Calculus 2 Workbook Solutions

Partial sums

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MATH

CALCULATING THE FIRST TERMS OF A SERIES OF PARTIAL SUMS

1. Approximate the first four terms of the series of partial sums.

$$\sum_{n=1}^{\infty} \frac{7n}{3n^2+2}$$

Solution:

Make a table and calculate the value of each term.

$n=1$	$a_1=\frac{7(1)}{3(1)^2+2}=\frac{7}{5}\approx 1.40$	$s_1=1.40$
$n=2$	$a_2=\frac{7(2)}{3(2)^2+2}=\frac{14}{14}=1=1.00$	$s_2=1.40+1.00=2.40$
$n=3$	$a_3=\frac{7(3)}{3(3)^2+2}=\frac{21}{29}\approx 0.72$	$s_3=2.40+0.72=3.12$
$n=4$	$a_4=\frac{7(4)}{3(4)^2+2}=\frac{28}{50}=\frac{14}{25}\approx 0.56$	$s_4=3.12+0.56=3.68$

2. Approximate the first four terms of the series of partial sums.

$$\sum_{n=1}^{\infty} \frac{5n^2}{7n+4}$$

Solution:

Make a table and calculate the value of each term.

$$n = 1 \qquad a_1 = \frac{5(1)^2}{7(1)+4} = \frac{5}{11} \approx 0.45 \qquad s_1 = 0.45$$

$$n = 2 \qquad a_2 = \frac{5(2)^2}{7(2)+4} = \frac{20}{18} = \frac{10}{9} \approx 1.11 \qquad s_2 = 0.45 + 1.11 = 1.56$$

$$n = 3 \qquad a_3 = \frac{5(3)^2}{7(3)+4} = \frac{45}{25} = \frac{9}{5} \approx 1.80 \qquad s_3 = 1.56 + 1.80 = 3.36$$

$$n = 4 \qquad a_4 = \frac{5(4)^2}{7(4)+4} = \frac{80}{32} = \frac{5}{2} \approx 2.50 \qquad s_4 = 3.36 + 2.50 = 5.86$$

■ 3. Approximate the first four terms of the series of partial sums.

$$\sum_{n=1}^{\infty} \frac{9n^3}{8n^2+13}$$

Solution:

Make a table and calculate the value of each term.

$$n = 1 \qquad a_1 = \frac{9(1)^3}{8(1)^2+13} = \frac{9}{21} = \frac{3}{7} \approx 0.43 \qquad s_1 = 0.43$$

$n = 2 \qquad a_2 = \frac{9(2)^3}{8(2)^2 + 13} = \frac{72}{45} = \frac{8}{5} = 1.60 \qquad s_2 = 0.43 + 1.60 = 2.03$

$n = 3 \qquad a_3 = \frac{9(3)^3}{8(3)^2 + 13} = \frac{243}{85} \approx 2.86 \qquad s_3 = 2.03 + 2.86 = 4.89$

$n = 4 \qquad a_4 = \frac{9(4)^3}{8(4)^2 + 13} = \frac{576}{141} \approx 4.09 \qquad s_4 = 4.89 + 4.09 = 8.98$

SUM OF THE SERIES OF PARTIAL SUMS

1. Use the partial sums equation to find the sum of the series.

$$s_n = 12 + \frac{9}{n}$$

Solution:

The sum of the series with terms a_n is given by

$$\sum_{n=1}^{\infty} a_n = \lim_{n\to\infty}\left(12 + \frac{9}{n}\right)$$

$$\sum_{n=1}^{\infty} a_n = 12 + \frac{9}{\infty}$$

$$\sum_{n=1}^{\infty} a_n = 12 + 0$$

$$\sum_{n=1}^{\infty} a_n = 12$$

2. Use the partial sums equation to find the sum of the series.

$$s_n = \frac{7n^2 + 9n}{n^2 - 6}$$

Solution:

The sum of the series with terms a_n is given by

$$\sum_{n=1}^{\infty} a_n = \lim_{n\to\infty} \left(\frac{7n^2 + 9n}{n^2 - 6} \right)$$

Because evaluating the limit gives an indeterminate form, use L'Hospital's Rule to evaluate the limit.

$$\lim_{n\to\infty} \left(\frac{7n^2 + 9n}{n^2 - 6} \right) = \lim_{n\to\infty} \left(\frac{14n + 9}{2n} \right) = \lim_{n\to\infty} \left(\frac{14}{2} \right) = 7$$

So the sum of the series a_n is 7.

3. Use the partial sums equation to find the sum of the series.

$$s_n = \frac{9n^3 + 7n + 9}{8n^3 + 2n^2 + 5}$$

Solution:

The sum of the series with terms a_n is given by

$$\sum_{n=1}^{\infty} a_n = \lim_{n\to\infty} \left(\frac{9n^3 + 7n + 9}{8n^3 + 2n^2 + 5} \right)$$

Because evaluating the limit gives an indeterminate form, use L'Hospital's Rule to evaluate the limit.

$$\lim_{n\to\infty}\left(\frac{9n^3+7n+9}{8n^3+2n^2+5}\right)=\lim_{n\to\infty}\left(\frac{27n^2+7}{24n^2+4n}\right)=\lim_{n\to\infty}\left(\frac{54n}{48n+4}\right)=\frac{54}{48}=\frac{9}{8}$$

So the sum of the series a_n is $9/8$.

4. Use the partial sums equation to find the sum of the series.

$$s_n=\frac{13}{15n^3}+\frac{12}{n}+5$$

Solution:

The sum of the series with terms a_n is given by

$$\sum_{n=1}^{\infty}a_n=\lim_{n\to\infty}\left(\frac{13}{15n^3}+\frac{12}{n}+5\right)$$

$$\sum_{n=1}^{\infty}a_n=\frac{13}{\infty}+\frac{12}{\infty}+5$$

$$\sum_{n=1}^{\infty}a_n=0+0+5$$

$$\sum_{n=1}^{\infty}a_n=5$$

5. Use the partial sums equation to find the sum of the series.

$$s_n = \frac{14n^2}{15n^3} - \frac{n}{16n^2} - \frac{1}{4n} + \frac{1}{3}$$

Solution:

The sum of the series with terms a_n is given by

$$\sum_{n=1}^{\infty} a_n = \lim_{n\to\infty}\left(\frac{14n^2}{15n^3} - \frac{n}{16n^2} - \frac{1}{4n} + \frac{1}{3}\right)$$

$$\sum_{n=1}^{\infty} a_n = \lim_{n\to\infty}\left(\frac{14}{15n} - \frac{1}{16n} - \frac{1}{4n} + \frac{1}{3}\right)$$

$$\sum_{n=1}^{\infty} a_n = \frac{14}{\infty} - \frac{1}{\infty} - \frac{1}{\infty} + \frac{1}{3}$$

$$\sum_{n=1}^{\infty} a_n = 0 + 0 + 0 + \frac{1}{3}$$

$$\sum_{n=1}^{\infty} a_n = \frac{1}{3}$$

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