Calculus 2 Workbook Solutions

Geometry

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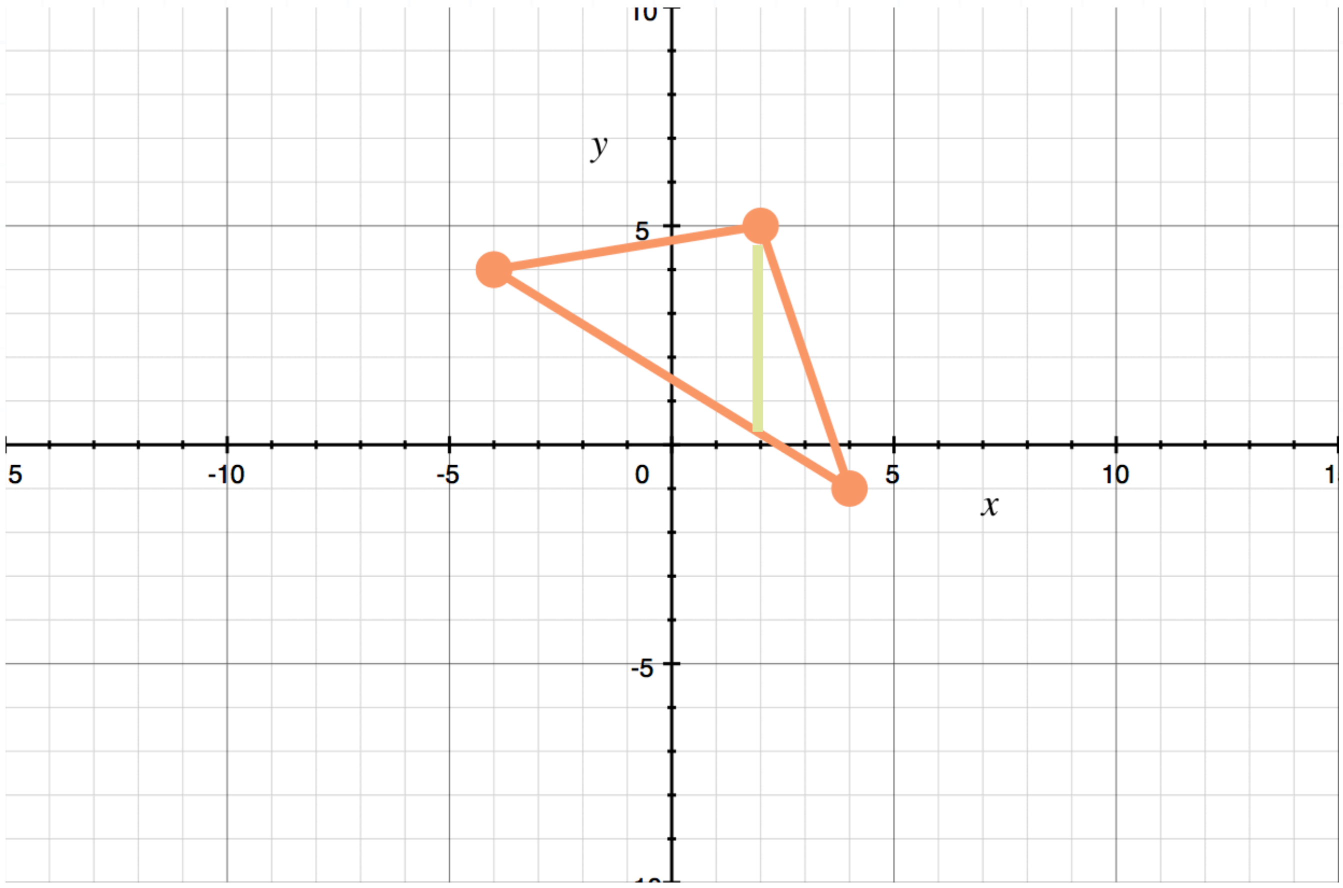
AREA OF A TRIANGLE WITH GIVEN VERTICES

■ 1. Find the area of the triangle with vertices $A(-4,4)$, $B(2,5)$, and $C(4,-1)$.

