Topic: Tangent line to the polar curve

Question: Find the tangent line to the polar curve at the given point.

$$r = \sin\theta$$

at $\theta = \dfrac{\pi}{3}$

Answer choices:

A $\quad y = \sqrt{3}x + \dfrac{3}{2}$

B $\quad y = -\sqrt{3}x + \dfrac{3}{2}$

C $\quad y = -\sqrt{3}x$

D $\quad y = \sqrt{3}x$

Solution: B

We'll find the equation of the tangent line to a polar curve by following these steps:

1. Find the **slope** of the tangent line m, using the formula

$$m = \frac{dy}{dx} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$$

remembering to plug the value of θ at the tangent point into dy/dx to get a real-number value for the slope m.

2. **Find** x_1 **and** y_1 by plugging the value of θ at the tangent point into the conversion formulas

$$x = r\cos\theta$$

$$y = r\sin\theta$$

3. Plug the slope m and the point (x_1, y_1) into the **point-slope formula** for the equation of a line

$$y - y_1 = m(x - x_1)$$

In order to find the slope, we need to first find $dr/d\theta$.

$$r = \sin\theta$$

$$\frac{dr}{d\theta} = \cos\theta$$

Plugging $dr/d\theta$ and the given polar equation $r=\sin\theta$ into the formula for the slope, then evaluating at $\theta=\pi/3$, we get

$$m=\frac{dy}{dx}=\frac{\cos\theta\sin\theta+\sin\theta\cos\theta}{\cos\theta\cos\theta-\sin\theta\sin\theta}$$

$$m=\frac{dy}{dx}=\frac{\sin\theta\cos\theta+\sin\theta\cos\theta}{\cos^2\theta-\sin^2\theta}$$

$$m=\frac{dy}{dx}=\frac{2\sin\theta\cos\theta}{\cos^2\theta-\sin^2\theta}$$

$$m=\frac{dy}{dx}=\frac{2\sin\frac{\pi}{3}\cos\frac{\pi}{3}}{\cos^2\frac{\pi}{3}-\sin^2\frac{\pi}{3}}$$

$$m=\frac{dy}{dx}=\frac{2\cdot\frac{\sqrt{3}}{2}\cdot\frac{1}{2}}{\left(\frac{1}{2}\right)^2-\left(\frac{\sqrt{3}}{2}\right)^2}$$

$$m=\frac{dy}{dx}=\frac{\frac{\sqrt{3}}{2}}{\frac{1}{4}-\frac{3}{4}}$$

$$m=\frac{dy}{dx}=\frac{\frac{\sqrt{3}}{2}}{-\frac{2}{4}}$$

$$m=\frac{dy}{dx}=\frac{\sqrt{3}}{2}\left(-\frac{4}{2}\right)$$

$$m=\frac{dy}{dx}=-\sqrt{3}$$

To find (x_1, y_1), we'll plug $\theta = \pi/3$ and the given polar equation into the conversion formulas

$$x = r\cos\theta$$

$$x_1 = \sin\frac{\pi}{3}\cos\frac{\pi}{3}$$

$$x_1 = \frac{\sqrt{3}}{2} \cdot \frac{1}{2}$$

$$x_1 = \frac{\sqrt{3}}{4}$$

and

$$y = r\sin\theta$$

$$y_1 = \sin\frac{\pi}{3}\sin\frac{\pi}{3}$$

$$y_1 = \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2}$$

$$y_1 = \frac{3}{4}$$

Plugging m and (x_1, y_1) into the point-slope formula for the equation of a line, we get

$$y - y_1 = m(x - x_1)$$

$$y - \frac{3}{4} = -\sqrt{3}\left(x - \frac{\sqrt{3}}{4}\right)$$

$$y = -\sqrt{3}x + \frac{3}{4} + \frac{3}{4}$$

$$y = -\sqrt{3}x + \frac{3}{2}$$

Topic: Tangent line to the polar curve

Question: Find the tangent line to the polar curve at the given point.

