Topic: Strategy for testing series

Question: Which convergence test should be used to determine whether or not the series converges?

$$\sum_{n=1}^{\infty} \frac{1}{n^{\frac{2}{3}}}$$

Answer choices:

A Geometric series test for convergence

B Alternating series test for convergence

C Comparison test for convergence

D p-series test for convergence

Solution: D

We should use the p-series test to say whether or not this series converges.

The p-series test states that a series

$$a_n = \sum_{n=1}^{\infty} \frac{1}{n^p}$$

converges when $p > 1$

diverges when $p \leq 1$

For the given series,

$$p = \frac{2}{3}$$

and

$$\frac{2}{3} \leq 1$$

so the series diverges by the p-series test for convergence.

Topic: Strategy for testing series

Question: Which convergence test should be used to determine whether or not the series converges?

$$\sum_{n=1}^{\infty} \frac{n}{n^2 - 2}$$

Answer choices:

A Geometric series test for convergence

B Ratio series test for convergence

C Comparison test for convergence

D p-series test for convergence

Solution: C

We should use the comparison test to say whether or not this series converges.

The comparison test for convergence lets us determine the convergence or divergence of the given series a_n by *comparing* it to a similar, but simpler comparison series b_n.

We're usually trying to find a comparison series that's a geometric or p-series, since it's very easy to determine the convergence of a geometric or p-series.

We can use the comparison test to show that

the original series a_n is **diverging** if

the original series a_n is greater than or equal to the comparison series b_n and both series are positive, $a_n \geq b_n \geq 0$, and

the comparison series b_n is diverging

Note: If $a_n < b_n$, the test is inconclusive

the original series is **converging** if

the original series a_n is less than or equal to the comparison series b_n and both series are positive, $0 \leq a_n \leq b_n$, and

the comparison series b_n is converging

Note: If $b_n < a_n$, the test is inconclusive

For the given series,

$$a_n = \frac{n}{n^2 - 2}$$

and

$$b_n = \frac{n}{n^2}$$

$$b_n = \frac{1}{n}$$

which is the harmonic series, which we know diverges.

Topic: Strategy for testing series

Question: Which convergence test should be used to determine whether or not the series converges?

$$\sum_{n=1}^{\infty} \frac{n!}{n^2+1}$$

Answer choices:

A Geometric series test for convergence

B Ratio series test for convergence

C Root test for convergence

D p-series test for convergence

Solution: B

We should use the ratio test to say whether or not this series converges.

The ratio test for convergence lets us calculate L as

$$L = \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right|$$

and then says that the series

converges if $L < 1$

diverges if $L > 1$

The test is inconclusive if $L = 1$.

The ratio test is great for series that include factorials, and we would set up and evaluate L as

$$L = \lim_{n \to \infty} \left| \frac{\frac{(n+1)!}{(n+1)^2+1}}{\frac{n!}{n^2+1}} \right|$$