Calculus 1 Workbook

Definition of the limit

krista king
MATH

IDEA OF THE LIMIT

■ 1. The table below shows some values of a function $g(x)$. What does the table show for the value of $\lim_{x \to 4} g(x)$?

x	$g(x)$
3.9	1.9748
3.99	1.9975
3.999	1.9997
4.001	2.0002
4.01	2.0025
4.1	2.0248

■ 2. How would we express, mathematically, the limit of the function $f(x) = x^2 - x + 2$ as x approaches 3?

■ 3. How would you write the limit of $g(x)$ as x approaches ∞, using correct mathematical notation?

$$g(x) = \frac{5x^2 - 7}{3x^2 + 8}$$

4. Explain what is meant by the equation.

$$\lim_{x \to -2} (x^3 + 2) = -6$$

5. Evaluate the limit.

$$\lim_{x \to -1} \frac{-x^2 + 3x - 1}{5}$$

6. Evaluate the limit.

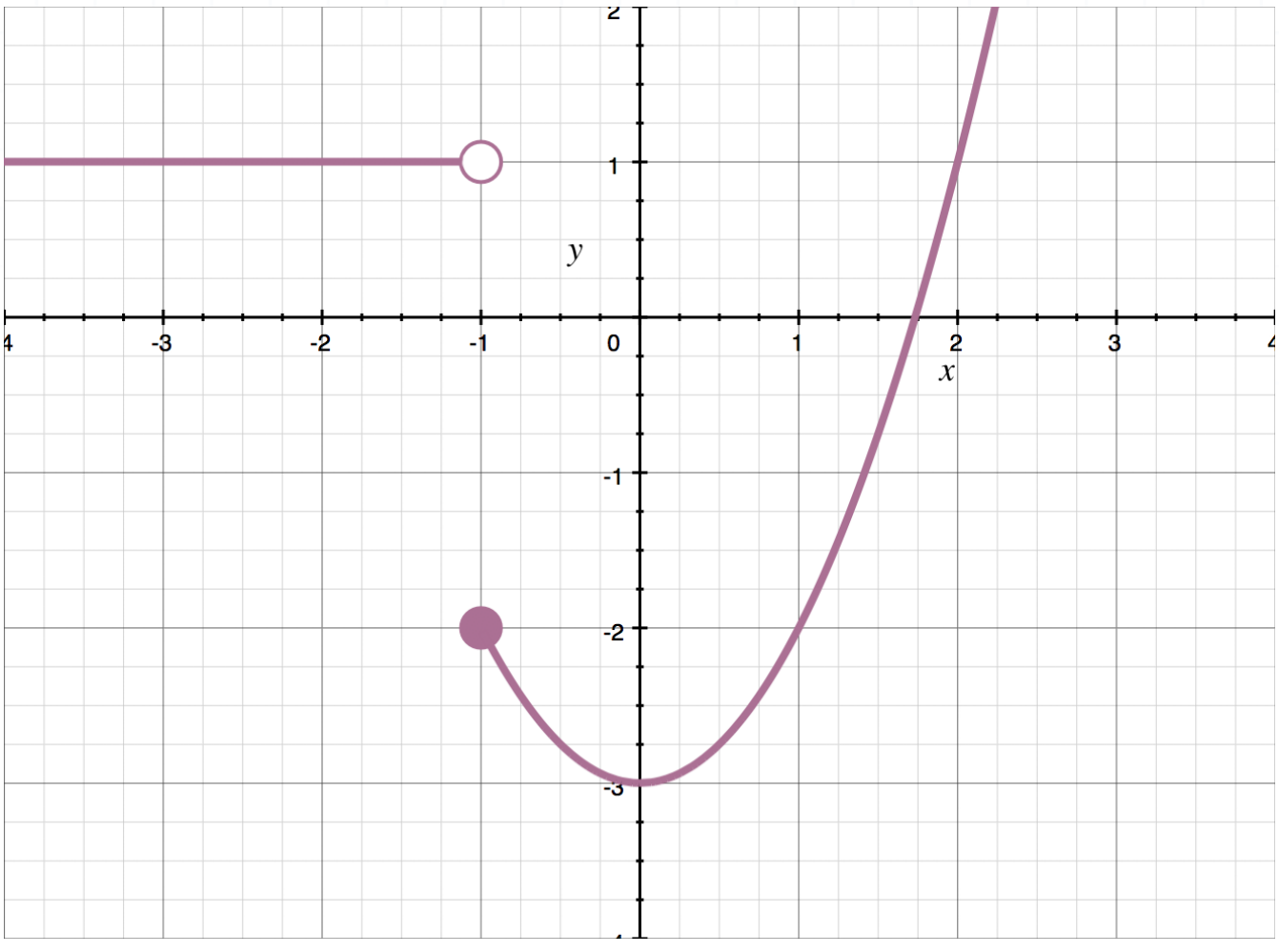
$$\lim_{x \to 0} \frac{x^2 - 5}{2}$$

ONE-SIDED LIMITS

■ 1. Find the limit.

$$\lim_{x \to -7^+} x^2\sqrt{x+7}$$

■ 2. What does the graph of $f(x)$ say about the value of $\lim_{x \to -1^+} f(x)$?



■ 3. The table shows values of $k(x)$. What is $\lim_{x \to -5^-} k(x)$?

x	-5.1	-5.01	-5.0001	-5	-4.999	-4.99	-4.9
k(x)	-392.1	-3,812	-38,012	?	37,988	3,788	368.1

■ 4. What is $\lim_{x \to -2^-} h(x)$?

$$h(x) = \begin{cases} -2x - 1 & x < -2 \\ x & -2 \leq x < 2 \\ 2x - 3 & x \geq 2 \end{cases}$$

■ 5. What is $\lim_{x \to 6^+} g(x)$?

$$g(x) = \frac{x^2 + x - 42}{x - 6}$$

■ 6. Find the left- and right-hand limits of the function at $x = 3$.

$$f(x) = \frac{|x - 3|}{x - 3}$$

PROVING THAT THE LIMIT DOES NOT EXIST

■ 1. Prove that the limit does not exist.

