Topic: Area of a triangle with given vertices

Question: Find the area of the triangle with the given vertices.

$$(-3,2)$$

$$(5,4)$$

$$(7,1)$$

Answer choices:

A $\quad 70$ square units

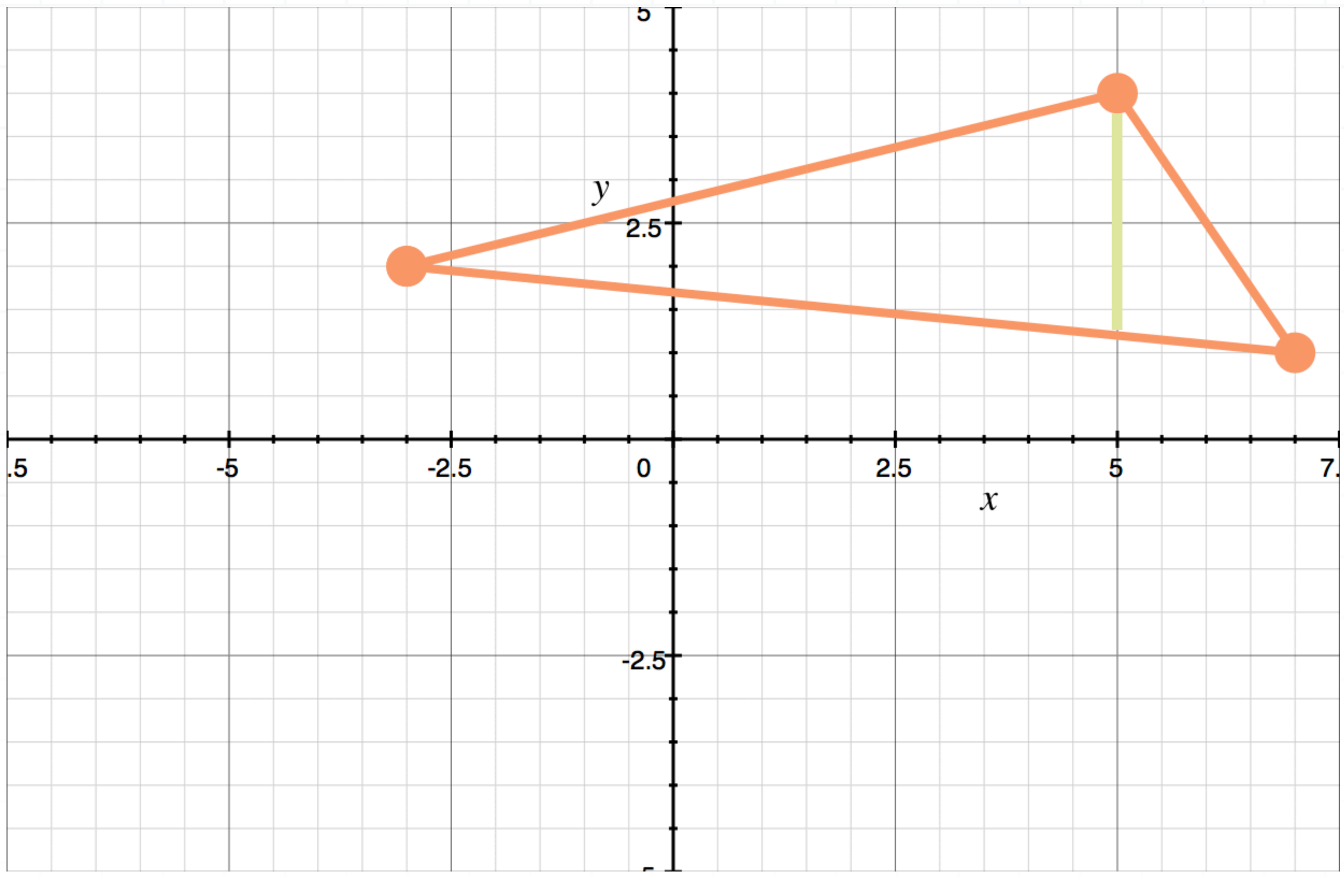
B $\quad \frac{56}{5}$ square units

C $\quad \frac{14}{5}$ square units

D $\quad 14$ square units

Solution: D

The triangle with vertices $(-3,2)$, $(5,4)$, and $(7,1)$ is



Finding the area of the triangle is really the same as finding the area between two curves. Based on the orientation of the triangle, we will integrate the difference between the side on top of the triangle and the side on the bottom of the triangle in two different intervals. We'll deal with the area to the left of the dashed line first, then find the area to the right of the dashed line, and then add the areas together to find total area.

