Topic: Formula for the general term

Question: Find a formula for the general term of the sequence.

$$1, 4, 7, 10, 13, \ldots$$

Answer choices:

A $a_n = 3n - 2$

B $a_n = 3n + 2$

C $a_n = 5n - 4$

D $a_n = 5n + 4$

Solution: A

The general term of a sequence is a single term that can represent every term in the sequence, based on the value of n that we pick. In other words, if the general term of a sequence is $1/n$, and the sequence starts at $n=1$, then we start plugging $n=1$, $n=2$, $n=3$, $n=4$, etc. into the general term, and we get the expanded sequence

$$\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ \dots$$

Oftentimes we're given the expanded sequence and asked to find the general term that represents it, so it's like we're working backwards.

The easiest way to find the general term is to look at each part of our sequence, and find its relationship to the corresponding n value.

The first thing we want to do is match each term in our expanded sequence with its n value. For the sequence we've been given in this problem, we'll get

$n=1$	$n=2$	$n=3$	$n=4$	$n=5$
1	4	7	10	13

You want to check first to see if there's a value you can add to n, a value you can subtract from n, or a value you can subtract n from. But in this case, the difference between n and each of the terms is different.

$$n=1 \qquad 1=1+0=n$$

$$n=2 \qquad 4=2+2=n+2$$

$$n = 3 \qquad 7 = 3 + 4 = n + 4$$

$$n = 4 \qquad 10 = 4 + 6 = n + 6$$

$$n = 5 \qquad 13 = 5 + 8 = n + 8$$

Since that doesn't work, we'll look to see if we can multiply n or divide n by some value that will give us the denominators from our expanded sequence. Unfortunately, that won't work either. However, if we look at $3n$, we can see a consistent difference between $3n$ and our terms.

$$n = 1 \qquad 1 = 3(1) - 2 = 3n - 2$$

$$n = 2 \qquad 4 = 3(2) - 2 = 3n - 2$$

$$n = 3 \qquad 7 = 3(3) - 2 = 3n - 2$$

$$n = 4 \qquad 10 = 3(4) - 2 = 3n - 2$$

$$n = 5 \qquad 13 = 3(5) - 2 = 3n - 2$$

Since every term can be represented by $3n - 2$, this will be the formula for the general term.

$$a_n = 3n - 2$$

We can always double-check ourselves by testing the general term at $n = 1, 2, 3, 4, 5, \ \dots$. If the formula for the general term returns the terms we were given in the original expanded sequence, then we know that our general term accurately represents the sequence.

Another thing to note: If you encounter this type of problem in a multiple choice test, keep in mind that you can always just plug $n = 1, 2, 3, 4, 5, \ \dots$

into each of the answer choices, to see if the values you get match the original expanded sequence.

Topic: Formula for the general term

Question: Find a formula for the general term of the sequence.

$$-1, \frac{2}{3}, -\frac{3}{5}, \frac{4}{7}, -\frac{5}{9}, \ldots$$

Answer choices:

A $\quad a_n = (-1)^n \frac{1}{2n-1}$

B $\quad a_n = (-1)^{n+1} \frac{1}{2n-1}$

C $\quad a_n = (-1)^n \frac{n}{2n-1}$

D $\quad a_n = (-1)^{n+1} \frac{n-1}{2n-1}$

Solution: C

The general term of a sequence is a single term that can represent every term in the sequence, based on the value of n that we pick. In other words, if the general term of a sequence is $1/n$, and the sequence starts at $n = 1$, then we start plugging $n = 1$, $n = 2$, $n = 3$, $n = 4$, etc. into the general term, and we get the expanded sequence

$$\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ \dots$$

Oftentimes we're given the expanded sequence and asked to find the general term that represents it, so it's like we're working backwards.

The easiest way to find the general term is to look at each part of our sequence, and find its relationship to the corresponding n value.

