Topic: Sum of the geometric series

Question: Find the sum of the geometric series.

$$\sum_{n=0}^{\infty} \frac{3}{4^n}$$

Answer choices:

A 4

B 3

C 2

D 1

Solution: A

We need to get the series into standard form for a geometric series to make sure the series is geometric. Since the index starts at $n = 0$, standard form is

$$\sum_{n=0}^{\infty} ar^n$$

so we'll rewrite the series as

$$\sum_{n=0}^{\infty} \frac{3}{4^n}$$

$$\sum_{n=0}^{\infty} 3\frac{1}{4^n}$$

$$\sum_{n=0}^{\infty} 3\frac{1^n}{4^n}$$

$$\sum_{n=0}^{\infty} 3\left(\frac{1}{4}\right)^n$$

Comparing this to the standard form, we'll say that

$$a = 3$$

and

$$r = \frac{1}{4}$$

Since

$$\left|\frac{1}{4}\right| = \frac{1}{4} < 1$$

the series converges by the geometric series test for convergence, which means we can find the sum of the series using

$$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$$

$$\sum_{n=0}^{\infty} ar^n = \frac{3}{1-\frac{1}{4}}$$

$$\sum_{n=0}^{\infty} ar^n = \frac{3}{\frac{4}{4}-\frac{1}{4}}$$

$$\sum_{n=0}^{\infty} ar^n = \frac{3}{\frac{3}{4}}$$

$$\sum_{n=0}^{\infty} ar^n = 3 \cdot \frac{4}{3}$$

$$\sum_{n=0}^{\infty} ar^n = 4$$

The sum of the series is 4.

Topic: Sum of the geometric series

Question: Find the sum of the geometric series.

$$\sum_{n=1}^{\infty} 2\left(\frac{1}{2}\right)^{n-1}$$

Answer choices:

A 2

B 1

C 4

D $\frac{1}{2}$

Solution: C

We need to get the series into standard form for a geometric series to make sure the series is geometric. Since the index starts at $n = 1$, standard form is

$$\sum_{n=1}^{\infty} ar^{n-1}$$

Comparing our series to this standard form, we'll say that

$$a = 2$$

and

$$r = \frac{1}{2}$$

Since

$$\left| \frac{1}{2} \right| = \frac{1}{2} < 1$$

the series converges by the geometric series test for convergence, which means we can find the sum of the series using

$$\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r}$$

$$\sum_{n=1}^{\infty} ar^{n-1} = \frac{2}{1-\frac{1}{2}}$$

$$\sum_{n=1}^{\infty} ar^{n-1} = \frac{2}{\frac{2}{2} - \frac{1}{2}}$$

$$\sum_{n=1}^{\infty} ar^{n-1} = \frac{2}{\frac{1}{2}}$$

$$\sum_{n=1}^{\infty} ar^{n-1} = 2 \cdot \frac{2}{1}$$

$$\sum_{n=1}^{\infty} ar^{n-1} = 4$$

The sum of the series is 4.

Topic: Sum of the geometric series

Question: Find the sum of the geometric series.

$$\sum_{n=0}^{\infty} \frac{3^{n-1}}{2^n}$$

Answer choices:

A $-\frac{1}{2}$

B 1

C $\frac{1}{2}$

D The sum can't be found because the series diverges.

Solution: D

We need to get the series into standard form for a geometric series to make sure the series is geometric. Since the index starts at $n = 0$, standard form is

$$\sum_{n=0}^{\infty} ar^n$$

so we'll rewrite the series as

$$\sum_{n=0}^{\infty} \frac{3^{n-1}}{2^n}$$

$$\sum_{n=0}^{\infty} \frac{3^n 3^{-1}}{2^n}$$

$$\sum_{n=0}^{\infty} 3^{-1} \cdot \frac{3^n}{2^n}$$

$$\sum_{n=0}^{\infty} \frac{1}{3} \left(\frac{3}{2} \right)^n$$

Comparing this to the standard form, we'll say that

$$a = \frac{1}{3}$$

and

$$r = \frac{3}{2}$$

Since

$$\left|\frac{3}{2}\right| = \frac{3}{2} \geq 1$$

the series diverges by the geometric series test for convergence, which means we can't find the sum.