Topic: Limit of a convergent sequence

Question: Find the limit of the convergent sequence.

$$a_n = \frac{2}{n^2}$$

Answer choices:

A $\sqrt{2}$

B 0

C 2

D ∞

Solution: B

We've already been told in the problem that this sequence converges.

We normally determine the convergence or divergence of a sequence by taking the limit of the sequence as $n \to \infty$, and we know that

The sequence **converges** if the limit exists and is finite

The sequence **diverges** if the limit does not exist or is infinite

Based on this definition, we should expect a finite answer when we take the limit of our sequence, since we know already that our sequence converges.

$$\lim_{n \to \infty} \frac{2}{n^2} = \frac{2}{\infty}$$

$$\lim_{n \to \infty} \frac{2}{n^2} = 0$$

The limit of the sequence is 0. Therefore, we can say that the sequence converges and that the limit is 0.

Topic: Limit of a convergent sequence

Question: Say whether the sequence converges. If it does, find its limit.

$$a_n = \frac{n^2}{2n^2 - 1}$$

Answer choices:

A -1

B 0

C $\frac{1}{2}$

D ∞

Solution: C

We determine the convergence or divergence of a sequence by taking the limit of the sequence as $n \to \infty$.

The sequence **converges** if the limit exists and is finite

The sequence **diverges** if the limit does not exist or is infinite

Taking the limit of the sequence we've been given, we get

$$\lim_{n \to \infty} \frac{n^2}{2n^2 - 1} = \frac{\infty}{\infty}$$

Since we get an indeterminate form, we need to back up a step and simplify the function.

$$\lim_{n \to \infty} \frac{n^2}{2n^2 - 1}$$

$$\lim_{n \to \infty} \frac{n^2}{2n^2 - 1} \left(\frac{\frac{1}{n^2}}{\frac{1}{n^2}} \right)$$

$$\lim_{n \to \infty} \frac{\frac{n^2}{n^2}}{\frac{2n^2}{n^2} - \frac{1}{n^2}}$$

$$\lim_{n \to \infty} \frac{1}{2 - \frac{1}{n^2}}$$

Evaluating our simplified function as $n \to \infty$, we get

$$\frac{1}{2 - \frac{1}{\infty}}$$

$$\frac{1}{2 - 0}$$

$$\frac{1}{2}$$

The limit of the sequence is $1/2$, which means the limit exists and is finite. Therefore, we can say that the sequence converges and that its limit is $1/2$.

Topic: Limit of a convergent sequence

Question: Say whether the sequence converges. If it does, find its limit.

$$a_n = \ln\left(6n^3 - n\right) - \ln\left(2n^3 + 4\right)$$

Answer choices:

A $\ln 3$

B 0

C $-\ln \frac{1}{4}$

D ∞

Solution: A

We determine the convergence or divergence of a sequence by taking the limit of the sequence as $n \to \infty$.

The sequence **converges** if the limit exists and is finite

The sequence **diverges** if the limit does not exist or is infinite

Taking the limit of the sequence we've been given, we get

$$\lim_{n \to \infty} \ln \frac{6n^3 - n}{2n^3 + 4} = \ln \frac{\infty}{\infty}$$

Since we get an indeterminate form, we need to back up a step and simplify the function.

$$\lim_{n \to \infty} \ln \frac{6n^3 - n}{2n^3 + 4}$$

$$\lim_{n \to \infty} \ln \frac{6n^3 - n}{2n^3 + 4} \left(\frac{\frac{1}{n^3}}{\frac{1}{n^3}} \right)$$

$$\lim_{n \to \infty} \ln \frac{\frac{6n^3}{n^3} - \frac{n}{n^3}}{\frac{2n^3}{n^3} + \frac{4}{n^3}}$$

$$\lim_{n \to \infty} \ln \frac{6 - \frac{1}{n^2}}{2 + \frac{4}{n^3}}$$

Evaluating our simplified function as $n \to \infty$, we get

$$\ln \frac{6 - \frac{1}{\infty}}{2 + \frac{4}{\infty}}$$

$$\ln \frac{6 + 0}{2 + 0}$$

$$\ln 3$$

The limit of the sequence is $\ln 3$, which means the limit exists and is finite. Therefore, we can say that the sequence converges and that its limit is $\ln 3$.