Topic: Taylor's inequality

Question: Taylor’s inequality can be applied to which of the following series?

Answer choices:

A Taylor series only

B Any series

C Maclaurin series only

D Taylor or Maclaurin series

Solution: D

Taylor's inequality states that

If	$\left\lvert f^{n+1}(x) \right\rvert \le M$	for	$\lvert x-a \rvert \le d$
then	$\left\lvert R_n(x) \right\rvert \le \dfrac{M}{(n+1)!} \lvert x-a \rvert^{n+1}$	for	$\lvert x-a \rvert \le d$

and it can be applied to both Taylor series and Maclaurin series. Remember that a Maclaurin series is just a Taylor series in which $a=0$.

Topic: Taylor's inequality

Question: Which of these is an accurate statement of Taylor's inequality?

Answer choices:

A If $\left|f^{n+1}(x)\right| > M$ for $|x-a| \leq d$

then $\left|R_n(x)\right| > \frac{M}{(n+1)!}|x-a|^{n+1}$ for $|x-a| \leq d$

B If $\left|f^{n+1}(x)\right| < M$ for $|x-a| \leq d$

then $\left|R_n(x)\right| < \frac{M}{(n+1)!}|x-a|^{n+1}$ for $|x-a| \leq d$

C If $\left|f^{n+1}(x)\right| \geq M$ for $|x-a| \leq d$

then $\left|R_n(x)\right| \geq \frac{M}{(n+1)!}|x-a|^{n+1}$ for $|x-a| \leq d$

D If $\left|f^{n+1}(x)\right| \leq M$ for $|x-a| \leq d$

then $\left|R_n(x)\right| \leq \frac{M}{(n+1)!}|x-a|^{n+1}$ for $|x-a| \leq d$

Solution: D

Taylor's inequality states that

If	$\left\| f^{n+1}(x) \right\| \leq M$	for	$\|x-a\| \leq d$
then	$\left\| R_n(x) \right\| \leq \frac{M}{(n+1)!}\|x-a\|^{n+1}$	for	$\|x-a\| \leq d$

The inequality just means that, if we can show that a Taylor or Maclaurin series has no remainder $R_n(x)$ in its power series representation, then the representation is a true and accurate reflection of the original series.

Topic: Taylor's inequality

Question: Which of these does Taylor's inequality prove?

Answer choices:

A If a Taylor or Maclaurin series has a remainder $R_n(x)$ in its power series representation, then the representation is a true and accurate reflection of the original series.

B If a Taylor or Maclaurin series has no remainder $R_n(x)$ in its power series representation, then the representation is a true and accurate reflection of the original series.

C If a Taylor or Maclaurin series has no remainder $R_n(x)$ in its power series representation, then the representation is not a true and accurate reflection of the original series.

D If a Taylor or Maclaurin series has no remainder $R_n(x)$ in its power series representation, then the representation might be a true and accurate reflection of the original series but needs further testing.

Solution: B

Taylor's inequality states that

If	$\left\| f^{n+1}(x) \right\| \le M$	for	$\|x-a\| \le d$
then	$\left\| R_n(x) \right\| \le \dfrac{M}{(n+1)!}\|x-a\|^{n+1}$	for	$\|x-a\| \le d$

The inequality just means that, if we can show that a Taylor or Maclaurin series has no remainder $R_n(x)$ in its power series representation, then the representation is a true and accurate reflection of the original series.