Topic: Surface area of revolution

Question: Find the surface area generated by revolving the curve around the given axis over the given interval.

$$y = 3x + 1$$

on the interval $0 \le x \le 1$

about the x-axis

Answer choices:

A 50π

B $5\pi\sqrt{10}$

C $2\pi\sqrt{10}$

D $\pi\sqrt{50}$

Solution: B

Because our curve is defined in the form $y = f(x)$ and our limits of integration are defined as $x = 0$ and $x = 1$, the formula we use to find the surface area of revolution is

$$A = \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \ dx$$

The derivative of our function is

$$\frac{dy}{dx} = 3$$

Plugging the derivative and our limits of integration into the surface area of revolution formula gives

$$A = \int_0^1 2\pi(3x + 1)\sqrt{1 + (3)^2} \ dx$$

$$A = 2\pi\sqrt{10} \int_0^1 3x + 1 \ dx$$

$$A = 2\pi\sqrt{10} \left(\frac{3}{2}x^2 + x\right)\Bigg|_0^1$$

$$A = 2\pi\sqrt{10} \left[\left(\frac{3}{2}(1)^2 + (1)\right) - \left(\frac{3}{2}(0)^2 + (0)\right)\right]$$

$$A = 2\pi\sqrt{10} \left(\frac{3}{2} + 1\right)$$

$$A = 2\pi\sqrt{10}\left(\frac{5}{2}\right)$$

$$A = 5\pi\sqrt{10}$$

Topic: Surface area of revolution

Question: Find the surface area generated by revolving the curve around the given axis over the given interval.

$$y = \sqrt{25 - x^2}$$

on the interval $-2 \leq x \leq 3$

about the x-axis

Answer choices:

A 50

B 25

C 25π

D 50π

Solution: D

Because our curve is defined in the form $y = f(x)$ and our limits of integration are defined as $x = -2$ and $x = 3$, the formula we use to find surface area of revolution is

$$A = \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\ dx$$

The derivative of our function is

$$\frac{dy}{dx} = \frac{-x}{\sqrt{25 - x^2}}$$

Plugging the derivative, and our limits of integration into the surface area of revolution formula gives

$$A = \int_{-2}^{3} 2\pi\sqrt{25 - x^2}\sqrt{1 + \left(\frac{-x}{\sqrt{25 - x^2}}\right)^2}\ dx$$

$$A = 2\pi \int_{-2}^{3} \sqrt{25 - x^2}\sqrt{1 + \frac{x^2}{25 - x^2}}\ dx$$

Find a common denominator and combine fractions.

$$A = 2\pi \int_{-2}^{3} \sqrt{25 - x^2}\sqrt{\frac{25 - x^2}{25 - x^2} + \frac{x^2}{25 - x^2}}\ dx$$

$$A = 2\pi \int_{-2}^{3} \sqrt{25 - x^2}\sqrt{\frac{25}{25 - x^2}}\ dx$$

$$A = 2\pi \int_{-2}^{3} \sqrt{25 - x^2} \cdot \frac{\sqrt{25}}{\sqrt{25 - x^2}} \ dx$$

$$A = 10\pi \int_{-2}^{3} \ dx$$

$$A = 10\pi x \Big|_{-2}^{3}$$

$$A = 10\pi(3) - 10\pi(-2)$$

$$A = 30\pi + 20\pi$$

$$A = 50\pi$$

Topic: Surface area of revolution

Question: Find the surface area generated by revolving the curve around the given axis over the given interval.

$$y = \frac{1}{2}x^2 - 1$$

on the interval $0 \le x \le 2\sqrt{2}$

about the y-axis

Answer choices:

A $\dfrac{52\pi}{3}$

B 52π

C $\dfrac{54\pi}{3}$

D 26π

Solution: A

Because our curve is defined in the form $y=f(x)$ and our limits of integration are defined as $x=0$ and $x=2\sqrt{2}$, the formula we use to find surface area of revolution is

$$A=\int_{a}^{b} 2\pi x\sqrt{1+\left(\frac{dy}{dx}\right)^2}\ dx$$

The derivative of our function is

$$\frac{dy}{dx}=x$$

Plugging the derivative and our limits of integration into the surface area of revolution formula gives us

$$A=\int_{0}^{2\sqrt{2}} 2\pi x\sqrt{1+x^2}\ dx$$

We'll use u-substitution.

$$u=1+x^2$$

$$du=2x\ dx$$

$$dx=\frac{du}{2x}$$

Making the substitution into the integral gives

$$A=\int_{x=0}^{x=2\sqrt{2}} 2\pi x\sqrt{u}\ \frac{du}{2x}$$

$$A = \pi \int_{x=0}^{x=2\sqrt{2}} \sqrt{u}\ du$$

Integrate.

$$A = \pi \left(\frac{2}{3} u^{\frac{3}{2}} \right) \Bigg|_{x=0}^{x=2\sqrt{2}}$$

Back substitute to get the problem back in terms of x, then evaluate over the interval.

$$A = \pi \left[\frac{2}{3} \left(1 + x^2\right)^{\frac{3}{2}} \right] \Bigg|_0^{2\sqrt{2}}$$

$$A = \pi \left[\frac{2}{3} \left(1 \left(2\sqrt{2} \right)^2 \right)^{\frac{3}{2}} - \frac{2}{3} \left(1 + (0)^2\right)^{\frac{3}{2}} \right]$$

$$A = \pi \left[\frac{2}{3} (1 + 8)^{\frac{3}{2}} - \frac{2}{3} (1)^{\frac{3}{2}} \right]$$

$$A = \pi \left(\frac{54}{3} - \frac{2}{3} \right)$$

$$A = \frac{52\pi}{3}$$