Calculus 2 Workbook Solutions

Reduction formulas

krista king MATH

INTEGRALS USING REDUCTION FORMULAS

■ 1. Use a reduction formula to evaluate the integral.

$$\int \cot^4 x \, dx$$

Solution:

Split up the integrand and make a substitution.

$$\int \cot^4 x \, dx$$

$$\int \cot^2 x \cot^2 x \, dx$$

$$\int \cot^2 x \left(\csc^2 x - 1\right) \, dx$$

$$\int \cot^2 x \csc^2 x \, dx - \int \cot^2 x \, dx$$

For the first integral, use substitution.

$$u = \cot x$$

$$du = -\csc^2 x \, dx$$

$$dx = \frac{du}{-\csc^2 x}$$

Make the substitution.

$$\int u^2 \csc^2 x \cdot \frac{du}{-\csc^2 x} - \int \cot^2 x \; dx$$

$$-\int u^2 \; du - \int \cot^2 x \; dx$$

$$-\frac{1}{3}u^3 + C - \int \cot^2 x \; dx$$

$$-\frac{1}{3}\cot^3 x + C - \int \cot^2 x \; dx$$

Work on the second integral.

$$-\frac{1}{3}\cot^3 x + C - \int 1 + \cot^2 x - 1 \; dx$$

$$-\frac{1}{3}\cot^3 x + C - \int \csc^2 x - 1 \; dx$$

$$-\frac{1}{3}\cot^3 x + C - (-\cot x - x)$$

$$-\frac{1}{3}\cot^3 x + \cot x + x + C$$

2. Use a reduction formula to evaluate the integral.

$$\int \sec^4 x \; dx$$

Solution:

Split up the integrand and make a substitution.

$$\int \sec^4 x \ dx$$

$$\int \sec^2 x \sec^2 x \ dx$$

$$\int \sec^2 x \left(\tan^2 x + 1\right) \ dx$$

$$\int \sec^2 x \tan^2 x \ dx + \int \sec^2 x \ dx$$

For the first integral, use substitution.

$$u = \tan x$$

$$du = \sec^2 x \ dx$$

$$dx = \frac{du}{\sec^2 x}$$

Make the substitution.

$$\int \sec^2 x \cdot u^2 \frac{du}{\sec^2 x} + \int \sec^2 x \ dx$$

$$\int u^2 \ du + \int \sec^2 x \ dx$$

$$\frac{1}{3}u^3 + C + \int \sec^2 x \ dx$$

$$\frac{1}{3}\tan^3 x + C + \int \sec^2 x \ dx$$

Work on the second integral.

$$\frac{1}{3}\tan^3 x + C + \tan x$$

$$\frac{1}{3}\tan^3 x + \tan x + C$$

3. Use a reduction formula to evaluate the integral.

$$\int \csc^4 x \ dx$$

Solution:

Split up the integrand and make a substitution.

$$\int \csc^4 x \ dx$$

$$\int \csc^2 x \csc^2 x \ dx$$

$$\int \csc^2 x \left(\cot^2 x + 1\right) \ dx$$

$$\int \csc^2 x \cot^2 x \, dx + \int \csc^2 x \, dx$$

For the first integral, use substitution.

$$u = \cot x$$

$$du = -\csc^2 x \, dx$$

$$dx = \frac{du}{-\csc^2 x}$$

Make the substitution.

