Topic: Area under or enclosed by the curve

Question: Find the area under the graph of the function over the given interval.

$$\int_{-2}^{2} x^2 + 3 \ dx$$

Answer choices:

A $-\frac{26}{3}$

B $-\frac{52}{3}$

C $\frac{26}{3}$

D $\frac{52}{3}$

Solution: D

Sometimes in calculus we need to distinguish between the area *under* a graph between the area *enclosed* by a graph.

When we talk about the area under the graph, we're talking about net area, which means we treat the area above the x-axis as positive and the area below the x-axis as negative. When we add the two together, the negative area will reduce the positive area. If our result is positive, we know we have more area above the x-axis than below it. On the other hand, if our result if negative, we know we have more area below the x-axis than above it.

In contrast, when we talk about the area enclosed by the graph, we're talking about gross area, which means we treat the area above and below the x-axis as positive. Because of this, any area below the x-axis just increases the total area, and our result will always be positive.

In this particular problem we're asked to find the area under the graph, which just means that we can take the integral of the function over the interval.

$$\int_{-2}^{2} x^2 + 3\ dx$$

$$\left(\frac{x^3}{3} + 3x\right)\Bigg|_{-2}^{2}$$

$$\left[\frac{(2)^3}{3} + 3(2)\right] - \left[\frac{(-2)^3}{3} + 3(-2)\right]$$

$$\frac{26}{3} - \left(\frac{-26}{3}\right)$$

$$\frac{52}{3}$$

Topic: Area under or enclosed by the curve

Question: Find the area enclosed by the graph of the function over the given interval.

$$\int_{-1}^{1} x^3 \, dx$$

Answer choices:

A $\frac{1}{2}$

B $\frac{1}{4}$

C $\frac{3}{4}$

D 0

Solution: A

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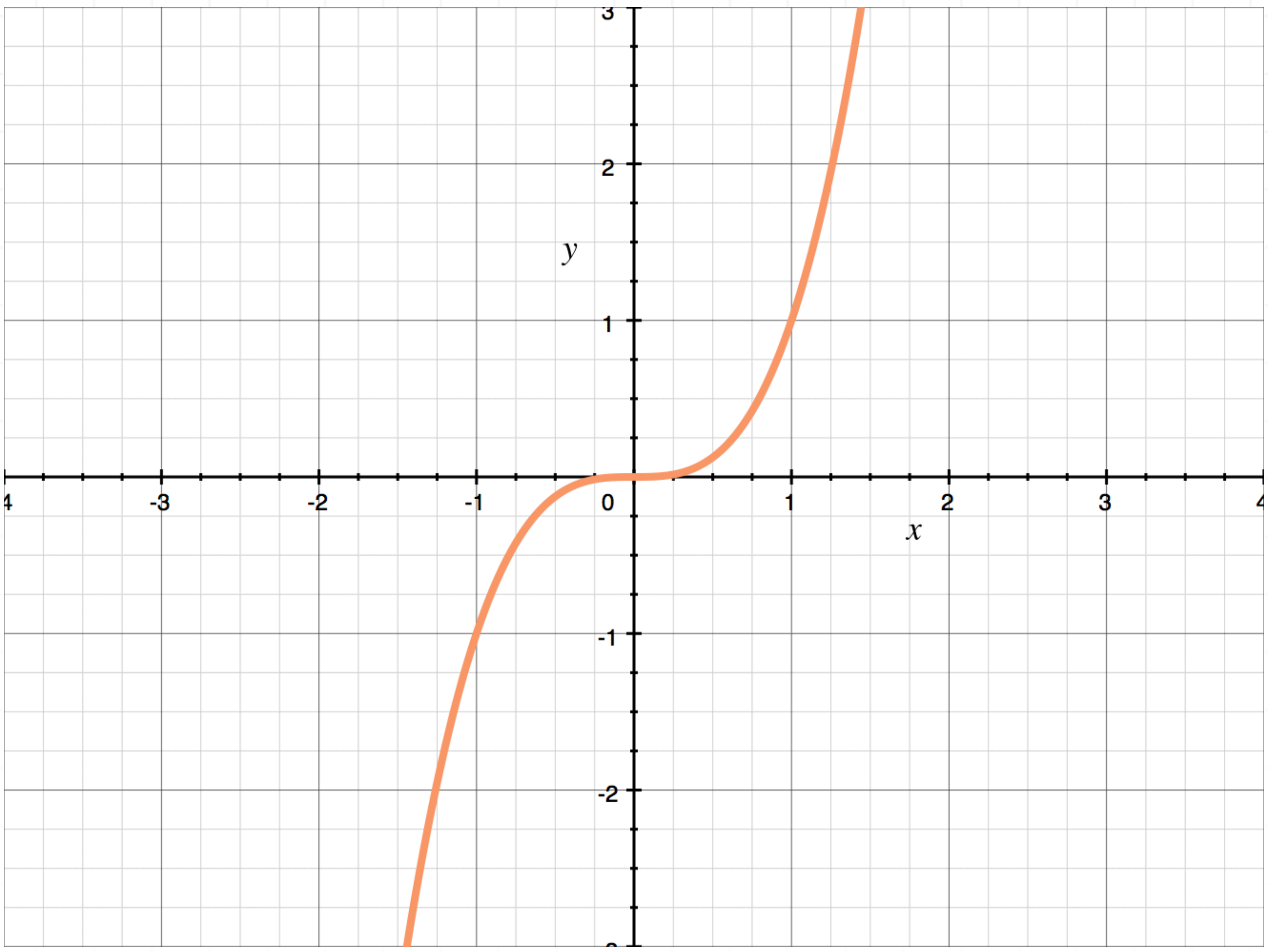
In contrast, when we talk about the area enclosed by the graph, we're talking about gross area, which means we treat the area above and below the x-axis as positive. Because of this, any area below the x-axis just increases the total area, and our result will always be positive.

