Topic: Simpson's rule

Question: Use Simpson's rule to approximate the area under the curve.

$$\int_1^2 x^3 \, dx$$

when $n = 2$

Answer choices:

A -3.75

B 3.57

C 3.75

D -3.57

Solution: C

Simpson's rule is another tool we can use to approximate the area under a function over a set interval $a \le x \le b$.

Just as we did with Riemann sums, we'll divide the area into rectangles and then sum the areas of all of the rectangles in order to get an approximation of area. When we use Simpson's rule, the number of rectangles n must be an even number. The greater the number of rectangles, the more accurate the approximation will be. Of course, if we use an infinite number of rectangles, taking the limit as $n \to \infty$ of the sum of the area of each rectangle, then we'd be taking the integral and calculating exact area.

When we approximate area with Simpson's rule we consider the area above the x-axis to be positive, and the area below the x-axis to be negative. If our final result is positive, it tells us that there's more area above the x-axis than below it. On the other hand, if our final result is negative, it means that there's more area below the x-axis than above it.

The Simpson's rule formula is

$$S_n = \frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + \ldots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\right]$$

where $\Delta x = (b - a)/n$ and Δx is the width of each rectangle, and where n is even and the number of rectangles we're using to approximate area, and where $[a, b]$ is the given interval.

Notice in the Simpson's rule formula that the first and last terms $f(x_0)$ and $f(x_n)$ are not multiplied by any coefficient. The second and second-to-last terms are multiplied by 4, and the third and third-to-last terms are multiplied by 2. All of the terms in between from $f(x_3)$ to $f(x_{n-3})$ alternate coefficients starting with 4, so

$$... + 4f(x_3) + 2f(x_4) + 4f(x_5) + ... + 4f(x_{n-5}) + 2f(x_{n-4}) + 4f(x_{n-3}) + ...$$

The number of rectangles n must be even in order for the pattern to work out correctly in this way.

Our plan is to solve for Δx, divide the interval into even segments that are each Δx wide, and then use the right endpoint of each segment as the values of x_n. Plugging the interval and the value of n we've been given into the formula for Δx, we get

$$\Delta x = \frac{b-a}{n}$$

$$\Delta x = \frac{2-1}{2}$$

$$\Delta x = \frac{1}{2}$$

Since the interval is $[1,2]$, we know that $x_0 = 1$ and that $x_n = 2$. Using $\Delta x = 1/2$ to find the subintervals, we get

$$x_0 = 1$$

$$x_1 = 1 + \frac{1}{2} \qquad x_1 = \frac{3}{2}$$

$$x_2 = \frac{3}{2} + \frac{1}{2} \qquad x_2 = 2$$

Plugging all of this into the Simpson's rule formula, remembering that $f(x) = x^3$, we get

$$S_n = \frac{\frac{1}{2}}{3}\left[f(x_0) + 4f(x_1) + f(x_2)\right]$$

$$S_n = \frac{\frac{1}{2}}{3}\left[(1)^3 + 4\left(\frac{3}{2}\right)^3 + (2)^3\right]$$

$$S_n = \frac{1}{6}\left(1 + \frac{27}{2} + 8\right)$$

$$S_n = \frac{1}{6}\left(\frac{45}{2}\right)$$

$$S_n = \frac{45}{12}$$

$$S_n = \frac{15}{4}$$

$$S_n = 3.75$$

Topic: Simpson's rule

Question: Use Simpson's rule to approximate the area under the curve.

$$\int_{2}^{6} 4\sqrt{x} - 1 \ dx$$

when $n = 4$

Answer choices:

A -27.684

B 27.684

C -27.648

D 27.648

Solution: D

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where $\Delta x = (b - a)/n$ and Δx is the width of each rectangle, and where n is even and the number of rectangles we're using to approximate area, and where $[a, b]$ is the given interval.

Notice in the Simpson's rule formula that the first and last terms $f(x_0)$ and $f(x_n)$ are not multiplied by any coefficient. The second and second-to-last terms are multiplied by 4, and the third and third-to-last terms are multiplied by 2. All of the terms in between from $f(x_3)$ to $f(x_{n-3})$ alternate coefficients starting with 4, so

$$\ldots + 4f(x_3) + 2f(x_4) + 4f(x_5) + \ldots + 4f(x_{n-5}) + 2f(x_{n-4}) + 4f(x_{n-3}) + \ldots$$

The number of rectangles n must be even in order for the pattern to work out correctly in this way.

