Topic: Trigonometric substitution setup

Question: Set up this integral for trigonometric substitution. Simplify, but don't evaluate the integral.

$$\int \frac{x}{\sqrt{4-9x^2}}\ dx$$

Answer choices:

A $\quad \dfrac{2}{9}\displaystyle\int \cos\theta\ d\theta$

B $\quad \displaystyle\int \sin\theta\ d\theta$

C $\quad \displaystyle\int \cos\theta\ d\theta$

D $\quad \dfrac{2}{9}\displaystyle\int \sin\theta\ d\theta$

Solution: D

The question asks us to set up this integral for trigonometric substitution.

$$\int \frac{x}{\sqrt{4-9x^2}}\ dx$$

The integral contains an expression of the form a^2-u^2 where a^2 is a number and u^2 is a function of x. This format requires the trigonometric substitution to be in the form $u=a\sin\theta$.

Now we'll use the values in the integral to find a and u.

$$a^2=4$$

$$a=2$$

and

$$u^2=9x^2$$

$$u=3x$$

This means that

$$u=a\sin\theta$$

$$3x=2\sin\theta$$

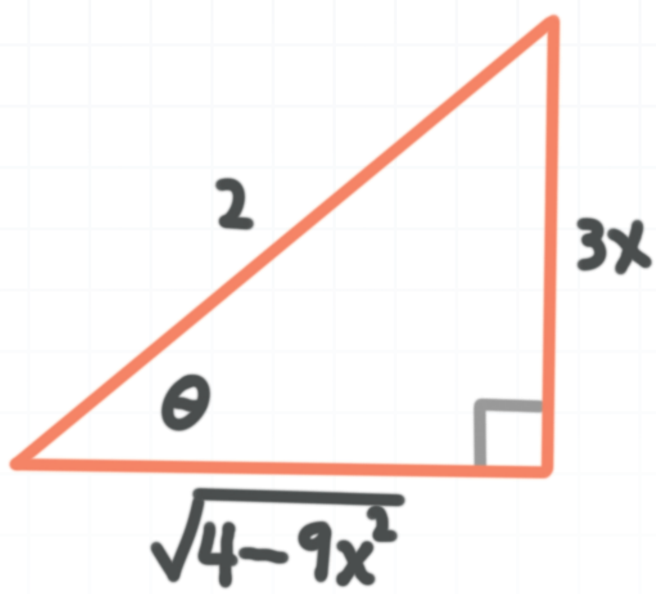
$$\sin\theta=\frac{3x}{2}$$

$$\theta=\arcsin\left(\frac{3x}{2}\right)$$

To put this in the perspective of right triangle trigonometry, recall that

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

This means we're dealing with a right triangle like this



We'll solve for x in terms of θ.

$$3x = 2\sin\theta$$

$$x = \frac{2}{3}\sin\theta$$

$$dx = \frac{2}{3}\cos\theta\ d\theta$$

We're finally ready to do the trigonometric substitution.

$$\int \frac{x}{\sqrt{4-9x^2}}\ dx$$

$$\int \frac{\left(\frac{2}{3}\sin\theta\right)}{\sqrt{4-9\left(\frac{2}{3}\sin\theta\right)^2}}\left(\frac{2}{3}\cos\theta\right)\ d\theta$$

$$\int \frac{\frac{2}{3}\sin\theta}{\sqrt{4-9\left(\frac{4}{9}\sin^2\theta\right)}}\frac{2}{3}\cos\theta\ d\theta$$

We can cancel the 9 in the radical. We can also remove the fractions, simplify them, and place them in front of the integral.

$$\frac{4}{9}\int \frac{\sin\theta\cos\theta}{\sqrt{4-4\sin^2\theta}}\ d\theta$$

$$\frac{4}{9}\int \frac{\sin\theta\cos\theta}{\sqrt{4\left(1-\sin^2\theta\right)}}\ d\theta$$

$$\frac{4}{9}\int \frac{\sin\theta\cos\theta}{\sqrt{4\left(\cos^2\theta\right)}}\ d\theta$$

$$\frac{4}{9}\int \frac{\sin\theta\cos\theta}{2\left(\cos\theta\right)}\ d\theta$$

$$\frac{2}{9}\int \sin\theta\ d\theta$$

Therefore,

$$\int \frac{x}{\sqrt{4-9x^2}}\ dx = \frac{2}{9}\int \sin\theta\ d\theta$$

Topic: Trigonometric substitution setup

Question: Set up this integral for trigonometric substitution. Simplify, but don't evaluate the integral.

