Calculus 2 Workbook

krista king
MATH

INDEFINITE INTEGRALS

1. Evaluate the indefinite integral.

$$\int 5x^4 - 4x^3 + 6x^2 - 2x + 1 \ dx$$

2. Evaluate the indefinite integral.

$$\int \frac{3x^3 + x^2 - 12x - 4}{x^2 - 4} \ dx$$

3. Evaluate the indefinite integral.

$$\int (5x - 7)(3x + 2) \ dx$$

4. Evaluate the indefinite integral.

$$\int \frac{x^3 - 3x + 2}{x^3} \ dx$$

PROPERTIES OF INTEGRALS

1. Given the value of each of these integrals,

$$\int_0^3 f(x)\ dx = 7 \qquad \int_3^6 f(x)\ dx = 9 \qquad \int_0^3 g(x)\ dx = 2 \qquad \int_3^6 g(x)\ dx = 5$$

what is the value of of the following integral?

$$\int_0^6 \left[2f(x) + 3g(x)\right]\ dx$$

FIND F GIVEN F''

1. Find $f(x)$ from its second derivative.

$$f''(x) = 3x^2 + 4x - 7$$

2. Find $g(x)$ from its second derivative.

$$g''(x) = \frac{x^4 - 4x^2 + 4}{x^2 - 2}$$

3. Find $h(x)$ from its second derivative.

$$h''(x) = \frac{8x^3 - 9x^2 + 6x}{x^7}$$

FIND F GIVEN F’’’

1. Find $f(x)$ given its third derivative.

$$f'''(x) = 2x + 3$$

2. Find $g(x)$ given its third derivative.

$$g'''(x) = 4x^3 + x^2 - 3$$

3. Find $h(x)$ given its third derivative.

$$h'''(x) = \frac{3}{x^5} - \frac{2}{x^4} + 4$$

INITIAL VALUE PROBLEMS

■ 1. Find $f(x)$ if $f'(x) = 7x - 5$ and $f(4) = 24$.

■ 2. Find $g(x)$ if $g'(x) = 2x^2 + 5x - 9$ and $g(-4) = 34$.

■ 3. Find $h(x)$ if $h'(x) = 3x^2 + 8x + 1$ and $h(2) = 31$.

■ 4. Find $f(x)$ if $f'(x) = x^3 + 4x + 3$ and $f(-2) = 15$.

FIND F GIVEN F’’ AND INITIAL CONDITIONS

■ 1. Find $g(x)$ if $g''(x) = 2x + 1$, $g'(1) = 5$, and $g(1) = 4$.

■ 2. Find $h(x)$ if $h''(x) = 2x - 7$, $h'(3) = -20$, and $h(6) = -98$.

■ 3. Find $f(x)$ if $f''(x) = 3x - 6$, $f'(2) = 2$, and $f(2) = 15$.

DEFINITE INTEGRALS

■ 1. Evaluate the definite integral.

$$\int_{0}^{3} x^3 + x^2 + x + 1 \ dx$$

■ 2. Evaluate the definite integral.

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{2}} 2 \sin x + 3 \cos x \ dx$$

■ 3. Evaluate the definite integral.

$$\int_{-4}^{4} 2x^3 - 4x^2 + 25 \ dx$$

■ 4. Evaluate the definite integral.

$$\int_{1}^{2} 6x^5 - 8x^3 + 4x + 3 \ dx$$

■ 5. Evaluate the definite integral.

$$\int_0^{\pi} 5\sin x \ dx$$

AREA UNDER OR ENCLOSED BY THE CURVE

■ 1. Find the area under the graph of $f(x) = 2x^2 - 3x + 5$ over the interval $[-2,6]$.

■ 2. Find the area enclosed by the graph of $g(x) = 2x(x + 4)(x - 2)$ over the interval $[-4,2]$.

■ 3. Find the area under the graph of $h(x) = 3\sqrt{x}$ over the interval $[4,16]$.

DEFINITE INTEGRALS OF EVEN AND ODD FUNCTIONS

1. Evaluate the definite integral.

$$\int_{-3}^{3} -x^4 + 19\ dx$$

2. Evaluate the definite integral.

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} 7\cos x\ dx$$

3. Evaluate the definite integral.

$$\int_{-2}^{2} \frac{3}{4}x^2 + 5\ dx$$

4. Evaluate the definite integral.

$$\int_{-2}^{2} 3x^5 - 4x^3 + 8x\ dx$$

5. Evaluate the definite integral.

$$\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} 9\sin x \ dx$$

■ 6. Evaluate the definite integral.

$$\int_{-2}^{2} 2x^3 - 4x \ dx$$

SUMMATION NOTATION, FINDING THE SUM

1. Calculate the exact sum.

$$\sum_{n=1}^{6} \frac{2n^2}{3^n}$$

2. Calculate the exact sum.

$$\sum_{n=1}^{5} \frac{2n}{3n+1}$$

3. Calculate the exact sum.

$$\sum_{n=0}^{6} 3n^2 - 5n + 7$$

SUMMATION NOTATION, EXPANDING

1. Expand the sum.

$$\sum_{n=1}^{6} \frac{5n+3}{2n-1}$$

2. Expand the sum.

$$\sum_{n=0}^{7} 2x^3 - 5x^2 + 9x + 3$$

3. Expand the sum.

$$\sum_{n=0}^{8} \frac{2n-8}{n+1}$$

SUMMATION NOTATION, COLLAPSING

■ 1. Use summation notation to rewrite the sum.

$$\frac{(x+3)^2}{3-1}+\frac{(x+3)^4}{9-2}+\frac{(x+3)^6}{27-3}+\frac{(x+3)^8}{81-4}+\frac{(x+3)^{10}}{243-5}+\frac{(x+3)^{12}}{729-6}$$

■ 2. Use summation notation to rewrite the sum.

$$\frac{3x+1}{7x}+\frac{6x+2}{14x^2}+\frac{9x+3}{21x^3}+\frac{12x+4}{28x^4}+\frac{15x+5}{35x^5}+\frac{18x+6}{42x^6}$$

■ 3. Use summation notation to rewrite the sum.

$$\frac{x^2-3x+1}{4x}+\frac{x^3-6x+2}{8x}+\frac{x^4-9x+3}{12x}+\frac{x^5-12x+4}{16x}$$

$$+\frac{x^6-15x+5}{20x}+\frac{x^7-18x+6}{24x}+\frac{x^8-21x+7}{28x}$$

RIEMANN SUMS, LEFT ENDPOINTS

1. Use a left endpoint Riemann Sum with $n = 5$ to find the area under $f(x)$ on the interval $[0,10]$.

x	0	1	2	3	4	5	6	7	8	9	10
f(x)	3	2	3	6	11	18	27	38	51	66	83

2. Use a left endpoint Riemann Sum with $n = 5$ to find the area under $g(x)$ on the interval $[0,20]$. Round the final answer to 2 decimal places.

$$g(x) = 2\sqrt{x} + 5$$

3. Use a left endpoint Riemann Sum with $n = 3$ to find the area under $h(x)$ on the interval $[-2,4]$.

$$h(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - x + 3$$

4. Use a left endpoint Riemann Sum with $n = 4$ to find the area under $k(x)$ on the interval $[0,28]$. Round the final answer to 2 decimal places.

$$k(x) = \frac{x^2 + 4x + 4}{x^2 + 4}$$

5. Use a left endpoint Riemann Sum with $n = 4$ to find the area under $f(x)$ on the interval $[0,2]$. Round the final answer to 2 decimal places.

$$f(x) = 2\ln(x+3) + 6$$

6. Use a left endpoint Riemann Sum with $n = 5$ to find the area under $g(x)$ on the interval $[0,1]$. Round the final answer to 2 decimal places.

$$g(x) = x^4 + 2x^3 - 3x^2 + 4x + 5$$

RIEMANN SUMS, RIGHT ENDPOINTS

1. Use a right endpoint Riemann Sum with $n = 5$ to find the area under $g(x)$ on the interval $[1,11]$.

x	1	2	3	4	5	6	7	8	9	10	11
g(x)	5	4	5	8	13	20	29	40	53	68	85

2. Use a right endpoint Riemann Sum with $n = 5$ to find the area under $f(x)$ on the interval $[5,25]$. Round the final answer to 2 decimal places.

$$f(x) = \sqrt{2x} - 1$$

3. Use a right endpoint Riemann Sum with $n = 3$ to find the area under $h(x)$ on the interval $[-2,4]$.

$$h(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - x + 3$$

4. Use a right endpoint Riemann Sum with $n = 4$ to find the area under $k(x)$ on the interval $[0,28]$. Round the final answer to 2 decimal places.

$$k(x) = \frac{x^2 + 4x + 4}{x^2 + 4}$$

5. Use a right endpoint Riemann Sum with $n=4$ to find the area under $f(x)$ on the interval $[0,2]$. Round the final answer to 2 decimal places.

$$f(x)=2\ln(x+3)+6$$

6. Use a right endpoint Riemann Sum with $n=5$ to find the area under $h(x)$ on the interval $[0,1]$. Round the final answer to 2 decimal places.

$$h(x)=x^4+2x^3-3x^2+4x+5$$

RIEMANN SUMS, MIDPOINTS

■ 1. Use a midpoint Riemann Sum with $n = 5$ to find the area under $h(x)$ on the interval $[6,16]$.

x	6	7	8	9	10	11	12	13	14	15	16
h(x)	84	67	52	39	26	17	10	7	4	3	4

■ 2. Use a midpoint Riemann Sum with $n = 5$ to find the area under $k(x)$ on the interval $[2,22]$. Round the final answer to 2 decimal places.

$$k(x) = 3\sqrt{7x} - 8$$

■ 3. Use a midpoint Riemann Sum with $n = 3$ to find the area under $h(x)$ on the interval $[-2,4]$.

$$h(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - x + 3$$

■ 4. Use a midpoint Riemann Sum with $n = 4$ to find the area under $k(x)$ on the interval $[0,28]$. Round the final answer to 2 decimal places.

$$k(x) = \frac{x^2 + 4x + 4}{x^2 + 4}$$

■ 5. Use a midpoint Riemann Sum with $n = 4$ to find the area under $f(x)$ on the interval $[0,2]$. Round the final answer to 2 decimal places.

$$f(x) = 2\ln(x + 3) + 6$$

■ 6. Use a midpoint Riemann Sum with $n = 5$ to find the area under $g(x)$ on the interval $[0,1]$. Round the final answer to 2 decimal places.

$$g(x) = x^4 + 2x^3 - 3x^2 + 4x + 5$$

MOVING FROM SUMMATION NOTATION TO THE INTEGRAL

■ 1. Convert the Riemann sum to a definite integral over the interval $[1,8]$.