First, we'll find the equation of the line that defines each side of the triangle. Let's start with the line connecting $(-3,2)$ and $(5,4)$. The slope of that line is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-2}{5-(-3)} = \frac{1}{4}$$

Use the point $(5,4)$ on that line, and the slope of the line $m = 1/4$ that we just found to plug into the point-slope formula for the equation of the line.

$$y - y_1 = m(x - x_1)$$

$$y - 4 = \frac{1}{4}(x - 5)$$

$$y = \frac{1}{4}x - \frac{5}{4} + 4$$

$$y = \frac{1}{4}x + \frac{11}{4}$$

Next, let's find the equation of the side connecting $(5,4)$ with $(7,1)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 4}{7 - 5} = -\frac{3}{2}$$

Use the point $(5,4)$ on that line, and the slope of the line $m = -3/2$ that we just found to plug into the point-slope formula for the equation of the line.

$$y - y_1 = m(x - x_1)$$

$$y - 4 = -\frac{3}{2}(x - 5)$$

$$y = -\frac{3}{2}x + \frac{15}{2} + 4$$

$$y = -\frac{3}{2}x + \frac{23}{2}$$

Next, let's find the equation of the side connecting $(-3,2)$ with $(7,1)$.

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{1-2}{7-(-3)}=-\frac{1}{10}$$

Use the point $(7,1)$ on that line, and the slope of the line $m=-1/10$ that we just found to plug into the point-slope formula for the equation of the line.

$$y-y_1=m(x-x_1)$$

$$y-1=-\frac{1}{10}(x-7)$$

$$y=-\frac{1}{10}x+\frac{7}{10}+1$$

$$y=-\frac{1}{10}x+\frac{17}{10}$$

Now that we have the equations for all three sides, this is simply an area between curves problem. The area to the left of the dashed line is

$$A=\int_{-3}^{5}\left(\frac{1}{4}x+\frac{11}{4}\right)-\left(-\frac{1}{10}x+\frac{17}{10}\right)\ dx$$

$$A=\int_{-3}^{5}\frac{1}{4}x+\frac{11}{4}+\frac{1}{10}x-\frac{17}{10}\ dx$$

$$A=\int_{-3}^{5}\frac{7}{20}x+\frac{21}{20}\ dx$$

$$A=\frac{7}{40}x^2+\frac{21}{20}x\Big|_{-3}^{5}$$

$$A = \frac{7}{40}(5)^2 + \frac{21}{20}(5) - \left(\frac{7}{40}(-3)^2 + \frac{21}{20}(-3)\right)$$

$$A = \frac{35}{8} + \frac{21}{4} - \frac{63}{40} + \frac{63}{20}$$

$$A = \frac{175}{40} + \frac{210}{40} - \frac{63}{40} + \frac{126}{40}$$

$$A = \frac{56}{5}$$

The area to the right of the dashed line is

$$A = \int_5^7 \left(-\frac{3}{2}x + \frac{23}{2}\right) - \left(-\frac{1}{10}x + \frac{17}{10}\right)\ dx$$

$$A = \int_5^7 -\frac{3}{2}x + \frac{23}{2} + \frac{1}{10}x - \frac{17}{10}\ dx$$

$$A = \int_5^7 -\frac{7}{5}x + \frac{49}{5}\ dx$$

$$A = -\frac{7}{10}x^2 + \frac{49}{5}x\Big|_5^7$$

$$A = -\frac{7}{10}(7)^2 + \frac{49}{5}(7) - \left(-\frac{7}{10}(5)^2 + \frac{49}{5}(5)\right)$$

$$A = -\frac{343}{10} + \frac{343}{5} + \frac{35}{2} - 49$$

$$A = -\frac{343}{10} + \frac{686}{10} + \frac{175}{10} - \frac{490}{10}$$

$$A=\frac{14}{5}$$

To find total area, we'll just add these two regions together.

$$A=\frac{56}{5}+\frac{14}{5}=14$$

Topic: Area of a triangle with given vertices

Question: Find the area of the triangle with the given vertices.

