Topic: Initial value problems

Question: Solve the initial value problem.

$$\frac{dy}{dx} = 2x + 3$$

$y = 5$ when $x = 0$

Answer choices:

A $\quad y = x^2 + 3x + 5$

B $\quad y = 5$

C $\quad y = 4x^2 + 3x + 5$

D $\quad y = x^2 + 3x - 40$

Solution: A

In order to find y, we multiply both sides of the equation by dx and then integrate both sides.

$$dy = (2x+3)\ dx$$

$$\int dy = \int 2x+3\ dx$$

$$y = x^2 + 3x + C$$

Now, in order to find the specific equation that passes through $y = 5$ when $x = 0$, we substitute these values into the general equation we found and solve for C.

$$5 = 0^2 + 3(0) + C$$

$$5 = C$$

Therefore, the specific equation we are looking for it

$$y = x^2 + 3x + 5$$

Topic: Initial value problems

Question: Solve the initial value problem.

$$f''(x) = \cos x$$

$$f'(0) = 1 \text{ and } f(0) = 3$$

Answer choices:

A $\quad f(x) = \sin x + 1$

B $\quad f(x) = -\cos x + x + 4$

C $\quad f(x) = -\sin x + 1$

D $\quad f(x) = \cos x + x + 2$

Solution: B

Before we can find the equation for $f(x)$, we must first find the equation for $f'(x)$, which we do by integrating $f''(x)$.

$$f'(x) = \int \cos x \ dx$$

$$f'(x) = \sin x + C$$

Now we find the specific equation for $f'(x)$ by solving for C with the initial condition given.

$$f'(0) = \sin 0 + C = 1$$

$$C = 1$$

$$f'(x) = \sin x + 1$$

In order to find $f(x)$, we integrate $f'(x)$ and find C by using the initial condition for $f(x)$.

$$f(x) = \int (\sin x + 1) \ dx$$

$$f(x) = -\cos x + x + C$$

$$f(0) = -\cos 0 + 0 + C = 3$$

$$-1 + C = 3$$

$$C = 4$$

Therefore,

$$f(x) = -\cos x + x + 4$$

Topic: Initial value problems

Question: Solve the initial value problem.

$$\frac{dy}{dx} = 11x^2 - 5x + 6$$

$$y(0) = 7$$

Answer choices:

A $\quad y = \frac{11}{3}x^3 - \frac{5}{2}x^2 + 6x$

B $\quad y = \frac{11}{3}x^3 - \frac{5}{2}x^2 + 6x + C$

C $\quad y = x^3 - x^2 + 6x + 7$

D $\quad y = \frac{11}{3}x^3 - \frac{5}{2}x^2 + 6x + 7$

Solution: D

The question asks us to solve the initial value problem.

$$\frac{dy}{dx} = 11x^2 - 5x + 6$$

$$y(0) = 7$$

In an initial value problem, you're given two things; a differential equation, and a function value at a specific input value. We know that the given equation is a differential equation because it begins with dy/dx, which is the notation for the first derivative of a function with respect to x.

To solve a differential equation, we separate the variables and integrate. The result of the integration gives us a general function because the function "could" contain a constant term, which becomes zero when we differentiate the function. Thus, when we find the anti-derivative, we add a constant labeled "C" to add the possibility of a constant term in the function, although we do not know what that constant is. When we use the initial condition, we will find the specific value of "C". The initial value enables us to find the value of "C".

First, we'll rewrite the differential equation, separating the variables, and then integrate.

$$\frac{dy}{dx} = 11x^2 - 5x + 6$$

$$dy = 11x^2 - 5x + 6\ dx$$

$$\int dy = \int 11x^2 - 5x + 6 \ dx$$

Since the integrand is a polynomial, we can change its terms so each term has an exponent. Then we'll perform the integration using the exponent rule.

$$\int y^0 \ dy = \int 11x^2 - 5x^1 + 6x^0 \ dx$$

$$y = \frac{11}{3}x^3 - \frac{5}{2}x^2 + 6x + C$$

Now we use the initial value $y(0) = 7$ to find "C".

$$7 = \frac{11}{3}(0)^3 - \frac{5}{2}(0)^2 + 6(0) + C$$

We can see that $C = 7$. Replace the "C" with 7. The answer to the initial value problem is

$$y = \frac{11}{3}x^3 - \frac{5}{2}x^2 + 6x + 7$$