

$$M_x = 6(0) + 3(0) + 9(0) = 0$$

■ 2. Calculate the moments of the system.



Solution:

The moments of the system are

$$M_y = m_1(x_1) + m_2(x_2) + m_3(x_3)$$

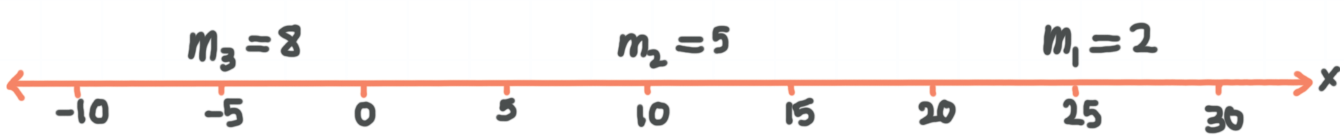
$$M_y = 4(12) + 7(-2) + 2(4) = 42$$

and

$$M_x = m_1(y_1) + m_2(y_2) + m_3(y_3)$$

$$M_x = 4(0) + 7(0) + 2(0) = 0$$

3. Calculate the moments of the system.



Solution:

The moments of the system are

$$M_y = m_1(x_1) + m_2(x_2) + m_3(x_3)$$

$$M_y = 2(25) + 5(10) + 8(-5) = 60$$

and

$$M_x = m_1(y_1) + m_2(y_2) + m_3(y_3)$$

$$M_x = 2(0) + 5(0) + 8(0) = 0$$

CENTER OF MASS OF THE SYSTEM

1. Find the center of mass of the system if $M_y = 16$ and $M_x = 22$ and the total mass is $m_T = 14$.

Solution:

The center of mass is the point

$$(\bar{x}, \bar{y}) = \left(\frac{M_y}{m_T}, \frac{M_x}{m_T}\right) = \left(\frac{16}{14}, \frac{22}{14}\right) = \left(\frac{8}{7}, \frac{11}{7}\right)$$

2. Find the center of mass of the system if $M_y = 32.5$ and $M_x = 28.5$ and the total mass is $m_T = 7.5$.

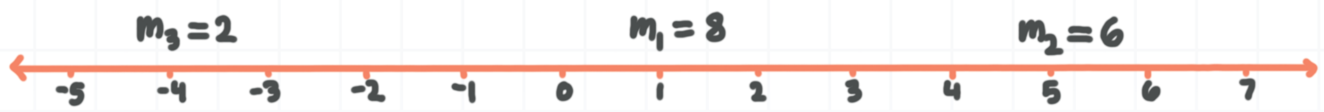
Solution:

The center of mass is the point

$$(\bar{x}, \bar{y}) = \left(\frac{M_y}{m_T}, \frac{M_x}{m_T}\right) = \left(\frac{32.5}{7.5}, \frac{28.5}{7.5}\right) = \left(\frac{13}{3}, \frac{19}{5}\right)$$

CENTER OF MASS OF THE SYSTEM, X-AXIS

1. Find the center of mass of the system.



Solution:

The moments of the system are

$$M_y = m_1(x_1) + m_2(x_2) + m_3(x_3)$$

$$M_y = 8(1) + 6(5) + 2(-4) = 30$$

and

$$M_x = m_1(y_1) + m_2(y_2) + m_3(y_3)$$

$$M_x = 8(0) + 6(0) + 2(0) = 0$$

The total mass in the system is

$$m_T = m_1 + m_2 + m_3$$

$$m_T = 8 + 6 + 2 = 16$$

So the center of mass of the system is

$$(\bar{x}, \bar{y}) = \left(\frac{M_y}{m_T}, \frac{M_x}{m_T}\right) = \left(\frac{30}{16}, \frac{0}{16}\right) = \left(\frac{15}{8}, 0\right)$$

■ 2. Find the center of mass of the system.

Solution:

The moments of the system are

$$M_y = m_1(x_1) + m_2(x_2) + m_3(x_3)$$

$$M_y = 3(6) + 8(-6) + 5(10) = 20$$

and

$$M_x = m_1(y_1) + m_2(y_2) + m_3(y_3)$$

$$M_x = 3(0) + 8(0) + 5(0) = 0$$

The total mass in the system is

$$m_T = m_1 + m_2 + m_3$$

$$m_T = 3 + 8 + 5 = 16$$

So the center of mass of the system is

$$(\bar{x}, \bar{y}) = \left(\frac{M_y}{m_T}, \frac{M_x}{m_T}\right) = \left(\frac{20}{16}, \frac{0}{16}\right) = \left(\frac{5}{4}, 0\right)$$

■ 3. Find the center of mass of the system.

