Topic: Sketching parametric curves by plotting points

Question: A parametric curve is defined by $x = 2\cos 2\theta - 3$ and $y = \sin^2 2\theta - 4$. Which statement is true about the position of the graph of the function?

Answer choices:

A The graph is a parabola that opens down around the vertex $(-3, -3)$.

B The graph is a parabola that opens down around the vertex $(-3, -4)$.

C The graph is a parabola that opens up around the vertex $(-3, -3)$.

D The graph is a parabola that opens up around the vertex $(-3, -4)$.

Solution: A

Rearrange the given equations.

$$x = 2\cos 2\theta - 3$$

$$x + 3 = 2\cos 2\theta$$

$$\frac{x+3}{2} = \cos 2\theta$$

$$\cos^2 2\theta = \frac{(x+3)^2}{4}$$

and

$$y = \sin^2 2\theta - 4$$

$$\sin^2 2\theta = y + 4$$

Now add these equations together.

$$\sin^2 2\theta + \cos^2 2\theta = \frac{(x+3)^2}{4} + y + 4$$

$$1 = \frac{(x+3)^2}{4} + y + 4$$

$$y = -\frac{(x+3)^2}{4} - 3$$

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Question: A parametric curve is defined by the functions $x = 3\tan t$ and $y = 2\sec t$. Which statement describes the type and position of the sketch of the given curve?

Answer choices:

A The graph of the curve is a rectangular hyperbola with vertices at $(0,-2)$ and $(0,2)$, and with asymptotes defined by $y = \pm \frac{3}{2}x$.

B The graph of the curve is a rectangular hyperbola with vertices at $(0,-2)$ and $(0,2)$, and with asymptotes defined by $y = \pm \frac{2}{3}x$.

C The graph of the curve is a rectangular hyperbola with vertices at $(0,-3)$ and $(0,3)$, and with asymptotes defined by $y = \pm \frac{2}{3}x \pm 2$.

D The graph of the curve is a rectangular hyperbola with vertices at $(0,-2)$ and $(0,2)$, and with asymptotes defined by $y = \pm \frac{2}{3}x \pm 1$.

Solution: B

Given the parametric equations $x = 3\tan t$ and $y = 2\sec t$, we'll square both sides, and then solve each for the trigonometric function.

$$x = 3\tan t$$

$$x^2 = 9\tan^2 t$$

$$\frac{x^2}{9} = \tan^2 t$$

and

$$y = 2\sec t$$

$$y^2 = 4\sec^2 t$$

$$\frac{y^2}{4} = \tan^2 t + 1$$

Now subtract the first equation from the second.

$$\frac{y^2}{4} - \frac{x^2}{9} = \tan^2 t + 1 - \tan^2 t$$

$$\frac{y^2}{4} - \frac{x^2}{9} = 1$$

The graph of the curve is a rectangular hyperbola with vertices at $(0,-2)$ and $(0,2)$, and with asymptotes defined by $y = \pm\frac{2}{3}x$.

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Question: The sketch of a parabola indicates that its vertex is at $(2,3)$, and it's open to the right. The graph of the line $y=3$ divides the curve into two symmetric curves. Which pair of parametric functions represents the sketch of the given parabola?

Answer choices:

A $x=(t-3)^2+2(2t+1)$ and $y=t+3$

B $x=(t-2)^2+2(2t-1)$ and $y=t-3$

C $x=(t+2)^2+2(3t-1)$ and $y=t+3$

D $x=(t-2)^2+2(2t-1)$ and $y=t+3$

Solution: D

Choose the equations from answer choice D.

$$x = (t-2)^2 + 2(2t-1)$$

$$y = t + 3$$

Simplify the first equation.

$$x = (t-2)^2 + 2(2t-1)$$

$$x = t^2 - 4t + 4 + 4t - 2$$

$$x = t^2 + 2$$

Rearrange the second equation.

$$y = t + 3$$

$$y - 3 = t$$

$$(y-3)^2 = t^2$$

Substitute $(y-3)^2 = t^2$ into $x = t^2 + 2$.

$$x = (y-3)^2 + 2$$

The vertex of this parabola is at $(2,3)$, and it opens to the right.