$$\lim_{x\to 0}\frac{-2|3x|}{3x}$$

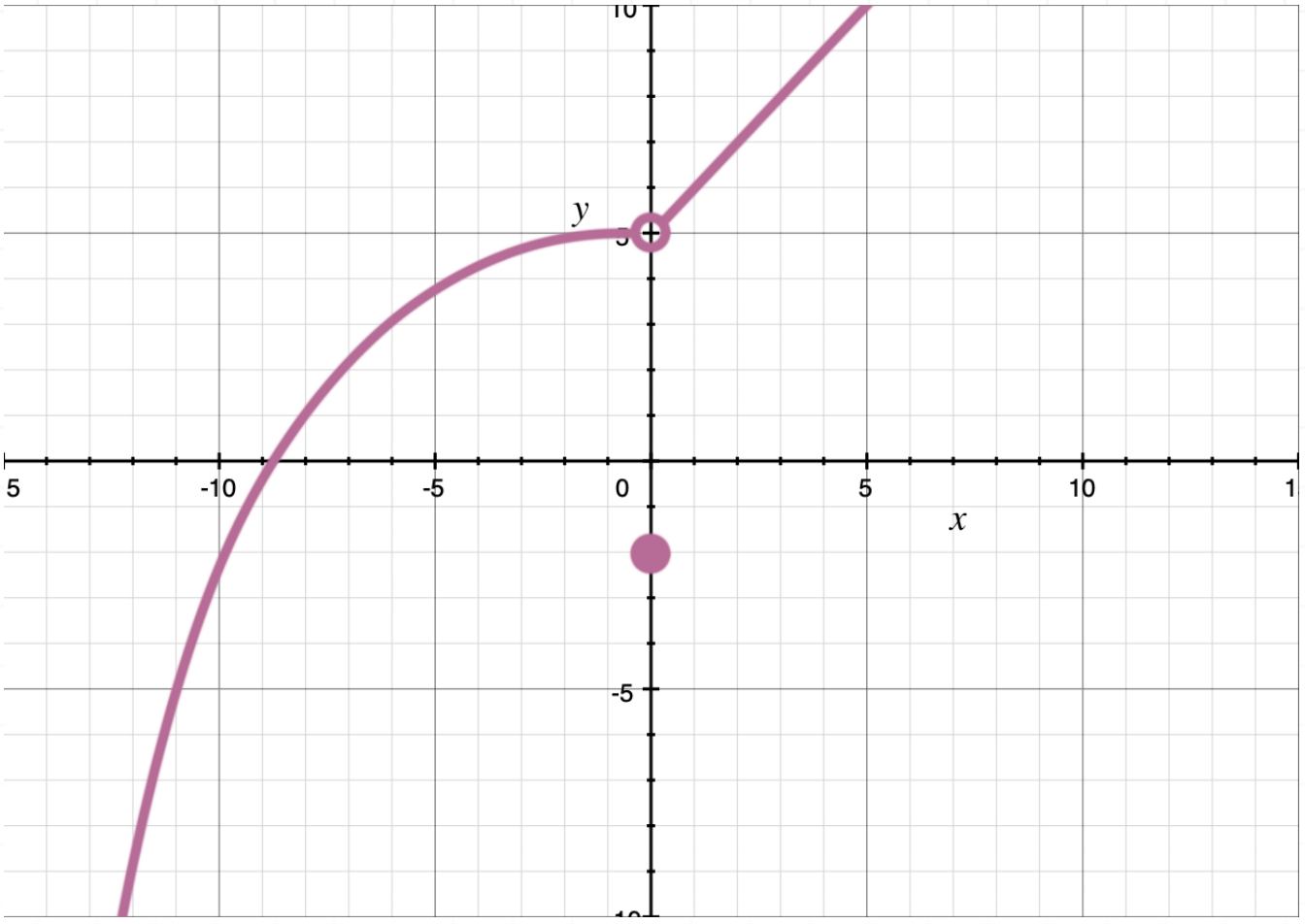
■ 2. Prove that the limit does not exist.

$$\lim_{x\to -5}\frac{x^2+7x+9}{x^2-25}$$

■ 3. Prove that $\lim_{x\to 1} f(x)$ does not exist.

$$f(x)=\begin{cases}-3x+2 & x<1\\ 3x-2 & x\geq 1\end{cases}$$

■ 4. Use the graph to determine whether or not the limit exists at $x=0$.



■ 5. Suppose we know that $\lim_{x \to 5} f(x) = 12$. If possible, determine the values of the one-sided limits.

$$\lim_{x \to 5^-} f(x)$$

$$\lim_{x \to 5^+} f(x)$$

■ 6. Prove that the limit does not exist.

$$\lim_{x \to -2} \frac{x^2 - 4}{(x+2)^2}$$

PRECISE DEFINITION OF THE LIMIT

■ 1. Use the precise definition of the limit to prove the value of the limit.

$$\lim_{x \to 4}(5x - 16) = 4$$

■ 2. Use the precise definition of the limit to prove the value of the limit.

$$\lim_{x \to -7}(-2x + 15) = 29$$

■ 3. Use the precise definition of the limit to prove the value of the limit.

