Topic: Integral test

Question: Use the integral test to say whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{2e^{3n}}{1+e^{6n}}$$

Answer choices:

A Convergent because the value of the integral is $\frac{1}{3}\left(\pi - 2\tan^{-1} e^3\right)$

B Convergent because the value of the integral is $\frac{1}{3}\left(\pi + 2\tan^{-1} e^3\right)$

C Divergent because the value of the integral is ∞

D Divergent because the value of the integral is $-\infty$

Solution: A

The integral test for convergence is only valid for series that are

Positive: all of the terms in the series are positive

Decreasing: every term is less than the one before it, $a_{n-1} > a_n$

Continuous: the series is defined everywhere in its domain

If the given series meets these three criteria, then we can use the integral test for convergence to integrate the series and say whether the series is converging or diverging.

Given the series

$$\sum_{n=1}^{\infty} a_n$$

we set $f(x) = a_n$ and evaluate the integral

$$\int_{1}^{\infty} f(x)\ dx$$

According to the integral test, the series and the integral always have the same result, meaning that they either both converge or they both diverge. This means that if the value of the of the integral

converges to a **real number**, then the series also **converges**

diverges to **infinity**, then the series also **diverges**

If we expand the series through the first few terms, we can see that the series is always positive, decreasing, and continuous.

$$\sum_{n=1}^{\infty} \frac{2e^{3n}}{1+e^{6n}} = \frac{2e^{3(1)}}{1+e^{6(1)}} + \frac{2e^{3(2)}}{1+e^{6(2)}} + \frac{2e^{3(3)}}{1+e^{6(3)}} + \frac{2e^{3(4)}}{1+e^{6(4)}} + \frac{2e^{3(5)}}{1+e^{6(5)}} + \dots$$

$$\sum_{n=1}^{\infty} \frac{2e^{3n}}{1+e^{6n}} = \frac{2e^{3}}{1+e^{6}} + \frac{2e^{6}}{1+e^{12}} + \frac{2e^{9}}{1+e^{18}} + \frac{2e^{12}}{1+e^{24}} + \frac{2e^{15}}{1+e^{30}} + \dots$$

$$\sum_{n=1}^{\infty} \frac{2e^{3n}}{1+e^{6n}} = 0.099328 + 0.004957 + 0.000247 + 0.000012 + 0.000000 + \dots$$

Now that we know we can use the integral test to say whether or not the series converges, we'll set $f(x) = a_n$ for our series, and we'll get

$$f(x) = \frac{2e^{3n}}{1+e^{6n}}$$

Now we'll plug $f(x)$ into the integral above, and use u-substitution to evaluate the integral.

$$\int_1^{\infty} \frac{2e^{3x}}{1+e^{6x}}\ dx$$

$$u = e^{3x}$$

$$du = 3e^{3x}\ dx$$

$$dx = \frac{du}{3e^{3x}}$$

$$\int_{x=1}^{x=\infty} \frac{2u}{1+u^2} \cdot \frac{du}{3e^{3x}}$$

$$\int_{x=1}^{x=\infty} \frac{2u}{1+u^2} \cdot \frac{du}{3u}$$

$$\frac{2}{3}\int_{x=1}^{x=\infty} \frac{1}{1+u^2}\ du$$

$$\frac{2}{3}\tan^{-1} u \Big|_{x=1}^{x=\infty}$$

$$\frac{2}{3}\tan^{-1} e^{3x} \Big|_{1}^{\infty}$$

$$\lim_{b\to\infty}\left(\frac{2}{3}\tan^{-1} e^{3x}\right)\Big|_{1}^{b}$$

$$\lim_{b\to\infty}\left[\frac{2}{3}\tan^{-1} e^{3(b)} - \frac{2}{3}\tan^{-1} e^{3(1)}\right]$$

$$\lim_{b\to\infty}\left(\frac{2}{3}\tan^{-1} e^{3b} - \frac{2}{3}\tan^{-1} e^{3}\right)$$

$$\frac{2}{3}\cdot\frac{\pi}{2} - \frac{2}{3}\tan^{-1} e^{3}$$

$$\frac{\pi}{3} - \frac{2}{3}\tan^{-1} e^{3}$$

$$\frac{1}{3}\left(\pi - 2\tan^{-1} e^{3}\right)$$

Since we got a finite value for the integral, it means that the integral converges, which proves that the series also converges.

Topic: Integral test

Question: Use the integral test to say whether the series converges or diverges.

$$\sum_{n=3}^{\infty} \frac{\ln(n+3)}{n+3}$$

Answer choices:

A Convergent because the value of the integral is 3

B Convergent because the value of the integral is -3

C Divergent because the value of the integral is ∞

D Divergent because the value of the integral is $-\infty$

Solution: C

The integral test for convergence is only valid for series that are

Positive: all of the terms in the series are positive

Decreasing: every term is less than the one before it, $a_{n-1} > a_n$

Continuous: the series is defined everywhere in its domain

If the given series meets these three criteria, then we can use the integral test for convergence to integrate the series and say whether the series is converging or diverging.

