Calculus 2 Workbook Solutions

Economics

krista king MATH

SINGLE DEPOSIT, COMPOUNDED N TIMES, FUTURE VALUE

■ 1. Find the future value of $\$9{,}500$ after 7 years, at an annual interest rate of $2.25\,\%$, compounded quarterly.

Solution:

Using the future value formula, the future value is

$$FV = PV\left(1+\frac{r}{n}\right)^{nt}$$

$$FV = 9{,}500\left(1+\frac{0.0225}{4}\right)^{4\times 7}$$

$$FV = 9{,}500(1.005625)^{28}$$

$$FV = \$11{,}115.61$$

■ 2. Find the future value of $\$14{,}550$ after 3 years, at an annual interest rate of $1.95\,\%$, compounded monthly.

Solution:

Using the future value formula, the future value is

$$FV = PV\left(1 + \frac{r}{n}\right)^{nt}$$

$$FV = 14{,}550\left(1 + \frac{0.0195}{12}\right)^{12 \times 3}$$

$$FV = 14{,}550(1.001625)^{36}$$

$$FV = \$15{,}425.83$$

3. Find the future value of $\$7{,}595$ after 5 years, at an annual interest rate of $3.25\,\%$, compounded weekly.

Solution:

Using the future value formula, the future value is

$$FV = PV\left(1 + \frac{r}{n}\right)^{nt}$$

$$FV = 7{,}595\left(1 + \frac{0.0325}{52}\right)^{52 \times 5}$$

$$FV = 7{,}595(1.000625)^{260}$$

$$FV = \$8{,}934.67$$

SINGLE DEPOSIT, COMPOUNDED N TIMES, PRESENT VALUE

■ 1. Find the present value of a deposit that, after 9 years, at an annual interest rate of 4.75%, compounded monthly, will have a value of $\$24{,}514.01$.

Solution:

Use the future value formula, then solve for present value.

$$FV = PV\left(1 + \frac{r}{n}\right)^{nt}$$

$$24514.01 = PV\left(1 + \frac{0.0475}{12}\right)^{12\times 9}$$

$$24514.01 = PV(1.003958333)^{108}$$

$$PV = \frac{24514.01}{1.003958333^{108}}$$

$$PV = \$16{,}000.00$$

■ 2. Find the present value of a deposit that, after 3 years, at an annual interest rate of 7.85%, compounded weekly, will have a value of $\$948.99$.

Solution:

Use the future value formula, then solve for present value.

$$FV = PV\left(1+\frac{r}{n}\right)^{nt}$$

$$948.99 = PV\left(1+\frac{0.0785}{52}\right)^{52\times 3}$$

$$948.99 = PV(1.001509615)^{156}$$

$$PV = \frac{948.99}{1.001509615^{156}}$$

$$PV = \$750.00$$

■ 3. Find the present value of a deposit that, after 6 years, at an annual interest rate of $3.95\,\%$, compounded quarterly, will have a value of $\$1{,}582{,}46$.

Solution:

Use the future value formula, then solve for present value.

$$FV = PV\left(1+\frac{r}{n}\right)^{nt}$$

$$1{,}582.46 = PV\left(1+\frac{0.0395}{4}\right)^{4\times 6}$$

$$1{,}582.46 = PV(1.009875)^{24}$$

$$PV = \frac{1{,}582.46}{1.009875^{24}}$$

$$PV = \$1{,}250.00$$

SINGLE DEPOSIT, COMPOUNDED CONTINUOUSLY, FUTURE VALUE

1. Find the future value of $\$2{,}850$, after 8 years, at an annual interest rate of $1.55\,\%$, compounded continuously.

Solution:

Use the future value formula for continuously compounded interest.

$$FV = PVe^{rt}$$

$$FV = 2{,}850e^{0.0155 \times 8}$$

$$FV = 2{,}850e^{0.124}$$

$$FV = \$3{,}226.25$$

2. Find the future value of $\$9{,}875$, after 15 years, at an annual interest rate of $4.15\,\%$, compounded continuously.

