Topic: Calculating the first terms of a sequence of partial sums

Question: Approximate to three decimal places the first four terms of the sequence of partial sums.

$$\sum_{n=1}^{\infty} \frac{2n}{n^2+1}$$

Answer choices:

A $1.000,\ 1.800,\ 1.600,\ 1.471$

B $1.000,\ 0.200,\ -0.400,\ -0.871$

C $1.000,\ 1.800,\ 2.400,\ 2.871$

D $1.000,\ 0.800,\ 0.600,\ 0.471$

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Solution: C

If we're given a series a_n, and we find the first few terms of a_n, the terms will be

$$a_1$$

$$a_2$$

$$a_3$$

$$a_4$$

$$\ldots$$

If instead we want to calculate the first few terms of the sequence of partial sums s_n, we simply add our terms together, such that

$$s_1 = a_1$$

$$s_2 = a_1 + a_2$$

$$s_3 = a_1 + a_2 + a_3$$

$$s_4 = a_1 + a_2 + a_3 + a_4$$

$$\ldots$$

To calculate the first four terms of the sequence of partial sums, we'll calculate each term of the series a_n individually, then add that term to any previous terms to get the term of the partial sum sequence.

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$n = 1$ $\quad a_1 = \dfrac{2(1)}{(1)^2 + 1} = \dfrac{2}{2} = 1.000$ $\quad s_1 = a_1$

$s_1 = 1.000$

$n = 2$ $\quad a_2 = \dfrac{2(2)}{(2)^2 + 1} = \dfrac{4}{5} = 0.800$ $\quad s_2 = a_1 + a_2$

$s_2 = 1.000 + 0.800$

$s_2 = 1.800$

$n = 3$ $\quad a_3 = \dfrac{2(3)}{(3)^2 + 1} = \dfrac{6}{10} = 0.600$ $\quad s_3 = a_1 + a_2 + a_3$

$s_3 = 1.000 + 0.800 + 0.600$

$s_3 = 2.400$

$n = 4$ $\quad a_4 = \dfrac{2(4)}{(4)^2 + 1} = \dfrac{8}{17} = 0.471$ $\quad s_4 = a_1 + a_2 + a_3 + a_4$

$s_4 = 1.000 + 0.800 + 0.600 + 0.471$

$s_4 = 2.871$

The first four terms of the sequence of partial sums, approximated to three decimal places, are

$$1.000,\ 1.800,\ 2.400,\ 2.871$$

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Topic: Calculating the first terms of a sequence of partial sums

Question: Approximate to three decimal places the first four terms of the sequence of partial sums.

$$\sum_{n=1}^{\infty} \frac{3-n}{n^3}$$

Answer choices:

A $2.000,\ 2.125,\ 2.125,\ 2.016$

B $2.000,\ 2.125,\ 2.000,\ 1.984$

C $2.000,\ 0.125,\ 0.000,\ -0.016$

D $2.000,\ 2.125,\ 2.125,\ 2.109$

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Solution: D

If we're given a series a_n, and we find the first few terms of a_n, the terms will be

$$a_1$$

$$a_2$$

$$a_3$$

$$a_4$$

$$\ldots$$

If instead we want to calculate the first few terms of the sequence of partial sums s_n, we simply add our terms together, such that

$$s_1 = a_1$$

$$s_2 = a_1 + a_2$$

$$s_3 = a_1 + a_2 + a_3$$

$$s_4 = a_1 + a_2 + a_3 + a_4$$

$$\ldots$$

To calculate the first four terms of the sequence of partial sums, we'll calculate each term of the series a_n individually, then add that term to any previous terms to get the term of the partial sum sequence.

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$n = 1$ $\quad a_1 = \dfrac{3-1}{(1)^3} = \dfrac{2}{1} = 2.000$ $\quad s_1 = a_1$

$s_1 = 2.000$

$n = 2$ $\quad a_2 = \dfrac{3-2}{(2)^3} = \dfrac{1}{8} = 0.125$ $\quad s_2 = a_1 + a_2$

$s_2 = 2.000 + 0.125$

$s_2 = 2.125$

$n = 3$ $\quad a_3 = \dfrac{3-3}{(3)^3} = \dfrac{0}{27} = 0.000$ $\quad s_3 = a_1 + a_2 + a_3$

$s_3 = 2.000 + 0.125 + 0.000$

$s_3 = 2.125$

$n = 4$ $\quad a_4 = \dfrac{3-4}{(4)^3} = \dfrac{-1}{64} = -0.016$ $\quad s_4 = a_1 + a_2 + a_3 + a_4$

$s_4 = 2.000 + 0.125 + 0.000 - 0.016$

$s_4 = 2.109$

The first four terms of the sequence of partial sums, approximated to three decimal places, are

$$2.000,\ 2.125,\ 2.125,\ 2.109$$

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Topic: Calculating the first terms of a sequence of partial sums

Question: Approximate to three decimal places the first four terms of the sequence of partial sums.

$$\sum_{n=1}^{\infty} \frac{n}{e^n}$$

Answer choices:

A 0.368, 0.271, 0.149, 0.073

B 0.368, 0.639, 0.788, 0.873

C 0.368, 0.639, 0.517, 0.441

D 0.368, 0.639, 0.788, 0.861

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Solution: D

If we're given a series a_n, and we find the first few terms of a_n, the terms will be

$$a_1$$

$$a_2$$

$$a_3$$

$$a_4$$

$$\ldots$$

If instead we want to calculate the first few terms of the series of partial sums s_n, we simply add our terms together, such that

$$s_1 = a_1$$

$$s_2 = a_1 + a_2$$

$$s_3 = a_1 + a_2 + a_3$$

$$s_4 = a_1 + a_2 + a_3 + a_4$$

$$\ldots$$

To calculate the first four terms of the sequence of partial sums, we'll calculate each term of the series a_n individually, then add that term to any previous terms to get the term of the partial sum sequence.

$$n = 1 \qquad a_1 = \frac{1}{e^1} = \frac{1}{e} = 0.368 \qquad s_1 = a_1$$

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$$s_1 = 0.368$$

$n = 2$ $\quad a_2 = \frac{2}{e^2} = 0.271$ $\quad s_2 = a_1 + a_2$

$$s_2 = 0.368 + 0.271$$

$$s_2 = 0.639$$

$n = 3$ $\quad a_3 = \frac{3}{e^3} = 0.149$ $\quad s_3 = a_1 + a_2 + a_3$

$$s_3 = 0.368 + 0.271 + 0.149$$

$$s_3 = 0.788$$

$n = 4$ $\quad a_4 = \frac{4}{e^4} = 0.073$ $\quad s_4 = a_1 + a_2 + a_3 + a_4$

$$s_4 = 0.368 + 0.271 + 0.149 + 0.073$$

$$s_4 = 0.861$$

The first four terms of the sequence of partial sums, approximated to three decimal places, are

$$0.368,\ 0.639,\ 0.788,\ 0.861$$

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