

**Topic:** Consumer and producer surplus**Question:** Find equilibrium quantity and equilibrium price.

$$S(q) = 0.04q^2 + 4$$

$$D(q) = -0.3q + 11$$

**Answer choices:**

A       $q_e = 8$       and       $p_e = 10$

B       $q_e = 7$       and       $p_e = 9.1$

C       $q_e = 9.1$       and       $p_e = 7$

D       $q_e = 10$       and       $p_e = 8$



**Solution: D**

In any supply and demand system, there's an equilibrium point where the supply curve  $S(q)$  and the demand curve  $D(q)$  intersect each other.

The intersection point is associated with a specific *equilibrium quantity* and *equilibrium price*:

The equilibrium quantity is the  $x$ -value of the intersection point

The equilibrium price is the  $y$ -value of the intersection point

In order to find these values, we'll set the demand and supply curves equal to one another and solve for  $q$ , the equilibrium quantity. It's possible to get more than one value for  $q$ , but keep in mind that the equilibrium quantity must be a positive number, so we can discard any negative answers.

$$-0.3q + 11 = 0.04q^2 + 4$$

$$0.04q^2 + 0.3q - 7 = 0$$

$$(0.04q^2 + 0.3q - 7 = 0) \cdot 100$$

$$4q^2 + 30q - 700 = 0$$

$$(4q + 70)(q - 10) = 0$$

$$4q + 70 = 0$$

$$4q = -70$$

$$q = -\frac{35}{2}$$



or

$$q - 10 = 0$$

$$q = 10$$

Since  $q$  must be positive, we can discard  $q = -35/2$ . Equilibrium quantity must be  $q = 10$ .

Now that we know that equilibrium quantity is  $q = 10$ , we can find equilibrium price by plugging  $q = 10$  into either the demand function or the supply function. Remember, because the equilibrium point represents the intersection of the two functions, both functions should give us the same value for equilibrium price.

$$D(10) = -0.3(10) + 11$$

$$D(10) = 8$$

$$p = 8$$

We denote equilibrium quantity as  $q_e$  and equilibrium price as  $p_e$ , so we can say

$$q_e = 10$$

and

$$p_e = 8$$



**Topic:** Consumer and producer surplus

**Question:** Find consumer surplus.

$$S(q) = 0.02q^2 + 10$$

$$D(q) = -0.34q + 14$$

**Answer choices:**

A      18.08

B      10.88

C      18.80

D      10.08



**Solution: B**

The formula we use to find consumer surplus is

$$CS = \int_0^{q_e} D(q) \, dq - p_e q_e$$

where  $D(q)$  is the demand curve,  $q_e$  is equilibrium quantity and  $p_e$  is our equilibrium price.

We've been given the demand curve, but we don't know equilibrium quantity or equilibrium price.

In any supply and demand system, there's an equilibrium point where the supply curve  $S(q)$  and the demand curve  $D(q)$  intersect each other.

The intersection point is associated with a specific *equilibrium quantity* and *equilibrium price*:

The equilibrium quantity is the  $x$ -value of the intersection point

The equilibrium price is the  $y$ -value of the intersection point

In order to find these values, we'll set the demand and supply curves equal to one another and solve for  $q$ , the equilibrium quantity. It's possible to get more than one value for  $q$ , but keep in mind that the equilibrium quantity must be a positive number, so we can discard any negative answers.

$$-0.34q + 14 = 0.02q^2 + 10$$

$$0.02q^2 + 0.34q - 4 = 0$$

$$(0.02q^2 + 0.34q - 4 = 0) \quad 100$$



$$2q^2 + 34q - 400 = 0$$

$$(2q + 50)(q - 8) = 0$$

$$2q + 50 = 0$$

$$2q = -50$$

$$q = -25$$

or

$$q - 8 = 0$$

$$q = 8$$

Since  $q$  must be positive, we can discard  $q = -25$ . Equilibrium quantity must be  $q = 8$ .