Solution:

Use the future value formula for continuously compounded interest.

$$FV = PVe^{rt}$$

$$FV = 9{,}875e^{0.0415 \times 15}$$

$$FV = 9{,}875e^{0.6225}$$

$$FV = \$18{,}402.86$$

■ 3. Find the future value of $\$15{,}000$, after 18 years, at an annual interest rate of $8.5\,\%$, compounded continuously.

Solution:

Use the future value formula for continuously compounded interest.

$$FV = PVe^{rt}$$

$$FV = 15{,}000e^{0.085 \times 18}$$

$$FV = 15{,}000e^{1.53}$$

$$FV = \$69{,}272.65$$

SINGLE DEPOSIT, COMPOUNDED CONTINUOUSLY, PRESENT VALUE

■ 1. Find the present value of a deposit that, after 11 years, at an annual interest rate of 2.75%, compounded continuously, will have a value of $\$11,631.08$.

Solution:

Use the future value formula for continuously compounded interest, then solve for present value.

$$FV = PVe^{rt}$$

$$11,631.08 = PVe^{0.0275 \times 11}$$

$$11,631.08 = PVe^{0.3025}$$

$$PV = \frac{11,631.08}{e^{0.3025}}$$

$$PV = \$8,595.00$$

■ 2. Find the present value of a deposit that, after 7 years, at an annual interest rate of 6.17%, compounded continuously, will have a value of $\$3,850.45$.

Solution:

Use the future value formula for continuously compounded interest, then solve for present value.

$$FV = PVe^{rt}$$

$$3{,}850.45 = PVe^{0.0617\times7}$$

$$3{,}850.45 = PVe^{0.4319}$$

$$PV = \frac{3{,}850.45}{e^{0.4319}}$$

$$PV = \$2{,}500.00$$

■ 3. Find the present value of a deposit that, after 4 years, at an annual interest rate of $5.95\,\%$, compounded continuously, will have a value of $\$6{,}343.55$.

Solution:

Use the future value formula for continuously compounded interest, then solve for present value.

$$FV = PVe^{rt}$$

$$6{,}343.55 = PVe^{0.0595\times4}$$

$$6{,}343.55 = PVe^{0.238}$$

$$PV = \frac{6,343.55}{e^{0.238}}$$

$$PV = \$5,000.00$$

INCOME STREAM, COMPOUNDED CONTINUOUSLY, FUTURE VALUE

■ 1. Money is invested at a rate of $\$10,000$ annually and the bank pays $8.85\,\%$ interest, compounded continuously. How many years will it take for the investment to grow to a balance of $\$300,000$?

Solution:

Use the future value formula for an income stream.

$$FV = \int_0^N S(t)e^{r(N-t)}\ dt$$

$$300{,}000 = \int_0^N 10{,}000e^{0.0885(N-t)}\ dt$$

$$300{,}000 = 10{,}000\int_0^N e^{0.0885(N-t)}\ dt$$

$$30 = \int_0^N e^{0.0885(N-t)}\ dt$$

$$30 = \int_0^N e^{0.0885N-0.0885t}\ dt$$

$$30 = \int_0^N e^{0.0885N}e^{-0.0885t}\ dt$$

$$30 = e^{0.0885N} \int_0^N e^{-0.0885t} \ dt$$

Integrate, then evaluate over the interval.

$$30 = e^{0.0885N} \left(\frac{1}{-0.0885} e^{-0.0885t} \right) \Bigg|_0^N$$

$$30 = e^{0.0885N} \left(\frac{1}{-0.0885} e^{-0.0885N} \right) - e^{0.0885N} \left(\frac{1}{-0.0885} e^{-0.0885(0)} \right)$$

$$30 = -e^{0.0885N} \left(\frac{e^{-0.0885N}}{0.0885} \right) + \frac{e^{0.0885N}}{0.0885}$$

$$30(0.0885) = -e^{0.0885N} e^{-0.0885N} + e^{0.0885N}$$

$$30(0.0885) = -1 + e^{0.0885N}$$

$$30(0.0885) + 1 = e^{0.0885N}$$

$$3.655 = e^{0.0885N}$$

Solve using the natural logarithm.

$$\ln 3.655 = \ln e^{0.0885N}$$

$$\ln 3.655 = 0.0885N \ln e$$

$$\ln 3.655 = 0.0885N$$

$$\frac{\ln 3.655}{0.0885} = N$$

$$N = 14.645$$

It will take the investment approximately 14.645 years to grow to a balance of $\$300{,}000$.

