Topic: Find f given f'''

Question: Find $f(x)$ if $f'''(x) = 24x$.

Answer choices:

A $\quad f(x) = x^4 + Cx^2 + Dx + E$

B $\quad f(x) = x^4$

C $\quad f(x) = x^3 - 2x^2 + Cx + D$

D $\quad f(x) = \frac{x^4}{4} - \frac{x^2}{2} + x + C$

Solution: A

The question asks us to find the function $f(x)$ if the third derivative of the function is $f'''(x) = 24x$.

Note that the question does not provide initial values of $f(x)$, $f'(x)$ or $f''(x)$ so our answer will be a family of possible $f(x)$ functions that could have the same third derivative.

We are given the third derivative of the function. To find the second derivative of the function, find the anti-derivative of the third derivative. To find the first derivative of the function, find the anti-derivative of the second derivative. Then, to find the function, we repeat the process by finding the anti-derivative of the first derivative.

The third derivative is a polynomial monomial function. To find the anti-derivative, add 1 to the exponent and divide the term by the new exponent.

Additionally, all functions "could" contain a constant term, which becomes zero when we take the derivative of the function. Thus, when we find the anti-derivative, we add a constant labeled "C" to add the possibility of a constant term in the function, although we do not know what that constant is.

First we'll write the third derivative showing the exponent.

$$f'''(x) = 24x = 24x^1$$

Then, we integrate to find the anti-derivative.

$$f''(x) = \int 24x^1 \ dx$$

$$f''(x) = \frac{24x^{1+1}}{2} + C$$

Simplify to finish finding the second derivative.

$$f''(x) = 12x^2 + C$$

Next, find the first derivative by repeating the process.

$$f''(x) = 12x^2 + Cx^0$$

Once again, we will add a constant to cover the likely event that the original function had a constant term that became zero when the first derivative was taken. We do not know that the new constant is the same as the old constant so we will call it "D".

$$f'(x) = \int 12x^2 + Cx^0 \ dx$$

$$f'(x) = \frac{12}{3}x^{2+1} + \frac{Cx^{0+1}}{1} + D$$

$$f'(x) = 4x^3 + Cx + D$$

Now, find the function by repeating the process again.

$$f'(x) = 4x^3 + Cx + D$$

$$f'(x) = 4x^3 + Cx^1 + Dx^0$$

Once again, we will add a constant to cover the likely event that the original function had a constant term that became zero when the first derivative was taken. We do not know that the new constant is the same as the old constants so we will call it “E”.

$$f(x) = \int 4x^3 + cx^1 + dx^0\ dx$$

$$f(x) = \frac{4x^{3+1}}{4} + \frac{Cx^{1+1}}{2} + \frac{Dx^{0+1}}{1} + E$$

Since the letter “C” is an arbitrary constant, we can ignore its division by 2. After we simplify each term, the function is

$$f(x) = x^4 + Cx^2 + Dx + E$$

Topic: Find f given f'''

Question: Find $f(x)$ if $f'''(x) = 336x^5 - 120x^3 + 24x$.

Answer choices:

A $\quad f(x) = \dfrac{x^8}{7} - \dfrac{x^6}{3} + \dfrac{x^4}{2} + Cx^2 + Dx + E$

B $\quad f(x) = x^8 - x^6 + x^4$

C $\quad f(x) = x^8 - x^6 + x^4 + C$

D $\quad f(x) = x^8 - x^6 + x^4 + Cx^2 + Dx + E$

Solution: D

The question asks us to find the function $f(x)$ if the third derivative of the function is $f'''(x) = 336x^5 - 120x^3 + 24x$.

Note that the question does not provide initial values of $f(x)$, $f'(x)$, or $f''(x)$ so our answer will be a family of possible $f(x)$ functions that could have the same third derivative.

We are given the third derivative of the function. To find the second derivative of the function, find the anti-derivative of the third derivative. To find the first derivative of the function, find the anti-derivative of the second derivative. Then, to find the function, we repeat the process by finding the anti-derivative of the first derivative.

The third derivative is a polynomial function. To find the anti-derivative, add 1 to the exponent and divide the term by the new exponent.

