Topic: Vertical motion

Question: Find the particle's position function.

A particle is moving along a straight line.

Its acceleration function is $a(t) = 6t + 5$

Its velocity is 25 m/s when $t = 2$

Its position is 0 m when $t = 0$

Answer choices:

A $x(t) = t^3 + 5t^2 + 3t$

B $x(t) = t^3 + \frac{5}{2}t^2 + 3t$

C $x(t) = t^3 + \frac{5}{2}t^2 + 3t + C$

D $x(t) = t^3 - \frac{5}{2}t^2 + 3t$

Solution: B

Given the position function $x(t)$ of an object, the derivative of the position function is the velocity function, and the derivative of the velocity function is the acceleration function.

$$x(t)$$

$$x'(t) = v(t)$$

$$x''(t) = v'(t) = a(t)$$

Given the above relationship, you can also conclude that velocity is the antiderivative of acceleration, and position is the antiderivative of velocity.

$$a(t)$$

$$\int a(t) = v(t)$$

$$\int \int a(t) = \int v(t) = x(t)$$

This problem asks us to find the position function $x(t)$ given the acceleration function, so we'll need to start by integrating acceleration to find velocity.

$$v(t) = \int a(t) = \int 6t + 5 \ dt$$

$$v(t) = 3t^2 + 5t + C$$

We've been told that velocity is 25 m/s when $t = 2$, which is the initial condition $v(2) = 25$. Plugging this into our velocity function, we get

$$3(2)^2 + 5(2) + C = 25$$

$$12 + 10 + C = 25$$

$$C = 3$$

So the velocity function is

$$v(t) = 3t^2 + 5t + 3$$

To find position, we'll just integrate velocity.

$$x(t) = \int v(t) = \int 3t^2 + 5t + 3 \ dt$$

$$x(t) = t^3 + \frac{5}{2}t^2 + 3t + C$$

We've been told that position is 0 m when $t = 0$, which is the initial condition $x(0) = 0$. Plugging this into the position function, we get

$$(0)^3 + \frac{5}{2}(0)^2 + 3(0) + C = 0$$

$$C = 0$$

Therefore, the position function is

$$x(t) = t^3 + \frac{5}{2}t^2 + 3t$$

Topic: Vertical motion

Question: Find the maximum height of the ball.

A basketball is thrown straight up from the ground.

Its velocity function is $v(t) = -9.8t + 30$

Answer choices:

A 187.2 m

B 9.5 m

C 140.1 m

D 45.9 m

Solution: D

Given the position function $x(t)$ of an object, the derivative of the position function is the velocity function, and the derivative of the velocity function is the acceleration function.

$$x(t)$$

$$x'(t) = v(t)$$

$$x''(t) = v'(t) = a(t)$$

Given the above relationship, you can also conclude that velocity is the antiderivative of acceleration, and position is the antiderivative of velocity.

$$a(t)$$

$$\int a(t) = v(t)$$

$$\int \int a(t) = \int v(t) = x(t)$$

This problem asks us to find the maximum height of the ball. As the ball leaves the ground and approaches it's highest point, velocity will be positive. At the ball's maximum height, velocity is 0, and then it becomes negative as the ball starts heading back down toward the ground.

Therefore, to find the maximum height, we need to find the point at which the velocity function is equal to 0.

$$-9.8t + 30 = 0$$

$$-9.8t = -30$$

$$t = 3.1$$

The position function is the one that will tell us how high the ball is at a specific time. Since we know from the velocity function that the ball reaches maximum height at $t = 3.1$, we can plug this time into the position function, and the result will be the maximum height of the ball.

Before we can plug $t = 3.1$ into the position function, we need to find it by integrating the velocity function.

$$x(t) = \int v(t) = \int -9.8t + 30 \ dt$$

$$x(t) = -4.9t^2 + 30t + C$$

Since we know that the ball is thrown up from the ground, we know that it's initial position when $t = 0$ is 0, in other words, $x(0) = 0$. Plugging this initial condition into the position function to solve for C, we get

$$-4.9(0)^2 + 30(0) + C = 0$$

$$C = 0$$

Which means the position function is

$$x(t) = -4.9t^2 + 30t$$

To find the maximum height, we'll plug $t = 3.1$ into the position function.

$$x(3.1) = -4.9(3.1)^2 + 30(3.1)$$

$$x(3.1) = 45.9$$

The maximum height of the ball is 45.9 m.

Topic: Vertical motion

Question: How long is the rock in the air?

A rock is thrown straight from the ground.

Its velocity function is $v(t) = -9.8t + 45$

Answer choices:

A 9.2 s

B 4.9 s

C 9.8 s

D 4.6 s

Solution: A

Given the position function $x(t)$ of an object, the derivative of the position function is the velocity function, and the derivative of the velocity function is the acceleration function.

$$x(t)$$

$$x'(t) = v(t)$$

$$x''(t) = v'(t) = a(t)$$

Given the above relationship, you can also conclude that velocity is the antiderivative of acceleration, and position is the antiderivative of velocity.

$$a(t)$$

$$\int a(t) = v(t)$$

$$\int \int a(t) = \int v(t) = x(t)$$

This problem asks us to find the amount of time that the rock is in the air, which means we need to find the time at which the rock comes back down and hits the ground.

The position of the rock is 0 at two separate times: when it's on the ground before being thrown into the air, and when it's on the ground after it falls back down again. Therefore, if we can find the points at which the position function is equal to 0, we'll know how much time the rock is in the air.

To find the position function, we'll integrate the velocity function.

$$x(t) = \int v(t) = \int -9.8t + 45 \ dt$$

$$x(t) = -4.9t^2 + 45t + C$$

We know the initial position at $t = 0$ is 0, since the rock is thrown up from the ground, so $x(0) = 0$. Plugging this into the position function, we get

$$-4.9(0)^2 + 45(0) + C = 0$$

$$C = 0$$

The position function is

$$x(t) = -4.9t^2 + 45t$$

We'll find the points in time when the rock is on the ground by setting the position function equal to 0.

$$-4.9t^2 + 45t = 0$$

$$-4.9t(t - 9.2) = 0$$

$t = 0$ and $t = 9.2$

We already know that $t = 0$ is associated with the rock's initial position before it was thrown into the air. That means that $t = 9.2$ must be associated with the rock's final position when it lands on the ground again. So we can conclude that the rock is in the air for 9.2 seconds.