Topic: Cylindrical shells, horizontal axis

Question: Use cylindrical shells to find the volume of the solid generated by revolving the region bounded by the curves about the given axis.

$x = \sqrt{y}$ and $x = \frac{y^3}{32}$

about the x-axis

Answer choices:

A $\frac{64\pi}{5}$

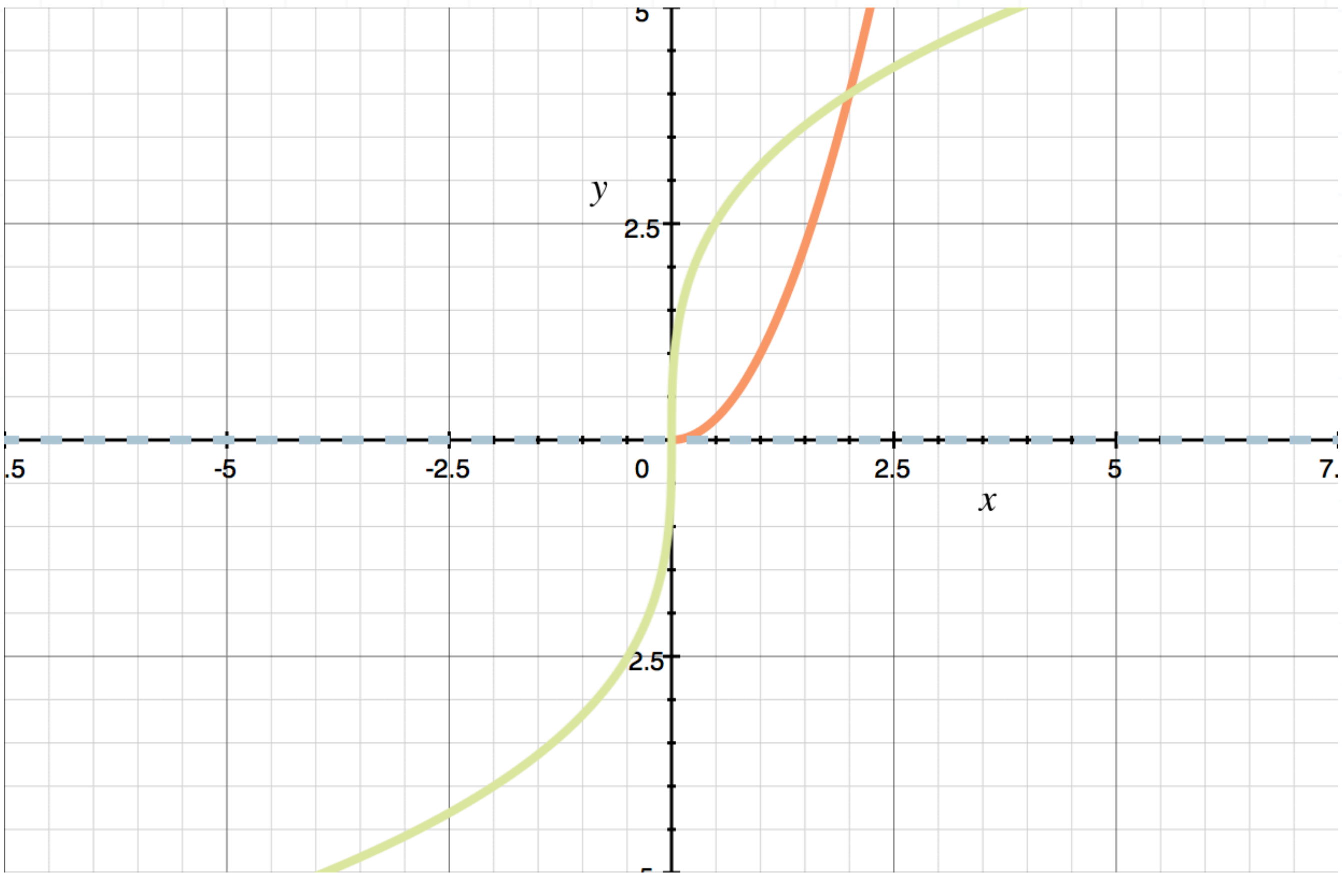
B 64π

C $\frac{5\pi}{64}$

D 5π

Solution: A

Before doing anything else, we always want to draw a picture of the area. If you don't know how to graph the function, just plug in values for x or y to get individual coordinate points, and plot them until you have a picture of each function.



