Calculus 2 Workbook Solutions

Other approximation methods

krista king
MATH

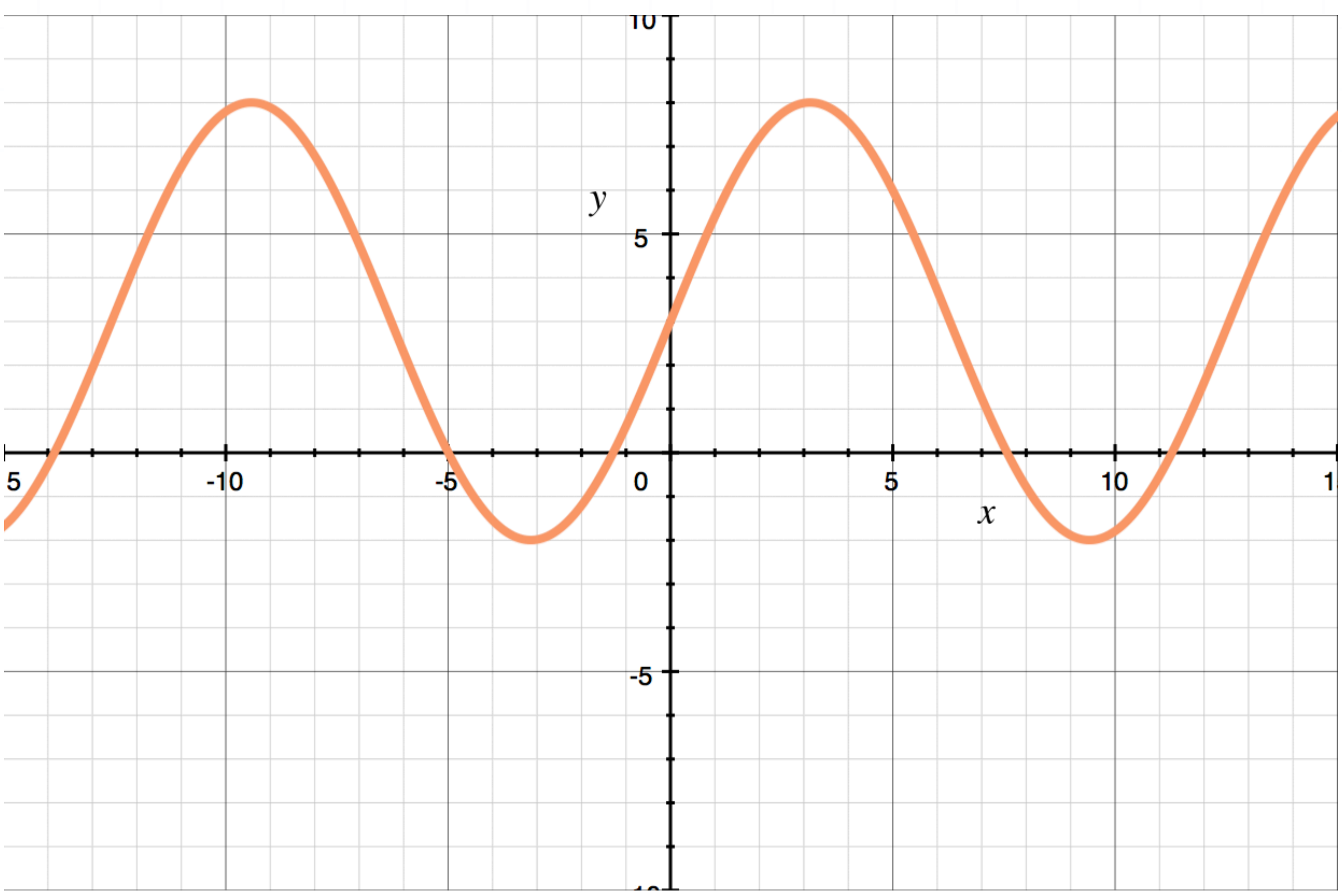
OVER AND UNDERESTIMATION

1. Use a Riemann sum to estimate the maximum and minimum area under the curve on $[0,\pi]$. Use rectangular approximation methods with 4 equal subintervals. Round the answer to 2 decimal places.

$$f(x) = 5\sin\frac{x}{2} + 3$$

Solution:

The graph of $f(x)$ is increasing on $[0,\pi]$.



So LRAM will underestimate the area and RRAM will overestimate the area. With 4 equal subintervals, calculate the value of $f(x)$ at

$$x = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \text{ and } \pi.$$

The function's values are

$$f(0) = 5\sin\left(\frac{0}{2}\right) + 3 = 3$$

$$f\left(\frac{\pi}{4}\right) = 5\sin\left(\frac{\pi}{8}\right) + 3 \approx 4.9134$$

$$f\left(\frac{\pi}{2}\right) = 5\sin\left(\frac{\pi}{4}\right) + 3 \approx 6.5355$$

$$f\left(\frac{3\pi}{4}\right) = 5\sin\left(\frac{3\pi}{8}\right) + 3 \approx 7.6194$$

