Topic: Volume of revolution of a parametric curve

Question: Find the volume of revolution of the parametric curve.

$$x = 3e^{3t} - 9t$$

$$y = 12e^{\frac{3t}{2}}$$

$$1 \le t \le 2$$

about the x-axis

Answer choices:

A $\quad 216\pi e^3\left(e^9 - 3e^3 + 2\right)$

B $\quad 216\pi e^3\left(e^9 + 3e^3 - 2\right)$

C $\quad 342\pi\left(e^{12} - 3e^6 + 2e^3\right)$

D $\quad 342\pi\left(e^{12} + 3e^6 + 2e^3\right)$

Solution: A

Since we're rotating around the x-axis, we'll use the formula

$$V_x = \int_{\alpha}^{\beta} \pi y^2 \frac{dx}{dt}\ dt$$

The problem gave the interval $1 \le t \le 2$, so $\alpha = 1$ and $\beta = 2$. Now we need to find dx/dt so that we can plug it into the volume formula.

$$x = 3e^{3t} - 9t$$

$$\frac{dx}{dt} = 9e^{3t} - 9$$

Plugging everything into the volume formula, we get

$$V_x = \int_1^2 \pi \left(12e^{\frac{3t}{2}}\right)^2 \left(9e^{3t} - 9\right)\ dt$$

$$V_x = 1{,}296\pi \int_1^2 \left(e^{\frac{3t}{2}}\right)^2 \left(e^{3t} - 1\right)\ dt$$

$$V_x = 1{,}296\pi \int_1^2 e^{3t}\left(e^{3t} - 1\right)\ dt$$

$$V_x = 1{,}296\pi \int_1^2 e^{6t} - e^{3t}\ dt$$

$$V_x = 1{,}296\pi \left(\frac{1}{6}e^{6t} - \frac{1}{3}e^{3t}\right)\Bigg|_1^2$$

$$V_x = 1{,}296\pi \left[\left(\frac{1}{6}e^{6(2)} - \frac{1}{3}e^{3(2)} \right) - \left(\frac{1}{6}e^{6(1)} - \frac{1}{3}e^{3(1)} \right) \right]$$

$$V_x = 1{,}296\pi \left[\frac{e^{12}}{6} - \frac{e^6}{3} - \left(\frac{e^6}{6} - \frac{e^3}{3} \right) \right]$$

$$V_x = 1{,}296\pi \left(\frac{e^{12}}{6} - \frac{e^6}{3} - \frac{e^6}{6} + \frac{e^3}{3} \right)$$

$$V_x = 1{,}296\pi \left(\frac{e^{12}}{6} - \frac{2e^6}{6} - \frac{e^6}{6} + \frac{2e^3}{6} \right)$$

$$V_x = \frac{1{,}296}{6}\pi \left(e^{12} - 2e^6 - e^6 + 2e^3 \right)$$

$$V_x = 216\pi \left(e^{12} - 3e^6 + 2e^3 \right)$$

$$V_x = 216\pi e^3 \left(e^9 - 3e^3 + 2 \right)$$

Topic: Volume of revolution of a parametric curve

Question: Find the volume of revolution of the parametric curve.

$$x = t^2$$

$$y = 3t^2$$

$$0 \le t \le 1$$

about the x-axis

Answer choices:

A 3π

B 2π

C -3π

D -2π

Solution: A

Since we're rotating around the x-axis, we'll use the formula

$$V_x = \int_{\alpha}^{\beta} \pi y^2 \frac{dx}{dt}\ dt$$

The problem gave the interval $0 \le t \le 1$, so $\alpha = 0$ and $\beta = 1$. Now we need to find dx/dt so that we can plug it into the volume formula.

$$x = t^2$$

$$\frac{dx}{dt} = 2t$$

Plugging everything into the volume formula, we get

$$V_x = \int_0^1 \pi \left(3t^2\right)^2 (2t)\ dt$$

$$V_x = 18\pi \int_0^1 t^5\ dt$$

$$V_x = 18\pi \left(\frac{1}{6}t^6\right)\Bigg|_0^1$$

$$V_x = 3\pi t^6 \Big|_0^1$$

$$V_x = 3\pi(1)^6 - 3\pi(0)^6$$

$$V_x = 3\pi$$

Topic: Volume of revolution of a parametric curve

Question: Find the volume of revolution of the parametric curve.

$$x = 4t^2$$

$$y = t^2 + 1$$

on the interval $0 \le t \le 1$

about the y-axis

Answer choices:

A $-\frac{16\pi}{3}$

B 6π

C $\frac{16\pi}{3}$

D -6π

Solution: C

Since we're rotating around the y-axis, we'll use the formula

$$V_y = \int_{\alpha}^{\beta} \pi x^2 \frac{dy}{dt}\ dt$$

The problem gave the interval $0 \le t \le 1$, so $\alpha = 0$ and $\beta = 1$. Now we need to find dy/dt so that we can plug it into the volume formula.

$$y = t^2 + 1$$

$$\frac{dy}{dt} = 2t$$

Plugging everything into the volume formula, we get

$$V_y = \int_0^1 \pi\left(4t^2\right)^2(2t)\ dt$$

$$V_y = 32\pi \int_0^1 t^5\ dt$$

$$V_y = 32\pi \left(\frac{1}{6}t^6\right)\Bigg|_0^1$$

$$V_y = \frac{16\pi}{3}t^6\Bigg|_0^1$$

$$V_y = \frac{16\pi}{3}(1)^6 - \frac{16\pi}{3}(0)^6$$

$$V_y = \frac{16\pi}{3}$$