U-substitution in definite integrals

U-substitution in definite integrals is just like substitution in indefinite integrals except that, since the variable is changed, the limits of integration must be changed as well.

Example

Use u-substitution to evaluate the integral.

$$\int_0^{\frac{\pi}{2}} \frac{\cos x}{1+\sin^2 x}\ dx$$

Let

$$u=\sin x$$

$$du=\cos x\ dx$$

$$dx=\frac{du}{\cos x}$$

Since we're dealing with a definite integral, we need to use the equation $u=\sin x$ to find limits of integration in terms of u, instead of x.

when $x=0$, $\qquad u=\sin 0$

$$u=0$$

when $x = \frac{\pi}{2}$, $\quad u = \sin\frac{\pi}{2}$

$$u = 1$$

Substituting back into the integral (including for our limits of integration), we get

$$\int_0^1 \frac{\cos x}{1+u^2} \cdot \frac{du}{\cos x}$$

$$\int_0^1 \frac{1}{1+u^2}\ du$$

Using this very common formula,

$$\int \frac{1}{1+x^2}\ dx = \tan^{-1} x + C$$

take the integral.

$$\int_0^1 \frac{1}{1+u^2}\ du = \tan^{-1} u \Big|_0^1$$

$$\int_0^1 \frac{1}{1+u^2}\ du = \tan^{-1} 1 - \tan^{-1} 0$$

$$\int_0^1 \frac{1}{1+u^2}\ du = \frac{\pi}{4}$$