Topic: Radius of convergence

Question: Find the radius of convergence of the series.

$$\sum_{n=1}^{\infty} \frac{(-1)^n n(x-1)^n}{2^n}$$

Answer choices:

A $\quad 2$

B $\quad 1$

C $\quad \frac{1}{2}$

D $\quad -1$

Solution: A

We can use the ratio test for convergence to find the radius of convergence of a series. The ratio test tells us that, for a series a_n, if

$$L=\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|$$

then the series converges absolutely if $L<1$. Therefore, we'll find the value of L for the given series, plug it into $L<1$, and then solve for the variable.

In order to get L, we'll need a_n and a_{n+1}.

$$a_n=\frac{(-1)^n n(x-1)^n}{2^n}$$

$$a_{n+1}=\frac{(-1)^{n+1}(n+1)(x-1)^{n+1}}{2^{n+1}}$$

Plugging these into the formula for L, we get

$$L=\lim_{n\to\infty}\left|\frac{\frac{(-1)^{n+1}(n+1)(x-1)^{n+1}}{2^{n+1}}}{\frac{(-1)^n n(x-1)^n}{2^n}}\right|$$

$$L=\lim_{n\to\infty}\left|\frac{(-1)^{n+1}(n+1)(x-1)^{n+1}}{2^{n+1}}\cdot\frac{2^n}{(-1)^n n(x-1)^n}\right|$$

Pairing similar numerators and denominators together, we get

$$L=\lim_{n\to\infty}\left|\frac{2^n}{2^{n+1}}\cdot\frac{(-1)^{n+1}(n+1)(x-1)^{n+1}}{(-1)^n n(x-1)^n}\right|$$

$$L=\lim_{n\to\infty}\left|\frac{2^n}{2^{n+1}}\cdot\frac{(-1)^{n+1}}{(-1)^n}\cdot\frac{(x-1)^{n+1}}{(x-1)^n}\cdot\frac{n+1}{n}\right|$$

$$L=\lim_{n\to\infty}\left|2^{n-(n+1)}\cdot(-1)^{n+1-n}\cdot(x-1)^{n+1-n}\cdot\frac{n+1}{n}\right|$$

$$L=\lim_{n\to\infty}\left|2^{n-n-1}\cdot(-1)\cdot(x-1)\cdot\frac{n+1}{n}\right|$$

$$L=\lim_{n\to\infty}\left|2^{-1}\cdot(-1)\cdot(x-1)\cdot\frac{n+1}{n}\right|$$

$$L=\lim_{n\to\infty}\left|\frac{1}{2}\cdot(-1)\cdot(x-1)\cdot\frac{n+1}{n}\right|$$

$$L=\lim_{n\to\infty}\left|\frac{-(x-1)}{2}\cdot\frac{n+1}{n}\right|$$

Since the limit only effects n, we can pull x out in front of the limit, as long as we keep it inside absolute value brackets.

$$L=\left|\frac{x-1}{2}\right|\lim_{n\to\infty}\left|-\frac{n+1}{n}\right|$$

$$L=\left|\frac{x-1}{2}\right|\lim_{n\to\infty}\left|-\frac{n+1}{n}\left(\frac{\frac{1}{n}}{\frac{1}{n}}\right)\right|$$

$$L=\left|\frac{x-1}{2}\right|\lim_{n\to\infty}\left|-\frac{\frac{n}{n}+\frac{1}{n}}{\frac{n}{n}}\right|$$

$$L=\left|\frac{x-1}{2}\right|\lim_{n\to\infty}\left|-\frac{1+\frac{1}{n}}{1}\right|$$

$$L=\left|\frac{x-1}{2}\right|\lim_{n\to\infty}\left|-1-\frac{1}{n}\right|$$

$$L=\left|\frac{x-1}{2}\right|\left|-1-\frac{1}{\infty}\right|$$

$$L=\left|\frac{x-1}{2}\right|\left|-1-0\right|$$

$$L=\left|\frac{x-1}{2}\right|\left|-1\right|$$

$$L=\left|\frac{x-1}{2}\right|(1)$$

$$L=\left|\frac{x-1}{2}\right|$$

The ratio test tells us that the series converges when $L<1$. Plugging L into this inequality, we get

$$\left|\frac{x-1}{2}\right|<1$$

$$\frac{1}{2}\left|x-1\right| < 1$$

$$\left|x-1\right| < 2$$

With the interval of convergence in the form $|x-a| < R$, the radius of convergence is R. Therefore, we can say that the radius of convergence is $R = 2$.

Topic: Radius of convergence

Question: Find the radius of convergence of the series.

$$\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{3^n}$$

Answer choices:

A $\quad -\frac{1}{3}$

B $\quad -3$

C $\quad 3$

D $\quad \frac{1}{3}$

Solution: C

We can use the root test for convergence to find the radius of convergence of a series. The root test tells us that, for a series a_n, if

$$L = \lim_{n\to\infty} \sqrt[n]{\left|a_n\right|}$$

then the series converges absolutely if $L < 1$. Therefore, we'll find the value of L for the given series, plug it into $L < 1$, and then solve for the variable.

