Topic: Converting rectangular equations

Question: Convert the rectangular equation to a polar equation.

$$x^2 + y^2 - 4y = 0$$

Answer choices:

A $r = 4 + 4\sin\theta$

B $r = 4\cos\theta$

C $r = 4 - 4\sin\theta$

D $r = 4\sin\theta$

Solution: D

Using the equations

$$x = r\cos\theta$$

$$y = r\sin\theta$$

$$r^2 = x^2 + y^2$$

we can convert the equation from rectangular coordinates to polar coordinates.

For this particular problem, we'll use $x = r\cos\theta$ and $y = r\sin\theta$, even though we could just as easily use $r^2 = x^2 + y^2$.

$$x^2 + y^2 - 4y = 0$$

$$(r\cos\theta)^2 + (r\sin\theta)^2 - 4(r\sin\theta) = 0$$

$$r^2\cos^2\theta + r^2\sin^2\theta - 4r\sin\theta = 0$$

$$r^2\left(\cos^2\theta + \sin^2\theta\right) - 4r\sin\theta = 0$$

$$r^2(1) - 4r\sin\theta = 0$$

$$r^2 - 4r\sin\theta = 0$$

$$r - 4\sin\theta = 0$$

$$r = 4\sin\theta$$

Topic: Converting rectangular equations

Question: Convert the rectangular equation to a polar equation.

$$x^2 + y^2 = x$$

Answer choices:

A $r = -\sin\theta$

B $r = \sin\theta$

C $r = -\cos\theta$

D $r = \cos\theta$

Solution: D

Using the equations

$$x = r\cos\theta$$

$$y = r\sin\theta$$

$$r^2 = x^2 + y^2$$

we can convert the equation from rectangular coordinates to polar coordinates.

For this particular problem, we'll use $r^2 = x^2 + y^2$ to change the left-hand side.

$$r^2 = x$$

Now we'll use $x = r\cos\theta$ to change the right-hand side.

$$r^2 = r\cos\theta$$

$$r = \cos\theta$$

Topic: Converting rectangular equations

Question: Convert the rectangular equation to a polar equation.

$$2x^2 + 2y^2 = -4y$$

Answer choices:

A $r = -4\sin\theta$

B $r = -2\sin\theta$

C $r = 4\sin\theta$

D $r = 2\sin\theta$

Solution: B

Using the equations

$$x = r\cos\theta$$

$$y = r\sin\theta$$

$$r^2 = x^2 + y^2$$

we can convert the equation from rectangular coordinates to polar coordinates.

For this particular problem, we'll divide through by 2 to simplify the equation, then use $r^2 = x^2 + y^2$ to change the left-hand side.

$$x^2 + y^2 = -2y$$

$$r^2 = -2y$$

Now we'll use $y = r\sin\theta$ to change the right-hand side.

$$r^2 = -2r\sin\theta$$

$$r = -2\sin\theta$$