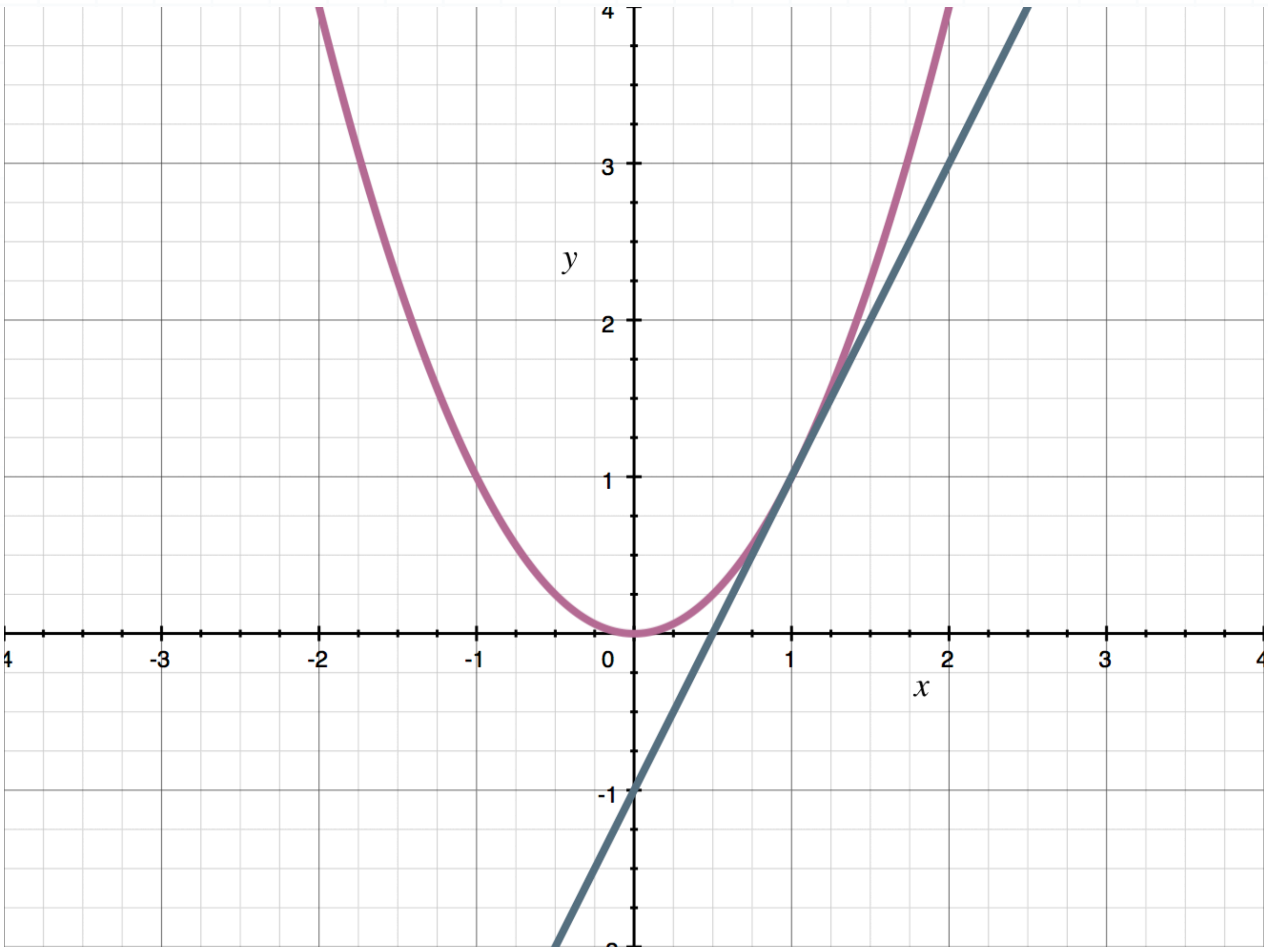
Tangent lines

We briefly talked about the tangent line when we first introduced the derivative, but now we want to spend more time talking about how to find its equation.

Remember that a **tangent line** is a line that touches the graph of a function at exactly one point. For instance, a tangent line might look like



The line comes alongside the graph, and touches the curve at exactly $x = 1$. The tangent line doesn't cross the graph, but stays on the same side of the curve.

As long as it's defined, we can find the equation of the tangent line to any curve, at any point on the curve. No matter which curve we're using, or where along that curve we're finding the tangent line, the equation of the tangent line will be

$$y = f(a) + f'(a)(x - a)$$

In this equation, $x = a$ is the x-value at which the tangent line intersects the curve. So $f(a)$ is the y-value where the tangent line intersects the curve, and $f'(a)$ is the value of the function's derivative at $x = a$, or the slope of the tangent line to $f(x)$ at $x = a$.

Vertical and horizontal tangent lines

If $f'(a)$ is undefined, then the tangent line is vertical. If $f'(a) = 0$, then the tangent line is horizontal.

- To find the equation of a vertical tangent line, first find the derivative of $f(x)$, $f'(x)$. If $f'(a)$ is undefined, then the equation of the vertical tangent line will be $x = a$.

- To find the equation of a horizontal tangent line, first find the derivative of $f(x)$, $f'(x)$, then solve $f'(a) = 0$. Find the y-value where the tangent line intersects the curve, $f(a)$, then the equation of the horizontal tangent line will be $y = f(a)$.

Let's walk through a full example, so that we can see step-by-step how to find the equation of a tangent line.

Example

Find the equation of the tangent line to $f(x)$ at $x = 4$.

$$f(x) = 6x^2 - 2x + 5$$

First, plug $x = 4$ into the original function.

$$f(4) = 6(4)^2 - 2(4) + 5$$

$$f(4) = 96 - 8 + 5$$

$$f(4) = 93$$

Next, take the derivative and then evaluate the derivative at $x = 4$.

$$f'(x) = 12x - 2$$

$$f'(4) = 12(4) - 2$$

$$f'(4) = 46$$

Finally, substitute both $f(4)$ and $f'(4)$ into the tangent line formula, along with $a = 4$, since this is the value at which we're finding the equation of the tangent line.

$$y = f(a) + f'(a)(x - a)$$

$$y = 93 + 46(x - 4)$$

We can either leave the equation in this form, or we can simplify it further.

$$y = 93 + 46x - 184$$

$$y = 46x - 91$$

Let's do one more example, but with a different type of function.

Example

Find the equation of the tangent line to $f(x)$ at $x = 0$.

$$f(x) = 3\sin x$$

First, plug $x = 0$ into the original function.

$$f(0) = 3\sin(0)$$

$$f(0) = 3(0)$$

$$f(0) = 0$$

Next, take the derivative and then evaluate the derivative at $x = 0$.

$$f'(x) = 3\cos x$$

$$f'(0) = 3\cos(0)$$

$$f'(0) = 3(1)$$

$$f'(0) = 3$$

Finally, substitute both $f(0)$ and $f'(0)$ into the tangent line formula, along with $a = 0$, since this is the value at which we're finding the equation of the tangent line.

$$y = f(a) + f'(a)(x - a)$$

$$y = 0 + 3(x - 0)$$

$$y = 3x$$