Topic: Linear approximation

Question: Find the linear approximation of the function at $a = 0$.

$$f(x) = \cos x$$

Answer choices:

A $L(x) = 1$

B $L(x) = x$

C $L(x) = x - 1$

D $L(x) = -x$

Solution: A

Take the derivative.

$$f(x) = \cos x$$

$$f'(x) = -\sin x$$

Evaluate the original function at $a = 0$.

$$f(0) = \cos 0$$

$$f(0) = 1$$

Evaluate the derivative at $a = 0$.

$$f'(0) = -\sin 0$$

$$f'(0) = 0$$

Substitute all of these pieces into the linear approximation formula.

$$L(x) = f(a) + f'(a)(x - a)$$

$$L(x) = 1 + 0(x - 0)$$

$$L(x) = 1$$

Topic: Linear approximation

Question: Use linear approximation to estimate $e^{-0.1}$.

Answer choices:

A 0.1

B 0

C 0.9

D 1.1

Solution: C

Given the value of the function we're asked to estimate, it's clear that the function should be e^x. Instead of trying to find $f(-0.1)$, let's use a linear approximation equation and $a = 0$ to get an approximation for $f(-0.1)$.

Take the derivative.

$$f(x) = e^x$$

$$f'(x) = e^x$$

Evaluate the original function at $a = 0$.

$$f(0) = e^0$$

$$f(0) = 1$$

Evaluate the derivative at $a = 0$.

$$f'(0) = e^0$$

$$f'(0) = 1$$

Substitute all of these pieces into the linear approximation formula.

$$L(x) = f(a) + f'(a)(x - a)$$

$$L(x) = 1 + 1(x - 0)$$

$$L(x) = 1 + x$$

Now that we've built the linear approximation equation, we can substitute $x = -0.1$.

$$L(-0.1) = 1 - 0.1$$

$$L(-0.1) = 0.9$$

Topic: Linear approximation

Question: Find the linear approximation of the function at $a = 2$.

$$f(x) = (x + 4)^2$$

Answer choices:

A $L(x) = 1 + x$

B $L(x) = 12 + 12x$

C $L(x) = -12 - 12x$

D $L(x) = 1 - x$

Solution: B

Take the derivative.

$$f(x) = (x+4)^2$$

$$f'(x) = 2(x+4)(1)$$

$$f'(x) = 2x + 8$$

Evaluate the original function at $a = 2$.

$$f(2) = (2+4)^2$$

$$f(2) = 36$$

Evaluate the derivative at $a = 2$.

$$f'(2) = 2(2) + 8$$

$$f'(2) = 12$$

Substitute all of these pieces into the linear approximation formula.

$$L(x) = f(a) + f'(a)(x - a)$$

$$L(x) = 36 + 12(x - 2)$$

$$L(x) = 36 + 12x - 24$$

$$L(x) = 12 + 12x$$