Solution:

The moments of the system are

$$M_y = m_1(x_1) + m_2(x_2) + m_3(x_3)$$

$$M_y = 6(-4) + 5(1) + 7(4) = 9$$

and

$$M_x = m_1(y_1) + m_2(y_2) + m_3(y_3)$$

$$M_x = 6(0) + 5(0) + 7(0) = 0$$

The total mass in the system is

$$m_T = m_1 + m_2 + m_3$$

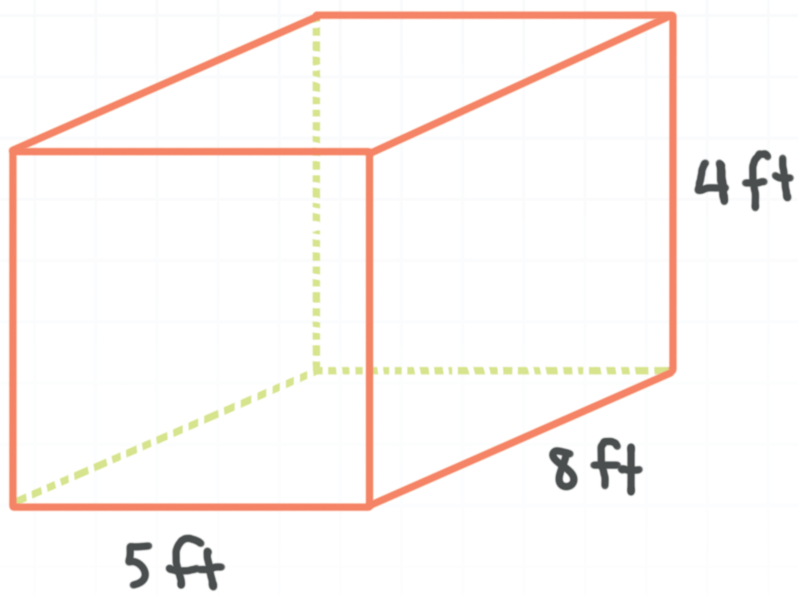
$$m_T = 6 + 5 + 7 = 18$$

So the center of mass of the system is

$$(\bar{x}, \bar{y}) = \left(\frac{M_y}{m_T}, \frac{M_x}{m_T}\right) = \left(\frac{9}{18}, \frac{0}{18}\right) = \left(\frac{1}{2}, 0\right)$$

HYDROSTATIC PRESSURE

1. Find the hydrostatic pressure per square foot on the bottom of the tank, which is filled to the top with gasoline. Assume the weight of a gallon of gasoline is 6.073 pounds per gallon.



Solution:

A gallon of gasoline weighs approximately 6.073 pounds. A cubic foot of the tank holds approximately 7.4805 gallons. So the density of a cubic foot of gasoline is

$$\delta = 6.073 \times 7.4805 = 45.4291$$

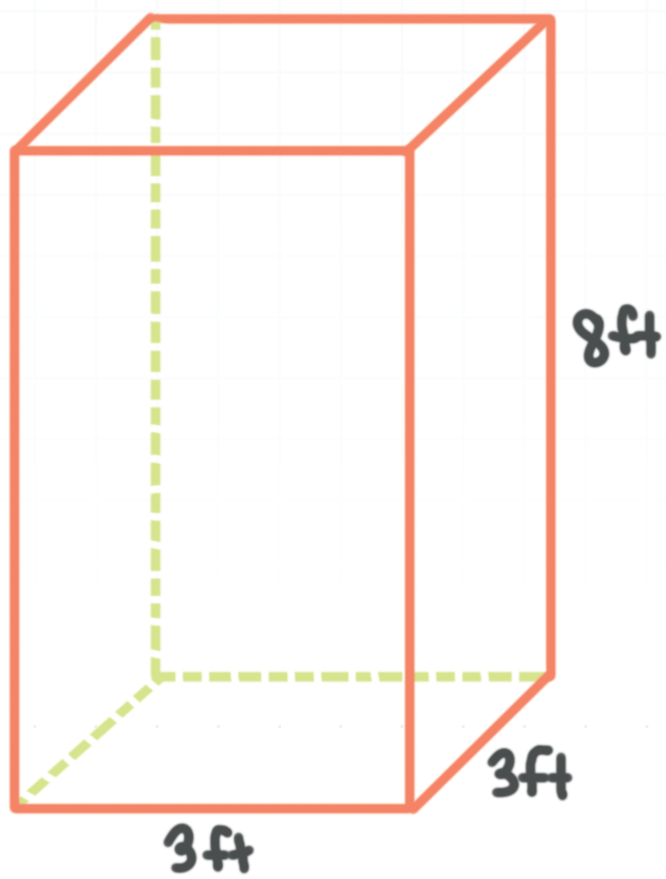
The depth of the gasoline in the tank is 4 feet, and pressure is the product of density and depth, so

