Sequences vs. series

Sequences and series are almost always studied together, because they're so closely related.

A **sequence** is just a list of terms in a specific order, and is denoted by

$$a_n$$

A **series** is the sum of a sequence, and is denoted by

$$\sum_{n=b}^{c} a_n$$

where b is the beginning of the interval you're calculating, typically $b = 1$, where c is the end of the interval being calculated, and where a_n is the sequence we're taking the sum of.

Both sequences and series can be defined over a closed or infinite interval.

Example

Calculate the series.

$$\sum_{n=1}^{\infty} \frac{1}{2^n}$$

This question is asking us to calculate an infinite series. The first step will be to calculate the first few terms. Let's calculate $n = 1$, $n = 2$, $n = 3$ and $n = 4$.

When $n = 1$, $\qquad a_1 = \dfrac{1}{2^1}$ $\qquad$ so $\qquad a_1 = \dfrac{1}{2}$

When $n = 2$, $\qquad a_2 = \dfrac{1}{2^2}$ $\qquad$ so $\qquad a_2 = \dfrac{1}{4}$

When $n = 3$, $\qquad a_3 = \dfrac{1}{2^3}$ $\qquad$ so $\qquad a_3 = \dfrac{1}{8}$

When $n = 4$, $\qquad a_4 = \dfrac{1}{2^4}$ $\qquad$ so $\qquad a_4 = \dfrac{1}{16}$

Now that we have the first four terms we can start our summation. Let's add the first four terms to see what we get

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}$$

$$\frac{8}{16} + \frac{4}{16} + \frac{2}{16} + \frac{1}{16}$$

$$\frac{15}{16}$$

From our answer, it looks like our series is approaching 1 as $n \to \infty$. Let's calculate the fifth term to see if our hypothesis holds.

When $n = 5$ $\qquad a_5 = \dfrac{1}{2^5}$ $\qquad$ so $\qquad a_5 = \dfrac{1}{32}$

Now let's add the fifth term to the sum of the first four terms

$$\frac{15}{16} + \frac{1}{32}$$

$$\frac{30}{32} + \frac{1}{32}$$

$$\frac{31}{32}$$

This number is even closer to 1 therefore our hypothesis is correct. We can write our answer as

$$\sum_{n=1}^{\infty} \frac{1}{2^n} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \ldots + \frac{1}{2^{\infty}}$$

$$\sum_{n=1}^{\infty} \frac{1}{2^n} = 1$$

The series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2^n}$ is equal to 1.