Riemann sums

Left endpoints, right endpoints, and midpoints

Since we don't know yet how to calculate the exact area under a curve (we'll learn how to do this later with integration), we'll need to use Riemann Sums to find an approximation of the area. The basic idea behind a Riemann Sum approximation is to use rectangles to approximate the area under the curve. The more rectangles we draw, the better our area approximation will be.

To use a Riemann Sum approximation, you'll simply calculate the area of each rectangle under the graph, and then add the areas together to get the total area under that section of the graph. The accuracy of the approximation will depend on whether we use the left-endpoints of the rectangles, the right-endpoints, or their midpoints. All three choices will give us different approximations of the area. Midpoint approximations are usually the most accurate.

Overestimation and underestimation

1. If the function is increasing everywhere in the interval, left endpoints will give an underestimation and right endpoints give an overestimation.

2. If the function is decreasing everywhere in the interval, left endpoints will give an overestimation and right endpoints give an underestimation.

3. Midpoints will usually give us a better approximation of both increasing and decreasing functions.

4. If the function is both increasing and decreasing at different points in the interval, it's going to be difficult for us to determine whether or not we'll get an under or overestimation from using left or right endpoints.

Calculating a Riemann sum

In order to calculate a Riemann Sum, follow these steps:

1. Find Δx if $\Delta x = \dfrac{b-a}{n}$.

2. Divide your interval and separate the x-axis into increments of Δx.

3. Decide whether you'll use the left endpoints, right endpoints, or midpoints of each of the rectangles indicated by the increments you created in Step 2.

4. Evaluate your function at these points, add all of your answers together, and then multiply your final answer by Δx. This is the Riemann sum for the interval.

$$A = \sum_{i=1}^{n} f(x_i)\Delta x$$

A note about integration

There is no limit to the number of rectangles you use to evaluate the area under the graph. The more rectangles you use, the more accurate your approximation will be. If you used an infinite number of rectangles, and therefore the value of Δx approached 0, you'd be integrating and finding the exact value of the area, instead of just an approximation.