Now that we know that equilibrium price is  $q = 8$ , we can find equilibrium price by plugging  $q = 8$  into either the demand function or the supply function. Remember, because the equilibrium point represents the intersection of the two functions, both functions should give us the same value for equilibrium price.

$$D(8) = -0.34(8) + 14$$

$$D(8) = 11.28$$

$$p = 11.28$$

We denote equilibrium quantity as  $q_e$  and equilibrium price as  $p_e$ , so we can say



$$q_e = 8$$

and

$$p_e = 11.28$$

With the equilibrium point and the demand curve, we now have everything we need to solve for consumer surplus. Plugging these values into the formula, we get

$$CS = \int_0^8 -0.34q + 14 \, dq - (11.28)(8)$$

$$CS = \int_0^8 -0.34q + 14 \, dq - 90.24$$

$$CS = \left( -\frac{0.34}{2}q^2 + 14q \right) \Big|_0^8 - 90.24$$

$$CS = \left[ -\frac{0.34}{2}(8)^2 + 14(8) \right] - \left[ -\frac{0.34}{2}(0)^2 + 14(0) \right] - 90.24$$

$$CS = -10.88 + 112 - 90.24$$

$$CS = 10.88$$



**Topic:** Consumer and producer surplus**Question:** Find producer surplus.

$$S(q) = 0.05q^2 + 9$$

$$D(q) = -0.1q + 15$$

**Answer choices:**

A      33.33

B      30

C      5.50

D      5





**Solution: A**

The formula we use to find consumer surplus is

$$PS = p_e q_e - \int_0^{q_e} S(q) \, dq$$

where  $S(q)$  is the supply curve,  $q_e$  is equilibrium quantity and  $p_e$  is our equilibrium price.

We've been given the supply curve, but we don't know equilibrium quantity or equilibrium price.

In any supply and demand system, there's an equilibrium point where the supply curve  $S(q)$  and the demand curve  $D(q)$  intersect each other.

The intersection point is associated with a specific *equilibrium quantity* and *equilibrium price*:

The equilibrium quantity is the  $x$ -value of the intersection point

The equilibrium price is the  $y$ -value of the intersection point

In order to find these values, we'll set the demand and supply curves equal to one another and solve for  $q$ , the equilibrium quantity. It's possible to get more than one value for  $q$ , but keep in mind that the equilibrium quantity must be a positive number, so we can discard any negative answers.

$$-0.1q + 15 = 0.05q^2 + 9$$

$$0.05q^2 + 0.1q - 6 = 0$$

$$(0.05q^2 + 0.1q - 6 = 0) \quad 100$$



$$5q^2 + 10q - 600 = 0$$

$$(5q + 60)(q - 10) = 0$$

$$5q + 60 = 0$$

$$5q = -60$$

$$q = -12$$

or

$$q - 10 = 0$$

$$q = 10$$

Since  $q$  must be positive, we can discard  $q = -12$ . Equilibrium quantity must be  $q = 10$ .

Now that we know that equilibrium price is  $q = 10$ , we can find equilibrium price by plugging  $q = 10$  into either the demand function or the supply function. Remember, because the equilibrium point represents the intersection of the two functions, both functions should give us the same value for equilibrium price.

$$D(10) = -0.1(10) + 15$$

$$D(10) = 14$$

$$p = 14$$

We denote equilibrium quantity as  $q_e$  and equilibrium price as  $p_e$ , so we can say



$$q_e = 10$$

and

$$p_e = 14$$

With the equilibrium point and the supply curve, we now have everything we need to solve for producer surplus. Plugging these values into the formula, we get

$$PS = (14)(10) - \int_0^{10} 0.05q^2 + 9 \, dq$$

$$PS = 140 - \int_0^{10} 0.05q^2 + 9 \, dq$$

$$PS = 140 - \left( \frac{0.05}{3}q^3 + 9q \right) \Big|_0^{10}$$

$$PS = 140 - \left[ \left[ \frac{0.05}{3}(10)^3 + 9(10) \right] - \left[ \frac{0.05}{3}(0)^3 + 9(0) \right] \right]$$

$$PS = 140 - (16.67 + 90)$$

$$PS = 33.33$$