Solution:

A sketch of the region, separated by a vertical line from B is



The slope of the line connecting $A(-4,4)$ and $B(2,5)$ is

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{5-4}{2-(-4)}=\frac{1}{6}$$

Then using $B(2,5)$ and the slope $m=1/6$, the equation of that line is

$$y = \frac{1}{6}(x-2)+5$$

$$y = \frac{1}{6}x - \frac{2}{6} + 5$$

$$y = \frac{1}{6}x + \frac{14}{3}$$

The slope of the line connecting $A(-4,4)$ and $C(4,-1)$ is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-4}{4-(-4)} = -\frac{5}{8}$$

Then using $A(-4,4)$ and the slope $m = -5/8$, the equation of that line is

$$y = -\frac{5}{8}(x+4)+4$$

$$y = -\frac{5}{8}x - \frac{5}{2} + 4$$

$$y = -\frac{5}{8}x + \frac{3}{2}$$

The slope of the line connecting $B(2,5)$ and $C(4,-1)$ is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-5}{4-2} = -3$$

Then using $B(2,5)$ and the slope $m = -3$, the equation of that line is

$$y = -3(x-2)+5$$

$$y = -3x + 6 + 5$$

$$y = -3x + 11$$

Then the area to the left of the vertical line A_L is

$$A_L = \int_{-4}^{2} \left(\frac{1}{6}x + \frac{14}{3}\right) - \left(-\frac{5}{8}x + \frac{3}{2}\right)\ dx$$

$$A_L = \int_{-4}^{2} \frac{1}{6}x + \frac{14}{3} + \frac{5}{8}x - \frac{3}{2}\ dx$$

$$A_L = \int_{-4}^{2} \frac{4}{24}x + \frac{15}{24}x + \frac{28}{6} - \frac{9}{6}\ dx$$

$$A_L = \int_{-4}^{2} \frac{19}{24}x + \frac{19}{6}\ dx$$

Integrate, then evaluate over the interval.

$$A_L = \frac{19}{48}x^2 + \frac{19}{6}x\Big|_{-4}^{2}$$

$$A_L = \frac{19}{48}(2)^2 + \frac{19}{6}(2) - \left(\frac{19}{48}(-4)^2 + \frac{19}{6}(-4)\right)$$

$$A_L = \frac{76}{48} + \frac{38}{6} - \frac{304}{48} + \frac{76}{6}$$

$$A_L = -\frac{228}{48} + \frac{114}{6}$$

$$A_L = -\frac{57}{12} + \frac{228}{12}$$

$$A_L = \frac{171}{12}$$

$$A_L = \frac{57}{4}$$

The area to the right of the vertical line A_R is

$$A_R = \int_2^4 (-3x+11) - \left(-\frac{5}{8}x + \frac{3}{2}\right)\ dx$$

$$A_R = \int_2^4 -3x + 11 + \frac{5}{8}x - \frac{3}{2}\ dx$$

$$A_R = \int_2^4 -\frac{24}{8}x + \frac{5}{8}x + \frac{22}{2} - \frac{3}{2}\ dx$$

$$A_R = \int_2^4 -\frac{19}{8}x + \frac{19}{2}\ dx$$

Integrate, then evaluate over the interval.