$$\sum_{i=1}^{n} \left(6x_i^5 - 4x_i^{\frac{4}{3}} + 2x_i^{-3}\right)\Delta x$$

■ 2. Convert the Riemann sum to a definite integral over the interval $[-2,4]$.

$$\sum_{i=1}^{n} \left((5x_i + 3)(2x_i^2 + x_i)^5\right)\Delta x$$

■ 3. Convert the Riemann sum to a definite integral over the interval $[5,11]$.

$$\sum_{i=1}^{n} \left((4 - x_i)\sqrt{x_i - 5}\right)\Delta x$$

OVER AND UNDERESTIMATION

1. Use a Riemann sum to estimate the maximum and minimum area under the curve on $[0,\pi]$. Use rectangular approximation methods with 4 equal subintervals. Round the answer to 2 decimal places.

$$f(x) = 5\sin\frac{x}{2} + 3$$

2. Use a Riemann sum to estimate the maximum and minimum area under the curve on $[0,4]$. Use rectangular approximation methods with 4 equal subintervals.

$$g(x) = \frac{1}{4}(x-4)^2 + 1$$

3. Use a Riemann sum to estimate the maximum and minimum area under the curve on $[0,9]$. Use rectangular approximation methods with 3 equal subintervals. Round the answer to 2 decimal places.

$$h(x) = \frac{1}{2}\sqrt{7x} + 2$$

LIMIT PROCESS TO FIND AREA ON [A,B]

■ 1. Use the limit process to find the area of the region between the graph of $f(x)$ and the x-axis on the interval $[3,7]$.

$$f(x) = x^2 + 2$$

■ 2. Use the limit process to find the area of the region between the graph of $g(x)$ and the x-axis on the interval $[2,6]$.

$$f(x) = x^2 - x + 3$$

■ 3. Use the limit process to find the area of the region between the graph of $h(x)$ and the x-axis on the interval $[2,5]$.

$$h(x) = x^2 - 3x + 7$$

LIMIT PROCESS TO FIND AREA ON [-A,A]

■ 1. Use the limit process to find the area of the region between the graph of $f(x)$ and the x-axis on the interval $[-5,5]$.

$$f(x) = x^2 + 1$$

■ 2. Use the limit process to find the area of the region between the graph of $g(x)$ and the x-axis on the interval $[-3,3]$.

$$g(x) = 3x^2 - 4$$

■ 3. Use the limit process to find the area of the region between the graph of $h(x)$ and the x-axis on the interval $[-1,1]$.

$$h(x) = 4x^2 - x + 1$$

TRAPEZOIDAL RULE

1. Using $n=6$ and the Trapezoidal rule, approximate the value of the integral. Round the answer to 2 decimal places.

$$\int_4^{16} 2\sqrt[3]{x}+3\ dx$$

2. Using $n=6$ and the Trapezoidal rule, approximate the value of the integral.

$$\int_0^6 \frac{1}{4}x^4-\frac{1}{2}x^3+2x^2-5x+8\ dx$$

3. Using $n=4$ and the Trapezoidal rule, approximate the value of the integral.

$$\int_0^8 \frac{1}{2}x^2-3x+6\ dx$$

4. Using $n=4$ and the Trapezoidal rule, approximate the value of the integral.

$$\int_0^{16} \frac{1}{16}x^4-\frac{1}{2}x^3-x^2-x+1\ dx$$

SIMPSON'S RULE

1. Use Simpson's Rule with $n = 6$ to approximate the value of the integral. Round the answer to 2 decimal places.

$$\int_{2}^{8} 6\sqrt{3x} + 5 \ dx$$

2. Use Simpson's Rule with $n = 8$ to approximate the value of the integral. Round the answer to 2 decimal places.

$$\int_{4}^{28} 120(0.95)^x \ dx$$

3. Use Simpson's Rule with $n = 4$ to approximate the value of the integral. Round the answer to 2 decimal places.

$$\int_{5}^{7} 3\ln(x+5) - 2 \ dx$$

4. Use Simpson's Rule with $n = 4$ to approximate the value of the integral.

$$\int_{-3}^{9} x^2 + 3x + 2 \ dx$$

5. Use Simpson's Rule with $n=6$ to approximate the value of the integral. Round the answer to 2 decimal places.

$$\int_{0.4}^{1.6} \frac{1}{3}x^3 - x^2 + 5x + 4\ dx$$

MIDPOINT RULE ERROR BOUND

■ 1. Calculate the area under the curve. Then use the Midpoint Rule with $n=3$ to approximate the same area. Compare the actual area to the result to determine the error of the of the Midpoint Rule approximation.

$$\int_0^6 3x^2-2x+5\ dx$$

■ 2. Calculate the area under the curve. Then use the Midpoint Rule with $n=4$ to approximate the same area. Compare the actual area to the result to determine the error of the of the Midpoint Rule approximation. Round your answer to the nearest 3 decimal places.

$$\int_5^{13} 4\sqrt{x-2}\ dx$$

■ 3. Calculate the area under the curve. Then use the Midpoint Rule with $n=4$ to approximate the same area. Compare the actual area to the result to determine the error of the of the Midpoint Rule approximation.

$$\int_2^{10} 4x^3-3x^2+2x-1\ dx$$

TRAPEZOIDAL RULE ERROR BOUND

1. Calculate the area under the curve. Then use the Trapezoidal Rule with $n = 4$ to approximate the same area. Compare the actual area to the result to determine the error of the of the Trapezoidal Rule approximation.

$$\int_1^5 6x^2 - 8x + 5 \ dx$$

2. Calculate the area under the curve. Then use the Trapezoidal Rule with $n = 5$ to approximate the same area. Compare the actual area to the result to determine the error of the of the Trapezoidal Rule approximation. Round your answer to the nearest 3 decimal places.

$$\int_2^{12} e^{-x} + 3 \ dx$$

3. Calculate the area under the curve. Then use the Trapezoidal Rule with $n = 4$ to approximate the same area. Compare the actual area to the result to determine the error of the of the Trapezoidal Rule approximation. Round your answer to the nearest three decimal places.

$$\int_0^2 4\sqrt{x} + 1 \ dx$$

SIMPSON'S RULE ERROR BOUND

1. Calculate the area under the curve. Then use Simpson's Rule with $n = 6$ to approximate the same area. Compare the actual area to the result to determine the error of the of Simpson's Rule approximation. Round your answer to the nearest three decimal places.

$$\int_{2.2}^{3.4} x^2 - x + 2\ dx$$

2. Calculate the area under the curve. Then use Simpson's Rule with $n = 4$ to approximate the same area. Compare the actual area to the result to determine the error of the of Simpson's Rule approximation. Round your answer to the nearest four decimal places.

$$\int_{0}^{1.2} e^x - 2x + 3\ dx$$

3. Calculate the area under the curve. Then use Simpson's Rule with $n = 4$ to approximate the same area. Compare the actual area to the result to determine the error of the of Simpson's Rule approximation. Round your answer to the nearest three decimal places.

$$\int_{-4}^{4} 2x^2 + 3x + 4\ dx$$

PART 1 OF THE FTC

■ 1. Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of $f(x)$.

$$f(x) = \int_{0}^{x^2} 7t\cos(2t)\ dt$$

■ 2. Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of $g(x)$.

$$g(x) = \int_{2}^{x^3} \frac{5}{3+e^t}\ dt$$

■ 3. Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of $h(x)$.

$$h(x) = \int_{\cos(3x)}^{7} 8t+1\ dt$$

■ 4. Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of $f(x)$.

$$f(x) = \int_{1}^{3x^2} \frac{\sin t}{t^3 + 5}\ dt$$

5. Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of $g(x)$.

$$g(x) = \int_{3x}^{2x^2} t^2 - 5t + 4\ dt$$

PART 2 OF THE FTC

■ 1. Use Part 2 of the Fundamental Theorem of Calculus to evaluate the integral.

$$\int_{2}^{5} 5 - \frac{3}{x} \ dx$$

■ 2. Use Part 2 of the Fundamental Theorem of Calculus to evaluate the integral.

$$\int_{4}^{9} 4x^3 - \sqrt{x} \ dx$$

■ 3. Use Part 2 of the Fundamental Theorem of Calculus to evaluate the integral.

$$\int_{-3}^{-1} \frac{3}{x^3} \ dx$$

■ 4. Use Part 2 of the Fundamental Theorem of Calculus to evaluate the integral.

$$\int_{25}^{36} \frac{2-\sqrt{x}}{\sqrt{x}}\ dx$$

NET CHANGE THEOREM

■ 1. Suppose the position of a particle moving along the horizontal s-axis is at $s=-2$ when $t=0$. The velocity of the particle is given by $v(t)$ with $0 \le t \le 10$, where t is time in seconds since the particle began moving. Use the Net Change Theorem to determine the position of the particle on the s-axis after the particle has been moving for 5 seconds.

$$v(t)=\frac{1}{4}t^2-\frac{9}{(t+1)^2}$$

■ 2. Water is being pumped from a tank at a rate (in gallons per minute) given by $w(t)=80-4\sqrt{t+3}$, with $0 \le t \le 60$, where t is the time in minutes since the pumping began. The tank had $5{,}000$ gallons of water in it when pumping began. Use the Net Change Theorem to determine how many gallons of water will be in the tank after 30 minutes of pumping.

■ 3. From 1990 to 2010, the rate of rice consumption in a particular country was $R(t)=5.8+1.07^t$ million pounds per year, with t being years since the beginning of the year 1990. The country had 7.2 million pounds of rice on hand at the beginning of 1994 and produced 7.5 million pounds of rice every year. Use the Net Change Theorem to determine how many millions of pounds of rice were on hand in that country at the end of 1998.

■ 4. A cooling pump connected to a power plant operates at a varying rate, depending on how much cooling is needed by the power plant. The rate (in gallons per second) at which the pump is operated is modeled by $r(t) = 0.003t^3 - 0.02t^2 + 0.29t + 59.81$, where t is defined in seconds for $0 \le t \le 120$. The pump has already pumped $1{,}508$ gallons during the first 25 seconds. Use the Net Change Theorem to determine how many gallons the pump will have pumped after 2 minutes.

■ 5. A rocket is launched upward from a cliff that's 86 feet above ground level. The velocity of the rocket is modeled by $v(t) = -32t + 88$, in feet per second, where t is seconds after the launch. Use the Net Change Theorem to determine the height in feet of the rocket 2 seconds after it's launched.

