Calculus 2 Workbook

Riemann sums

krista king MATH

SUMMATION NOTATION, FINDING THE SUM

■ 1. Calculate the exact sum.

$$\sum_{n=1}^{6} \frac{2n^2}{3^n}$$

■ 2. Calculate the exact sum.

$$\sum_{n=1}^{5} \frac{2n}{3n+1}$$

■ 3. Calculate the exact sum.

$$\sum_{n=0}^{6} 3n^2 - 5n + 7$$

SUMMATION NOTATION, EXPANDING

1. Expand the sum.

$$\sum_{n=1}^{6} \frac{5n+3}{2n-1}$$

2. Expand the sum.

$$\sum_{n=0}^{7} 2x^3 - 5x^2 + 9x + 3$$

3. Expand the sum.

$$\sum_{n=0}^{8} \frac{2n-8}{n+1}$$

SUMMATION NOTATION, COLLAPSING

1. Use summation notation to rewrite the sum.

$$\frac{(x+3)^2}{3-1}+\frac{(x+3)^4}{9-2}+\frac{(x+3)^6}{27-3}+\frac{(x+3)^8}{81-4}+\frac{(x+3)^{10}}{243-5}+\frac{(x+3)^{12}}{729-6}$$

2. Use summation notation to rewrite the sum.

$$\frac{3x+1}{7x}+\frac{6x+2}{14x^2}+\frac{9x+3}{21x^3}+\frac{12x+4}{28x^4}+\frac{15x+5}{35x^5}+\frac{18x+6}{42x^6}$$

3. Use summation notation to rewrite the sum.

$$\frac{x^2-3x+1}{4x}+\frac{x^3-6x+2}{8x}+\frac{x^4-9x+3}{12x}+\frac{x^5-12x+4}{16x}$$

$$+\frac{x^6-15x+5}{20x}+\frac{x^7-18x+6}{24x}+\frac{x^8-21x+7}{28x}$$

RIEMANN SUMS, LEFT ENDPOINTS

■ 1. Use a left endpoint Riemann Sum with $n = 5$ to find the area under $f(x)$ on the interval $[0,10]$.

x	0	1	2	3	4	5	6	7	8	9	10
f(x)	3	2	3	6	11	18	27	38	51	66	83

■ 2. Use a left endpoint Riemann Sum with $n = 5$ to find the area under $g(x)$ on the interval $[0,20]$. Round the final answer to 2 decimal places.

$$g(x) = 2\sqrt{x} + 5$$

■ 3. Use a left endpoint Riemann Sum with $n = 3$ to find the area under $h(x)$ on the interval $[-2,4]$.

$$h(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - x + 3$$

■ 4. Use a left endpoint Riemann Sum with $n = 4$ to find the area under $k(x)$ on the interval $[0,28]$. Round the final answer to 2 decimal places.

$$k(x) = \frac{x^2 + 4x + 4}{x^2 + 4}$$

5. Use a left endpoint Riemann Sum with $n=4$ to find the area under $f(x)$ on the interval $[0,2]$. Round the final answer to 2 decimal places.

$$f(x)=2\ln(x+3)+6$$

6. Use a left endpoint Riemann Sum with $n=5$ to find the area under $g(x)$ on the interval $[0,1]$. Round the final answer to 2 decimal places.

$$g(x)=x^4+2x^3-3x^2+4x+5$$

RIEMANN SUMS, RIGHT ENDPOINTS

■ 1. Use a right endpoint Riemann Sum with $n = 5$ to find the area under $g(x)$ on the interval $[1,11]$.

x	1	2	3	4	5	6	7	8	9	10	11
g(x)	5	4	5	8	13	20	29	40	53	68	85

■ 2. Use a right endpoint Riemann Sum with $n = 5$ to find the area under $f(x)$ on the interval $[5,25]$. Round the final answer to 2 decimal places.

$$f(x) = \sqrt{2x} - 1$$

■ 3. Use a right endpoint Riemann Sum with $n = 3$ to find the area under $h(x)$ on the interval $[-2,4]$.

$$h(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - x + 3$$

■ 4. Use a right endpoint Riemann Sum with $n = 4$ to find the area under $k(x)$ on the interval $[0,28]$. Round the final answer to 2 decimal places.

$$k(x) = \frac{x^2 + 4x + 4}{x^2 + 4}$$

■ 5. Use a right endpoint Riemann Sum with $n = 4$ to find the area under $f(x)$ on the interval $[0,2]$. Round the final answer to 2 decimal places.

$$f(x) = 2\ln(x+3) + 6$$

■ 6. Use a right endpoint Riemann Sum with $n = 5$ to find the area under $h(x)$ on the interval $[0,1]$. Round the final answer to 2 decimal places.

$$h(x) = x^4 + 2x^3 - 3x^2 + 4x + 5$$

RIEMANN SUMS, MIDPOINTS

■ 1. Use a midpoint Riemann Sum with $n = 5$ to find the area under $h(x)$ on the interval $[6,16]$.

x	6	7	8	9	10	11	12	13	14	15	16
h(x)	84	67	52	39	26	17	10	7	4	3	4

■ 2. Use a midpoint Riemann Sum with $n = 5$ to find the area under $k(x)$ on the interval $[2,22]$. Round the final answer to 2 decimal places.

$$k(x) = 3\sqrt{7x} - 8$$

■ 3. Use a midpoint Riemann Sum with $n = 3$ to find the area under $h(x)$ on the interval $[-2,4]$.

$$h(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - x + 3$$

■ 4. Use a midpoint Riemann Sum with $n = 4$ to find the area under $k(x)$ on the interval $[0,28]$. Round the final answer to 2 decimal places.

$$k(x) = \frac{x^2 + 4x + 4}{x^2 + 4}$$

5. Use a midpoint Riemann Sum with $n = 4$ to find the area under $f(x)$ on the interval $[0,2]$. Round the final answer to 2 decimal places.

$$f(x) = 2\ln(x+3) + 6$$

6. Use a midpoint Riemann Sum with $n = 5$ to find the area under $g(x)$ on the interval $[0,1]$. Round the final answer to 2 decimal places.

$$g(x) = x^4 + 2x^3 - 3x^2 + 4x + 5$$

MOVING FROM SUMMATION NOTATION TO THE INTEGRAL

■ 1. Convert the Riemann sum to a definite integral over the interval $[1,8]$.

$$\sum_{i=1}^{n}\left(6x_i^5 - 4x_i^{\frac{4}{3}} + 2x_i^{-3}\right)\Delta x$$

■ 2. Convert the Riemann sum to a definite integral over the interval $[-2,4]$.

$$\sum_{i=1}^{n}\left((5x_i + 3)(2x_i^2 + x_i)^5\right)\Delta x$$

■ 3. Convert the Riemann sum to a definite integral over the interval $[5,11]$.

$$\sum_{i=1}^{n}\left((4 - x_i)\sqrt{x_i - 5}\right)\Delta x$$

W W W . K R I S T A K I N G M A T H . C O M