Topic: Improper integrals, case 4

Question: Evaluate the improper integral.

$$\int_{-3}^{5} \frac{6}{x-5}\, dx$$

Answer choices:

A $6\ln 8$

B ∞

C $-\infty$

D $-6\ln 8$

Solution: C

The integral in this problem is considered to be an improper integral, case 4, because the integrand is undefined at the upper limit limit of integration. Evaluating this type of improper integral follows this general rule:

$$\int_a^b f(x)\ dx = \lim_{c \to b^-} \int_a^c f(x)\ dx$$

Let's begin by re-writing the integral using this rule.

$$\int_{-3}^{5} \frac{6}{x-5}\ dx = \lim_{c \to 5^-} \int_{-3}^{c} \frac{6}{x-5}\ dx$$

$$6 \lim_{c \to 5^-} \int_{-3}^{c} \frac{1}{x-5}\ dx$$

Integrate.

$$6 \lim_{c \to 5^-} \ln \left| x-5 \right| \Big|_{-3}^{c}$$

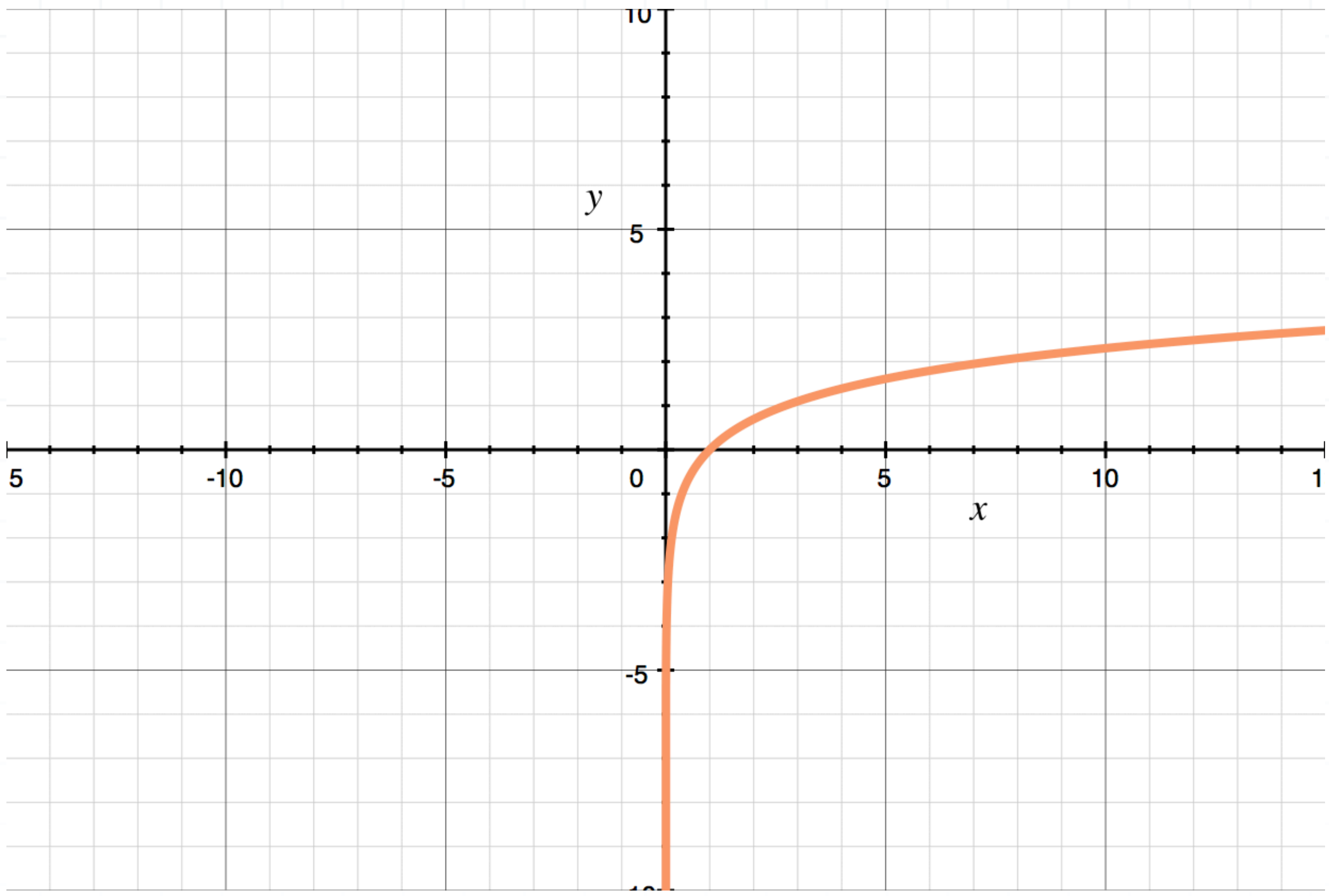
Evaluate over the interval.