The first thing we want to do is match each term in our expanded sequence with its n value. For the sequence we've been given in this problem, we'll get

$$\begin{array}{ccccc} n = 1 & n = 2 & n = 3 & n = 4 & n = 5 \\ \\ -1 & \dfrac{2}{3} & -\dfrac{3}{5} & \dfrac{4}{7} & -\dfrac{5}{9} \end{array}$$

We can start by noticing that the numerator of each term is equal to its n value. So we'll put n in the numerator of the formula for the general term.

$$a_n = \frac{n}{}$$

The denominator of the general term isn't quite so obvious. You want to check first to see if there's a value you can add to n, a value you can subtract from n, or a value you can subtract n from. But in this case, the difference between n and each of the denominators is different.

$$n = 1 \qquad 1 = 1 + 0 = n$$

$$n = 2 \qquad 3 = 2 + 1 = n + 1$$

$$n = 3 \qquad 5 = 3 + 2 = n + 2$$

$$n = 4 \qquad 7 = 4 + 3 = n + 3$$

$$n = 5 \qquad 9 = 5 + 4 = n + 4$$

Since that doesn't work, we'll look to see if we can multiply n or divide n by some value that will give us the denominators from our expanded sequence. Unfortunately, that won't work either. However, if we look at $2n$, we can see a consistent difference between $2n$ and our denominators.

$$n = 1 \qquad 1 = 2(1) - 1 = 2n - 1$$

$$n = 2 \qquad 3 = 2(2) - 1 = 2n - 1$$

$$n = 3 \qquad 5 = 2(3) - 1 = 2n - 1$$

$$n = 4 \qquad 7 = 2(4) - 1 = 2n - 1$$

$$n = 5 \qquad 9 = 2(5) - 1 = 2n - 1$$

Since every denominator can be represented by $2n - 1$, we'll put this into the formula for the general term, along with the numerator we already found, and we'll get the formula for the general term:

$$a_n = \frac{n}{2n-1}$$

Before we're done though, we need to analyze our alternating negative signs. When a negative sign alternates in a sequence and the first term is negative, the sequence must be multiplied by $(-1)^n$. Once we add this multiplier to the general term, we'll have the full formula:

$$a_n = (-1)^n \frac{n}{2n-1}$$

We can always double-check ourselves by testing the general term at $n = 1, 2, 3, 4, 5, \ ...$. If the formula for the general term returns the terms we were given in the original expanded sequence, then we know that our general term accurately represents the sequence.

Another thing to note: If you encounter this type of problem in a multiple choice test, keep in mind that you can always just plug $n = 1, 2, 3, 4, 5, \ ...$ into each of the answer choices, to see if the values you get match the original expanded sequence.

Topic: Formula for the general term

Question: Find a formula for the general term of the sequence.

$$\left\{\frac{1}{2},\frac{4}{2},\frac{9}{2},\frac{16}{2}\right\}$$

Answer choices:

A $\quad a_n=\frac{n^2}{n+1}$

B $\quad a_n=\frac{n^2+2}{4}$

C $\quad a_n=\frac{n^2}{2}$

D $\quad a_n=\frac{n+2}{2}$

Solution: C

The general term of a sequence is a single term that can represent every term in the sequence, based on the value of n that we pick. In other words, if the general term of a sequence is $1/n$, and the sequence starts at $n=1$, then we start plugging $n=1$, $n=2$, $n=3$, $n=4$, etc. into the general term, and we get the expanded sequence

$$\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ \dots$$

Oftentimes we're given the expanded sequence and asked to find the general term that represents it, so it's like we're working backwards.

The easiest way to find the general term is to look at each part of our sequence, and find its relationship to the corresponding n value.

The first thing we want to do is match each term in our expanded sequence with its n value. For the sequence we've been given in this problem, we'll get

$n=1$	$n=2$	$n=3$	$n=4$
$\frac{1}{2}$	$\frac{4}{2}$	$\frac{9}{2}$	$\frac{16}{2}$

We can start by noticing that the denominator of each term, regardless of its n value, is equal to 2. So we'll put a 2 in the denominator of the formula for the general term.

$$a_n = \frac{}{2}$$

Turning to the numerator, we can see that each numerator is the square of its n value.

$$n = 1 \qquad 1 = 1^2 = n^2$$

$$n = 2 \qquad 4 = 2^2 = n^2$$

$$n = 3 \qquad 9 = 3^3 = n^2$$

$$n = 4 \qquad 16 = 4^2 = n^2$$

Since every numerator can be represented by n^2, we'll put this into the formula for the general term, along with the denominator we already found, and we'll get the formula for the general term:

$$a_n = \frac{n^2}{2}$$

We can always double-check ourselves by testing the general term at $n = 1, 2, 3, 4, 5, \ldots$. If the formula for the general term returns the terms we were given in the original expanded sequence, then we know that our general term accurately represents the sequence.

Another thing to note: If you encounter this type of problem in a multiple choice test, keep in mind that you can always just plug $n = 1, 2, 3, 4, 5, \ldots$ into each of the answer choices, to see if the values you get match the original expanded sequence.