Topic: Converting polar equations

Question: Convert the polar equation to a rectangular equation.

$$r = \frac{6}{\sin\theta - 3\cos\theta}$$

Answer choices:

A $y = -3x + 6$

B $y = 3x + 6$

C $y = 3x - 6$

D $y = -3x - 6$

Solution: B

Converting a polar equation to a rectangular equation requires us to get r and θ out of the equation and get x and y into it. The following equations are needed for the conversion:

$$r^2 = x^2 + y^2$$

$$x = r\cos\theta$$

$$y = r\sin\theta$$

In this particular example, only the last two equations above are useful. Before we make substitutions, we'll simplify the given polar equation.

$$r = \frac{6}{\sin\theta - 3\cos\theta}$$

$$r(\sin\theta - 3\cos\theta) = 6$$

$$r\sin\theta - 3r\cos\theta = 6$$

Now we'll make the substitutions.

$$y - 3x = 6$$

$$y = 3x + 6$$

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Question: Convert the polar equation to a rectangular equation.

$$r = (\csc\theta)2e^{3r\cos\theta}$$

Answer choices:

A $\quad y = 2e^3$

B $\quad y = 3e^{2x}$

C $\quad y = 2e^{3x}$

D $\quad y = -2e^{3x}$

Solution: C

In order to convert our polar equation to a rectangular equation, we'll need the following conversion formulas.

$$x = r\cos\theta$$

$$y = r\sin\theta$$

Using the trigonometric identity

$$\csc\theta = \frac{1}{\sin\theta}$$

the polar equation is first reduced to

$$r = \left(\frac{1}{\sin\theta}\right)2e^{3r\cos\theta}$$

Multiplying the equation by $\sin\theta$ and then using our conversion formulas gives us

$$r\sin\theta = 2e^{3r\cos\theta}$$

$$y = 2e^{3x}$$

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Question: The parametric coordinates $x(t) = f(t)\cos t$ and $y(t) = g(t)\sin t$ are given, where $f(t) = t^2 - 3$ and $g(t) = \sqrt{9 - 6t^2 + t^4}$. Which statement describes the polar coordinates of the given coordinates?

Answer choices:

A The given coordinates define a circle with radius $t^2 - 3$ centered at the origin, where $t > \sqrt{3}$.

B The given coordinates define a circle with a radius $t^2 + 3$ centered at the origin, where $t > \sqrt{3}$.

C The given coordinates define a circle with a radius $t - 3$ centered at the origin, where $t > \sqrt{3}$.

D The given coordinates define a circle with a radius $t + 3$ centered at the origin, where $t < \sqrt{3}$.

Solution: A

Transform the polar coordinates to rectangular coordinates.

Replace $f(t)$ and $g(t)$ in the given coordinates and square both sides of each equation.

$$x(t) = f(t)\cos t$$

$$x(t) = \left(t^2 - 3\right)\cos t$$

$$\left[x(t)\right]^2 = \left(t^2 - 3\right)^2 \cos^2 t$$

$$x^2 = \left(t^4 - 6t^2 + 9\right)\cos^2 t$$

and

$$y(t) = g(t)\sin t$$

$$y(t) = \sqrt{9 - 6t^2 + t^4}\,\sin t$$

$$y^2 = \left(9 - 6t^2 + t^4\right)\sin^2 t$$

$$y^2 = \left(t^4 - 6t^2 + 9\right)\sin^2 t$$

Now add $x^2 = \left(t^4 - 6t + 9\right)\cos^2 t$ and $y^2 = \left(t^4 - 6t^2 + 9\right)\sin^2 t$.

$$x^2 + y^2 = \left(t^4 - 6t^2 + 9\right)\cos^2 t + \left(t^4 - 6t^2 + 9\right)\sin^2 t$$

$$x^2 + y^2 = \left(t^4 - 6t^2 + 9\right)\left(\cos^2 t + \sin^2 t\right)$$

$$x^2 + y^2 = t^4 - 6t^2 + 9$$

$$x^2+y^2=\left(t^2-3\right)^2$$

Because we know that in polar coordinates $x^2+y^2=r^2$, the radius of the polar coordinate is t^2-3 centered at the origin, where $t>\sqrt{3}$.