Topic: Find f given f'' and initial conditions

Question: Find $f(x)$.

$$f''(x) = 18x + 8$$

$$f'(2) = 46 \text{ and } f(0) = 8$$

Answer choices:

A $\quad f(x) = x^3 + x^2 - x + 8$

B $\quad f(x) = 3x^3 + 4x^2 - 6x + 8$

C $\quad f(x) = 3x^3 + 4x^2 - 6x - 8$

D $\quad f(x) = \frac{x^3}{3} + \frac{x^2}{4} - \frac{x}{6} + 8$

Solution: B

The question asks us to find the function $f(x)$ if the second derivative of the function is $f''(x) = 18x + 8$, $f'(2) = 46$, and $f(0) = 8$.

Note that the question provides initial values of $f(x)$ and $f'(x)$ so our answer will be a specific $f(x)$ function with the given second derivative.

We are given the second derivative of the function. To find the first derivative of the function, find the anti-derivative of the second derivative. Then, to find the function, we repeat the process by finding the anti-derivative of the first derivative. Once we find the general first derivative and the general function, we will use the initial conditions to find the specific function.

The second derivative is a polynomial function. To find the anti-derivative, in each term, add 1 to the exponent and divide the term by the new exponent.

Additionally, all functions "could" contain a constant term, which becomes zero when we take the derivative of the function. Thus, when we find the anti-derivative, we add a constant labeled "C" to add the possibility of a constant term in the function, although we do not know what that constant is. When we use the initial condition, we will find the specific value of "C".

We will, first, write the second derivative showing all exponents.

$$f''(x) = 18x + 8 = 18x^1 + 8x^0$$

$$f'(x) = \int 18x^1 + 8x^0 \ dx$$

$$f'(x) = \frac{18x^{1+1}}{2} + \frac{8x^{0+1}}{1} + C$$

Simplify each term to finish finding the general first derivative.

$$f'(x) = 9x^2 + 8x + C$$

The question states that $f'(2) = 46$, so to find "C" let's make the derivative equal to 46 when $x = 2$.

$$46 = 9(2)^2 + 8\,(2) + C$$

$$46 = 36 + 16 + C$$

$$46 = 52 + C$$

$$-6 = C$$

Therefore,

$$f'(x) = 9x^2 + 8x - 6$$

Now, find the function by repeating the process.

$$f'(x) = 9x^2 + 8x - 6 = 9x^2 + 8x^1 - 6x^0$$

Once again, we will add a constant to cover the likely event that the original function had a constant term that became zero when the first derivative was taken. We do not know that the new constant is the same as the old constant so we will call it "D", but we will find its value using the initial condition.

$$f(x) = \int 9x^2 + 8x^1 - 6x^0\ dx$$

$$f(x) = \frac{9x^{2+1}}{3} + \frac{8x^{1+1}}{2} - \frac{6x^{0+1}}{1} + D$$

After we simplify each term, the general function is

$$f(x) = 3x^3 + 4x^2 - 6x + D$$

The question further states that $f(0) = 8$, so to find "D" let's make the function equal to 8 when $x = 0$.

$$8 = 3(0)^3 + 4(0)^2 - 6(0) + D$$

$$D = 8$$

The specific function in this problem is

$$f(x) = 3x^3 + 4x^2 - 6x + 8$$

Topic: Find f given f'' and initial conditions

Question: Find $f(x)$.

$$f''(x) = 60x^2 - 36x + 6$$

$$f'(1) = 0 \text{ and } f(1) = 8$$

Answer choices:

A $\quad f(x) = 5x^4 - 6x^3 + 3x^2 + Cx + D$

B $\quad f(x) = \dfrac{x^4}{5} - \dfrac{x^3}{6} + \dfrac{x^2}{3} - \dfrac{x}{8} + 14$

C $\quad f(x) = x^4 - x^3 + x^2 - x + 8$

D $\quad f(x) = 5x^4 - 6x^3 + 3x^2 - 8x + 14$

Solution: D

The question asks us to find the function $f(x)$ if the second derivative of the function is $f''(x) = 60x^2 - 36x + 6$, and if $f'(1) = 0$ and $f(1) = 8$.

Note that the question provides initial values of $f(x)$ and $f'(x)$ so our answer will be a specific $f(x)$ function with the given second derivative.

We are given the second derivative of the function. To find the first derivative of the function, find the anti-derivative of the second derivative. Then, to find the function, we repeat the process by finding the anti-derivative of the first derivative. Once we find the general first derivative and the general function, we will use the initial conditions to find the specific function.

The second derivative is a polynomial function. To find the anti-derivative, in each term, add 1 to the exponent and divide the term by the new exponent.

