Topic: Summation notation, expanding

Question: Expand the summation.

$$\sum_{n=0}^{4} \frac{4n + 1}{3^n}$$

Answer choices:

A $\quad \dfrac{5}{3} + 1 + \dfrac{13}{27} + \dfrac{17}{81}$

B $\quad 1 + \dfrac{5}{3} + 1 + \dfrac{13}{27} + \dfrac{17}{81} + \dfrac{7}{81}$

C $\quad 1 + \dfrac{5}{3} + 1 + \dfrac{13}{27} + \dfrac{17}{81}$

D $\quad \dfrac{353}{81}$

Solution: C

To expand the summation

$$\sum_{n=0}^{4} \frac{4n + 1}{3^n}$$

write each term with the value of n for that term. Since the summation shows that n begins with a value of 0 and ends with 4, there are five terms.

$n = 0$ $\qquad \frac{4(0) + 1}{3^0} = \frac{1}{1} = 1$

$n = 1$ $\qquad \frac{4(1) + 1}{3^1} = \frac{4 + 1}{3} = \frac{5}{3}$

$n = 2$ $\qquad \frac{4(2) + 1}{3^2} = \frac{8 + 1}{9} = \frac{9}{9} = 1$

$n = 3$ $\qquad \frac{4(3) + 1}{3^3} = \frac{12 + 1}{27} = \frac{13}{27}$

$n = 4$ $\qquad \frac{4(4) + 1}{3^4} = \frac{16 + 1}{81} = \frac{17}{81}$

Write the terms as a sum to expand the summation.

$$1 + \frac{5}{3} + 1 + \frac{13}{27} + \frac{17}{81}$$

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Question: Expand the summation.

$$\sum_{x=1}^{7} \left(x^2 + 4x + 3 \right)$$

Answer choices:

A   273

B $8 + 15 + 24 + 35 + 48 + 63 + 80$

C $3 + 8 + 15 + 24 + 35 + 48 + 63 + 80$

D $3 + 8 + 15 + 24 + 35 + 48 + 63$

Solution: B

To expand the summation

$$\sum_{x=1}^{7} \left(x^2 + 4x + 3\right)$$

write each term with the value of x for that term. Since the summation shows that x begins with a value of 1, and ends with 7, there are seven terms.

$x = 1$ $(1)^2 + 4(1) + 3 = 1 + 4 + 3 = 8$

$x = 2$ $(2)^2 + 4(2) + 3 = 4 + 8 + 3 = 15$

$x = 3$ $(3)^2 + 4(3) + 3 = 9 + 12 + 3 = 24$

$x = 4$ $(4)^2 + 4(4) + 3 = 16 + 16 + 3 = 35$

$x = 5$ $(5)^2 + 4(5) + 3 = 25 + 20 + 3 = 48$

$x = 6$ $(6)^2 + 4(6) + 3 = 36 + 24 + 3 = 63$

$x = 7$ $(7)^2 + 4(7) + 3 = 49 + 28 + 3 = 80$

Write the terms as a sum to expand the summation.

$$8 + 15 + 24 + 35 + 48 + 63 + 80$$

Topic: Summation notation, expanding

Question: Expand the summation.

$$\sum_{x=0}^{4} \left(\frac{1}{2}x^3 - \frac{1}{3}x^2 + \frac{1}{4}x - \frac{1}{5} \right)$$

Answer choices:

A $\quad \frac{13}{60} + \frac{89}{30} + \frac{221}{20} + \frac{412}{15} + \frac{3,313}{60}$

B $\quad \frac{83}{2}$

C $\quad \frac{1,163}{12}$

D $\quad -\frac{1}{5} + \frac{13}{60} + \frac{89}{30} + \frac{221}{20} + \frac{412}{15}$

Solution: D

To expand the summation

$$\sum_{x=0}^{4} \left(\frac{1}{2}x^3 - \frac{1}{3}x^2 + \frac{1}{4}x - \frac{1}{5} \right)$$

write each term with the value of x for that term. Since the summation shows that x begins with a value of 0 and ends with 4, there are five terms.

$x = 0$ $\qquad\qquad \frac{1}{2}(0)^3 - \frac{1}{3}(0)^2 + \frac{1}{4}(0) - \frac{1}{5}$

$$= -\frac{1}{5}$$

$x = 1$ $\qquad\qquad \frac{1}{2}(1)^3 - \frac{1}{3}(1)^2 + \frac{1}{4}(1) - \frac{1}{5}$

$$= \frac{30}{60} - \frac{20}{60} + \frac{15}{60} - \frac{12}{60} = \frac{13}{60}$$

$x = 2$ $\qquad\qquad \frac{1}{2}(2)^3 - \frac{1}{3}(2)^2 + \frac{1}{4}(2) - \frac{1}{5}$

$$= \frac{8}{2} - \frac{4}{3} + \frac{2}{4} - \frac{1}{5} = 4 - \frac{4}{3} + \frac{1}{2} - \frac{1}{5}$$

$$= \frac{120}{30} - \frac{40}{30} + \frac{15}{30} - \frac{6}{30} = \frac{89}{30}$$

$x = 3$ $\qquad\qquad \frac{1}{2}(3)^3 - \frac{1}{3}(3)^2 + \frac{1}{4}(3) - \frac{1}{5} = \frac{27}{2} - \frac{9}{3} + \frac{3}{4} - \frac{1}{5}$

$$= \frac{27}{2} - 3 + \frac{3}{4} - \frac{1}{5} = \frac{270}{20} - \frac{60}{20} + \frac{15}{20} - \frac{4}{20} = \frac{221}{20}$$

$$x = 4 \qquad \frac{1}{2}(4)^3 - \frac{1}{3}(4)^2 + \frac{1}{4}(4) - \frac{1}{5} = \frac{64}{2} - \frac{16}{3} + \frac{4}{4} - \frac{1}{5}$$

$$= 32 - \frac{16}{3} + 1 - \frac{1}{5} = \frac{480}{15} - \frac{80}{15} + \frac{15}{15} - \frac{3}{15} = \frac{412}{15}$$

Write the terms as a sum to expand the summation.

$$-\frac{1}{5} + \frac{13}{60} + \frac{89}{30} + \frac{221}{20} + \frac{412}{15}$$