Topic: Absolute, relative, and percentage error

Question: Use a linear approximation to estimate the value of $\sqrt{99}$, then find the absolute error of the estimate.

Answer choices:

A $E_A(100) \approx 0.0050$

B $E_A(100) \approx 0.0001$

C $E_A(99) \approx 0.0050$

D $E_A(99) \approx 0.0001$

Solution: D

The root $\sqrt{99}$ is very close to $\sqrt{100}$, which we already know is 10. So instead of thinking specifically about $\sqrt{99}$, let's think about $\sqrt{x}$ and use the function $f(x) = \sqrt{x}$.

Differentiate the function,

$$f'(x) = \frac{1}{2}x^{-\frac{1}{2}}$$

$$f'(x) = \frac{1}{2x^{\frac{1}{2}}}$$

$$f'(x) = \frac{1}{2\sqrt{x}}$$

then evaluate the derivative at $x = 100$.

$$f'(100) = \frac{1}{2\sqrt{100}}$$

$$f'(100) = \frac{1}{2(10)}$$

$$f'(100) = \frac{1}{20}$$

So the linear approximation line intersects $f(x) = \sqrt{x}$ at the point of tangency $(100,10)$, and has a slope of $m = 1/20$. Substitute these values into the linear approximation equation.

$$L(x) = f(a) + f'(a)(x - a)$$

$$L(x) = 10 + \frac{1}{20}(x - 100)$$

$$L(x) = 10 + \frac{1}{20}x - \frac{100}{20}$$

$$L(x) = \frac{1}{20}x - \frac{100}{20} + \frac{200}{20}$$

$$L(x) = \frac{1}{20}x + \frac{100}{20}$$

Then the linear approximation at $x = 99$ is

$$L(99) = \frac{1}{20}(99) + \frac{100}{20}$$

$$L(99) = \frac{99}{20} + \frac{100}{20}$$

$$L(99) = \frac{199}{20}$$

$$L(99) = 9.95$$

In comparison, the actual value of $\sqrt{99}$ is

$$f(x) = \sqrt{x}$$

$$f(99) = \sqrt{99}$$

$$f(99) \approx 9.9499$$

Therefore, the absolute error of the approximation is

$$E_A(a) = |f(a) - L(a)|$$

$$E_A(99) = |f(99) - L(99)|$$

$$E_A(99) \approx |9.9499 - 9.95|$$

$$E_A(99) \approx |-0.0001|$$

$$E_A(99) \approx 0.0001$$

Topic: Absolute, relative, and percentage error

Question: If the absolute error of a linear approximation estimate of $\sqrt{99}$ is $E_A(99) = 0.0001$, then find the relative error of the estimate.

Answer choices:

A $\quad E_R(99) \approx 0.00005001$

B $\quad E_R(99) \approx 0.00001005$

C $\quad E_R(100) \approx 0.00005001$

D $\quad E_R(100) \approx 0.00001005$

Solution: B

The actual value of $\sqrt{99}$ is

$$f(x) = \sqrt{x}$$

$$f(99) = \sqrt{99}$$

$$f(99) \approx 9.9499$$

and we've been told that the absolute error of the estimate is

$$E_A(99) = 0.0001$$

The relative error is

$$E_R(a) = \frac{E_A(a)}{f(a)}$$

$$E_R(99) = \frac{E_A(99)}{f(99)}$$

$$E_R(99) \approx \frac{0.0001}{9.9499}$$

$$E_R(99) \approx 0.00001005$$

Topic: Absolute, relative, and percentage error

Question: If the absolute error of a linear approximation estimate of $\sqrt{99}$ is $E_A(99) = 0.0001$ and the relative error is $E_R(99) \approx 0.00001005$, then find the percentage error of the estimate.

Answer choices:

A $E_P(100) \approx 0.001005$

B $E_P(100) \approx 0.00001005$

C $E_P(99) \approx 0.001005$

D $E_P(99) \approx 0.00001005$

Solution: C

The actual value of $\sqrt{99}$ is

$$f(x)=\sqrt{x}$$

$$f(99)=\sqrt{99}$$

$$f(99)\approx 9.9499$$

and we've been told that the absolute error of the estimate is $E_A(99)=0.0001$ and that the relative error of the estimate is $E_R(99)\approx 0.00001005$.

The percentage error is

$$E_P(a)=100\%\cdot E_R(a)$$

$$E_P(99)=100\%\cdot E_R(99)$$

$$E_P(99)\approx 100\%\cdot 0.00001005$$

$$E_P(99)\approx 0.001005$$