Topic: Tabular integration

Question: Use tabular integration to evaluate the integral.

$$\int \left(3x^2 - 4x\right) e^{5x}\ dx$$

Answer choices:

A $\quad \frac{1}{125}xe^{5x}\left(75x^2 - 130x\right) + C$

B $\quad \frac{1}{125}e^{5x}\left(75x^2 - 130x + 26\right) + C$

C $\quad \frac{1}{125}xe^{5x}\left(75x^2 + 130x\right) + C$

D $\quad \frac{1}{25}e^{5x}\left(75x^2 - 130x + 26\right) + C$

Solution: B

First, split the integrand into two functions $f(x)$ and $g(x)$.

$$f(x) = 3x^2 - 4x$$

$$g(x) = e^{5x}$$

Create the table by differentiating $f(x)$ successively until the derivative is 0 or is no longer differentiable and integrating $g(x)$ successively as many times as $f(x)$ was differentiated, the table becomes

Derivatives of $f(x)$	**Integrals of $g(x)$**
$f(x) = 3x^2 - 4x$	$g(x) = e^{5x}$
$f'(x) = 6x - 4$	$\int e^{5x}\ dx = \frac{1}{5}e^{5x}$
$f''(x) = 6$	$\int \frac{1}{5}e^{5x}\ dx = \frac{1}{25}e^{5x}$
$f'''(x) = 0$	$\int \frac{1}{25}e^{5x}\ dx = \frac{1}{125}e^{5x}$

The integral is given by

$$\int \left(3x^2 - 4x\right) e^{5x}\ dx = \frac{1}{5}e^{5x}\left(3x^2 - 4x\right) - \frac{1}{25}e^{5x}(6x - 4) + \frac{1}{125}e^{5x}(6) + C$$

$$\frac{1}{125}e^{5x}\left[25\left(3x^2 - 4x\right) - 5(6x - 4) + (6)\right] + C$$

$$\frac{1}{125}e^{5x}\left(75x^2 - 100x - 30x + 20 + 6\right) + C$$

$$\frac{1}{125}e^{5x}\left(75x^2-130x+26\right)+C$$

Topic: Tabular integration

Question: Use tabular integration to evaluate the integral.

$$\int_0^{2\pi} 2x^2 \sin\left(\frac{x}{2}\right)\, dx$$

Answer choices:

A $16\pi^2$

B $16\pi^2 + 64$

C $\pi^2 - 64$

D $16\pi^2 - 64$

Solution: D

First, split the integrand into two functions $f(x)$ and $g(x)$.

$$f(x) = 2x^2$$

$$g(x) = \sin\left(\frac{x}{2}\right)$$

Create the table containing the derivatives of $f(x)$ and the integrals of $g(x)$ as shown below.

Derivatives of $f(x)$	**Integrals of** $g(x)$
$f(x) = 2x^2$	$g(x) = \sin\left(\frac{x}{2}\right)$
$f'(x) = 4x$	$\int \sin\left(\frac{x}{2}\right)\ dx = -2\cos\left(\frac{x}{2}\right)$
$f''(x) = 4$	$\int -2\cos\left(\frac{x}{2}\right)\ dx = -4\sin\left(\frac{x}{2}\right)$
$f'''(x) = 0$	$\int -4\sin\left(\frac{x}{2}\right)\ dx = 8\cos\left(\frac{x}{2}\right)$