Using cylindrical shells means we'll take slices of our area that are parallel to the axis of rotation. Therefore, since the axis of rotation is horizontal, we'll take horizontal slices of our area and rotate each of them around the axis to form cylindrical shells.

Using cylindrical shells around a horizontal axis, specifically the x-axis, tells us that we'll use the volume formula

$$V = \int_c^d 2\pi y \left[f(y) - g(y)\right] \ dy$$

We can see from the formula that we need our curves and our limits of integration defined in terms of y. The given curves are already defined for x in terms of y, so now we just need to find limits of integration, which will be the smallest and largest y-values for which the area is defined. Since these are just the two points of intersection, we can do this by looking at the graph, or we can set the curves equal to one another and solve for y.

$$\sqrt{y} = \frac{y^3}{32}$$

$$32\sqrt{y} = y^3$$

$$1{,}024y = y^6$$

$$y^6 - 1{,}024y = 0$$

$$y\left(y^5 - 1{,}024\right) = 0$$

$$y = 0$$

and

$$y^5 - 1{,}024 = 0$$

$$y^5 = 1{,}024$$

$$\left(y^5\right)^{\frac{1}{5}} = 1{,}024^{\frac{1}{5}}$$

$$y = 4$$

Now we know that our limits of integration are $c = 0$ and $d = 4$. Since the axis of revolution is the x-axis, our radius is y.

$f(y) - g(y)$ is the height of the approximating cylinder, which means we need to subtract the curve on the left $g(y)$ from the curve on the right $f(y)$. To figure out which curve is on the right and which is on the left, we can look at the graph or we can plug a y-value between the points of intersection (between $y = 0$ and $y = 4$) into both curves to see which function returns a larger value (this will be the right curve) and which one returns a smaller value (this will be the left curve). Let's plug in $y = 1$ to check.

$$x = \sqrt{y}$$

$$x = \sqrt{1}$$

$$x = 1$$

and

$$x = \frac{y^3}{32}$$

$$x = \frac{(1)^3}{32}$$

$$x = \frac{1}{32}$$

Since $x = \sqrt{y}$ returns a larger value than $x = y^3/32$, we can say

$$f(y) = \sqrt{y}$$

and

$$g(y) = \frac{y^3}{32}$$

Plugging everything we know into the volume formula, we get

$$V=\int_0^4 2\pi y\left(\sqrt{y}-\frac{y^3}{32}\right)\ dy$$

$$V=2\pi\int_0^4 y^{\frac{3}{2}}-\frac{y^4}{32}\ dy$$

$$V=2\pi\left(\frac{2}{5}y^{\frac{5}{2}}-\frac{y^5}{160}\right)\Bigg|_0^4$$

$$V=2\pi\left[\left(\frac{2}{5}(4)^{\frac{5}{2}}-\frac{(4)^5}{160}\right)-\left(\frac{2}{5}(0)^{\frac{5}{2}}-\frac{(0)^5}{160}\right)\right]$$

$$V=2\pi\left(\frac{64}{5}-\frac{1{,}024}{160}\right)$$

$$V=2\pi\left(\frac{64}{5}-\frac{32}{5}\right)$$

$$V=\frac{64\pi}{5}$$

Topic: Cylindrical shells, horizontal axis

Question: Use cylindrical shells to find the volume of the solid generated by revolving the region bounded by the curves about the given axis.

$x = y^{\frac{1}{2}}$ and $x = 1$ and $y = 0$

about $y = 1$

Answer choices:

A $\frac{2}{3}$

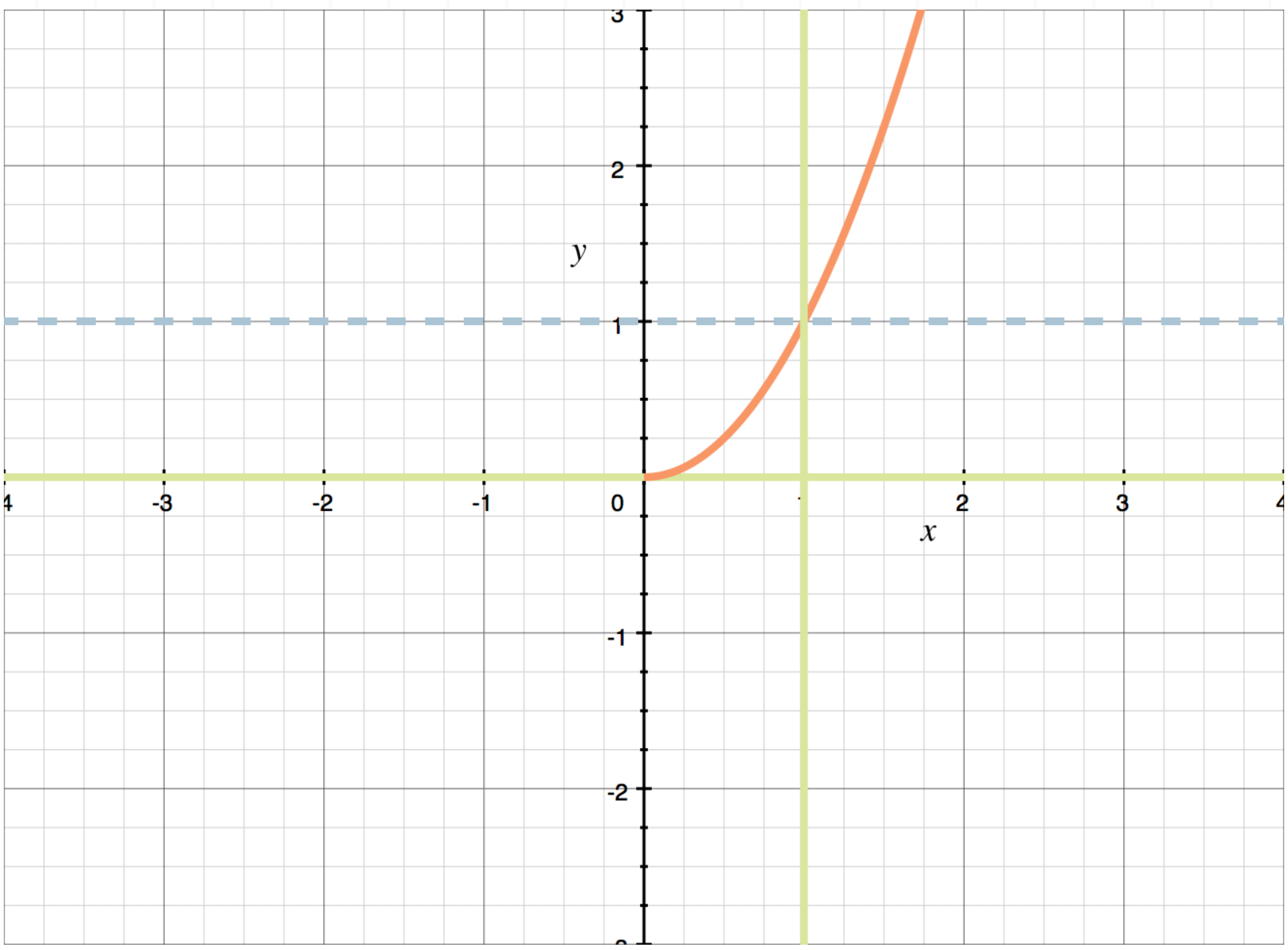
B $\frac{7\pi}{30}$

C $\frac{7\pi}{15}$

D $\frac{3}{2}$

Solution: C

Before doing anything else, we always want to draw a picture of the area. If you don't know how to graph the function, just plug in values for x or y to get individual coordinate points, and plot them until you have a picture of each function.



Using cylindrical shells means we'll take slices of our area that are parallel to the axis of rotation. Therefore, since the axis of rotation is horizontal, we'll take horizontal slices of our area and rotate each of them around the axis to form cylindrical shells.

Using cylindrical shells around a horizontal axis, specifically $y = 1$, tells us that we'll use the volume formula

$$V = \int_c^d 2\pi(k - y)\left[f(y) - g(y)\right] \ dy$$

We can see from the formula that we need our curves and our limits of integration defined in terms of y. The given curves are already defined for x in terms of y, so now we just need to find limits of integration, which will be the smallest and largest y-values for which the area is defined.