$$f(\pi) = 5\sin\left(\frac{\pi}{2}\right) + 3 = 8$$

Then the LRAM and RRAM are

$$LRAM = \frac{\pi}{4}(3 + 4.9134 + 6.5355 + 7.6194) \approx 17.3324$$

$$RRAM = \frac{\pi}{4}(4.9134 + 6.5355 + 7.6194 + 8) \approx 21.2594$$

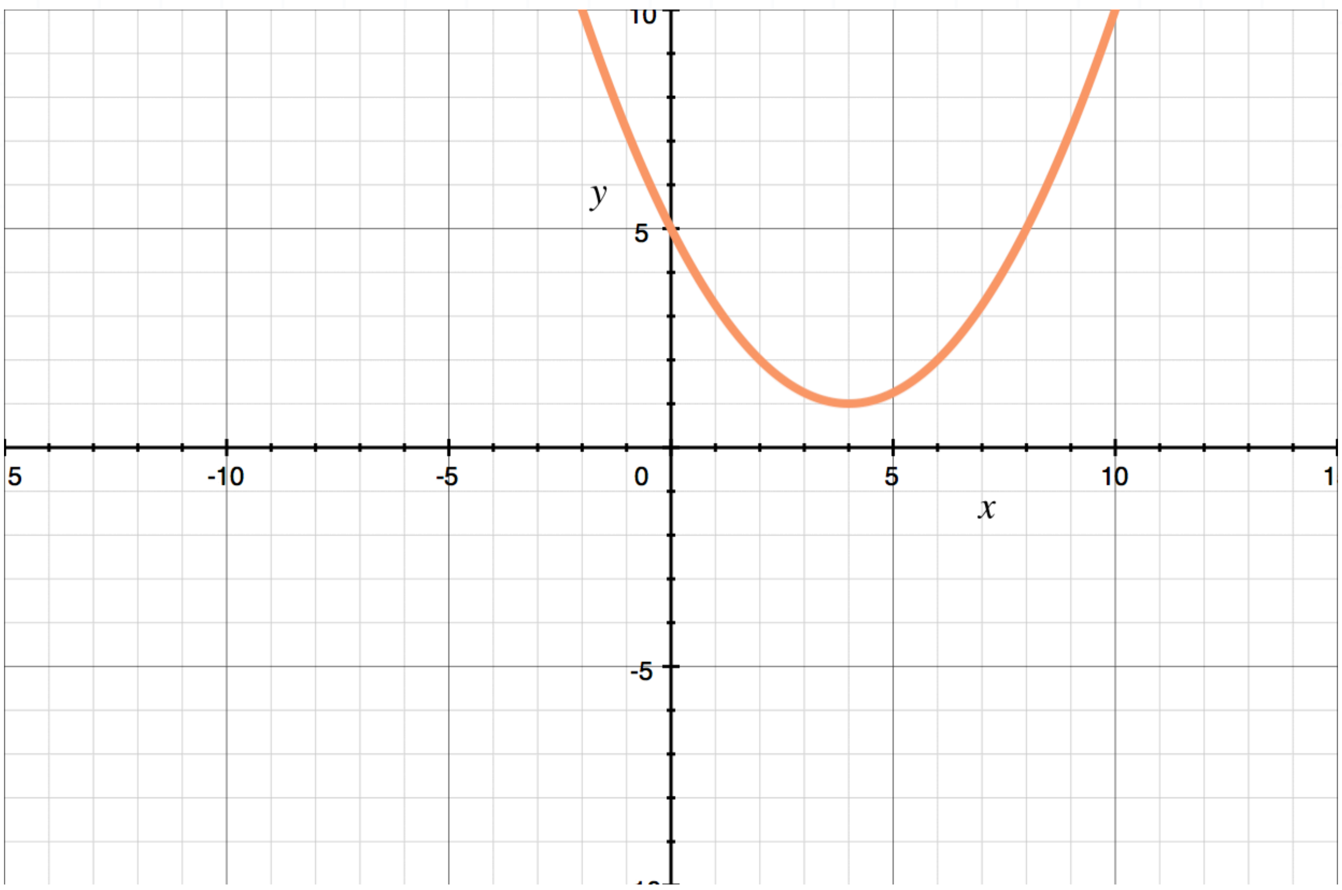
So the minimum area under the curve is 17.33 and the maximum area under the curve is 21.26.

2. Use a Riemann sum to estimate the maximum and minimum area under the curve on $[0,4]$. Use rectangular approximation methods with 4 equal subintervals.

$$g(x) = \frac{1}{4}(x-4)^2 + 1$$

Solution:

The graph of $g(x)$ is decreasing on $[0,4]$.



So RRAM will underestimate the area and LRAM will overestimate the area. With 4 equal subintervals, calculate the value of $g(x)$ at