U-SUBSTITUTION IN DEFINITE INTEGRALS

1. Use u-substitution to evaluate the integral.

$$\int_2^4 8x^3\sqrt{7+x^4}\ dx$$

INTEGRATION BY PARTS

1. Use integration by parts to evaluate the integral.

$$\int 9x \sin x \ dx$$

2. Use integration by parts to evaluate the integral.

$$\int 5xe^x \ dx$$

3. Use integration by parts to evaluate the integral.

$$\int 7x \ln x \ dx$$

4. Use integration by parts to evaluate the integral.

$$\int 2x \cos x \ dx$$

5. Use integration by parts to evaluate the integral.

$$\int 3\sqrt{x}\ln x \, dx$$

INTEGRATION BY PARTS TWO TIMES

1. Apply integration by parts two times to evaluate the integral.

$$\int 3x^2 e^x \, dx$$

2. Use integration by parts to evaluate the integral.

$$\int e^{3x} \cos(5x) \, dx$$

INTEGRATION BY PARTS THREE TIMES

1. Apply integration by parts three times to evaluate the integral.

$$\int 7x^3e^x\ dx$$

2. Apply integration by parts three times to evaluate the integral.

$$\int \left(2x^3 + x^2\right) e^x\ dx$$

3. Use integration by parts three times to evaluate the integral.

$$\int (\ln x)^3\ dx$$

INTEGRATION BY PARTS WITH U-SUBSTITUTION

1. Use integration by parts and substitution to evaluate the integral.

$$\int \tan^{-1} x \ dx$$

2. Use integration by parts and substitution to evaluate the integral.

$$\int 7x \cos(9x) \ dx$$

3. Use integration by parts and substitution to evaluate the integral.

$$\int \ln(3x + 5) \ dx$$

PROVE THE REDUCTION FORMULA

■ 1. Use integration by parts, and $n = 8$, to prove the reduction formula for the integral.

$$\int x^n \sin x \ dx = -x^n \cos x + n \int x^{n-1} \cos x \ dx$$

■ 2. Use integration by parts, and $n = 11$, to prove the reduction formula for the integral.

$$\int x^n \cos x \ dx = x^n \sin x - n \int x^{n-1} \sin x \ dx$$

■ 3. Use integration by parts, $a = 5$, and $n = 9$, to prove the reduction formula for the integral.

$$\int x^n a^x \ dx = \frac{x^n a^x}{\ln a} - \frac{n}{\ln a} \int x^{n-1} a^x \ dx$$

TABULAR INTEGRATION

■ 1. Use tabular integration to evaluate the integral.

$$\int \left(5x^2+4x-3\right)e^{2x}\ dx$$

■ 2. Use tabular integration to evaluate the integral.

$$\int x^3\cos(3x)\ dx$$

■ 3. Use tabular integration to evaluate the integral.

$$\int \frac{x^4e^x}{6}\ dx$$

DISTINCT LINEAR FACTORS

■ 1. Use partial fractions to evaluate the integral.

$$\int \frac{4x+5}{x^2+5x+6}\,dx$$

DISTINCT QUADRATIC FACTORS

■ 1. Use partial fractions to evaluate the integral.

$$\int \frac{3x+6}{(x^2+2)(x^2+1)}\ dx$$

REPEATED LINEAR FACTORS

1. Use partial fractions to evaluate the integral.

$$\int \frac{5x-3}{(x+2)^2}\,dx$$

2. Use partial fractions to evaluate the integral.

$$\int \frac{x+12}{(3x-2)^2}\,dx$$

3. Use partial fractions to evaluate the integral.

$$\int \frac{7x-4}{(5x+1)^2}\,dx$$

4. Use partial fractions to evaluate the integral.

$$\int \frac{12x+9}{(2x+7)^2}\,dx$$

5. Use partial fractions to evaluate the integral.

$$\int \frac{24x+41}{(3x+4)^2}\,dx$$

REPEATED QUADRATIC FACTORS

1. Rewrite the integral using partial fractions, but do not evaluate it.

$$\int \frac{x^2 - 3x + 2}{(x^2 + 2)^2}\, dx$$

2. Rewrite the integral using partial fractions, but do not evaluate it.

$$\int \frac{x^2 - 4x + 6}{(x^2 + 3)^2}\, dx$$

3. Rewrite the integral using partial fractions, but do not evaluate it.

$$\int \frac{4x^3 - 2x^2 + x + 1}{(2x^2 + 1)^2}\, dx$$

4. Rewrite the integral using partial fractions, but do not evaluate it.

$$\int \frac{x^3 - 2x^2 + 3x + 5}{(x^2 + 1)^3}\, dx$$

RATIONALIZING SUBSTITUTIONS

■ 1. Use a rationalizing substitution to rewrite the integral in terms of u, but don't integrate it.

$$\int \frac{\sqrt{x+16}}{x}\ dx$$

■ 2. Use a rationalizing substitution to rewrite the integral in terms of u, but don't integrate it.

$$\int \frac{\sqrt{3x+5}}{x}\ dx$$

■ 3. Use a rationalizing substitution to rewrite the integral in terms of u, but don't integrate it.

$$\int \frac{\sqrt{7x-2}}{x}\ dx$$

HOW TO FACTOR DIFFICULT DENOMINATORS

■ 1. Use partial fractions to factor the denominator and evaluate the integral.

$$\int \frac{2x^2 - 5x + 4}{4x^3 - x^2 - 4x + 1}\ dx$$

■ 2. Use partial fractions to factor the denominator and evaluate the integral.

$$\int \frac{3x^2 + 7x - 2}{x^5 - 34x^3 + 225x}\ dx$$

■ 3. Use partial fractions to factor the denominator and evaluate the integral.

$$\int \frac{4x^2 + 3x + 1}{x^5 + x^4 - 13x^3 - 13x^2 + 36x + 36}\ dx$$

TWO WAYS TO FIND THE CONSTANTS

■ 1. Use an alternative approach for partial fractions to evaluate the integral by setting the factors of the constants equal to 0 to find the other constants.

$$\int \frac{3x-2}{x^2+9x+18}\ dx$$

■ 2. Use an alternative approach for partial fractions to evaluate the integral by setting the factors of the constants equal to 0 to find the other constants.

$$\int \frac{8x+13}{x^2+x-12}\ dx$$

■ 3. Use an alternative approach for partial fractions to evaluate the integral by setting the factors of the constants equal to 0 to find the other constants.

$$\int \frac{x-21}{9x^2+9x+2}\ dx$$

SIN^M COS^N, ODD M

1. Evaluate the trigonometric integral.

$$\int \sin^5(3x^2 + 2x + 1)\cos(3x^2 + 2x + 1)(6x + 2)\ dx$$

SIN^M COS^N, ODD N

1. Evaluate the trigonometric integral.

$$\int_{-\frac{\pi}{6}}^{\frac{\pi}{3}} (4 + \cos x)\sin x \ dx$$

2. Evaluate the trigonometric integral.

$$\int \sin(2x)\cos^3(2x) \ dx$$

SIN^M COS^N, M AND N EVEN

1. Evaluate the trigonometric integral.

$$\int \sin^2(2x+3)\cos^2(2x+3)\ dx$$

2. Evaluate the trigonometric integral.

$$\int \sin^4(2x)\cos^2(2x)\ dx$$

3. Evaluate the trigonometric integral.

$$\int \sin^6(3x)\cos^4(3x)\ dx$$

TAN^M SEC^N, ODD M

1. Evaluate the trigonometric integral.

$$\int \tan^3(2x)\sec(2x)\ dx$$

2. Evaluate the trigonometric integral.

$$\int \tan^5(3x)\sec(3x)\ dx$$

TAN^M SEC^N, EVEN N

1. Evaluate the trigonometric integral.

$$\int \tan^2(4x)\sec^4(4x)\ dx$$

2. Evaluate the trigonometric integral.

$$\int \tan^4(2x)\sec^4(2x)\ dx$$

3. Evaluate the trigonometric integral.

$$\int \tan^4(3x-1)\sec^4(3x-1)\ dx$$

SIN(MX) COS(NX)

1. Evaluate the trigonometric integral.

$$\int 5\sin(6x)\cos(3x)\ dx$$

2. Evaluate the trigonometric integral.

$$\int 2\sin(9x)\cos(4x)\ dx$$

3. Evaluate the trigonometric integral.

$$\int \frac{1}{3}\sin(12x)\cos(7x)\ dx$$

SIN(MX) SIN(NX)

■ 1. Evaluate the trigonometric integral.

$$\int 6\sin(9x)\sin(2x)\ dx$$

■ 2. Evaluate the trigonometric integral.

$$\int \frac{1}{2}\sin(8x)\sin(4x)\ dx$$

■ 3. Evaluate the trigonometric integral.

$$\int 8\sin(14x)\sin(7x)\ dx$$

COS(MX) COS(NX)

■ 1. Evaluate the trigonometric integral.

$$\int 7\cos(8x)\cos(3x)\ dx$$

■ 2. Evaluate the trigonometric integral.

$$\int 5\cos(15x)\cos(5x)\ dx$$

■ 3. Evaluate the trigonometric integral.

$$\int 49\cos(21x)\cos(14x)\ dx$$

INVERSE HYPERBOLIC INTEGRALS

1. Evaluate the hyperbolic integral.

$$\int x \sinh(3x^2 + 7)\ dx$$

2. Evaluate the hyperbolic integral using the substitution $x = \sqrt{3} \cosh u$.

$$\int \frac{\sqrt{x^2 - 3}}{x^2}\ dx$$

TRIGONOMETRIC SUBSTITUTION WITH SECANT

■ 1. Set up and simplify the integral for trig substitution, but don't integrate.

$$\int \frac{3}{\sqrt{9x^2+6x}}\ dx$$

■ 2. Set up and simplify the integral for trig substitution, but don't integrate.

$$\int \frac{5}{\sqrt{4x^2+4x}}\ dx$$

■ 3. Set up and simplify the integral for trig substitution, but don't integrate.

$$\int \frac{dx}{x^2\sqrt{x^2-9}}$$

■ 4. Set up and simplify the integral for trig substitution, but don't integrate.

$$\int \frac{4\ dx}{x^2\sqrt{x^2-25}}$$

TRIGONOMETRIC SUBSTITUTION WITH SINE

1. Set up and simplify the integral for trig substitution, but don't integrate.

$$\int \frac{3x}{\sqrt{64-49x^2}}\ dx$$

2. Set up and simplify the integral for trig substitution, but don't integrate.

$$\int \frac{2x}{\sqrt{121-144x^2}}\ dx$$

3. Set up and simplify the integral for trig substitution, but don't integrate.

$$\int \frac{6x}{\sqrt{81-36x^2}}\ dx$$

4. Set up and simplify the integral for trig substitution, but don't integrate.

$$\int \frac{35x}{\sqrt{25-100x^2}}\,dx$$

TRIGONOMETRIC SUBSTITUTION WITH TANGENT

■ 1. Set up and simplify the integral for trig substitution, but don’t integrate.

