Topic: Integrals using reduction formulas

Question: Use the reduction formula to find the integral.

$$\int \tan^6 x \ dx$$

Use $\int \tan^n x \ dx = \frac{1}{n-1}\tan^{n-1}x - \int \tan^{n-2}x \ dx$

Answer choices:

A $\quad \frac{1}{5}\tan^5 x - \frac{1}{3}\tan^3 x + \tan x - x + C$

B $\quad \tan^5 x - \tan^3 x + \tan x - x + C$

C $\quad \frac{1}{5}\tan^5 x - \frac{1}{3}\tan^3 x + \tan x - x$

D $\quad \frac{1}{7}\tan^7 x + C$

Solution: A

The reduction formula

$$\int \tan^n x \ dx = \frac{1}{n-1}\tan^{n-1}x - \int \tan^{n-2}x \ dx$$

simply gives us a method of evaluating the integral without trying to figure out how to do the integration. In our integral, $n = 6$. We use the reduction formula, repeatedly until we have the final result of the integration. The formula tells us to reduce the exponents each time.

Let's do the first iteration of the reduction formula.

$$\int \tan^6 x \ dx = \frac{1}{6-1}\tan^{6-1}x - \int \tan^{6-2}x \ dx$$

$$\int \tan^6 x \ dx = \frac{1}{5}\tan^5 x - \int \tan^4 x \ dx$$

Let's do the second iteration of the reduction formula.

$$\int \tan^6 x \ dx = \frac{1}{5}\tan^5 x - \left(\frac{1}{4-1}\tan^{4-1}x - \int \tan^{4-2}x \ dx\right)$$

$$\int \tan^6 x \ dx = \frac{1}{5}\tan^5 x - \frac{1}{3}\tan^3 x + \int \tan^2 x \ dx$$

Let's do the third iteration of the reduction formula.

$$\int \tan^6 x \ dx = \frac{1}{5}\tan^5 x - \frac{1}{3}\tan^3 x + \frac{1}{2-1}\tan^{2-1}x - \int \tan^{2-2}x \ dx$$

$$\int \tan^6 x \ dx = \frac{1}{5}\tan^5 x - \frac{1}{3}\tan^3 x + \tan x - \int dx$$

Do the final integration without the reduction formula.

$$\int \tan^6 x \ dx = \frac{1}{5}\tan^5 x - \frac{1}{3}\tan^3 x + \tan x - x + C$$

Topic: Integrals using reduction formulas

Question: Use the reduction formula to find the integral.

$$\int x^4 e^x \, dx$$

Use $\int x^n e^x \, dx = x^n e^x - \int n x^{n-1} e^x \, dx$

Answer choices:

A $\quad x^4e^x - x^3e^x + x^2e^x - xe^x + e^x + C$

B $\quad x^4e^x - 4x^3e^x + 12x^2e^x - 24xe^x + 24e^x$

C $\quad x^4e^x - 4x^3e^x + 12x^2e^x - 24xe^x + 24e^x + x + C$

D $\quad x^4e^x - 4x^3e^x + 12x^2e^x - 24xe^x + 24e^x + C$

Solution: D

The reduction formula

$$\int x^n e^x \, dx = x^n e^x - \int n x^{n-1} e^x \, dx$$

simply gives us a method of evaluating the integral without trying to figure out how to do the integration. In our integral, $n = 4$. We use the reduction formula, repeatedly until we have the final result of the integration. The formula tells us to reduce the exponents each time.

Let's do the first iteration of the reduction formula.

$$\int x^4 e^x \, dx = x^4 e^x - \int 4x^{4-1} e^x \, dx$$

$$\int x^4 e^x \, dx = x^4 e^x - 4 \int x^3 e^x \, dx$$

Let's do the second iteration of the reduction formula.

$$\int x^4 e^x \, dx = x^4 e^x - 4 \left[x^3 e^x - \int 3x^{3-1} e^x \, dx \right]$$

$$\int x^4 e^x \, dx = x^4 e^x - 4x^3 e^x + 4 \int 3x^{3-1} e^x \, dx$$

$$\int x^4 e^x \, dx = x^4 e^x - 4x^3 e^x + 12 \int x^2 e^x \, dx$$

Let's do the third iteration of the reduction formula.