$$x = 0,\ 1,\ 2,\ 3, \text{ and } 4$$

The function's values are

$$g(0)=\frac{1}{4}(0-4)^2+1=5$$

$$g(1)=\frac{1}{4}(1-4)^2+1=\frac{13}{4}$$

$$g(2)=\frac{1}{4}(2-4)^2+1=2$$

$$g(3)=\frac{1}{4}(3-4)^2+1=\frac{5}{4}$$

$$g(4)=\frac{1}{4}(3-4)^2+1=1$$

Then the LRAM and RRAM are

$$LRAM=1\left(5+\frac{13}{4}+2+\frac{5}{4}\right)=11.5$$

$$RRAM=1\left(\frac{13}{4}+2+\frac{5}{4}+1\right)=7.5$$

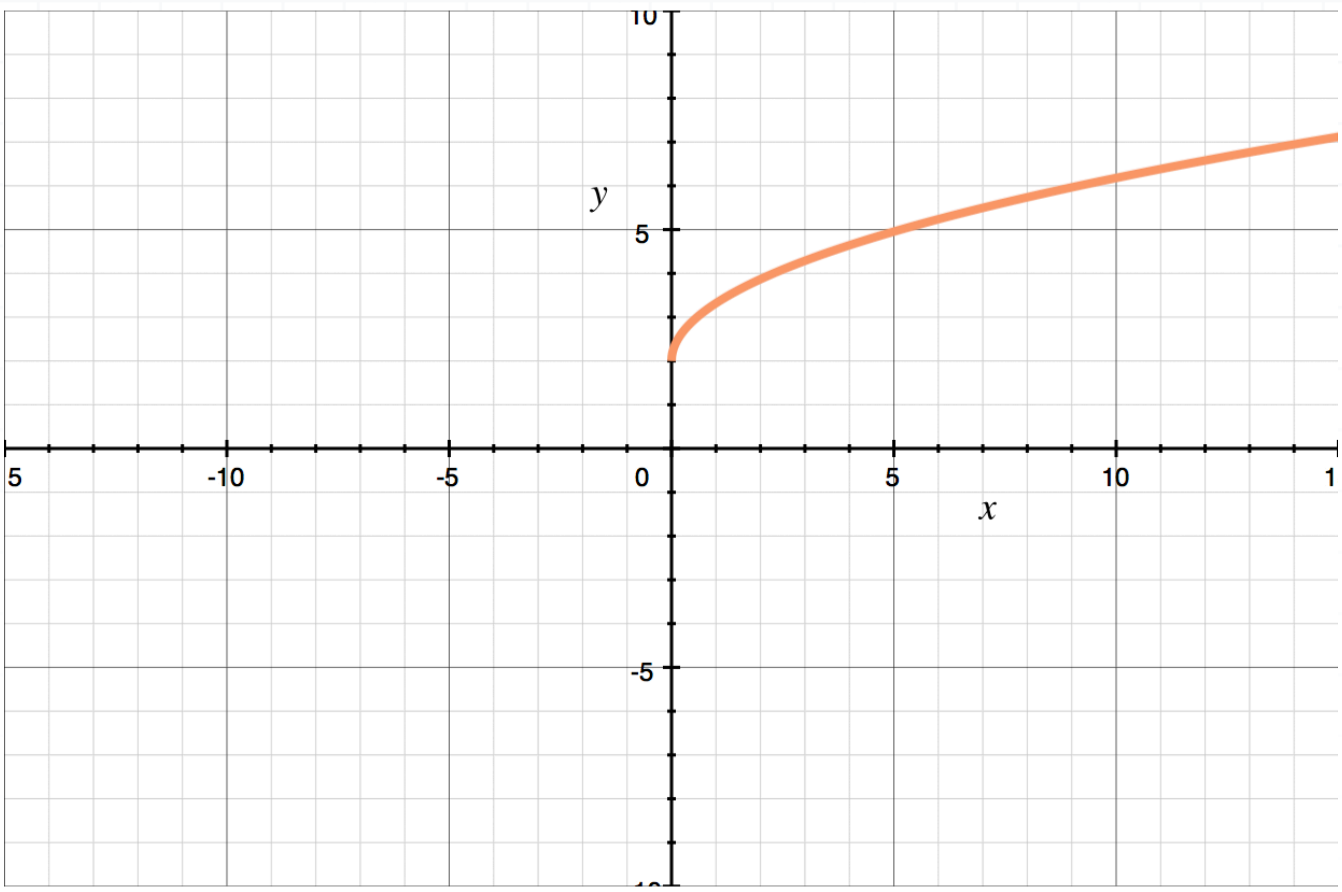
So the minimum area under the curve is 7.5 and the maximum area under the curve is 11.5.

■ 3. Use a Riemann sum to estimate the maximum and minimum area under the curve on $[0,9]$. Use rectangular approximation methods with 3 equal subintervals. Round the answer to 2 decimal places.

$$h(x)=\frac{1}{2}\sqrt{7x}+2$$

Solution:

The graph of $h(x)$ is increasing on $[0,9]$.



So LRAM will underestimate the area and RRAM will overestimate the area. With 3 equal subintervals, calculate the value of $h(x)$ at

$x=0$, 3, 6, and 9

The function's values are

$$h(0)=\frac{1}{2}\sqrt{7(0)}+2=2$$

$$h(3)=\frac{1}{2}\sqrt{7(3)}+2\approx 4.2913$$

$$h(6) = \frac{1}{2}\sqrt{7(6)} + 2 \approx 5.2404$$

$$h(9) = \frac{1}{2}\sqrt{7(9)} + 2 \approx 5.9686$$

Then the LRAM and RRAM are

$$LRAM = 3(2 + 4.2913 + 5.2404) \approx 34.5951$$

$$RRAM = 3(4.2913 + 5.2404 + 5.9686) \approx 46.5009$$

So the minimum area under the curve is 34.60 and the maximum area under the curve is 46.50.

LIMIT PROCESS TO FIND AREA ON [A,B]

■ 1. Use the limit process to find the area of the region between the graph of $f(x)$ and the x-axis on the interval $[3,7]$.

$$f(x) = x^2 + 2$$

Solution:

Find Δx.

$$\Delta x = \frac{b-a}{n} = \frac{7-3}{n} = \frac{4}{n}$$

With Δx, find x_i.

$$x_i = 3 + i\Delta x = 3 + i \cdot \frac{4}{n} = 3 + \frac{4i}{n}$$

Then you get

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} f\left(3 + \frac{4i}{n}\right)\frac{4}{n}$$

$$\lim_{n\to\infty} \frac{4}{n} \sum_{i=1}^{n} \left(3 + \frac{4i}{n}\right)^2 + 2$$

$$\lim_{n\to\infty} \frac{4}{n} \sum_{i=1}^{n} 9 + \frac{24i}{n} + \frac{16i^2}{n^2} + 2$$

$$\lim_{n\to\infty} \frac{4}{n} \sum_{i=1}^{n} 11 + \frac{24i}{n} + \frac{16i^2}{n^2}$$