Plugging a_n into the formula for L, we get

$$L = \lim_{n\to\infty} \sqrt[n]{\left|\frac{(-1)^n x^n}{3^n}\right|}$$

$$L = \lim_{n\to\infty} \sqrt[n]{\left|\left(\frac{-x}{3}\right)^n\right|}$$

$$L = \lim_{n\to\infty} \left|\left(\frac{-x}{3}\right)^n\right|^{\frac{1}{n}}$$

$$L = \lim_{n\to\infty} \left|\left(\frac{-x}{3}\right)^{n\cdot\frac{1}{n}}\right|$$

$$L = \lim_{n\to\infty} \left|\frac{-x}{3}\right|$$

Since there are no ns remaining, and the limit only effects n, we can eliminate the limit.

$$L = \left| \frac{-x}{3} \right|$$

$$L = \left| \frac{x}{3} \right|$$

The root test tells us that the series converges when $L < 1$. Plugging L into this inequality, we get

$$\left| \frac{x}{3} \right| < 1$$

$$\frac{1}{3} |x| < 1$$

$$|x| < 3$$

$$|x - 0| < 3$$

With the interval of convergence in the form $|x - a| < R$, the radius of convergence is R. Therefore, we can say that the radius of convergence is $R = 3$.

Topic: Radius of convergence

Question: Find the radius of convergence of the series.

$$\sum_{n=1}^{\infty} \frac{(-1)^n n(x+1)^n}{5^n}$$

Answer choices:

A 1

B 5

C $\frac{1}{5}$

D -1

Solution: B

We can use the ratio test for convergence to find the radius of convergence of a series. The ratio test tells us that, for a series a_n, if

$$L = \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right|$$

then the series converges absolutely if $L < 1$. Therefore, we'll find the value of L for the given series, plug it into $L < 1$, and then solve for the variable.

In order to get L, we'll need a_n and a_{n+1}.

$$a_n = \frac{(-1)^n n(x+1)^n}{5^n}$$

$$a_{n+1} = \frac{(-1)^{n+1}(n+1)(x+1)^{n+1}}{5^{n+1}}$$

Plugging these into the formula for L, we get

$$L = \lim_{n \to \infty} \left| \frac{\frac{(-1)^{n+1}(n+1)(x+1)^{n+1}}{5^{n+1}}}{\frac{(-1)^n n(x+1)^n}{5^n}} \right|$$

$$L = \lim_{n \to \infty} \left| \frac{(-1)^{n+1}(n+1)(x+1)^{n+1}}{5^{n+1}} \cdot \frac{5^n}{(-1)^n n(x+1)^n} \right|$$

Pairing similar numerators and denominators together, we get

$$L = \lim_{n \to \infty} \left| \frac{5^n}{5^{n+1}} \cdot \frac{(-1)^{n+1}(n+1)(x+1)^{n+1}}{(-1)^n n(x+1)^n} \right|$$

$$L=\lim_{n\to\infty}\left|\frac{5^n}{5^{n+1}}\cdot\frac{(-1)^{n+1}}{(-1)^n}\cdot\frac{(x+1)^{n+1}}{(x+1)^n}\cdot\frac{n+1}{n}\right|$$

$$L=\lim_{n\to\infty}\left|5^{n-(n+1)}\cdot(-1)^{n+1-n}\cdot(x+1)^{n+1-n}\cdot\frac{n+1}{n}\right|$$

$$L=\lim_{n\to\infty}\left|5^{n-n-1}\cdot(-1)\cdot(x+1)\cdot\frac{n+1}{n}\right|$$

$$L=\lim_{n\to\infty}\left|5^{-1}\cdot(-1)\cdot(x+1)\cdot\frac{n+1}{n}\right|$$

$$L=\lim_{n\to\infty}\left|\frac{1}{5}\cdot(-1)\cdot(x+1)\cdot\frac{n+1}{n}\right|$$

$$L=\lim_{n\to\infty}\left|\frac{-(x+1)}{5}\cdot\frac{n+1}{n}\right|$$

Since the limit only effects n, we can pull x out in front of the limit, as long as we keep it inside absolute value brackets.

$$L=\left|\frac{x+1}{5}\right|\lim_{n\to\infty}\left|-\frac{n+1}{n}\right|$$

$$L=\left|\frac{x+1}{5}\right|\lim_{n\to\infty}\left|-\frac{n+1}{n}\left(\frac{\frac{1}{n}}{\frac{1}{n}}\right)\right|$$

$$L=\left|\frac{x+1}{5}\right|\lim_{n\to\infty}\left|-\frac{\frac{n}{n}+\frac{1}{n}}{\frac{n}{n}}\right|$$

$$L=\left|\frac{x+1}{5}\right|\lim_{n\to\infty}\left|-\frac{1+\frac{1}{n}}{1}\right|$$

$$L=\left|\frac{x+1}{5}\right|\lim_{n\to\infty}\left|-1-\frac{1}{n}\right|$$

$$L=\left|\frac{x+1}{5}\right|\left|-1-\frac{1}{\infty}\right|$$

$$L=\left|\frac{x+1}{5}\right|\left|-1-0\right|$$

$$L=\left|\frac{x+1}{5}\right|\left|-1\right|$$

$$L=\left|\frac{x+1}{5}\right|(1)$$

$$L=\left|\frac{x+1}{5}\right|$$

The ratio test tells us that the series converges when $L<1$. Plugging L into this inequality, we get

$$\left|\frac{x+1}{5}\right|<1$$

$$\frac{1}{5}\left|x+1\right| < 1$$

$$\left|x+1\right| < 5$$

$$\left|x-(-1)\right| < 5$$

With the interval of convergence in the form $|x-a| < R$, the radius of convergence is R. Therefore, we can say that the radius of convergence is $R = 5$.