Calculus 1
Final Exam Solutions

krista king MATH

Calculus 1 Final Exam Answer Key

1. (5 pts)	■	B	C	D	E
2. (5 pts)	A	B	C	D	■
3. (5 pts)	A	■	C	D	E
4. (5 pts)	A	B	■	D	E
5. (5 pts)	■	B	C	D	E
6. (5 pts)	A	B	C	■	E
7. (5 pts)	A	B	■	D	E
8. (5 pts)	A	B	C	■	E

9. (15 pts) $\frac{1}{108\pi}$ cm/s

10. (15 pts) $t \approx 30.3$ years

11. (15 pts) -28 m/s

12. (15 pts) $y = -\frac{49}{304}x + \frac{657}{152}$

Calculus 1 Final Exam Solutions

1. A. Use the chain rule to find the derivative.

$$f(x) = \sqrt{14 - 2x}$$

$$f'(x) = \frac{1}{2}(14 - 2x)^{-\frac{1}{2}} \cdot -2$$

$$f'(x) = -\frac{1}{\sqrt{14 - 2x}}$$

2. E. Factor the numerator and denominator as completely as possible.

$$f(x) = \frac{x^2 + 7x + 10}{x^2 - 3x - 10}$$

$$f(x) = \frac{(x+2)(x+5)}{(x+2)(x-5)}$$

The factor $x + 2$ can be canceled from the numerator and denominator.

$$f(x) = \frac{x+5}{x-5}$$

Because $x = -2$ is a value that *would have* made the denominator 0, but we canceled it out when we canceled $x + 2$, we know that the function has a point discontinuity at $x = -2$.

3. B. Break down the limit

$$\lim_{x\to 4} x^2 + 1 = 17$$

into its component parts.

- x approaches 4
- the function is $f(x) = x^2 + 1$
- the value of the limit is 17

Putting these pieces together gives a full statement of the limit:

"The limit as x approaches 4 of the function $f(x) = x^2 + 1$ is 17."

4. C. Find the limit.

$$\lim_{x\to 4} g(x)$$

$$\lim_{x\to 4} 2x + 3$$

$$2(4) + 3$$

$$11$$

Find $f(11)$.

$$f(11) = 2(11)^3$$

$$f(11) = 2(1{,}331)$$

$$f(11) = 2{,}662$$

$$\lim_{x \to 4} f[g(x)] = 2{,}662$$

5. A. The half-life equation is

$$\frac{1}{2} = e^{kt}$$

Solve this for the decay constant k.

$$\ln \frac{1}{2} = \ln e^{kt}$$

$$\ln \frac{1}{2} = kt$$

$$k = \frac{\ln \frac{1}{2}}{t}$$

Use laws of logarithms to rewrite the log.

$$k = \frac{\ln 1 - \ln 2}{t}$$

$$k = \frac{0 - \ln 2}{t}$$

$$k = -\frac{\ln 2}{t}$$

Because k is a constant, we can absorb the negative sign into it.

$$k = \frac{\ln 2}{t}$$

Substitute $t = 68.9$.

$$k = \frac{\ln 2}{68.9}$$

$$k \approx 0.0101$$

6. D. If $f(x) = x^2 + 4x + 3$ and $g(x) = 2x - 1$, evaluate the limit.

$$\lim_{x \to -1} \frac{f(x)}{4g(x)}$$

We'll start by plugging $f(x)$ and $g(x)$ into the limit.

$$\lim_{x \to -1} \frac{f(x)}{4g(x)}$$

$$\lim_{x \to -1} \frac{x^2 + 4x + 3}{4(2x - 1)}$$

Now we'll substitute the value we're approaching into the function.

$$\frac{(-1)^2 + 4(-1) + 3}{4(2(-1) - 1)}$$

$$\frac{1 - 4 + 3}{4(-2 - 1)}$$

$$\frac{0}{-12}$$

$$0$$

7. C. To find marginal revenue, take the derivative of the revenue formula $R(x) = -0.4x^2 + 500x$.

$$R'(x) = -0.8x + 500$$

To maximize revenue, set the marginal revenue equal to 0 and then solve for x.

$$0 = -0.8x + 500$$

$$0.8x = 500$$

$$x = 625$$

The sandwich shop needs to sell 625 sandwiches each week to maximize weekly revenue.