■ 2. Money is invested at a rate of $\$5{,}000$ annually and the bank pays $6.75\ \%$ interest, compounded continuously. How many years will it take for the investment to grow to a balance of $\$100{,}000$?

Solution:

Use the future value formula for an income stream.

$$FV = \int_0^N S(t)e^{r(N-t)}\ dt$$

$$100{,}000 = \int_0^N 5{,}000e^{0.0675(N-t)}\ dt$$

$$100{,}000 = 5{,}000 \int_0^N e^{0.0675(N-t)}\ dt$$

$$20 = \int_0^N e^{0.0675(N-t)}\ dt$$

$$20 = \int_0^N e^{0.0675N-0.0675t}\ dt$$

$$20 = \int_0^N e^{0.0675N} e^{-0.0675t} \ dt$$

$$20 = e^{0.0675N} \int_0^N e^{-0.0675t} \ dt$$

Integrate, then evaluate over the interval.

$$20 = e^{0.0675N} \left(\frac{1}{-0.0675} e^{-0.0675t} \right) \Bigg|_0^N$$

$$20 = e^{0.0675N} \left(\frac{1}{-0.0675} e^{-0.0675N} \right) - e^{0.0675N} \left(\frac{1}{-0.0675} e^{-0.0675(0)} \right)$$

$$20 = -e^{0.0675N} \left(\frac{e^{-0.0675N}}{0.0675} \right) + \frac{e^{0.0675N}}{0.0675}$$

$$20(0.0675) = -e^{0.0675N} e^{-0.0675N} + e^{0.0675N}$$

$$20(0.0675) = -1 + e^{0.0675N}$$

$$20(0.0675) + 1 = e^{0.0675N}$$

$$2.35 = e^{0.0675N}$$

Solve using the natural logarithm.

$$\ln 2.35 = \ln e^{0.0675N}$$

$$\ln 2.35 = 0.0675N \ln e$$

$$\ln 2.35 = 0.0675N$$

$$\frac{\ln 2.35}{0.0675} = N$$

$$N = 12.658$$

It will take the investment approximately 12.658 years to grow to a balance of $\$100{,}000$.

■ 3. Money is invested at a rate of $\$2{,}500$ annually and the bank pays $5.25\,\%$ interest, compounded continuously. How many years will it take for the investment to grow to a balance of $\$25{,}000$?

Solution:

Use the future value formula for an income stream.

$$FV = \int_0^N S(t)e^{r(N-t)}\ dt$$

$$25{,}000 = \int_0^N 2{,}500e^{0.0525(N-t)}\ dt$$

$$25{,}000 = 2{,}500\int_0^N e^{0.0525(N-t)}\ dt$$

$$10 = \int_0^N e^{0.0525(N-t)}\ dt$$

$$10 = \int_0^N e^{0.0525N - 0.0525t} \ dt$$

$$10 = \int_0^N e^{0.0525N} e^{-0.0525t} \ dt$$

$$10 = e^{0.0525N} \int_0^N e^{-0.0525t} \ dt$$

Integrate, then evaluate over the interval.

$$10 = e^{0.0525N} \left(\frac{1}{-0.0525} e^{-0.0525t} \right) - e^{0.0525N} \left(\frac{1}{-0.0525} e^{-0.0525t} \right) \Bigg|_0^N$$

$$10 = e^{0.0525N} \left(\frac{1}{-0.0525} e^{-0.0525N} \right) - e^{0.0525N} \left(\frac{1}{-0.0525} e^{-0.0525(0)} \right)$$

$$10 = -e^{0.0525N} \left(\frac{e^{-0.0525N}}{0.0525} \right) + \frac{e^{0.0525N}}{0.0525}$$

$$10(0.0525) = -e^{0.0525N} e^{-0.0525N} + e^{0.0525N}$$

$$10(0.0525) = -1 + e^{0.0525N}$$

$$10(0.0525) + 1 = e^{0.0525N}$$

$$1.525 = e^{0.0525N}$$

Solve using the natural logarithm.