$$\int \sqrt{36x^2+25}\ dx$$

■ 2. Set up and simplify the integral for trig substitution, but don’t integrate.

$$\int \sqrt{4x^2+81}\ dx$$

■ 3. Set up and simplify the integral for trig substitution, but don’t integrate.

$$\int \frac{7}{\sqrt{x^2+4x+8}}\ dx$$

IMPROPER INTEGRALS, CASE 1

1. Evaluate the improper integral.

$$\int_1^\infty \frac{5}{x^3}\ dx$$

2. Evaluate the improper integral.

$$\int_3^\infty \frac{7}{(x-2)^2}\ dx$$

3. Evaluate the improper integral.

$$\int_0^\infty 2e^{-2x}\ dx$$

4. Evaluate the improper integral.

$$\int_0^\infty \frac{3x}{2+2x^2}\ dx$$

IMPROPER INTEGRALS, CASE 2

1. Evaluate the improper integral.

$$\int_{-\infty}^{0} e^{3x}\ dx$$

2. Evaluate the improper integral.

$$\int_{-\infty}^{1} xe^{x^2}\ dx$$

3. Evaluate the improper integral.

$$\int_{-\infty}^{-2} \frac{2}{x-1} - \frac{2}{x+1}\ dx$$

4. Evaluate the improper integral.

$$\int_{-\infty}^{3} \frac{3}{x^2+9}\ dx$$

5. Evaluate the improper integral.

$$\int_{-\infty}^{0} \frac{2\ dx}{e^x}$$

6. Evaluate the improper integral.

$$\int_{-\infty}^{0} 4e^{-4x}\ dx$$

IMPROPER INTEGRALS, CASE 3

1. Evaluate the improper integral.

$$\int_{-\infty}^{\infty} 2xe^{-x^2}\ dx$$

2. Evaluate the improper integral.

$$\int_{-\infty}^{\infty} \frac{3\ dx}{x^2+1}$$

3. Evaluate the improper integral.

$$\int_{-\infty}^{\infty} x^2+7x+1\ dx$$

4. Evaluate the improper integral.

$$\int_{-\infty}^{\infty} 3x^2e^{-x^3}\ dx$$

IMPROPER INTEGRALS, CASE 4

1. Evaluate the improper integral.

$$\int_{-\frac{\pi}{2}}^{0} \frac{3\cos x}{2\sin x}\ dx$$

2. Evaluate the improper integral.

$$\int_{-8}^{0} \frac{e^x\ dx}{e^x - 1}$$

3. Evaluate the improper integral.

$$\int_{-9}^{0} \frac{e^{\sqrt{-x}}\ dx}{\sqrt{-x}}$$

4. Evaluate the improper integral.

$$\int_{1}^{3} \frac{2x - 3}{\sqrt{3x - x^2}}\ dx$$

5. Evaluate the improper integral.

$$\int_{0}^{2\sqrt{2}} \frac{x}{\sqrt{8-x^2}}\ dx$$

■ 6. Evaluate the improper integral.

$$\int_{1}^{3} \frac{x-1}{x^2-4x+3}\ dx$$

IMPROPER INTEGRALS, CASE 5

1. Evaluate the improper integral.

$$\int_0^2 \frac{3}{\sqrt[3]{x}}\ dx$$

2. Evaluate the improper integral.

$$\int_{-1}^5 \frac{3}{\sqrt{x+1}}\ dx$$

3. Evaluate the improper integral.

$$\int_3^7 \frac{5}{x-3}\ dx$$

4. Evaluate the improper integral.

$$\int_0^6 \frac{9}{5\sqrt[4]{x^3}}\ dx$$

5. Evaluate the improper integral.

$$\int_{-1}^{7} \frac{x^2}{x^3+1}\,dx$$

6. Evaluate the improper integral.

$$\int_{-4}^{4} \frac{x+4}{x^2+8x+16}\,dx$$

IMPROPER INTEGRALS, CASE 6

1. Evaluate the improper integral.

$$\int_{-2}^{2} \frac{3}{2\sqrt[5]{x^3}}\ dx$$

2. Evaluate the improper integral.

$$\int_{0}^{4} \frac{7\ dx}{2(x-2)^2}$$

3. Evaluate the improper integral.

$$\int_{-27}^{8} \frac{3\ dx}{\sqrt[3]{x}}$$

4. Evaluate the improper integral.

$$\int_{-3}^{3} \frac{x+2}{x^2-4}\ dx$$

5. Evaluate the improper integral.

$$\int_0^6 \frac{4}{x-3} - \frac{4}{x+3}\ dx$$

COMPARISON THEOREM

1. Use the Comparison Theorem to say whether the integral converges or diverges.

$$\int_1^\infty \frac{1}{2+2x^2}\ dx$$

2. Use the Comparison Theorem to say whether the integral converges or diverges.

$$\int_1^\infty \frac{1}{5x+e^x}\ dx$$

3. Can we use the harmonic series $1/x$ as a comparison series to say whether or not the integral converges?

$$\int_1^\infty \frac{x}{x^2+1}\ dx$$

INTEGRALS USING REDUCTION FORMULAS

■ 1. Use a reduction formula to evaluate the integral.

$$\int \cot^4 x \ dx$$

■ 2. Use a reduction formula to evaluate the integral.

$$\int \sec^4 x \ dx$$

■ 3. Use a reduction formula to evaluate the integral.

$$\int \csc^4 x \ dx$$

AREA BETWEEN UPPER AND LOWER CURVES

1. Find the area, in square units, between the two curves. Round your answer to two decimal places.

$$f(x) = -2x^2 + 7$$

$$g(x) = -x + 3$$

2. Find the area, in square units, between the two curves.

$$f(x) = -3x^2 + 9x$$

$$g(x) = 3x^2 - 9x$$

AREA BETWEEN LEFT AND RIGHT CURVES

■ 1. Find the area, in square units, between the two curves. Round your answer to two decimal places.

$$f(y) = 2y^2 + 12y + 15$$

$$g(y) = -2y^2 - 12y - 15$$

■ 2. Find the area, in square units, between the two curves, and between $y = -2$ and $y = -5$.

$$f(y) = 2y^2 + 12y + 19$$

$$g(y) = -\frac{y^2}{2} - 4y - 10$$

■ 3. Find the area, in square units, between the two curves.

$$f(y) = -y^3 + 6y$$

$$g(y) = -y^2$$

■ 4. Find the area, in square units, between the two curves.

$$f(y) = \frac{y^2}{2} - 3y - \frac{1}{2}$$

$$g(y) = 3$$

■ 5. Find the area, in square units, between the two curves, and between $y = 0$ and $y = 4$.

$$f(y) = 2y^2 - 8y + 9$$

$$g(y) = \frac{y^2}{2} - 2y - 1$$

SKETCHING THE AREA BETWEEN CURVES

1. Find the area of the region in the first quadrant that's enclosed by the graphs of the curves.

$$y = \sqrt{x}$$

$$y = x - 2$$

2. Find the area of the region that's enclosed by the graphs of the curves.

$$y = x^3$$

$$y = \sqrt{x + 2}$$

$$y = -\sqrt{x + 2}$$

3. Find the area of the region that's enclosed by the graphs of the curves.

$$y = 2x^2$$

$$y = x^4 - 2x^2$$

DIVIDING THE AREA BETWEEN CURVES INTO EQUAL PARTS

■ 1. The line $x = k$ divides the area bounded by the curves into two equal parts. Find k.

$$f(x) = 4x - x^2$$

$$g(x) = 5 - 2x$$

■ 2. The line $x = k$ divides the area bounded by the curves into two equal parts, for $x > 0$. Find k. Round your answer to the nearest three decimal places.

$$f(x) = x^3 - 12x$$

$$g(x) = x^2$$

■ 3. The line $x = k$ divides the area bounded by the curves on $\pi/4 \le x \le 5\pi/4$ into two equal parts. Find k.

$$f(x) = \sin x$$

$$g(x) = \cos x$$

ARC LENTH OF Y=F(X)

1. Find the arc length of the curve over $[0,2]$.

$$y = \frac{4\sqrt{2}}{3}x^{\frac{3}{2}} + 6$$

2. Find the arc length of the curve over $[-3,3]$. Round your answer to the nearest three decimal places.

$$y = x^2 - 3$$

3. Set up the arc length integral of the curve over $[-1,2]$. Do not evaluate the integral.

$$y = \frac{x^3}{3} + x^2 + 5$$

4. Set up the arc length integral of the curve over $[-\pi, \pi]$. Do not evaluate the integral.

$$y = \sin x - 5$$

5. Set up the arc length integral of the curve over $[-\pi/4, \pi/4]$. Do not evaluate the integral.

$$y = \tan x \sec x + 2$$

ARC LENTH OF X=G(Y)

1. Find the arc length of the curve on the interval $1 \le y \le 6$.

$$x = \frac{y^2}{2} - \frac{\ln y}{4} - 8$$

2. Find the arc length of the curve on the interval $0 \le y \le 4$.

$$x = \frac{1}{3}(y^2 + 2)^{\frac{3}{2}} + 5$$

3. Find the arc length of the curve on the interval $4 \le y \le 16$.

$$x = y^{\frac{3}{2}} + 15$$

4. Find the arc length of the curve on the interval $1 \le y \le 8$.

$$x = \left(1 - y^{\frac{2}{3}}\right)^{\frac{3}{2}}$$

5. Find the arc length of the curve on the interval $1 \le y \le 5$.

$$x = \frac{y^2}{8} - \ln y$$

AVERAGE VALUE

1. Find the average value of $f(x)$ over the interval $[-3,5]$.

$$f(x) = -3x^3 - 5x^2 + x + 4$$

2. Find the average value of $g(x)$ over the interval $[-4,3]$.

$$g(x) = \frac{1}{3}x^3 + \frac{3}{2}x^2 + \frac{2}{5}x - 2$$

3. Find the average value of $h(x)$ over the interval $[-2,3]$.

$$h(x) = 3(2x - 5)^2$$

4. Set up the average value formula for $f(x)$ over the interval $[-4,4]$. Do not evaluate the integral.

$$f(x) = \sqrt{16 - x^2}$$

MEAN VALUE THEOREM FOR INTEGRALS

1. Use the Mean Value Theorem for integrals to find a value for $f(c)$.

$$\int_{4}^{20} f(x)\ dx = 26$$

2. Use the Mean Value Theorem for integrals to find a value for $g(c)$.

$$\int_{-15}^{35} g(x)\ dx = -20$$

3. Use the Mean Value Theorem for integrals to find a value for $h(c)$.

$$\int_{-1}^{5} h(x)\ dx = 48$$

SURFACE AREA OF REVOLUTION

■ 1. Find the surface area of the object generated by revolving the curve around the x-axis on the interval $2 \leq x \leq 7$.

