Topic: Geometric series test

Question: Use the geometric series test to say whether the geometric series converges or diverges.

$$\sum_{n=0}^{\infty} \frac{1^{n-1}}{2^n}$$

Answer choices:

A The series is convergent and $r = \frac{1}{4}$.

B The series is divergent and $r = 1$.

C The series is convergent and $r = \frac{1}{2}$.

D The series is divergent and $r = 2$.

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Solution: C

We need to get the series into standard form for a geometric series to make sure the series is geometric. Since the index starts at $n = 0$, standard form is

$$\sum_{n=0}^{\infty} ar^n$$

so we'll rewrite the series as

$$\sum_{n=0}^{\infty} \frac{1^{n-1}}{2^n}$$

$$\sum_{n=0}^{\infty} \frac{1^n 1^{-1}}{2^n}$$

$$\sum_{n=0}^{\infty} 1^{-1} \cdot \frac{1^n}{2^n}$$

$$\sum_{n=0}^{\infty} \frac{1}{1} \left(\frac{1}{2}\right)^n$$

$$\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n$$

Comparing this to the standard form, we'll say that

$$a = 1$$

and

$$r = \frac{1}{2}$$

We'll use the geometric series test to determine whether this geometric series converges or diverges. Since

$$\left|\frac{1}{2}\right| = \frac{1}{2} < 1$$

we can say that $|r| < 1$ and therefore that the series converges.

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Topic: Geometric series test

Question: Use the geometric series test to say whether the geometric series converges or diverges.

$$\sum_{n=0}^{\infty} \frac{8^{n-1}}{2^n}$$

Answer choices:

A The series is convergent and $r = \frac{1}{2}$.

B The series is divergent and $r = 2$.

C The series is convergent and $r = \frac{1}{8}$.

D The series is divergent and $r = 4$.

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Solution: D

We need to get the series into standard form for a geometric series to make sure the series is geometric. Since the index starts at $n = 0$, standard form is

$$\sum_{n=0}^{\infty} ar^n$$

so we'll rewrite the series as

$$\sum_{n=0}^{\infty} \frac{8^{n-1}}{2^n}$$

$$\sum_{n=0}^{\infty} \frac{8^n 8^{-1}}{2^n}$$

$$\sum_{n=0}^{\infty} 8^{-1} \cdot \frac{8^n}{2^n}$$

$$\sum_{n=0}^{\infty} \frac{1}{8} \left(\frac{8}{2}\right)^n$$

$$\sum_{n=0}^{\infty} \frac{1}{8} (4)^n$$

Comparing this to the standard form, we'll say that

$$a = \frac{1}{8}$$

and

$$r = 4$$

We'll use the geometric series test to determine whether this geometric series converges or diverges. Since

$$|4| = 4 \geq 1$$

we can say that $|r| \geq 1$ and therefore that the series diverges.

Topic: Geometric series test

Question: Use the geometric series test to say whether the geometric series converges or diverges.

$$\sum_{n=0}^{\infty} \frac{(-1)^{n-1}}{3^n}$$

Answer choices:

A The series is convergent and $r = -\frac{1}{3}$.

B The series is divergent and $r = -1$.

C The series is convergent and $r = \frac{1}{3}$.

D The series is divergent and $r = 1$.

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Solution: A

We need to get the series into standard form for a geometric series to make sure the series is geometric. Since the index starts at $n = 0$, standard form is

$$\sum_{n=0}^{\infty} ar^n$$

so we'll rewrite the series as

$$\sum_{n=0}^{\infty} \frac{(-1)^{n-1}}{3^n}$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n(-1)^{-1}}{3^n}$$

$$\sum_{n=0}^{\infty} (-1)^{-1} \cdot \frac{(-1)^n}{3^n}$$

$$\sum_{n=0}^{\infty} \frac{1}{(-1)} \left(\frac{-1}{3}\right)^n$$

$$\sum_{n=0}^{\infty} -\left(-\frac{1}{3}\right)^n$$

Comparing this to the standard form, we'll say that

$$a = -1$$

and

$$r = -\frac{1}{3}$$

We'll use the geometric series test to determine whether this geometric series converges or diverges. Since

$$\left| -\frac{1}{3} \right| = \frac{1}{3} < 1$$

we can say that $|r| < 1$ and therefore that the series converges.

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