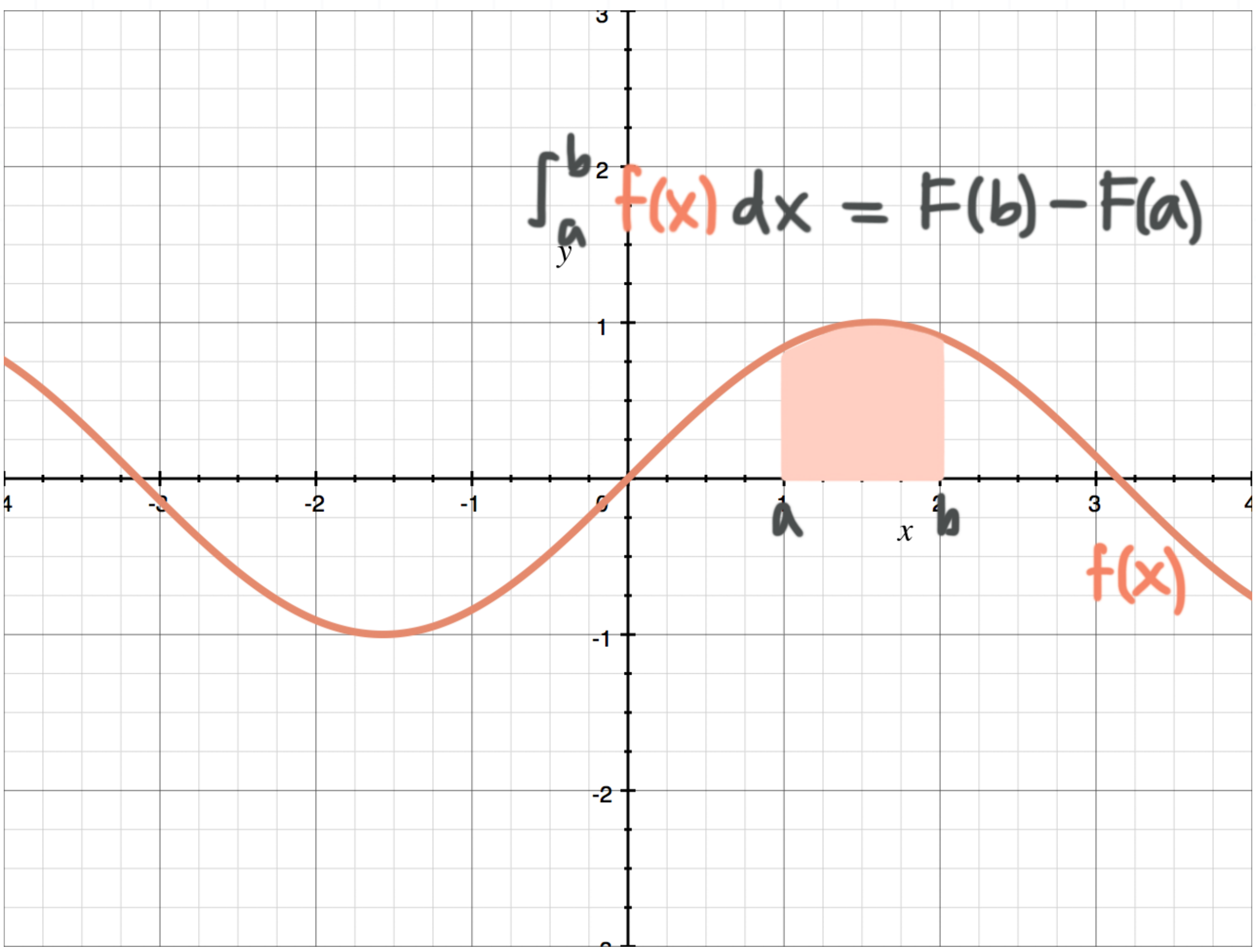
Definite integrals

Evaluating a definite integral means finding the area enclosed by the graph of the function and the x-axis, over the given interval $[a, b]$.

In the graph below, the shaded area is the integral of $f(x)$ on the interval $[a, b]$. Finding this area means taking the integral of $f(x)$, plugging the upper limit b into the result, and then subtracting from that whatever you get when you plug in the lower limit a.



Let's do an example where we evaluate a definite integral.

Example

Evaluate the integral.

$$\int_0^2 3x^2 - 5x + 2\ dx$$

If we let $f(x) = 3x^2 - 5x + 2$ and then integrate the polynomial, we get

$$F(x) = \left(x^3 - \frac{5}{2}x^2 + 2x + C\right)\Bigg|_0^2$$

where C is the constant of integration.

Evaluating on the interval $[0,2]$, we get

$$F(x) = \left[(2)^3 - \frac{5}{2}(2)^2 + 2(2) + C\right] - \left[(0)^3 - \frac{5}{2}(0)^2 + 2(0) + C\right]$$

$$F(x) = (8 - 10 + 4 + C) - (0 - 0 + 0 + C)$$

$$F(x) = 8 - 10 + 4 + C - C$$

$$F(x) = 2$$

As you can see, the constant of integration "cancels out" in the end, leaving a definite value as the final answer, not just a function for y defined in terms of x.

