Topic: Inverse hyperbolic derivatives

Question: Find the derivative of the inverse hyperbolic function.

$$y = \cosh^{-1}(x^3)$$

Answer choices:

A $y' = \dfrac{3x^2}{\sqrt{x^6 - 1}}$ with $x^3 < 1$

B $y' = \dfrac{3x^2}{\sqrt{x^6 + 1}}$ with $x^3 < 1$

C $y' = \dfrac{3x^2}{\sqrt{x^6 + 1}}$ with $x^3 > 1$

D $y' = \dfrac{3x^2}{\sqrt{x^6 - 1}}$ with $x^3 > 1$

Solution: D

Apply the formula for the derivative of inverse hyperbolic cosine, with $g(x) = x^3$ and $g'(x) = 3x^2$.

$$y' = \frac{g'(x)}{\sqrt{[g(x)]^2 - 1}} \qquad \text{with } x^3 > 1$$

$$y' = \frac{3x^2}{\sqrt{(x^3)^2 - 1}} \qquad \text{with } x^3 > 1$$

$$y' = \frac{3x^2}{\sqrt{x^6 - 1}} \qquad \text{with } x^3 > 1$$

Topic: Inverse hyperbolic derivatives

Question: Find the derivative of the inverse hyperbolic function.

$$y = \tanh^{-1}(2x^5 - 1)$$

Answer choices:

A $y' = \dfrac{5}{2x + 2x^6}$ with $|2x^5 - 1| < 1$

B $y' = \dfrac{1}{2x - 2x^6}$ with $|2x^5 - 1| < 1$

C $y' = \dfrac{5}{x - x^6}$ with $|2x^5 - 1| < 1$

D $y' = \dfrac{5}{2x - 2x^6}$ with $|2x^5 - 1| < 1$

Solution: D

Apply the formula for the derivative of inverse hyperbolic tangent, with $g(x) = 2x^5 - 1$ and $g'(x) = 10x^4$.

$$y' = \frac{g'(x)}{1 - [g(x)]^2} \quad \text{with } |g(x)| < 1$$

$$y' = \frac{10x^4}{1 - (2x^5 - 1)^2} \quad \text{with } |2x^5 - 1| < 1$$

Simplify the derivative.

$$y' = \frac{10x^4}{1 - (4x^{10} - 4x^5 + 1)} \quad \text{with } |2x^5 - 1| < 1$$

$$y' = \frac{10x^4}{1 - 4x^{10} + 4x^5 - 1} \quad \text{with } |2x^5 - 1| < 1$$

$$y' = \frac{10x^4}{-4x^{10} + 4x^5} \quad \text{with } |2x^5 - 1| < 1$$

$$y' = \frac{5}{-2x^6 + 2x} \quad \text{with } |2x^5 - 1| < 1$$

$$y' = \frac{5}{2x - 2x^6} \quad \text{with } |2x^5 - 1| < 1$$

Topic: Inverse hyperbolic derivatives

Question: Find the derivative of the inverse hyperbolic function.

$$f(x) = (\sinh^{-1}(2x^3))^4$$

Answer choices:

A $$f'(x) = \frac{24x^2(\sinh^{-1}(2x^3))^3}{\sqrt{x^2+1}}$$

B $$f'(x) = \frac{24x^2(\sinh^{-1}(2x^3))^3}{\sqrt{4x^6+1}}$$

C $$f'(x) = \frac{6x^2(\sinh^{-1}(2x^3))^3}{\sqrt{x^2+1}}$$

D $$f'(x) = \frac{6x^2(\sinh^{-1}(2x^3))^3}{\sqrt{4x^6+1}}$$

Solution: B

Use a substitution with $u = \sinh^{-1}(2x^3)$ and

$$u' = \frac{6x^2}{\sqrt{(2x^3)^2 + 1}}$$

$$u' = \frac{6x^2}{\sqrt{4x^6 + 1}}$$

Then the function is

$$f(x) = u^4$$

and the derivative is

$$f'(x) = 4u^3 \cdot u'$$

$$f'(x) = 4(\sinh^{-1}(2x^3))^3 \cdot \frac{6x^2}{\sqrt{4x^6 + 1}}$$

$$f'(x) = \frac{24x^2(\sinh^{-1}(2x^3))^3}{\sqrt{4x^6 + 1}}$$