Calculus 2 Workbook Solutions

Geometric series

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MATH

GEOMETRIC SERIES TEST

1. Use the geometric series test to say whether the geometric series converges or diverges, then give the value of the common ratio r.

$$\sum_{n=1}^{\infty} 6\left(\frac{2}{3}\right)^{n-1}$$

Solution:

In the series, $a = 6$ and $r = 2/3$. Since $|r| < 1$, the series converges.

2. Use the geometric series test to say whether the geometric series converges or diverges, then give the value of the common ratio r.

$$\sum_{n=1}^{\infty} \left(\frac{3}{7}\right)^{n-1}$$

Solution:

In the series, $a = 1$ and $r = 3/7$. Since $|r| < 1$, the series converges.

■ 3. Use the geometric series test to say whether the geometric series converges or diverges, then give the value of the common ratio r.

$$\frac{\pi}{2}+\frac{\pi^2}{6}+\frac{\pi^3}{18}+\frac{\pi^4}{54}+\cdots$$

Solution:

In the series, $a=\pi/2$ and $r=\pi/3$. Since $|r|>1$, the series diverges.

■ 4. Use the geometric series test to say whether the geometric series converges or diverges, then give the value of the common ratio r.

$$1-\frac{1}{3}+\frac{1}{9}-\frac{1}{27}+\cdots+\left(-\frac{1}{3}\right)^{n-1}+\cdots$$

Solution:

In the series, $a=1$ and $r=-1/3$. Since $|r|<1$, the series converges.

■ 5. Use the geometric series test to say whether the geometric series converges or diverges, then give the value of the common ratio r.

$$\sum_{n=1}^{\infty}\left(\frac{e}{\pi}\right)^n$$

Solution:

In the series, $a = e/\pi$ and $r = e/\pi$. Since $|r| < 1$, the series converges.

SUM OF THE GEOMETRIC SERIES

1. Find the sum of the geometric series.

$$\sum_{n=1}^{\infty} 7\left(\frac{3}{8}\right)^{n-1}$$

Solution:

In the series, $a = 7$ and $r = 3/8$, so $|r| < 1$. Then the series converges to the sum

$$S = \frac{a}{1-r} = \frac{7}{1-\frac{3}{8}} = \frac{\frac{7}{1}}{\frac{5}{8}} = \frac{7}{1} \cdot \frac{8}{5} = \frac{56}{5}$$

2. Find the sum of the geometric series.

$$\sum_{n=1}^{\infty} 9\left(\frac{5}{14}\right)^{n-1}$$

Solution:

In the series, $a = 9$ and $r = 5/14$, so $|r| < 1$. Then the series converges to the sum

$$S = \frac{a}{1-r} = \frac{9}{1-\frac{5}{14}} = \frac{\frac{9}{1}}{\frac{9}{14}} = \frac{9}{1} \cdot \frac{14}{9} = 14$$

■ 3. Find the sum of the geometric series.

$$\frac{1}{3} - \frac{2}{9} + \frac{4}{27} - \frac{8}{81} + \cdots$$

Solution:

In the series, $a = 1/3$ and $r = -2/3$, so $|r| < 1$. Then the series converges to the sum

$$S = \frac{a}{1-r} = \frac{\frac{1}{3}}{1-\left(-\frac{2}{3}\right)} = \frac{\frac{1}{3}}{\frac{5}{3}} = \frac{1}{3} \cdot \frac{3}{5} = \frac{1}{5}$$

■ 4. Find the sum of the geometric series.

$$\sum_{n=1}^{\infty} \left(\frac{e}{\pi}\right)^n$$

Solution:

In the series, $a = e/\pi$ and $r = e/\pi$, so $|r| < 1$. Then the series converges to the sum

$$S = \frac{a}{1-r} = \frac{\frac{e}{\pi}}{1-\frac{e}{\pi}} = \frac{\frac{e}{\pi}}{\frac{\pi}{\pi}-\frac{e}{\pi}} = \frac{\frac{e}{\pi}}{\frac{\pi-e}{\pi}} = \frac{e}{\pi} \cdot \frac{\pi}{\pi-e} = \frac{e}{\pi-e}$$

VALUES FOR WHICH THE SERIES CONVERGES

1. Find the values of x for which the geometric series converges.

$$\sum_{n=1}^{\infty} \frac{17}{3} x^{n-1}$$

Solution:

Expand the series.

$$\sum_{n=1}^{\infty} \frac{17}{3} x^{n-1} = \frac{17}{3} + \frac{17}{3}x + \frac{17}{3}x^2 + \frac{17}{3}x^3 + \frac{17}{3}x^4 + \cdots$$

The common ratio between each term is x. So we'll set up the inequality $|r| < 1$ to solve for the values where the series converges.

$$|x| < 1$$

$$-1 < x < 1$$

2. Find the values of x for which the geometric series converges.

$$\sum_{n=1}^{\infty} 5 \left(\frac{x-2}{3} \right)^{n-1}$$

Solution:

Expand the series.

$$\sum_{n=1}^{\infty} 5\left(\frac{x-2}{3}\right)^{n-1} = 5 + 5\left(\frac{x-2}{3}\right) + 5\left(\frac{x-2}{3}\right)^2 + 5\left(\frac{x-2}{3}\right)^3$$

$$+5\left(\frac{x-2}{3}\right)^4 + 5\left(\frac{x-2}{3}\right)^5 + \cdots$$

The common ratio between each term is $(x-2)/3$. So we'll set up the inequality $|r| < 1$ to solve for the values where the series converges.