$$\int x^4e^x\ dx = x^4e^x - 4x^3e^x + 12\left[x^2e^x - \int 2x^{2-1}e^x\ dx\right]$$

$$\int x^4e^x\ dx = x^4e^x - 4x^3e^x + 12x^2e^x - 12\int 2x^{2-1}e^x\ dx$$

$$\int x^4e^x\ dx = x^4e^x - 4x^3e^x + 12x^2e^x - 24\int xe^x\ dx$$

Let's do the fourth iteration of the reduction formula.

$$\int x^4e^x\ dx = x^4e^x - 4x^3e^x + 12x^2e^x - 24\left[xe^x - \int 1x^{1-1}e^x\ dx\right]$$

$$\int x^4e^x\ dx = x^4e^x - 4x^3e^x + 12x^2e^x - 24xe^x + 24\int e^x\ dx$$

Do the final integration without the reduction formula.

$$\int x^4e^x\ dx = x^4e^x - 4x^3e^x + 12x^2e^x - 24xe^x + 24e^x$$

$$\int x^4e^x\ dx = x^4e^x - 4x^3e^x + 12x^2e^x - 24xe^x + 24e^x + C$$

Topic: Integrals using reduction formulas

Question: Use the reduction formula to find the integral.

$$\int (\ln x)^4 \ dx$$

Use $\int (\ln x)^n \ dx = x(\ln x)^n - n \int (\ln x)^{n-1} \ dx$

Answer choices:

A $\quad (\ln x)^3 - 4(\ln x)^3 + 12(\ln x)^2 - 24 \ln x + 24 + C$

B $\quad x(\ln x)^4 - 4x(\ln x)^3 + 12x(\ln x)^2 - 24x(\ln x) + 24x + C$

C $\quad x\left[(\ln x)^3 - 4(\ln x)^2 + 12(\ln x)\right]$

D $\quad (\ln x)^3 - 4(\ln x)^3 + 12(\ln x)^2 - 24 \ln x + 24$

Solution: B

The reduction formula

$$\int (\ln x)^n \ dx = x(\ln x)^n - n \int (\ln x)^{n-1} \ dx$$

simply gives us a method of evaluating the integral without trying to figure out how to do the integration. In our integral, $n = 4$. We use the reduction formula, repeatedly until we have the final result of the integration. The formula tells us to reduce the exponents each time.

Let's do the first iteration of the reduction formula.

$$\int (\ln x)^4 \ dx = x(\ln x)^4 - 4 \int (\ln x)^{4-1} \ dx$$

$$\int (\ln x)^4 \ dx = x(\ln x)^4 - 4 \int (\ln x)^3 \ dx$$

Let's do the second iteration of the reduction formula.

$$\int (\ln x)^4 \ dx = x(\ln x)^4 - 4 \left[x(\ln x)^3 - 3 \int (\ln x)^{3-1} \ dx \right]$$

$$\int (\ln x)^4 \ dx = x(\ln x)^4 - 4x(\ln x)^3 + 12 \int (\ln x)^2 \ dx$$

Let's do the third iteration of the reduction formula.

$$\int (\ln x)^4 \ dx = x(\ln x)^4 - 4x(\ln x)^3 + 12 \left[x(\ln x)^2 - 2 \int (\ln x)^{2-1} \ dx \right]$$

$$\int (\ln x)^4 \ dx = x(\ln x)^4 - 4x(\ln x)^3 + 12x(\ln x)^2 - 24 \int \ln x \ dx$$

Let's do the fourth iteration of the reduction formula.

$$\int (\ln x)^4 \ dx = x(\ln x)^4 - 4x(\ln x)^3 + 12x(\ln x)^2 - 24 \left[x(\ln x)^1 - 1 \int (\ln x)^{1-1} \ dx \right]$$

$$\int (\ln x)^4 \ dx = x(\ln x)^4 - 4x(\ln x)^3 + 12x(\ln x)^2 - 24x(\ln x) + 24 \int \ dx$$

Do the final integration without the reduction formula.

$$\int (\ln x)^4 \ dx = x(\ln x)^4 - 4x(\ln x)^3 + 12x(\ln x)^2 - 24x(\ln x) + 24x + C$$