$$P = \delta d$$

$$P = 45.4291 \times 4$$

$$P = 181.7164 \text{ lbs/ft}^2$$

■ 2. Find the hydrostatic pressure per square foot on the bottom of the tank, which is filled to the top with water. Assume the weight of a gallon of water is 8.3454 pounds per gallon.



Solution:

A gallon of water weighs approximately 8.3454 pounds. A cubic foot of the tank holds approximately 7.4805 gallons. So the density of a cubic foot of water is

$$\delta = 8.3454 \times 7.4805 = 62.4278$$

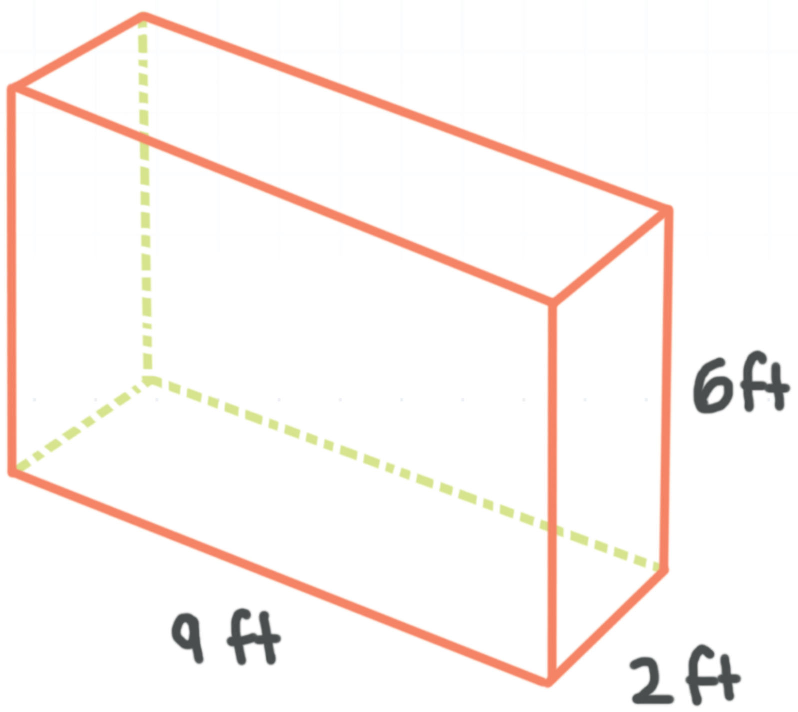
The depth of the water in the tank is 8 feet, and pressure is the product of density and depth, so

$$P = \delta d$$

$$P = 62.4278 \times 8$$

$$P = 499.4224 \text{ lbs/ft}^2$$

3. Find the hydrostatic pressure per square foot on the bottom of the tank, which is filled to the top with diesel fuel. Assume the weight of a gallon of diesel is 7.1089 pounds per gallon.



Solution:

A gallon of diesel fuel weighs approximately 7.1089 pounds. A cubic foot of the tank holds approximately 7.4805 gallons. So the density of a cubic foot of diesel is

$$\delta = 7.1089 \times 7.4805 = 53.1781$$

The depth of the fuel in the tank is 6 feet, and pressure is the product of density and depth, so