Making substitutions for i and i^2 gives

$$\lim_{n\to\infty} \frac{4}{n} \left[11n + \frac{24}{n} \cdot \frac{n(n+1)}{2} + \frac{16}{n^2} \cdot \frac{n(n+1)(2n+1)}{6} \right]$$

$$\lim_{n\to\infty} \frac{4}{n} \left[11n + 12(n+1) + \frac{8(2n^2+3n+1)}{3n} \right]$$

$$\lim_{n\to\infty} \frac{4}{n} \left[11n + 12n + 12 + \frac{16n^2}{3n} + \frac{24n}{3n} + \frac{8}{3n} \right]$$

$$\lim_{n\to\infty} \frac{4}{n} \left[23n + 12 + \frac{16n}{3} + 8 + \frac{8}{3n} \right]$$

$$\lim_{n\to\infty} \left[92 + \frac{48}{n} + \frac{64n}{3n} + \frac{32}{n} + \frac{32}{3n^2} \right]$$

$$\lim_{n\to\infty} \left[92 + \frac{48}{n} + \frac{64}{3} + \frac{32}{n} + \frac{32}{3n^2} \right]$$

Evaluate the limit.

$$92 + 0 + \frac{64}{3} + 0 + 0$$

$$\frac{276}{3} + \frac{64}{3}$$

$$\frac{340}{3}$$

■ 2. Use the limit process to find the area of the region between the graph of $g(x)$ and the x-axis on the interval $[2,6]$.

$$f(x) = x^2 - x + 3$$

Solution:

Find Δx.

$$\Delta x = \frac{b-a}{n} = \frac{6-2}{n} = \frac{4}{n}$$

With Δx, find x_i.

$$x_i = 2 + i\Delta x = 2 + i \cdot \frac{4}{n} = 2 + \frac{4i}{n}$$

Then you get

$$\lim_{n\to\infty} \sum_{i=1}^{n} g(x_i)\Delta x$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} g\left(2 + \frac{4i}{n}\right)\frac{4}{n}$$

$$\lim_{n\to\infty} \frac{4}{n} \sum_{i=1}^{n} \left(2 + \frac{4i}{n}\right)^2 - \left(2 + \frac{4i}{n}\right) + 3$$

$$\lim_{n\to\infty} \frac{4}{n} \sum_{i=1}^{n} 4 + \frac{16i}{n} + \frac{16i^2}{n^2} - 2 - \frac{4i}{n} + 3$$

$$\lim_{n\to\infty} \frac{4}{n} \sum_{i=1}^{n} 5 + \frac{12i}{n} + \frac{16i^2}{n^2}$$

Making substitutions for i and i^2 gives

$$\lim_{n\to\infty} \frac{4}{n} \left[5n + \frac{12}{n} \cdot \frac{n(n+1)}{2} + \frac{16}{n^2} \cdot \frac{n(n+1)(2n+1)}{6}\right]$$

$$\lim_{n\to\infty} \frac{4}{n} \left[5n + 6(n+1) + \frac{8(2n^2+3n+1)}{3n}\right]$$

$$\lim_{n\to\infty} \frac{4}{n} \left[5n + 6n + 6 + \frac{16n^2}{3n} + \frac{24n}{3n} + \frac{8}{3n}\right]$$

$$\lim_{n\to\infty} \frac{4}{n} \left[11n + 6 + \frac{16n}{3} + 8 + \frac{8}{3n}\right]$$

$$\lim_{n\to\infty} \left[44 + \frac{24}{n} + \frac{64n}{3n} + \frac{32}{n} + \frac{32}{3n^2}\right]$$

$$\lim_{n\to\infty} \left[44 + \frac{24}{n} + \frac{64}{3} + \frac{32}{n} + \frac{32}{3n^2}\right]$$

Evaluate the limit.

$$44 + 0 + \frac{64}{3} + 0 + 0$$

$$\frac{132}{3} + \frac{64}{3}$$

$$\frac{196}{3}$$

■ 3. Use the limit process to find the area of the region between the graph of $h(x)$ and the x-axis on the interval $[2,5]$.

$$h(x) = x^2 - 3x + 7$$

Solution:

Find Δx.

$$\Delta x = \frac{b-a}{n} = \frac{5-2}{n} = \frac{3}{n}$$

With Δx, find x_i.

$$x_i = 2 + i\Delta x = 2 + i \cdot \frac{3}{n} = 2 + \frac{3i}{n}$$

Then you get

$$\lim_{n\to\infty} \sum_{i=1}^{n} h(x_i)\Delta x$$

$$\lim_{n\to\infty}\sum_{i=1}^{n} h\left(2+\frac{3i}{n}\right)\frac{3}{n}$$

$$\lim_{n\to\infty}\frac{3}{n}\sum_{i=1}^{n}\left(2+\frac{3i}{n}\right)^2-3\left(2+\frac{3i}{n}\right)+7$$

$$\lim_{n\to\infty}\frac{3}{n}\sum_{i=1}^{n}4+\frac{12i}{n}+\frac{9i^2}{n^2}-6-\frac{9i}{n}+7$$

$$\lim_{n\to\infty}\frac{3}{n}\sum_{i=1}^{n}5+\frac{3i}{n}+\frac{9i^2}{n^2}$$

Making substitutions for i and i^2 gives

$$\lim_{n\to\infty}\frac{3}{n}\left[5n+\frac{3}{n}\cdot\frac{n(n+1)}{2}+\frac{9}{n^2}\cdot\frac{n(n+1)(2n+1)}{6}\right]$$

