Topic: Ratio test with factorials

Question: Use the ratio test to determine the convergence of the series.

$$\sum_{n=1}^{\infty} \frac{x^n}{n!}$$

Answer choices:

A The series converges

B The series conditionally converges

C The series diverges

D None of these

Solution: A

The ratio test for convergence lets us calculate L as

$$L = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|$$

and then says that the series

converges if $L < 1$

diverges if $L > 1$

The test is inconclusive if $L = 1$.

To find L, we'll need a_n and a_{n+1}.

$$a_n = \frac{x^n}{n!}$$

$$a_{n+1} = \frac{x^{n+1}}{(n+1)!}$$

Plugging these into the formula for L from the ratio test, we get

$$L = \lim_{n\to\infty} \left| \frac{\frac{x^{n+1}}{(n+1)!}}{\frac{x^n}{n!}} \right|$$

$$L = \lim_{n\to\infty} \left| \frac{x^{n+1}}{(n+1)!} \cdot \frac{n!}{x^n} \right|$$

Pairing similar numerators and denominators together, we get

$$L = \lim_{n\to\infty} \left| \frac{x^{n+1}}{x^n} \cdot \frac{n!}{(n+1)!} \right|$$

Expanding the factorials so that we can get an idea of what we can cancel, and then canceling terms, we get

$$L = \lim_{n\to\infty} \left| x^{n+1-n} \cdot \frac{n(n-1)(n-2)(n-3)\ldots}{(n+1)(n+1-1)(n+1-2)(n+1-3)(n+1-4)\ldots} \right|$$

$$L = \lim_{n\to\infty} \left| x \cdot \frac{n(n-1)(n-2)(n-3)\ldots}{(n+1)(n)(n-1)(n-2)(n-3)\ldots} \right|$$

$$L = \lim_{n\to\infty} \left| x \cdot \frac{1}{n+1} \right|$$

Since the limit only effects n, we can pull x outside of the limit.

$$L = x \lim_{n\to\infty} \left| \frac{1}{n+1} \right|$$

$$L = x(0)$$

$$L = 0$$

or

$$L = 0 < 1$$

Therefore, the series is convergent for all $x \in R$.

Topic: Ratio test with factorials

Question: Use the ratio test to determine the convergence of the series.

$$\sum_{n=1}^{\infty} \frac{3^n}{n!}$$

Answer choices:

A The series converges

B The series conditionally converges

C The series diverges

D None of these

Solution: A

The ratio test for convergence lets us calculate L as

$$L = \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right|$$

and then says that the series

converges if $L < 1$

diverges if $L > 1$

The test is inconclusive if $L = 1$.

To find L, we'll need a_n and a_{n+1}.

$$a_n = \frac{3^n}{n!}$$

$$a_{n+1} = \frac{3^{n+1}}{(n+1)!}$$

Plugging these into the formula for L from the ratio test, we get

$$L = \lim_{n \to \infty} \left| \frac{\frac{3^{n+1}}{(n+1)!}}{\frac{3^n}{n!}} \right|$$

$$L = \lim_{n \to \infty} \left| \frac{3^{n+1}}{(n+1)!} \cdot \frac{n!}{3^n} \right|$$

Pairing similar numerators and denominators together, we get

$$L = \lim_{n \to \infty} \left| \frac{3^{n+1}}{3^n} \cdot \frac{n!}{(n+1)!} \right|$$

Expanding the factorials so that we can get an idea of what we can cancel, and then canceling terms, we get

$$L = \lim_{n \to \infty} \left| 3^{n+1-n} \cdot \frac{n(n-1)(n-2)(n-3)\dots}{(n+1)(n+1-1)(n+1-2)(n+1-3)(n+1-4)\dots} \right|$$

$$L = \lim_{n \to \infty} \left| 3^1 \cdot \frac{n(n-1)(n-2)(n-3)\dots}{(n+1)(n)(n-1)(n-2)(n-3)\dots} \right|$$

$$L = 3 \lim_{n \to \infty} \left| \frac{1}{n+1} \right|$$

$$L = 3(0)$$

$$L = 0$$

or

$$L = 0 < 1$$

Therefore, the series is convergent for all $x \in R$.

Topic: Ratio test with factorials

Question: Use the ratio test to determine the convergence of the series.

$$\sum_{n=1}^{\infty} \frac{n!}{4^n}$$

Answer choices:

A The series converges

B The series conditionally converges

C The series diverges

D None of these

Solution: C

The ratio test for convergence lets us calculate L as

$$L = \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right|$$

and then says that the series

converges if $L < 1$

diverges if $L > 1$

The test is inconclusive if $L = 1$.

To find L, we'll need a_n and a_{n+1}.

$$a_n = \frac{n!}{4^n}$$

$$a_{n+1} = \frac{(n+1)!}{4^{n+1}}$$

Plugging these into the formula for L from the ratio test, we get

$$L = \lim_{n \to \infty} \left| \frac{\frac{(n+1)!}{4^{n+1}}}{\frac{n!}{4^n}} \right|$$

$$L = \lim_{n \to \infty} \left| \frac{(n+1)!}{4^{n+1}} \cdot \frac{4^n}{n!} \right|$$

Pairing similar numerators and denominators together, we get

$$L = \lim_{n\to\infty} \left| \frac{4^n}{4^{n+1}} \cdot \frac{(n+1)!}{n!} \right|$$

Expanding the factorials so that we can get an idea of what we can cancel, and then canceling terms, we get

$$L = \lim_{n\to\infty} \left| 4^{n-(n+1)} \cdot \frac{(n+1)(n+1-1)(n+1-2)(n+1-3)(n+1-4)\ldots}{n(n-1)(n-2)(n-3)\ldots} \right|$$

$$L = \lim_{n\to\infty} \left| 4^{n-n-1} \cdot \frac{(n+1)(n)(n-1)(n-2)(n-3)\ldots}{n(n-1)(n-2)(n-3)\ldots} \right|$$

$$L = \lim_{n\to\infty} \left| 4^{-1} \cdot \frac{n+1}{1} \right|$$

$$L = \lim_{n\to\infty} \left| \frac{1}{4} \cdot (n+1) \right|$$

$$L = \frac{1}{4} \lim_{n\to\infty} \left| n+1 \right|$$

$$L = \frac{1}{4}(\infty + 1)$$

$$L = \frac{\infty}{4}$$

$$L = \infty$$

or

$$L = \infty > 1$$

Therefore, the series is divergent for all $x \in R$.