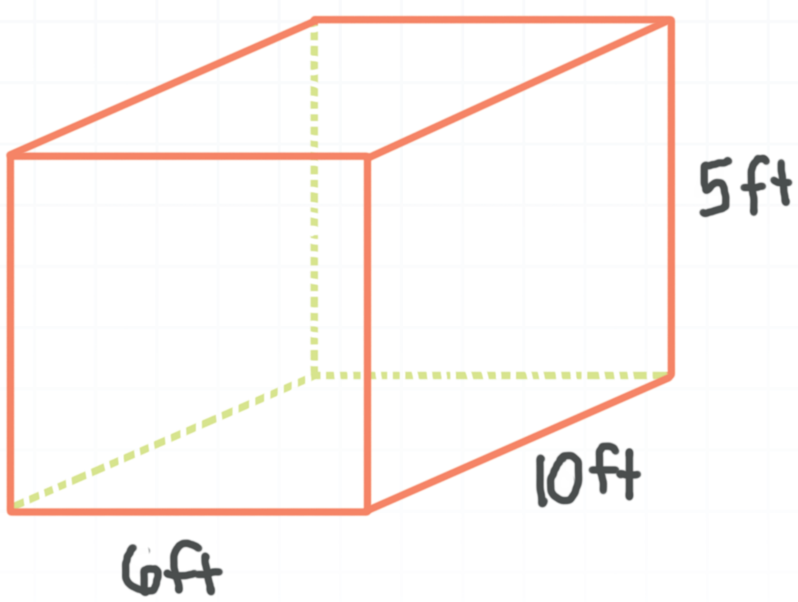
$$P = \delta d$$

$$P = 53.1781 \times 6$$

$$P = 319.0686 \text{ lbs/ft}^2$$

HYDROSTATIC FORCE

1. Find the hydrostatic force on the bottom of the tank, which is filled to the top with gasoline. Assume the weight of a gallon of gasoline is 6.073 pounds per gallon.



Solution:

A gallon of gasoline weighs approximately 6.073 pounds. A cubic foot of the tank holds approximately 7.4805 gallons. So the density of a cubic foot of gasoline is

$$\delta = 6.073 \times 7.4805 = 45.4291$$

The depth of the gasoline in the tank is 5 feet, and pressure is the product of density and depth, so

$$P = \delta d$$

$$P = 45.4291 \times 5$$

$$P = 227.1455 \text{ lbs/ft}^2$$

The area of the bottom of the tank is

$$A = L \cdot W = 6 \cdot 10 = 60$$

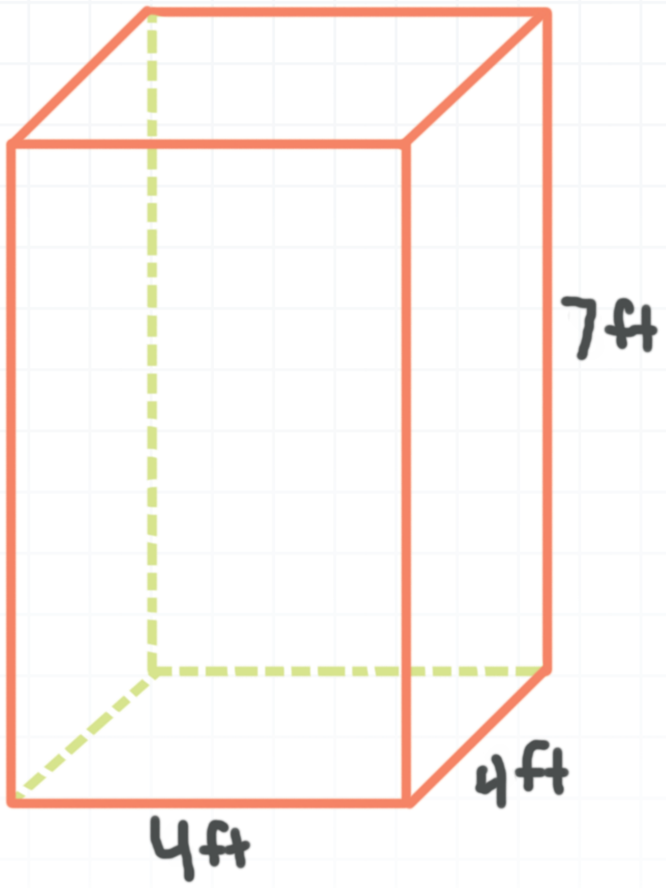
So the force on the bottom of the tank is

$$F = PA$$

$$F = 227.1455 \cdot 60$$

$$F = 13{,}628.73 \text{ pounds}$$

■ 2. Find the hydrostatic force on the bottom of the tank, which is filled to the top with water. Assume the weight of a gallon of water is 8.3454 pounds per gallon.