We can see from the graph that the smallest y-value for which the area is defined is $y = 0$. This was given in the original problem. We can see that the largest value for which it's defined is a point of intersection, so we can set the curves equal to one another and solve for y.

$$y^{\frac{1}{2}} = 1$$

$$\sqrt{y} = 1$$

$$y = 1$$

Now we know that our limits of integration are $c = 0$ and $d = 1$. Since the axis of revolution is $y = 1$, our radius is $1 - y$.

$f(y) - g(y)$ is the height of the approximating cylinder, which means we need to subtract the curve on the left $g(y)$ from the curve on the right $f(y)$. To figure out which curve is on the right and which is on the left, we can look at the graph or we can plug a y-value between the points of intersection (between $y = 0$ and $y = 1$) into both curves to see which function returns a larger value (this will be the right curve) and which one returns a smaller value (this will be the left curve). Let's plug in $y = 1/4$ to check.

$$x = y^{\frac{1}{2}}$$

$$x = \sqrt{\frac{1}{4}}$$

$$x = \frac{1}{2}$$

and

$$x = 1$$

Since $x = 1$ returns a larger value than $x = y^{\frac{1}{2}}$, we can say

$$g(y) = y^{\frac{1}{2}}$$

and

$$f(y) = 1$$

Plugging everything we know into the volume formula, we get

$$V = \int_0^1 2\pi(1 - y)\left(1 - y^{\frac{1}{2}}\right)\ dy$$

$$V = 2\pi \int_0^1 1 - y^{\frac{1}{2}} - y + y^{\frac{3}{2}}\ dy$$

$$V = 2\pi \left(y - \frac{2}{3}y^{\frac{3}{2}} - \frac{1}{2}y^2 + \frac{2}{5}y^{\frac{5}{2}}\right)\Bigg|_0^1$$

$$V = 2\pi \left[\left((1) - \frac{2}{3}(1)^{\frac{3}{2}} - \frac{1}{2}(1)^2 + \frac{2}{5}(1)^{\frac{5}{2}}\right) - \left((0) - \frac{2}{3}(0)^{\frac{3}{2}} - \frac{1}{2}(0)^2 + \frac{2}{5}(0)^{\frac{5}{2}}\right)\right]$$

$$V = 2\pi\left(1 - \frac{2}{3} - \frac{1}{2} + \frac{2}{5}\right)$$

$$V = 2\pi \left(\frac{30 - 20 - 15 + 12}{30} \right)$$

$$V = 2\pi \left(\frac{7}{30} \right)$$

$$V = \frac{7\pi}{15}$$

Topic: Cylindrical shells, horizontal axis

Question: Use cylindrical shells to find the volume of the solid generated by revolving the region bounded by the curves about the given axis.

$y = x^3$ and $x = 2$ and $y = 0$

about the line $y = 2$

Answer choices:

A $\frac{28\pi + 9\pi\sqrt[3]{2}}{7}$

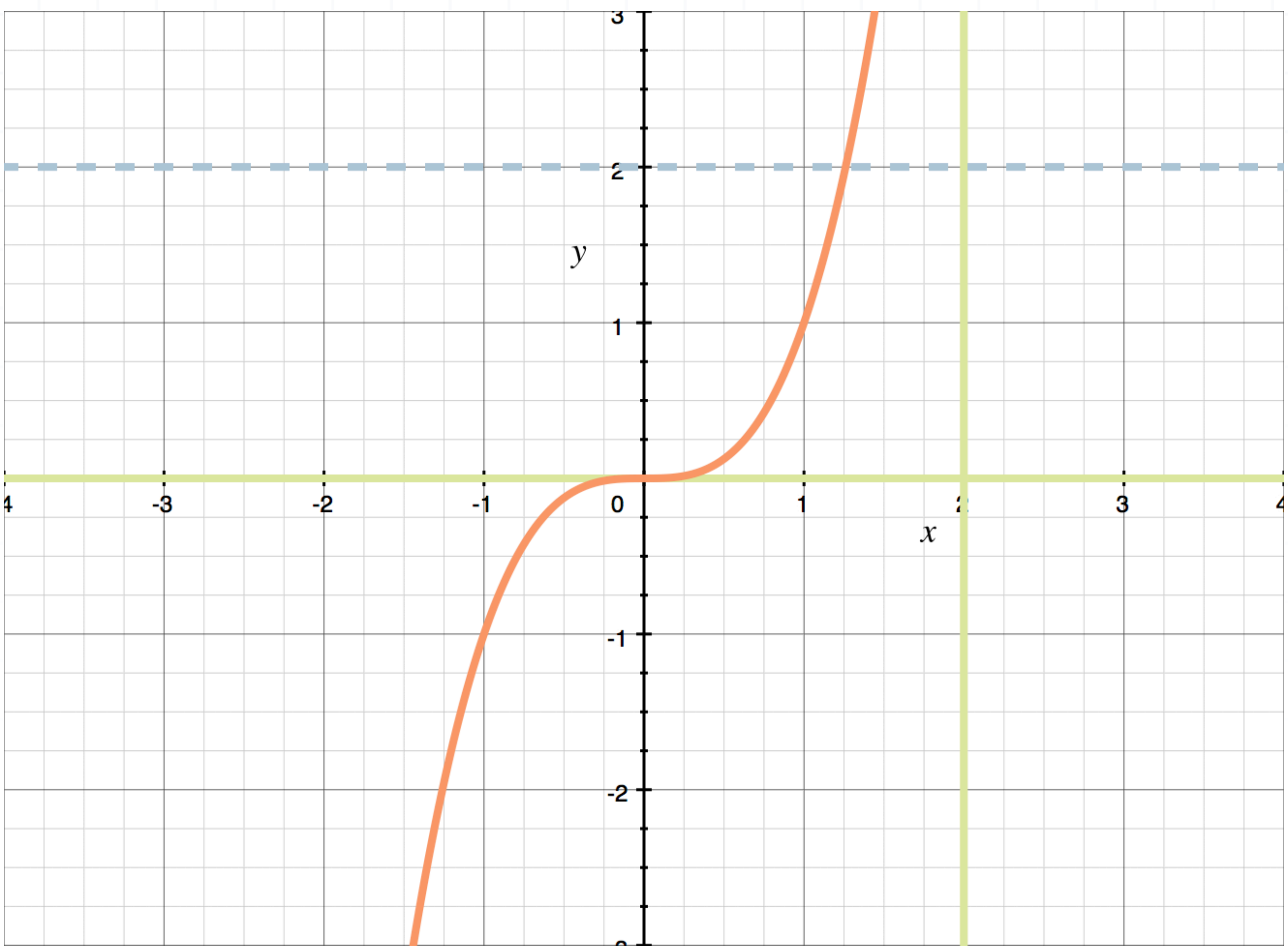
B $\frac{28\pi - 9\pi\sqrt[3]{2}}{7}$

C $\frac{56\pi + 18\pi\sqrt[3]{2}}{7}$

D $\frac{56\pi - 18\pi\sqrt[3]{2}}{7}$

Solution: D

Before doing anything else, we always want to draw a picture of the area. If you don't know how to graph the function, just plug in values for x or y to get individual coordinate points, and plot them until you have a picture of each function.



Using cylindrical shells means we'll take slices of our area that are parallel to the axis of rotation. Therefore, since the axis of rotation is horizontal, we'll take horizontal slices of our area and rotate each of them around the axis to form cylindrical shells.

Using cylindrical shells around a horizontal axis, specifically $y = 2$, tells us that we'll use the volume formula

$$V = \int_c^d 2\pi(k - y)\left[f(y) - g(y)\right] \ dy$$

We can see from the formula that we need our curves and our limits of integration defined in terms of y. Our curve $x=2$ is already defined for x in terms of y, so now we just need to define $y=x^3$ for x in terms of y.

$$y=x^3$$

$$y^{\frac{1}{3}}=\left(x^3\right)^{\frac{1}{3}}$$

$$y^{\frac{1}{3}}=x$$

$$x=y^{\frac{1}{3}}$$

Now we'll find the limits of integration, which will be the smallest and largest y-values for which the area is defined.

