Topic: Trapezoidal rule

Question: Use trapezoidal rule to approximate the integral.

$$\int_0^1 \sqrt{2 - x^2}\ dx$$

when $n = 4$

Answer choices:

A 7.66

B 1.28

C 4.20

D 1.67

Solution: B

Trapezoidal rule is another tool we can use to approximate the area under a function over a set interval $a \le x \le b$.

Instead of dividing the area into rectangles, as we did with Riemann sums, we'll divide the area into trapezoids and then sum the areas of all of the trapezoids in order to get an approximation of area. The greater the number of trapezoids, the more accurate the approximation will be. Of course, if we use an infinite number of trapezoids, taking the limit as $n \to \infty$ of the sum of the area of each trapezoid, then we'd be taking the integral and calculating exact area.

When we approximate area with Trapezoidal rule we consider the area above the x-axis to be positive, and the area below the x-axis to be negative. If our final result is positive, it tells us that there's more area above the x-axis than below it. On the other hand, if our final result is negative, it means that there's more area below the x-axis than above it.

The Trapezoidal rule formula is

$$\int_a^b f(x)\ dx \approx \frac{\Delta x}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + \dots + 2f(x_{n-2}) + 2f(x_{n-1}) + f(x_n)\right]$$

where $\Delta x = (b - a)/n$ and Δx is the width of each trapezoid, and where n is the number of trapezoids we're using to approximate area, and where $[a, b]$ is the given interval.

Notice in the Trapezoidal rule formula that the first and last terms $f(x_0)$ and $f(x_n)$ are not multiplied by any coefficient. All the other terms between them are multiplied by 2. That's because the formula for the area of a trapezoid uses the length of both bases of the trapezoid. Since the right base of every trapezoid in our approximation is the same line as the left base of the trapezoid next to it, every base except the first and last ones get used twice in our approximation.

Our plan is to solve for Δx, divide the interval into even segments that are each Δx wide, and then use the right endpoint of each segment as the values of x_n. Plugging the interval and the value of n we've been given into the formula for Δx, we get

$$\Delta x = \frac{b-a}{n}$$

$$\Delta x = \frac{1-0}{4}$$

$$\Delta x = \frac{1}{4}$$

Since the interval is $[0,1]$, we know that $x_0 = 0$ and that $x_n = 1$. Using $\Delta x = 1/4$ to find the subintervals, we get

$$x_0 = 0$$

$$x_1 = 0 + \frac{1}{4} \qquad x_1 = \frac{1}{4}$$

$$x_2 = \frac{1}{4} + \frac{1}{4} \qquad x_2 = \frac{1}{2}$$

$$x_3 = \frac{1}{2} + \frac{1}{4} \qquad x_3 = \frac{3}{4}$$

$$x_4 = \frac{3}{4} + \frac{1}{4} \qquad x_4 = \frac{4}{4} \qquad x_4 = 1$$

Plugging all of this into the Trapezoidal rule formula, remembering that $f(x) = \sqrt{2 - x^2}$, we get

$$\frac{\frac{1}{4}}{2}\left[\sqrt{2-0^2} + 2\sqrt{2-\left(\frac{1}{4}\right)^2} + 2\sqrt{2-\left(\frac{1}{2}\right)^2} + 2\sqrt{2-\left(\frac{3}{4}\right)^2} + \sqrt{2-1^2}\right]$$

$$\frac{1}{8}\left(\sqrt{2} + 2\sqrt{2-\frac{1}{16}} + 2\sqrt{2-\frac{1}{4}} + 2\sqrt{2-\frac{9}{16}} + \sqrt{2-1}\right)$$

$$\frac{1}{8}\left(\sqrt{2} + 2\sqrt{\frac{32}{16}-\frac{1}{16}} + 2\sqrt{\frac{8}{4}-\frac{1}{4}} + 2\sqrt{\frac{32}{16}-\frac{9}{16}} + \sqrt{1}\right)$$

$$\frac{1}{8}\left(\sqrt{2} + 2\sqrt{\frac{31}{16}} + 2\sqrt{\frac{7}{4}} + 2\sqrt{\frac{23}{16}} + 1\right)$$

$$\frac{1}{8}\left(\sqrt{2} + \frac{2\sqrt{31}}{4} + \frac{2\sqrt{7}}{2} + \frac{2\sqrt{23}}{4} + 1\right)$$

$$\frac{1}{8}\left(\frac{2\sqrt{2}}{2} + \frac{\sqrt{31}}{2} + \frac{2\sqrt{7}}{2} + \frac{\sqrt{23}}{2} + \frac{2}{2}\right)$$

$$\frac{1}{8}\left(\frac{2\sqrt{2} + \sqrt{31} + 2\sqrt{7} + \sqrt{23} + 2}{2}\right)$$

$$\frac{2\sqrt{2}+\sqrt{31}+2\sqrt{7}+\sqrt{23}+2}{16}$$

1.28

Topic: Trapezoidal rule

Question: Use trapezoidal rule to approximate the integral.

$$\int_0^2 x^2\, dx$$

when $n = 4$

Answer choices:

A $\frac{11}{4}$

B $\frac{8}{3}$

C $-\frac{11}{4}$

D $-\frac{8}{3}$

Solution: A

Trapezoidal rule is another tool we can use to approximate the area under a function over a set interval $a \leq x \leq b$.

