**Topic**: Inflection points and the second derivative test

**Question**: Find the function's inflection points.

$$f(x) = x^4 - 6x^2 - 3x + 2$$

**Answer choices**:

A $(1,0)$ and $(-1,-6)$

B $(-1,0)$ and $(1,6)$

C $(-1,0)$ and $(1,-6)$

D $(-1,-6)$ and $(0,1)$

**Solution**: C

Find the second derivative of the function.

$$f(x) = x^4 - 6x^2 - 3x + 2$$

$$f'(x) = 4x^3 - 12x - 3$$

$$f''(x) = 12x^2 - 12$$

Set the second derivative equal to $0$ and solve for $x$.

$$12x^2 - 12 = 0$$

$$12x^2 = 12$$

$$x^2 = 1$$

$$x = \pm 1$$

There are two possible inflection points at $x = -1$ and $x = 1$. Investigate $x = -1$ by testing $x = -2$ and $x = 0$ in the second derivative.

$$f''(-2) = 12(-2)^2 - 12$$

$$f''(-2) = 36$$

and

$$f''(0) = 12(0)^2 - 12$$

$$f''(0) = -12$$

Since $f''(-2) = 36 > 0$, the function is concave up to the left of $x = -1$, and since $f''(0) = -12 < 0$, the function is concave down to the right of $x = -1$.

Because the function changes concavity at $x = -1$ and $f''(x)$ is continuous, there's an inflection point there. We'll get the $y$-coordinate of the inflection point by substituting $x = -1$ into $f(x)$.

$$f(-1) = (-1)^4 - 6(-1)^2 - 3(-1) + 2$$

$$f(-1) = 0$$

The function has an inflection point at $(-1,0)$.

Investigate $x = 1$ by testing $x = 0$ and $x = 2$ into the second derivative.

$$f''(0) = 12(0)^2 - 12$$

$$f''(0) = -12$$

and

$$f''(2) = 12(2)^2 - 12$$

$$f''(2) = 36$$

Since $f''(0) = -12 < 0$, the function is concave down to the left of $x = 1$, and since $f''(2) = 36 > 0$, the function is concave up to the right of $x = 1$.

Because the function changes concavity at $x = 1$ and $f''(x)$ is continuous, there's an inflection point there. We'll get the $y$-coordinate of the inflection point by substituting $x = 1$ into $f(x)$.

$$f(1) = (1)^4 - 6(1)^2 - 3(1) + 2$$

$$f(1) = -6$$

The function has a second inflection point at $(1, -6)$.

**Topic**: Inflection points and the second derivative test

**Question**: Use the Second Derivative Test to classify the critical points at $x = 0$ and $x = 2$.

$$f''(x) = -6x + 6$$

**Answer choices**:

A Relative minimum at $x = 0$; Relative maximum at $x = 2$

B Relative minimum at $x = 2$; Relative maximum at $x = 0$

C Relative minima at $x = 0$ and $x = 2$

D Relative maxima at $x = 0$ and $x = 2$

**Solution**: A

The second derivative is positive at $x = 0$,

$$f''(0) = -6(0) + 6 = 6 > 0$$

so the function is concave up at that critical point, which means there's a relative minimum there.

The second derivative is negative at $x = 2$,

$$f''(2) = -6(2) + 6 = -6 < 0$$

so the function is concave down at that critical point, which means there's a relative maximum there.

**Topic**: Inflection points and the second derivative test

**Question**: Use the second derivative test to find the function's extrema?

$$f(x) = x^2 + x + 4$$

**Answer choices**:

A The function has a local minimum at $x = -1/2$.

B The function has a local maximum at $x = -1/2$.

C The function has a local minimum at $x = 1/2$.

D The function has a local maximum at $x = 1/2$.

**Solution**: A

Take the first derivative.

$$f(x) = x^2 + x + 4$$

$$f'(x) = 2x + 1$$

Set the derivative equal to $0$ and solve for $x$.

$$2x + 1 = 0$$

$$2x = -1$$

$$x = -\frac{1}{2}$$

There's one critical point at $x = -1/2$. Take the second derivative.

$$f'(x) = 2x + 1$$

$$f''(x) = 2$$

Substitute the critical point $x = -1/2$ into the second derivative.

$$f''\left(-\frac{1}{2}\right) = 2$$

Because the second derivative is positive at the critical point, it means there's a local minimum at $x = -1/2$.