Topic: Mean value theorem for integrals

Question: Choose the correct formula.

Given the function $f(x)$ over the interval $[a, b]$, where c is a point in the interval, the mean value theorem for integrals says that...

Answer choices:

A $\int_a^b f(x)\ dx = f(c)(b + a)$

B $\int_c^c f(x)\ dx = f(c)(b - a)$

C $\int_a^b f(x)\ dx = f(c)(b - a)$

D $\int_c^c f(x)\ dx = f(c)(b + a)$

Solution: C

The mean value theorem states that a point c must exist on the given interval $[a, b]$ for the function $f(x)$ such that

$$\int_a^b f(x)\ dx = f(c)(b-a)$$

If you have trouble remembering this formula, remember that it's just a rearrangement of the average value formula

$$f_{avg} = \frac{1}{b-a}\int_a^b f(x)\ dx$$

If we say that $f(c) = f_{avg}$, then we make a substitution and get

$$f(c) = \frac{1}{b-a}\int_a^b f(x)\ dx$$

$$f(c)(b-a) = \int_a^b f(x)\ dx$$

and we're back to the mean value theorem.

Topic: Mean value theorem for integrals

Question: Apply the mean value theorem.

Use the mean value theorem to find the value of the function at an unknown point c.

$$\int_0^2 f(x)\, dx = 6$$

Answer choices:

A 3

B 0

C 6

D 2

Solution: A

The mean value theorem states that a point c must exist on the given interval $[a, b]$ for the function $f(x)$ such that

$$\int_a^b f(x)\ dx = f(c)(b - a)$$

The given integral tells us that $a = 0$, $b = 2$ and $f(c)(b - a) = 6$. We can plug the values of a and b into $f(c)(b - a) = 6$ and then solve for $f(c)$, which is the value of the function at the unknown point c.

$$f(c)(2 - 0) = 6$$

$$f(c) = 3$$

Topic: Mean value theorem for integrals

Question: Apply the mean value theorem.

Use the mean value theorem to find the value of the function at an unknown point c.

$$\int_1^5 x^2\ dx$$

Answer choices:

A $\frac{62}{9}$

B $\frac{31}{3}$

C $\frac{31}{12}$

D $\frac{124}{3}$

Solution: B

The Mean Value Theorem states that a point c must exist on the given interval $[a, b]$ for the function $f(x)$ such that

$$\int_a^b f(x)\ dx = f(c)(b - a)$$

The given integral tells us that $a = 1$, $b = 5$. We need to solve the definite integral and then we can solve for $f(c)$.

$$\int_1^5 x^2\ dx = \frac{1}{3}x^3\Big|_1^5$$

$$\int_1^5 x^2\ dx = \frac{1}{3}(5)^3 - \frac{1}{3}(1)^3$$

$$\int_1^5 x^2\ dx = \frac{124}{3}$$

Using the mean value theorem this means that

$$f(c)(b - a) = \frac{124}{3}$$

Now we can solve for $f(c)$, which is the value of the function at the unknown point c.

$$f(c)(5 - 1) = \frac{124}{3}$$

$$f(c) = \frac{124}{12}$$

$$f(c)=\frac{31}{3}$$