Sum of the sequence of partial sums

Remember, a normal series is given by

$$\sum_{n=1}^{\infty} a_n$$

where a_n is a sequence whose n values increase by increments of 1. For example, this series could be

$$\sum_{n=1}^{\infty} a_n = 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ \dots\ a_n$$

On the other hand, a partial sums sequence is called s_n, and its n values increase by additive increments. This means that the first term in a partial sums sequence is the $n=1$ term, the second term is the $n=1$ term plus the $n=2$ term, the third term is $(n=1)+(n=2)+(n=3)$, etc.

A normal series is related to its corresponding partial sums sequence by

$$\sum_{n=1}^{\infty} a_n = \lim_{n\to\infty} s_n$$

This equation is critical, because it allows us to work backwards from the partial sums sequence to the original series, a_n.

Example

Find the sum of the sequence of the partial sums.

$$s_n = 1 - 2(0.4)^n$$

This question is asking us to find the sum of the series a_n, given its corresponding sequence of partial sums, so we can use

$$\sum_{n=1}^{\infty} a_n = \lim_{n \to \infty} s_n$$

$$\sum_{n=1}^{\infty} a_n = \lim_{n \to \infty} 1 - 2(0.4)^n$$

Now we can evaluate the limit.

$$\sum_{n=1}^{\infty} a_n = 1 - 2(0.4)^{\infty}$$

When 0.4 is raised to the power of ∞, it'll become smaller and smaller and eventually approach 0.

$$\sum_{n=1}^{\infty} a_n = 1 - 2(0)$$

$$\sum_{n=1}^{\infty} a_n = 1$$

The sum of the series a_n given the sequence of the partial sums $s_n = 1 - 2(0.4)^n$ is 1.