Topic: Logarithmic differentiation

Question: Use logarithmic differentiation to find the derivative.

$$y = 3^{5x}$$

Answer choices:

A $y' = 15^{5x}(\ln 3)$

B $y' = 3^{4x}(5 \ln 3)$

C $y' = 3^{5x}(5 \ln 3)$

D $y' = 3^{5x}(\ln 15)$

Solution: C

Apply the natural log to both sides of the equation.

$$y = 3^{5x}$$

$$\ln y = \ln(3^{5x})$$

Use laws of logs to rewrite the equation.

$$\ln y = 5x \ln 3$$

Take the derivative of both sides of the equation and remembering to multiply by y' when we take the derivative of y.

$$\frac{1}{y}y' = 5 \ln 3$$

Solve for y' in terms of x by isolating y' and substituting for y.

$$y' = 5y \ln 3$$

$$y' = 5(3^{5x})\ln 3$$

$$y' = 3^{5x}(5 \ln 3)$$

Topic: Logarithmic differentiation

Question: Use logarithmic differentiation to find the derivative.

$$y = 3\ln(5x)$$

Answer choices:

A $y' = \frac{5}{x}$

B $y' = \frac{3}{x}$

C $y' = \frac{3}{5x}$

D $y' = \frac{15}{x}$

Solution: B

Apply the natural log to both sides of the equation.

$$y = 3\ln(5x)$$

$$\ln y = \ln(3\ln(5x))$$

Use laws of logs to rewrite the equation.

$$\ln y = \ln 3 + \ln(\ln(5x))$$

Take the derivative of both sides of the equation, remembering to multiply by y' when we take the derivative of y.

$$\frac{1}{y}y' = 0 + \frac{1}{\ln(5x)}\left(\frac{1}{5x}(5)\right)$$

$$\frac{1}{y}y' = \frac{1}{x\ln(5x)}$$

Solve for y' in terms of x by isolating y' and substituting for y.

$$y' = \frac{y}{x\ln(5x)}$$

$$y' = \frac{3\ln(5x)}{x\ln(5x)}$$

$$y' = \frac{3}{x}$$

Topic: Logarithmic differentiation

Question: Use logarithmic differentiation to find the derivative.

$$y = \ln(6x^2 - 3x + 9)^{4x}$$

Answer choices:

A $y' = 4\ln(6x^2 - 3x + 9)$

B $y' = \dfrac{4x(4x - 1)}{2x^2 - x + 3}$

C $y' = 4\ln(6x^2 - 3x + 9) + \dfrac{4x(4x - 1)}{2x^2 - x + 3}$

D $y' = 4\ln(6x^2 - 3x + 9) + \dfrac{1}{6x^2 - 3x + 9}$

Solution: C

Apply the natural log to both sides of the equation.

$$y = \ln(6x^2 - 3x + 9)^{4x}$$

$$\ln y = \ln(\ln(6x^2 - 3x + 9)^{4x})$$

Use laws of logs to rewrite the equation.

$$\ln y = \ln(4x \ln(6x^2 - 3x + 9))$$

$$\ln y = \ln(4x) + \ln(\ln(6x^2 - 3x + 9))$$

Take the derivative of both sides of the equation, remembering to multiply by y' when we take the derivative of y.

$$\frac{1}{y}y' = \frac{1}{4x}(4) + \frac{1}{\ln(6x^2 - 3x + 9)}\left(\frac{1}{6x^2 - 3x + 9}(12x - 3)\right)$$

$$\frac{1}{y}y' = \frac{1}{x} + \frac{1}{\ln(6x^2 - 3x + 9)}\left(\frac{12x - 3}{6x^2 - 3x + 9}\right)$$

$$\frac{1}{y}y' = \frac{1}{x} + \frac{12x - 3}{(6x^2 - 3x + 9)\ln(6x^2 - 3x + 9)}$$

Solve for y' in terms of x by isolating y' and substituting for y.

$$y' = y\left[\frac{1}{x} + \frac{12x - 3}{(6x^2 - 3x + 9)\ln(6x^2 - 3x + 9)}\right]$$

$$y' = \ln(6x^2 - 3x + 9)^{4x}\left[\frac{1}{x} + \frac{12x - 3}{(6x^2 - 3x + 9)\ln(6x^2 - 3x + 9)}\right]$$

$$y' = \frac{\ln(6x^2 - 3x + 9)^{4x}}{x} + \frac{(12x - 3)\ln(6x^2 - 3x + 9)^{4x}}{(6x^2 - 3x + 9)\ln(6x^2 - 3x + 9)}$$

Use laws of logarithms to simplify the derivative.

$$y' = \frac{4x\ln(6x^2 - 3x + 9)}{x} + \frac{4x(12x - 3)\ln(6x^2 - 3x + 9)}{(6x^2 - 3x + 9)\ln(6x^2 - 3x + 9)}$$

$$y' = 4\ln(6x^2 - 3x + 9) + \frac{4x(12x - 3)}{6x^2 - 3x + 9}$$

$$y' = 4\ln(6x^2 - 3x + 9) + \frac{12x(4x - 1)}{3(2x^2 - x + 3)}$$

$$y' = 4\ln(6x^2 - 3x + 9) + \frac{4x(4x - 1)}{2x^2 - x + 3}$$