$$\int \sqrt{9x^2 + 16} \ dx$$

Answer choices:

A $\quad \dfrac{16}{3} \int \sec^2\theta \ d\theta$

B $\quad \dfrac{16}{3} \int \sec^3\theta \ d\theta$

C $\quad \dfrac{16}{3} \int \tan^3\theta \ d\theta$

D $\quad \dfrac{16}{3} \int \tan^2\theta \ d\theta$

Solution: B

The question asks us to set up this integral for trigonometric substitution.

$$\int \sqrt{9x^2 + 16}\ dx$$

The integral contains an expression of the form $u^2 + a^2$ where a^2 is a number and u^2 is a function of x. This format requires the trigonometric substitution to be in the form $u = a \tan \theta$.

Now we'll use the values in the integral to find a and u.

$$a^2 = 16$$

$$a = 4$$

and

$$u^2 = 9x^2$$

$$u = 3x$$

This means that

$$u = a \tan \theta$$

$$3x = 4 \tan \theta$$

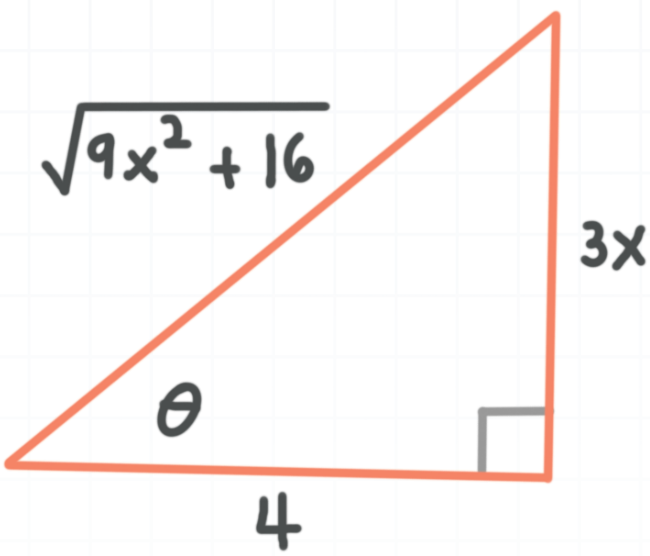
$$\tan \theta = \frac{3x}{4}$$

$$\theta = \arctan \left(\frac{3x}{4} \right)$$

To put this in the perspective of right triangle trigonometry, recall that

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

This means we're dealing with a right triangle like this



We'll solve for x in terms of θ.

$$3x = 4\tan\theta$$

$$x = \frac{4}{3}\tan\theta$$

$$dx = \frac{4}{3}\sec^2\theta\ d\theta$$

We're finally ready to do the trigonometric substitution.

$$\int\sqrt{9x^2+16}\ dx$$

$$\int\sqrt{9\left(\frac{4}{3}\tan\theta\right)^2+16}\left(\frac{4}{3}\sec^2\theta\right)\ d\theta$$

$$\frac{4}{3}\int \sec^2\theta\sqrt{9\left(\frac{16}{9}\tan^2\theta\right)+16}\ d\theta$$

$$\frac{4}{3}\int \sec^2\theta\sqrt{16\tan^2\theta+16}\ d\theta$$

$$\frac{4}{3}\int \sec^2\theta\sqrt{16\left(\tan^2\theta+1\right)}\ d\theta$$

$$\frac{4}{3}\int \sec^2\theta\sqrt{16\sec^2\theta}\ d\theta$$

$$\frac{16}{3}\int \sec^3\theta\ d\theta$$

Therefore,

$$\int \sqrt{9x^2+16}\ dx=\frac{16}{3}\int \sec^3\theta\ d\theta$$

Topic: Trigonometric substitution setup

Question: Setup this integral for trigonometric substitution.

$$\int \frac{\sqrt{4x^2-25}}{x}\ dx$$

Answer choices:

A $5\int \tan^2\theta\ d\theta$

B $\int \tan^2\theta\ d\theta$

C $5\int \sec^2\theta\ d\theta$

D $5\int \sec^2\theta\ d\theta$

Solution: A

The question asks us to set up this integral for trigonometric substitution.

$$\int \frac{\sqrt{4x^2 - 25}}{x}\ dx$$

The integral contains an expression of the form $u^2 - a^2$ where a^2 is a number and u^2 is a function of x. This format requires the trigonometric substitution to be in the form $u = a \sec \theta$.