8. D. The temperature of the object after 1 hour is given by T when $t = 1$. So substitute $t = 1$ into the temperature function.

$$T(t) = 6e^{-t}$$

$$T(t) = 6e^{-1}$$

$$T(t) \approx 6(0.37)$$

$$T(t) \approx 2.2 \approx 2^\circ \ C$$

9. The formula for the volume of a sphere is

$$V = \frac{4}{3}\pi r^3$$

Use implicit differentiation to take the derivative of both sides.

$$(1)\frac{dV}{dt} = \frac{4}{3}\pi(3r^2)\frac{dr}{dt}$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

From the question, we know that $dV/dt = 3$ and that $r = 9$, so we'll plug those in.

$$3 = 4\pi(9)^2\frac{dr}{dt}$$

$$3 = 324\pi\frac{dr}{dt}$$

Solve for dr/dt, which is the rate we were asked to find.

$$\frac{dr}{dt} = \frac{3}{324\pi}$$

$$\frac{dr}{dt} = \frac{1}{108\pi}$$

The length of the radius is increasing at a rate of $1/108\pi$ cm/s.

10. Both the interest rate and time have units in years, so we have matching units, which means we can plug directly into the exponential growth formula to find the number of years we've held the investment.

$$A(t) = A_0e^{rt}$$

$$6{,}430 = 1{,}500e^{(0.048)t}$$

$$4.2867 = e^{0.048t}$$

Apply the natural logarithm to both sides.

$$\ln 4.2867 = \ln(e^{0.048t})$$

$$\ln 4.2867 = 0.048t$$

$$t = \frac{\ln 4.2867}{0.048}$$

$$t \approx 30.3$$

11. Take the derivative of the position function to get the velocity function.

$$s(t) = -8t^2 + 4t - 7$$

$$s'(t) = -16t + 4$$

$$v(t) = -16t + 4$$

Substitute $t = 2$ to find instantaneous velocity at that time.

$$v(2) = -16(2) + 4$$

$$v(3) = -28$$

The instantaneous velocity at $t = 2$ is -28 m/s. Because the velocity is negative, it means that the water balloon is falling toward the ground.

12. Use quotient rule to take the derivative of the function,

$$f'(x) = \frac{(12x^2)(x+5) - (4x^3)(1)}{(x+5)^2}$$

$$f'(x) = \frac{12x^3 + 60x^2 - 4x^3}{(x+5)^2}$$

$$f'(x) = \frac{8x^3 + 60x^2}{(x+5)^2}$$

and then evaluate it at $(2,4)$.

$$f'(2) = \frac{8(2)^3 + 60(2)^2}{(2+5)^2}$$

$$f'(2) = \frac{8(8) + 60(4)}{7^2}$$

$$f'(2) = \frac{64 + 240}{49}$$

$$f'(2) = \frac{304}{49}$$

This is the slope of the tangent line at $(2,4)$. Since $m = 304/49$, we'll take the negative reciprocal to find n, the slope of the normal line.

$$n = -\frac{49}{304}$$

We'll plug $n = -49/304$ and the point $(2,4)$ into the point-slope formula for the equation of the line. Once we simplify, we'll have the equation of the normal line to the function at $(2,4)$.

$$y - y_1 = n(x - x_1)$$

$$y - 4 = -\frac{49}{304}(x - 2)$$

$$y - 4 = -\frac{49}{304}x + \frac{49}{152}$$

$$y = -\frac{49}{304}x + \frac{49}{152} + \frac{608}{152}$$

$$y = -\frac{49}{304}x + \frac{657}{152}$$

WWW.KRISTAKINGMATH.COM