Topic: Riemann sums, midpoints

Question: Use Riemann Sums and midpoints to approximate area.

$$f(x) = -x^2 - x$$

on the interval $-1 \leq x \leq 2$

when $n = 5$

Answer choices:

A $-\frac{69}{50}$

B $\frac{441}{100}$

C $-\frac{441}{100}$

D $\frac{69}{50}$

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Solution: C

The Riemann sum is a tool we can use to approximate the area under a function over a set interval $a \le x \le b$.

We'll divide the area into rectangles and then sum the areas of all of the rectangles in order to get an approximation of area. The greater the number of rectangles, the more accurate the approximation will be. Of course, if we use an infinite number of rectangles, taking the limit as $n \to \infty$ of the sum of the area of each rectangle, then we'd be taking the integral and calculating exact area.

When we approximate area with Riemann sums we consider the area above the x-axis to be positive, and the area below the x-axis to be negative. If our final result is positive, it tells us that there's more area above the x-axis than below it. On the other hand, if our final result is negative, it means that there's more area below the x-axis than above it.

The Riemann sum formula is

$$R_n = \sum_{i=1}^{n} f(x_i)\Delta x$$

where $\Delta x = (b - a)/n$ and Δx is the width of each rectangle, and where n is the number of rectangles we're using to approximate area. If we expand the Riemann sum, we get the formula

$$R_n = \Delta x \left[f(x_1) + f(x_2) + \dots + f(x_n) \right]$$

Our plan is to solve for Δx, divide the interval into even segments that are each Δx wide, and then use an endpoint of each segment as the values of

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x_n. When we're using a Riemann sum to approximate area, we can choose the left endpoints, right endpoints, or midpoints of our rectangles.

Plugging the interval and the value of n we've been given into the formula for Δx, we get

$$\Delta x = \frac{b-a}{n}$$

$$\Delta x = \frac{2-(-1)}{5}$$

$$\Delta x = \frac{3}{5}$$

Since the interval is $[-1,2]$, we know that $x_0 = -1$ and that $x_n = 2$. Using $\Delta x = 3/5$ to find the subintervals, we get

$$x_0 = -1$$

$$x_1 = -1 + \frac{3}{5} \qquad x_1 = -\frac{2}{5}$$

$$x_2 = -\frac{2}{5} + \frac{3}{5} \qquad x_2 = \frac{1}{5}$$

$$x_3 = \frac{1}{5} + \frac{3}{5} \qquad x_3 = \frac{4}{5}$$

$$x_4 = \frac{4}{5} + \frac{3}{5} \qquad x_4 = \frac{7}{5}$$

$$x_5 = \frac{7}{5} + \frac{3}{5} \qquad x_5 = \frac{10}{5} \qquad x_5 = 2$$

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Since we're using midpoints, we need to find the value of x that's halfway between each of the x_n values above.

The first interval is $[-1, -2/5]$, so

$$M_1 = \frac{-1 + \left(-\frac{2}{5}\right)}{2}$$

$$M_1 = -\frac{7}{10}$$

The second interval is $[-2/5, 1/5]$, so

$$M_2 = \frac{-\frac{2}{5} + \frac{1}{5}}{2}$$

$$M_2 = -\frac{1}{10}$$

The third interval is $[1/5, 4/5]$, so

$$M_3 = \frac{\frac{1}{5} + \frac{4}{5}}{2}$$

$$M_3 = \frac{1}{2}$$

The fourth interval is $[4/5, 7/5]$, so

$$M_4 = \frac{\frac{4}{5} + \frac{7}{5}}{2}$$

$$M_4 = \frac{11}{10}$$

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The fifth interval is [7/5,2], so

$$M_5 = \frac{\frac{7}{5} + 2}{2}$$

$$M_5 = \frac{17}{10}$$

Plugging all of this into our Riemann sum formula, remembering that $f(x) = -x^2 - x$, we get

$$R_5 = \frac{3}{5}\left[f\left(-\frac{7}{10}\right) + f\left(-\frac{1}{10}\right) + f\left(\frac{1}{2}\right) + f\left(\frac{11}{10}\right) + f\left(\frac{17}{10}\right)\right]$$

$$R_5 = \frac{3}{5}\left\{\left[-\left(-\frac{7}{10}\right)^2 - \left(-\frac{7}{10}\right)\right] + \left[-\left(-\frac{1}{10}\right)^2 - \left(-\frac{1}{10}\right)\right] + \left[-\left(\frac{1}{2}\right)^2 - \frac{1}{2}\right]\right.$$

$$\left. + \left[-\left(\frac{11}{10}\right)^2 - \frac{11}{10}\right] + \left[-\left(\frac{17}{10}\right)^2 - \frac{17}{10}\right]\right\}$$

$$R_5 = \frac{3}{5}\left(-\frac{49}{100} + \frac{7}{10} - \frac{1}{100} + \frac{1}{10} - \frac{1}{4} - \frac{1}{2} - \frac{121}{100} - \frac{11}{10} - \frac{289}{100} - \frac{17}{10}\right)$$