$$\lim_{n\to\infty}\frac{3}{n}\left[5n+\frac{3}{2}(n+1)+\frac{3(2n^2+3n+1)}{2n}\right]$$

$$\lim_{n\to\infty}\frac{3}{n}\left[5n+\frac{3}{2}n+\frac{3}{2}+\frac{6n^2}{2n}+\frac{9n}{2n}+\frac{3}{2n}\right]$$

$$\lim_{n\to\infty}\frac{3}{n}\left[\frac{13}{2}n+\frac{3}{2}+3n+\frac{9}{2}+\frac{3}{2n}\right]$$

$$\lim_{n\to\infty}\left[\frac{39}{2}+\frac{9}{2n}+9+\frac{27}{2n}+\frac{9}{2n^2}\right]$$

Evaluate the limit.

$$\frac{39}{2}+0+9+0+0$$

$$\frac{39}{2}+\frac{18}{2}$$

$$\frac{57}{2}$$

LIMIT PROCESS TO FIND AREA ON [-A,A]

1. Use the limit process to find the area of the region between the graph of $f(x)$ and the x-axis on the interval $[-5,5]$.

$$f(x) = x^2 + 1$$

Solution:

Find Δx.

$$\Delta x = \frac{b-a}{n} = \frac{5-(-5)}{n} = \frac{10}{n}$$

With Δx, find x_i.

$$x_i = -5 + i\Delta x = -5 + i \cdot \frac{10}{n} = -5 + \frac{10i}{n}$$

Then you get

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} f\left(-5 + \frac{10i}{n}\right)\frac{10}{n}$$

$$\lim_{n\to\infty} \frac{10}{n} \sum_{i=1}^{n} \left(-5 + \frac{10i}{n}\right)^2 + 1$$

$$\lim_{n\to\infty}\frac{10}{n}\sum_{i=1}^{n}25-\frac{100i}{n}+\frac{100i^2}{n^2}+1$$

$$\lim_{n\to\infty}\frac{10}{n}\sum_{i=1}^{n}26-\frac{100i}{n}+\frac{100i^2}{n^2}$$

Making substitutions for i and i^2 gives

$$\lim_{n\to\infty}\frac{10}{n}\left[26n-\frac{100}{n}\cdot\frac{n(n+1)}{2}+\frac{100}{n^2}\cdot\frac{n(n+1)(2n+1)}{6}\right]$$

$$\lim_{n\to\infty}\frac{10}{n}\left[26n-50(n+1)+\frac{50(2n^2+3n+1)}{3n}\right]$$

$$\lim_{n\to\infty}\frac{10}{n}\left[26n-50n-50+\frac{100n^2}{3n}+\frac{150n}{3n}+\frac{50}{3n}\right]$$

$$\lim_{n\to\infty}\frac{10}{n}\left[-24n-50+\frac{100n}{3}+50+\frac{50}{3n}\right]$$

$$\lim_{n\to\infty}\frac{10}{n}\left[-24n+\frac{100n}{3}+\frac{50}{3n}\right]$$

$$\lim_{n\to\infty}\left[-240+\frac{1{,}000}{3}+\frac{500}{3n^2}\right]$$

Evaluate the limit.

$$-240+\frac{1{,}000}{3}+0$$

$$\frac{280}{3}$$

2. Use the limit process to find the area of the region between the graph of $g(x)$ and the x-axis on the interval $[-3,3]$.

$$g(x) = 3x^2 - 4$$

Solution:

Find Δx.

$$\Delta x = \frac{b-a}{n} = \frac{3-(-3)}{n} = \frac{6}{n}$$

With Δx, find x_i.

$$x_i = -3 + i\Delta x = -3 + i \cdot \frac{6}{n} = -3 + \frac{6i}{n}$$

Then you get

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} g\left(-3 + \frac{6i}{n}\right)\frac{6}{n}$$

$$\lim_{n\to\infty} \frac{6}{n}\sum_{i=1}^{n} 3\left(-3 + \frac{6i}{n}\right)^2 - 4$$

$$\lim_{n\to\infty} \frac{6}{n} \sum_{i=1}^{n} 3\left(9 - \frac{36i}{n} + \frac{36i^2}{n^2}\right) - 4$$

$$\lim_{n\to\infty} \frac{6}{n} \sum_{i=1}^{n} 23 - \frac{108i}{n} + \frac{108i^2}{n^2}$$

Making substitutions for i and i^2 gives

$$\lim_{n\to\infty} \frac{6}{n} \left[23n - \frac{108}{n} \cdot \frac{n(n+1)}{2} + \frac{108}{n^2} \cdot \frac{n(n+1)(2n+1)}{6}\right]$$

$$\lim_{n\to\infty} \frac{6}{n} \left[23n - 54(n+1) + \frac{18(2n^2+3n+1)}{n}\right]$$

$$\lim_{n\to\infty} \frac{6}{n} \left[23n - 54n - 54 + \frac{36n^2}{n} + \frac{54n}{n} + \frac{18}{n}\right]$$

$$\lim_{n\to\infty} \frac{6}{n} \left[-31n - 54 + 36n + 54 + \frac{18}{n}\right]$$

$$\lim_{n\to\infty} \frac{6}{n} \left[5n + \frac{18}{n}\right]$$

$$\lim_{n\to\infty} \left[30 + \frac{108}{n^2}\right]$$

Evaluate the limit.

