Topic: Tangent line to the parametric curve

Question: Find the equation of the tangent line to the parametric curve.

$$x = t^2$$

$$y = t^3$$

$$t = 2$$

Answer choices:

A $\quad y = 3x + 4$

B $\quad y = 3x - 4$

C $\quad y = -3x - 4$

D $\quad y = -3x + 4$

Solution: B

To define the tangent line, we use the point-slope formula for the equation of the line.

$$y - y_1 = m(x - x_1)$$

where m is the slope and (x_1, y_1) is the point where the tangent line intersects the curve.

At $t = 2$, the slope m is

$$m = \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{3t^2}{2t}$$

$$m = \frac{3t}{2}$$

$$m = \frac{3(2)}{2}$$

$$m = 3$$

and the point (x_1, y_1) is

$$x_1 = 2^2 = 4$$

$$y_1 = 2^3 = 8$$

Plugging the slope and the point into our formula gives

$$y - 8 = 3(x - 4)$$

$$y - 8 = 3x - 12$$

$$y = 3x - 4$$

Topic: Tangent line to the parametric curve

Question: Find the equation of the tangent line to the parametric curve.

$$x = 2t^2 + 6$$

$$y = t^4$$

$$t = -1$$

Answer choices:

A $y = x + 7$

B $y = x - 7$

C $y = -x - 7$

D $7y = x - 1$

Solution: B

To define the tangent line, we use the point-slope formula for the equation of the line.

$$y - y_1 = m(x - x_1)$$

where m is the slope and (x_1, y_1) is the point where the tangent line intersects the curve.

At $t = -1$, the slope m is

$$m = \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{4t^3}{4t}$$

$$m = t^2$$

$$m = (-1)^2$$

$$m = 1$$

and the point (x_1, y_1) is

$$x_1 = 2(-1)^2 + 6 = 8$$

$$y_1 = (-1)^4 = 1$$

Plugging the slope and the point into our formula gives

$$y - 1 = 1(x - 8)$$

$$y - 1 = x - 8$$

$$y = x - 7$$

Topic: Tangent line to the parametric curve

Question: At which point is $9x + 7y - 126 = 0$ the equation of the tangent line to the parametric curve?

$$x = 7e^t$$

$$y = 9e^{-t}$$

Answer choices:

A $t = 0$

B $t = 1$

C $t = \frac{1}{e}$

D $t = -\frac{1}{e}$

Solution: A

Differentiate both functions.

$$\frac{dx}{dt} = 7e^t$$

$$\frac{dy}{dt} = -9e^{-t}$$

Divide dy/dt by dx/dt to get dy/dx.

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-9e^{-t}}{7e^t}$$

$$\frac{dy}{dx} = -\frac{9}{7}e^{-2t}$$

At the point $t = 0$, we get:

$$x = 7e^t$$

$$x = 7e^0$$

$$x = 7(1)$$

$$x = 7$$

and

$$y = 9e^{-t}$$

$$y = 9e^{-0}$$

$$y = 9(1)$$

$$y = 9$$

and

$$\frac{dy}{dx} = -\frac{9}{7}e^{-2t}$$

$$\frac{dy}{dx} = -\frac{9}{7}e^{-2(0)}$$

$$\frac{dy}{dx} = -\frac{9}{7}(1)$$

$$\frac{dy}{dx} = -\frac{9}{7}$$

Use the information we just found to write the equation of the tangent line.

$$y - 9 = -\frac{9}{7}(x - 7)$$

$$7y - 63 = -9(x - 7)$$

$$7y - 63 = -9x + 63$$

$$9x + 7y - 126 = 0$$

This is the equation of the tangent line we were given. We found it by plugging in $t = 0$, which means answer choice A is correct.