Topic: Radius and interval of convergence of a Maclaurin series

Question: Find the radius of convergence of the Maclaurin series.

$$f(x) = \frac{3}{1 - x^2}$$

Answer choices:

A -1

B 3

C 1

D -3

Solution: C

To find the radius of convergence of a Maclaurin series, we'll use the ratio test to say where the power series representation of the function converges. But first, we'll have to find the power series representation of the given function.

The easiest way to find the power series representation of the given series is to identify a similar Maclaurin series and then manipulate the known series until it matches the given series.

In this case, the given series is similar to the known series

$$\frac{1}{1-x}=\sum_{n=1}^{\infty} x^n$$

We'll manipulate this series until it matches the given series. We'll just replace each x with x^2, then multiply the series by 3, and then simplify.

$$\frac{1}{1-x^2}=\sum_{n=1}^{\infty} \left(x^2\right)^n$$

$$\frac{1}{1-x^2}=\sum_{n=1}^{\infty} x^{2n}$$

$$\frac{3}{1-x^2}=\sum_{n=1}^{\infty} 3x^{2n}$$

Since the left side of this manipulation now matches the given series, we can say that the power series representation of the function is $a_n=3x^{2n}$.

To use the ratio test tells us that a series converges when $L < 1$, where

$$L = \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right|$$

In order to find L, we'll need to identify a_n and a_{n+1}.

$$a_n = 3x^{2n}$$

$$a_{n+1} = 3x^{2(n+1)} = 3x^{2n+2}$$

Now we'll plug these values into the formula for L.

$$L = \lim_{n \to \infty} \left| \frac{3x^{2n+2}}{3x^{2n}} \right|$$

$$L = \lim_{n \to \infty} \left| \frac{x^{2n+2}}{x^{2n}} \right|$$

$$L = \lim_{n \to \infty} \left| x^{2n+2-2n} \right|$$

$$L = \lim_{n \to \infty} \left| x^2 \right|$$

Since the limit only effects n, and there are no terms involving n, we can eliminate the limit.

$$L = \left| x^2 \right|$$

The ratio test tells us that the series converges when $L < 1$.

$$\left| x^2 \right| < 1$$

We need to get this into the form $|x-a| < R$, so that we can identify R as the radius of convergence.

$$-1 < x < 1$$

$$-1 < x - 0 < 1$$

Now we can say that the radius of convergence of the Maclaurin series is $R = 1$.

Topic: Radius and interval of convergence of a Maclaurin series

Question: Find the radius of convergence of the Maclaurin series.

$$f(x) = 2e^{3x-1}$$

Answer choices:

A $\quad 0$

B $\quad \infty$

C $\quad \frac{1}{3}$

D $\quad 1$

Solution: B

To find the radius of convergence of a Maclaurin series, we'll use the ratio test to say where the power series representation of the function converges. But first, we'll have to find the power series representation of the given function.

The easiest way to find the power series representation of the given series is to identify a similar Maclaurin series and then manipulate the known series until it matches the given series.

In this case, the given series is similar to the known series

$$e^x = \sum_{n=1}^{\infty} \frac{x^n}{n!}$$

We'll manipulate this series until it matches the given series. We'll just replace each x with $3x - 1$, then multiply the series by 2, and then simplify.

$$e^{3x-1} = \sum_{n=1}^{\infty} \frac{(3x-1)^n}{n!}$$

$$2e^{3x-1} = \sum_{n=1}^{\infty} \frac{2(3x-1)^n}{n!}$$

Since the left side of this manipulation now matches the given series, we can say that the power series representation of the function is

$$a_n = \frac{2(3x-1)^n}{n!}$$

To use the ratio test tells us that a series converges when $L < 1$, where

$$L = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|$$

In order to find L, we'll need to identify a_n and a_{n+1}.

$$a_n = \frac{2(3x-1)^n}{n!}$$

$$a_{n+1} = \frac{2(3x-1)^{n+1}}{(n+1)!}$$

Now we'll plug these values into the formula for L.

$$L = \lim_{n\to\infty} \left| \frac{\frac{2(3x-1)^{n+1}}{(n+1)!}}{\frac{2(3x-1)^n}{n!}} \right|$$

$$L = \lim_{n\to\infty} \left| \frac{2(3x-1)^{n+1}}{(n+1)!} \cdot \frac{n!}{2(3x-1)^n} \right|$$

$$L = \lim_{n\to\infty} \left| \frac{2(3x-1)^{n+1}}{2(3x-1)^n} \cdot \frac{n!}{(n+1)!} \right|$$

$$L = \lim_{n\to\infty} \left| (3x-1)^{n+1-n} \cdot \frac{n!}{(n+1)!} \right|$$

$$L = \lim_{n\to\infty} \left| \frac{(3x-1)(n)(n-1)(n-2)\dots}{(n+1)(n)(n-1)(n-2)\dots} \right|$$

$$L = \lim_{n \to \infty} \left| \frac{3x - 1}{n + 1} \right|$$

Since the limit only effects n, we can pull out $3x - 1$, as long as we keep it inside absolute value brackets.