$$30 + 0$$

$$30$$

3. Use the limit process to find the area of the region between the graph of $h(x)$ and the x-axis on the interval $[-1,1]$.

$$h(x) = 4x^2 - x + 1$$

Solution:

Find Δx.

$$\Delta x = \frac{b-a}{n} = \frac{1-(-1)}{n} = \frac{2}{n}$$

With Δx, find x_i.

$$x_i = -1 + i\Delta x = -1 + i \cdot \frac{2}{n} = -1 + \frac{2i}{n}$$

Then you get

$$\lim_{n\to\infty} \sum_{i=1}^{n} h(x_i)\Delta x$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} h\left(-1 + \frac{2i}{n}\right)\frac{2}{n}$$

$$\lim_{n\to\infty} \frac{2}{n} \sum_{i=1}^{n} 4\left(-1 + \frac{2i}{n}\right)^2 - \left(-1 + \frac{2i}{n}\right) + 1$$

$$\lim_{n\to\infty} \frac{2}{n} \sum_{i=1}^{n} 4\left(1 - \frac{4i}{n} + \frac{4i^2}{n^2}\right) - \left(-1 + \frac{2i}{n}\right) + 1$$

$$\lim_{n\to\infty} \frac{2}{n} \sum_{i=1}^{n} 4 - \frac{16i}{n} + \frac{16i^2}{n^2} + 1 - \frac{2i}{n} + 1$$

$$\lim_{n\to\infty} \frac{2}{n} \sum_{i=1}^{n} 6 - \frac{18i}{n} + \frac{16i^2}{n^2}$$

Making substitutions for i and i^2 gives

$$\lim_{n\to\infty} \frac{2}{n} \left[6n - \frac{18}{n} \cdot \frac{n(n+1)}{2} + \frac{16}{n^2} \cdot \frac{n(n+1)(2n+1)}{6}\right]$$

$$\lim_{n\to\infty} \frac{2}{n} \left[6n - 9(n+1) + \frac{8(2n^2+3n+1)}{3n}\right]$$

$$\lim_{n\to\infty} \frac{2}{n} \left[6n - 9n - 9 + \frac{16n^2}{3n} + \frac{24n}{3n} + \frac{8}{3n}\right]$$

$$\lim_{n\to\infty} \frac{2}{n} \left[-3n - 9 + \frac{16n}{3} + 8 + \frac{8}{3n}\right]$$

$$\lim_{n\to\infty} \frac{2}{n} \left[-3n - 1 + \frac{16n}{3} + \frac{8}{3n}\right]$$

$$\lim_{n\to\infty} \left[-6 - \frac{2}{n} + \frac{32}{3} + \frac{16}{3n^2}\right]$$

Evaluate the limit.

$$-6 - 0 + \frac{32}{3} + 0$$

$$-\frac{18}{3} + \frac{32}{3}$$

$$\frac{14}{3}$$

TRAPEZOIDAL RULE

1. Using $n=6$ and the Trapezoidal rule, approximate the value of the integral. Round the answer to 2 decimal places.

$$\int_4^{16} 2\sqrt[3]{x}+3\ dx$$

Solution:

Evaluating the integral with $n=6$ means the interval of $[4,16]$ is split into 6 subintervals.

$[4,6]$, $[6,8]$, $[8,10]$, $[10,12]$, $[12,14]$, and $[14,16]$

Evaluate the integrand at the endpoints of each subinterval.

$$f(4)=2\sqrt[3]{4}+3\approx 6.1748$$

$$f(6)=2\sqrt[3]{6}+3\approx 6.6342$$

$$f(8)=2\sqrt[3]{8}+3=7$$

$$f(10)=2\sqrt[3]{10}+3\approx 7.3089$$

$$f(12)=2\sqrt[3]{12}+3\approx 7.5789$$

$$f(14)=2\sqrt[3]{14}+3\approx 7.8203$$

$$f(16)=2\sqrt[3]{16}+3\approx 8.0397$$

Use these values in the Trapezoidal rule with $\Delta x = 2$.

$$\frac{2}{2}\left[6.1748 + 2(6.6342) + 2(7) + 2(7.3089) + 2(7.5789) + 2(7.8203) + 8.0397\right]$$

$$6.1748 + 13.2684 + 14 + 14.6178 + 15.1578 + 15.6406 + 8.0397$$

$$86.8991$$

This answer rounds to 86.90.

■ 2. Using $n = 6$ and the Trapezoidal rule, approximate the value of the integral.

$$\int_0^6 \frac{1}{4}x^4 - \frac{1}{2}x^3 + 2x^2 - 5x + 8\ dx$$

Solution:

Evaluating the integral with $n = 6$ means the interval of $[0,6]$ is split into 6 subintervals.