Solution:

A gallon of water weighs approximately 8.3454 pounds. A cubic foot of the tank holds approximately 7.4805 gallons. So the density of a cubic foot of water is

$$8.3454 \times 7.4805 = 62.4278$$

The depth of the water in the tank is 7 feet, and pressure is the product of density and depth, so

$$P = \delta d$$

$$P = 62.4278 \times 7$$

$$P = 436.9946 \text{ lbs/ft}^2$$

The area of the bottom of the tank is

$$A = L \cdot W = 4 \cdot 4 = 16$$

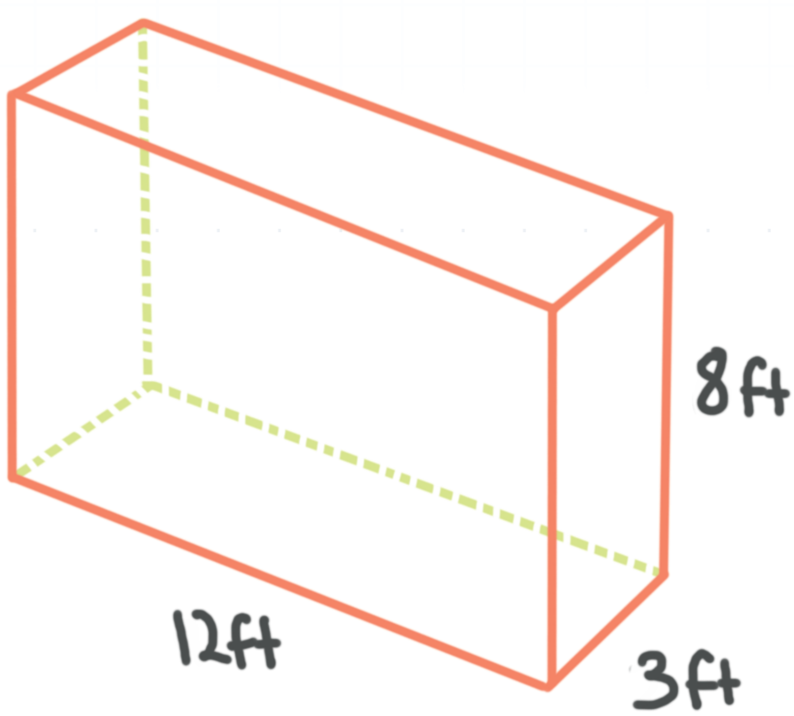
So the force on the bottom of the tank is

$$F = PA$$

$$F = 436.9946 \cdot 16$$

$$F = 6{,}991.9136 \text{ pounds}$$

■ 3. Find the hydrostatic force on the bottom of the tank, which is filled to the top with diesel fuel. Assume the weight of a gallon of diesel is 7.1089 pounds per gallon.



Solution:

A gallon of diesel weighs approximately 7.1089 pounds. A cubic foot of the tank holds approximately 7.4805 gallons. So the density of a cubic foot of water is

$$7.1089 \times 7.4805 = 53.1781$$

The depth of the diesel in the tank is 8 feet, and pressure is the product of density and depth, so

$$P = \delta d$$

$$P = 53.1781 \times 8$$

$$P = 425.4250 \text{ lbs/ft}^2$$

The area of the bottom of the tank is

$$A = L \cdot W = 12 \cdot 3 = 36$$

So the force on the bottom of the tank is

$$F = PA$$

$$F = 425.4250 \cdot 36$$

$$F = 15{,}315.3 \text{ pounds}$$

VERTICAL MOTION

1. What is the maximum height of a baseball that's thrown straight up from a position 6 feet above the ground with an initial velocity of $v(t) = -32t + 88$ ft/sec?

Solution:

The baseball will reach its maximum height when the velocity is 0, so we'll need to find t when $v(t) = 0$.

$$-32t + 88 = 0$$

$$-32t = -88$$

$$t = \frac{-88}{-32}$$

$$t = 2.75$$

The baseball will reach its maximum height at $t = 2.75$ seconds. To find a function for height, integrate velocity.

$$h(t) = \int v(t)\ dt = \int -32t + 88\ dt$$

$$h(t) = -16t^2 + 88t + C$$

The fact that the baseball was thrown from an initial height of 6 feet means we have the initial condition $h(0) = 6$. Substitute the initial condition into the height function.

$$6 = -16(0)^2 + 88(0) + C$$

$$C = 6$$

So the height function is

$$h(t) = -16t^2 + 88t + 6$$

Then at $t = 2.75$ seconds, the height of the baseball is

$$h(2.75) = -16(2.75)^2 + 88(2.75) + 6 = 127 \text{ feet}$$

■ 2. What is the maximum height of a football that's thrown straight up from 1.67 yards above the ground with an initial velocity of $v(t) = -10.67t + 40$ yards/sec?