$$\ln 1.525 = \ln e^{0.0525N}$$

$$\ln 1.525 = 0.0525N \ln e$$

$$\ln 1.525 = 0.0525N$$

$$\frac{\ln 1.525}{0.0525} = N$$

$$N = 8.038$$

It will take the investment approximately 8.038 years to grow to a balance of $\$25{,}000$.

INCOME STREAM, COMPOUNDED CONTINUOUSLY, PRESENT VALUE

1. Suppose that money is deposited steadily into an account at a constant rate of $\$15{,}000$ per year for 13 years. Find the present value of this income stream if the account pays $7.35\,\%$, compounded continuously.

Solution:

Use the present value formula for an income stream.

$$PV = \int_0^T S(t)e^{-rt}\ dt$$

$$PV = \int_0^{13} 15{,}000e^{-0.0735t}\ dt$$

Integrate, then evaluate over the interval.

$$PV = 15{,}000\left(\frac{e^{-0.0735t}}{-0.0735}\right)\Bigg|_0^{13}$$

$$PV = 15{,}000\left(\frac{e^{-0.0735(13)}}{-0.0735} - \frac{e^{-0.0735(0)}}{-0.0735}\right)$$

$$PV = 15{,}000(8.372519911)$$

$$PV = \$125{,}587.80$$

2. Suppose that money is deposited steadily into a college fund at a constant rate of $\$3{,}000$ per year for 18 years. Find the present value of this income stream if the account pays $5.15\,\%$, compounded continuously.

Solution:

Use the present value formula for an income stream.

$$PV = \int_0^T S(t)e^{-rt}\ dt$$

$$PV = \int_0^{18} 3{,}000e^{-0.0515t}\ dt$$

Integrate, then evaluate over the interval.

$$PV = 3{,}000\left(\frac{e^{-0.0515t}}{-0.0515}\right)\Bigg|_0^{18}$$

$$PV = 3{,}000\left(\frac{e^{-0.0515(18)}}{-0.0515} - \frac{e^{-0.0515(0)}}{-0.0515}\right)$$

$$PV = 3{,}000(11.73322041)$$

$$PV = \$35{,}199.66$$

3. Suppose that money is deposited steadily into a new car account at a constant rate of $\$2{,}500$ per year for 8 years. Find the present value of this income stream if the account pays $7.5\,\%$, compounded continuously.

Solution:

Use the present value formula for an income stream.

$$PV = \int_0^T S(t)e^{-rt}\ dt$$

$$PV = \int_0^8 2{,}500e^{-0.075t}\ dt$$

Integrate, then evaluate over the interval.

$$PV = 2{,}500\left(\frac{e^{-0.075t}}{-0.075}\right)\Bigg|_0^8$$

$$PV = 2{,}500\left(\frac{e^{-0.075(8)}}{-0.075} - \frac{e^{-0.075(0)}}{-0.075}\right)$$

$$PV = 2{,}500(6.015844852)$$

$$PV = \$15{,}039.61$$

CONSUMER AND PRODUCER SURPLUS

1. Find the equilibrium quantity q_e and the equilibrium price p_e.

$$S(q) = 0.06q^2 + 5$$

$$D(q) = 0.1q + 17$$

Solution:

The equilibrium point is where the supply curve $S(q)$ and the demand curve $D(q)$ intersect. The equilibrium quantity is the x-value of the intersection point and the equilibrium price is the y-value of the intersection point. Set the supply equation equal to the demand equation and find their intersection point.

