

Topic: Infinite limits and vertical asymptotes**Question:** Evaluate the limit.

$$\lim_{x \rightarrow 2} \frac{\sqrt{x^2 + 1}}{(x - 2)^2}$$

Answer choices:

- A 0
- B 1
- C ∞
- D $-\infty$



Solution: C

We can't use substitution to evaluate the limit, because we get a 0 in the denominator and therefore an undefined value.

$$\frac{\sqrt{2^2 + 1}}{(2 - 2)^2}$$

$$\frac{\sqrt{5}}{0^2}$$

$$\frac{\sqrt{5}}{0}$$

So we'll test values close to, and on either side of, $x = 2$ to see how the function is behaving close to that point.

$$f(1.9999) = \frac{\sqrt{1.9999^2 + 1}}{(1.9999 - 2)^2} = \frac{4.999}{0.00000001} = 499,900,000$$

$$f(2.0001) = \frac{\sqrt{2.0001^2 + 1}}{(2.0001 - 2)^2} = \frac{5.0004}{0.00000001} = 500,040,001$$

The function is approaching ∞ on both sides of $x = 2$, so we can say that the value of the limit is ∞ .

$$\lim_{x \rightarrow 2} \frac{\sqrt{x^2 + 1}}{(x - 2)^2} = \infty$$



Topic: Infinite limits and vertical asymptotes**Question:** Evaluate the limit.

$$\lim_{x \rightarrow \frac{\pi}{2}} \tan x$$

Answer choices:

A $\lim_{x \rightarrow \frac{\pi}{2}^-} \tan x = -\infty$

$$\lim_{x \rightarrow \frac{\pi}{2}^+} \tan x = \infty$$

B $\lim_{x \rightarrow \frac{\pi}{2}^-} \tan x = \infty$

$$\lim_{x \rightarrow \frac{\pi}{2}^+} \tan x = -\infty$$

C $\lim_{x \rightarrow \frac{\pi}{2}^-} \tan x = -\infty$

$$\lim_{x \rightarrow \frac{\pi}{2}^+} \tan x = -\infty$$

D $\lim_{x \rightarrow \frac{\pi}{2}^-} \tan x = \infty$

$$\lim_{x \rightarrow \frac{\pi}{2}^+} \tan x = \infty$$



Solution: B

We can't use substitution to evaluate the limit, because we get a 0 in the denominator and therefore an undefined value.

$$\tan \frac{\pi}{2}$$

$$\frac{\sin \frac{\pi}{2}}{\cos \frac{\pi}{2}}$$

$$\frac{1}{0}$$

So we'll test values close to, and on either side of, $x = \pi/2$ to see how the function is behaving close to that point.

$$f\left(\frac{49\pi}{100}\right) = \tan \frac{49\pi}{100} \approx 31.82$$

$$f\left(\frac{51\pi}{100}\right) = \tan \frac{51\pi}{100} \approx -31.82$$

The function is approaching ∞ to the left of $x = \pi/2$ and $-\infty$ to the right of $x = \pi/2$, so the general limit does not exist. But the one-sided limits are

$$\lim_{x \rightarrow \frac{\pi}{2}^-} \tan x = \infty$$

$$\lim_{x \rightarrow \frac{\pi}{2}^+} \tan x = -\infty$$



Topic: Infinite limits and vertical asymptotes**Question:** Evaluate the limit.

$$\lim_{x \rightarrow \pi} \cot x$$

Answer choices:

- A 0
- B 1
- C ∞
- D Does not exist (DNE)



Solution: D

We can't use substitution to evaluate the limit, because we get a 0 in the denominator and therefore an undefined value.

$$\cot \pi$$

$$\frac{\cos \pi}{\sin \pi}$$

$$\frac{-1}{0}$$

So we'll test values close to, and on either side of, $x = \pi$ to see how the function is behaving close to that point.

$$f\left(\frac{99\pi}{100}\right) = \cot \frac{99\pi}{100} \approx -31.82$$

$$f\left(\frac{101\pi}{100}\right) = \cot \frac{101\pi}{100} \approx 31.82$$

The function is approaching $-\infty$ to the left of $x = \pi$ and ∞ to the right of $x = \pi$, so the general limit does not exist.

