Topic: Maclaurin series to evaluate a limit

Question: Use a Maclaurin series to evaluate the limit.

$$\lim_{x \to 0} \frac{e^x - 1 - x}{x^2}$$

Answer choices:

A $\quad 2$

B $\quad -\frac{1}{2}$

C $\quad -2$

D $\quad \frac{1}{2}$

Solution: D

When we're asked to use a Maclaurin series to evaluate a limit, we're supposed to use a known Maclaurin series expansion in place of part of the function, such that we turn the function into a polynomial expression.

The Maclaurin series expansion of e^x is

$$e^x = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \dots$$

so we'll substitute the first few terms of this expansion into the limit we've been given.

$$\lim_{x \to 0} \frac{e^x - 1 - x}{x^2} = \lim_{x \to 0} \frac{1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \dots - 1 - x}{x^2}$$

$$\lim_{x \to 0} \frac{e^x - 1 - x}{x^2} = \lim_{x \to 0} \frac{\frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \dots}{x^2}$$

$$\lim_{x \to 0} \frac{e^x - 1 - x}{x^2} = \lim_{x \to 0} \frac{1}{2} + \frac{1}{6}x + \frac{1}{24}x^2 + \dots$$

$$\lim_{x \to 0} \frac{e^x - 1 - x}{x^2} = \frac{1}{2} + \frac{1}{6}(0) + \frac{1}{24}(0)^2 + \dots$$

$$\lim_{x \to 0} \frac{e^x - 1 - x}{x^2} = \frac{1}{2}$$

The limit of the function is $1/2$.

Topic: Maclaurin series to evaluate a limit

Question: Use a Maclaurin series to evaluate the limit.

$$\lim_{x \to 0} \frac{\ln(1+2x) - 2x}{x^2}$$

Answer choices:

A 2

B $-\frac{1}{2}$

C -2

D $\frac{1}{2}$

Solution: C

When we're asked to use a Maclaurin series to evaluate a limit, we're supposed to use a known Maclaurin series expansion in place of part of the function, such that we turn the function into a polynomial expression.

The Maclaurin series expansion of $\ln(1+x)$ is

$$\ln(1+x)=x-\frac{1}{2}x^2+\frac{1}{3}x^3-\frac{1}{4}x^4+\dots$$

This is really similar to the part of the function we've been given, $\ln(1+2x)$. We just have to substitute $2x$ for x.

$$\ln(1+2x)=2x-\frac{1}{2}(2x)^2+\frac{1}{3}(2x)^3-\frac{1}{4}(2x)^4+\dots$$

$$\ln(1+2x)=2x-2x^2+\frac{8}{3}x^3-4x^4+\dots$$

Now we'll substitute the first few terms of this expansion into the limit we've been given.

$$\lim_{x\to 0}\frac{\ln(1+2x)-2x}{x^2}=\lim_{x\to 0}\frac{2x-2x^2+\frac{8}{3}x^3-4x^4+\dots-2x}{x^2}$$

$$\lim_{x\to 0}\frac{\ln(1+2x)-2x}{x^2}=\lim_{x\to 0}\frac{-2x^2+\frac{8}{3}x^3-4x^4+\dots}{x^2}$$

$$\lim_{x\to 0}\frac{\ln(1+2x)-2x}{x^2}=\lim_{x\to 0}-2+\frac{8}{3}x-4x^2+\dots$$

$$\lim_{x\to 0}\frac{\ln(1+2x)-2x}{x^2}=-2+\frac{8}{3}(0)-4(0)^2+\dots$$

$$\lim_{x\to 0} \frac{\ln(1+2x)-2x}{x^2} = -2$$

The limit of the function is -2.

Topic: Maclaurin series to evaluate a limit

Question: Use a Maclaurin series to evaluate the limit.

$$\lim_{x \to 0} \frac{2 \sin x - 2x}{x^3}$$

Answer choices:

A $\frac{1}{3}$

B $-\frac{1}{2}$

C $-\frac{1}{3}$

D $\frac{1}{2}$

Solution: C

When we're asked to use a Maclaurin series to evaluate a limit, we're supposed to use a known Maclaurin series expansion in place of part of the function, such that we turn the function into a polynomial expression.

The Maclaurin series expansion of $\sin x$ is

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$$

This is really similar to the part of the function we've been given, $2\sin x$. We just have to multiply by 2.

$$2\sin x = 2x - \frac{2x^3}{3!} + \frac{2x^5}{5!} - \dots$$

$$2\sin x = 2x - \frac{x^3}{3} + \frac{x^5}{60} - \dots$$

Now we'll substitute the first few terms of this expansion into the limit we've been given.

$$\lim_{x \to 0} \frac{2\sin x - 2x}{x^3} = \lim_{x \to 0} \frac{2x - \frac{x^3}{3} + \frac{x^5}{60} - \dots - 2x}{x^3}$$

$$\lim_{x \to 0} \frac{2\sin x - 2x}{x^3} = \lim_{x \to 0} \frac{-\frac{x^3}{3} + \frac{x^5}{60} - \dots}{x^3}$$

$$\lim_{x \to 0} \frac{2\sin x - 2x}{x^3} = \lim_{x \to 0} -\frac{1}{3} + \frac{x^2}{60} - \dots$$

$$\lim_{x\to 0} \frac{2\sin x - 2x}{x^3} = -\frac{1}{3} + \frac{0^2}{60} - \dots$$

$$\lim_{x\to 0} \frac{2\sin x - 2x}{x^3} = -\frac{1}{3}$$

The limit of the function is $-1/3$.