$$6 \lim_{c \to 5^-} \left[\ln \left| c-5 \right| - \ln \left| -3-5 \right| \right]$$

$$6 \lim_{c \to 5^-} \left[\ln \left| c-5 \right| - \ln 8 \right]$$

$$6 \ln \left| 5-5 \right| - 6 \ln 8$$

When we look at $\ln|5-5| = \ln 0$, we know that $\ln 0$ is undefined. If we look at the graph of the natural logarithm, we can see that the value approaches $-\infty$.



Therefore, we can evaluate the limit using the graph.

$$6\ln 0 - 6\ln 8$$

$$-\infty - 6\ln 8$$

$$-\infty$$

Topic: Improper integrals, case 4

Question: Evaluate the improper integral.

$$\int_{-4}^{6} \frac{x-8}{x^2-14x+48}\, dx$$

Answer choices:

A ∞

B $-\infty$

C $\ln 10$

D $-\ln 10$

Solution: B

The integral in this problem is considered to be an improper integral, case 4, because the integrand is undefined at the upper limit limit of integration. Evaluating this type of improper integral follows this general rule:

$$\int_a^b f(x)\ dx = \lim_{c \to b^-} \int_a^c f(x)\ dx$$

Since the denominator in the given integrand is factorable, let's factor it and see if the integrand can be simplified.

$$\int_{-4}^{6} \frac{x-8}{x^2-14x+48}\ dx = \int_{-4}^{6} \frac{x-8}{(x-6)(x-8)}\ dx$$

$$\int_{-4}^{6} \frac{\cancel{x-8}}{(x-6)\cancel{(x-8)}}\ dx$$

$$\int_{-4}^{6} \frac{1}{x-6}\ dx$$

Now let's begin by re-writing the integral using the rule above.

$$\lim_{c \to 6^-} \int_{-4}^{c} \frac{1}{x-6}\ dx$$

Integrate.

