Calculus 2 Workbook Solutions

Probability

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MATH

PROBABILITY DENSITY FUNCTIONS

■ 1. Given $f(x)$, find $P(0 \leq x \leq 2)$.

$$f(x) = \begin{cases} \frac{1}{32} & 0 \leq x \leq 32 \\ 0 & x < 0 \text{ or } x > 32 \end{cases}$$

Solution:

First ensure that the function meets the criteria to be a probability density function, in that $f(x) \geq 0$ on $-\infty \leq x \leq \infty$, and the integral of $f(x)$ on $-\infty \leq x \leq \infty$ equals 1.

The given function $f(x)$ is a piecewise constant function, and based on the function's definition, $f(x) \geq 0$ for all x.

The integral of $f(x)$ on $-\infty \leq x \leq \infty$ is

$$\int_{-\infty}^{\infty} f(x)\ dx$$

$$\int_{-\infty}^{0} f(x)\ dx + \int_{0}^{32} f(x)\ dx + \int_{32}^{\infty} f(x)\ dx$$

$$0 + \int_{0}^{32} \frac{1}{32}\ dx + 0$$

Integrate, then evaluate over the interval.

$$\frac{1}{32}x\Big|_0^{32}$$

$$\frac{1}{32}(32) - \frac{1}{32}(0)$$

$$1$$

Then $P(0 \leq x \leq 2)$ is

$$\int_0^2 f(x)\ dx = \int_0^2 \frac{1}{32}\ dx = \frac{1}{32}x\Big|_0^2 = \frac{1}{32}(2) - \frac{1}{32}(0) = \frac{2}{32} = \frac{1}{16}$$

■ 2. Given $g(x)$, find $P(1 \leq x \leq 5)$.

$$g(x) = \begin{cases} e^{-x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

Solution:

First ensure that the function meets the criteria to be a probability density function, in that $g(x) \geq 0$ on $-\infty \leq x \leq \infty$, and the integral of $g(x)$ on $-\infty \leq x \leq \infty$ equals 1.

The given function $g(x)$ is a piecewise exponential function. So based on the function's definition, $g(x) \geq 0$ for all x.

The integral of $g(x)$ on $-\infty \leq x \leq \infty$ is

$$\int_{-\infty}^{\infty} g(x)\ dx$$

$$\int_{-\infty}^{0} g(x)\ dx + \int_{0}^{\infty} g(x)\ dx$$

$$\int_{-\infty}^{0} 0\ dx + \int_{0}^{\infty} e^{-x}\ dx$$

$$\lim_{a \to -\infty} \int_{a}^{0} 0\ dx + \lim_{b \to \infty} \int_{0}^{b} e^{-x}\ dx$$

Integrate, then evaluate over the interval.

$$\lim_{a \to -\infty} 0 + \lim_{b \to \infty} (-e^{-x}) \Big|_{0}^{b}$$

$$\lim_{b \to \infty} -e^{-b} - (-e^{-0})$$

$$0 + 1$$

$$1$$

Then $P(1 \le x \le 5)$ is

$$\int_{1}^{5} e^{-x}\ dx = -e^{-x} \Big|_{1}^{5} = -e^{-5} - (-e^{-1}) = -\frac{1}{e^5} + \frac{1}{e} = \frac{1}{e} - \frac{1}{e^5}$$

■ 3. Given $h(x)$, find $P(-1 \le x \le 1)$.

$$h(x) = \begin{cases} \frac{1}{6} & -2 \le x \le 4 \\ 0 & x < -2 \text{ or } x > 4 \end{cases}$$

Solution:

First ensure that the function meets the criteria to be a probability density function, in that $h(x) \ge 0$ on $-\infty \le x \le \infty$, and the integral of $h(x)$ on $-\infty \le x \le \infty$ equals 1.

The given function $f(x)$ is a piecewise constant function, and based on the function's definition, $f(x) \ge 0$ for all x.

The integral of $h(x)$ on $-\infty \le x \le \infty$ is

$$\int_{-\infty}^{\infty} h(x)\ dx$$

$$\int_{-\infty}^{-2} h(x)\ dx + \int_{-2}^{4} h(x)\ dx + \int_{4}^{\infty} h(x)\ dx$$

$$\lim_{a \to -\infty} \int_{a}^{-2} h(x)\ dx + \int_{-2}^{4} h(x)\ dx + \lim_{b \to \infty} \int_{4}^{b} h(x)\ dx$$

$$\lim_{a \to -\infty} \int_{a}^{-2} 0\ dx + \int_{-2}^{4} \frac{1}{6}\ dx + \lim_{b \to \infty} \int_{4}^{b} 0\ dx$$

Integrate, then evaluate over the interval.

$$0 + \frac{1}{6}x \Big|_{-2}^{4} + 0$$

$$\frac{1}{6}(4) - \frac{1}{6}(-2)$$

$$\frac{4}{6} + \frac{2}{6}$$

$$\frac{6}{6}$$

$$1$$

Then $P(-1 \leq x \leq 1)$ is

$$\int_{-1}^{1} h(x)\ dx = \int_{-1}^{1} \frac{1}{6}\ dx$$

$$\frac{1}{6}x\Big|_{-1}^{1}$$

$$\frac{1}{6}(1) - \frac{1}{6}(-1)$$

$$\frac{1}{6} + \frac{1}{6}$$

$$\frac{2}{6}$$

$$\frac{1}{3}$$

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