$(1,5)$

$(5,1)$

$(7,4)$

Answer choices:

A $\frac{20}{3}$ square units

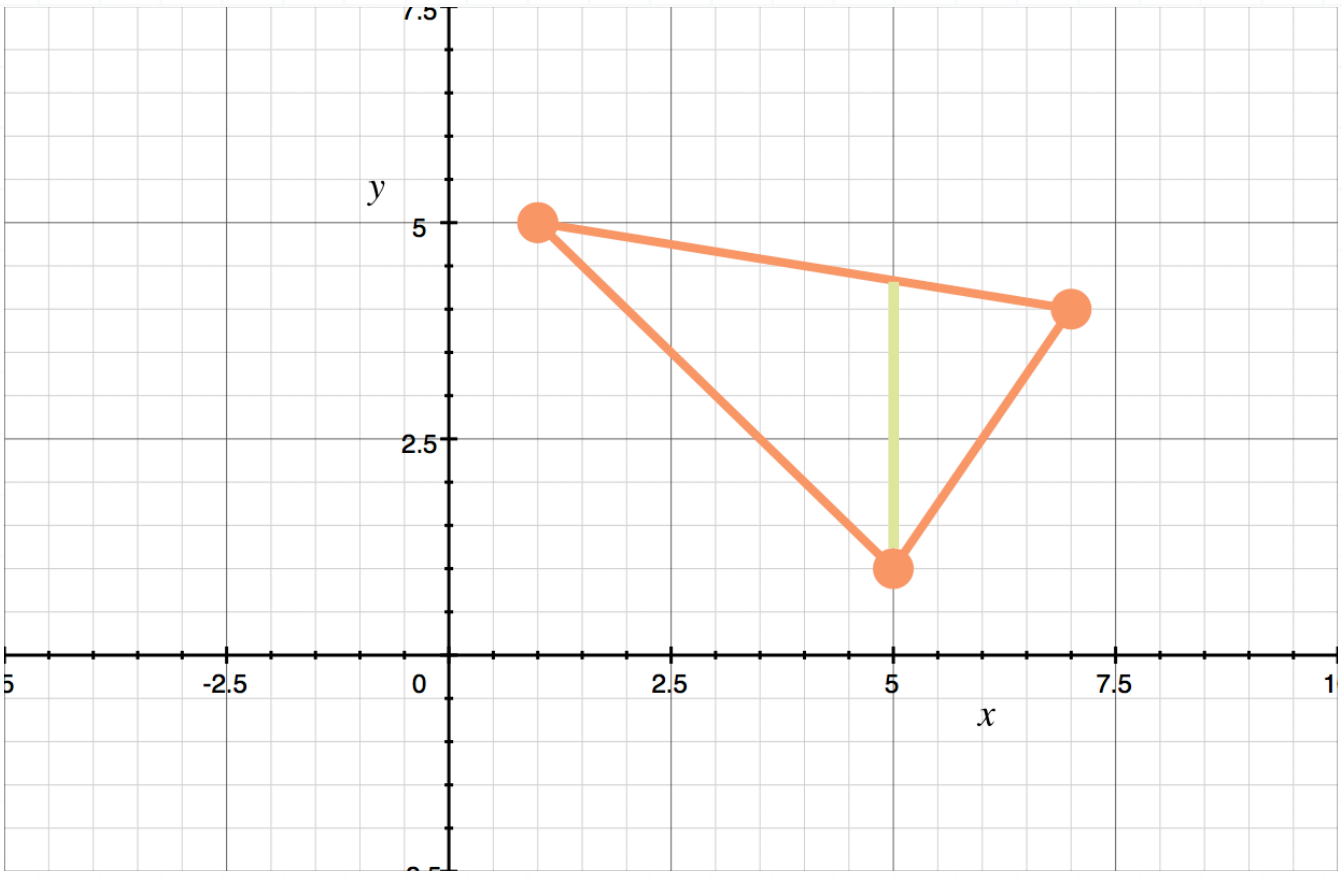
B 30 square units

C 10 square units

D $\frac{10}{3}$ square units

Solution: C

The triangle with vertices $(1,5)$, $(5,1)$, and $(7,4)$ is



Finding the area of the triangle is really the same as finding the area between two curves. Based on the orientation of the triangle, we will integrate the difference between the side on top of the triangle and the side on the bottom of the triangle in two different intervals. We'll deal with the area to the left of the dashed line first, then find the area to the right of the dashed line, and then add the areas together to find total area.

First, we'll find the equation of the line that defines each side of the triangle. Let's start with the line connecting $(1,5)$ and $(5,1)$. The slope of that line is

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{5-1}{1-5}=-1$$

Use the point $(1,5)$ on that line, and the slope of the line $m = -1$ that we just found to plug into the point-slope formula for the equation of the line.

$$y - y_1 = m(x - x_1)$$

$$y - 5 = -1(x - 1)$$

$$y = -x + 1 + 5$$

$$y = -x + 6$$

Next, let's find the equation of the side connecting $(5,1)$ with $(7,4)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 4}{5 - 7} = \frac{3}{2}$$

Use the point $(5,1)$ on that line, and the slope of the line $m = 3/2$ that we just found to plug into the point-slope formula for the equation of the line.

$$y - y_1 = m(x - x_1)$$

$$y - 1 = \frac{3}{2}(x - 5)$$

$$y = \frac{3}{2}x - \frac{15}{2} + 1$$

$$y = \frac{3}{2}x - \frac{13}{2}$$