Additionally, all functions "could" contain a constant term, which becomes zero when we take the derivative of the function. Thus, when we find the anti-derivative, we add a constant labeled "C" to add the possibility of a constant term in the function, although we do not know what that constant is.

We will, first, write the third derivative showing the exponent.

$$f'''(x) = 336x^5 - 120x^3 + 24x^1$$

Next, we integrate to find the anti-derivative.

$$f''(x) = \int 336x^5 - 120x^3 + 24x^1 \ dx$$

$$f''(x) = \frac{336x^{5+1}}{6} - \frac{120x^{3+1}}{4} + \frac{24x^{1+1}}{2} + C$$

Simplify to finish finding the second derivative.

$$f''(x) = 56x^6 - 30x^4 + 12x^2 + C$$

Next, find the first derivative by repeating the process.

$$f''(x) = 56x^6 - 30x^4 + 12x^2 + Cx^0$$

Once again, we will add a constant to cover the likely event that the original function had a constant term that became zero when the first derivative was taken. We do not know that the new constant is the same as the old constant so we will call it "D".

$$f'(x) = \int 56x^6 - 30x^4 + 12x^2 + Cx^0\ dx$$

$$f'(x) = \frac{56x^{6+1}}{7} - \frac{30x^{4+1}}{5} + \frac{12x^{2+1}}{3} + \frac{Cx^{0+1}}{1} + D$$

$$f'(x) = 8x^7 - 6x^5 + 4x^3 + Cx + D$$

Now, find the function by repeating the process.

$$f'(x) = 8x^7 - 6x^5 + 4x^3 + Cx + D$$

$$f'(x) = 8x^7 - 6x^5 + 4x^3 + Cx^1 + Dx^0$$

Once again, we will add a constant to cover the likely event that the original function had a constant term that became zero when the first derivative was taken. We do not know that the new constant is the same as the old constants so we will call it "E".

$$f(x) = \int 8x^7 - 6x^5 + 4x^3 + Cx^1 + Dx^0 \ dx$$

$$f(x) = \frac{8x^{7+1}}{8} - \frac{6x^{5+1}}{6} + \frac{4x^{3+1}}{4} + \frac{Cx^{1+1}}{2} + \frac{Dx^{0+1}}{1} + E$$

Since the letter "C" is an arbitrary constant, we can ignore its division by 2. After we simplify each term, the function is

$$f(x) = x^8 - x^6 + x^4 + Cx^2 + Dx + E$$

Topic: Find f given f’’’

Question: Find $f(x)$.

$$f'''(x) = -\frac{2}{9}x^{-\frac{4}{3}} - \frac{5}{16}x^{-\frac{9}{4}}$$

Answer choices:

A $\quad \frac{3}{5}x^{\frac{5}{3}} - \frac{4}{3}x^{-\frac{3}{4}}$

B $\quad x^{\frac{5}{3}} - x^{-\frac{3}{4}} + Cx^2 + Dx + E$

C $\quad \frac{3}{5}x^{\frac{5}{3}} - \frac{4}{3}x^{\frac{3}{4}} + Cx^2 + Dx + E$

D $\quad \frac{3}{5}x^{\frac{5}{3}} - \frac{4}{3}x^{-\frac{3}{4}} + C$

Solution: C

The question asks us to find the function $f(x)$ if the third derivative of the function is

$$f'''(x) = -\frac{2}{9}x^{-\frac{4}{3}} - \frac{5}{16}x^{-\frac{9}{4}}$$

Note that the question does not provide initial values of $f(x)$, $f'(x)$, or $f''(x)$ so our answer will be a family of possible $f(x)$ functions that could have the same third derivative.

We are given the third derivative of the function. To find the second derivative of the function, find the anti-derivative of the third derivative. To find the first derivative of the function, find the anti-derivative of the second derivative. Then, to find the function, we repeat the process by finding the anti-derivative of the first derivative.

The third derivative is a function with rational exponents. To find the anti-derivative, add 1 to the exponent and divide the term by the new exponent.