$$r = 4\cos 2\theta$$

at $\theta = \dfrac{2\pi}{3}$

Answer choices:

A $\quad y = \dfrac{7\sqrt{3}}{3}x - \dfrac{4\sqrt{3}}{3}$

B $\quad y = -\dfrac{7\sqrt{3}}{3}x - \dfrac{4\sqrt{3}}{3}$

C $\quad 7\sqrt{3}x - 3y = 4\sqrt{3}$

D $\quad 7\sqrt{3}x + 3y = 4\sqrt{3}$

Solution: D

We'll find the equation of the tangent line to a polar curve by following these steps:

1. Find the slope of the tangent line m, using the formula

$$m=\frac{dy}{dx}=\frac{\frac{dr}{d\theta}\sin\theta+r\cos\theta}{\frac{dr}{d\theta}\cos\theta-r\sin\theta}$$

remembering to plug the value of θ at the tangent point into dy/dx to get a real-number value for the slope m.

2. Find x_1 and y_1 by plugging the value of θ at the tangent point into the conversion formulas

$$x=r\cos\theta$$

$$y=r\sin\theta$$

3. Plug the slope m and the point (x_1,y_1) into the point-slope formula for the equation of a line

$$y-y_1=m(x-x_1)$$

In order to find the slope, we need to first find $dr/d\theta$.

$$r=4\cos 2\theta$$

$$\frac{dr}{d\theta}=-8\sin 2\theta$$

Plugging $dr/d\theta$ and the given polar equation $r = 4\cos 2\theta$ into the formula for the slope, then evaluating at $\theta = \pi/3$, we get

$$m = \frac{dy}{dx} = \frac{(-8\sin 2\theta)\sin\theta + (4\cos 2\theta)\cos\theta}{(-8\sin 2\theta)\cos\theta - (4\cos 2\theta)\sin\theta}$$

$$m = \frac{dy}{dx} = \frac{-8\sin 2\theta\sin\theta + 4\cos 2\theta\cos\theta}{-8\sin 2\theta\cos\theta - 4\cos 2\theta\sin\theta}$$

$$m = \frac{dy}{dx} = \frac{-8\sin\left(2\cdot\frac{2\pi}{3}\right)\sin\frac{2\pi}{3} + 4\cos\left(2\cdot\frac{2\pi}{3}\right)\cos\frac{2\pi}{3}}{-8\sin\left(2\cdot\frac{2\pi}{3}\right)\cos\frac{2\pi}{3} - 4\cos\left(2\cdot\frac{2\pi}{3}\right)\sin\frac{2\pi}{3}}$$

$$m = \frac{dy}{dx} = \frac{-8\sin\frac{4\pi}{3}\sin\frac{2\pi}{3} + 4\cos\frac{4\pi}{3}\cos\frac{2\pi}{3}}{-8\sin\frac{4\pi}{3}\cos\frac{2\pi}{3} - 4\cos\frac{4\pi}{3}\sin\frac{2\pi}{3}}$$

$$m = \frac{dy}{dx} = \frac{-8\left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + 4\left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{-8\left(-\frac{\sqrt{3}}{2}\right)\left(-\frac{1}{2}\right) - 4\left(-\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)}$$

$$m = \frac{dy}{dx} = \frac{6+1}{-2\sqrt{3}+\sqrt{3}}$$

$$m = \frac{dy}{dx} = \frac{7}{-\sqrt{3}}$$

$$m = \frac{dy}{dx} = -\frac{7\sqrt{3}}{3}$$

To find (x_1, y_1), we'll plug $\theta = 2\pi/3$ and the given polar equation into the conversion formulas

$$x = r\cos\theta$$

$$x_1 = 4\cos\left(2 \cdot \frac{2\pi}{3}\right)\cos\frac{2\pi}{3}$$

$$x_1 = 4\cos\frac{4\pi}{3}\cos\frac{2\pi}{3}$$

$$x_1 = 4\left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right)$$

$$x_1 = 1$$

and

$$y = r\sin\theta$$

$$y_1 = 4\cos\left(2 \cdot \frac{2\pi}{3}\right)\sin\frac{2\pi}{3}$$

$$y_1 = 4\cos\frac{4\pi}{3}\sin\frac{2\pi}{3}$$

$$y_1 = 4\left(-\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$y_1 = -\sqrt{3}$$