Additionally, all functions "could" contain a constant term, which becomes zero when we take the derivative of the function. Thus, when we find the anti-derivative, we add a constant labeled "C" to add the possibility of a constant term in the function, although we do not know what that constant is. When we use the initial condition, we will find the specific value of "C".

We will, first, write the second derivative showing all exponents.

$$f''(x) = 60x^2 - 36x + 6 = 60x^2 - 36x^1 + 6x^0$$

$$f'(x) = \int 60x^2 - 36x^1 + 6x^0 \ dx$$

$$f'(x) = \frac{60x^{2+1}}{3} - \frac{36x^{1+1}}{2} + \frac{6x^{0+1}}{1} + C$$

Simplify each term to finish finding the general first derivative.

$$f'(x) = 20x^3 - 18x^2 + 6x + C$$

The question states that $f'(1) = 0$, so to find "C" let's make the derivative equal to 0 when $x = 1$.

$$0 = 20(1)^3 - 18(1)^2 + 6(1) + C$$

$$0 = 20 - 18 + 6 + C$$

$$0 = 8 + C$$

$$C = -8$$

So

$$f'(x) = 20x^3 - 18x^2 + 6x - 8$$

Now find the function by repeating the process.

$$f'(x) = 20x^3 - 18x^2 + 6x - 8$$

$$f'(x) = 20x^3 - 18x^2 + 6x^1 - 8x^0$$

Once again, we will add a constant to cover the likely event that the original function had a constant term that became zero when the first derivative was taken. We do not know that the new constant is the same as the old constant so we will call it "D", but we will find its value using the initial condition.

$$f(x) = \int 20x^3 - 18x^2 + 6x^1 - 8x^0 \ dx$$

$$f(x) = \frac{20x^{3+1}}{4} - \frac{18x^{2+1}}{3} + \frac{6x^{1+1}}{2} - \frac{8x^{0+1}}{1} + D$$

After we simplify each term, the general function is

$$f(x) = 5x^4 - 6x^3 + 3x^2 - 8x + D$$

The question further states that $f(1) = 8$, so to find "D" let's make the function equal to 8 when $x = 1$.

$$8 = 5(1)^4 - 6(1)^3 + 3(1)^2 - 8(1) + D$$

$$8 = 5 - 6 + 3 - 8 + D$$

$$8 = -6 + D$$

$$D = 14$$

The specific function in this problem is

$$f(x) = 5x^4 - 6x^3 + 3x^2 - 8x + 14$$

Topic: Find f given f'' and initial conditions

Question: Find $f(x)$.

$$f''(x) = \frac{35}{4}x^{\frac{3}{2}} + \frac{15}{2}x^{\frac{1}{2}} - \frac{3}{4}x^{-\frac{1}{2}} - \frac{1}{2}x^{-\frac{3}{2}}$$

$f'(1) = 12$ and $f(4) = 189$

Answer choices:

A $\quad f(x) = x^{\frac{7}{2}} + 2x^{\frac{5}{2}} - x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + 4x - 15$

B $\quad f(x) = x^{\frac{7}{2}} + x^{\frac{5}{2}} - x^{\frac{3}{2}} + x^{\frac{1}{2}} + 1$

C $\quad f(x) = x^{\frac{7}{2}} + 2x^{\frac{5}{2}} - x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + C$

D $\quad f(x) = x^{\frac{7}{2}} + 2x^{\frac{5}{2}} - x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + Cx + D$

Solution: A

The question asks us to find the function $f(x)$ if the second derivative of the function is

$$f''(x) = \frac{35}{4}x^{\frac{3}{2}} + \frac{15}{2}x^{\frac{1}{2}} - \frac{3}{4}x^{-\frac{1}{2}} - \frac{1}{2}x^{-\frac{3}{2}}$$

and if $f'(1) = 12$ and $f(4) = 189$.

Note that the question provides initial values of $f(x)$ and $f'(x)$, so our answer will be a specific $f(x)$ function with the given second derivative.

We are given the second derivative of the function. To find the first derivative of the function, find the anti-derivative of the second derivative. Then, to find the function, we repeat the process by finding the anti-derivative of the first derivative. Once we find the general first derivative and the general function, we will use the initial conditions to find the specific function.

The second derivative is a polynomial function. To find the anti-derivative, in each term, add 1 to the exponent and divide the term by the new exponent.

Additionally, all functions "could" contain a constant term, which becomes 0 when we take the derivative of the function. Thus, when we find the anti-derivative, we add a constant labeled "C" to add the possibility of a constant term in the function, although we do not know what that constant is. When we use the initial condition, we will find the specific value of "C".