Solution:

The football will reach its maximum height when the velocity is 0, so we'll need to find t when $v(t) = 0$.

$$-10.67t + 40 = 0$$

$$-10.67t = -40$$

$$t = \frac{-40}{-10.67}$$

$$t = 3.75$$

The football will reach its maximum height at $t = 3.75$ seconds. To find a function for height, integrate velocity.

$$h(t) = \int v(t)\ dt = \int -10.67t + 40\ dt$$

$$h(t) = -\frac{10.67t^2}{2} + 40t + C$$

The fact that the football was thrown from an initial height of 1.67 yards means we have the initial condition $h(0) = 1.67$. Substitute the initial condition into the height function.

$$1.67 = -\frac{10.67(0)^2}{2} + 40(0) + C$$

$$C = 1.67$$

So the height function is

$$h(t) = -\frac{10.67t^2}{2} + 40t + 1.67$$

Then at $t = 3.75$ seconds, the height of the football is

$$h(3.75) = -5.33(3.75)^2 + 40(3.75) + 1.67 = 76.716875 \approx 77 \text{ yards}$$

3. What is the maximum height of a model rocket that's launched straight up from the ground with an initial velocity of $v(t) = -32t + 200$ ft/sec?

Solution:

The rocket will reach its maximum height when the velocity is 0, so we'll need to find t when $v(t) = 0$.

$$-32t + 200 = 0$$

$$-32t = -200$$

$$t = \frac{-200}{-32}$$

$$t = 6.25$$

The rocket will reach its maximum height at $t = 6.25$ seconds. To find a function for height, integrate velocity.

$$h(t) = \int v(t)\ dt = \int -32t + 200\ dt$$

$$h(t) = -16t^2 + 200t + C$$

The fact that the rocket was launched from ground level means we have the initial condition $h(0) = 0$. Substitute the initial condition into the height function.

$$0 = -16(0)^2 + 200(0) + C$$

$$C = 0$$

So the height function is

$$h(t) = -16t^2 + 200t$$

Then at $t = 6.25$ seconds, the height of the rocket is

$$h(6.25) = -16(6.25)^2 + 200(6.25) = 625 \text{ feet}$$

■ 4. What is the maximum height of a bottle rocket that's launched straight up from the ground with an initial velocity of $v(t) = -19.6t + 29.4$ m/sec?

Solution:

The rocket will reach its maximum height when the velocity is 0, so we'll need to find t when $v(t) = 0$.

$$-19.6t + 29.4 = 0$$

$$-19.6t = -29.4$$

$$t = \frac{-29.4}{-19.6}$$

$$t = 1.5$$

The rocket will reach its maximum height at $t = 1.5$ seconds. To find a function for height, integrate velocity.

$$h(t) = \int v(t)\ dt = \int -19.6t + 29.4\ dt$$

$$h(t) = -\frac{19.6t^2}{2} + 29.4t + C$$

The fact that the rocket was launched from ground level means we have the initial condition $h(0) = 0$. Substitute the initial condition into the height function.

$$0 = -\frac{19.6(0)^2}{2} + 29.4(0) + C$$

$$C = 0$$

So the height function is

$$h(t) = -\frac{19.6t^2}{2} + 29.4t$$

Then at $t = 1.5$ seconds, the height of the rocket is

$$h(1.5) = -9.8(1.5)^2 + 29.4(1.5) = 22.05 \approx 22 \text{ meters}$$

■ 5. What is the maximum height of a golf ball that's hit straight up from the ground with an initial velocity of $v(t) = -19.6t + 68.208$ m/sec?

