Trigonometric integrals

Trigonometric functions follow standard rules for integration. The integrals of the six standard trigonometric functions are

$$\int \sin(ax)\ dx = \frac{-\cos(ax)}{a} + C$$

$$\int \cos(ax)\ dx = \frac{\sin(ax)}{a} + C$$

$$\int \tan(ax)\ dx = \frac{-\ln\left|\cos(ax)\right|}{a} + C$$

$$\int \cot(ax)\ dx = \frac{\ln\left|\sin(ax)\right|}{a} + C$$

$$\int \sec(ax)\ dx = \frac{\ln\left|\sec(ax)+\tan(ax)\right|}{a} + C$$

$$\int \csc(ax)\ dx = \frac{-\ln\left|\csc(ax)+\cot(ax)\right|}{a} + C$$

We also have a few other standard trigonometric integrals that are based on the standard trigonometric derivatives.

$$\int \sec^2(ax)\ dx = \frac{\tan(ax)}{a} + C$$

$$\int \csc^2(ax)\ dx = \frac{-\cot(ax)}{a} + C$$

$$\int \sec(ax)\tan(ax)\ dx = \frac{\sec(ax)}{a} + C$$

$$\int \csc(ax)\cot(ax)\ dx = \frac{-\csc(ax)}{a} + C$$

Example

Evaluate the integral.

$$\int 6\tan(2x)\ dx$$

We'll simplify the integral by pulling the constant out front.

$$6\int \tan(2x)\ dx$$

Using the formula for the integral of tangent,

$$\int \tan(ax)\ dx = \frac{-\ln\left|\cos(ax)\right|}{a} + C$$

we get

$$6\int \tan(2x)\ dx = 6 \cdot \frac{-\ln\left|\cos(2x)\right|}{2} + C$$

$$6\int \tan(2x)\ dx = -3\ln\left|\cos(2x)\right| + C$$

Let's try a more complex example.

Example

Evaluate the integral.

$$\int e^{4x} - \sec(7x)\tan(7x) + 5\sin x \; dx$$

First we'll separate the terms of the function into different integrals.

$$\int e^{4x} \; dx + \int -\sec(7x)\tan(7x) \; dx + \int 5\sin x \; dx$$

$$\int e^{4x} \; dx - \int \sec(7x)\tan(7x) \; dx + 5\int \sin x \; dx$$

Now we're ready to integrate using the formulas we defined earlier,

$$\int \sin(ax) \; dx = \frac{-\cos(ax)}{a} + C$$

$$\int \sec(ax)\tan(ax) \; dx = \frac{\sec(ax)}{a} + C$$

We get

$$\frac{e^{4x}}{4} - \frac{\sec(7x)}{7} + 5\left(\frac{-\cos x}{1}\right) + C$$

$$\frac{e^{4x}}{4} - \frac{\sec(7x)}{7} - 5\cos x + C$$