$$A_R = -\frac{19}{16}x^2 + \frac{19}{2}x\Big|_2^4$$

$$A_R = -\frac{19}{16}(4)^2 + \frac{19}{2}(4) - \left(-\frac{19}{16}(2)^2 + \frac{19}{2}(2)\right)$$

$$A_R = -19 + 38 + \frac{19}{4} - 19$$

$$A_R = \frac{19}{4}$$

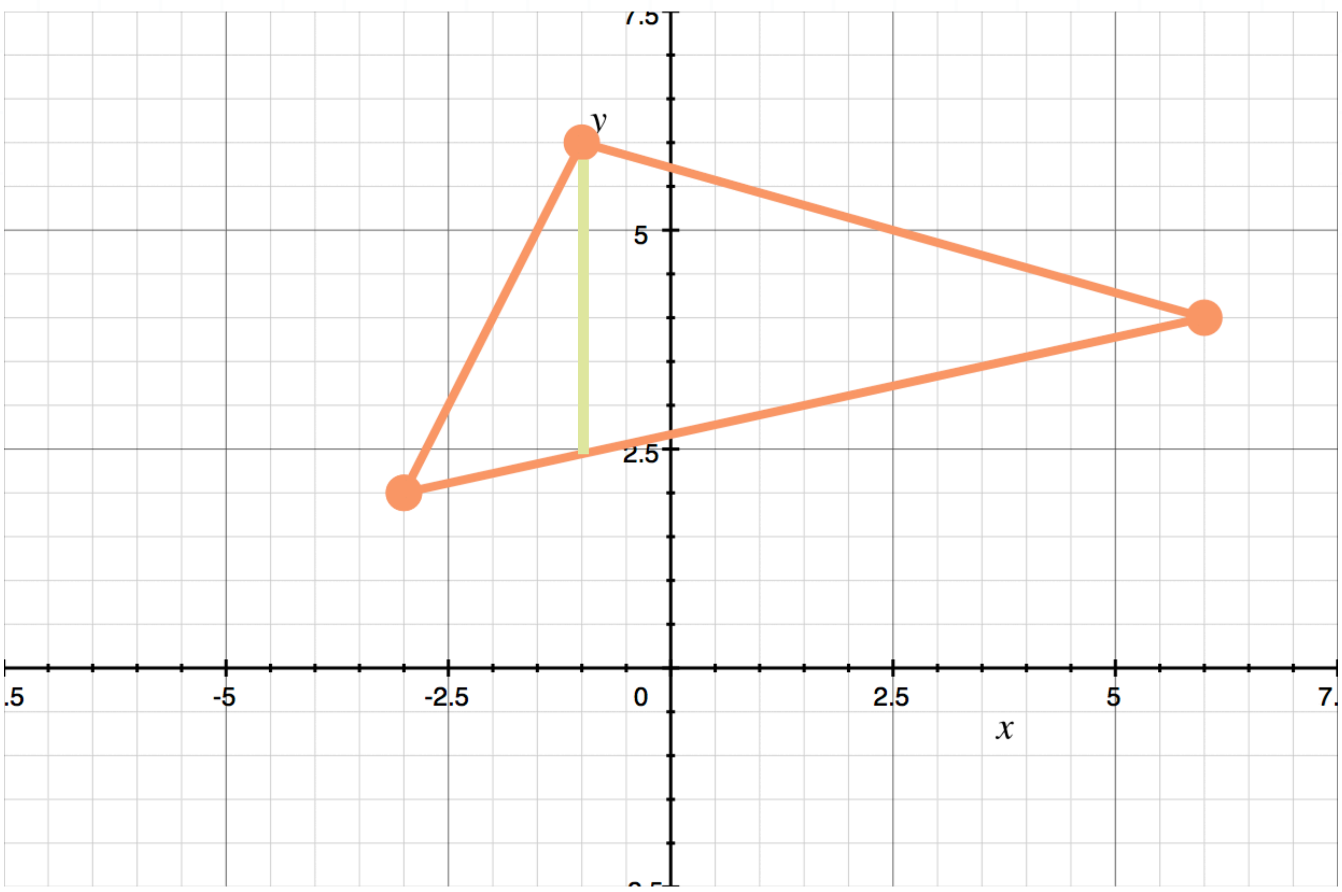
The area of the triangle is

$$A_L + A_R = \frac{57}{4} + \frac{19}{4} = \frac{76}{4} = 19$$

2. Find the area of the triangle with vertices $D(-3,2)$, $E(-1,6)$, and $F(6,4)$.

Solution:

A sketch of the region, separated by a vertical line from E is



The slope of the line connecting $D(-3,2)$ and $E(-1,6)$ is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-2}{-1-(-3)} = \frac{4}{2} = 2$$

Then using $D(-3,2)$ and the slope $m=2$, the equation of that line is

$$y=2(x+3)+2$$

$$y=2x+6+2$$

$$y=2x+8$$

The slope of the line connecting $E(-1,6)$ and $F(6,4)$ is

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{4-6}{6-(-1)}=-\frac{2}{7}$$

Then using $E(-1,6)$ and the slope $m=-2/7$, the equation of that line is

$$y=-\frac{2}{7}(x+1)+6$$

$$y=-\frac{2}{7}x-\frac{2}{7}+6$$

$$y=-\frac{2}{7}x+\frac{40}{7}$$

The slope of the line connecting $D(-3,2)$ and $F(6,4)$ is

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{4-2}{6-(-3)}=\frac{2}{9}$$

Then using $F(6,4)$ and the slope $m=2/9$, the equation of that line is

$$y=\frac{2}{9}(x-6)+4$$

$$y = \frac{2}{9}x - \frac{4}{3} + 4$$

$$y = \frac{2}{9}x + \frac{8}{3}$$

Then the area to the left of the vertical line A_L is

$$A_L = \int_{-3}^{-1} (2x + 8) - \left(\frac{2}{9}x + \frac{8}{3}\right)\ dx$$

$$A_L = \int_{-3}^{-1} 2x + 8 - \frac{2}{9}x - \frac{8}{3}\ dx$$

$$A_L = \int_{-3}^{-1} \frac{18}{9}x - \frac{2}{9}x + \frac{24}{3} - \frac{8}{3}\ dx$$

$$A_L = \int_{-3}^{-1} \frac{16}{9}x + \frac{16}{3}\ dx$$

Integrate, then evaluate over the interval.

$$A_L = \frac{16}{18}x^2 + \frac{16}{3}x\Big|_{-3}^{-1}$$

$$A_L = \frac{16}{18}(-1)^2 + \frac{16}{3}(-1) - \left(\frac{16}{18}(-3)^2 + \frac{16}{3}(-3)\right)$$

$$A_L = \frac{16}{18} - \frac{16}{3} - (8 - 16)$$

$$A_L = \frac{8}{9} - \frac{48}{9} - \frac{72}{9} + \frac{144}{9}$$

$$A_L = \frac{32}{9}$$

The area to the right of the vertical line A_R is

$$A_R = \int_{-1}^{6} \left(-\frac{2}{7}x + \frac{40}{7}\right) - \left(\frac{2}{9}x + \frac{8}{3}\right)\ dx$$

$$A_R = \int_{-1}^{6} -\frac{2}{7}x + \frac{40}{7} - \frac{2}{9}x - \frac{8}{3}\ dx$$

$$A_R = \int_{-1}^{6} -\frac{18}{63}x - \frac{14}{63}x + \frac{120}{21} - \frac{56}{21}\ dx$$

$$A_R = \int_{-1}^{6} -\frac{32}{63}x + \frac{64}{21}\ dx$$

Integrate, then evaluate over the interval.

$$A_R = -\frac{16}{63}x^2 + \frac{64}{21}x \Big|_{-1}^{6}$$

$$A_R = -\frac{16}{63}(6)^2 + \frac{64}{21}(6) - \left(-\frac{16}{63}(-1)^2 + \frac{64}{21}(-1)\right)$$

