Topic: Values for which the series converges

Question: Find the values for which the geometric series converges.

$$\sum_{n=1}^{\infty} 3x^n$$

Answer choices:

A $-1 < x < 1$

B $-\frac{1}{3} < x < \frac{1}{3}$

C $-3 < x < 3$

D $-\sqrt{3} < x < \sqrt{3}$

Solution: A

From the expanded form of a geometric series,

$$\sum_{n=1}^{\infty} ar^{n-1} = a\left\{1 + r + r^2 + r^3 + \dots\right\}$$

we can use the value of r and the geometric series test for convergence to determine the interval over which the geometric series converges.

The geometric series test says that

if $|r| < 1$ then the series converges

if $|r| \geq 1$ then the series diverges

Therefore, in order to find the values for which the geometric series converges, we just expand the series to identify the value of r and then use it in the geometric series test.

We'll start by expanding the series, calculating its first few terms.

$n = 1 \qquad a_1 = 3x^1 = 3x$

$n = 2 \qquad a_2 = 3x^2$

$n = 3 \qquad a_3 = 3x^3$

$n = 4 \qquad a_4 = 3x^4$

Writing these terms into our expanded series, we get

$$\sum_{n=1}^{\infty} 3x^n = 3x + 3x^2 + 3x^3 + 3x^4 + \dots$$

The first term in a geometric series is always 1, which means this series is only geometric if we can factor out $3x$.

$$\sum_{n=1}^{\infty} 3x^n = 3x\left(1 + x + x^2 + x^3 + \dots\right)$$

Comparing this to the expanded form of the general geometric series, we can see that

$$a = 3x$$

$$r = x$$

Since the geometric series test tells us that the series converges when $|r| < 1$, we plug the value we found for r into this inequality, and we get

$$|x| < 1$$

$$-1 < x < 1$$

The series converges on the interval $-1 < x < 1$.

Topic: Values for which the series converges

Question: Find the values for which the geometric series converges.

$$\sum_{n=1}^{\infty} \frac{x^{n-1}}{2^n}$$

Answer choices:

A $-1 < x < 1$

B $-\frac{1}{2} < x < \frac{1}{2}$

C $-2 < x < 2$

D $-\sqrt{2} < x < \sqrt{2}$

Solution: C

From the expanded form of a geometric series,

$$\sum_{n=1}^{\infty} ar^{n-1} = a\left\{1 + r + r^2 + r^3 + \dots\right\}$$

we can use the value of r and the geometric series test for convergence to determine the interval over which the geometric series converges.

The geometric series test says that

if $|r| < 1$ then the series converges

if $|r| \geq 1$ then the series diverges

Therefore, in order to find the values for which the geometric series converges, we just expand the series to identify the value of r and then use it in the geometric series test.

We'll start by expanding the series, calculating its first few terms.

$n = 1$ $\qquad a_1 = \dfrac{x^{1-1}}{2^1} = \dfrac{1}{2}$

$n = 2$ $\qquad a_2 = \dfrac{x^{2-1}}{2^2} = \dfrac{x}{4}$

$n = 3$ $\qquad a_3 = \dfrac{x^{3-1}}{2^3} = \dfrac{x^2}{8}$

$n = 4$ $\qquad a_4 = \dfrac{x^{4-1}}{2^4} = \dfrac{x^3}{16}$

Writing these terms into our expanded series, we get

$$\sum_{n=1}^{\infty} \frac{x^{n-1}}{2^n} = \frac{1}{2} + \frac{x}{4} + \frac{x^2}{8} + \frac{x^3}{16} + \dots$$

The first term in a geometric series is always 1, which means this series is only geometric if we can factor out $1/2$.

$$\sum_{n=1}^{\infty} \frac{x^{n-1}}{2^n} = \frac{1}{2}\left(1 + \frac{x}{2} + \frac{x^2}{4} + \frac{x^3}{8} + \dots\right)$$

Comparing this to the expanded form of the general geometric series, we can see that

$$a = \frac{1}{2}$$

$$r = \frac{x}{2}$$

Since the geometric series test tells us that the series converges when $|r| < 1$, we plug the value we found for r into this inequality, and we get

$$\left|\frac{x}{2}\right| < 1$$

$$-1 < \frac{x}{2} < 1$$

$$-2 < x < 2$$

The series converges on the interval $-2 < x < 2$.

Topic: Values for which the series converges

Question: Find the values for which the geometric series converges.

$$\sum_{n=1}^{\infty} \frac{(5x)^{n-1}}{3^n}$$

Answer choices:

A $\quad -\frac{3}{25} < x < \frac{3}{25}$

B $\quad -\frac{5}{3} < x < \frac{5}{3}$

C $\quad -\frac{25}{3} < x < \frac{25}{3}$

D $\quad -\frac{3}{5} < x < \frac{3}{5}$

Solution: D

From the expanded form of a geometric series,

$$\sum_{n=1}^{\infty} ar^{n-1} = a\left\{1 + r + r^2 + r^3 + \dots\right\}$$

we can use the value of r and the geometric series test for convergence to determine the interval over which the geometric series converges.

The geometric series test says that

if $|r| < 1$ then the series converges

if $|r| \geq 1$ then the series diverges

Therefore, in order to find the values for which the geometric series converges, we just expand the series to identify the value of r and then use it in the geometric series test.

We'll start by expanding the series, calculating its first few terms.

$$n = 1 \qquad a_1 = \frac{(5x)^{1-1}}{3^1} = \frac{1}{3}$$

$$n = 2 \qquad a_2 = \frac{(5x)^{2-1}}{3^2} = \frac{5x}{9}$$

$$n = 3 \qquad a_3 = \frac{(5x)^{3-1}}{3^3} = \frac{25x^2}{27}$$

$$n = 4 \qquad a_4 = \frac{(5x)^{4-1}}{3^4} = \frac{125x^3}{81}$$

Writing these terms into our expanded series, we get

$$\sum_{n=1}^{\infty} \frac{(5x)^{n-1}}{3^n} = \frac{1}{3} + \frac{5x}{9} + \frac{25x^2}{27} + \frac{125x^3}{81} + \dots$$

The first term in a geometric series is always 1, which means this series is only geometric if we can factor out $1/3$.

$$\sum_{n=1}^{\infty} \frac{(5x)^{n-1}}{3^n} = \frac{1}{3}\left(1 + \frac{5x}{3} + \frac{25x^2}{9} + \frac{125x^3}{27} + \dots\right)$$

Comparing this to the expanded form of the general geometric series, we can see that

$$a = \frac{1}{3}$$

$$r = \frac{5x}{3}$$

Since the geometric series test tells us that the series converges when $|r| < 1$, we plug the value we found for r into this inequality, and we get

$$\left|\frac{5x}{3}\right| < 1$$

$$-1 < \frac{5x}{3} < 1$$

$$-3 < 5x < 3$$

$$-\frac{3}{5} < x < \frac{3}{5}$$

The series converges on the interval $-\frac{3}{5} < x < \frac{3}{5}$.