Topic: sin(mx) cos(nx)

Question: Evaluate the trigonometric integral.

$$\int \sin 4x \cos 3x \ dx$$

Answer choices:

A $\quad \frac{1}{2}\cos x + \frac{1}{14}\cos 7x + C$

B $\quad \frac{1}{2}\sin x + \frac{1}{14}\sin 7x + C$

C $\quad -\frac{1}{2}\cos x - \frac{1}{14}\cos 7x + C$

D $\quad -\frac{1}{2}\sin x - \frac{1}{14}\sin 7x + C$

Solution: C

In the specific case where our function is the product of

one **sine** factor and

one **cosine** factor,

our plan is to

1. use the identity $\sin A \cos B = \frac{1}{2}\left[\sin(A-B)+\sin(A+B)\right]$

We'll use the identity to simplify the integral.

$$\int \sin 4x \cos 3x \ dx$$

$$\int \frac{1}{2}\left[\sin(4x-3x)+\sin(4x+3x)\right] \ dx$$

$$\frac{1}{2}\int \sin x + \sin 7x \ dx$$

$$\frac{1}{2}\left(-\cos x - \frac{1}{7}\cos 7x\right)+C$$

$$-\frac{1}{2}\cos x - \frac{1}{14}\cos 7x + C$$

Topic: sin(mx) cos(nx)

Question: Evaluate the trigonometric integral.

$$\int \sin 7x \cos 2x \ dx$$

Answer choices:

A $\frac{1}{10}\sin 5x + \frac{1}{18}\sin 9x + C$

B $-\frac{1}{10}\sin 5x - \frac{1}{18}\sin 9x + C$

C $\frac{1}{10}\cos 5x + \frac{1}{18}\cos 9x + C$

D $-\frac{1}{10}\cos 5x - \frac{1}{18}\cos 9x + C$

Solution: D

In the specific case where our function is the product of

one **sine** factor and

one **cosine** factor,

our plan is to

1. use the identity $\sin A \cos B = \frac{1}{2}\left[\sin(A-B)+\sin(A+B)\right]$

We'll use the identity to simplify the integral.

$$\int \sin 7x \cos 2x \ dx$$

$$\int \frac{1}{2}\left[\sin(7x-2x)+\sin(7x+2x)\right] \ dx$$

$$\frac{1}{2}\int \sin 5x + \sin 9x \ dx$$

$$\frac{1}{2}\left(-\frac{1}{5}\cos 5x - \frac{1}{9}\cos 9x\right)+C$$

$$-\frac{1}{10}\cos 5x - \frac{1}{18}\cos 9x + C$$

Topic: sin(mx) cos(nx)

Question: Evaluate the trigonometric integral.

$$\int_0^{\frac{\pi}{2}} \sin 5x \cos 2x \ dx$$

Answer choices:

A $\frac{21}{5}$

B $\frac{5}{21}$

C $-\frac{21}{5}$

D $-\frac{5}{21}$

Solution: B

In the specific case where our function is the product of

one **sine** factor and

one **cosine** factor,

our plan is to

1. use the identity $\sin A \cos B = \frac{1}{2}\left[\sin(A-B)+\sin(A+B)\right]$

We'll use the identity to simplify the integral.

$$\int_0^{\frac{\pi}{2}} \sin 5x \cos 2x \ dx$$

$$\int_0^{\frac{\pi}{2}} \frac{1}{2}\left[\sin(5x-2x)+\sin(5x+2x)\right] \ dx$$

$$\frac{1}{2}\int_0^{\frac{\pi}{2}} \sin 3x + \sin 7x \ dx$$

$$\frac{1}{2}\left(-\frac{1}{3}\cos 3x - \frac{1}{7}\cos 7x\right)\Bigg|_0^{\frac{\pi}{2}}$$

$$\left(-\frac{1}{6}\cos 3x - \frac{1}{14}\cos 7x\right)\Bigg|_0^{\frac{\pi}{2}}$$

$$\left[-\frac{1}{6}\cos 3\left(\frac{\pi}{2}\right) - \frac{1}{14}\cos 7\left(\frac{\pi}{2}\right)\right] - \left[-\frac{1}{6}\cos 3(0) - \frac{1}{14}\cos 7(0)\right]$$

$$\left[-\frac{1}{6}\cos\frac{3\pi}{2}-\frac{1}{14}\cos\frac{7\pi}{2}\right]-\left(-\frac{1}{6}\cos 0-\frac{1}{14}\cos 0\right)$$

$$\left[-\frac{1}{6}(0)-\frac{1}{14}(0)\right]-\left[-\frac{1}{6}(1)-\frac{1}{14}(1)\right]$$

$$\frac{1}{6}+\frac{1}{14}$$

$$\frac{14}{84}+\frac{6}{84}$$

$$\frac{20}{84}$$

$$\frac{5}{21}$$