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where $\Delta x = (b - a)/n$ and Δx is the width of each trapezoid, and where n is the number of trapezoids we're using to approximate area, and where $[a, b]$ is the given interval.

Notice in the Trapezoidal rule formula that the first and last terms $f(x_0)$ and $f(x_n)$ are not multiplied by any coefficient. All the other terms between them are multiplied by 2. That's because the formula for the area of a trapezoid uses the length of both bases of the trapezoid. Since the right base of every trapezoid in our approximation is the same line as the left base of the trapezoid next to it, every base except the first and last ones get used twice in our approximation.

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$$\Delta x = \frac{b-a}{n}$$

$$\Delta x = \frac{2-0}{4}$$

$$\Delta x = \frac{1}{2}$$

Since the interval is $[0,2]$, we know that $x_0 = 0$ and that $x_n = 2$. Using $\Delta x = 1/2$ to find the subintervals, we get

$$x_0 = 0$$

$$x_1 = 0 + \frac{1}{2} \qquad x_1 = \frac{1}{2}$$

$$x_2 = \frac{1}{2} + \frac{1}{2} \qquad x_2 = 1$$

$$x_3 = 1 + \frac{1}{2} \qquad x_3 = \frac{3}{2}$$

$$x_4 = \frac{3}{2} + \frac{1}{2} \qquad x_4 = 2$$

Plugging all of this into the Trapezoidal rule formula, remembering that $f(x) = x^2$, we get

$$\frac{\frac{1}{2}}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + f(x_4)\right]$$

$$\frac{\frac{1}{2}}{2}\left[f(0) + 2f\left(\frac{1}{2}\right) + 2f(1) + 2f\left(\frac{3}{2}\right) + f(2)\right]$$

$$\frac{\frac{1}{2}}{2}\left[(0)^2 + 2\left(\frac{1}{2}\right)^2 + 2(1)^2 + 2\left(\frac{3}{2}\right)^2 + (2)^2\right]$$

$$\frac{1}{4}\left[2\left(\frac{1}{4}\right) + 2 + 2\left(\frac{9}{4}\right) + 4\right]$$

$$\frac{1}{4}\left(\frac{1}{2} + 2 + \frac{9}{2} + 4\right)$$

$$\frac{1}{4}\left(6 + \frac{10}{2}\right)$$

$$\frac{6}{4} + \frac{10}{8}$$

$$\frac{6}{4} + \frac{5}{4}$$

$$\frac{11}{4}$$

Topic: Trapezoidal rule

Question: Use trapezoidal rule to approximate the integral.

$$\int_1^4 x^3 - 4\ dx$$

when $n = 2$

Answer choices:

A $-\frac{963}{16}$

B 16

C -16

D $\frac{963}{16}$

Solution: D

Trapezoidal rule is another tool we can use to approximate the area under a function over a set interval $a \le x \le b$.

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where $\Delta x = (b - a)/n$ and Δx is the width of each trapezoid, and where n is the number of trapezoids we're using to approximate area, and where $[a, b]$ is the given interval.

Notice in the Trapezoidal rule formula that the first and last terms $f(x_0)$ and $f(x_n)$ are not multiplied by any coefficient. All the other terms between them are multiplied by 2. That's because the formula for the area of a trapezoid uses the length of both bases of the trapezoid. Since the right base of every trapezoid in our approximation is the same line as the left base of the trapezoid next to it, every base except the first and last ones get used twice in our approximation.

Our plan is to solve for Δx, divide the interval into even segments that are each Δx wide, and then use the right endpoint of each segment as the values of x_n. Plugging the interval and the value of n we've been given into the formula for Δx, we get

$$\Delta x = \frac{b-a}{n}$$

$$\Delta x = \frac{4-1}{2}$$

$$\Delta x = \frac{3}{2}$$

Since the interval is $[1,4]$, we know that $x_0 = 1$ and that $x_n = 4$. Using $\Delta x = 3/2$ to find the subintervals, we get

$$x_0 = 1$$

$$x_1 = 1 + \frac{3}{2} \qquad x_1 = \frac{5}{2}$$

$$x_2 = \frac{5}{2} + \frac{3}{2} \qquad x_2 = \frac{8}{2} \qquad x_2 = 4$$

Plugging all of this into the Trapezoidal rule formula, remembering that $f(x) = x^3 - 4$, we get

$$\frac{3}{4}\left[f(x_0)+2f(x_1)+f(x_2)\right]$$

$$\frac{3}{4}\left[f(1)+2f\left(\frac{5}{2}\right)+f(4)\right]$$

$$\frac{3}{4}\left[(1)^3-4+2\left[\left(\frac{5}{2}\right)^3-4\right]+(4)^3-4\right]$$

$$\frac{3}{4}\left[1-4+2\left(\frac{125}{8}-4\right)+64-4\right]$$

$$\frac{3}{4}\left(1-4+\frac{125}{4}-8+64-4\right)$$

$$\frac{3}{4}\left(49+\frac{125}{4}\right)$$

$$\frac{147}{4}+\frac{375}{16}$$

$$\frac{588}{16}+\frac{375}{16}$$

$$\frac{963}{16}$$