Topic: Calculating the first terms

Question: Write the first five terms of the sequence and find the limit.

$$a_n = \frac{4n^2 + 1}{n^2 - 2n + 3}$$

$$\lim_{n \to \infty} a_n$$

Answer choices:

A $\quad \frac{5}{2}, \frac{17}{3}, \frac{37}{4}, \frac{65}{5}, \frac{101}{6}$ and $\lim_{n \to \infty} a_n = 3$

B $\quad \frac{7}{2}, \frac{17}{3}, \frac{37}{6}, \frac{67}{11}, \frac{107}{18}$ and $\lim_{n \to \infty} a_n = 0$

C $\quad \frac{5}{2}, \frac{17}{3}, \frac{37}{6}, \frac{65}{11}, \frac{101}{18}$ and $\lim_{n \to \infty} a_n = 4$

D $\quad \frac{5}{2}, \frac{17}{3}, \frac{37}{6}, \frac{65}{12}, \frac{101}{18}$ and $\lim_{n \to \infty} a_n = \text{DNE}$

Solution: C

To get the first five terms of the sequence, just plug $n = 1, 2, 3, 4$ into the formula for a_{n+1} as follows.

$$n = 1 \qquad a_1 = \frac{4(1)^2 + 1}{(1)^2 - 2(1) + 3} \qquad a_1 = \frac{5}{2}$$

$$n = 2 \qquad a_2 = \frac{4(2)^2 + 1}{(2)^2 - 2(2) + 3} \qquad a_2 = \frac{17}{3}$$

$$n = 3 \qquad a_3 = \frac{4(3)^2 + 1}{(3)^2 - 2(3) + 3} \qquad a_3 = \frac{37}{6}$$

$$n = 4 \qquad a_4 = \frac{4(4)^2 + 1}{(4)^2 - 2(4) + 3} \qquad a_4 = \frac{65}{11}$$

$$n = 5 \qquad a_5 = \frac{4(5)^2 + 1}{(5)^2 - 2(5) + 3} \qquad a_5 = \frac{101}{18}$$

The first five terms of the sequence are

$$\frac{5}{2}, \frac{17}{3}, \frac{37}{6}, \frac{65}{11}, \frac{101}{18}$$

To find the limit of the sequence, divide both the numerator and denominator by the highest power of n.

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{4n^2 + 1}{n^2 - 2n + 3}\left(\frac{\frac{1}{n^2}}{\frac{1}{n^2}}\right)$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{\frac{4n^2}{n^2} + \frac{1}{n^2}}{\frac{n^2}{n^2} - \frac{2n}{n^2} + \frac{3}{n^2}}$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{4 + \frac{1}{n^2}}{1 - \frac{2}{n} + \frac{3}{n^2}}$$

$$\lim_{n\to\infty} a_n = \frac{4+0}{1-0+0}$$

$$\lim_{n\to\infty} a_n = 4$$

Therefore, the limit of the series is 4.

Topic: Calculating the first terms

Question: Write the first five terms of the sequence and find the limit.

$$a_n = \frac{e^n}{n^2}$$

$$\lim_{n\to\infty} a_n$$

Answer choices:

A $\quad \frac{e}{2}, \frac{e^2}{4}, \frac{e^3}{9}, \frac{e^4}{16}, \frac{e^5}{25}$ and $\lim_{n\to\infty} a_n = 3$

B $\quad e, \frac{e^2}{4}, \frac{e^3}{9}, \frac{e^4}{16}, \frac{e^5}{25}$ and $\lim_{n\to\infty} a_n =$ DNE

C $\quad e, \frac{e^3}{9}, \frac{e^4}{16}, \frac{e^5}{25}, \frac{e^6}{36}$ and $\lim_{n\to\infty} a_n = 4$

D $\quad e, \frac{e^2}{2}, \frac{e^3}{3}, \frac{e^4}{4}, \frac{e^5}{5}$ and $\lim_{n\to\infty} a_n = 0$

Solution: B

To get the first five terms of the sequence, just plug $n=1, 2, 3, 4, 5$ into the formula for a_n as follows.

$$n=1 \qquad a_1=\frac{e^1}{1^2} \qquad a_1=e$$

$$n=2 \qquad a_2=\frac{e^2}{2^2} \qquad a_2=\frac{e^2}{4}$$

$$n=3 \qquad a_3=\frac{e^3}{3^2} \qquad a_3=\frac{e^3}{9}$$

$$n=4 \qquad a_4=\frac{e^4}{4^2} \qquad a_4=\frac{e^4}{16}$$

$$n=5 \qquad a_5=\frac{e^5}{5^2} \qquad a_5=\frac{e^5}{25}$$

The first five terms of the sequence are

$$e, \frac{e^2}{4}, \frac{e^3}{9}, \frac{e^4}{16}, \frac{e^5}{25}$$

To find the limit of the sequence, apply L'Hospital's rule twice by replacing the numerator and denominator with their derivatives.

$$\lim_{n\to\infty} a_n=\lim_{n\to\infty}\frac{e^n}{n^2}$$

$$\lim_{n\to\infty} a_n=\lim_{n\to\infty}\frac{e^n}{2n}$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{e^n}{2}$$

$$\lim_{n\to\infty} a_n = \infty$$

Therefore, the limit of the series does not exist (DNE).

Topic: Calculating the first terms

Question: Write the first three terms of the sequence.

$$a_n = 2^n$$

Answer choices:

A $\quad 2$, 8 and 16

B $\quad 2$, 4 and 8

C $\quad 4$, 8 and 12

D $\quad 2$, 4 and 6

Solution: B

To get the first three terms of the sequence, just plug $n = 1, 2, 3$ into the formula for a_n as follows.

$$n = 1 \qquad a_1 = 2^1 \qquad a_1 = 2$$

$$n = 2 \qquad a_2 = 2^2 \qquad a_2 = 4$$

$$n = 3 \qquad a_3 = 2^3 \qquad a_3 = 8$$

The first three terms of the sequence are

$$2, 4, 8$$