Coin dropped from the roof

In the previous lesson, we were looking at a vertical motion pattern in which we threw an object up from the ground, or some other height, and the object traveled upward, eventually reached a maximum height, and then fell back down to earth, eventually stopping when it hits the ground.

In this lesson, we're looking at a different vertical motion pattern. This time, we're dropping a coin, or some other other object, from a height, letting it fall straight to the ground, eventually stopping when it hits the ground.

Let's work through an example of how to find different values from a position function that models this pattern of vertical motion.

Example

A watermelon is dropped from the top of a building 28 meters high. Find instantaneous velocity at $t = 2$, average velocity between $t = 0$ and $t = 2$, and find the time when the watermelon hits the ground.

Plugging everything we know into the formula for standard projectile motion, we get

$$x(t) = -\frac{1}{2}gt^2 + v_0 t + y_0$$

$$x(t) = -\frac{1}{2}(9.8)t^2 + 0t + 28$$

$$x(t) = -4.9t^2 + 28$$

Find the velocity function by differentiating the position function.

$$v(t) = x'(t) = -9.8t$$

To find instantaneous velocity at $t = 2$, substitute $t = 2$ into the velocity function.

$$v(4) = -9.8(2)$$

$$v(4) = -19.6$$

This is the instantaneous velocity at $t = 2$. Find average velocity over $t = [0,2]$.

$$\Delta v(a, b) = \frac{x(b) - x(a)}{b - a}$$

$$\Delta v(0,2) = \frac{x(2) - x(0)}{2 - 0}$$

$$\Delta v(0,2) = \frac{-4.9(2)^2 + 28 - (-4.9(0)^2 + 28)}{2}$$

$$\Delta v(0,2) = \frac{-19.6 + 28 - 28}{2}$$

$$\Delta v(0,2) = \frac{-19.6}{2}$$

$$\Delta v(0,2) = -9.8$$

This is the average velocity of the watermelon between $t = 0$ and $t = 2$.

The watermelon will hit the ground when $x(t) = 0$, so we'll set the position function equal to 0 and then solve for t.

$$-4.9t^2 + 28 = 0$$

$$4.9t^2 = 28$$

$$t^2 = \frac{28}{4.9}$$

$$t \approx \sqrt{5.71}$$

$$t \approx 2.39$$

The watermelon hits the ground after $t \approx 2.39$ seconds.