Topic: Power series division

Question: Use power series division to find the first three non-zero terms of the Maclaurin series.

$$y = \frac{x}{e^{2x}}$$

Answer choices:

A $\quad \dfrac{x}{e^{2x}} = x + 2x^2 + 2x^3$

B $\quad \dfrac{x}{e^{2x}} = x - 2x^2 + 2x^3$

C $\quad \dfrac{x}{e^{2x}} = x - 2x^2 - 2x^3$

D $\quad \dfrac{x}{e^{2x}} = -x - 2x^2 - 2x^3$

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Solution: B

When we divide one power series by another, we want to find the expansion of the sum of each series, so that we essentially have polynomial representations. Then finding the quotient of the series will be like dividing polynomials.

The numerator of the given function, x, is already in polynomial form, so we just need a series expansion for the denominator.

There's a common Maclaurin series that's similar to e^{2x}.

$$e^x = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \dots$$

We want to modify this common series to match the given series.

For $y = e^{2x}$, we'll substitute $2x$ for x:

$$e^{2x} = 1 + 2x + \frac{1}{2}(2x)^2 + \frac{1}{6}(2x)^3 + \dots$$

$$e^{2x} = 1 + 2x + 2x^2 + \frac{4x^3}{3} + \dots$$

Dividing the numerator, x, by the modified series, we get

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$$
\begin{array}{r|l}
 & x-2x^2+2x^3-\frac{4x^4}{3} \\
\hline
1+2x+2x^2+\frac{4x^3}{3} & x+0x^2+0x^3+0x^4 \\
 & -\left(x+2x^2+2x^3+\frac{4x^4}{3}\right) \\
\cline{2-2}
 & -2x^2-2x^3-\frac{4x^4}{3} \\
 & -\left(-2x^2-4x^3-4x^4\right) \\
\cline{2-2}
 & 2x^3+\frac{8x^4}{3} \\
 & -\left(2x^3+4x^4\right) \\
\cline{2-2}
 & -\frac{4x^4}{3} \\
 & -\left(-\frac{4x^4}{3}\right) \\
\cline{2-2}
 & 0
\end{array}
$$

Since we only need the first three non-zero terms, our answer will be

$$\frac{x}{e^{2x}} = x - 2x^2 + 2x^3$$

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Topic: Power series division

Question: Use power series division to find the first three non-zero terms of the Maclaurin series.

$$y = \frac{2x}{\ln(1 + 2x)}$$

Answer choices:

A $\quad \frac{2x}{\ln(1+2x)} = 1 - x + \frac{x^2}{3}$

B $\quad \frac{2x}{\ln(1+2x)} = 1 + x + \frac{x^2}{3}$

C $\quad \frac{2x}{\ln(1+2x)} = 1 + x - \frac{x^2}{3}$

D $\quad \frac{2x}{\ln(1+2x)} = 1 - x - \frac{x^2}{3}$

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Solution: C

When we divide one power series by another, we want to find the expansion of the sum of each series, so that we essentially have polynomial representations. Then finding the quotient of the series will be like dividing polynomials.

The numerator of the given function, $2x$, is already in polynomial form, so we just need a series expansion for the denominator.

There's a common Maclaurin series that's similar to $\ln(1+2x)$.

$$\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 + \dots$$

We want to modify this common series to match the given series.

For $y = \ln(1+2x)$, we'll substitute $2x$ for x:

$$\ln(1+2x) = 2x - \frac{1}{2}(2x)^2 + \frac{1}{3}(2x)^3 + \dots$$

$$\ln(1+2x) = 2x - \frac{4x^2}{2} + \frac{8x^3}{3} + \dots$$

$$\ln(1+2x) = 2x - 2x^2 + \frac{8x^3}{3} + \dots$$

Dividing the numerator, $2x$, by the modified series, we get

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$$
\begin{array}{r|l}
 & 1 + x - \dfrac{x^2}{3} \\
\hline
2x - 2x^2 + \dfrac{8x^3}{3} & 2x + 0x^2 + 0x^3 + 0x^4 \\
 & -\left(2x - 2x^2 + \dfrac{8x^3}{3}\right) \\
\hline
 & 2x^2 - \dfrac{8x^3}{3} \\
 & -\left(2x^2 - 2x^3 + \dfrac{8x^4}{3}\right) \\
\hline
 & -\dfrac{2x^3}{3} - \dfrac{8x^4}{3} \\
 & -\left(-\dfrac{2x^3}{3} + \dfrac{2x^4}{3} - \dfrac{8x^5}{9}\right) \\
\hline
 & -\dfrac{10x^4}{3} + \dfrac{8x^5}{9}
\end{array}
$$

Since we only need the first three non-zero terms, our answer will be

$$\frac{2x}{\ln(1+2x)} = 1 + x - \frac{x^2}{3}$$

Topic: Power series division

Question: Use power series division to find the first three non-zero terms of the Maclaurin series.

$$y = \frac{e^{3x}}{\frac{1}{1-3x}}$$

Answer choices:

A $\quad \dfrac{e^{3x}}{\frac{1}{1-3x}} = 1 + \dfrac{9x}{2} + 9x^2$

B $\quad \dfrac{e^{3x}}{\frac{1}{1-3x}} = 1 + \dfrac{9x^2}{2} - 9x^3$

C $\quad \dfrac{e^{3x}}{\frac{1}{1-3x}} = 1 - \dfrac{9x}{2} + 9x^2$

D $\quad \dfrac{e^{3x}}{\frac{1}{1-3x}} = 1 - \dfrac{9x^2}{2} - 9x^3$

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Solution: D

When we divide one power series by another, we want to find the expansion of the sum of each series, so that we essentially have polynomial representations. Then finding the quotient of the series will be like dividing polynomials.

We need to recognize that the given series is the quotient of two other series

$$y = e^{3x}$$

$$y = \frac{1}{1 - 3x}$$

There are common Maclaurin series that are similar to each of these.

$$e^x = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \dots$$

$$\frac{1}{1 - x} = 1 + x + x^2 + x^3 + \dots$$

We want to modify each of these common series to match the given series.

For $y = e^{3x}$, we'll substitute $3x$ for x:

$$e^{3x} = 1 + 3x + \frac{1}{2}(3x)^2 + \frac{1}{6}(3x)^3 + \dots$$

$$e^{3x} = 1 + 3x + \frac{9x^2}{2} + \frac{9x^3}{2} + \dots$$

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For $y = 1/(1 - 3x)$, we'll substitute $3x$ for x:

$$\frac{1}{1 - 3x} = 1 + 3x + (3x)^2 + (3x)^3 + \dots$$

$$\frac{1}{1 - 3x} = 1 + 3x + 9x^2 + 27x^3 + \dots$$

Dividing these modified series, we get

$$1 - \frac{9x^2}{2} - 9x^3$$

$$1 + 3x + 9x^2 + 27x^3 \,\overline{\big)\, 1 + 3x + \frac{9x^2}{2} + \frac{9x^3}{2}}$$

$$-(1 + 3x + 9x^2 + 27x^3)$$

$$-\frac{9x^2}{2} - \frac{45x^3}{2}$$

$$-\left(-\frac{9x^2}{2} - \frac{27x^3}{2}\right)$$

$$-9x^3$$

$$-(-9x^3)$$

$$0$$

Since we only need the first three non-zero terms, our answer will be

$$\frac{e^{3x}}{\frac{1}{1 - 3x}} = 1 - \frac{9x^2}{2} - 9x^3$$

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