We can see from the graph that the smallest y-value for which the area is defined is $y=0$. This was given in the original problem. Since the area overlaps the axis of rotation, we have to cut the area off at the axis and say that the largest y-value for which the area is defined is $y=2$.

Now we know that our limits of integration are $c=0$ and $d=2$. Since the axis of revolution is $y=2$, our radius is $2-y$.

$f(y)-g(y)$ is the height of the approximating cylinder, which means we need to subtract the curve on the left $g(y)$ from the curve on the right $f(y)$. To figure out which curve is on the right and which is on the left, we can look at the graph or we can plug a y-value between the points of intersection (between $y=0$ and $y=2$) into both curves to see which function returns a larger value (this will be the right curve) and which one returns a smaller value (this will be the left curve). Let's plug in $y=1$ to check.

$$x = y^{\frac{1}{3}}$$

$$x = (1)^{\frac{1}{3}}$$

$$x = 1$$

and

$$x = 2$$

Since $x = 2$ returns a larger value than $x = y^{\frac{1}{3}}$, we can say

$$g(y) = y^{\frac{1}{3}}$$

and

$$f(y) = 2$$

Plugging everything we know into the volume formula, we get

$$V = \int_0^2 2\pi(2 - y)\left(2 - y^{\frac{1}{3}}\right)\ dy$$

$$V = 2\pi \int_0^2 4 - 2y^{\frac{1}{3}} - 2y + y^{\frac{4}{3}}\ dy$$

We'll integrate and then evaluate over the interval.

$$V = 2\pi\left(4y - \frac{6}{4}y^{\frac{4}{3}} - y^2 + \frac{3}{7}y^{\frac{7}{3}}\right)\Bigg|_0^2$$

$$V = 2\pi\left[\left(4(2) - \frac{6}{4}(2)^{\frac{4}{3}} - (2)^2 + \frac{3}{7}(2)^{\frac{7}{3}}\right) - \left(4(0) - \frac{6}{4}(0)^{\frac{4}{3}} - (0)^2 + \frac{3}{7}(0)^{\frac{7}{3}}\right)\right]$$

$$V=2\pi\left[8-\frac{3}{2}\left[(2)^4\right]^{\frac{1}{3}}-4+\frac{3}{7}\left[(2)^7\right]^{\frac{1}{3}}\right]$$

$$V=2\pi\left[8-\frac{3}{2}(16)^{\frac{1}{3}}-4+\frac{3}{7}(128)^{\frac{1}{3}}\right]$$

$$V=2\pi\left[8-\frac{3}{2}(8\cdot 2)^{\frac{1}{3}}-4+\frac{3}{7}(64\cdot 2)^{\frac{1}{3}}\right]$$

$$V=2\pi\left[8-\frac{3}{2}\left(8^{\frac{1}{3}}\cdot 2^{\frac{1}{3}}\right)-4+\frac{3}{7}(8\cdot 8\cdot 2)^{\frac{1}{3}}\right]$$

$$V=2\pi\left[8-\frac{3}{2}\left(2\cdot 2^{\frac{1}{3}}\right)-4+\frac{3}{7}\left(8^{\frac{1}{3}}\cdot 8^{\frac{1}{3}}\cdot 2^{\frac{1}{3}}\right)\right]$$

$$V=2\pi\left[8-3\left(2^{\frac{1}{3}}\right)-4+\frac{3}{7}\left(2\cdot 2\cdot 2^{\frac{1}{3}}\right)\right]$$

$$V=2\pi\left(8-3\sqrt[3]{2}-4+\frac{12}{7}\sqrt[3]{2}\right)$$

$$V=2\pi\left(4-3\sqrt[3]{2}+\frac{12}{7}\sqrt[3]{2}\right)$$

$$V=2\pi\left(\frac{28}{7}-\frac{21\sqrt[3]{2}}{7}+\frac{12\sqrt[3]{2}}{7}\right)$$

$$V=2\pi\left(\frac{28-21\sqrt[3]{2}+12\sqrt[3]{2}}{7}\right)$$

$$V=2\pi\left(\frac{28-9\sqrt[3]{2}}{7}\right)$$

$$V=\frac{56\pi-18\pi\sqrt[3]{2}}{7}$$