If we graph the given function,



we can see that from -1 to 0 the function is below the x-axis and from 0 to 1 the function is above the x-axis. We'll break the integral into these two sections.

The area of the first section is

$$\int_{-1}^{0} x^3\ dx = \frac{x^4}{4}\Big|_{-1}^{0}$$

$$\int_{-1}^{0} x^3\ dx = \frac{(0)^4}{4} - \frac{(-1)^4}{4}$$

$$\int_{-1}^{0} x^3\ dx = -\frac{1}{4}$$

The area of the second section is

$$\int_0^1 x^3\,dx = \frac{x^4}{4}\Big|_0^1$$

$$\int_0^1 x^3\,dx = \frac{(1)^4}{4} - \frac{(0)^4}{4}$$

$$\int_0^1 x^3\,dx = \frac{1}{4}$$

Because we're looking for area enclosed by the graph (gross area), we take the absolute value of each area, and then sum the areas together.

$$A = \left|-\frac{1}{4}\right| + \left|\frac{1}{4}\right|$$

$$A = \frac{2}{4}$$

$$A = \frac{1}{2}$$

Topic: Area under or enclosed by the curve

Question: Find the area enclosed by the graph of the function over the given interval.

$$\int_{-\pi}^{2\pi} \sin x \ dx$$

Answer choices:

A -6

B 4

C 6

D -4

Solution: C

Sometimes in calculus we need to distinguish between the area *under* a graph between the area *enclosed* by a graph.

When we talk about the area under the graph, we're talking about net area, which means we treat the area above the x-axis as positive and the area below the x-axis as negative. When we add the two together, the negative area will reduce the positive area. If our result is positive, we know we have more area above the x-axis than below it. On the other hand, if our result if negative, we know we have more area below the x-axis than above it.