Next, let's find the equation of the side connecting $(7,4)$ with $(1,5)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 5}{7 - 1} = -\frac{1}{6}$$

Use the point $(7,4)$ on that line, and the slope of the line $m = -1/6$ that we just found to plug into the point-slope formula for the equation of the line.

$$y - y_1 = m(x - x_1)$$

$$y - 4 = -\frac{1}{6}(x - 7)$$

$$y = -\frac{1}{6}x + \frac{7}{6} + 4$$

$$y = -\frac{1}{6}x + \frac{31}{6}$$

Now that we have the equations for all three sides, this is simply an area between curves problem. The area to the left of the dashed line is

$$A = \int_1^5 \left(-\frac{1}{6}x + \frac{31}{6}\right) - \left(-x + 6\right)\ dx$$

$$A = \int_1^5 -\frac{1}{6}x + \frac{31}{6} + x - 6\ dx$$

$$A = \int_1^5 \frac{5}{6}x - \frac{5}{6}\ dx$$

$$A = \frac{5}{12}x^2 - \frac{5}{6}x\Big|_1^5$$

$$A = \frac{5}{12}(5)^2 - \frac{5}{6}(5) - \left(\frac{5}{12}(1)^2 - \frac{5}{6}(1)\right)$$

$$A = \frac{125}{12} - \frac{25}{6} - \frac{5}{12} + \frac{5}{6}$$

$$A = \frac{125}{12} - \frac{50}{12} - \frac{5}{12} + \frac{10}{12}$$

$$A = \frac{20}{3}$$

The area to the right of the dashed line is

$$A = \int_5^7 \left(-\frac{1}{6}x + \frac{31}{6}\right) - \left(\frac{3}{2}x - \frac{13}{2}\right)\ dx$$

$$A = \int_5^7 -\frac{1}{6}x + \frac{31}{6} - \frac{3}{2}x + \frac{13}{2}\ dx$$

$$A = \int_5^7 -\frac{5}{3}x + \frac{35}{3}\ dx$$

$$A = -\frac{5}{6}x^2 + \frac{35}{3}x\Big|_5^7$$

$$A = -\frac{5}{6}(7)^2 + \frac{35}{3}(7) - \left(-\frac{5}{6}(5)^2 + \frac{35}{3}(5)\right)$$

$$A = -\frac{245}{6} + \frac{245}{3} + \frac{125}{6} - \frac{175}{3}$$

$$A = -\frac{245}{6} + \frac{490}{6} + \frac{125}{6} - \frac{350}{6}$$

$$A = \frac{10}{3}$$

To find total area, we'll just add these two regions together.

