Topic: Normal lines

Question: Find the equation of the normal line to the function at $(1,2)$.

$$f(x) = 2x^4$$

Answer choices:

A $\quad y = 8x - 6$

B $\quad y = -\frac{1}{8}x - \frac{17}{8}$

C $\quad y = -\frac{1}{8}x + \frac{17}{8}$

D $\quad y = 8x - 10$

Solution: C

Take the derivative of the function,

$$f'(x) = 8x^3$$

and then evaluate it at $(1,2)$.

$$f'(1) = 8(1)^3$$

$$f'(1) = 8$$

This is the slope of the tangent line at $(1,2)$. Since $m = 8$, we'll take the negative reciprocal to find n, the slope of the normal line.

$$n = -\frac{1}{8}$$

We'll plug $n = -1/8$ and the point $(1,2)$ into the point-slope formula for the equation of the line. Once we simplify, we'll have the equation of the normal line to the function at $(1,2)$.

$$y - y_1 = n(x - x_1)$$

$$y - 2 = -\frac{1}{8}(x - 1)$$

$$y - 2 = -\frac{1}{8}x + \frac{1}{8}$$

$$y = -\frac{1}{8}x + \frac{1}{8} + \frac{16}{8}$$

$$y = -\frac{1}{8}x + \frac{17}{8}$$

Topic: Normal lines

Question: Find the equation of the normal line to the function at $(3,6)$.

$$f(x) = x\sqrt{x+1}$$

Answer choices:

A $\quad y = -\frac{4}{11}x + \frac{78}{11}$

B $\quad y = \frac{11}{4}x - \frac{57}{4}$

C $\quad y = \frac{11}{4}x - \frac{9}{4}$

D $\quad y = -\frac{4}{11}x - \frac{54}{11}$

Solution: A

Take the derivative of the function,

$$f'(x) = (1)(\sqrt{x+1}) + (x)\left(\frac{1}{2}(x+1)^{-\frac{1}{2}}\right)$$

$$f'(x) = \sqrt{x+1} + \frac{x}{2\sqrt{x+1}}$$

and then evaluate it at $(3,6)$.

$$f'(3) = \sqrt{3+1} + \frac{3}{2\sqrt{3+1}}$$

$$f'(3) = 2 + \frac{3}{2(2)}$$

$$f'(3) = \frac{8}{4} + \frac{3}{4}$$

$$f'(3) = \frac{11}{4}$$

This is the slope of the tangent line at $(3,6)$. Since $m = 11/4$, we'll take the negative reciprocal to find n, the slope of the normal line.

$$n = -\frac{4}{11}$$

We'll plug $n = -4/11$ and the point $(3,6)$ into the point-slope formula for the equation of the line. Once we simplify, we'll have the equation of the normal line to the function at $(3,6)$.

$$y - y_1 = n(x - x_1)$$

$$y - 6 = -\frac{4}{11}(x - 3)$$

$$y - 6 = -\frac{4}{11}x + \frac{12}{11}$$

$$y = -\frac{4}{11}x + \frac{12}{11} + \frac{66}{11}$$

$$y = -\frac{4}{11}x + \frac{78}{11}$$

Topic: Normal lines

Question: Find the equation of the normal line to the function at $(2,2)$.

$$f(x) = \frac{2x^2}{x+2}$$

Answer choices:

A $\quad y = -\frac{2}{3}x - \frac{2}{3}$

B $\quad y = \frac{3}{2}x - 1$

C $\quad y = \frac{3}{2}x - 5$

D $\quad y = -\frac{2}{3}x + \frac{10}{3}$

Solution: D

Take the derivative of the function,

$$f'(x) = \frac{(4x)(x+2) - (2x^2)(1)}{(x+2)^2}$$

$$f'(x) = \frac{4x^2 + 8x - 2x^2}{(x+2)^2}$$

$$f'(x) = \frac{2x^2 + 8x}{(x+2)^2}$$

and then evaluate it at $(2,2)$.

$$f'(2) = \frac{2(2)^2 + 8(2)}{(2+2)^2}$$

$$f'(2) = \frac{2(4) + 16}{4^2}$$

$$f'(2) = \frac{8 + 16}{16}$$

$$f'(2) = \frac{3}{2}$$

This is the slope of the tangent line at $(2,2)$. Since $m = 3/2$, we'll take the negative reciprocal to find n, the slope of the normal line.

$$n = -\frac{2}{3}$$

We'll plug $n = -2/3$ and the point $(2,2)$ into the point-slope formula for the equation of the line. Once we simplify, we'll have the equation of the normal line to the function at $(2,2)$.

$$y - y_1 = n(x - x_1)$$

$$y - 2 = -\frac{2}{3}(x - 2)$$

$$y - 2 = -\frac{2}{3}x + \frac{4}{3}$$

$$y = -\frac{2}{3}x + \frac{4}{3} + \frac{6}{3}$$

$$y = -\frac{2}{3}x + \frac{10}{3}$$