Topic: Hyperbolic integrals

Question: Evaluate the hyperbolic integral.

$$\int x\cosh\left(x^2+3\right)\ dx$$

Answer choices:

A $\quad \sinh\left(x^2-3\right)+C$

B $\quad \sinh\left(x^2\right)+C$

C $\quad \dfrac{1}{2}\sinh\left(x^2\right)+C$

D $\quad \dfrac{1}{2}\sinh\left(x^2+3\right)+C$

Solution: D

Let

$$u = x^2 + 3$$

$$du = 2x\ dx$$

$$\frac{du}{2x} = dx$$

When we plug these values into the integral, we get

$$\int x \cosh \left(x^2 + 3\right)\ dx$$

$$\int x \cosh u\ \frac{du}{2x}$$

$$\frac{1}{2} \int \cosh u\ du$$

Knowing that

$$\int \cosh x\ dx = \sinh x + C$$

we get

$$\frac{1}{2} \sinh \left(x^2 + 3\right) + C$$

Topic: Hyperbolic integrals

Question: Evaluate the hyperbolic integral.

$$\int x^2 \text{sech}\left(\frac{1}{3}x^3\right)\ dx$$

Answer choices:

A $\sin^{-1}\left[\sinh\left(\frac{1}{3}x^3\right)\right]+C$

B $\cos^{-1}\left[\sinh\left(\frac{1}{3}x^3\right)\right]+C$

C $\tan^{-1}\left[\sinh\left(\frac{1}{3}x^3\right)\right]+C$

D $\cot^{-1}\left[\sinh\left(\frac{1}{3}x^3\right)\right]+C$

Solution: C

Let

$$u = \frac{1}{3}x^3$$

$$du = x^2\ dx$$

When we plug these values into the integral, we get

$$\int x^2 \text{sech}\left(\frac{1}{3}x^3\right)\ dx$$

$$\int \text{sech}\left(\frac{1}{3}x^3\right)\ \left(x^2\ dx\right)$$

$$\int \text{sech}\ u\ du$$

Knowing that

$$\int \text{sech}\ u\ du = \tan^{-1}\left(\sinh u\right) + C$$

we get

$$\tan^{-1}\left(\sinh u\right) + C$$

$$\tan^{-1}\left[\sinh\left(\frac{1}{3}x^3\right)\right] + C$$

Topic: Hyperbolic integrals

Question: Evaluate the hyperbolic integral.

$$\int_{-\ln 4}^{-\ln 12} 2e^t \sinh t \ dt$$

Answer choices:

A $\frac{2}{5} + \ln 2$

B $\ln 5 - \frac{1}{12}$

C $\frac{1}{36} - \ln 3$

D $\ln 3 - \frac{1}{36}$

Solution: D

At first, the integral may look like a hyperbolic function problem. However, the exponential expression in the integral complicates the problem. So we decompose the hyperbolic expression into its exponential definition

$$\sinh u = \frac{e^u - e^{-u}}{2}$$

and solve the integral as an exponential function.

$$\int_{-\ln 4}^{-\ln 12} 2e^t \sinh t\ dt$$

$$\int_{-\ln 4}^{-\ln 12} 2e^t \left(\frac{e^t - e^{-t}}{2}\right)\ dt$$

$$\int_{-\ln 4}^{-\ln 12} \left(e^{2t} - e^0\right)\ dt$$

$$\int_{-\ln 4}^{-\ln 12} e^{2t} - 1\ dt$$

$$\frac{1}{2}e^{2t} - t\Big|_{-\ln 4}^{-\ln 12}$$

$$\left[\frac{1}{2}e^{2(-\ln 12)} - (-\ln 12)\right] - \left[\frac{1}{2}e^{2(-\ln 4)} - (-\ln 4)\right]$$

$$\frac{1}{2}e^{2(-\ln 12)} + \ln 12 - \frac{1}{2}e^{2(-\ln 4)} - \ln 4$$

$$\frac{1}{2}e^{\ln 12^{-2}} + \ln 12 - \frac{1}{2}e^{\ln 4^{-2}} - \ln 4$$

$$\frac{1}{2}12^{-2} + \ln 12 - \frac{1}{2}4^{-2} - \ln 4$$

$$\frac{1}{288} - \frac{1}{32} + \ln 12 - \ln 4$$

$$\frac{1}{288} - \frac{9}{288} + \ln\frac{12}{4}$$

$$-\frac{8}{288} + \ln 3$$

$$\ln 3 - \frac{1}{36}$$