Plugging m and (x_1, y_1) into the point-slope formula for the equation of a line, we get

$$y - y_1 = m(x - x_1)$$

$$y - (-\sqrt{3}) = -\frac{7\sqrt{3}}{3}(x - 1)$$

$$3y + 3\sqrt{3} = -7\sqrt{3}(x - 1)$$

$$3y + 3\sqrt{3} = -7\sqrt{3}x + 7\sqrt{3}$$

$$7\sqrt{3}x + 3y = 4\sqrt{3}$$

Topic: Tangent line to the polar curve

Question: Find the tangent line to the polar curve at the given point.

$$r = 3 + \sin\theta$$

at $\theta = \dfrac{5\pi}{4}$

Answer choices:

A $\quad 6y - 2\left(\sqrt{2} - 3\right)x = 12 - 19\sqrt{2}$

B $\quad 6y + 2\left(\sqrt{2} - 3\right)x = 12 + 19\sqrt{2}$

C $\quad y = \dfrac{2\sqrt{2} - 6}{6}x + \dfrac{\sqrt{2} - 6}{6}$

D $\quad y = -\dfrac{2\sqrt{2} - 6}{6}x + \dfrac{\sqrt{2} - 6}{6}$

Solution: A

We'll find the equation of the tangent line to a polar curve by following these steps:

1. Find the **slope** of the tangent line m, using the formula

$$m = \frac{dy}{dx} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$$

remembering to plug the value of θ at the tangent point into dy/dx to get a real-number value for the slope m.

2. **Find** x_1 **and** y_1 by plugging the value of θ at the tangent point into the conversion formulas

$$x = r\cos\theta$$

$$y = r\sin\theta$$

3. Plug the slope m and the point (x_1, y_1) into the **point-slope formula** for the equation of a line

$$y - y_1 = m(x - x_1)$$

In order to find the slope, we need to first find $dr/d\theta$.

$$r = 3 + \sin\theta$$

$$\frac{dr}{d\theta} = \cos\theta$$

Plugging $dr/d\theta$ and the given polar equation $r = 3 + \sin\theta$ into the formula for the slope, then evaluating at $\theta = 5\pi/4$, we get

$$m = \frac{dy}{dx} = \frac{\cos\theta\sin\theta + (3+\sin\theta)\cos\theta}{\cos\theta\cos\theta - (3+\sin\theta)\sin\theta}$$

$$m = \frac{dy}{dx} = \frac{\cos\theta\sin\theta + 3\cos\theta + \cos\theta\sin\theta}{\cos\theta\cos\theta - 3\sin\theta - \sin\theta\sin\theta}$$

$$m = \frac{dy}{dx} = \frac{2\cos\theta\sin\theta + 3\cos\theta}{\cos^2\theta - 3\sin\theta - \sin^2\theta}$$

$$m = \frac{dy}{dx} = \frac{2\cos\frac{5\pi}{4}\sin\frac{5\pi}{4} + 3\cos\frac{5\pi}{4}}{\cos^2\frac{5\pi}{4} - 3\sin\frac{5\pi}{4} - \sin^2\frac{5\pi}{4}}$$

$$m = \frac{dy}{dx} = \frac{2\left(-\frac{\sqrt{2}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) + 3\left(-\frac{\sqrt{2}}{2}\right)}{\left(-\frac{\sqrt{2}}{2}\right)^2 - 3\left(-\frac{\sqrt{2}}{2}\right) - \left(-\frac{\sqrt{2}}{2}\right)^2}$$

$$m = \frac{dy}{dx} = \frac{1 - \frac{3\sqrt{2}}{2}}{\frac{2}{4} + \frac{3\sqrt{2}}{2} - \frac{2}{4}}$$

