Topic: Find f given f''

Question: Find $f(x)$ if $f''(x) = 6x - 4$.

Answer choices:

A $\quad f(x) = \frac{x^4}{4} - \frac{2x^3}{3} + x^2 + x$

B $\quad f(x) = 3x^2 - 4x + C$

C $\quad f(x) = x^3 - 2x^2 + Cx + D$

D $\quad f(x) = \frac{x^3}{4} - \frac{2x^2}{3} + \frac{Cx}{D} + \frac{D}{C}$

Solution: C

The question asks us to find the function $f(x)$ if the second derivative of the function is $f''(x) = 6x - 4$.

Note that the question does not provide initial values of $f(x)$ or $f'(x)$ so our answer will be a family of possible $f(x)$ functions that could have the same second derivative.

We are given the second derivative of the function. To find the first derivative of the function, find the anti-derivative of the second derivative. Then, to find the function, we repeat the process by finding the anti-derivative of the first derivative.

The second derivative is a polynomial function. To find the anti-derivative, in each term, add 1 to the exponent and divide the term by the new exponent.

Additionally, all functions "could" contain a constant term, which becomes zero when we take the derivative of the function. Thus, when we find the anti-derivative, we add a constant labeled "C" to add the possibility of a constant term in the function, although we do not know what that constant is.

First we'll write the second derivative showing all exponents.

$$f''(x) = 6x - 4 = 6x^1 - 4x^0$$

$$f'(x) = \int 6x - 4 \ dx$$

$$f'(x) = \frac{6x^{1+1}}{2} - \frac{4x^{0+1}}{1} + C$$

Simplify each term to finish finding the first derivative.

$$f'(x) = 3x^2 - 4x + C$$

Now, find the function by repeating the process.

$$f'(x) = 3x^2 - 4x + C = 3x^2 - 4x^1 + Cx^0$$

Once again, we'll add a constant to cover the likely event that the original function had a constant term that became zero when the first derivative was taken. We do not know that the new constant is the same as the old constant so we will call it "D".

$$f(x) = \int 3x^2 - 4x^1 + Cx^0 \ dx$$

$$f(x) = \frac{3x^{2+1}}{3} - \frac{4x^{1+1}}{2} + \frac{Cx^{0+1}}{1} + D$$

After we simplify each term, the function is

$$f(x) = x^3 - 2x^2 + Cx + D$$

Topic: Find f given f’’

Question: Find $f(x)$ if $f''(x) = 42x^5 - 40x^3 + 12x^2 - 12x + 2$.

Answer choices:

A $\quad f(x) = \frac{x^8}{7} - \frac{x^6}{3} + \frac{x^5}{5} - \frac{x^4}{2} + \frac{x^3}{3} + \frac{x^2}{2} + x + C$

B $\quad f(x) = x^7 - 2x^5 + x^4 - 2x^3 + x^2 + 2x + 1$

C $\quad f(x) = 7x^6 - 10x^4 + 4x^3 - 6x^2 + 2x + C$

D $\quad f(x) = x^7 - 2x^5 + x^4 - 2x^3 + x^2 + Cx + D$

Solution: D

The question asks us to find the function $f(x)$ if the second derivative of the function is

$$f''(x) = 42x^5 - 40x^3 + 12x^2 - 12x + 2$$

Note that the question does not provide initial values of $f(x)$ or $f'(x)$ so our answer will be a family of possible $f(x)$ functions that could have the same second derivative.

We are given the second derivative of the function. To find the first derivative of the function, find the anti-derivative of the second derivative. Then, to find the function, we repeat the process by finding the anti-derivative of the first derivative.

The second derivative is a polynomial function. To find the anti-derivative, in each term, add 1 to the exponent and divide the term by the new exponent.

Additionally, all functions "could" contain a constant term, which becomes zero when we take the derivative of the function. Thus, when we find the anti-derivative, we add a constant labeled "C" to add the possibility of a constant term in the function, although we do not know what that constant is.

We will, first, write the second derivative showing all exponents.

$$f''(x) = 42x^5 - 40x^3 + 12x^2 - 12x + 2 = 42x^5 - 40x^3 + 12x^2 - 12x^1 + 2x^0$$

$$f'(x) = \int 42x^5 - 40x^3 + 12x^2 - 12x^1 + 2x^0 \ dx$$

$$f'(x) = \frac{42x^{5+1}}{6} - \frac{40x^{3+1}}{4} + \frac{12x^{2+1}}{3} - \frac{12x^{1+1}}{2} + \frac{2x^{0+1}}{1} + C$$

After we simplify each term, the first derivative function is

$$f'(x) = 7x^6 - 10x^4 + 4x^3 - 6x^2 + 2x + C$$

Now, find the function by repeating the process.

$$f'(x) = 7x^6 - 10x^4 + 4x^3 - 6x^2 + 2x + C$$

$$f'(x) = 7x^6 - 10x^4 + 4x^3 - 6x^2 + 2x^1 + Cx^0$$

Once again, we will add a constant to cover the likely event that the original function had a constant term that became zero when the first derivative was taken. We do not know that the new constant is the same as the old constant so we will call it "D".

$$f(x) = \int 7x^6 - 10x^4 + 4x^3 - 6x^2 + 2x^1 + Cx^0 \ dx$$

$$f(x) = \frac{7x^{6+1}}{7} - \frac{10x^{4+1}}{5} + \frac{4x^{3+1}}{4} - \frac{6x^{2+1}}{3} + \frac{2x^{1+1}}{2} + \frac{Cx^{0+1}}{1} + D$$

