Topic: nth term test

Question: Use the nth term test to say whether or not the series divergence.

$$\sum_{n=1}^{\infty} 2+\frac{1}{n}$$

Answer choices:

A The series converges.

B The series diverges.

C The test is inconclusive.

D The series is infinite.

Solution: B

The nth term test, also called the divergence test, or the zero test,

says that a series a_n **diverges** if $\lim_{n\to\infty} a_n \neq 0$

is **inconclusive** if $\lim_{n\to\infty} a_n = 0$

The nth term test can't tell us that a series converges, only that it diverges. Otherwise, the test is inconclusive.

To use it, we just take the limit as $n \to \infty$ of the series a_n that we've been given.

$$\lim_{n\to\infty} 2 + \frac{1}{n}$$

$$2 + \frac{1}{\infty}$$

$$2 + 0$$

$$2$$

Since $2 \neq 0$, the nth term test tells us that the series diverges.

Topic: nth term test

Question: Use the nth term test to say whether or not the series diverges.

$$\sum_{n=1}^{\infty} \frac{3n^2 - 2}{5n^2 + 8}$$

Answer choices:

A The series converges.

B The test is inconclusive.

C The series diverges.

D The series is infinite.

Solution: C

The nth term test, also called the divergence test, or the zero test,

says that a series a_n **diverges** if $\lim_{n\to\infty} a_n \neq 0$

is **inconclusive** if $\lim_{n\to\infty} a_n = 0$

The nth term test can't tell us that a series converges, only that it diverges. Otherwise, the test is inconclusive.

To use it, we just take the limit as $n \to \infty$ of the series a_n that we've been given.

$$\lim_{n\to\infty} \frac{3n^2-2}{5n^2+8}$$

$$\frac{\infty}{\infty}$$

Since we can an indeterminate form, we'll go back a step and manipulate our function.

$$\lim_{n\to\infty} \frac{3n^2-2}{5n^2+8}\left(\frac{\frac{1}{n^2}}{\frac{1}{n^2}}\right)$$

$$\lim_{n\to\infty} \frac{\frac{3n^2}{n^2}-\frac{2}{n^2}}{\frac{5n^2}{n^2}+\frac{8}{n^2}}$$

$$\lim_{n\to\infty} \frac{3 - \frac{2}{n^2}}{5 + \frac{8}{n^2}}$$

$$\frac{3 - \frac{2}{\infty^2}}{5 + \frac{8}{\infty^2}}$$

$$\frac{3 - 0}{5 + 0}$$

$$\frac{3}{5}$$

Since $3/5 \neq 0$, the nth term test tells us that the series diverges.

Topic: nth term test

Question: Use the nth term test to say whether or not the series diverges.

$$\sum_{n=1}^{\infty} \frac{n^2}{e^n}$$

Answer choices:

A The series converges.

B The series is infinite.

C The series diverges.

D The test is inconclusive.

Solution: D

The nth term test, also called the divergence test, or the zero test,

says that a series a_n **diverges** if $\lim_{n\to\infty} a_n \neq 0$

is **inconclusive** if $\lim_{n\to\infty} a_n = 0$

The nth term test can't tell us that a series converges, only that it diverges. Otherwise, the test is inconclusive.

To use it, we just take the limit as $n \to \infty$ of the series a_n that we've been given.

$$\lim_{n\to\infty} \frac{n^2}{e^n}$$

$$\frac{\infty}{\infty}$$

Since we can an indeterminate form, we'll go back a step and use L'Hospital's rule to simplify our function by replacing both the numerator and the denominator with their derivatives.

$$\lim_{n\to\infty} \frac{2n}{e^n}$$

$$\frac{\infty}{\infty}$$

We'll back up a step and use L'Hospital's rule again.

$$\lim_{n\to\infty} \frac{2}{e^n}$$

$$\frac{2}{\infty}$$

$$0$$

Since our answer is 0, the nth term test is inconclusive.