Calculus 2 Workbook Solutions

Fundamental theorem of calculus

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PART 1 OF THE FTC

1. Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of $f(x)$.

$$f(x)=\int_0^{x^2} 7t\cos(2t)\ dt$$

Solution:

Evaluate the integrand at the upper bound, x^2, multiplying by the derivative of x^2. That will give the derivative of the function $f(x)$.

$$f'(x)=7x^2\cos(2x^2)\cdot\frac{d}{dx}(x^2)$$

$$f'(x)=7x^2\cos(2x^2)\cdot 2x$$

$$f'(x)=14x^3\cos(2x^2)$$

2. Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of $g(x)$.

$$g(x)=\int_2^{x^3}\frac{5}{3+e^t}\ dt$$

Solution:

Evaluate the integrand at the upper bound, x^3, multiplying by the derivative of x^3. That will give the derivative of the function $g(x)$.

$$g'(x) = \frac{5}{3+e^{x^3}} \cdot \frac{d}{dx}(x^3)$$

$$g'(x) = \frac{5}{3+e^{x^3}} \cdot 3x^2$$

$$g'(x) = \frac{15x^2}{3+e^{x^3}}$$

■ 3. Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of $h(x)$.

$$h(x) = \int_{\cos(3x)}^{7} 8t+1 \ dt$$

Solution:

Flip the upper and lower bound, multiplying the integral by -1.

$$h(x) = -\int_{7}^{\cos(3x)} 8t+1 \ dt$$

Evaluate the integrand at the upper bound, $\cos(3x)$, multiplying by the derivative of $\cos(3x)$. That will give the derivative of the function $h(x)$.

$$h'(x) = -(8\cos(3x)+1)\cdot\frac{d}{dx}(\cos(3x))$$

$$h'(x) = -(8\cos(3x)+1)(-3\sin(3x))$$

$$h'(x) = (8\cos(3x)+1)(3\sin(3x))$$

$$h'(x) = 24\sin(3x)\cos(3x)+3\sin(3x)$$

4. Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of $f(x)$.

$$f(x) = \int_1^{3x^2} \frac{\sin t}{t^3+5}\ dt$$

Solution:

Evaluate the integrand at the upper bound, $3x^2$, multiplying by the derivative of $3x^2$. That will give the derivative of the function $f(x)$.

$$f'(x) = \frac{\sin(3x^2)}{(3x^2)^3+5}\cdot\frac{d}{dx}(3x^2)$$

$$f'(x) = \frac{\sin(3x^2)}{27x^6+5}\cdot 6x$$

$$f'(x) = \frac{6x\sin(3x^2)}{27x^6+5}$$

5. Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of $g(x)$.

$$g(x)=\int_{3x}^{2x^2} t^2-5t+4\ dt$$

Solution:

Split the interval at 0.

$$g(x)=\int_{3x}^{0} t^2-5t+4\ dt+\int_{0}^{2x^2} t^2-5t+4\ dt$$

Flip the bounds on the first integral, multiplying by -1.

$$g(x)=-\int_{0}^{3x} t^2-5t+4\ dt+\int_{0}^{2x^2} t^2-5t+4\ dt$$

For the first integral, evaluate the integrand at the upper bound, $3x$, multiplying by the derivative of $3x$. For the second integral, evaluate at the upper bound, $2x^2$, multiplying by the derivative of $2x^2$. That will give the derivative of the function $g(x)$.

$$g'(x)=-\left((3x)^2-5(3x)+4\right)\cdot\frac{d}{dx}(3x)+\left((2x^2)^2-5(2x^2)+4\right)\cdot\frac{d}{dx}\left(2x^2\right)$$

$$g'(x)=-(9x^2-15x+4)\cdot 3+(4x^4-10x^2+4)\cdot 4x$$

$$g'(x)=-(27x^2-45x+12)+(16x^5-40x^3+16x)$$

$$g'(x)=-27x^2+45x-12+16x^5-40x^3+16x$$

$$g'(x) = 16x^5 - 40x^3 - 27x^2 + 61x - 12$$

PART 2 OF THE FTC

1. Use Part 2 of the Fundamental Theorem of Calculus to evaluate the integral.

$$\int_2^5 5-\frac{3}{x}\ dx$$

Solution:

Integrate, then evaluate over the interval.

$$5x-3\ln x\Big|_2^5$$

$$5(5)-3\ln 5-(5(2)-3\ln 2)$$

$$25-3\ln 5-10+3\ln 2$$

$$15+3\ln 2-3\ln 5$$

$$15+3(\ln 2-\ln 5)$$

$$15+3\ln\frac{2}{5}$$

2. Use Part 2 of the Fundamental Theorem of Calculus to evaluate the integral.

$$\int_4^9 4x^3 - \sqrt{x}\ dx$$

Solution:

Integrate, then evaluate over the interval.

$$x^4 - \frac{2x^{\frac{3}{2}}}{3} \Bigg|_4^9$$

$$9^4 - \frac{2(9)^{\frac{3}{2}}}{3} - \left(4^4 - \frac{2(4)^{\frac{3}{2}}}{3}\right)$$

$$6{,}561 - \frac{2(27)}{3} - 256 + \frac{2(8)}{3}$$

$$6{,}305 - \frac{54}{3} + \frac{16}{3}$$

$$\frac{18{,}915}{3} - \frac{38}{3}$$

$$\frac{18{,}877}{3}$$

■ 3. Use Part 2 of the Fundamental Theorem of Calculus to evaluate the integral.

$$\int_{-3}^{-1} \frac{3}{x^3}\ dx$$

Solution:

Integrate, then evaluate over the interval.

$$-\frac{3}{2}x^{-2}\Big|_{-3}^{-1}$$

$$-\frac{3}{2x^2}\Big|_{-3}^{-1}$$

$$-\frac{3}{2(-1)^2}-\left(-\frac{3}{2(-3)^2}\right)$$

$$-\frac{3}{2}+\frac{3}{2(9)}$$

$$\frac{3}{18}-\frac{3}{2}$$

$$\frac{3}{18}-\frac{27}{18}$$

$$-\frac{24}{18}$$

$$-\frac{4}{3}$$

■ 4. Use Part 2 of the Fundamental Theorem of Calculus to evaluate the integral.

$$\int_{25}^{36} \frac{2-\sqrt{x}}{\sqrt{x}}\ dx$$

Solution:

Rewrite the integral.

$$\int_{25}^{36} \frac{2}{\sqrt{x}} - \frac{\sqrt{x}}{\sqrt{x}}\ dx$$

$$\int_{25}^{36} 2x^{-\frac{1}{2}} - 1\ dx$$

Integrate, then evaluate over the interval.

$$4x^{\frac{1}{2}} - x \Big|_{25}^{36}$$

$$4(36)^{\frac{1}{2}} - 36 - \left(4(25)^{\frac{1}{2}} - 25\right)$$

$$4(6) - 36 - 4(5) + 25$$

$$24 - 36 - 20 + 25$$

$$-7$$

NET CHANGE THEOREM

1. Suppose the position of a particle moving along the horizontal s-axis is at $s = -2$ when $t = 0$. The velocity of the particle is given by $v(t)$ with $0 \le t \le 10$, where t is time in seconds since the particle began moving. Use the Net Change Theorem to determine the position of the particle on the s-axis after the particle has been moving for 5 seconds.

$$v(t) = \frac{1}{4}t^2 - \frac{9}{(t+1)^2}$$

Solution:

The interval of time from the time the particle starts moving until 5 seconds later is $0 \le t \le 5$, and that will be the bounds for the integral of $v(t)$. Because the particle starts at $s = -2$, we need to subtract 2 from the value of the integral. So the particle's position is given by

$$-2 + \int_0^5 \frac{1}{4}t^2 - \frac{9}{(t+1)^2}\ dt$$

$$-2 + \int_0^5 \frac{1}{4}t^2 - 9(t+1)^{-2}\ dt$$

Integrate, then evaluate over the interval.

$$-2 + \left(\frac{1}{12}t^3 + 9(t+1)^{-1}\right)\Big|_0^5$$

$$-2+\left(\frac{1}{12}(5)^3+9(5+1)^{-1}\right)-\left(\frac{1}{12}(0)^3+9(0+1)^{-1}\right)$$

$$-2+\frac{1}{12}(125)+\frac{9}{6}-9$$

$$-11+\frac{125}{12}+\frac{3}{2}$$

$$-\frac{132}{12}+\frac{125}{12}+\frac{18}{12}$$

$$\frac{11}{12}$$

■ 2. Water is being pumped from a tank at a rate (in gallons per minute) given by $w(t)=80-4\sqrt{t+3}$, with $0 \le t \le 60$, where t is the time in minutes since the pumping began. The tank had $5{,}000$ gallons of water in it when pumping began. Use the Net Change Theorem to determine how many gallons of water will be in the tank after 30 minutes of pumping.

