Calculus 2 Workbook Solutions

Definite integrals

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MATH

DEFINITE INTEGRALS

1. Evaluate the definite integral.

$$\int_0^3 x^3 + x^2 + x + 1 \ dx$$

Solution:

$$\int_0^3 x^3 + x^2 + x + 1 \ dx$$

$$\frac{x^4}{4} + \frac{x^3}{3} + \frac{x^2}{2} + x \Big|_0^3$$

$$\left(\frac{3^4}{4} + \frac{3^3}{3} + \frac{3^2}{2} + 3\right) - \left(\frac{0^4}{4} + \frac{0^3}{3} + \frac{0^2}{2} + 0\right)$$

$$\frac{81}{4} + 9 + \frac{9}{2} + 3$$

$$\frac{81}{4} + \frac{36}{4} + \frac{18}{4} + \frac{12}{4}$$

$$\frac{147}{4}$$

2. Evaluate the definite integral.

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{2}} 2\sin x + 3\cos x \ dx$$

Solution:

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{2}} 2\sin x + 3\cos x \ dx$$

$$-2\cos x + 3\sin x \Big|_{-\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$\left(-2\cos\frac{\pi}{2} + 3\sin\frac{\pi}{2}\right) - \left(-2\cos\left(-\frac{\pi}{4}\right) + 3\sin\left(-\frac{\pi}{4}\right)\right)$$

$$\left(-2(0) + 3(1)\right) - \left(-2\left(\frac{\sqrt{2}}{2}\right) + 3\left(-\frac{\sqrt{2}}{2}\right)\right)$$

$$3 + \sqrt{2} + \frac{3\sqrt{2}}{2}$$

■ 3. Evaluate the definite integral.

$$\int_{-4}^{4} 2x^3 - 4x^2 + 25 \ dx$$

Solution:

$$\int_{-4}^{4} 2x^3 - 4x^2 + 25 \ dx$$

$$\frac{x^4}{2} - \frac{4x^3}{3} + 25x \Big|_{-4}^{4}$$

$$\left(\frac{4^4}{2} - \frac{4(4)^3}{3} + 25(4)\right) - \left(\frac{(-4)^4}{2} - \frac{4(-4)^3}{3} + 25(-4)\right)$$

$$\left(128 - \frac{256}{3} + 100\right) - \left(128 + \frac{256}{3} - 100\right)$$

$$-\frac{512}{3} + 200$$

$$-\frac{512}{3} + \frac{600}{3}$$

$$\frac{88}{3}$$

4. Evaluate the definite integral.

$$\int_{1}^{2} 6x^5 - 8x^3 + 4x + 3 \ dx$$

Solution:

$$\int_1^2 6x^5 - 8x^3 + 4x + 3\ dx$$

$$x^6 - 2x^4 + 2x^2 + 3x\Big|_1^2$$

$$\left(2^6 - 2(2)^4 + 2(2)^2 + 3(2)\right) - \left(1^6 - 2(1)^4 + 2(1)^2 + 3(1)\right)$$

$$(64 - 32 + 8 + 6) - (1 - 2 + 2 + 3)$$

$$46 - 4$$

$$42$$

5. Evaluate the definite integral.

$$\int_0^{\pi} 5 \sin x\ dx$$

Solution:

$$\int_0^{\pi} 5 \sin x\ dx$$

$$-5 \cos x\Big|_0^{\pi}$$

$$-5 \cos \pi - (-5 \cos 0)$$

$$5 - (-5)$$

$$10$$

AREA UNDER OR ENCLOSED BY THE CURVE

1. Find the area under the graph of $f(x)=2x^2-3x+5$ over the interval $[-2,6]$.

Solution:

Because we were asked for area "under" the graph, we're looking for net area, which we find by evaluating the integral of the function over the given interval.

