Limit of a convergent sequence

Remember that a sequence is convergent if its limit exists as $n \to \infty$. So it makes sense that once we know that a sequence is convergent, we should be able to evaluate the limit as $n \to \infty$ and get a real-number answer.

The way that we simplify and evaluate the limit will depend on the kind of functions we have in our sequence (trigonometric, exponential, etc.), but we know that the limit as $n \to \infty$ exists.

Example

Find the limit of the convergent sequence.

$$a_n = \ln\left(4n^3 + 3\right) - \ln\left(3n^3 - 5\right)$$

We've been told the sequence converges, so we already know that the limit will exist as $n \to \infty$.

$$\lim_{n \to \infty} a_n = \lim_{n \to \infty} \ln\left(4n^3 + 3\right) - \ln\left(3n^3 - 5\right)$$

If we remember our laws of logarithms, we know that

$$\ln a - \ln b = \ln \frac{a}{b}$$

so we can simplify the limit to

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$$\lim_{n\to\infty} \ln\left(4n^3+3\right) - \ln\left(3n^3-5\right) = \lim_{n\to\infty} \ln\left(\frac{4n^3+3}{3n^3-5}\right)$$

We'll divide each term in our rational function by the variable of the highest degree, n^3.

$$\lim_{n\to\infty} \ln\left(4n^3+3\right) - \ln\left(3n^3-5\right) = \lim_{n\to\infty} \ln\left(\frac{\frac{4n^3}{n^3}+\frac{3}{n^3}}{\frac{3n^3}{n^3}-\frac{5}{n^3}}\right)$$

$$\lim_{n\to\infty} \ln\left(4n^3+3\right) - \ln\left(3n^3-5\right) = \lim_{n\to\infty} \ln\left(\frac{4+\frac{3}{n^3}}{3-\frac{5}{n^3}}\right)$$

Now we will evaluate the limit.

$$\lim_{n\to\infty} \ln\left(4n^3+3\right) - \ln\left(3n^3-5\right) = \ln\left(\frac{4+\frac{3}{\infty}}{3-\frac{5}{\infty}}\right)$$

We know that any fraction that has a constant in the numerator and an infinitely large denominator will approach 0, so

$$\lim_{n\to\infty} \ln\left(4n^3+3\right) - \ln\left(3n^3-5\right) = \ln\left(\frac{4+0}{3-0}\right)$$

$$\lim_{n\to\infty} \ln\left(4n^3+3\right) - \ln\left(3n^3-5\right) = \ln\frac{4}{3}$$

The limit of the convergent sequence $a_n = \ln\left(4n^3+3\right) - \ln\left(3n^3-5\right)$ is $\ln\frac{4}{3}$.

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