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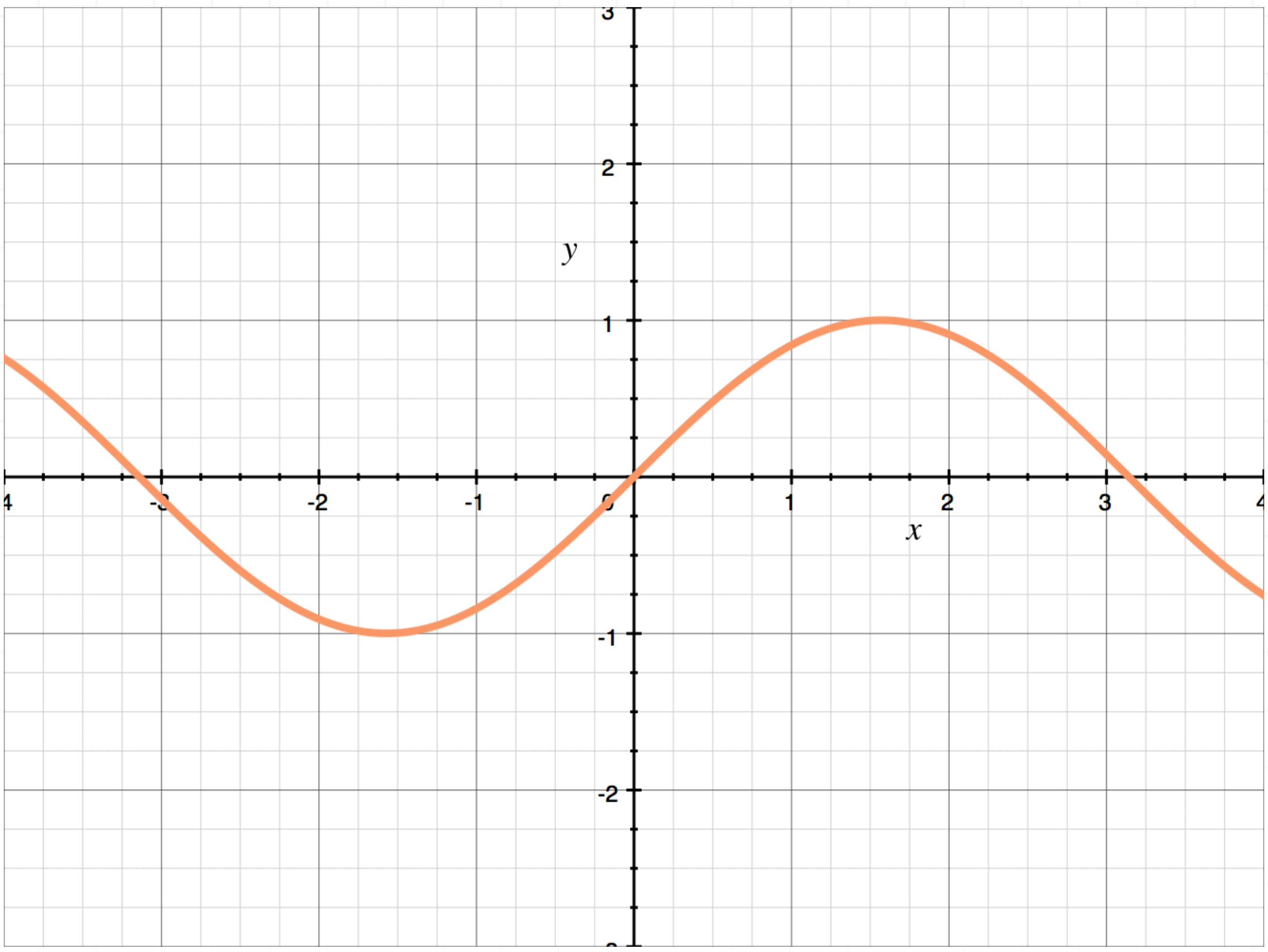
If we graph the given function,



we can see that from $-\pi$ to 0 the function is below the x-axis and from 0 to π the function is above the x-axis. From π to 2π, the function is below the x-axis again. We'll break the integral into these three sections.

The area of the first section is

$$\int_{-\pi}^{0} \sin x \ dx$$

$$-\cos x \Big|_{-\pi}^{0}$$

$$-\cos 0 - \left[-\cos(-\pi)\right]$$

$$-2$$

The area of the second section is

$$\int_0^\pi \sin x \ dx$$

$$-\cos x \Big|_0^\pi$$

$$-\cos \pi - (-\cos 0)$$

$$2$$

The area of the third section is

$$\int_\pi^{2\pi} \sin x \ dx$$

$$-\cos x \Big|_\pi^{2\pi}$$

$$-\cos(2\pi) - (-\cos \pi)$$

$$-2$$

Because we're looking for area enclosed by the graph (gross area), we take the absolute value of each area, and then sum the areas together.

$$A = |-2| + |2| + |-2|$$

$$A = 6$$