$$f(x) = \frac{1}{3}x + 4$$

■ 2. Find the surface area of the object generated by revolving the curve around the x-axis on the interval $1 \leq x \leq 5$.

$$g(x) = \frac{2}{3}x + 5$$

■ 3. Set up the integral that approximates the surface area of the object generated by revolving the curve around the x-axis on the interval $-3 \leq x \leq 3$. Do not evaluate the integral.

$$h(x) = x^2 + 3$$

■ 4. Find the surface area of the object generated by revolving the curve around the line $y = -1$ on the interval $3 \leq x \leq 9$.

$$g(x) = 2\sqrt{2}x + 7$$

SURFACE OF REVOLUTION EQUATION

1. Find an equation for the surface generated by revolving the curve around the x-axis.

$$3x^2 + 2y^2 = 8$$

2. Find an equation for the surface generated by revolving the curve around the y-axis.

$$5x^2 = 8y^2$$

3. Find an equation for the surface generated by revolving the curve around the x-axis.

$$9x^2 + 25y^2 = 36$$

DISKS, HORIZONTAL AXIS

■ 1. Use disks to find the volume of the solid that's formed by rotating the region enclosed by the curves about the x-axis.

$$y = x^2 + 2x + 3$$

$x = -3$ and $x = 1$

■ 2. Use disks to find the volume of the solid that's formed by rotating the region enclosed by the curves about the x-axis.

$$y = \sqrt{x - 1}$$

$x = 1$ and $x = 10$

■ 3. Use disks to find the volume of the solid that's formed by rotating the region enclosed by the curves about the x-axis.

$$y = 2\sec x$$

$x = -\frac{\pi}{3}$ and $x = \frac{\pi}{3}$

■ 4. Set up the integral that approximates the volume of the solid that's formed by rotating the region enclosed by the curves about the x-axis. Do not evaluate the integral.

$$y = \arctan x$$

$x = 0$ and $x = 5$

■ 5. Use disks to find the volume of the solid that's formed by rotating the region enclosed by the curves about the x-axis.

$$y = \sqrt{25 - x^2}$$

$x = -4$ and $x = 4$

DISKS, VERTICAL AXIS

■ 1. Use disks to find the volume of the solid that's formed by rotating the region enclosed by the curves about the y-axis.

$x = \frac{1}{6}y - 2$ and $x = 0$

$y = 1$ and $y = 6$

■ 2. Use disks to find the volume of the solid that's formed by rotating the region enclosed by the curves about the y-axis.

$x = \frac{3}{7}\sqrt{y} + 2$ and $x = 0$

$y = 2$ and $y = 5$

■ 3. Use disks to find the volume of the solid that's formed by rotating the region enclosed by the curves about the y-axis.

$x = y^2 + 1$ and $x = 0$

$y = -2$ and $y = 2$

4. Use disks to find the volume of the solid that's formed by rotating the region enclosed by the curves about the y-axis. Set up the integral, but do not evaluate it.

$x = \sin y$

$y = 0$ and $y = \pi$

DISKS, VOLUME OF THE FRUSTUM

■ 1. Use disks to find the volume of the frustum of a right circular cone with height $h = 18$ inches, a lower base radius $R = 9$ inches, and an upper radius of $r = 6$ inches.

■ 2. Use disks to find the volume of the frustum of a right circular cone with height $h = 16$ inches, a lower base radius $R = 12$ inches, and an upper radius of $r = 9$ inches.

■ 3. Use disks to find the volume of the frustum of a right circular cone with height $h = 7$ inches, a lower base radius $R = 8\sqrt{3}$ inches, and an upper radius of $r = \sqrt{3}$ inches.

WASHERS, HORIZONTAL AXIS

1. Use washers to find the volume of the solid that's formed by rotating the region enclosed by the curves about the x-axis.

$y = x^{\frac{2}{3}}$ and $y = 4$

$x = 0$ and $x = 8$

2. Use washers to find the volume of the solid that's formed by rotating the region enclosed by the curves about the x-axis.

$y = x^2$ and $y = \sqrt{x}$

3. Use washers to find the volume of the solid that's formed by rotating the region enclosed by the curves about the x-axis.

$y = x^2$ and $y = x^3$

WASHERS, VERTICAL AXIS

■ 1. Use washers to find the volume of the solid that's formed by rotating the region enclosed by the curves about the y-axis.

$x = y^2 - 4y + 6$ and $x = 6$

$y = 2$ and $y = 4$

■ 2. Use washers to find the volume of the solid that's formed by rotating the region enclosed by the curves about the y-axis.

$x = 12(y^2 - y^3) + 2$ and $x = 2$

$y = 0$ and $y = 1$

■ 3. Use washers to find the volume of the solid that's formed by rotating the region enclosed by the curves about the y-axis.

$x = \frac{y^4}{4} - \frac{y^2}{2} + 2$ and $x = \frac{y^2}{2} + 2$

$y = -2$ and $y = 2$

CYLINDRICAL SHELLS, HORIZONTAL AXIS

1. Use cylindrical shells to find the volume of the solid that's formed by rotating the region enclosed by the curves about the x-axis.

$$x = \left(\frac{y}{2}\right)^2 \text{ and } x = 4$$

$$y = 0$$

2. Use cylindrical shells to find the volume of the solid that's formed by rotating the region enclosed by the curves about the x-axis.

$$x = \frac{y}{3} \text{ and } x = \sqrt{y}$$

3. Use cylindrical shells to find the volume of the solid that's formed by rotating the region enclosed by the curves about the x-axis.

$$x = \sqrt[3]{\frac{y}{3}} \text{ and } x = \sqrt{\frac{y}{6}}$$

$$y = 3$$

4. Use cylindrical shells to find the volume of the solid that's formed by rotating the region enclosed by the curves about the x-axis.

$x = 4 - \sqrt{y}$ and $x = 2 - \sqrt{\frac{y}{6}}$

$y = 0$ and $y = 3$

WORK DONE TO LIFE A WEIGHT OR MASS

■ 1. Find the work required to lift a 50-pound load from ground level up into a tree house that's 60 feet above the ground, if the chain being used to lift the weight itself weighs 1 pound per foot.

■ 2. Find the work required to lift a 40-pound box of roofing nails from ground level up onto a roof that's 35 feet above the ground, if the rope being used to lift the weight itself weighs 2 ounces per foot.

■ 3. Find the work required to lift a $5{,}500$-pound load of concrete from ground level up onto a construction platform that's 75 feet above the ground, if the cable being used to lift the weight itself weighs 8 pounds per foot.

■ 4. Find the work required to lift a 5-gallon bucket of water, with each gallon of water weighing 6.75 pounds and the bucket weighing 2 pounds, from ground level up onto a scaffold that's 14 feet above the ground, if the rope being used to lift the weight itself weighs 8 ounces per foot.

5. Find the work required to lift a $7,200$-pound load of rocks from ground level up into a dump truck that's 13 feet above the ground, if the chain being used to lift the weight itself weighs 12 pounds per foot.

WORK DONE ON ELASTIC SPRINGS

■ 1. Find the work required to stretch a spring 3 feet beyond its normal length, if a force of $5s$ lbs is required to stretch the spring s feet beyond its normal length.

■ 2. Find the work required to stretch a spring 7 inches beyond its normal length, if a force of $9s$ lbs is required to stretch the spring s inches beyond its normal length.

■ 3. Find the work required to stretch a spring 6 feet beyond its normal length, if a force of $15s$ lbs is required to stretch the spring s feet beyond its normal length.

■ 4. Find the work required to stretch a spring 1 foot beyond its normal length, if a force of $3.5s$ lbs is required to stretch the spring s feet beyond its normal length.

■ 5. Find the work required, in foot pounds, to stretch a spring 58 inches beyond its normal length, if a force of $4s$ lbs is required to stretch the spring s feet beyond its normal length.

WORK DONE TO EMPTY A TANK

■ 1. Find the work required to empty a tank that is 6 feet wide, 8 feet tall, 12 feet long, and completely full. The tank will be emptied by pumping the liquid in the tank through a hose to a height of 2 feet above the top of the tank. The liquid in the tank has a density of 58.9 lbs/ft^3.

■ 2. Find the work required to empty an in-ground swimming pool that is 20 feet wide, 4 feet deep, 18 feet long, and completely full. The pool will be emptied by pumping the water in the pool through a hose over the top of the pool. The water in the pool has a density of 62.43 lbs/ft^3.

■ 3. Find the work required to empty a cylindrical tank that is 12 feet tall, has a radius of 6 feet, and is half full of diesel fuel. The tank will be emptied by pumping the fuel in the tank through a hose to a height of 6 feet above the top of the tank. The diesel fuel in the tank has a density of 53.5 lbs/ft^3.

■ 4. Find the work required to empty an above-ground child's pool that is 2 feet tall, has a diameter of 8 feet, and is three-fourths full. The pool will be emptied by pumping the water in the pool through a hose over the top of the pool. The water in the pool has a density of 62.4 lbs/ft^3.

■ 5. Find the work required to empty a cylindrical tank that is 8 feet tall, has a radius of 9 feet, and is three-fourths full of gasoline. The tank will be emptied by pumping the gas in the tank through a hose into a truck that's 8 feet above the top of the tank. The gasoline in the tank has a density of 54.5 lbs/ft^3.