$$\lim_{c \to 6^-} \ln|x-6| \Big|_{-4}^{c}$$

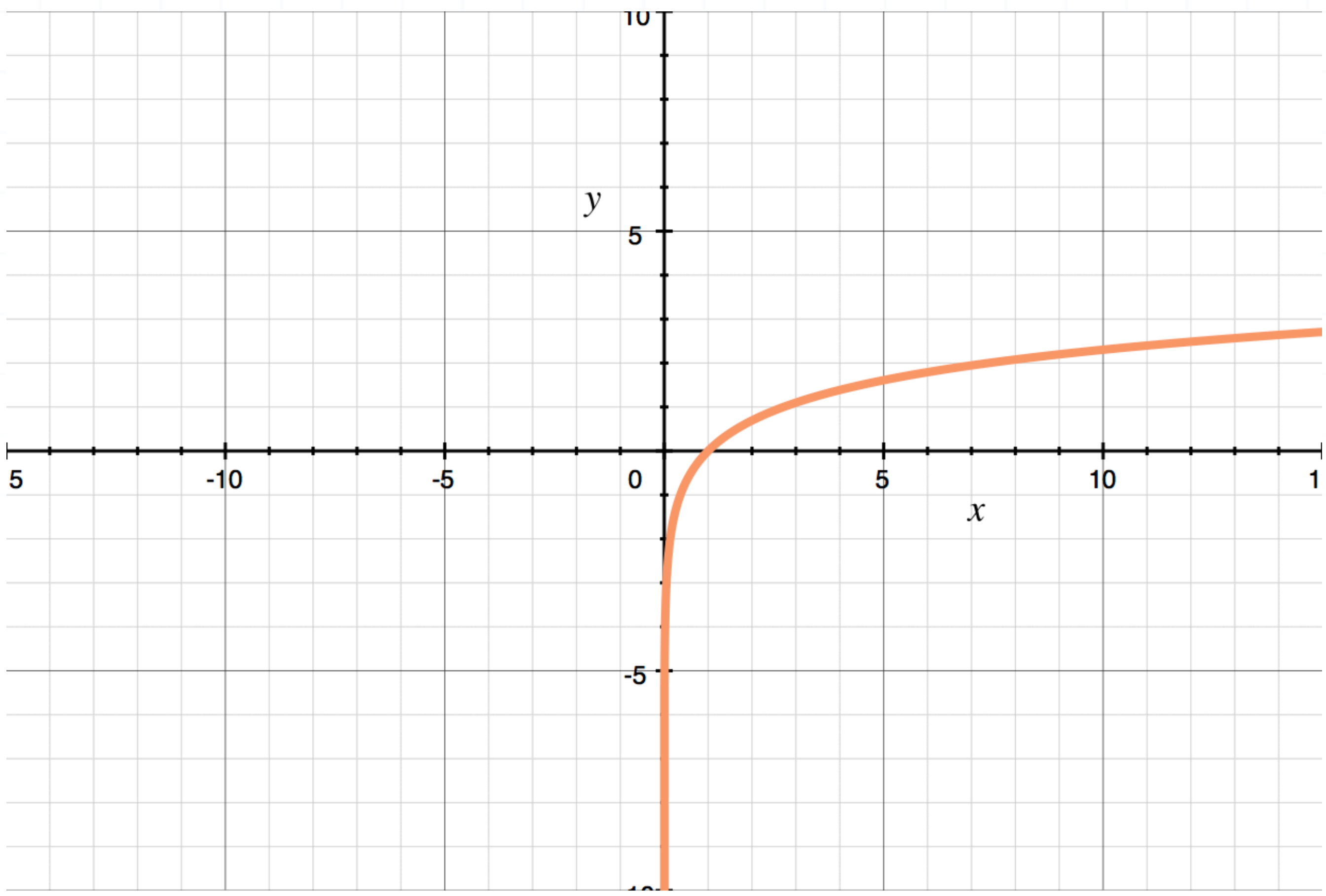
Evaluate over the interval.

$$\lim_{c \to 6^-} \left[\ln|c-6| - \ln|-4-6| \right]$$

$$\lim_{c \to 6^-} \left[\ln|c-6| - \ln 10 \right]$$

$$\ln|6-6| - \ln 10$$

When we look at $\ln|6-6| = \ln 0$, we know that $\ln 0$ is undefined. If we look at the graph of the natural logarithm, we can see that the value approaches $-\infty$.



Therefore, we can evaluate the limit using the graph.

$$-\infty - \ln 10$$

$$-\infty$$

Topic: Improper integrals, case 4

Question: Evaluate the improper integral.

$$\int_{-7}^{7} \frac{15}{7-x}\, dx$$

Answer choices:

A ∞

B $-\infty$

C $15 \ln 14$

D $-15 \ln 14$

Solution: A

The integral in this problem is considered to be an improper integral, case 4, because the integrand is undefined at the upper limit limit of integration. Evaluating this type of improper integral follows this general rule:

$$\int_a^b f(x)\ dx = \lim_{c\to b^-} \int_a^c f(x)\ dx$$

Let's begin by re-writing the integral using the rule.

$$\int_{-7}^{7} \frac{15}{7-x}\ dx = \lim_{c\to 7^-} \int_{-7}^{c} \frac{15}{7-x}\ dx$$

Integrate, remembering that chain rule tells us to multiply by -1, since the derivative of $7-x$ is -1.