Now we'll use the values in the integral to find a and u.

$$a^2 = 25$$

$$a = 5$$

and

$$u^2 = 4x^2$$

$$u = 2x$$

This means that

$$u = a \sec \theta$$

$$2x = 5 \sec \theta$$

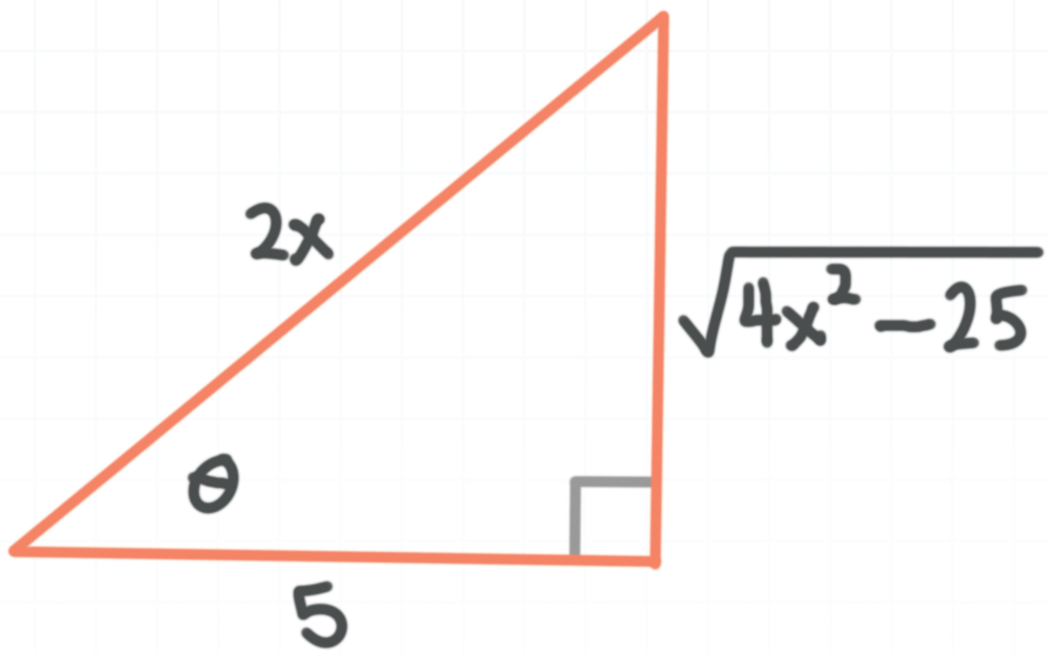
$$\sec \theta = \frac{2x}{5}$$

$$\theta = \text{arcsec}\left(\frac{2x}{5}\right)$$

To put this in the perspective of right triangle trigonometry, recall that

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

This means we're dealing with a right triangle like this



We'll solve for x in terms of θ.

$$2x = 5\sec\theta$$

$$x = \frac{5}{2}\sec\theta$$

$$dx = \frac{5}{2}\sec\theta\tan\theta\ d\theta$$

We're finally ready to do the trigonometric substitution.

$$\int \frac{\sqrt{4x^2-25}}{x}\ dx$$

$$\int \frac{\sqrt{4\left(\frac{5}{2}\sec\theta\right)^2 - 25}}{\frac{5}{2}\sec\theta}\left(\frac{5}{2}\sec\theta\tan\theta\ d\theta\right)$$

$$\int \sqrt{4\left(\frac{5}{2}\sec\theta\right)^2 - 25}\ (\tan\theta\ d\theta)$$

$$\int \sqrt{4\left(\frac{25}{4}\sec^2\theta\right) - 25}\ (\tan\theta\ d\theta)$$

$$\int \sqrt{25\sec^2\theta - 25}\ (\tan\theta\ d\theta)$$

$$\int \tan\theta\sqrt{25\left(\sec^2\theta - 1\right)}\ d\theta$$

Remembering the trig identity $1 + \tan^2 x = \sec^2 x$, we can make the substitution for $\sec^2\theta - 1$.

$$\int \tan\theta\sqrt{25\tan^2\theta}\ d\theta$$

$$\int \tan\theta(5\tan\theta)\ d\theta$$

$$5\int \tan^2\theta\ d\theta$$