$$L = \left| 3x - 1 \right| \lim_{n \to \infty} \left| \frac{1}{n + 1} \right|$$

$$L = \left| 3x - 1 \right| \lim_{n \to \infty} \left| \frac{1}{n + 1} \left(\frac{\frac{1}{n}}{\frac{1}{n}} \right) \right|$$

$$L = \left| 3x - 1 \right| \lim_{n \to \infty} \left| \frac{\frac{1}{n}}{\frac{n}{n} + \frac{1}{n}} \right|$$

$$L = \left| 3x - 1 \right| \lim_{n \to \infty} \left| \frac{\frac{1}{n}}{1 + \frac{1}{n}} \right|$$

$$L = \left| 3x - 1 \right| \left| \frac{\frac{1}{\infty}}{1 + \frac{1}{\infty}} \right|$$

$$L = \left| 3x - 1 \right| \left| \frac{0}{1 + 0} \right|$$

$$L = \left| 3x - 1 \right| \left| 0 \right|$$

$$L = 0$$

The ratio test tells us that the series converges when $L < 1$. Since 0 is always less than 1, it means that the series converges everywhere, which means that the radius of convergence is infinite and $R = \infty$.

Topic: Radius and interval of convergence of a Maclaurin series

Question: Find the radius of convergence of the Maclaurin series.

$$f(x) = \ln(1 + 3x)$$

Answer choices:

A $\frac{1}{3}$

B 1

C ∞

D 3

Solution: A

To find the radius of convergence of a Maclaurin series, we'll use the ratio test to say where the power series representation of the function converges. But first, we'll have to find the power series representation of the given function.

The easiest way to find the power series representation of the given series is to identify a similar Maclaurin series and then manipulate the known series until it matches the given series.

In this case, the given series is similar to the known series

$$\ln(1+x)=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}x^n$$

We'll manipulate this series until it matches the given series. We'll just replace each x with $3x$, and then simplify.

$$\ln(1+3x)=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}(3x)^n$$

Since the left side of this manipulation now matches the given series, we can say that the power series representation of the function is

$$a_n=\frac{(-1)^{n+1}}{n}(3x)^n$$

To use the ratio test tells us that a series converges when $L<1$, where

$$L=\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|$$

In order to find L, we'll need to identify a_n and a_{n+1}.

$$a_n = \frac{(-1)^{n+1}}{n}(3x)^n$$

$$a_{n+1} = \frac{(-1)^{n+1+1}}{n+1}(3x)^{n+1}$$

$$a_n = \frac{(-1)^{n+2}}{n+1}(3x)^{n+1}$$

Now we'll plug these values into the formula for L.

$$L = \lim_{n\to\infty}\left|\frac{\frac{(-1)^{n+2}(3x)^{n+1}}{n+1}}{\frac{(-1)^{n+1}(3x)^n}{n}}\right|$$

$$L = \lim_{n\to\infty}\left|\frac{(-1)^{n+2}(3x)^{n+1}}{n+1}\cdot\frac{n}{(-1)^{n+1}(3x)^n}\right|$$

$$L = \lim_{n\to\infty}\left|\frac{(-1)^{n+2}(3x)^{n+1}}{(-1)^{n+1}(3x)^n}\cdot\frac{n}{n+1}\right|$$

$$L = \lim_{n\to\infty}\left|(-1)^{n+2-(n+1)}(3x)^{n+1-n}\cdot\frac{n}{n+1}\right|$$

$$L = \lim_{n\to\infty}\left|(-1)^{n+2-n-1}(3x)\cdot\frac{n}{n+1}\right|$$

$$L = \lim_{n\to\infty}\left|(-1)(3x)\cdot\frac{n}{n+1}\right|$$

$$L = \lim_{n \to \infty} \left| (-3x) \cdot \frac{n}{n+1} \right|$$

Since the limit only effects n, we can pull out $-3x$, as long as we keep it inside absolute value brackets.

$$L = \left| -3x \right| \lim_{n \to \infty} \left| \frac{n}{n+1} \right|$$

$$L = \left| -3x \right| \lim_{n \to \infty} \left| \frac{n}{n+1} \left(\frac{\frac{1}{n}}{\frac{1}{n}} \right) \right|$$

$$L = \left| -3x \right| \lim_{n \to \infty} \left| \frac{\frac{n}{n}}{\frac{n}{n} + \frac{1}{n}} \right|$$

$$L = \left| -3x \right| \lim_{n \to \infty} \left| \frac{1}{1 + \frac{1}{n}} \right|$$

$$L = \left| -3x \right| \left| \frac{1}{1 + \frac{1}{\infty}} \right|$$

$$L = \left| -3x \right| \left| \frac{1}{1+0} \right|$$

$$L = \left| -3x \right| \left| 1 \right|$$

$$L = \left| -3x \right|$$

$$L = \left|3x\right|$$

The ratio test tells us that the series converges when $L < 1$.

$$\left|3x\right| < 1$$

We need to get this into the form $|x - a| < R$, so that we can identify R as the radius of convergence.

$$|x| < \frac{1}{3}$$

$$\left|x - 0\right| < \frac{1}{3}$$

Now we can say that the radius of convergence of the Maclaurin series is $R = 1/3$.