$$-15 \lim_{c\to 7^-} \ln|7-x| \Big|_{-7}^{c}$$

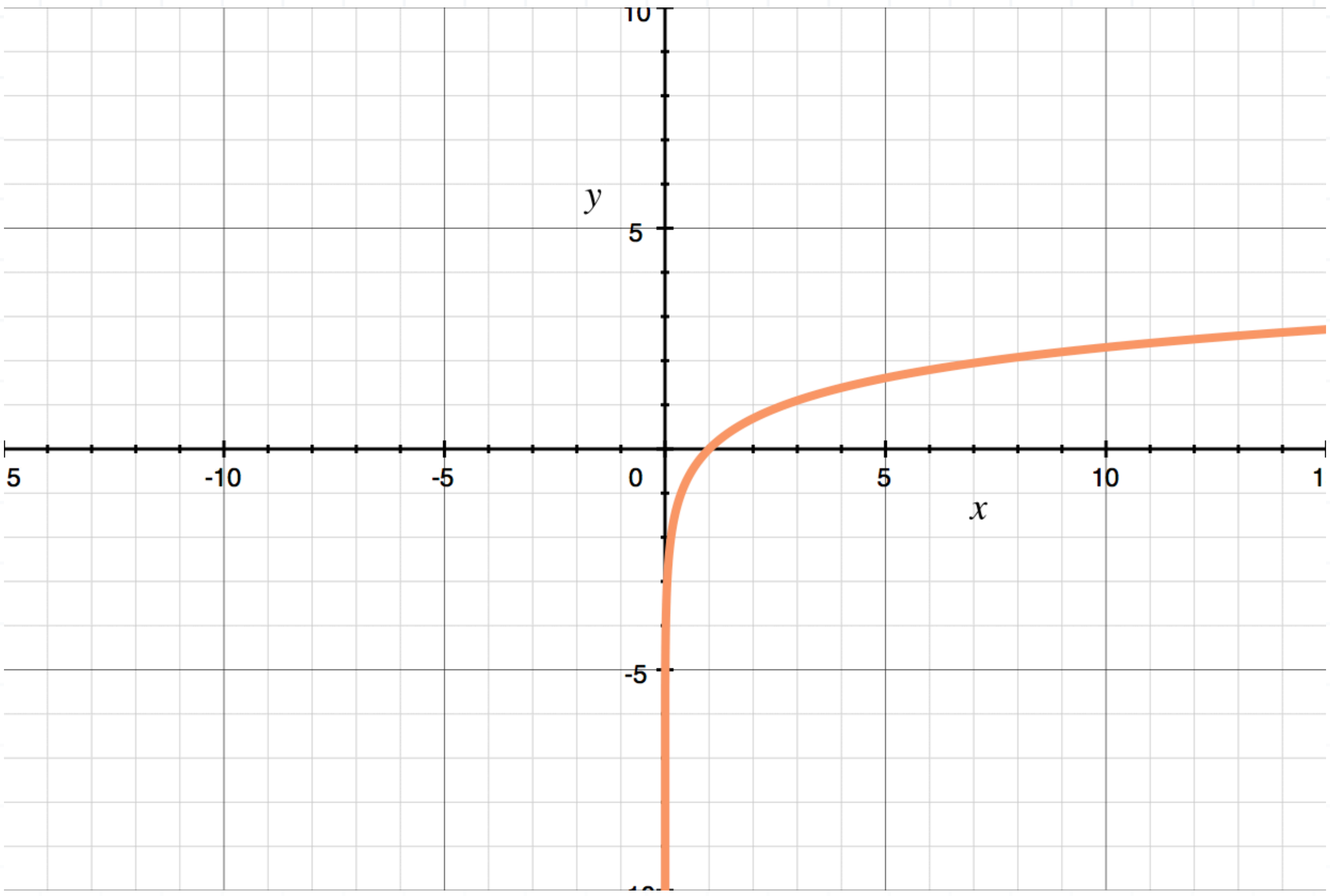
Evaluate over the interval.

$$-15 \lim_{c\to 7^-} \left[\ln|7-c| - \ln|7-(-7)|\right]$$

$$-15 \lim_{c\to 7^-} \left[\ln|7-c| - \ln 14\right]$$

$$-15 \left[\ln|7-7| - \ln 14\right]$$

When we look at $\ln|7-7| = \ln 0$, we know that $\ln 0$ is undefined. If we look at the graph of the natural logarithm, we can see that the value approaches $-\infty$.



Therefore, we can evaluate the limit using the graph.

$$-15\left[-\infty - \ln 14\right]$$

$$\infty$$