Topic: Summation notation, finding the sum

Question: Calculate the exact sum.

$$\sum_{n=1}^{5} \frac{n^2}{2^n}$$

Answer choices:

A 4.4

B $\frac{141}{32}$

C 27.5

 $D \qquad \frac{57}{32}$

Solution: B

The question asks you to find

$$\sum_{n=1}^{5} \frac{n^2}{2^n}$$

To find the sum, write each term with the value of n for that term. Since the summation shows that n begins with a value of 1 and ends with a value of 5, there are five terms.

$$n = 1$$

$$\frac{1^2}{2^1} = \frac{1}{2}$$

$$n = 2$$

$$\frac{2^2}{2^2} = \frac{4}{4} = 1$$

$$n = 3$$

$$\frac{3^2}{2^3} = \frac{9}{8}$$

$$n = 4$$

$$\frac{4^2}{2^4} = \frac{16}{16} = 1$$

$$n = 5$$

$$\frac{5^2}{2^5} = \frac{25}{32}$$

Add all the terms together.

$$\frac{1}{2} + 1 + \frac{9}{8} + 1 + \frac{25}{32}$$

Rewrite the terms with a least common denominator of 32 and combine the five terms.

16	32	36	32	25	141
32	$\overline{32}$	32	$\overline{32}$	${32}$	32



Topic: Summation notation, finding the sum

Question: Calculate the exact sum.

$$\sum_{n=1}^{4} \frac{3n}{2n+1}$$

Answer choices:

A
$$\frac{1,788}{910}$$

B
$$\frac{2,081}{910}$$

C
$$\frac{482}{105}$$

D
$$\frac{506}{105}$$

Solution: D

The question asks you to find

$$\sum_{n=1}^{4} \frac{3n}{2n+1}$$

To find the sum, write each term with the value of n for that term. Since the summation shows that n begins with a value of 1 and ends with 4, there are four terms.

$$n = 1 \qquad \frac{3 \times 1}{2 \times 1 + 1} = \frac{3}{3} = 1$$

$$n=2 \qquad \qquad \frac{3\times 2}{2\times 2+1} = \frac{6}{5}$$

$$n = 3 \qquad \frac{3 \times 3}{2 \times 3 + 1} = \frac{9}{7}$$

$$n = 4 \qquad \frac{3 \times 4}{2 \times 4 + 1} = \frac{12}{9} = \frac{4}{3}$$

Add the terms.

$$1 + \frac{6}{5} + \frac{9}{7} + \frac{4}{3}$$

Rewrite the terms with a least common denominator of 105 and combine the four terms.

$$\frac{105}{105} + \frac{126}{105} + \frac{135}{105} + \frac{140}{105} = \frac{506}{105}$$

Topic: Summation notation, finding the sum

Question: Calculate the exact sum.

$$\sum_{n=0}^{6} (-1)^n \frac{5}{n+2}$$

Answer choices:

A
$$\frac{481}{56}$$

B
$$-\frac{481}{56}$$

C
$$\frac{307}{168}$$

D
$$-\frac{307}{168}$$

Solution: C

The question asks you to find

$$\sum_{n=0}^{6} (-1)^n \frac{5}{n+2}$$

To find the sum, write each term with the value of n for that term. Since the summation shows that n begins with a value of 0, and ends with 6, there are seven terms.

$$n = 0$$

$$(-1)^{0} \times \frac{5}{0+2} = 1 \times \frac{5}{2} = \frac{5}{2}$$

$$n = 1$$

$$(-1)^{1} \times \frac{5}{1+2} = (-1) \times \frac{5}{3} = -\frac{5}{3}$$

$$n = 2$$

$$(-1)^{2} \times \frac{5}{2+2} = 1 \times \frac{5}{4} = \frac{5}{4}$$

$$n = 3$$

$$(-1)^{3} \times \frac{5}{3+2} = (-1) \times \frac{5}{5} = -\frac{5}{5} = -1$$

$$n = 4$$

$$(-1)^{4} \times \frac{5}{4+2} = 1 \times \frac{5}{6} = \frac{5}{6}$$

$$n = 5$$

$$(-1)^{5} \times \frac{5}{5+2} = (-1) \times \frac{5}{7} = -\frac{5}{7}$$

$$n = 6$$

$$(-1)^{6} \times \frac{5}{6+2} = 1 \times \frac{5}{8} = \frac{5}{8}$$

Add the terms.

$$\frac{5}{2} - \frac{5}{3} + \frac{5}{4} - 1 + \frac{5}{6} - \frac{5}{7} + \frac{5}{8}$$

Rewrite the terms with a least common denominator of 168 and combine the seven terms.

$$\frac{420}{168} - \frac{280}{168} + \frac{210}{168} - \frac{168}{168} + \frac{140}{168} - \frac{120}{168} + \frac{105}{168} = \frac{307}{168}$$