WORK DONE BY A VARIABLE FORCE

1. Calculate the variable force on the interval $[0,2]$.

$$F(x) = 3x^2 + 2x$$

2. Calculate the variable force on the interval $[0,\pi/2]$.

$$F(x) = 3\sin(2x) + x$$

3. Calculate the variable force on the interval $[1,6]$.

$$F(x) = x^2 + x + 1$$

4. Calculate the variable force on the interval $[0,\pi/3]$.

$$F(x) = 2\tan^2 x$$

5. Calculate the variable force on the interval $[1.2,3.5]$.

$$F(x) = 4(x-2)^3 - 2(x-2) + 1$$

MOMENTS OF THE SYSTEM

1. Calculate the moments of the system.

$$m_1 = 3; \;\; P_1(2,5)$$

$$m_2 = 4; \;\; P_2(-2,6)$$

$$m_3 = 6; \;\; P_3(4,-5)$$

2. Calculate the moments of the system.

$$m_1 = 7; \;\; P_1(5,2)$$

$$m_2 = 3; \;\; P_2(-4,3)$$

$$m_3 = 5; \;\; P_3(-3,4)$$

3. Calculate the moments of the system.

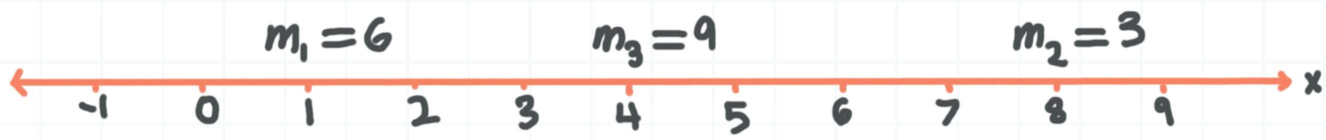
$$m_1 = 9; \;\; P_1(7,5)$$

$$m_2 = -5; \;\; P_2(3,8)$$

$$m_3 = 4; \;\; P_3(5,4)$$

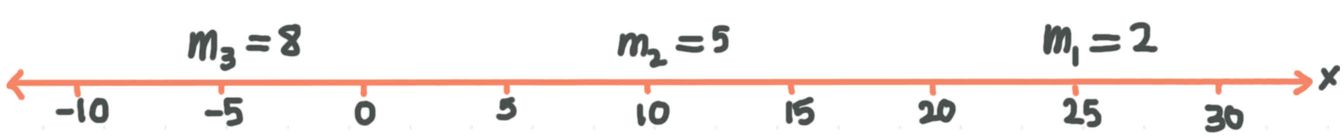
MOMENTS OF THE SYSTEM, X-AXIS

■ 1. Calculate the moments of the system.



■ 2. Calculate the moments of the system.

■ 3. Calculate the moments of the system.



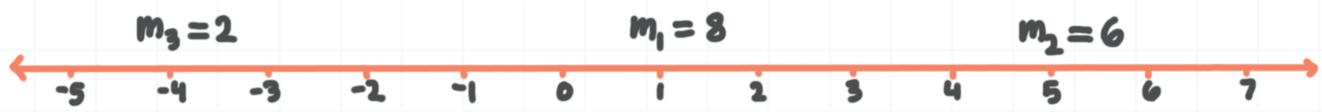
CENTER OF MASS OF THE SYSTEM

■ 1. Find the center of mass of the system if $M_y = 16$ and $M_x = 22$ and the total mass is $m_T = 14$.

■ 2. Find the center of mass of the system if $M_y = 32.5$ and $M_x = 28.5$ and the total mass is $m_T = 7.5$.

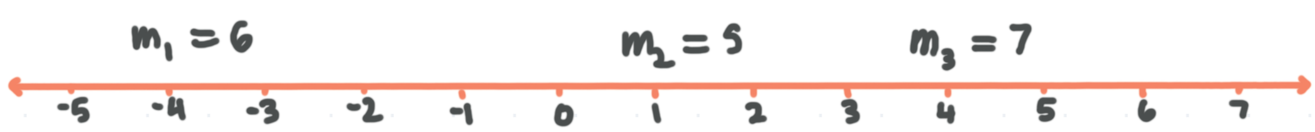
CENTER OF MASS OF THE SYSTEM, X-AXIS

■ 1. Find the center of mass of the system.



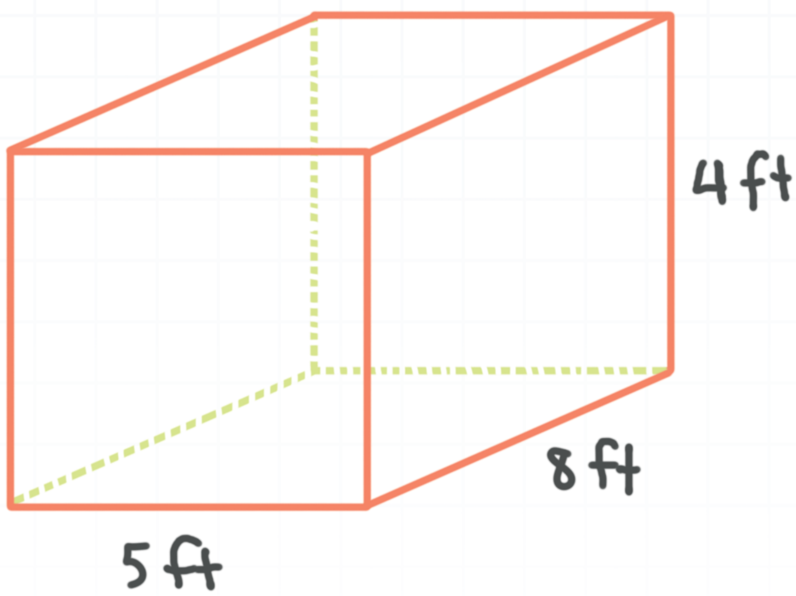
■ 2. Find the center of mass of the system.

■ 3. Find the center of mass of the system.

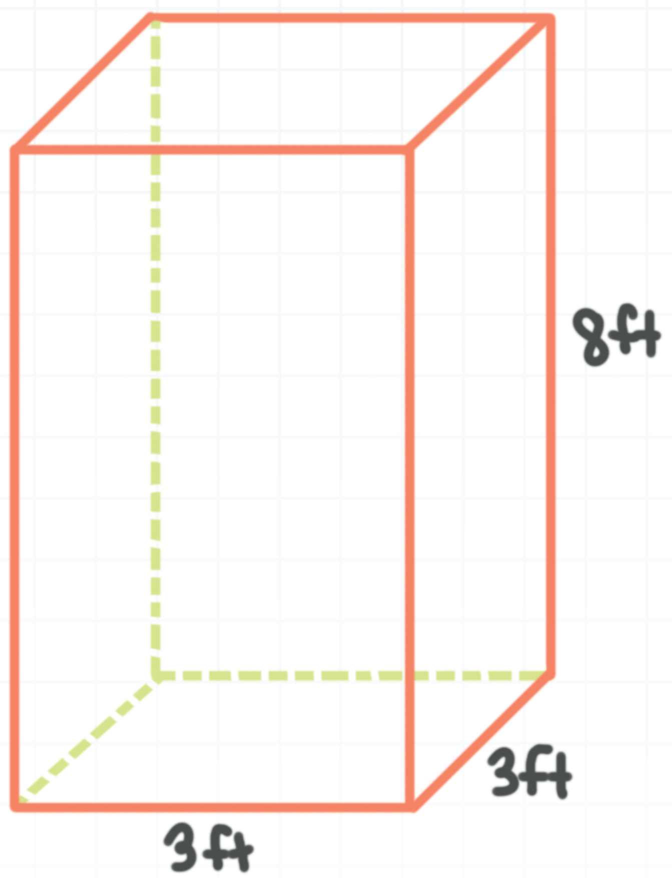


HYDROSTATIC PRESSURE

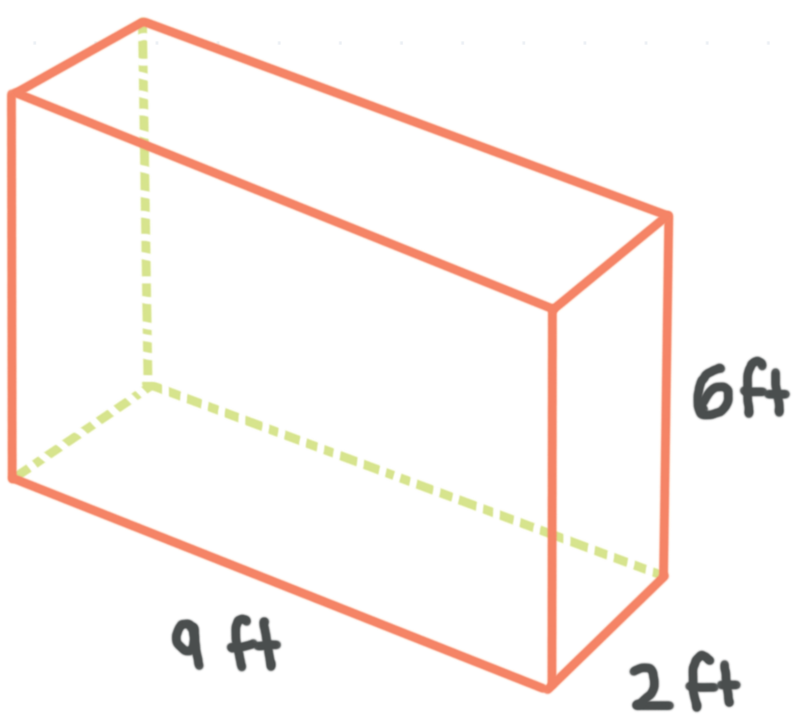
1. Find the hydrostatic pressure per square foot on the bottom of the tank, which is filled to the top with gasoline. Assume the weight of a gallon of gasoline is 6.073 pounds per gallon.



2. Find the hydrostatic pressure per square foot on the bottom of the tank, which is filled to the top with water. Assume the weight of a gallon of water is 8.3454 pounds per gallon.

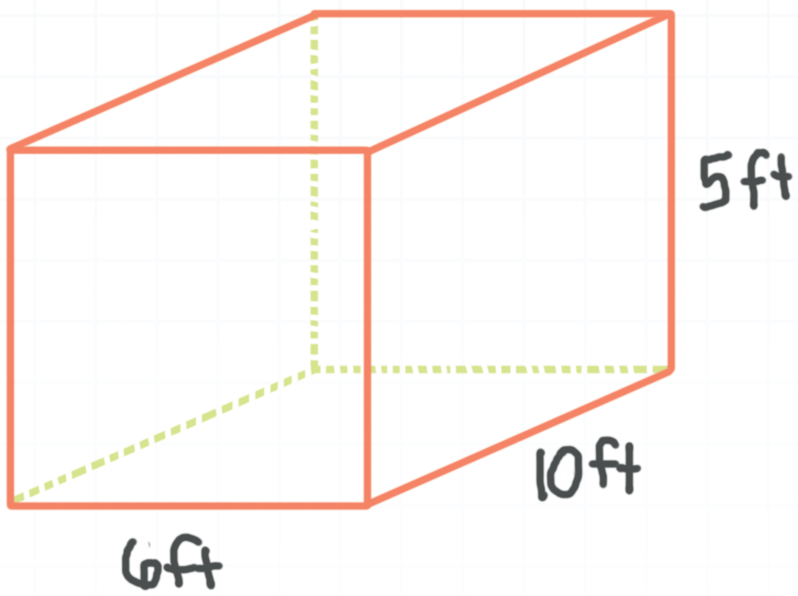


■ 3. Find the hydrostatic pressure per square foot on the bottom of the tank, which is filled to the top with diesel fuel. Assume the weight of a gallon of diesel is 7.1089 pounds per gallon.