Additionally, all functions "could" contain a constant term, which becomes zero when we take the derivative of the function. Thus, when we find the anti-derivative, we add a constant labeled "C" to add the possibility of a constant term in the function, although we do not know what that constant is.

First, we integrate to find the anti-derivative.

$$f''(x) = \int -\frac{2}{9}x^{-\frac{4}{3}} - \frac{5}{16}x^{-\frac{9}{4}}\ dx$$

$$f''(x) = -\left(\frac{2}{9}\right)\frac{x^{-\frac{4}{3}+1}}{-\frac{1}{3}} - \left(\frac{5}{16}\right)\frac{x^{-\frac{9}{4}+1}}{-\frac{5}{4}} + C$$

Since we are dividing fractions by fractions, let's multiply the fraction in the numerator by the reciprocal of the fraction in the denominator and simplify the exponents.

$$f''(x) = -\left(\frac{2}{9}\right)\left(-\frac{3}{1}\right)x^{-\frac{1}{3}} - \left(\frac{5}{16}\right)\left(-\frac{4}{5}\right)x^{-\frac{5}{4}} + C$$

Simplify to finish finding the second derivative.

$$f''(x) = \frac{2}{3}x^{-\frac{1}{3}} + \frac{1}{4}x^{-\frac{5}{4}} + C$$

Next, find the first derivative by repeating the process.

$$f''(x) = \frac{2}{3}x^{-\frac{1}{3}} + \frac{1}{4}x^{-\frac{5}{4}} + Cx^0$$

Once again, we will add a constant to cover the likely event that the original function had a constant term that became zero when the first derivative was taken. We do not know that the new constant is the same as the old constant so we will call it "D".

$$f'(x) = \int \frac{2}{3}x^{-\frac{1}{3}} + \frac{1}{4}x^{-\frac{5}{4}} + Cx^0\ dx$$

$$f'(x) = \left(\frac{2}{3}\right)\frac{x^{-\frac{1}{3}+1}}{\frac{2}{3}} + \left(\frac{1}{4}\right)\frac{x^{-\frac{5}{4}+1}}{-\frac{1}{4}} + \frac{Cx^{0+1}}{1} + D$$

Once again, we are dividing fractions by fractions, so let's multiply the fraction in the numerator by the reciprocal of the fraction in the denominator and simplify the exponents.

$$f'(x) = \left(\frac{2}{3}\right)\left(\frac{3}{2}\right)x^{\frac{2}{3}} + \left(\frac{1}{4}\right)\left(-\frac{4}{1}\right)x^{-\frac{1}{4}} + Cx + D$$

$$f'(x) = x^{\frac{2}{3}} - x^{-\frac{1}{4}} + Cx + D$$

Now, find the function by repeating the process again.

$$f'(x) = x^{\frac{2}{3}} - x^{-\frac{1}{4}} + Cx + D$$

$$f'(x) = x^{\frac{2}{3}} - x^{-\frac{1}{4}} + Cx^1 + Dx^0$$

Once again, we will add a constant to cover the likely event that the original function had a constant term that became zero when the first derivative was taken. We do not know that the new constant is the same as the old constants so we will call it "E".

$$f(x) = \int x^{\frac{2}{3}} - x^{-\frac{1}{4}} + Cx^1 + Dx^0 \ dx$$

$$f(x) = \frac{x^{\frac{2}{3}+1}}{\frac{5}{3}} - \frac{x^{-\frac{1}{4}+1}}{\frac{3}{4}} + \frac{Cx^{1+1}}{2} + \frac{Dx^{0+1}}{1} + E$$

We are, once again, dividing fractions by fractions, so let's multiply by the reciprocal of the denominator again and simplify the exponents.

$$f(x) = \frac{3}{5}x^{\frac{5}{3}} - \frac{4}{3}x^{\frac{3}{4}} + Cx^2 + Dx + E$$

Since the letter "C" is an arbitrary constant, we can ignore its division by 2. After we simplify each term, the function is

$$f(x) = \frac{3}{5}x^{\frac{5}{3}} - \frac{4}{3}x^{\frac{3}{4}} + Cx^2 + Dx + E$$