Topic: Indefinite integral as infinite series

Question: Use an infinite series to evaluate the indefinite integral.

$$\int xe^{2x}\ dx$$

Answer choices:

A $\quad C+\sum_{n=0}^{\infty}\frac{2^n x^{n+2}}{n!(n+2)}$

B $\quad C+\sum_{n=0}^{\infty}\frac{2^n x^{n+2}}{n(n+2)}$

C $\quad C+\sum_{n=0}^{\infty}\frac{2x^{n+2}}{n!(n+2)}$

D $\quad C+\sum_{n=0}^{\infty}\frac{2^n x^{n+1}}{n!(n+1)}$

Solution: A

When we're asked to use an infinite series to evaluate an indefinite integral, it means we're supposed to find a power series representation for the function we've been asked to integrate, and then integrate that power series instead of the original function.

To find the power series representation of the given function, we'll start with the known Maclaurin series

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

and then manipulate it until it matches the given series. To get it to match the given series, we'll replace x with $2x$, and then multiply by x.

$$e^{2x} = \sum_{n=0}^{\infty} \frac{(2x)^n}{n!}$$

$$xe^{2x} = \sum_{n=0}^{\infty} \frac{(2x)^n}{n!} \cdot x$$

$$xe^{2x} = \sum_{n=0}^{\infty} \frac{2^n x^n x^1}{n!}$$

$$xe^{2x} = \sum_{n=0}^{\infty} \frac{2^n x^{n+1}}{n!}$$

Now we can integrate the power series instead of the original function.

$$\int xe^{2x}\ dx = \int \sum_{n=0}^{\infty} \frac{2^n x^{n+1}}{n!}\ dx$$

Since we're integrating with respect to x, we can remove from the integral on the right any term that doesn't involve x.

$$\int xe^{2x}\ dx = \sum_{n=0}^{\infty} \frac{2^n}{n!} \int x^{n+1}\ dx$$

$$\int xe^{2x}\ dx = \sum_{n=0}^{\infty} \frac{2^n}{n!} \cdot \frac{x^{n+1+1}}{n+1+1} + C$$

$$\int xe^{2x}\ dx = \sum_{n=0}^{\infty} \frac{2^n x^{n+2}}{n!(n+2)} + C$$

Topic: Indefinite integral as infinite series

Question: Use an infinite series to evaluate the indefinite integral.

$$\int \frac{2x}{1-3x}\ dx$$

Answer choices:

A $\quad C + \sum_{n=0}^{\infty} \frac{6x^{n+2}}{n+2}$

B $\quad C + \sum_{n=0}^{\infty} \frac{2(3^n)x^{n+1}}{n+1}$

C $\quad C + \sum_{n=0}^{\infty} \frac{2(3^n)x^{n+2}}{n+2}$

D $\quad C + \sum_{n=0}^{\infty} \frac{6^n x^{n+2}}{n+2}$

Solution: C

When we're asked to use an infinite series to evaluate an indefinite integral, it means we're supposed to find a power series representation for the function we've been asked to integrate, and then integrate that power series instead of the original function.

To find the power series representation of the given function, we'll start with the known Maclaurin series

$$\frac{1}{1-x}=\sum_{n=0}^{\infty}x^n$$

and then manipulate it until it matches the given series. To get it to match the given series, we'll replace x with $3x$, and then multiply by $2x$.

$$\frac{1}{1-3x}=\sum_{n=0}^{\infty}(3x)^n$$

$$\frac{2x}{1-3x}=\sum_{n=0}^{\infty}2x(3x)^n$$

$$\frac{2x}{1-3x}=\sum_{n=0}^{\infty}2x3^nx^n$$

$$\frac{2x}{1-3x}=\sum_{n=0}^{\infty}2(3^n)x^{n+1}$$

Now we can integrate the power series instead of the original function.

$$\int \frac{2x}{1-3x}\ dx = \int \sum_{n=0}^{\infty} 2(3^n)x^{n+1}\ dx$$

Since we're integrating with respect to x, we can remove from the integral on the right any term that doesn't involve x.

$$\int \frac{2x}{1-3x}\ dx = \sum_{n=0}^{\infty} 2(3^n) \int x^{n+1}\ dx$$

$$\int \frac{2x}{1-3x}\ dx = \sum_{n=0}^{\infty} 2(3^n)\frac{x^{n+1+1}}{n+1+1}$$

$$\int \frac{2x}{1-3x}\ dx = \sum_{n=0}^{\infty} \frac{2(3^n)x^{n+2}}{n+2} + C$$

Topic: Indefinite integral as infinite series

Question: Use an infinite series to evaluate the indefinite integral.

$$\int x^2 \ln(1+4x)\ dx$$

Answer choices:

A $\quad C + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}4^n x^{n+3}}{n!(n+3)}$

B $\quad C + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}4^n x^{n+3}}{n(n+3)}$

C $\quad C + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}4^n x^{n+2}}{n(n+2)}$

D $\quad C + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}4^n x^{n+2}}{n!(n+2)}$

Solution: B

When we're asked to use an infinite series to evaluate an indefinite integral, it means we're supposed to find a power series representation for the function we've been asked to integrate, and then integrate that power series instead of the original function.

To find the power series representation of the given function, we'll start with the known Maclaurin series

$$\ln(1+x)=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}x^n$$

and then manipulate it until it matches the given series. To get it to match the given series, we'll replace x with $4x$, and then multiply by x^2.

$$\ln(1+4x)=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}(4x)^n$$

$$x^2\ln(1+4x)=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}(4x)^n\cdot x^2$$

$$x^2\ln(1+4x)=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\cdot 4^nx^nx^2$$

$$x^2\ln(1+4x)=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}4^nx^{n+2}}{n}$$

Now we can integrate the power series instead of the original function.

$$\int x^2 \ln(1+4x)\ dx = \int \sum_{n=1}^{\infty} \frac{(-1)^{n+1}4^n x^{n+2}}{n}\ dx$$

Since we're integrating with respect to x, we can remove from the integral on the right any term that doesn't involve x.

$$\int x^2 \ln(1+4x)\ dx = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}4^n}{n} \int x^{n+2}\ dx$$

$$\int x^2 \ln(1+4x)\ dx = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}4^n}{n} \cdot \frac{x^{n+2+1}}{n+2+1} + C$$

$$\int x^2 \ln(1+4x)\ dx = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}4^n x^{n+3}}{n(n+3)} + C$$