Solution:

The interval of time from the time the pumping begins until 30 minutes after pumping starts is $0 \le t \le 30$, and that will be the bounds for the integral of $w(t)$. Because the tank starts with $5{,}000$ gallons, we need to add $5{,}000$ to the value of the integral. So the gallons in the tank is given by

$$5{,}000 - \int_0^{30} 80 - 4\sqrt{t+3}\ dt$$

$$5{,}000 - \left(80t - \frac{8}{3}(t+3)^{\frac{3}{2}}\right)\Bigg|_0^{30}$$

$$= 5{,}000 - \left(80(30) - \frac{8}{3}(30+3)^{\frac{3}{2}}\right) + \left(80(0) - \frac{8}{3}(0+3)^{\frac{3}{2}}\right)$$

$$5{,}000 - 2{,}400 + \frac{8}{3}(33)^{\frac{3}{2}} - \frac{8}{3}(3)^{\frac{3}{2}}$$

$$2{,}600 + \frac{264\sqrt{33}}{3} - \frac{24\sqrt{3}}{3}$$

$$\frac{7{,}800 + 264\sqrt{33} - 24\sqrt{3}}{3}$$

■ 3. From 1990 to 2010, the rate of rice consumption in a particular country was $R(t) = 5.8 + 1.07^t$ million pounds per year, with t being years since the beginning of the year 1990. The country had 7.2 million pounds of rice on hand at the beginning of 1994 and produced 7.5 million pounds of rice every year. Use the Net Change Theorem to determine how many millions of pounds of rice were on hand in that country at the end of 1998.

Solution:

Since 1990 is the beginning of the time frame, that's when $t=0$. Therefore, the interval of time from 1994 to 1998 is $4 \le t \le 8$, and that will be the bounds for the integral of $R(t)$. Because the country starts with 7.2 million pounds of rice, we need to add 7.2 to the value of the integral. Because the country produces 7.5 million pounds of rice per year, but consumes $R(t)$ pounds per year, the amount of rice at the end of 1998 is

$$7.2+\int_4^8 7.5-(5.8+1.07^t)\ dt$$

$$7.2+\int_4^8 1.7-1.07^t\ dt$$

$$7.2+\left(1.7t-\frac{1.07^t}{\ln 1.07}\right)\Bigg|_4^8$$

$$7.2+\left(1.7(8)-\frac{1.07^8}{\ln 1.07}\right)-\left(1.7(4)-\frac{1.07^4}{\ln 1.07}\right)$$

$$7.2-11.794923+12.573665$$

$$7.978742$$

The country has slightly less than 8 million pounds of rice on hand at the end of 1998.

■ 4. A cooling pump connected to a power plant operates at a varying rate, depending on how much cooling is needed by the power plant. The rate (in gallons per second) at which the pump is operated is modeled by

$r(t) = 0.003t^3 - 0.02t^2 + 0.29t + 59.81$, where t is defined in seconds for $0 \le t \le 120$. The pump has already pumped $1{,}508$ gallons during the first 25 seconds. Use the Net Change Theorem to determine how many gallons the pump will have pumped after 2 minutes.

Solution:

The interval of time from 25 seconds after the pump starts until 2 minutes after pumping starts is $25 \le t \le 120$ (since 2 minutes is 120 seconds), and that will be the bounds for the integral of $r(t)$. Because the pump had already pumped $1{,}508$ gallons, we need to add $1{,}508$ to the value of the integral. So the gallons pumped after 2 minutes is given by

$$1{,}508 + \int_{25}^{120} 0.003t^3 - 0.02t^2 + 0.29t + 59.81\ dt$$

$$1{,}508 + \left(\frac{0.003}{4}t^4 - \frac{0.02}{3}t^3 + \frac{0.29}{2}t^2 + 59.81t\right)\Bigg|_{25}^{120}$$

$$1{,}508 + \left(\frac{0.003}{4}(120)^4 - \frac{0.02}{3}(120)^3 + \frac{0.29}{2}(120)^2 + 59.81(120)\right)$$

$$-\left(\frac{0.003}{4}(25)^4 - \frac{0.02}{3}(25)^3 + \frac{0.29}{2}(25)^2 + 59.81(25)\right)$$

$$152{,}998.5229$$

After 2 minutes, the pump has pumped just about $153{,}000$ gallons.

5. A rocket is launched upward from a cliff that's 86 feet above ground level. The velocity of the rocket is modeled by $v(t) = -32t + 88$, in feet per second, where t is seconds after the launch. Use the Net Change Theorem to determine the height in feet of the rocket 2 seconds after it's launched.

Solution:

The interval of time from the time the rocket is launched until 2 seconds later is $0 \le t \le 2$, and that will be the bounds for the integral of $v(t)$. Because the rocket starts at 86 feet above the ground, we need to add 86 to the value of the integral. So the height of the rocket is given by

$$86 + \int_0^2 -32t + 88 \ dt$$

$$86 + (-16t^2 + 88t)\Big|_0^2$$

$$86 + (-16(2)^2 + 88(2)) - (-16(0)^2 + 88(0))$$

$$86 + (-64 + 176)$$

$$86 - 64 + 176$$

$$198$$

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