$$R_5 = \frac{3}{5}\left(-\frac{460}{100} - \frac{20}{10} - \frac{1}{4} - \frac{1}{2}\right)$$

$$R_5 = \frac{3}{5}\left(-\frac{46}{10} - \frac{20}{10} - \frac{1}{4} - \frac{1}{2}\right)$$

$$R_5 = \frac{3}{5}\left(-\frac{33}{5} - \frac{1}{4} - \frac{1}{2}\right)$$

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$$R_5 = \frac{3}{5}\left(-\frac{132}{20} - \frac{5}{20} - \frac{10}{20}\right)$$

$$R_5 = -\frac{441}{100}$$

Topic: Riemann sums, midpoints

Question: Approximate the area under the curve using a midpoint rectangular approximation method, and five equal subintervals.

$$f(x) = \frac{1}{20}x^2 + x - 7$$

on the interval $[6,26]$

Answer choices:

A $\quad 576$

B $\quad 472$

C $\quad 468$

D $\quad 368$

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Solution: C

The term rectangular approximation method means we will approximate the area under the curve using rectangles. We calculate the area of each rectangle by multiplying the height of the rectangle (the function value) times the width of the rectangle (the length of the subinterval).

Because we are using a midpoint rectangular approximation method, we will find the height of the rectangle by calculating the function value at the midpoint of each subinterval.

The five equal subintervals in the interval $[6,26]$ are $[6,10]$, $[10,14]$, $[14,18]$, $[18,22]$, and $[22,26]$. We will next find the midpoint of each subinterval by adding the two endpoints and dividing that sum by 2. We will calculate the function values at 8, 12, 16, 20, and 24. Each subdivision is 4 units wide, so we will find the area of each triangle by multiplying the function values by 4.

$$f(8) = \frac{1}{20}(8)^2 + 8 - 7 = \frac{1}{20}(64) + 1 = 3.2 + 1 = 4.2 \qquad \text{Area: } 4.2 \times 4 = 16.8$$

$$f(12) = \frac{1}{20}(12)^2 + 12 - 7 = \frac{1}{20}(144) + 5 = 7.2 + 5 = 12.2 \qquad \text{Area:}$$

$$12.2 \times 4 = 48.8$$

$$f(16) = \frac{1}{20}(16)^2 + 16 - 7 = \frac{1}{20}(256) + 9 = 12.8 + 9 = 21.8 \qquad \text{Area:}$$

$$21.8 \times 4 = 87.2$$

$$f(20) = \frac{1}{20}(20)^2 + 20 - 7 = \frac{1}{20}(400) + 13 = 20 + 13 = 33 \qquad \text{Area:}$$

$$33 \times 4 = 132$$

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$$f(24) = \frac{1}{20}(24)^2 + 24 - 7 = \frac{1}{20}(576) + 17 = 28.8 + 17 = 45.8$$ Area: $45.8 \times 4 = 183.2$

Now that we know the area of each rectangle calculated at the midpoints of the subintervals, we will add the areas together to get the final approximation.

$$16.8 + 48.8 + 87.2 + 132 + 183.2 = 468$$

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Topic: Riemann sums, midpoints

Question: Approximate the area under the curve using a midpoint rectangular approximation method, and four equal subintervals.

$$g(x) = -\frac{1}{2}x^3 + 6x^2 - 4x + 2$$

on the interval $[1,9]$

Answer choices:

A 568

B 496

C 484

D 400

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Solution: B

The term rectangular approximation method means we will approximate the area under the curve using rectangles. We calculate the area of each rectangle by multiplying the height of the rectangle (the function value) times the width of the rectangle (the length of the subinterval).

Because we are using a midpoint rectangular approximation method, we will find the height of the rectangle by calculating the function value at the midpoint of each subinterval.

The four equal subintervals in the interval $[1,9]$ are $[1,3]$, $[3,5]$, $[7,7]$ and $[7,9]$. Each subinterval is 2 units wide. We will calculate the midpoint of each subinterval by finding the average of the endpoints.

We will calculate the function values at 2, 4, 6, and 8, and then multiply each value by 2 to find the area of the rectangles.

$$g(2) = -\frac{1}{2}(2)^3 + 6(2)^2 - 4(2) + 2 = 14$$

Area: $14 \times 2 = 28$

$$g(4) = -\frac{1}{2}(4)^3 + 6(4)^2 - 4(4) + 2 = 50$$

Area: $50 \times 2 = 100$

$$g(6) = -\frac{1}{2}(6)^3 + 6(6)^2 - 4(6) + 2 = 86$$

Area: $86 \times 2 = 172$

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$$g(8) = -\frac{1}{2}(8)^3 + 6(8)^2 - 4(8) + 2 = 98$$

Area: $98 \times 2 = 196$

Now that we know the area of each rectangle calculated at the midpoints of the subintervals, we will add the areas together to get the final approximation.

$$28 + 100 + 172 + 196 = 496$$

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