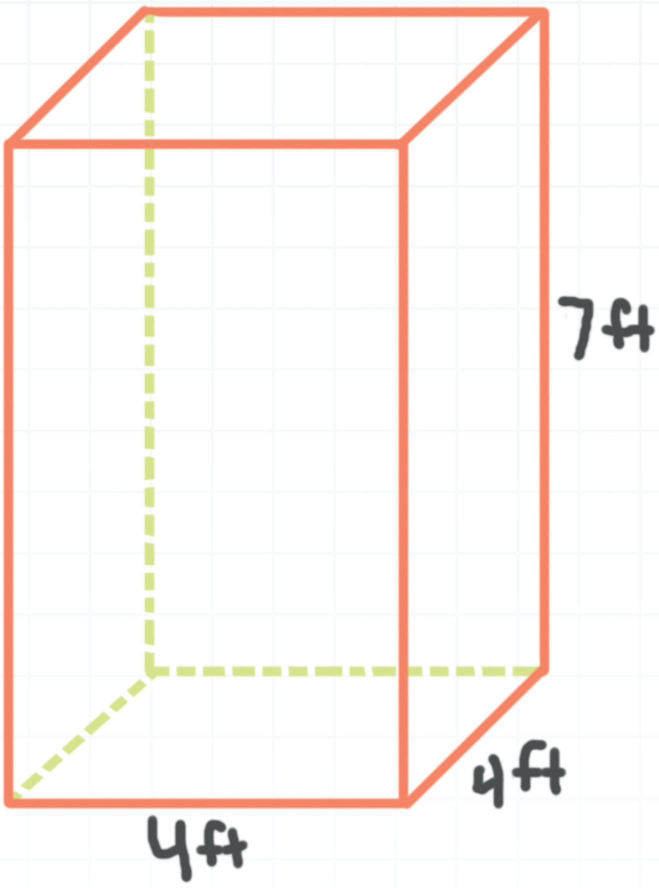


HYDROSTATIC FORCE

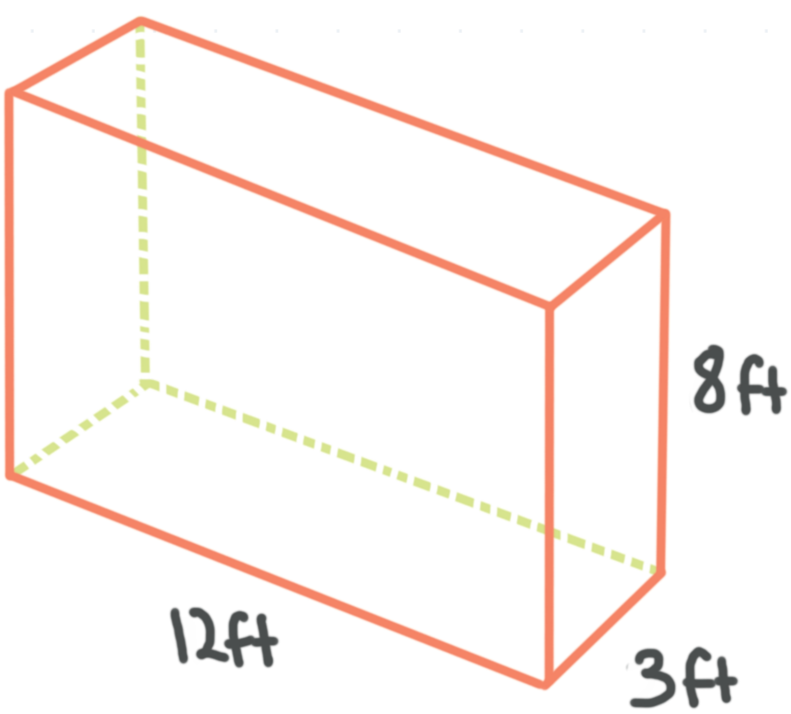
■ 1. Find the hydrostatic force on the bottom of the tank, which is filled to the top with gasoline. Assume the weight of a gallon of gasoline is 6.073 pounds per gallon.



■ 2. Find the hydrostatic force on the bottom of the tank, which is filled to the top with water. Assume the weight of a gallon of water is 8.3454 pounds per gallon.



■ 3. Find the hydrostatic force on the bottom of the tank, which is filled to the top with diesel fuel. Assume the weight of a gallon of diesel is 7.1089 pounds per gallon.



VERTICAL MOTION

■ 1. What is the maximum height of a baseball that's thrown straight up from a position 6 feet above the ground with an initial velocity of $v(t) = -32t + 88$ ft/sec?

■ 2. What is the maximum height of a football that's thrown straight up from 1.67 yards above the ground with an initial velocity of $v(t) = -10.67t + 40$ yards/sec?

■ 3. What is the maximum height of a model rocket that's launched straight up from the ground with an initial velocity of $v(t) = -32t + 200$ ft/sec?

■ 4. What is the maximum height of a bottle rocket that's launched straight up from the ground with an initial velocity of $v(t) = -19.6t + 29.4$ m/sec?

■ 5. What is the maximum height of a golf ball that's hit straight up from the ground with an initial velocity of $v(t) = -19.6t + 68.208$ m/sec?

RECTILINEAR MOTION

1. Find the position function $x(t)$ that models the rectilinear motion of a particle moving along the x-axis.

$$a(t) = 10 - t$$

$$v(0) = -1$$

$$x(0) = 6$$

2. Find the position function $x(t)$ that models the rectilinear motion of a particle moving along the x-axis.

$$a(t) = 9t^2 - 4t + 6$$

$$v(-1) = 0$$

$$x(0) = 2$$

3. Find the position function $x(t)$ that models the rectilinear motion of a particle moving along the x-axis.

$$a(t) = 2 - 6t$$

$$v(0) = 4$$

$$x(0) = 3$$

AREA OF A TRIANGLE WITH GIVEN VERTICES

■ 1. Find the area of the triangle with vertices $A(-4,4)$, $B(2,5)$, and $C(4,-1)$.

■ 2. Find the area of the triangle with vertices $D(-3,2)$, $E(-1,6)$, and $F(6,4)$.

■ 3. Find the area of the triangle with vertices $G(-3,-2)$, $H(1,2)$, and $I(4,-3)$.

SINGLE DEPOSIT, COMPOUNDED N TIMES, FUTURE VALUE

■ 1. Find the future value of $\$9,500$ after 7 years, at an annual interest rate of 2.25%, compounded quarterly.

■ 2. Find the future value of $\$14,550$ after 3 years, at an annual interest rate of 1.95%, compounded monthly.

■ 3. Find the future value of $\$7,595$ after 5 years, at an annual interest rate of 3.25%, compounded weekly.

SINGLE DEPOSIT, COMPOUNDED N TIMES, PRESENT VALUE

■ 1. Find the present value of a deposit that, after 9 years, at an annual interest rate of 4.75%, compounded monthly, will have a value of $\$24{,}514.01$.

■ 2. Find the present value of a deposit that, after 3 years, at an annual interest rate of 7.85%, compounded weekly, will have a value of $\$948.99$.

■ 3. Find the present value of a deposit that, after 6 years, at an annual interest rate of 3.95%, compounded quarterly, will have a value of $\$1{,}582{,}46$.

SINGLE DEPOSIT, COMPOUNDED CONTINUOUSLY, FUTURE VALUE

■ 1. Find the future value of $\$2{,}850$, after 8 years, at an annual interest rate of $1.55\ \%$, compounded continuously.

■ 2. Find the future value of $\$9{,}875$, after 15 years, at an annual interest rate of $4.15\ \%$, compounded continuously.

■ 3. Find the future value of $\$15{,}000$, after 18 years, at an annual interest rate of $8.5\ \%$, compounded continuously.

SINGLE DEPOSIT, COMPOUNDED CONTINUOUSLY, PRESENT VALUE

■ 1. Find the present value of a deposit that, after 11 years, at an annual interest rate of 2.75%, compounded continuously, will have a value of $\$11{,}631.08$.

■ 2. Find the present value of a deposit that, after 7 years, at an annual interest rate of 6.17%, compounded continuously, will have a value of $\$3{,}850.45$.

■ 3. Find the present value of a deposit that, after 4 years, at an annual interest rate of 5.95%, compounded continuously, will have a value of $\$6{,}343.55$.

INCOME STREAM, COMPOUNDED CONTINUOUSLY, FUTURE VALUE

■ 1. Money is invested at a rate of $\$10{,}000$ annually and the bank pays $8.85\,\%$ interest, compounded continuously. How many years will it take for the investment to grow to a balance of $\$300{,}000$?

■ 2. Money is invested at a rate of $\$5{,}000$ annually and the bank pays $6.75\,\%$ interest, compounded continuously. How many years will it take for the investment to grow to a balance of $\$100{,}000$?

■ 3. Money is invested at a rate of $\$2{,}500$ annually and the bank pays $5.25\,\%$ interest, compounded continuously. How many years will it take for the investment to grow to a balance of $\$25{,}000$?

INCOME STREAM, COMPOUNDED CONTINUOUSLY, PRESENT VALUE

■ 1. Suppose that money is deposited steadily into an account at a constant rate of $\$15,000$ per year for 13 years. Find the present value of this income stream if the account pays 7.35%, compounded continuously.

■ 2. Suppose that money is deposited steadily into a college fund at a constant rate of $\$3,000$ per year for 18 years. Find the present value of this income stream if the account pays 5.15%, compounded continuously.

■ 3. Suppose that money is deposited steadily into a new car account at a constant rate of $\$2,500$ per year for 8 years. Find the present value of this income stream if the account pays 7.5%, compounded continuously.

CONSUMER AND PRODUCER SURPLUS

1. Find the equilibrium quantity q_e and the equilibrium price p_e.

$$S(q) = 0.06q^2 + 5$$

$$D(q) = 0.1q + 17$$

2. Find the consumer surplus.

$$S(q) = 0.05q^2 + 7$$

$$D(q) = -0.2q + 11.8$$

3. Find the equilibrium quantity q_e and the equilibrium price p_e.

$$S(q) = 0.09q^2 + 8$$

$$D(q) = 1.55q + 25.5$$

PROBABILITY DENSITY FUNCTIONS

■ 1. Given $f(x)$, find $P(0 \le x \le 2)$.

$$f(x) = \begin{cases} \frac{1}{32} & 0 \le x \le 32 \\ 0 & x < 0 \text{ or } x > 32 \end{cases}$$

■ 2. Given $g(x)$, find $P(1 \le x \le 5)$.

$$g(x) = \begin{cases} e^{-x} & x \ge 0 \\ 0 & x < 0 \end{cases}$$

■ 3. Given $h(x)$, find $P(-1 \le x \le 1)$.

$$h(x) = \begin{cases} \frac{1}{6} & -2 \le x \le 4 \\ 0 & x < -2 \text{ or } x > 4 \end{cases}$$

CARDIAC OUTPUT

■ 1. Find the cardiac output, in liters/second, if 8 mg of dye is injected into the heart and the amount of dye remaining in the heart t seconds after the injection is modeled by $C(t) = 14te^{-0.6t}$. Assume $0 \leq t \leq 20$.

