Maclaurin series

A Maclaurin series is the specific instance of the Taylor series when $a=0$. Remember that we can choose any value of a in order to find a Taylor polynomial. Maclaurin series eliminate that choice and force us to choose $a=0$.

Remember that we would always use the formula

$$\frac{f^{(n)}(a)}{n!}(x-a)^n$$

to build each term in the Taylor series. Since $a=0$ in every Maclaurin series, this formula simplifies to

$$\frac{f^{(n)}(0)}{n!}(x-0)^n$$

$$\frac{f^{(n)}(0)}{n!}x^n$$

Everything else about the Maclaurin series is the same.

Example

Find the seventh-degree Maclaurin series of the function.

$$f(x)=\sin(3x)$$

We'll start by creating the chart we've always made for Taylor polynomials. Since we're finding the series to the seventh-degree, we'll use n from 0 to 7. Since it's a Maclaurin series, we'll use $a = 0$.

n	$n!$	$f^{(n)}(x)$	$f^{(n)}(0)$	$\frac{f^{(n)}(0)}{n!}$
0	1	$\sin(3x)$	$\sin(3 \cdot 0) = 0$	$\frac{0}{1} = 0$
1	1	$3\cos(3x)$	$3\cos(3 \cdot 0) = 3$	$\frac{3}{1} = 3$
2	2	$-9\sin(3x)$	$-9\sin(3 \cdot 0) = 0$	$\frac{0}{2} = 0$
3	6	$-27\cos(3x)$	$-27\cos(3 \cdot 0) = -27$	$\frac{-27}{6} = -\frac{27}{6}$
4	24	$81\sin(3x)$	$81\sin(3 \cdot 0) = 0$	$\frac{0}{24} = 0$
5	120	$243\cos(3x)$	$243\cos(3 \cdot 0) = 243$	$\frac{243}{120} = \frac{81}{40}$
6	720	$-729\sin(3x)$	$-729\sin(3 \cdot 0) = 0$	$\frac{0}{720} = 0$
7	$5{,}040$	$-2{,}187\cos(3x)$	$-2{,}187\cos(3 \cdot 0) = -2{,}187$	$\frac{-2{,}187}{5{,}040} = -\frac{243}{560}$

With the whole chart filled in, we can build each term of the Maclaurin series.

$n=0$	$\frac{f^{(n)}(0)}{n!}x^n = 0x^0$	0
$n=1$	$\frac{f^{(n)}(0)}{n!}x^n = 3x^1$	$3x$
$n=2$	$\frac{f^{(n)}(0)}{n!}x^n = 0x^2$	0
$n=3$	$\frac{f^{(n)}(0)}{n!}x^n = -\frac{27}{6}x^3$	$-\frac{27}{6}x^3$
$n=4$	$\frac{f^{(n)}(0)}{n!}x^n = 0x^4$	0
$n=5$	$\frac{f^{(n)}(0)}{n!}x^n = \frac{81}{40}x^5$	$\frac{81}{40}x^5$
$n=6$	$\frac{f^{(n)}(0)}{n!}x^n = 0x^6$	0
$n=7$	$\frac{f^{(n)}(0)}{n!}x^n = -\frac{243}{560}x^7$	$-\frac{243}{560}x^7$

Putting all of the terms together, we get the seventh-degree Maclaurin series.

$$0 + 3x + 0 - \frac{27}{6}x^3 + 0 + \frac{81}{40}x^5 + 0 - \frac{243}{560}x^7$$

$$3x - \frac{27}{6}x^3 + \frac{81}{40}x^5 - \frac{243}{560}x^7$$