$$\lim_{x \to 16}\left(\frac{2}{5}x - \frac{17}{5}\right) = 3$$

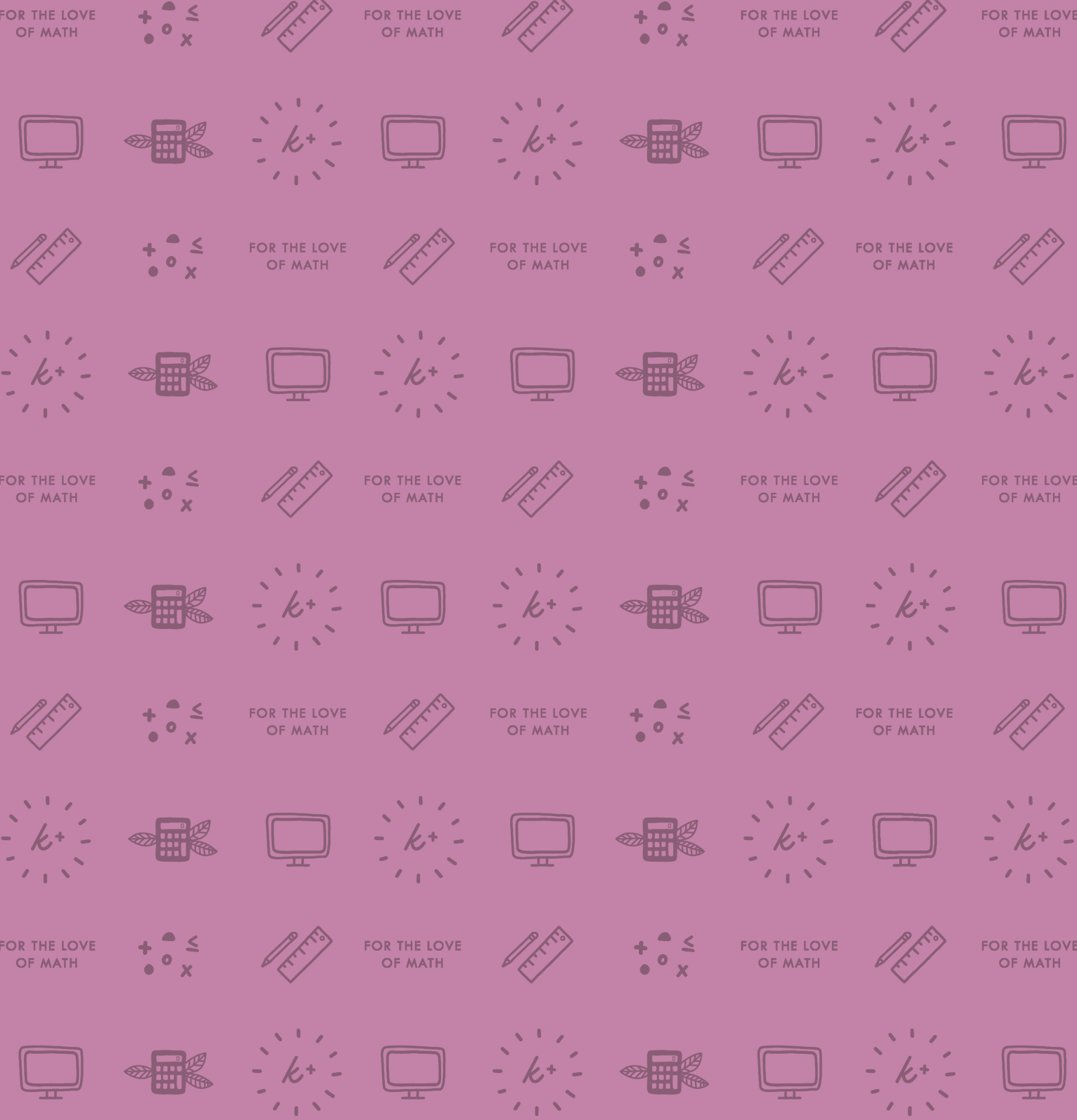
■ 4. Use the precise definition of the limit to prove the value of the limit.

$$\lim_{x \to 7}\frac{x^2 - 15x + 56}{x - 7} = -1$$

■ 5. Find δ when $f(x) = 2x - 5$, such that if $0 < |x - 1| < \delta$ then $|f(x) + 3| < 0.1$.

6. Find a value of δ given $\epsilon = 0.04$.

$$\lim_{x \to 2}(x-2)^2 = 0$$



WWW.KRISTAKINGMATH.COM