$$0.06q^2 + 5 = 0.1q + 17$$

$$0.06q^2 + 5 - 0.1q - 17 = 0$$

$$0.06q^2 - 0.1q - 12 = 0$$

$$6q^2 + 10q - 1{,}200 = 0$$

$$(6q + 80)(q - 15) = 0$$

Then the solutions for q are

$$6q + 80 = 0$$

$$6q = -80$$

$$q = -\frac{40}{3}$$

and

$$q - 15 = 0$$

$$q = 15$$

Since q is a quantity, the answer must be positive, so discard $q = -40/3$ as a possible solution, and accept the equilibrium quantity of $q = 15$. Use the equilibrium quantity to find the equilibrium price.

$$D(q) = 0.1q + 17$$

$$D(15) = 0.1(15) + 17$$

$$D(15) = 18.50$$

$$p = 18.50$$

Then the equilibrium quantity and equilibrium price are

$$q_e = 15$$

$$p_e = 18.50$$

2. Find the consumer surplus.

$$S(q) = 0.05q^2 + 7$$

$$D(q) = -0.2q + 11.8$$

Solution:

Find equilibrium quantity by setting the curves equal to one another.

$$0.05q^2 + 7 = -0.2q + 11.8$$

$$0.05q^2 + 7 + 0.2q - 11.8 = 0$$

$$0.05q^2 + 0.2q - 4.8 = 0$$

$$5q^2 + 20q - 480 = 0$$

$$(5q - 40)(q + 12) = 0$$

Then the solutions for q are

$$5q - 40 = 0$$

$$5q = 40$$

$$q = 8$$

and

$$q + 12 = 0$$

$$q = -12$$

Since q is a quantity, the answer must be positive, so discard $q = -12$ as a possible solution, and accept the equilibrium quantity of $q = 8$. Use the equilibrium quantity to find the equilibrium price.

$$D(q) = -0.2q + 11.8$$

$$D(15) = -0.2(8) + 11.8$$

$$D(15) = 10.20$$

$$p = 10.20$$

Then the equilibrium quantity and equilibrium price are

$$q_e = 8$$

$$p_e = 10.20$$

Then the consumer surplus will be

$$CS = \int_0^{q_e} D(q)\ dq - p_e q_e$$

$$CS = \int_0^8 -0.2q + 11.8\ dq - (10.20)(8)$$

$$CS = -0.1q^2 + 11.8q \Big|_0^8 - 81.6$$

$$CS = -0.1(8)^2 + 11.8(8) - (-0.1(0)^2 + 11.8(0)) - 81.6$$

$$CS = -6.4 + 94.4 - 81.6$$

$$CS = -6.4 + 94.4 - 81.6$$

$$CS = 6.4$$

■ 3. Find the equilibrium quantity q_e and the equilibrium price p_e.

$$S(q) = 0.09q^2 + 8$$

$$D(q) = 1.55q + 25.5$$

Solution:

The equilibrium point is where the supply curve $S(q)$ and the demand curve $D(q)$ intersect. The equilibrium quantity is the x-value of the intersection point and the equilibrium price is the y-value of the intersection point. Set the supply equation equal to the demand equation and find their intersection point.

$$0.09q^2 + 8 = 1.55q + 25.5$$

$$0.09q^2 + 8 - 1.55q - 25.5 = 0$$

$$0.09q^2 - 1.55q - 17.5 = 0$$

$$9q^2 - 155q - 1{,}750 = 0$$

$$(9q + 70)(q - 25) = 0$$

Then the solutions for q are

$$9q + 70 = 0$$

$$9q = -70$$

$$q = -\frac{70}{9}$$

and

$$q - 25 = 0$$

$$q = 25$$

Since q is a quantity, the answer must be positive, so discard $q = -70/9$ as a possible solution, and accept the equilibrium quantity of $q = 25$. Use the equilibrium quantity to find the equilibrium price.

$$D(q) = 1.55q + 25.5$$

$$D(15) = 1.55(25) + 25.5$$

$$D(15) = 64.25$$

$$p = 64.25$$

Then the equilibrium quantity and equilibrium price are

$$q_e = 25$$

$$p_e = 64.25$$

W W W . K R I S T A K I N G M A T H . C O M