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$$\Delta x = \frac{b-a}{n}$$

$$\Delta x = \frac{6-2}{4}$$

$$\Delta x = 1$$

Since the interval is $[2,6]$, we know that $x_0 = 2$ and that $x_n = 6$. Using $\Delta x = 1$ to find the subintervals, we get

$$x_0 = 2$$

$$x_1 = 2 + 1 \qquad x_1 = 3$$

$$x_2 = 3 + 1 \qquad x_2 = 4$$

$$x_3 = 4 + 1 \qquad x_3 = 5$$

$$x_4 = 5 + 1 \qquad x_4 = 6$$

Plugging all of this into the Simpson's rule formula, remembering that $f(x) = 4\sqrt{x} - 1$, we get

$$S_n = \frac{1}{3}\left[\left(4\sqrt{2} - 1\right) + 4\left(4\sqrt{3} - 1\right) + 2\left(4\sqrt{4} - 1\right) + 4\left(4\sqrt{5} - 1\right) + \left(4\sqrt{6} - 1\right)\right]$$

$$S_n = \frac{1}{3}\left[4\sqrt{2} - 1 + 16\sqrt{3} - 4 + 8\sqrt{4} - 2 + 16\sqrt{5} - 4 + 4\sqrt{6} - 1\right]$$

$$S_n = \frac{1}{3}\left[4\sqrt{2} + 16\sqrt{3} + 16\sqrt{5} + 4\sqrt{6} + 4\right]$$

$$S_n = 27.648$$

Topic: Simpson's rule

Question: Use Simpson's rule to approximate the area under the curve.

$$\int_1^4 -x^2+3\ dx$$

when $n=4$

Answer choices:

A -12

B 13

C 12

D -13

Solution: A

Simpson's rule is another tool we can use to approximate the area under a function over a set interval $a \leq x \leq b$.

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where $\Delta x = (b-a)/n$ and Δx is the width of each rectangle, and where n is even and the number of rectangles we're using to approximate area, and where $[a, b]$ is the given interval.

Notice in the Simpson's rule formula that the first and last terms $f(x_0)$ and $f(x_n)$ are not multiplied by any coefficient. The second and second-to-last terms are multiplied by 4, and the third and third-to-last terms are multiplied by 2. All of the terms in between from $f(x_3)$ to $f(x_{n-3})$ alternate coefficients starting with 4, so

$$... + 4f(x_3) + 2f(x_4) + 4f(x_5) + ... + 4f(x_{n-5}) + 2f(x_{n-4}) + 4f(x_{n-3}) + ...$$

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$$\Delta x = \frac{b-a}{n}$$

$$\Delta x = \frac{4-1}{4}$$

$$\Delta x = \frac{3}{4}$$

Since the interval is $[1,4]$, we know that $x_0 = 1$ and that $x_n = 4$. Using $\Delta x = 3/4$ to find the subintervals, we get

$$x_0 = 1$$

$$x_1 = 1 + \frac{3}{4} \qquad x_1 = \frac{7}{4}$$

$$x_2 = \frac{7}{4} + \frac{3}{4} \qquad x_2 = \frac{10}{4} \qquad x_2 = \frac{5}{2}$$

$$x_3 = \frac{10}{4} + \frac{3}{4} \qquad x_3 = \frac{13}{4}$$

$$x_4 = \frac{13}{4} + \frac{3}{4} \qquad x_4 = \frac{16}{4} \qquad x_4 = 4$$

Plugging all of this into the Simpson's rule formula, remembering that $f(x) = -x^2 + 3$, we get

$$S_n = \frac{\frac{3}{4}}{3}\left\{\left[-(1)^2 + 3\right] + 4\left[-\left(\frac{7}{4}\right)^2 + 3\right] + 2\left[-\left(\frac{10}{4}\right)^2 + 3\right] + 4\left[-\left(\frac{13}{4}\right)^2 + 3\right] + \left[-(4)^2 + 3\right]\right\}$$

$$S_n = \frac{1}{4}\left(2 - \frac{1}{4} - \frac{13}{2} - \frac{121}{4} - 13\right)$$

$$S_n = \frac{1}{4}(-48)$$

$$S_n = -12$$