First we'll integrate the second derivative to find the first derivative.

$$f'(x) = \int \frac{35}{4}x^{\frac{3}{2}} + \frac{15}{2}x^{\frac{1}{2}} - \frac{3}{4}x^{-\frac{1}{2}} - \frac{1}{2}x^{-\frac{3}{2}}\ dx$$

$$f'(x) = \frac{\frac{35}{4}x^{\frac{3}{2}+1}}{\frac{5}{2}} + \frac{\frac{15}{2}x^{\frac{1}{2}+1}}{\frac{3}{2}} - \frac{\frac{3}{4}x^{-\frac{1}{2}+1}}{\frac{1}{2}} - \frac{\frac{1}{2}x^{-\frac{3}{2}+1}}{-\frac{1}{2}} + C$$

Since we're dividing each term by a fraction, change each term to multiplying by the reciprocal of the fraction in the denominator.

$$f'(x) = \left(\frac{35}{4}\right)\left(\frac{2}{5}\right)x^{\frac{5}{2}} + \left(\frac{15}{2}\right)\left(\frac{2}{3}\right)x^{\frac{3}{2}} - \left(\frac{3}{4}\right)\left(\frac{2}{1}\right)x^{\frac{1}{2}} - \left(\frac{1}{2}\right)\left(-\frac{2}{1}\right)x^{-\frac{1}{2}} + C$$

Simplify each term to finish finding the general first derivative.

$$f'(x) = \frac{7}{2}x^{\frac{5}{2}} + 5x^{\frac{3}{2}} - \frac{3}{2}x^{\frac{1}{2}} + x^{-\frac{1}{2}} + C$$

The question states that $f'(1) = 12$, so to find "C" let's make the derivative equal to 12 when $x = 1$.

$$12 = \frac{7}{2}(1)^{\frac{5}{2}} + 5(1)^{\frac{3}{2}} - \frac{3}{2}(1)^{\frac{1}{2}} + (1)^{-\frac{1}{2}} + C$$

$$12 = \frac{7}{2} + 5 - \frac{3}{2} + (1) + C$$

$$12 = 2 + 5 + 1 + C$$

$$12 = 8 + C$$

$$C = 4$$

So

$$f'(x) = \frac{7}{2}x^{\frac{5}{2}} + 5x^{\frac{3}{2}} - \frac{3}{2}x^{\frac{1}{2}} + x^{-\frac{1}{2}} + 4$$

Now find the function by repeating the process.

$$f'(x) = \frac{7}{2}x^{\frac{5}{2}} + 5x^{\frac{3}{2}} - \frac{3}{2}x^{\frac{1}{2}} + x^{-\frac{1}{2}} + 4x^0$$

Once again, we will add a constant to cover the likely event that the original function had a constant term that became 0 when the first derivative was taken. We do not know that the new constant is the same as the old constant so we will call it "D", but we will find its value using the initial condition.

$$f(x) = \int \frac{7}{2}x^{\frac{5}{2}} + 5x^{\frac{3}{2}} - \frac{3}{2}x^{\frac{1}{2}} + x^{-\frac{1}{2}} + 4x^0 \ dx$$

$$f(x) = \frac{\frac{7}{2}x^{\frac{5}{2}+1}}{\frac{7}{2}} + \frac{5x^{\frac{3}{2}+1}}{\frac{5}{2}} - \frac{\frac{3}{2}x^{\frac{1}{2}+1}}{\frac{3}{2}} + \frac{x^{-\frac{1}{2}+1}}{\frac{1}{2}} + \frac{4x^{0+1}}{1} + D$$

Again, since we are dividing each term by a fraction, change each term to multiplying by the reciprocal of the fraction in the denominator.

$$f(x) = \left(\frac{7}{2}\right)\left(\frac{2}{7}\right)x^{\frac{7}{2}} + (5)\left(\frac{2}{5}\right)x^{\frac{5}{2}} - \left(\frac{3}{2}\right)\left(\frac{2}{3}\right)x^{\frac{3}{2}} + (1)\left(\frac{2}{1}\right)x^{\frac{1}{2}} + 4x + D$$

$$f(x) = x^{\frac{7}{2}} + 2x^{\frac{5}{2}} - x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + 4x + D$$

The question further states that $f(4) = 189$, so to find "D" let's make the function equal to 189 when $x = 4$.

$$189 = (4)^{\frac{7}{2}} + 2(4)^{\frac{5}{2}} - (4)^{\frac{3}{2}} + 2(4)^{\frac{1}{2}} + 4(4) + D$$

$$189 = 128 + 64 - 8 + 4 + 16 + D$$

$$189 = 204 + D$$

$$D = -15$$

The specific function in this problem is

$$f(x) = x^{\frac{7}{2}} + 2x^{\frac{5}{2}} - x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + 4x - 15$$