Topic: Ratio test

Question: Use the ratio test to determine the convergence of the series.

$$\sum_{n=1}^{\infty} \frac{n}{2^n}$$

Answer choices:

A The series converges

B The series conditionally converges

C The series diverges

D None of these

Solution: A

The ratio test for convergence lets us calculate L as

$$L = \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right|$$

and then says that the series

converges if $L < 1$

diverges if $L > 1$

The test is inconclusive if $L = 1$.

To find L, we'll need a_n and a_{n+1}.

$$a_n = \frac{n}{2^n}$$

$$a_{n+1} = \frac{n+1}{2^{n+1}}$$

Plugging these into the formula for L from the ratio test, we get

$$L = \lim_{n \to \infty} \left| \frac{\frac{n+1}{2^{n+1}}}{\frac{n}{2^n}} \right|$$

$$L = \lim_{n \to \infty} \left| \frac{n+1}{2^{n+1}} \cdot \frac{2^n}{n} \right|$$

Pairing similar numerators and denominators together, we get

$$L = \lim_{n \to \infty} \left| \frac{n+1}{n} \cdot \frac{2^n}{2^{n+1}} \right|$$

$$L = \lim_{n \to \infty} \left| \frac{n+1}{n} \cdot \frac{2^n}{2^n 2^1} \right|$$

$$L = \lim_{n \to \infty} \left| \frac{n+1}{n} \cdot \frac{1}{2} \right|$$

$$L = \lim_{n \to \infty} \left| \frac{n+1}{2n} \right|$$

$$L = \lim_{n \to \infty} \left| \frac{n}{2n} + \frac{1}{2n} \right|$$

$$L = \lim_{n \to \infty} \left| \frac{1}{2} + \frac{1}{2n} \right|$$

$$L = \left| \frac{1}{2} + 0 \right|$$

$$L = \frac{1}{2}$$

or

$$L = \frac{1}{2} < 1$$

Therefore, the series is convergent for all $x \in R$.

Topic: Ratio test

Question: Use the ratio test to determine the convergence of the series.

$$\sum_{n=1}^{\infty} \frac{3^n}{n^2}$$

Answer choices:

A The series converges

B The series conditionally converges

C The series diverges

D None of these

Solution: C

The ratio test for convergence lets us calculate L as

$$L = \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right|$$

and then says that the series

converges if $L < 1$

diverges if $L > 1$

The test is inconclusive if $L = 1$.

To find L, we'll need a_n and a_{n+1}.

$$a_n = \frac{3^n}{n^2}$$

$$a_{n+1} = \frac{3^{n+1}}{(n+1)^2}$$

Plugging these into the formula for L from the ratio test, we get

$$L = \lim_{n \to \infty} \left| \frac{\frac{3^{n+1}}{(n+1)^2}}{\frac{3^n}{n^2}} \right|$$

$$L = \lim_{n \to \infty} \left| \frac{3^{n+1}}{(n+1)^2} \cdot \frac{n^2}{3^n} \right|$$

Pairing similar numerators and denominators together, we get

$$L = \lim_{n\to\infty} \left| \frac{n^2}{(n+1)^2} \cdot 3^{n+1-n} \right|$$

$$L = \lim_{n\to\infty} \left| \frac{n^2}{(n+1)^2} \cdot 3^1 \right|$$

$$L = 3 \lim_{n\to\infty} \left| \frac{n^2}{(n+1)^2} \right|$$

$$L = 3 \lim_{n\to\infty} \left| \frac{n^2}{n^2 + 2n + 1} \right|$$

$$L = 3 \lim_{n\to\infty} \left| \frac{n^2}{n^2 + 2n + 1} \left(\frac{\frac{1}{n^2}}{\frac{1}{n^2}} \right) \right|$$

$$L = 3 \lim_{n\to\infty} \left| \frac{\frac{n^2}{n^2}}{\frac{n^2}{n^2} + \frac{2n}{n^2} + \frac{1}{n^2}} \right|$$

$$L = 3 \lim_{n\to\infty} \left| \frac{1}{1 + \frac{2}{n} + \frac{1}{n^2}} \right|$$

$$L = 3 \left| \frac{1}{1 + 0 + 0} \right|$$

$$L = 3 \left| 1 \right|$$

$$L = 3$$

or

$$L = 3 > 1$$

Therefore, the series is divergent for all $x \in R$.

Topic: Ratio test

Question: Use the ratio test to determine the convergence of the series.

$$\sum_{n=1}^{\infty} \frac{2n^2}{10^n}$$

Answer choices:

A The series converges

B The series conditionally converges

C The series diverges

D None of these

Solution: A

The ratio test for convergence lets us calculate L as

$$L = \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right|$$

and then says that the series

converges if $L < 1$

diverges if $L > 1$

The test is inconclusive if $L = 1$.

To find L, we'll need a_n and a_{n+1}.

$$a_n = \frac{2n^2}{10^n}$$

$$a_{n+1} = \frac{2(n+1)^2}{10^{n+1}}$$

Plugging these into the formula for L from the ratio test, we get

$$L = \lim_{n \to \infty} \left| \frac{\frac{2(n+1)^2}{10^{n+1}}}{\frac{2n^2}{10^n}} \right|$$

$$L = \lim_{n \to \infty} \left| \frac{2(n+1)^2}{10^{n+1}} \cdot \frac{10^n}{2n^2} \right|$$

Pairing similar numerators and denominators together, we get

$$L=\lim_{n\to\infty}\left|\frac{10^n}{10^{n+1}}\cdot\frac{2(n+1)^2}{2n^2}\right|$$

$$L=\lim_{n\to\infty}\left|10^{n-(n+1)}\cdot\frac{2(n^2+2n+1)}{2n^2}\right|$$

$$L=\lim_{n\to\infty}\left|10^{n-n-1}\cdot\frac{n^2+2n+1}{n^2}\right|$$

$$L=\lim_{n\to\infty}\left|10^{-1}\cdot\frac{n^2+2n+1}{n^2}\right|$$

$$L=\lim_{n\to\infty}\left|\frac{1}{10}\cdot\frac{n^2+2n+1}{n^2}\right|$$

$$L=\frac{1}{10}\lim_{n\to\infty}\left|\frac{n^2+2n+1}{n^2}\right|$$

$$L=\frac{1}{10}\lim_{n\to\infty}\left|\frac{n^2+2n+1}{n^2}\left(\frac{\frac{1}{n^2}}{\frac{1}{n^2}}\right)\right|$$

$$L=\frac{1}{10}\lim_{n\to\infty}\left|\frac{\frac{n^2}{n^2}+\frac{2n}{n^2}+\frac{1}{n^2}}{\frac{n^2}{n^2}}\right|$$

$$L = \frac{1}{10} \lim_{n \to \infty} \left| \frac{1 + \frac{2}{n} + \frac{1}{n^2}}{1} \right|$$

$$L = \frac{1}{10} \lim_{n \to \infty} \left| 1 + \frac{2}{n} + \frac{1}{n^2} \right|$$

$$L = \frac{1}{10} \left| 1 + 0 + 0 \right|$$

$$L = \frac{1}{10} \left| 1 \right|$$

$$L = \frac{1}{10}$$

or

$$L = \frac{1}{10} < 1$$

Therefore, the series is convergent for all $x \in R$.