Topic: Increasing, decreasing, not monotonic

Question: Say whether the sequence is increasing, decreasing, or not monotonic.

$$a_n = \frac{3}{n^2 + 1}$$

Answer choices:

A The sequence is increasing

B The sequence is not monotonic

C The sequence is decreasing and monotonic

D The sequence is increasing and not monotonic

Solution: C

Monotonic sequences are those which head in the same direction throughout the entire sequence. In other words, they increase everywhere, or they decrease everywhere. If a sequence increases in some places and decreases in other places, then it's not monotonic.

To get an idea about what's happening with our sequence, we'll calculate its first few terms.

$$n = 1 \qquad a_1 = \frac{3}{(1)^2 + 1} = \frac{3}{2}$$

$$n = 2 \qquad a_2 = \frac{3}{(2)^2 + 1} = \frac{3}{5}$$

$$n = 3 \qquad a_3 = \frac{3}{(3)^2 + 1} = \frac{3}{10}$$

$$n = 4 \qquad a_4 = \frac{3}{(4)^2 + 1} = \frac{3}{17}$$

If we look at the first few terms, we can see that the terms of the sequence are getting smaller as n gets larger, which means the sequence is decreasing, and therefore it's also monotonic.

Topic: Increasing, decreasing, not monotonic

Question: Say whether the sequence is increasing, decreasing, or not monotonic.

$$a_n = \frac{2n^2 - 1}{n + 4}$$

Answer choices:

A The sequence is decreasing

B The sequence is not monotonic

C The sequence is decreasing and not monotonic

D The sequence is increasing and monotonic

Solution: D

Monotonic sequences are those which head in the same direction throughout the entire sequence. In other words, they increase everywhere, or they decrease everywhere. If a sequence increases in some places and decreases in other places, then it's not monotonic.

To get an idea about what's happening with our sequence, we'll calculate its first few terms.

$$n = 1 \qquad a_1 = \frac{2(1)^2 - 1}{1 + 4} = \frac{1}{5}$$

$$n = 2 \qquad a_2 = \frac{2(2)^2 - 1}{2 + 4} = \frac{7}{6}$$

$$n = 3 \qquad a_3 = \frac{2(3)^2 - 1}{3 + 4} = \frac{17}{7}$$

$$n = 4 \qquad a_4 = \frac{2(4)^2 - 1}{4 + 4} = \frac{31}{8}$$

If we look at the first few terms, we can see that the terms of the sequence are getting larger as n gets larger, which means the sequence is increasing, and therefore it's also monotonic.

Topic: Increasing, decreasing, not monotonic

Question: Say whether the sequence is increasing, decreasing, or not monotonic.

$$a_n = (-1)^{n+1}\frac{2}{n+2}$$

Answer choices:

A The sequence is decreasing

B The sequence is not monotonic

C The sequence is decreasing and not monotonic

D The sequence is increasing and monotonic

Solution: B

Monotonic sequences are those which head in the same direction throughout the entire sequence. In other words, they increase everywhere, or they decrease everywhere. If a sequence increases in some places and decreases in other places, then it's not monotonic.

To get an idea about what's happening with our sequence, we'll calculate its first few terms.

$$n = 1 \qquad a_1 = (-1)^{1+1}\frac{2}{1+2} = \frac{2}{3}$$

$$n = 2 \qquad a_2 = (-1)^{2+1}\frac{2}{2+2} = -\frac{2}{4} = -\frac{1}{2}$$

$$n = 3 \qquad a_3 = (-1)^{3+1}\frac{2}{3+2} = \frac{2}{5}$$

$$n = 4 \qquad a_4 = (-1)^{4+1}\frac{2}{4+2} = -\frac{2}{6} = -\frac{1}{3}$$

If we look at the first few terms, we can see that the terms of the sequence are neither consistently increasing or consistently decreasing as n gets larger, which means the sequence is not monotonic.