Recognizing types of series

Use this visual guide to train yourself to quickly recognize different types of series. Sometimes the best way to recognize the type of series is to write out the first few terms of the series and then match it to one of the types of series below.

Geometric series

$$2.3171717... = 2.3 + \frac{17}{10^3} + \frac{17}{10^5} + \frac{17}{10^7} + \ldots = 2.3 + \frac{17}{10^3}\left(1 + \frac{1}{10^2} + \frac{1}{10^4} + \ldots\right)$$

$$5 - \frac{10}{3} + \frac{20}{9} - \frac{40}{27} + \ldots = 5\left[1 - \frac{2}{3} + \left(\frac{2}{3}\right)^2 - \left(\frac{2}{3}\right)^3 + \ldots\right]$$

$$4 + 3 + \frac{9}{4} + \frac{27}{16} + \ldots = 4\left[1 + \frac{3}{4} + \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3 + \ldots\right]$$

$$3 - 4 + \frac{16}{3} - \frac{64}{9} + \ldots = 3\left[1 - \frac{4}{3} + \left(\frac{4}{3}\right)^2 - \left(\frac{4}{3}\right)^3 + \ldots\right]$$

$$10 - 2 + 0.4 - 0.08 + \ldots = 10\left[1 - 0.2 + (0.2)^2 - (0.2)^3 + \ldots\right]$$

$$2 + 0.5 + 0.125 + 0.03125 + = 2\left[1 + 0.25 + (0.25)^2 + (0.25)^3 + \ldots\right]$$

p-series

$$\sum_{n=1}^{\infty} \frac{1}{n^3} = \frac{1}{1^3} + \frac{1}{2^3} + \frac{1}{3^3} + \frac{1}{4^3} + \ldots$$

$$\sum_{n=1}^{\infty} \frac{1}{n^{\frac{1}{3}}} = \sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n}} = 1 + \frac{1}{\sqrt[3]{2}} + \frac{1}{\sqrt[3]{3}} + \frac{1}{\sqrt[3]{4}} + \dots$$

Telescoping series

$$\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1}\right) = \left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \dots + \left(\frac{1}{n} - \frac{1}{n+1}\right)$$

Alternating series

$(-1)^n$

Will start with −

$$\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n}}{2n+3}$$

$$\sum_{n=1}^{\infty} (-1)^n \frac{n}{\sqrt{n^3+2}}$$

$$\sum_{n=1}^{\infty} (-1)^n \frac{3n-1}{2n+1}$$

$$\sum_{n=1}^{\infty} (-1)^n e^{-n}$$

$(-1)^{n-1}$

Will start with +

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n+1}$$

$$\sum_{n=1}^{\infty} (-1)^{n-1} e^{\frac{2}{n}}$$

$$\sum_{n=1}^{\infty} (-1)^{n-1} \arctan n$$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\ln(n+4)}$$

$(-1)^{n+1}$

Will start with +

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^2}{n^3+4}$$

$$\sum_{n=1}^{\infty} (-1)^{n+1} n e^{-n}$$