$$A = \frac{20}{3} + \frac{10}{3} = 10$$

Topic: Area of a triangle with given vertices

Question: Find the area of the triangle with the given vertices.

$$(-3,1)$$

$$(0,6)$$

$$(3,4)$$

Answer choices:

A $\frac{21}{2}$ square units

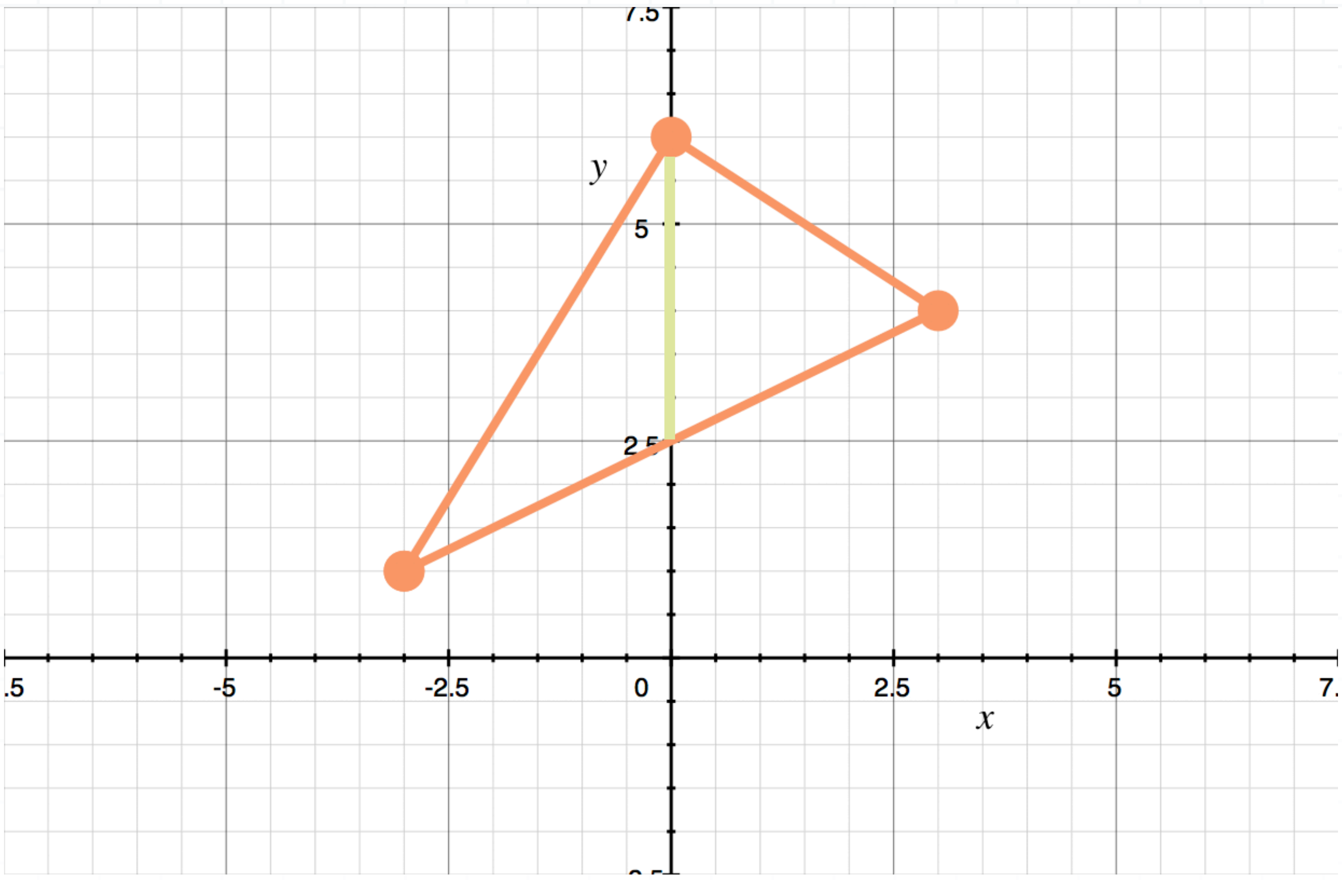
B 21 square units

C $\frac{21}{4}$ square units

D 14 square units

Solution: A

The triangle with vertices $(-3,1)$, $(0,6)$, and $(3,4)$ is



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First, we'll find the equation of the line that defines each side of the triangle. Let's start with the line connecting $(-3,1)$ and $(0,6)$. The slope of that line is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 6}{-3 - 0} = \frac{5}{3}$$

Use the point $(0,6)$ on that line, and the slope of the line $m=5/3$ that we just found to plug into the point-slope formula for the equation of the line.

$$y-y_1=m(x-x_1)$$

$$y-6=\frac{5}{3}(x-0)$$

$$y=\frac{5}{3}x+6$$

Next, let's find the equation of the side connecting $(-3,1)$ with $(3,4)$.

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{1-4}{-3-3}=\frac{1}{2}$$

Use the point $(3,4)$ on that line, and the slope of the line $m=1/2$ that we just found to plug into the point-slope formula for the equation of the line.

$$y-y_1=m(x-x_1)$$

$$y-4=\frac{1}{2}(x-3)$$

$$y=\frac{1}{2}x-\frac{3}{2}+4$$

$$y=\frac{1}{2}x+\frac{5}{2}$$

Next, let's find the equation of the side connecting $(0,6)$ with $(3,4)$.