$$\int \csc^2 x \cdot u^2 \frac{du}{-\csc^2 x} + \int \csc^2 x \, dx$$

$$-\int u^2 \, du + \int \csc^2 x \, dx$$

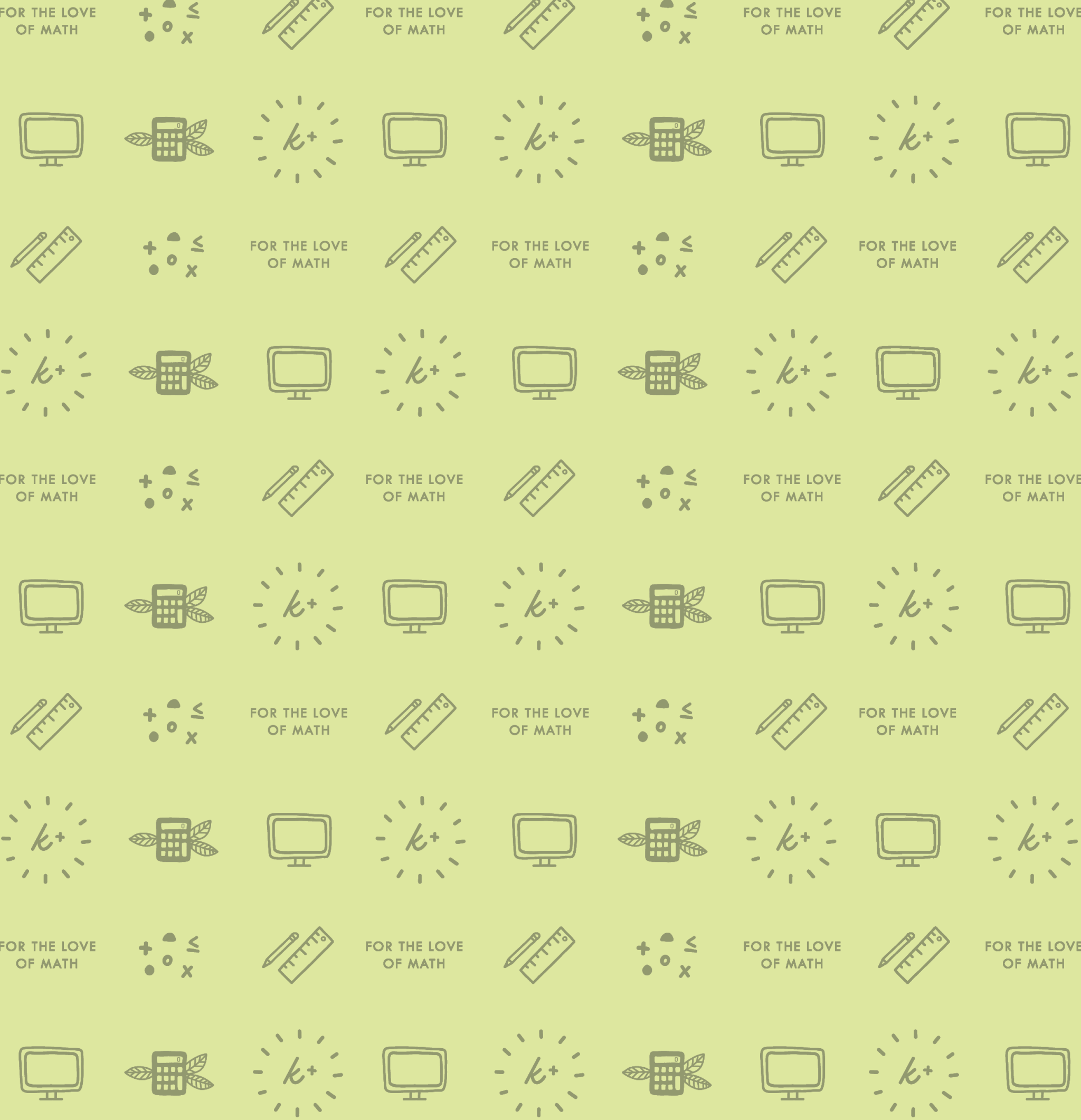
$$-\frac{1}{3}u^3 + C + \int \csc^2 x \, dx$$

$$-\frac{1}{3}\cot^3 x + C + \int \csc^2 x \, dx$$

Work on the second integral.

$$-\frac{1}{3}\cot^3 x + C + (-\cot x)$$

$$-\frac{1}{3}\cot^3 x - \cot x + C$$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