$$\left|\frac{x-2}{3}\right| < 1$$

$$-1 < \frac{x-2}{3} < 1$$

$$-3 < x - 2 < 3$$

$$-1 < x < 5$$

3. Find the values of x for which the geometric series converges.

$$\sum_{n=0}^{\infty} 4^n x^n$$

Solution:

Expand the series.

$$\sum_{n=0}^{\infty} 4^n x^n = 1 + 4x + 16x^2 + 64x^3 + 256x^4 + \cdots$$

The common ratio between each term is $4x$. So we'll set up the inequality $|r| < 1$ to solve for the values where the series converges.

$$|4x| < 1$$

$$-1 < 4x < 1$$

$$-\frac{1}{4} < x < \frac{1}{4}$$

GEOMETRIC SERIES FOR REPEATING DECIMALS

1. Express the repeating decimal $0.\overline{17}$ as a geometric series.

Solution:

The repeating decimal can be re-written as

$$0.\overline{17}$$

$$0.17171717171717...$$

$$0.17 + 0.0017 + 0.000017 + 0.00000017 + \dots$$

$$\frac{17}{100} + \frac{17}{10{,}000} + \frac{17}{1{,}000{,}000} + \frac{17}{100{,}000{,}000} + \dots$$

$$\frac{17}{100}\left(1 + \frac{1}{100} + \frac{1}{10{,}000} + \frac{1}{1{,}000{,}000} + \dots\right)$$

Now that the repeating decimal is written as a series, we can identify $a = 17/100$ and $r = 1/100$. So the series is

$$\sum_{n=1}^{\infty} a_1 r^{n-1}$$

$$\sum_{n=1}^{\infty} \frac{17}{100}\left(\frac{1}{100}\right)^{n-1}$$

2. Express the repeating decimal $23.\overline{23}$ as a geometric series.

Solution:

The repeating decimal can be re-written as

$$23.\overline{23}$$

$$23.23232323...$$

$$23 + 0.23 + 0.0023 + 0.000023 + 0.00000023 + \dots$$

$$23 + \frac{23}{100} + \frac{23}{10{,}000} + \frac{23}{1{,}000{,}000} + \frac{23}{100{,}000{,}000} + \dots$$

$$23 + \frac{23}{100}\left(1 + \frac{1}{100} + \frac{1}{10{,}000} + \frac{1}{1{,}000{,}000} + \dots\right)$$

Now that the repeating decimal is written as a series, we can identify $a = 23/100$ and $r = 1/100$. So the series is

$$\sum_{n=1}^{\infty} a_1 r^{n-1}$$

$$23 + \sum_{n=1}^{\infty} \frac{23}{100}\left(\frac{1}{100}\right)^{n-1}$$

3. Express the repeating decimal $6.7\overline{2}$ as a geometric series.

Solution:

The repeating decimal can be re-written as

$$6.7\overline{2}$$

$$6.722222222...$$

$$6.7 + 0.02 + 0.002 + 0.0002 + 0.00002 + \dots$$

$$6.7 + \frac{2}{100} + \frac{2}{1,000} + \frac{2}{10,000} + \frac{2}{100,000} + \dots$$

$$6.7 + \frac{1}{50} + \frac{1}{500} + \frac{1}{5,000} + \frac{1}{50,000} + \dots$$

$$6.7 + \frac{1}{50}\left(1 + \frac{1}{10} + \frac{1}{100} + \frac{1}{1,000} + \dots\right)$$

Now that the repeating decimal is written as a series, we can identify $a = 1/50$ and $r = 1/10$. So the series is

$$\sum_{n=1}^{\infty} a_1 r^{n-1}$$

$$6.7 + \sum_{n=1}^{\infty} \frac{1}{50}\left(\frac{1}{10}\right)^{n-1}$$

4. Express the repeating decimal $9.15\overline{65}$ as a geometric series.

Solution:

The repeating decimal can be re-written as

$$9.15\overline{65}$$

$$9.1565656565...$$

$$9.15 + 0.0065 + 0.000065 + 0.00000065 + 0.0000000065 + \ldots$$

$$9.15 + \frac{65}{10{,}000} + \frac{65}{1{,}000{,}000} + \frac{65}{100{,}000{,}000} + \frac{65}{10{,}000{,}000{,}000} + \ldots$$

$$9.15 + \frac{13}{2{,}000} + \frac{13}{200{,}000} + \frac{13}{20{,}000{,}000} + \frac{13}{2{,}000{,}000{,}000} + \ldots$$

$$9.15 + \frac{13}{2{,}000}\left(1 + \frac{1}{100} + \frac{1}{10{,}000} + \frac{1}{1{,}000{,}000} + \ldots\right)$$

Now that the repeating decimal is written as a series, we can identify $a = 13/2{,}000$ and $r = 1/100$. So the series is

$$\sum_{n=1}^{\infty} a_1 r^{n-1}$$

$$9.15 + \sum_{n=0}^{\infty} \frac{13}{2{,}000}\left(\frac{1}{100}\right)^{n-1}$$

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