$[0,1]$, $[1,2]$, $[2,3]$, $[3,4]$, $[4,5]$, and $[5,6]$

Evaluate the integrand at the endpoints of each subinterval.

$$f(0) = \frac{1}{4}(0)^4 - \frac{1}{2}(0)^3 + 2(0)^2 - 5(0) + 8 = 8$$

$$f(1)=\frac{1}{4}(1)^4-\frac{1}{2}(1)^3+2(1)^2-5(1)+8=\frac{19}{4}$$

$$f(2)=\frac{1}{4}(2)^4-\frac{1}{2}(2)^3+2(2)^2-5(2)+8=6$$

$$f(3)=\frac{1}{4}(3)^4-\frac{1}{2}(3)^3+2(3)^2-5(3)+8=\frac{71}{4}$$

$$f(4)=\frac{1}{4}(4)^4-\frac{1}{2}(4)^3+2(4)^2-5(4)+8=52$$

$$f(5)=\frac{1}{4}(5)^4-\frac{1}{2}(5)^3+2(5)^2-5(5)+8=\frac{507}{4}$$

$$f(6)=\frac{1}{4}(6)^4-\frac{1}{2}(6)^3+2(6)^2-5(6)+8=266$$

Use these values in the Trapezoidal rule with $\Delta x=1$.

$$\frac{1}{2}\left[8+2\left(\frac{19}{4}\right)+2\left(6\right)+2\left(\frac{71}{4}\right)+2\left(52\right)+2\left(\frac{507}{4}\right)+266\right]$$

$$\frac{1}{2}\left(8+\frac{19}{2}+12+\frac{71}{2}+104+\frac{507}{2}+266\right)$$

$$\frac{1}{2}\left(\frac{597}{2}+\frac{780}{2}\right)$$

$$\frac{1,377}{4}$$

3. Using $n = 4$ and the Trapezoidal rule, approximate the value of the integral.

$$\int_0^8 \frac{1}{2}x^2 - 3x + 6\ dx$$

Solution:

Evaluating the integral with $n = 4$ means the interval of $[0,8]$ is split into 4 subintervals.

$[0,2]$, $[2,4]$, $[4,6]$, and $[6,8]$

Evaluate the integrand at the endpoints of each subinterval.

$$f(0) = \frac{1}{2}(0)^2 - 3(0) + 6 = 6$$

$$f(2) = \frac{1}{2}(2)^2 - 3(2) + 6 = 2$$

$$f(4) = \frac{1}{2}(4)^2 - 3(4) + 6 = 2$$

$$f(6) = \frac{1}{2}(6)^2 - 3(6) + 6 = 6$$

$$f(8) = \frac{1}{2}(8)^2 - 3(8) + 6 = 14$$

Use these values in the Trapezoidal rule with $\Delta x = 2$.

$$\frac{2}{2}\left[6+2(2)+2(2)+2(6)+14\right]$$

$$6+4+4+12+14$$

$$40$$

4. Using $n=4$ and the Trapezoidal rule, approximate the value of the integral.

$$\int_0^{16} \frac{1}{16}x^4-\frac{1}{2}x^3-x^2-x+1\ dx$$

Solution:

Evaluating the integral with $n=4$ means the interval of $[0,16]$ is split into 4 subintervals.

$[0,4]$, $[4,8]$, $[8,12]$, and $[12,16]$

Evaluate the integrand at the endpoints of each subinterval.

$$f(0)=\frac{1}{16}(0)^4-\frac{1}{2}(0)^3-(0)^2-(0)+1=1$$

$$f(4)=\frac{1}{16}(4)^4-\frac{1}{2}(4)^3-(4)^2-(4)+1=-35$$

$$f(8)=\frac{1}{16}(8)^4-\frac{1}{2}(8)^3-(8)^2-(8)+1=-71$$

$$f(12)=\frac{1}{16}(12)^4-\frac{1}{2}(12)^3-(12)^2-(12)+1=277$$

$$f(16)=\frac{1}{16}(16)^4-\frac{1}{2}(16)^3-(16)^2-(16)+1=1{,}777$$

Use these values in the Trapezoidal rule with $\Delta x=4$.

$$\frac{4}{2}\left[1+2(-35)+2(-71)+2(277)+1{,}777\right]$$

$$2(1-70-142+554+1{,}777)$$

$$4{,}240$$

SIMPSON'S RULE

1. Use Simpson's Rule with $n = 6$ to approximate the value of the integral. Round the answer to 2 decimal places.

$$\int_2^8 6\sqrt{3x} + 5\ dx$$

Solution:

Since $n = 6$ and the interval is $[2,8]$, that means $\Delta x = 1$. So

$$x_1 = 2,\ x_2 = 3,\ x_3 = 4,\ x_3 = 5,\ x_4 = 6,\ x_5 = 7,\text{ and } x_6 = 8$$

Evaluate the integrand at each of these values.

$$f(2) = 6\sqrt{3(2)} + 5 \approx 19.6969$$

$$f(3) = 6\sqrt{3(3)} + 5 = 23$$

$$f(4) = 6\sqrt{3(4)} + 5 \approx 25.7846$$

$$f(5) = 6\sqrt{3(5)} + 5 \approx 28.2379$$

$$f(6) = 6\sqrt{3(6)} + 5 \approx 30.4558$$

$$f(7) = 6\sqrt{3(7)} + 5 \approx 32.4955$$

$$f(8) = 6\sqrt{3(8)} + 5 \approx 34.3939$$

Use these values in Simpson's rule with $\Delta x = 1$.

$$\frac{1}{3}\left[19.6969 + 4(23) + 2(25.7846) + 4(28.2379) + 2(30.4558) + 4(32.4955) + 34.3939\right]$$

$$\frac{1}{3}(19.6969 + 92 + 51.5692 + 112.9516 + 60.9116 + 129.9820 + 34.3939)$$

$$167.1684$$

This answer rounds to 167.17.