Solution:

The golf ball will reach its maximum height when the velocity is 0, so we'll need to find t when $v(t) = 0$.

$$-19.6t + 68.208 = 0$$

$$-19.6t = -68.208$$

$$t = \frac{-68.208}{-19.6}$$

$$t = 3.48$$

The golf ball will reach its maximum height at $t = 3.48$ seconds. To find a function for height, integrate velocity.

$$h(t) = \int v(t)\ dt = \int -19.6t + 68.208\ dt$$

$$h(t) = -\frac{19.6t^2}{2} + 68.208t + C$$

The fact that the golf ball was hit from ground level means we have the initial condition $h(0) = 0$. Substitute the initial condition into the height function.

$$0 = -\frac{19.6(0)^2}{2} + 68.208(0) + C$$

$$C = 0$$

So the height function is

$$h(t) = -\frac{19.6t^2}{2} + 68.208t$$

Then at $t = 3.48$ seconds, the height of the golf ball is

$$h(3.48) = -9.8(3.48)^2 + 68.208(3.48) = 118.68192 \approx 119 \text{ meters}$$

RECTILINEAR MOTION

■ 1. Find the position function $x(t)$ that models the rectilinear motion of a particle moving along the x-axis.

$$a(t) = 10 - t$$

$$v(0) = -1$$

$$x(0) = 6$$

Solution:

Integrate the acceleration function to find the velocity function.

$$v(t) = \int a(t)\ dt = \int 10 - t\ dt$$

$$v(t) = 10t - \frac{t^2}{2} + C$$

Substitute the initial condition $v(0) = -1$ to find C.

$$-1 = 10(0) - \frac{0^2}{2} + C$$

$$C = -1$$

So the velocity function is

$$v(t) = -\frac{t^2}{2} + 10t - 1$$

Then the position function is the integral of the velocity function.

$$x(t) = \int v(t)\ dt = \int -\frac{t^2}{2} + 10t - 1\ dt$$

$$x(t) = -\frac{t^3}{6} + \frac{10t^2}{2} - t + C$$

$$x(t) = -\frac{t^3}{6} + 5t^2 - t + C$$

Substitute the initial condition $x(0) = 6$ to find C.

$$6 = -\frac{0^3}{6} + 5(0)^2 - 0 + C$$

$$C = 6$$

So the position function is

$$x(t) = -\frac{t^3}{6} + 5t^2 - t + 6$$

■ 2. Find the position function $x(t)$ that models the rectilinear motion of a particle moving along the x-axis.

$$a(t) = 9t^2 - 4t + 6$$

$$v(-1) = 0$$

$$x(0) = 2$$

Solution:

Integrate the acceleration function to find the velocity function.

$$v(t) = \int a(t)\ dt = \int 9t^2 - 4t + 6\ dt$$

$$v(t) = 3t^3 - 2t^2 + 6t + C$$

Substitute the initial condition $v(-1) = 0$ to find C.

$$0 = 3(-1)^3 - 2(-1)^2 + 6(-1) + C$$

$$0 = -3 - 2 - 6 + C$$

$$C = 11$$

So the velocity function is

$$v(t) = 3t^3 - 2t^2 + 6t + 11$$

Then the position function is the integral of the velocity function.

$$x(t) = \int v(t)\ dt = \int 3t^3 - 2t^2 + 6t + 11\ dt$$

$$x(t) = \frac{3t^4}{4} - \frac{2t^3}{3} + 3t^2 + 11t + C$$

Substitute the initial condition $x(0) = 2$ to find C.

$$2 = \frac{3(0)^4}{4} - \frac{2(0)^3}{3} + 3(0)^2 + 11(0) + C$$

$$C = 2$$

So the position function is

$$x(t) = \frac{3t^4}{4} - \frac{2t^3}{3} + 3t^2 + 11t + 2$$

■ 3. Find the position function $x(t)$ that models the rectilinear motion of a particle moving along the x-axis.

$$a(t) = 2 - 6t$$

$$v(0) = 4$$

$$x(0) = 3$$

Solution:

Integrate the acceleration function to find the velocity function.

$$v(t) = \int a(t)\ dt = \int 2 - 6t\ dt$$

$$v(t) = 2t - 3t^2 + C$$

Substitute the initial condition $v(0) = 4$ to find C.

$$4 = -3(0)^2 + 2(0) + C$$

$$C = 4$$

So the velocity function is

$$v(t) = 2t - 3t^2 + 4$$

Then the position function is the integral of the velocity function.

$$x(t) = \int v(t)\ dt = \int 2t - 3t^2 + 4\ dt$$

$$x(t) = -t^3 + t^2 + 4t + C$$

Substitute the initial condition $x(0) = 3$ to find C.

$$3 = -(0)^3 + (0)^2 + 4(0) + C$$

$$C = 3$$

So the position function is

$$x(t) = -t^3 + t^2 + 4t + 3$$

W W W . K R I S T A K I N G M A T H . C O M