Now, simplify each term and the function is

$$f(x) = x^7 - 2x^5 + x^4 - 2x^3 + x^2 + Cx + D$$

Topic: Find f given f''

Question: Find $f(x)$.

$$f''(x) = \frac{45\sqrt{x}}{16} - \frac{1}{2\sqrt{x}} - \frac{1}{20}x^{-\frac{3}{2}}$$

Answer choices:

A $\quad \frac{45}{16}x^{\frac{5}{2}} - \frac{1}{2}x^{\frac{3}{2}} - \frac{1}{10}\sqrt{x}$

B $\quad \frac{3}{4}x^{\frac{5}{2}} - \frac{2}{3}x^{\frac{3}{2}} + \frac{\sqrt{x}}{5} + Cx + D$

C $\quad \frac{3}{4}x^{\frac{5}{2}} - \frac{2}{3}x^{\frac{3}{2}} + \frac{\sqrt{x}}{5} + x + 1$

D $\quad \frac{3}{4}x^{\frac{5}{2}} - \frac{2}{3}x^{\frac{3}{2}} + \frac{\sqrt{x}}{5}$

Solution: B

The question asks us to find the function $f(x)$ if the second derivative of the function is

$$f''(x) = \frac{45\sqrt{x}}{16} - \frac{1}{2\sqrt{x}} - \frac{1}{20}x^{-\frac{3}{2}}$$

Note that the question does not provide initial values of $f(x)$ or $f'(x)$ so our answer will be a family of possible $f(x)$ functions that could have the same second derivative.

We are given the second derivative of the function. To find the first derivative of the function, find the anti-derivative of the second derivative. Then, to find the function, we repeat the process by finding the anti-derivative of the first derivative.

The second derivative is a function with radicals and rational exponents. In order to use the exponent rule when finding the anti-derivative, we will convert all terms to rational exponent terms. To find the anti-derivative, in each term, add 1 to the exponent and divide the term by the new exponent.

Additionally, all functions "could" contain a constant term, which becomes zero when we take the derivative of the function. Thus, when we find the anti-derivative, we add a constant labeled "c" to add the possibility of a constant term in the function, although we do not know what that constant is.

First we'll write the second derivative, showing all exponents.

$$f''(x)=\frac{45\sqrt{x}}{16}-\frac{1}{2\sqrt{x}}-\frac{1}{20}x^{-\frac{3}{2}}$$

$$f''(x)=\frac{45}{16}x^{\frac{1}{2}}-\frac{1}{2}x^{-\frac{1}{2}}-\frac{1}{20}x^{-\frac{3}{2}}$$

$$f'(x)=\int\frac{45}{16}x^{\frac{1}{2}}-\frac{1}{2}x^{-\frac{1}{2}}-\frac{1}{20}x^{-\frac{3}{2}}\ dx$$

$$f'(x)=\frac{\frac{45}{16}x^{\frac{1}{2}+1}}{\frac{3}{2}}-\frac{\frac{1}{2}x^{-\frac{1}{2}+1}}{\frac{1}{2}}-\frac{\frac{1}{20}x^{-\frac{3}{2}+1}}{-\frac{1}{2}}+C$$

Since we are dividing fractions by fractions, let's change the expressions to multiplying by the reciprocal of the fraction in the denominator. We will also simplify the exponents of each term.

$$f'(x)=\frac{45}{16}\times\frac{2}{3}x^{\frac{3}{2}}-\frac{1}{2}\times\frac{2}{1}x^{\frac{1}{2}}-\frac{1}{20}\times-\frac{2}{1}x^{-\frac{1}{2}}+C$$

Simplify each term to finish finding the first derivative.

$$f'(x)=\frac{15}{8}x^{\frac{3}{2}}-x^{\frac{1}{2}}+\frac{1}{10}x^{-\frac{1}{2}}+C$$

Now, find the function by repeating the process.

$$f'(x)=\frac{15}{8}x^{\frac{3}{2}}-x^{\frac{1}{2}}+\frac{1}{10}x^{-\frac{1}{2}}+Cx^{0}$$

Once again, we will add a constant to cover the likely event that the original function had a constant term that became zero when the first derivative was taken. We do not know that the new constant is the same as the old constant so we will call it "D".

$$f(x)=\int \frac{15}{8}x^{\frac{3}{2}}-x^{\frac{1}{2}}+\frac{1}{10}x^{-\frac{1}{2}}+Cx^0\ dx$$

$$f(x)=\frac{\frac{15}{8}x^{\frac{3}{2}+1}}{\frac{5}{2}}-\frac{x^{\frac{1}{2}+1}}{\frac{3}{2}}+\frac{\frac{1}{10}x^{-\frac{1}{2}+1}}{\frac{1}{2}}+\frac{Cx^{0+1}}{1}+D$$

Since we are dividing fractions by fractions again, let's change the expressions to multiplying by the reciprocal of the fraction in the denominator, as we did before. We will also simplify the exponents of each term.

$$f(x)=\frac{15}{8}\times\frac{2}{5}x^{\frac{5}{2}}-\frac{2}{3}x^{\frac{3}{2}}+\frac{1}{10}\times\frac{2}{1}x^{\frac{1}{2}}+Cx+D$$

After we simplify each term, the function is

$$f(x)=\frac{3}{4}x^{\frac{5}{2}}-\frac{2}{3}x^{\frac{3}{2}}+\frac{\sqrt{x}}{5}+Cx+D$$