Distance between polar points

To find the distance between two polar coordinates, we have two options. We can

1. Convert our polar coordinates points into cartesian coordinate points using the conversion formulas

$$x = r\cos\theta$$

$$y = r\sin\theta$$

and then use the distance formula from the cartesian coordinate system.

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

where (x_1, y_1) and (x_2, y_2) are the converted rectangular points. It doesn't matter which point we use for (x_1, y_1) and (x_2, y_2).

2. Use the distance formula from the polar coordinate system

$$D = \sqrt{r_1^2 + r_2^2 - 2r_1 r_2 \cos(\theta_1 - \theta_2)}$$

where (r_1, θ_1) and (r_2, θ_2) are the given polar points. It doesn't matter which point we use for (r_1, θ_1) and (r_2, θ_2), but it's easier to make θ_1 the larger of the two θ values, since we subtract θ_2 from θ_1.

Let's try an example to test out the first method.

Example

Find the distance between the polar points.

$$\left(2,\frac{\pi}{2}\right) \text{ and } \left(3,\frac{\pi}{4}\right)$$

We'll convert the polar points into cartesian points.

Using the conversion formulas to change $\left(2,\frac{\pi}{2}\right)$ into polar, we get

$$x_1 = 2\cos\frac{\pi}{2}$$

$$x_1 = 0$$

and

$$y_1 = 2\sin\frac{\pi}{2}$$

$$y_1 = 2$$

The new point is $(0,2)$.

Using the conversion formulas to change $\left(3,\frac{\pi}{4}\right)$ into polar, we get

$$x_2 = 3\cos\frac{\pi}{4}$$

$$x_2 = \frac{3\sqrt{2}}{2}$$

and

$$y_2 = 3\sin\frac{\pi}{4}$$

$$y_2 = \frac{3\sqrt{2}}{2}$$

The new point is $\left(\frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}\right)$.

Setting

$$(x_1, y_1) = (0,2)$$

$$(x_2, y_2) = \left(\frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}\right)$$

and plugging these points into the distance formula from the cartesian coordinate system, we get

$$D = \sqrt{\left(\frac{3\sqrt{2}}{2} - 0\right)^2 + \left(\frac{3\sqrt{2}}{2} - 2\right)^2}$$

$$D = \sqrt{\frac{18}{4} + \frac{18}{4} - \frac{12\sqrt{2}}{2} + 4}$$

$$D = \sqrt{13 - 6\sqrt{2}}$$

The distance between the polar points is $D = \sqrt{13 - 6\sqrt{2}}$.

Let's look at a different example where we use the second method.

Example

Find the distance between the polar points.

$(1,2\pi)$ and $(2,\pi)$

Setting

$$(r_1, \theta_1) = (1,2\pi)$$

$$(r_2, \theta_2) = (2,\pi)$$

and plugging these points into the distance formula from the polar coordinate system, we get

$$D = \sqrt{(1)^2 + (2)^2 - 2(1)(2)\cos(2\pi - \pi)}$$

$$D = \sqrt{5 - 4\cos\pi}$$

$$D = \sqrt{5 - 4(-1)}$$

$$D = \sqrt{5 + 4}$$

$$D = \sqrt{9}$$

$$D = 3$$

The distance between the polar points is $D = 3$.