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{6-4}{0-3}=-\frac{2}{3}$$

Use the point $(0,6)$ on that line, and the slope of the line $m = -2/3$ that we just found to plug into the point-slope formula for the equation of the line.

$$y - y_1 = m(x - x_1)$$

$$y - 6 = -\frac{2}{3}(x - 0)$$

$$y = -\frac{2}{3}x + 6$$

Now that we have the equations for all three sides, this is simply an area between curves problem. The area to the left of the dashed line is

$$A = \int_{-3}^{0} \left(\frac{5}{3}x + 6\right) - \left(\frac{1}{2}x + \frac{5}{2}\right)\ dx$$

$$A = \int_{-3}^{0} \frac{5}{3}x + 6 - \frac{1}{2}x - \frac{5}{2}\ dx$$

$$A = \int_{-3}^{0} \frac{7}{6}x + \frac{7}{2}\ dx$$

$$A = \frac{7}{12}x^2 + \frac{7}{2}x\Big|_{-3}^{0}$$

$$A = \frac{7}{12}(0)^2 + \frac{7}{2}(0) - \left(\frac{7}{12}(-3)^2 + \frac{7}{2}(-3)\right)$$

$$A = -\frac{21}{4} + \frac{21}{2}$$

$$A = \frac{21}{4}$$

The area to the right of the dashed line is

$$A=\int_0^3\left(-\frac{2}{3}x+6\right)-\left(\frac{1}{2}x+\frac{5}{2}\right)\ dx$$

$$A=\int_0^3-\frac{2}{3}x+6-\frac{1}{2}x-\frac{5}{2}\ dx$$

$$A=\int_0^3-\frac{7}{6}x+\frac{7}{2}\ dx$$

$$A=-\frac{7}{12}x^2+\frac{7}{2}x\Big|_0^3$$

$$A=-\frac{7}{12}(3)^2+\frac{7}{2}(3)-\left(-\frac{7}{12}(0)^2+\frac{7}{2}(0)\right)$$

$$A=-\frac{21}{4}+\frac{21}{2}$$

$$A=\frac{21}{4}$$

To find total area, we'll just add these two regions together.

$$A=\frac{21}{4}+\frac{21}{4}=\frac{21}{2}$$