Topic: Root test

Question: Use the root test to determine the convergence of the series.

$$\sum_{n=2}^{\infty}\left(\frac{3n^3+4n^2-7}{\sqrt{4n^6+9n^4-10}}\right)^n$$

Answer choices:

A The series converges

B The series conditionally converges

C The series diverges

D None of these

Solution: C

Let

$$a_n = \left(\frac{3n^3 + 4n^2 - 7}{\sqrt{4n^6 + 9n^4 - 10}} \right)^n$$

Then by the root test,

$$R = \lim_{n \to \infty} \left| a_n \right|^{\frac{1}{n}}$$

$$R = \lim_{n \to \infty} \left| \left(\frac{3n^3 + 4n^2 - 7}{\sqrt{4n^6 + 9n^4 - 10}} \right)^n \right|^{\frac{1}{n}}$$

Taking the nth root gives us

$$R = \lim_{n \to \infty} \left| \frac{3n^3 + 4n^2 - 7}{\sqrt{4n^6 + 9n^4 - 10}} \right|$$

Dividing both the numerator and denominator by the highest power of n,

$$R = \lim_{n \to \infty} \left| \frac{3n^3 + 4n^2 - 7}{\sqrt{4n^6 + 9n^4 - 10}} \left(\frac{\frac{1}{n^3}}{\frac{1}{n^3}} \right) \right|$$

$$R = \lim_{n \to \infty} \left| \frac{3 + \frac{4}{n} - \frac{7}{n^3}}{\sqrt{4n^6 + 9n^4 - 10}} \cdot \frac{1}{\frac{1}{\sqrt{n^6}}} \right|$$

$$R = \lim_{n\to\infty} \left| \frac{3 + \frac{4}{n} - \frac{7}{n^3}}{\frac{\sqrt{4n^6 + 9n^4 - 10}}{\sqrt{n^6}}} \right|$$

$$R = \lim_{n\to\infty} \left| \frac{3 + \frac{4}{n} - \frac{7}{n^3}}{\sqrt{\frac{4n^6 + 9n^4 - 10}{n^6}}} \right|$$

$$R = \lim_{n\to\infty} \left| \frac{3 + \frac{4}{n} - \frac{7}{n^3}}{\sqrt{4 + \frac{9}{n^2} - \frac{10}{n^6}}} \right|$$

$$R = \left| \frac{3 + 0 - 0}{\sqrt{4 + 0 - 0}} \right|$$

$$R = \left| \frac{3}{\sqrt{4}} \right|$$

$$R = \frac{3}{2}$$

Since

$$R = \frac{3}{2} > 1$$

the series is divergent.

Topic: Root test

Question: Use the root test to determine the convergence of the series.

$$\sum_{n=1}^{\infty} \frac{n}{e^{n^2}}$$

Answer choices:

A The series converges

B The series conditionally converges

C The series diverges

D None of these

Solution: A

Let

$$a_n = \frac{n}{e^{n^2}}$$

Then by the root test,

$$R = \lim_{n\to\infty} \left| a_n \right|^{\frac{1}{n}}$$

$$R = \lim_{n\to\infty} \left| \frac{n}{e^{n^2}} \right|^{\frac{1}{n}}$$

$$R = \lim_{n\to\infty} \left| \frac{n^{\frac{1}{n}}}{\left(e^{n^2}\right)^{\frac{1}{n}}} \right|$$

$$R = \lim_{n\to\infty} \left| \frac{n^{\frac{1}{n}}}{e^n} \right|$$

$$R = \left| \frac{1}{\infty} \right| = 0 < 1$$

Since

$$R = 0 < 1$$

the series is convergent.

Topic: Root test

Question: Use the root test to determine the convergence of the series.

$$\sum_{n=1}^{\infty} \frac{2^n}{(n+1)^n}$$

Answer choices:

A The series converges

B The series conditionally converges

C The series diverges

D None of these

Solution: A

Let

$$a_n = \frac{2^n}{(n+1)^n}$$

Then by the root test,

$$R = \lim_{n\to\infty} \left| a_n \right|^{\frac{1}{n}}$$

$$R = \lim_{n\to\infty} \left| \frac{2^n}{(n+1)^n} \right|^{\frac{1}{n}}$$

$$R = \lim_{n\to\infty} \left| \left(\frac{2}{n+1} \right)^n \right|^{\frac{1}{n}}$$

$$R = \lim_{n\to\infty} \left| \left(\frac{2}{n+1} \right)^{n\cdot\frac{1}{n}} \right|$$

$$R = \lim_{n\to\infty} \left| \frac{2}{n+1} \right|$$

$$R = \left| \frac{2}{\infty+1} \right|$$

$$R = \left| \frac{2}{\infty} \right|$$

$$R = |0|$$

$$R = 0$$

Since

$$R = 0 < 1$$

the series is convergent.