Therefore, the integration is

$$\int_0^{2\pi} 2x^2 \sin\left(\frac{x}{2}\right)\ dx = \left[2x^2\left[-2\cos\left(\frac{x}{2}\right)\right] - 4x\left[-4\sin\left(\frac{x}{2}\right)\right] + 4\left[8\cos\left(\frac{x}{2}\right)\right]\right]\Bigg|_0^{2\pi}$$

$$-4x^2\cos\left(\frac{x}{2}\right)+16x\sin\left(\frac{x}{2}\right)+32\cos\left(\frac{x}{2}\right)\Big|_0^{2\pi}$$

$$\left[-4(2\pi)^2\cos\left(\frac{2\pi}{2}\right)+16(2\pi)\sin\left(\frac{2\pi}{2}\right)+32\cos\left(\frac{2\pi}{2}\right)\right]-$$

$$\left[-4(0)^2\cos\left(\frac{0}{2}\right)+16(0)\sin\left(\frac{0}{2}\right)+32\cos\left(\frac{0}{2}\right)\right]$$

$$\left(-16\pi^2\cos\pi+32\pi\sin\pi+32\cos\pi\right)-(0+0+32\cos 0)$$

$$\left[-16\pi^2(-1)+32\pi(0)+32(-1)\right]-32$$

$$16\pi^2-32-32$$

$$16\pi^2-64$$

Topic: Tabular integration

Question: Evaluate the integral using tabular integration.

$$\int \left(x^2 + 3x - 4\right) e^x \, dx$$

Answer choices:

A $e^x\left(x^2 + x - 5\right) + C$

B $x^2 + x - 5 + C$

C $e^x\left(x^2 + x - 5\right)$

D $x^2 + x - 5$

Solution: A

Tabular integration is a method of integrating by parts to evaluate an integral that cannot be evaluated using normal integration techniques, by using integration by substitution, or by using integration formulas. This method of integration is particularly useful when the integral requires multiple iterations of integration by parts. It works only when the portion of the integrand that we select as u eventually differentiates to 0.

The general formula for the first iterations of integration by parts is

$$\int u_1\ dv_1 = u_1v_1 - \int v_1\ du_1$$

If we use integration by parts a second time, it looks like

$$\int u_1\ dv_1 = u_1v_1 - u_2v_2 + \int v_2\ du_2$$

Then, if we use integration by parts a third time, it looks like

$$\int u_1\ dv_1 = u_1v_1 - u_2v_2 + u_3v_3 - \int v_3\ du_3$$

The process would continue until the integration is finished.

In this formula, we separate the integrand into two parts; one part is called u and the other part is called dv. In making these two parts, we must use all of the integrand.

Although there is sometimes flexibility in choosing u, we can generally use the following sequence of choices to select the best part of the integrand to be u. This method involves the acronym LIPET, where we select the first u in the sequence of the list below. The letters mean

L	Logarithmic expression
I	Inverse trigonometric expression
P	Polynomial expression
E	Exponential expression
T	Trigonometric function expression

In this problem, the integrand is $\left(x^2+3x-4\right)e^x$ where we have a polynomial and an exponential function. In the LIPET sequence, polynomial comes before the exponential function, so u is the polynomial.

Now, when we use tabular integration, we create a table with the first column containing u and the second column containing dv.

Next, differentiate u until the derivative is 0, and integrate dv until the number of rows in each column is the same. The table for this problem looks like this:

u	dv
x^2+3x-4	e^x
$2x+3$	e^x
2	e^x
0	e^x

Now that the table is finished, multiply the first row of the first column by the second row of the second column. Next, multiply the second row of the first column by the opposite of the third row of the second column. Then, multiply the third row of the first column by the fourth row of the second column. The fourth row of the first column is zero, so the process is finished. The result is

$$\left(x^2+3x-4\right)e^x-(2x+3)e^x+2e^x$$

This could be the final answer, but we can factor out e^x, as a GCF, and combine like terms.

$$\left(x^2+3x-4\right)e^x-(2x+3)e^x+2e^x$$

$$e^x\left(x^2+3x-4-2x-3+2\right)$$

$$e^x\left(x^2+x-5\right)$$

We add a C to accommodate the possibility of a constant in the function. The last step is to simplify the result of the integration. The final answer is

$$\int\left(x^2+3x-4\right)e^x\ dx=e^x\left(x^2+x-5\right)+C$$