Topic: Endpoint discontinuities

Question: Which of the following statements is true?

Answer choices:

A The endpoint of an interval is discontinuous because one of the one-sided limits will be 0.

B The endpoint of an interval is discontinuous because one of the one-sided limits will be ∞.

C The endpoint of an interval is discontinuous because one of the one-sided limits will not exist.

D The endpoint of an interval is discontinuous because both of the one-sided limits will not exist.

Solution: C

The endpoint of an interval is discontinuous because one of the one-sided limits does not exist.

Because the function stops at an endpoint, either the left-hand limit will exist while the right-hand limit does not, or the right-hand limit will exist while the left-hand limit does not.

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Question: If the function $f(x) = x^2$ is only defined on $[1,4]$, and does not extend beyond that interval, what are the discontinuities of the function?

Answer choices:

A Endpoint discontinuities at $x = 0,\ 4$.

B A jump discontinuity at $x = 0$.

C Endpoint discontinuities at $x = 1,\ 4$ and a jump discontinuity at $x = 0$.

D Endpoint discontinuities at $x = 1,\ 4$.

Solution: D

The endpoints of an interval are discontinuous for a function because one of the one-sided limits will not exist at each endpoint.

The function $f(x) = x^2$ is a continuous function, but the interval $[1,4]$ means that there will be endpoint discontinuities at $x = 1$ and $x = 4$.

At $x = 1$, only the right-hand limit exists. The left-hand limit would be outside the function's domain. By the definition of continuity (that the left-hand limit exists, the right-hand limit exists, and the left- and right-hand limits are equal), that means the function isn't continuous at $x = 1$, so there's an endpoint discontinuity there.

At $x = 4$, only the left-hand limit exists. The right-hand limit is outside the function's domain. By the definition of continuity, that means the function isn't continuous at $x = 4$, so there's an endpoint discontinuity there.

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Question: What are the discontinuities of the function on the interval $[2,5]$?

$$f(x) = \sqrt{x}$$

Answer choices:

A Endpoint discontinuities at $x = 2$ and $x = 5$ and when $x \geq 0$.

B Endpoint discontinuities at $x = 2$ and $x = 5$.

C Endpoint discontinuities at $x = 2$ and $x = 5$ and when $x \leq 0$.

D Endpoint discontinuities at $x = 0$ and $x = 5$.

Solution: B

The function $f(x) = \sqrt{x}$ is a continuous function when $x \geq 0$ but the interval $[2,5]$ means that there will be endpoint discontinuities at the points $x = 2$ and $x = 5$.

An endpoint discontinuity exists at $x = 2$ because the left-hand limit doesn't exist there, and an endpoint discontinuity exists at $x = 5$ because the right-hand limit doesn't exist there.