

Consumer and producer surplus

Consumer and producer surplus are values that a company can calculate to see when they have excess demand or production. If a company can better balance demand and production, they can be more profitable.

Consumer surplus is calculated using

$$CS = \int_0^{q_e} D(q) \, dq - p_e q_e$$

where CS is consumer surplus, $D(q)$ is the demand curve, p_e is the equilibrium price and q_e is the equilibrium quantity.

Producer surplus is calculated using

$$PS = p_e q_e - \int_0^{q_e} S(q) \, dq$$

where PS is producer surplus, $S(q)$ is the supply curve, p_e is the equilibrium price and q_e is the equilibrium quantity.

Example

Find equilibrium quantity and price, and then consumer and producer surplus.

$$D(q) = -0.25q + 13$$

$$S(q) = 0.05q^2 - 2$$



In order to find the equilibrium quantity, we need to remember that our system will achieve equilibrium when supply equals demand. In other words, if we set the supply curve equal to the demand curve, the resulting q value will be the equilibrium quantity q_e .

$$-0.25q + 13 = 0.05q^2 - 2$$

$$0 = 0.05q^2 + 0.25q - 15$$

$$0 = 5q^2 + 25q - 1500$$

$$0 = (5q + 100)(q - 15)$$

Setting each factor equal to 0 separately, we get

$$5q + 100 = 0$$

$$q = -20$$

or

$$q - 15 = 0$$

$$q = 15$$

Since the equilibrium quantity must be positive, $q_e = 15$ is the equilibrium quantity for the given demand and supply curves $D(q) = -0.25q + 13$ and $S(q) = 0.05q^2 - 2$.

Now we can solve for the equilibrium price p_e . We can find the equilibrium price by plugging equilibrium quantity into either the demand or supply



curve (they will both give us the same answer). Let's use the supply curve $S(q) = 0.05q^2 - 2$.

$$S(15) = 0.05(15)^2 - 2$$

$$S(15) = 9.25$$

The equilibrium price p_e for the demand curve $D(q) = -0.25q + 13$ and the supply curve $S(q) = 0.05q^2 - 2$ is $p_e = 9.25$, and that's the answer to the first part of the question.

To solve for consumer surplus, we'll plug the demand curve, plus the equilibrium price and quantity into the consumer surplus formula, and get

$$CS = \int_0^{15} -0.25q + 13 \, dq - (9.25)(15)$$

$$CS = \int_0^{15} -0.25q + 13 \, dq - 138.75$$

$$CS = \left(\frac{-0.25q^2}{2} + 13q \right) \Big|_0^{15} - 138.75$$

$$CS = (-0.125q^2 + 13q) \Big|_0^{15} - 138.75$$

$$CS = -0.125(15)^2 + 13(15) - [-0.125(0)^2 + 13(0)] - 138.75$$

$$CS = 28.125$$

The consumer surplus is 28.125.



Now we can solve for the producer surplus by plugging the supply curve and the equilibrium price and quantity into the producer surplus equation.

$$PS = (9.25)(15) - \int_0^{15} 0.05q^2 - 2 \, dq$$

$$PS = 138.75 - \int_0^{15} 0.05q^2 - 2 \, dq$$

$$PS = 138.75 - \left(\frac{0.05q^3}{3} - 2q \right) \Big|_0^{15}$$

$$PS = 138.75 - (0.017q^3 - 2q) \Big|_0^{15}$$

$$PS = 138.75 - \left[0.017(15)^3 - 2(15) - [0.017(0)^3 - 2(0)] \right]$$

$$PS = 111.375$$

The producer surplus is 111.375.