$$m = \frac{dy}{dx} = \frac{\frac{2}{2} - \frac{3\sqrt{2}}{2}}{\frac{3\sqrt{2}}{2}}$$

$$m = \frac{dy}{dx} = \frac{\frac{2 - 3\sqrt{2}}{2}}{\frac{3\sqrt{2}}{2}}$$

$$m=\frac{dy}{dx}=\frac{2-3\sqrt{2}}{2}\cdot\frac{2}{3\sqrt{2}}$$

$$m=\frac{dy}{dx}=\frac{2-3\sqrt{2}}{3\sqrt{2}}$$

$$m=\frac{dy}{dx}=\frac{2-3\sqrt{2}}{3\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}$$

$$m=\frac{dy}{dx}=\frac{2\sqrt{2}-3\cdot 2}{3\cdot 2}$$

$$m=\frac{dy}{dx}=\frac{\sqrt{2}-3}{3}$$

To find (x_1, y_1), we'll plug $\theta = 5\pi/4$ and the given polar equation into the conversion formulas

$$x = r\cos\theta$$

$$x_1 = (3+\sin\theta)\cos\theta$$

$$x_1 = 3\cos\theta + \sin\theta\cos\theta$$

$$x_1 = 3\cos\frac{5\pi}{4} + \sin\frac{5\pi}{4}\cos\frac{5\pi}{4}$$

$$x_1 = 3\left(-\frac{\sqrt{2}}{2}\right)+\left(-\frac{\sqrt{2}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right)$$

$$x_1 = -\frac{3\sqrt{2}}{2} + \frac{2}{4}$$

$$x_1 = -\frac{3\sqrt{2}}{2} + \frac{1}{2}$$

$$x_1 = \frac{1 - 3\sqrt{2}}{2}$$

and

$$y = r\sin\theta$$

$$y_1 = (3 + \sin\theta)\sin\theta$$

$$y_1 = 3\sin\theta + \sin\theta\sin\theta$$

$$y_1 = 3\sin\frac{5\pi}{4} + \sin\frac{5\pi}{4}\sin\frac{5\pi}{4}$$

$$y_1 = 3\left(-\frac{\sqrt{2}}{2}\right) + \left(-\frac{\sqrt{2}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right)$$

$$y_1 = -\frac{3\sqrt{2}}{2} + \frac{2}{4}$$

$$y_1 = -\frac{3\sqrt{2}}{2} + \frac{1}{2}$$

$$y_1 = \frac{1 - 3\sqrt{2}}{2}$$

Plugging m and (x_1, y_1) into the point-slope formula for the equation of a line, we get

$$y - y_1 = m(x - x_1)$$

$$y - \frac{1 - 3\sqrt{2}}{2} = \frac{\sqrt{2} - 3}{3}\left(x - \frac{1 - 3\sqrt{2}}{2}\right)$$

$$y - \frac{1 - 3\sqrt{2}}{2} = \frac{\sqrt{2} - 3}{3}x - \frac{\sqrt{2} - 3 \cdot 2 - 3 + 9\sqrt{2}}{6}$$

$$y - \frac{1 - 3\sqrt{2}}{2} = \frac{\sqrt{2} - 3}{3}x - \frac{10\sqrt{2} - 9}{6}$$

$$2y - \left(1 - 3\sqrt{2}\right) = \frac{2\sqrt{2} - 6}{3}x - \frac{10\sqrt{2} - 9}{3}$$

$$6y - 3\left(1 - 3\sqrt{2}\right) = \left(2\sqrt{2} - 6\right)x - \left(10\sqrt{2} - 9\right)$$

$$6y - 3 + 9\sqrt{2} = \left(2\sqrt{2} - 6\right)x - 10\sqrt{2} + 9$$

$$6y - \left(2\sqrt{2} - 6\right)x = -10\sqrt{2} - 9\sqrt{2} + 9 + 3$$

$$6y - \left(2\sqrt{2} - 6\right)x = -19\sqrt{2} + 12$$

$$6y - 2\left(\sqrt{2} - 3\right)x = 12 - 19\sqrt{2}$$