Topic: Convergence of a sequence

Question: If the sequence converges, find its limit.

$$a_n = \frac{6}{n}$$

Answer choices:

A The sequence diverges

B The sequence converges and the limit is 0

C The sequence converges and the limit is 6

D The sequence converges and the limit is 12

Solution: B

We determine the convergence or divergence of a sequence by taking the limit of the sequence as $n \to \infty$.

The sequence **converges** if the limit exists and is finite

The sequence **diverges** if the limit does not exist or is infinite

Taking the limit of the sequence we've been given, we get

$$\lim_{n\to\infty} \frac{6}{n} = \frac{6}{\infty}$$

$$\lim_{n\to\infty} \frac{6}{n} = 0$$

The limit of the sequence is 0, which means the limit exists and is finite.

Therefore, we can say that the sequence converges.

Topic: Convergence of a sequence

Question: If the sequence converges, find its limit.

$$a_n = \frac{n^2}{2}$$

Answer choices:

A The sequence diverges

B The sequence converges and the limit is 0

C The sequence converges and the limit is $1/2$

D The sequence converges and the limit is 1

Solution: A

We determine the convergence or divergence of a sequence by taking the limit of the sequence as $n \to \infty$.

The sequence **converges** if the limit exists and is finite

The sequence **diverges** if the limit does not exist or is infinite

Taking the limit of the sequence we've been given, we get

$$\lim_{n\to\infty} \frac{n^2}{2} = \frac{\infty}{2}$$

$$\lim_{n\to\infty} \frac{n^2}{2} = \infty$$

The limit of the sequence is infinite. Therefore, we can say that the sequence diverges.

Topic: Convergence of a sequence

Question: If the sequence converges, find its limit.

$$a_n = \frac{2n^2 + 4n}{3n^2 + 3}$$

Answer choices:

A The sequence diverges

B The sequence converges and the limit is 0

C The sequence converges and the limit is $2/3$

D The sequence converges and the limit is 1

Solution: C

We determine the convergence or divergence of a sequence by taking the limit of the sequence as $n \to \infty$.

The sequence **converges** if the limit exists and is finite

The sequence **diverges** if the limit does not exist or is infinite

Taking the limit of the sequence we've been given, we get

$$\lim_{n\to\infty} \frac{2n^2+4n}{3n^2+3} = \frac{\infty}{\infty}$$

Since we get an indeterminate form, we need to back up a step and simplify the function.

$$\lim_{n\to\infty} \frac{2n^2+4n}{3n^2+3}$$

$$\lim_{n\to\infty} \frac{2n^2+4n}{3n^2+3} \left(\frac{\frac{1}{n^2}}{\frac{1}{n^2}} \right)$$

$$\lim_{n\to\infty} \frac{\frac{2n^2}{n^2}+\frac{4n}{n^2}}{\frac{3n^2}{n^2}+\frac{3}{n^2}}$$

$$\lim_{n\to\infty} \frac{2+\frac{4}{n}}{3+\frac{3}{n^2}}$$

Evaluating our simplified function as $n \to \infty$, we get

$$\frac{2+\frac{4}{\infty}}{3+\frac{3}{\infty}}$$

$$\frac{2+0}{3+0}$$

$$\frac{2}{3}$$

The limit of the sequence is $2/3$, which means the limit exists and is finite. Therefore, we can say that the sequence converges.