$$A=\int_{-2}^{6} 2x^2-3x+5\ dx$$

$$A=\frac{2x^3}{3}-\frac{3x^2}{2}+5x\Big|_{-2}^{6}$$

$$A=\left(\frac{2(6)^3}{3}-\frac{3(6)^2}{2}+5(6)\right)-\left(\frac{2(-2)^3}{3}-\frac{3(-2)^2}{2}+5(-2)\right)$$

$$A=(144-54+30)-\left(-\frac{16}{3}-6-10\right)$$

$$A=120+\frac{64}{3}$$

$$A=141\frac{1}{3}$$

2. Find the area enclosed by the graph of $g(x) = 2x(x+4)(x-2)$ over the interval $[-4,2]$.

Solution:

Because we were asked for area "enclosed by" the graph, we're looking for gross area, which means we need to start by finding the zeros of the function.

The graph crosses the x-axis at $x = -4$, $x = 0$, and $x = 2$. Since we're looking for enclosed area over the interval $[-4,2]$, we'll take the absolute value of the area on the interval $[-4,0]$, and the absolute value of the area on the interval $[0,2]$, and then add the areas together.

$$A = \left| \int_{-4}^{0} 2x(x+4)(x-2)\ dx \right| + \left| \int_{0}^{2} 2x(x+4)(x-2)\ dx \right|$$

$$A = \left| \int_{-4}^{0} 2x^3 + 4x^2 - 16x\ dx \right| + \left| \int_{0}^{2} 2x^3 + 4x^2 - 16x\ dx \right|$$

$$A = \left| \left(\frac{x^4}{2} + \frac{4x^3}{3} - 8x^2 \right) \Bigg|_{-4}^{0} \right| + \left| \left(\frac{x^4}{2} + \frac{4x^3}{3} - 8x^2 \right) \Bigg|_{0}^{2} \right|$$

$$A = \left| \left(\frac{0^4}{2} + \frac{4(0)^3}{3} - 8(0)^2 \right) - \left(\frac{(-4)^4}{2} + \frac{4(-4)^3}{3} - 8(-4)^2 \right) \right|$$

$$+\left|\left(\frac{2^4}{2}+\frac{4(2)^3}{3}-8(2)^2\right)-\left(\frac{0^4}{2}+\frac{4(0)^3}{3}-8(0)^2\right)\right|$$

$$A=\left|-\left(\frac{256}{2}+\frac{4(-64)}{3}-8(16)\right)\right|+\left|\left(\frac{16}{2}+\frac{32}{3}-8(4)\right)\right|$$

$$A=\left|-128+\frac{256}{3}+128\right|+\left|8+\frac{32}{3}-32\right|$$

$$A=\left|\frac{256}{3}\right|+\left|\frac{32}{3}-24\right|$$

$$A=\frac{256}{3}+\left|\frac{32}{3}-\frac{72}{3}\right|$$

$$A=\frac{256}{3}+\left|-\frac{40}{3}\right|$$

$$A=\frac{256}{3}+\frac{40}{3}$$

$$A=\frac{296}{3}$$

■ 3. Find the area under the graph of $h(x)=3\sqrt{x}$ over the interval $[4,16]$.

Solution:

Because we were asked for area “under” the graph, we’re looking for net area, which we find by evaluating the integral of the function over the given interval.

$$A = \int_4^{16} 3\sqrt{x}\ dx$$

$$A = \int_4^{16} 3x^{\frac{1}{2}}\ dx$$

$$A = 3\left(\frac{2}{3}x^{\frac{3}{2}}\right)\Bigg|_4^{16}$$

$$A = 2x^{\frac{3}{2}}\Bigg|_4^{16}$$

$$A = (2(16)^{\frac{3}{2}}) - (2(4)^{\frac{3}{2}})$$

$$A = (2(4)^3) - (2(2)^3)$$

$$A = (2(64)) - (2(8))$$

$$A = 128 - 16$$

$$A = 112$$

DEFINITE INTEGRALS OF EVEN AND ODD FUNCTIONS

1. Evaluate the definite integral.

$$\int_{-3}^{3} -x^4+19\ dx$$

Solution:

In an even function, $f(x)=f(-x)$ and the graph is symmetric about the y -axis. The integral contains an even function.

$$x^4+19=(-x)^4+19=x^4+19$$

Because of the symmetry across the y-axis, we can rewrite the integral as

$$2\int_{0}^{3} -x^4+19\ dx$$

$$2\left(-\frac{x^5}{5}+19x\right)\Bigg|_0^3$$

$$2\left(-\frac{3^5}{5}+19(3)\right)-2\left(-\frac{0^5}{5}+19(0)\right)$$

$$2\left(-\frac{243}{5}+57\right)$$

$$\frac{84}{5} \approx 16.8$$

■ 2. Evaluate the definite integral.

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} 7\cos x \ dx$$

Solution:

In an even function, $f(x) = f(-x)$ and the graph is symmetric about the y-axis. The integral contains an even function.

$$7\cos x = 7\cos(-x) = 7\cos x$$

Because of the symmetry across the y-axis, we can rewrite the integral as

$$2\int_{0}^{\frac{\pi}{4}} 7\cos x \ dx$$

$$14\sin x\Big|_{0}^{\frac{\pi}{4}}$$

$$14\sin\left(\frac{\pi}{4}\right) - 14\sin(0)$$

$$14\left(\frac{\sqrt{2}}{2}\right)$$

$$7\sqrt{2}$$

3. Evaluate the definite integral.

$$\int_{-2}^{2} \frac{3}{4}x^2 + 5\ dx$$

Solution:

In an even function, $f(x) = f(-x)$ and the graph is symmetric about the y-axis. The integral contains an even function.

$$\frac{3}{4}x^2 + 5 = \frac{3}{4}(-x)^2 + 5 = \frac{3}{4}x^2 + 5$$

Because of the symmetry across the y-axis, we can rewrite the integral as

$$2\int_{0}^{2} \frac{3}{4}x^2 + 5\ dx$$

$$\left(\frac{x^3}{2} + 10x\right)\Big|_0^2$$

$$\left(\frac{2^3}{2} + 10(2)\right) - \left(\frac{0^3}{2} + 10(0)\right)$$

$$4 + 20$$

$$24$$

■ 4. Evaluate the definite integral.

$$\int_{-2}^{2} 3x^5 - 4x^3 + 8x \ dx$$

Solution:

In an odd function, $f(-x) = -f(x)$ and the graph is symmetric about the origin. Check to see if the function is odd.

$$3(-x)^5 - 4(-x)^3 + 8(-x) = -3x^5 + 4x^3 - 8x = -\left(3x^5 - 4x^3 + 8x\right)$$

The function is odd, and the interval $[-2,2]$ fits the format $[-a, a]$, so

$$\int_{-2}^{2} 3x^5 - 4x^3 + 8x \ dx = 0$$

■ 5. Evaluate the definite integral.

$$\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} 9 \sin x \ dx$$

Solution:

In an odd function, $f(-x) = -f(x)$ and the graph is symmetric about the origin. Check to see if the function is odd.

$$9\sin(-x) = -9\sin x = -(9\sin x)$$

The function is odd, and the interval $[-\pi/3, \pi/3]$ fits the format $[-a, a]$, so

$$\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} 9\sin x \ dx = 0$$

6. Evaluate the definite integral.

$$\int_{-2}^{2} 2x^3 - 4x \ dx$$

Solution:

In an odd function, $f(-x) = -f(x)$ and the graph is symmetric about the origin. Check to see if the function is odd.

$$2(-x)^3 - 4(-x) = -2x^3 + 4x = -\left(2x^3 - 4x\right)$$

The function is odd, and the interval $[-2, 2]$ fits the format $[-a, a]$, so

$$\int_{-2}^{2} 2x^3 - 4x \ dx = 0$$

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