Topic: Polar coordinates

Question: Convert the polar coordinates to rectangular coordinates.

$$\left(2, \frac{11}{6}\pi\right)$$

Answer choices:

A $\left(\sqrt{3}, -1\right)$

B $\left(\sqrt{3}, 1\right)$

C $\left(\sqrt{3}, -\frac{1}{2}\right)$

D $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

Solution: A

Using the equations $x = r\cos\theta$ and $y = r\sin\theta$, the rectangular coordinates are

$$x = 2\cos\frac{11}{6}\pi$$

$$y = 2\sin\frac{11}{6}\pi$$

From the unit circle, we know that the sine and cosine of $11\pi/6$ are $-1/2$ and $\sqrt{3}/2$ respectively. Therefore,

$$x = 2\left(\frac{\sqrt{3}}{2}\right)$$

$$x = \sqrt{3}$$

and

$$y = 2\sin\frac{11}{6}\pi$$

$$y = 2\left(-\frac{1}{2}\right)$$

$$y = -1$$

The polar coordinate $\left(2, \frac{11}{6}\pi\right)$ is equal to the rectangular coordinate $\left(\sqrt{3}, -1\right)$.

Topic: Polar coordinates

Question: What are the measures of θ if $r = 4$, given that an ellipse is defined by

$$\frac{1}{r^2} = \frac{\cos^2\theta}{9} + \frac{\sin^2\theta}{16}$$

Answer choices:

A $\theta = \frac{\pi}{2}$ and $\theta = \frac{2\pi}{3}$

B $\theta = \frac{\pi}{2}$ and $\theta = \frac{3\pi}{2}$

C $\theta = \arccos\left(\pm\frac{\sqrt{35}}{10}\right)$

D $\theta = \arccos\left(\pm\frac{\sqrt{35}}{5}\right)$

Solution: B

We were asked to use $r = 4$, so we'll plug that into the equation for the ellipse.

$$\frac{1}{r^2} = \frac{\cos^2\theta}{9} + \frac{\sin^2\theta}{16}$$

$$\frac{1}{4^2} = \frac{\cos^2\theta}{9} + \frac{\sin^2\theta}{16}$$

$$1 = \frac{16}{9}\cos^2\theta + \sin^2\theta$$

$$9 = 16\cos^2\theta + 9\sin^2\theta$$

$$9 = 16\left(1 - \sin^2\theta\right) + 9\sin^2\theta$$

$$9 = 16 - 16\sin^2\theta + 9\sin^2\theta$$

$$-7 = -7\sin^2\theta$$

$$1 = \sin^2\theta$$

$$\pm 1 = \sin\theta$$

$$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$$

Topic: Polar coordinates

Question: What is the length of r for the graph of the polar curve?

$$r = \left(\sin^6\theta + \cos^6\theta\right) - \left(\sin^4\theta + \cos^4\theta\right) - \sin^4\theta + \sin^2\theta + 1$$

Answer choices:

A $r = 9$

B $r = 6$

C $r = 4$

D $r = 1$

Solution: D

Using trigonometric identities to simplify the function.

$$r = \left(\sin^6\theta + \cos^6\theta\right) - \left(\sin^4\theta + \cos^4\theta\right) - \sin^4\theta + \sin^2\theta + 1$$

$$r = \left(\sin^2\theta + \cos^2\theta\right)\left(\sin^4\theta - \sin^2\theta\cos^2\theta + \cos^4\theta\right) - \left(\sin^4\theta + \cos^4\theta\right) - \sin^4\theta + \sin^2\theta + 1$$

$$r = \sin^4\theta - \sin^2\theta\cos^2\theta + \cos^4\theta - \sin^4\theta - \cos^4\theta - \sin^4\theta + \sin^2\theta + 1$$

$$r = -\sin^2\theta\cos^2\theta - \sin^4\theta + \sin^2\theta + 1$$

$$r = -\sin^2\theta\left(\cos^2\theta + \sin^2\theta\right) + \sin^2\theta + 1$$

$$r = -\sin^2\theta + \sin^2\theta + 1$$

$$r = 1$$