$$A_R = -\frac{576}{63} + \frac{384}{21} + \frac{16}{63} + \frac{64}{21}$$

$$A_R = -\frac{560}{63} + \frac{448}{21}$$

$$A_R = -\frac{80}{9} + \frac{64}{3}$$

$$A_R = -\frac{240}{27} + \frac{576}{27}$$

$$A_R = \frac{336}{27}$$

$$A_R = \frac{112}{9}$$

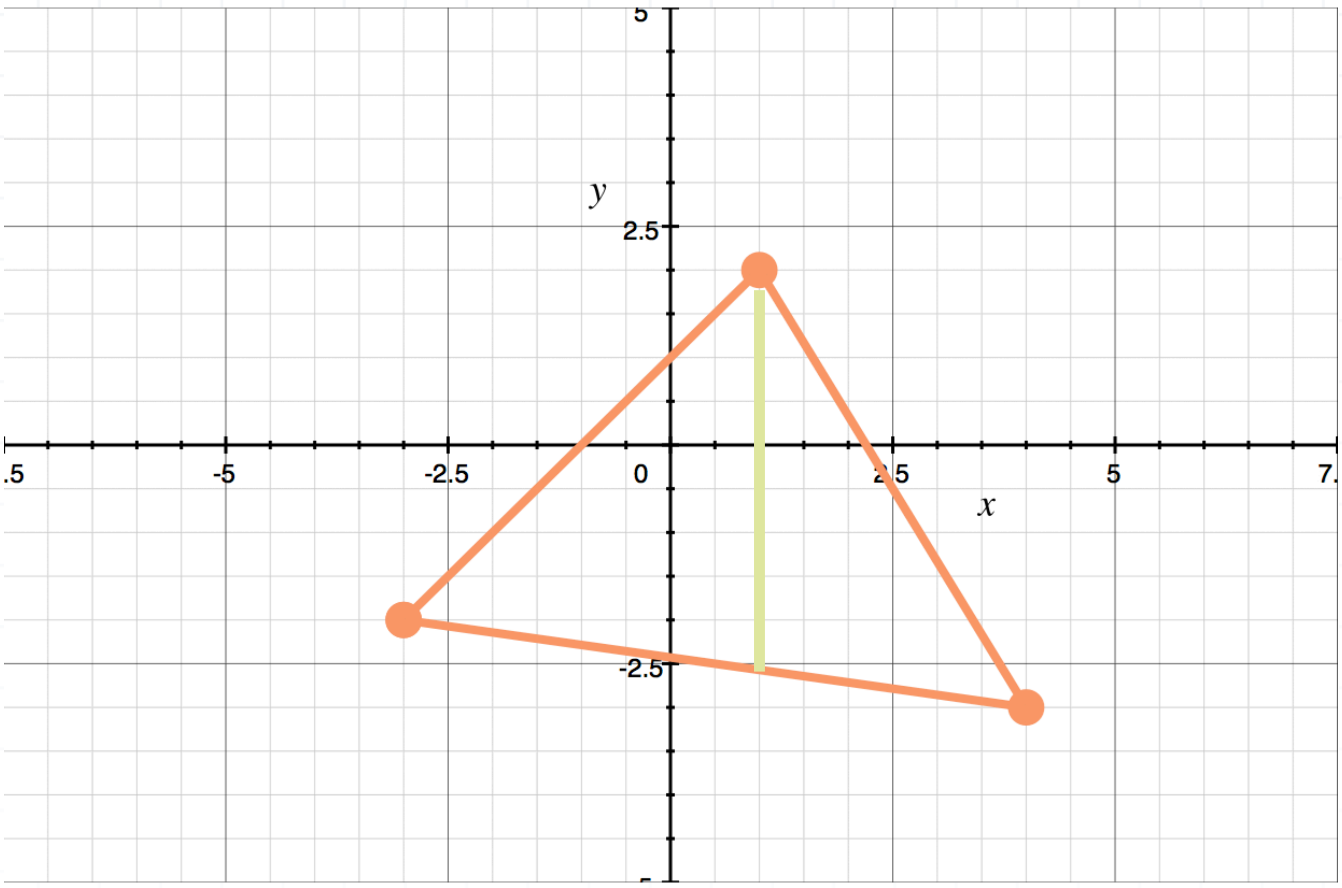
The area of the triangle is

$$A_L + A_R = \frac{32}{9} + \frac{112}{9} = \frac{144}{9} = 16$$

3. Find the area of the triangle with vertices $G(-3,-2)$, $H(1,2)$, and $I(4,-3)$.

Solution:

A sketch of the region, separated by a vertical line from H is



The slope of the line connecting $G(-3,-2)$ and $H(1,2)$ is

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{2-(-2)}{1-(-3)}=\frac{4}{4}=1$$

Then using $H(1,2)$ and the slope $m=1$, the equation of that line is

$$y=1(x-1)+2$$

$$y=x-1+2$$

$$y=x+1$$

The slope of the line connecting $H(1,2)$ and $I(4,-3)$ is

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{-3-2}{4-1}=-\frac{5}{3}$$

Then using $I(4,-3)$ and the slope $m=-5/3$, the equation of that line is

$$y=-\frac{5}{3}(x-4)-3$$

$$y=-\frac{5}{3}x+\frac{20}{3}-3$$

$$y=-\frac{5}{3}x+\frac{11}{3}$$

The slope of the line connecting $G(-3,-2)$ and $I(4,-3)$ is

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{-3-(-2)}{4-(-3)}=-\frac{1}{7}$$

Then using $G(-3,-2)$ and the slope $m=-1/7$, the equation of that line is

$$y=-\frac{1}{7}(x+3)-2$$

$$y=-\frac{1}{7}x-\frac{3}{7}-2$$

$$y=-\frac{1}{7}x-\frac{17}{7}$$

Then the area to the left of the vertical line A_L is

$$A_L=\int_{-3}^{1}(x+1)-\left(-\frac{1}{7}x-\frac{17}{7}\right)\ dx$$

$$A_L=\int_{-3}^{1}\frac{7}{7}x+\frac{1}{7}x+\frac{7}{7}+\frac{17}{7}\ dx$$

$$A_L=\int_{-3}^{1}\frac{8}{7}x+\frac{24}{7}\ dx$$

Integrate, then evaluate over the interval.

$$A_L = \frac{4}{7}x^2 + \frac{24}{7}x\Big|_{-3}^{1}$$

$$A_L = \frac{4}{7}(1)^2 + \frac{24}{7}(1) - \left(\frac{4}{7}(-3)^2 + \frac{24}{7}(-3)\right)$$

$$A_L = \frac{4}{7} + \frac{24}{7} - \frac{36}{7} + \frac{72}{7}$$

$$A_L = \frac{64}{7}$$

The area to the right of the vertical line A_R is

$$A_R = \int_1^4 \left(-\frac{5}{3}x + \frac{11}{3}\right) - \left(-\frac{1}{7}x - \frac{17}{7}\right)\ dx$$

$$A_R = \int_1^4 -\frac{5}{3}x + \frac{11}{3} + \frac{1}{7}x + \frac{17}{7}\ dx$$

$$A_R = \int_1^4 \frac{3}{21}x - \frac{35}{21}x + \frac{77}{21} + \frac{51}{21}\ dx$$

$$A_R = \int_1^4 -\frac{32}{21}x + \frac{128}{21}\ dx$$

Integrate, then evaluate over the interval.

$$A_R = -\frac{16}{21}x^2 + \frac{128}{21}x\Big|_1^4$$

$$A_R = -\frac{16}{21}(4)^2 + \frac{128}{21}(4) - \left(-\frac{16}{21}(1)^2 + \frac{128}{21}(1)\right)$$

$$A_R = -\frac{256}{21} + \frac{512}{21} + \frac{16}{21} - \frac{128}{21}$$

$$A_R = \frac{144}{21}$$

$$A_R = \frac{48}{7}$$

The area of the triangle is

$$A_L + A_R = \frac{64}{7} + \frac{48}{7} = \frac{112}{7} = 16$$

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