

# Sales decline

If sales of a product are consistently declining at an exponential rate, we can model that decline with the formula

$$F = Pe^{-rt}$$

where  $P$  is the number of items being sold,  $r$  is the rate of decline, and  $F$  is the number of items being sold after sales have continued to decline for some specified amount of time  $t$ .

It's important to remember that the rate  $r$  and the time  $t$  need to have complementary units. For instance, if  $r$  is given as some rate per *month*, then time  $t$  needs to be measured in *months*. If  $r$  is in years, then  $t$  should also be in years.

When the units of rate and time don't match, we'll need to convert one to match the other. For example, if  $r$  was given as a rate per year, and  $t$  was given in months, we could either divide  $t$  by 12 to convert it into years to match the rate, or we could divide  $r$  by 12 to convert it into months to match the time.

Let's work through an example where we solve for the rate of decline.

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## Example

Sales of Isaac Newton bobbleheads have decreased over the last 4 years. Four years ago, 285,674 bobbleheads were sold, but over the last year, sales were only 97,546 bobbleheads. Assuming that sales have declined at a steady exponential rate, what is the rate of decline?



We know that fourth year sales are  $F = 97,546$ , and that first year sales were  $P = 285,674$ . Time is  $t = 4$  and, we need to calculate  $r$ , so we'll substitute what we know into the sales decline formula.

$$F = Pe^{-rt}$$

$$97,546 = 285,674e^{-r(4)}$$

Solve for  $r$ .

$$\frac{97,546}{285,674} = e^{-4r}$$

$$0.3415 = e^{-4r}$$

$$\ln 0.3415 = \ln(e^{-4r})$$

$$-1.075 = -4r$$

$$r = 0.269$$

This result tells us that sales of Isaac Newton bobbleheads have declined at an annual rate of about  $r = 0.269$ , or  $r = 26.9\%$  per year.

We'll do one more example, and in this one we'll use the exponential model we come up with to predict the future sales decline of a product.

### Example



One year ago, a company sold 5,698 disposable sandwich bags in a month. But over the last 12 months, sales have decreased at a steady exponential rate of 15 % per month. How many disposable sandwich bags did the company sell last month, and how many are predicted to sell in 6 months from now?

The rate of decline is  $r = 0.15$ , the original amount from one year ago (12 months ago) was  $P = 5,698$ . Therefore, we could say that last month is at  $t_{\text{last month}} = 12$ , and 6 months from now is  $t_{\text{in 6 months}} = 18$ .

We'll substitute into the sales decline formula to find sales for last month.

$$F = Pe^{-rt}$$

$$F_{\text{last month}} = 5,698e^{-0.15(12)}$$

$$F_{\text{last month}} \approx 941.87$$

In the past month, about 942 units of disposable sandwich bags were sold. Now we'll calculate expected sales for 6 months from now.

$$F_{\text{in 6 months}} = 5,698e^{-0.15(18)}$$

$$F_{\text{in 6 months}} \approx 382.94$$

Assuming the rate of decline remains the same, in 6 months from now, the company will sell about 383 disposable sandwich bags.