Given the series

$$\sum_{n=1}^{\infty} a_n$$

we set $f(x) = a_n$ and evaluate the integral

$$\int_1^{\infty} f(x)\ dx$$

According to the integral test, the series and the integral always have the same result, meaning that they either both converge or they both diverge. This means that if the value of the of the integral

converges to a **real number**, then the series also **converges**

diverges to **infinity**, then the series also **diverges**

If we expand the series through the first few terms, we can see that the series is always positive, decreasing, and continuous.

$$\sum_{n=3}^{\infty} \frac{\ln(n+3)}{n+3} = \frac{\ln(3+3)}{3+3} + \frac{\ln(4+3)}{4+3} + \frac{\ln(5+3)}{5+3} + \frac{\ln(6+3)}{6+3} + \frac{\ln(7+3)}{7+3} + \dots$$

$$\sum_{n=3}^{\infty} \frac{\ln(n+3)}{n+3} = \frac{\ln(6)}{6} + \frac{\ln(7)}{7} + \frac{\ln(8)}{8} + \frac{\ln(9)}{9} + \frac{\ln(10)}{10} + \dots$$

$$\sum_{n=3}^{\infty} \frac{\ln(n+3)}{n+3} = 0.299 + 0.278 + 0.260 + 0.244 + 0.230 + \dots$$

Now that we know we can use the integral test to say whether or not the series converges, we'll set $f(x) = a_n$ for our series, and we'll get

$$f(x) = \frac{\ln(x+3)}{x+3}$$

Now we'll plug $f(x)$ into the integral above, and use u-substitution to evaluate the integral.

$$\int_1^{\infty} \frac{\ln(x+3)}{x+3}\ dx$$

$$u = \ln(x+3)$$

$$du = \frac{1}{x+3}\ dx$$

$$dx = (x+3)\ du$$

$$\int_{x=3}^{x=\infty} \frac{u}{x+3}(x+3)\ du$$

$$\int_{x=3}^{x=\infty} u\ du$$

$$\frac{1}{2}u^2\Big|_{x=3}^{x=\infty}$$

$$\frac{1}{2}\left[\ln(x+3)\right]^2\Big|_3^\infty$$

$$\lim_{b\to\infty}\frac{1}{2}\left[\ln(x+3)\right]^2\Big|_3^b$$

$$\lim_{b\to\infty}\frac{1}{2}\left[\ln(b+3)\right]^2-\frac{1}{2}\left[\ln(3+3)\right]^2$$

$$\frac{1}{2}\left[\ln(\infty+3)\right]^2-\frac{1}{2}\left[\ln(3+3)\right]^2$$

$$\frac{1}{2}\left[\ln(\infty)\right]^2-\frac{1}{2}\left[\ln(6)\right]^2$$

$$\frac{1}{2}(\infty)-\frac{1}{2}\left[\ln(6)\right]^2$$

$$\infty$$

Since we got an infinite value for the integral, it means that the integral diverges, which proves that the series also diverges.

Topic: Integral test

Question: Use the integral test to say whether the series converges or diverges.

$$\sum_{n=1}^{\infty} (0.1)^n$$

Answer choices:

A Convergent because the value of the integral is $\frac{1}{10\ln(10)}$

B Divergent because the value of the integral is $\frac{1}{10\ln(10)}$

C Convergent because the value of the integral is $10\ln(10)$

D Divergent because the value of the integral is $10\ln(10)$

Solution: A

The integral test for convergence is only valid for series that are

Positive: all of the terms in the series are positive

Decreasing: every term is less than the one before it, $a_{n-1} > a_n$

Continuous: the series is defined everywhere in its domain

If the given series meets these three criteria, then we can use the integral test for convergence to integrate the series and say whether the series is converging or diverging.

Given the series

$$\sum_{n=1}^{\infty} a_n$$

we set $f(x) = a_n$ and evaluate the integral

$$\int_1^{\infty} f(x)\ dx$$

According to the integral test, the series and the integral always have the same result, meaning that they either both converge or they both diverge. This means that if the value of the of the integral

converges to a **real number**, then the series also **converges**

diverges to **infinity**, then the series also **diverges**

If we expand the series through the first few terms, we can see that the series is always positive, decreasing, and continuous.

$$\sum_{n=1}^{\infty} (0.1)^n = (0.1)^1 + (0.1)^2 + (0.1)^3 + (0.1)^4 + (0.1)^5 + \dots$$

$$\sum_{n=1}^{\infty} (0.1)^n = 0.1 + 0.01 + 0.001 + 0.0001 + 0.00001 + \dots$$

Now that we know we can use the integral test to say whether or not the series converges, we'll set $f(x) = a_n$ for our series, and we'll get

$$f(x) = (0.1)^x$$

$$f(x) = \frac{1}{10^x}$$

Now we'll plug $f(x)$ into the integral above, and evaluate using substitution, with $u = -x$ and $-du = dx$.

$$\int_1^{\infty} \frac{1}{10^x}\ dx$$

$$-\int_{x=1}^{x=\infty} 10^u\ du$$

$$-\frac{10^u}{\ln 10}\Bigg|_{x=1}^{x=\infty}$$

$$-\frac{10^{-x}}{\ln 10}\Bigg|_1^{\infty}$$

$$-\frac{1}{10^x \ln 10}\Bigg|_1^{\infty}$$

$$\lim_{x\to\infty}\left(-\frac{1}{10^x \ln 10}\right)+\frac{1}{10^1 \ln 10}$$

$$\frac{1}{10\ln 10}$$

Since we got a finite value for the integral, it means that the integral converges, which proves that the series also converges.