Sum of a telescoping series

Telescoping series are series in which all but the first and last terms cancel out. If you think about the way that a long telescope collapses on itself, you can better understand how the middle of a telescoping series cancels itself.

To determine whether a series is telescoping, we'll need to calculate at least the first few terms to see whether the middle terms start canceling with each other.

Sum of the telescoping series

The sum of a telescoping series is given by the formula

$$\sum_{n=1}^{\infty} a_n = \lim_{n \to \infty} s_n$$

We know that s_n is the series of partial sums, so we can say that the sum of the telescoping series a_n is the limit as $n \to \infty$ of its corresponding series of partial sums s_n.

Example

Show that the series is a telescoping series, then find the sum of the series.

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$$\sum_{n=1}^{\infty} \frac{1}{n} - \frac{1}{n+1}$$

In order to show that the series is telescoping, we'll need to start by expanding the series. Let's use $n = 1$, $n = 2$, $n = 3$ and $n = 4$.

$n = 1$	$\frac{1}{1} - \frac{1}{1+1}$	$1 - \frac{1}{2}$
$n = 2$	$\frac{1}{2} - \frac{1}{2+1}$	$\frac{1}{2} - \frac{1}{3}$
$n = 3$	$\frac{1}{3} - \frac{1}{3+1}$	$\frac{1}{3} - \frac{1}{4}$
$n = 4$	$\frac{1}{4} - \frac{1}{4+1}$	$\frac{1}{4} - \frac{1}{5}$

Writing these terms into our expanded series and including the last term of the series, we get

$$\sum_{n=1}^{\infty} \frac{1}{n} - \frac{1}{n+1} = \left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \left(\frac{1}{4} - \frac{1}{5}\right) + \ldots + \left(\frac{1}{n} - \frac{1}{n+1}\right)$$

The series is telescoping if we can cancel all of the terms in the middle (every term but the first and last). When we look at our expanded series, we see that the second half of the first term will cancel with the first half of the second term, that the second half of the second term will cancel with the first half of the third term, and so on, so we can say that the series is telescoping.

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Canceling everything but the first half of the first term and the second half of the last term gives an expression for the series of partial sums.

$$s_n = 1 - \frac{1}{n+1}$$

To find the sum of the telescoping series, we'll take the limit as $n \to \infty$ of the series or partial sums s_n.

$$\sum_{n=1}^{\infty} a_n = \lim_{n\to\infty} s_n$$

$$\sum_{n=1}^{\infty} \frac{1}{n} - \frac{1}{n+1} = \lim_{n\to\infty} 1 - \frac{1}{n+1}$$

$$\sum_{n=1}^{\infty} \frac{1}{n} - \frac{1}{n+1} = 1 - \frac{1}{\infty+1}$$

$$\sum_{n=1}^{\infty} \frac{1}{n} - \frac{1}{n+1} = 1 - \frac{1}{\infty}$$

$$\sum_{n=1}^{\infty} \frac{1}{n} - \frac{1}{n+1} = 1 - 0$$

$$\sum_{n=1}^{\infty} \frac{1}{n} - \frac{1}{n+1} = 1$$

The sum of the series is 1.

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