Eliminating the parameter

Given a parametric curve where our function is defined by two equations, one for x and one for y, and both of them in terms of a parameter t,

$$x = f(t)$$

$$y = g(t)$$

we can eliminate the parameter value in a few different ways. We can

1. Solve each equation for the parameter t, then set the equations equal to one another, or

2. Solve one equation for the parameter t, then plug that value into the second equation, or

3. Solve each equation for part of an identity, then plug both values into the identity.

Let's try an example using the second method, where we eliminate the parameter by solving for t in one of our functions, and then plugging the value we find into the other function.

Example

Eliminate the parameter.

$$x = 2t^2 + 6$$

$$y = 5t$$

We'll solve $y = 5t$ for t, since this will be easier than solving $x = 2t^2 + 6$ for t.

$$y = 5t$$

$$t = \frac{y}{5}$$

Plugging this into the equation for x, we get

$$x = 2\left(\frac{y}{5}\right)^2 + 6$$

$$x = \frac{2y^2}{25} + 6$$

Removing the fraction, we get

$$25x = 2y^2 + 150$$

$$25x - 2y^2 = 150$$

Now let's try an example using the third method, where we solve each equation for part of an identity, and then plug both values into the identity.

Example

Eliminate the parameter.

$$x = e^t$$

$$y = e^{4t}$$

We know that $y = e^{ab}$ is the same as $y = (e^a)^b$. If we use this property, we can take $y = e^{4t}$ and rewrite it as $y = (e^t)^4$. Since $x = e^t$, we can substitute x into $y = (e^t)^4$ for e^t.

$$y = x^4$$

And because we have e^t in the original parametric equations, and $e^t > 0$ for all t, that requires that $x > 0$, and we have to transfer this condition to our final answer.

$$y = x^4, \text{ where } x > 0$$

Let's try another example using the third method.

Example

Eliminate the parameter.

$$x = 2\cos\theta$$

$$y = 3\sin\theta$$

$$0 \le \theta \le 2\pi$$

Rearranging $x = 2\cos\theta$ and $y = 3\sin\theta$ to isolate the trigonometric functions, we get

$$x = 2\cos\theta$$

$$\cos\theta = \frac{x}{2}$$

and

$$y = 3\sin\theta$$

$$\sin\theta = \frac{y}{3}$$

Since we know that $\sin^2\theta + \cos^2\theta = 1$, we can substitute the values we just found for $\cos\theta$ and $\sin\theta$.

$$\left(\frac{y}{3}\right)^2 + \left(\frac{x}{2}\right)^2 = 1$$

$$\frac{y^2}{9} + \frac{x^2}{4} = 1$$

$$y^2 + \frac{9x^2}{4} = 9$$

$$4y^2 + 9x^2 = 36$$

$$9x^2 + 4y^2 = 36$$