Since this will always be the case, you can just leave C out of your answer whenever you're solving a definite integral.

So, what do we mean when we say $F(x) = 2$? What does this value represent? When we say that $F(x) = 2$, it means that the area

1. below the graph of $f(x)$,

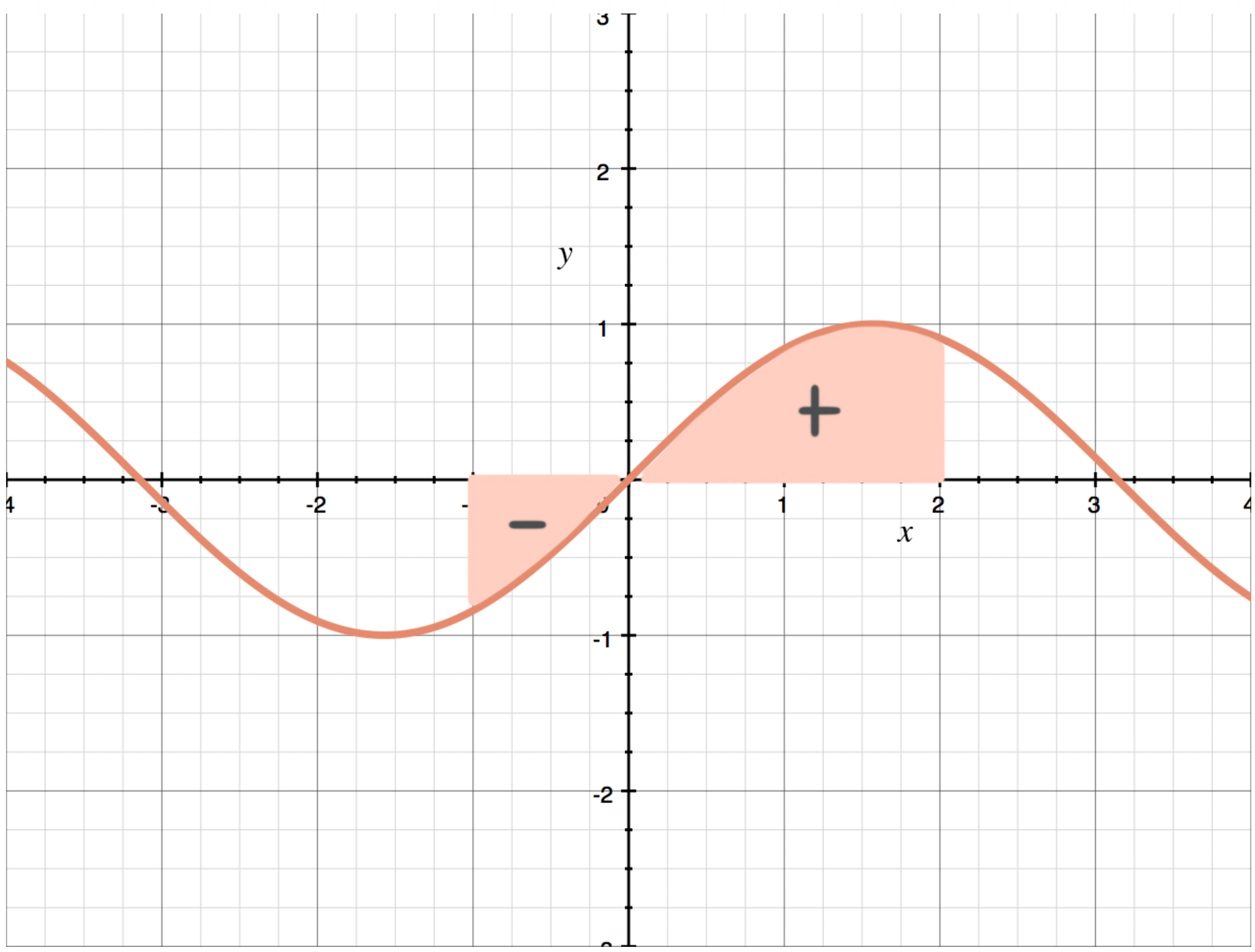
2. above the x-axis, and

3. between the lines $x = 0$ and $x = 2$

is 2 square units.

Keep in mind that we're talking about the area *enclosed* by the graph and the x-axis. If $f(x)$ drops below the x-axis inside $[a, b]$, we treat the area under the x-axis as negative area.

Then finding the value of $F(x)$ means subtracting the area enclosed by the graph under the x-axis from the area enclosed by the graph above the x-axis.



In other words, evaluating the definite integral of $f(x) = \sin x$ on $[-1,2]$ means subtracting the area enclosed by the graph below the x-axis from the area enclosed by the graph above the x-axis.

This means that, if the area enclosed by the graph below the x-axis is larger than the area enclosed by the graph above the x-axis, then the value of $F(x)$ will be negative ($F(x) < 0$).

If the area enclosed by the graph below the x-axis is exactly equal to the area enclosed by the graph above the x-axis, then $F(x) = 0$.