■ 2. Use Simpson's Rule with $n = 8$ to approximate the value of the integral. Round the answer to 2 decimal places.

$$\int_4^{28} 120(0.95)^x \ dx$$

Solution:

Since $n = 6$ and the interval is $[4,25]$, that means $\Delta x = 3$. So

$$x_1 = 4,\ x_2 = 7,\ x_3 = 10,\ x_4 = 13,\ x_5 = 16,\ x_6 = 19,\ x_7 = 22,\ x_8 = 25, \text{ and } x_9 = 28$$

Evaluate the integrand at each of these values.

$$f(4) = 120(0.95)^4 \approx 97.7408$$

$$f(7) = 120(0.95)^7 \approx 83.8005$$

$$f(10)=120(0.95)^{10}\approx 71.8484$$

$$f(13)=120(0.95)^{13}\approx 61.6010$$

$$f(16)=120(0.95)^{16}\approx 52.8152$$

$$f(19)=120(0.95)^{19}\approx 45.2824$$

$$f(22)=120(0.95)^{22}\approx 38.8240$$

$$f(25)=120(0.95)^{25}\approx 33.2867$$

$$f(28)=120(0.95)^{28}\approx 28.5392$$

Use these values in Simpson's rule with $\Delta x=3$.

$$\frac{3}{3}\Big[97.7408+4(83.8005)+2(71.8484)+4(61.6010)+2(52.8152)$$

$$+4(45.2824)+2(38.8240)+4(33.2867)+28.5392\Big]$$

$$97.7408+335.2020+142.6968+246.4040+105.6304$$

$$+181.1296+77.6480+133.1468+28.5392$$

$$1{,}349.1376$$

This answer rounds to $1{,}349.14$.

■ 3. Use Simpson's Rule with $n=4$ to approximate the value of the integral. Round the answer to 2 decimal places.

$$\int_{5}^{7} 3\ln(x+5) - 2\ dx$$

Solution:

Since $n = 4$ and the interval is $[5,7]$, that means $\Delta x = 0.5$. So

$x_1 = 5$, $x_2 = 5.5$, $x_3 = 6$, $x_4 = 6.5$, and $x_5 = 7$

Evaluate the integrand at each of these values.

$$f(5) = 3\ln(5+5) - 2 \approx 4.9078$$

$$f(5.5) = 3\ln(5.5+5) - 2 \approx 5.0541$$

$$f(6) = 3\ln(6+5) - 2 \approx 5.1937$$

$$f(6.5) = 3\ln(6.5+5) - 2 \approx 5.3270$$

$$f(7) = 3\ln(7+5) - 2 \approx 5.4547$$

Use these values in Simpson's rule with $\Delta x = 0.5$.

$$\frac{0.5}{3}\left[4.9078 + 4(5.0541) + 2(5.1937) + 4(5.3270) + 5.4547\right]$$

$$\frac{0.5}{3}\left(4.9078 + 20.2164 + 10.3874 + 21.3080 + 5.4547\right)$$

$$10.37905$$

This answer rounds to 10.38.

4. Use Simpson's Rule with $n=4$ to approximate the value of the integral.

$$\int_{-3}^{9} x^2+3x+2\ dx$$

Solution:

Since $n=4$ and the interval is $[-3,9]$, that means $\Delta x=3$. So

$$x_1=-3,\ x_2=0,\ x_3=3,\ x_4=6,\text{ and } x_5=9$$

Evaluate the integrand at each of these values.

$$f(-3)=(-3)^2+3(-3)+2=2$$

$$f(0)=(0)^2+3(0)+2=2$$

$$f(3)=(3)^2+3(3)+2=20$$

$$f(6)=(6)^2+3(6)+2=56$$

$$f(9)=(9)^2+3(9)+2=110$$

Use these values in Simpson's rule with $\Delta x=3$.

$$\frac{3}{3}\left[2+4(2)+2(20)+4(56)+110\right]$$

$$2+8+40+224+110$$

384

5. Use Simpson's Rule with $n = 6$ to approximate the value of the integral. Round the answer to 2 decimal places.

$$\int_{0.4}^{1.6} \frac{1}{3}x^3 - x^2 + 5x + 4 \ dx$$

Solution:

Since $n = 6$ and the interval is $[0.4,1.6]$, that means $\Delta x = 0.2$. So

$x_1 = 0.4$, $x_2 = 0.6$, $x_3 = 0.8$, $x_4 = 1.0$, $x_5 = 1.2$, $x_6 = 1.4$, and $x_7 = 1.6$

Evaluate the integrand at each of these values.

$$f(0.4) = \frac{1}{3}(0.4)^3 - (0.4)^2 + 5(0.4) + 4 \approx 5.8613$$

$$f(0.6) = \frac{1}{3}(0.6)^3 - (0.6)^2 + 5(0.6) + 4 \approx 6.712$$

$$f(0.8) = \frac{1}{3}(0.8)^3 - (0.8)^2 + 5(0.8) + 4 \approx 7.5307$$

$$f(1) = \frac{1}{3}(1)^3 - (1)^2 + 5(1) + 4 \approx 8.3333$$

$$f(1.2) = \frac{1}{3}(1.2)^3 - (1.2)^2 + 5(1.2) + 4 \approx 9.136$$

$$f(1.4) = \frac{1}{3}(1.4)^3 - (1.4)^2 + 5(1.4) + 4 \approx 9.9547$$

$$f(1.6) = \frac{1}{3}(1.6)^3 - (1.6)^2 + 5(1.6) + 4 \approx 10.8053$$

Use these values in Simpson's rule with $\Delta x = 0.2$.

$$\frac{0.2}{3}\Big[5.8613 + 4(6.712) + 2(7.5307) + 4(8.3333)$$

$$+2(9.136) + 4(9.9547) + 10.8053\Big]$$

$$\frac{0.2}{3}(5.8613 + 26.848 + 15.0614 + 33.3332 + 18.272 + 39.8188 + 10.8053)$$

$$10$$

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