Second derivative of a parametric curve

To find the second derivative of a parametric curve, we need to find its first derivative dy/dx, using the formula

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

and then plug it into this formula for the second derivative:

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$$

where d^2y/dx^2 is the second derivative of the parametric curve, dy/dx is its first derivative and dx/dt is the first derivative of the equation for x. The d/dt is notation that tells us to take the derivative of dy/dx with respect to t.

Example

Find the second derivative of the parametric curve.

$$x = 5t^3 + 6t$$

$$y = t^4 - 3$$

To find the first derivative, we'll solve for dx/dt and dy/dt.

This means we will have to solve for dy/dt, dx/dt and dy/dx first. Let's start with dy/dt.

$$x = 5t^3 + 6t$$

$$\frac{dx}{dt} = 15t^2 + 6$$

and

$$y = t^4 - 3$$

$$\frac{dy}{dt} = 4t^3$$

Plugging these two derivatives into the formula for the first derivative,

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

we get

$$\frac{dy}{dx} = \frac{4t^3}{15t^2 + 6}$$

Now plugging the first derivative dy/dx and the value we found earlier for dx/dt into the formula for the second derivative,

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$$

we get

$$\frac{d^2y}{dx^2}=\frac{\frac{d}{dt}\left(\frac{4t^3}{15t^2+6}\right)}{15t^2+6}$$

We'll use quotient rule to take the derivative of dy/dx with respect to t.

$$\frac{d^2y}{dx^2}=\frac{\frac{\left(12t^2\right)\left(15t^2+6\right)-\left(4t^3\right)(30t)}{\left(15t^2+6\right)^2}}{15t^2+6}$$

$$\frac{d^2y}{dx^2}=\frac{\left(12t^2\right)\left(15t^2+6\right)-\left(4t^3\right)(30t)}{\left(15t^2+6\right)^2}\cdot\frac{1}{15t^2+6}$$

$$\frac{d^2y}{dx^2}=\frac{180t^4+72t^2-120t^4}{\left(15t^2+6\right)^3}$$

$$\frac{d^2y}{dx^2}=\frac{60t^4+72t^2}{\left(15t^2+6\right)^3}$$

$$\frac{d^2y}{dx^2}=\frac{12t^2\left(5t^2+6\right)}{\left(15t^2+6\right)^3}$$