■ 2. Find the cardiac output, in liters/second, if 4 mg of dye is injected into the heart and the amount of dye remaining in the heart t seconds after the injection is modeled by $C(t) = 6te^{-0.2t}$. Assume $0 \leq t \leq 5$.

■ 3. Find the cardiac output, in liters/second, if 9 mg of dye is injected into the heart and the amount of dye remaining in the heart t seconds after the injection is modeled by $C(t) = 28te^{-0.85t}$. Assume $0 \leq t \leq 10$.

POISEUILLE'S LAW

■ 1. Use Poiseuille's law to find the flow of blood in the human artery in which $n = 0.031$, $R = 0.008$ cm, $L = 6$ cm, and $P = 3{,}900$ dynes/cm^2. Express the answer using scientific notation.

■ 2. Use Poiseuille's law to find the flow of blood in the human artery in which $n = 0.028$, $R = 0.007$ cm, $L = 3.5$ cm, and $P = 3{,}600$ dynes/cm^2. Express the answer using scientific notation.

■ 3. Use Poiseuille's law to find the flow of blood in the human artery in which $n = 0.027$, $R = 0.006$ cm, $L = 2.5$ cm, and $P = 3{,}800$ dynes/cm^2. Express the answer using scientific notation.

THEOREM OF PAPPUS

1. Use the Theorem of Pappus to find the exact volume of a right circular cone with radius 6 feet and height 18 feet.

2. Use the Theorem of Pappus to find the exact volume of a right circular cone with radius 8 inches and height 9 inches.

3. Use the Theorem of Pappus to find the exact volume of a right circular cone with radius 12 centimeters and height 7 centimeters.

ELIMINATING THE PARAMETER

■ 1. Eliminate the parameter.

$$x = t^2 - 2$$

$$y = 8 - 3t$$

$$t \geq 0$$

DERIVATIVES OF PARAMETRIC CURVES

■ 1. Find the derivative of the parametric curve.

$$x = 3 + \sqrt{t}$$

$$y = t^2 - 5t$$

■ 2. Find the derivative of the parametric curve.

$$x = 4\cos t$$

$$y = t - 5\sin t$$

■ 3. Find the derivative of the parametric curve.

$$x = 7\cos t$$

$$y = 3t^2 - t$$

■ 4. Find the derivative of the parametric curve.

$$x = e^t - 3t$$

$$y = e^{-t} + 2t$$

5. Find the derivative of the parametric curve.

$$x = 7t - 4$$

$$y = 5t^2 + 9t$$

SECOND DERIVATIVES OF PARAMETRIC CURVES

■ 1. Find the second derivative of the parametric curve.

$$x = 1 - \cos^2 t$$

$$y = \sin t$$

■ 2. Find the second derivative of the parametric curve.

$$x = e^{-3t}$$

$$y = e^{2t^2}$$

■ 3. Find the second derivative of the parametric curve.

$$x = t^2 + 2t + 1$$

$$y = 3t + 4$$

SKETCHING PARAMETRIC CURVES BY PLOTTING POINTS

■ 1. The graph of the parametric equation on the interval $0 \le t \le 2$ is a segment. What is the Cartesian equation in x and y? Find the left and right endpoints of the segment.

$$x = 2t + 3$$

$$y = 4t + 5$$

■ 2. What are the points on the curve for the parameter values $t = 1, 2, 3,$ and 4?

$$x = t^2 + t$$

$$y = t^2 - t$$

■ 3. What are the points on the curve for the parameter values $t = 0, 1, 2,$ and 3?

$$x = 3t^2 - 5$$

$$y = 2t^3 + 1$$

TANGENT LINES OF PARAMETRIC CURVES

■ 1. Find the equation of the tangent line to the parametric curve at $t = 3$.

$$x = 3t + 5$$

$$y = 7t - 2$$

■ 2. Find the equation of the tangent line to the parametric curve at $t = 4$.

$$x = 3t^2 - 12$$

$$y = 2t^3 + 6$$

■ 3. Find the equation of the tangent line to the parametric curve at $t = \pi/3$.

$$x = \cos^2 t$$

$$y = \sin^2 t$$

■ 4. Find the equation of the tangent line to the parametric curve at $t = 4$.

$$x = t^2 + t + 3$$

$$y = t^2 - 3t + 2$$

5. Find the equation of the tangent line to the parametric curve at $t = 9$.

$$x = 3\sqrt{t}$$

$$y = 5t\sqrt{t}$$

AREA UNDER A PARAMETRIC CURVE

■ 1. Find the area under the parametric curve.

$$x(t) = 3t^2$$

$$y(t) = t + 2$$

$$0 \le t \le 3$$

■ 2. Find the area under the parametric curve.

$$x(t) = 5t^2 - 3t + 4$$

$$y(t) = 6t - 1$$

$$0 \le t \le 5$$

■ 3. Find the area under the parametric curve.

$$x(t) = t + \sin t$$

$$y(t) = 4 + \cos t$$

$$0 \le t \le 2\pi$$

■ 4. Find the area under the parametric curve.

$$x(t) = t^2 + 5t - 8$$

$$y(t) = t^2 + 4t + 2$$

$$0 \le t \le 2$$

AREA UNDER ONE ARC OR LOOP

■ 1. Find the area in one loop of the parametric curve.

$$x(\theta) = 2\cos(2\theta)$$

$$y(\theta) = 4 + \sin(2\theta)$$

$$0 \le \theta \le \pi$$

■ 2. Find the area in one loop of the parametric curve.

$$x(\theta) = 2\sin\theta$$

$$y(\theta) = 5 + \cos\theta$$

$$0 \le \theta \le 2\pi$$

■ 3. Find the area in one loop of the parametric curve.

$$x(\theta) = 8 + 3\cos\theta$$

$$y(\theta) = 9 - 2\sin\theta$$

$$0 \le \theta \le 2\pi$$

■ 4. Find the area in one loop of the parametric curve.

$$x(\theta) = 12 + 6\sin\theta$$

$$y(\theta) = 12 - 6\cos\theta$$

$$0 \le \theta \le 2\pi$$

5. Find the area in one loop of the parametric curve.

$$x(\theta) = 15 - 5\cos\theta$$

$$y(\theta) = 5 + 15\sin\theta$$

$$0 \le \theta \le 2\pi$$

ARC LENGTH OF PARAMETRIC CURVES

1. Find the length of the parametric curve on the given interval.

$$x(t) = 7 - 3t$$

$$y(t) = 5 + 8t$$

$$-1 \le t \le 4$$

2. Find the length of the parametric curve on the given interval.

$$x(t) = \cos^3 t$$

$$y(t) = \sin^3 t$$

$$0 \le t \le \frac{3\pi}{4}$$

3. Find the length of the parametric curve on the given interval.

$$x(t) = 5t - 5\sin t$$

$$y(t) = -5\cos t$$

$$0 \le t \le 2\pi$$

4. Find the length of the parametric curve on the given interval.

$$x(t) = \cos t$$

$$y(t) = t + \sin t$$

$$0 \le t \le \pi$$

SURFACE AREA OF REVOLUTION, HORIZONTAL AXIS

■ 1. Find the surface area of revolution of the parametric curve on the interval $0 \leq t \leq 3$, rotated about the x-axis.

$$x = \frac{5}{3}t$$

$$y = 4t + 6$$

■ 2. Find the surface area of revolution of the parametric curve on the interval $0 \leq t \leq 2\pi$, rotated about the x-axis.

$$x = 3 + \cos t$$

$$y = 4 + \sin t$$

■ 3. Find the surface area of revolution of the parametric curve on the interval $0 \leq t \leq 2\pi$, rotated about the x-axis.

$$x = 7 - 3\sin t$$

$$y = 6 + 3\cos t$$

■ 4. Find the surface area of revolution of the parametric curve on the interval $0 \leq t \leq \pi$, rotated about the x-axis.

$$x = 5 - \cos(2t)$$

$$y = 3 + \sin(2t)$$

SURFACE AREA OF REVOLUTION, VERTICAL AXIS

■ 1. Find the surface area of revolution of the parametric curve on the interval $0 \le t \le \pi/3$, rotated about the y-axis.

$$x = 8 + \sin(6t)$$

$$y = 7 - \cos(6t)$$

■ 2. Find the surface area of revolution of the parametric curve on the interval $0 \le t \le 2\pi$, rotated about the y-axis.

$$x = 5 + 4\sin(t)$$

$$y = 5 + 4\cos(t)$$

■ 3. Find the surface area of revolution of the parametric curve on the interval $0 \le t \le 2\pi$, rotated about the y-axis.

$$x = 12 - \sin t$$

$$y = 2 + \cos t$$

■ 4. Find the surface area of revolution of the parametric curve on the interval $0 \le t \le \pi$, rotated about the y-axis.

$$x = 4 - 3\sin(2t)$$

$$y = 4 - 3\cos(2t)$$

5. Find the surface area of revolution of the parametric curve on the interval $0 \le t \le 4$, rotated about the y-axis.

$$x = 6t + 5$$

$$y = 8t + 7$$

VOLUME OF REVOLUTION, PARAMETRIC CURVES

■ 1. Find the volume of revolution of the parametric curve, rotated about the x-axis, over the interval $1 \leq t \leq 2$.

$$x(t) = 2t^2$$

$$y(t) = 4t^2$$

■ 2. Find the volume of revolution of the parametric curve, rotated about the y-axis, over the interval $1 \leq t \leq 3$.

$$x(t) = 3t$$

$$y(t) = 4t^2$$

■ 3. Find the volume of revolution of the parametric curve, rotated about the x-axis, over the interval $1 \leq t \leq 3$.

$$x(t) = 2e^{2t} - 4t$$

$$y(t) = 6e^{\frac{5t}{2}}$$

■ 4. Find the volume of revolution of the parametric curve, rotated about the y-axis, over the interval $0 \leq t \leq 1$.

$$x(t) = 3e^t$$

$$y(t) = e^t$$

POLAR COORDINATES

■ 1. Convert the rectangular point $(2,-2)$ to a polar point.

■ 2. Convert the polar point $(3,\pi/4)$ to a rectangular point.

■ 3. Convert the rectangular point $\left(-5\sqrt{3},5\right)$ to a polar point.

■ 4. Convert the polar point $(8,11\pi/6)$ to a rectangular point.

CONVERTING RECTANGULAR EQUATIONS

1. Convert the rectangular equation to an equivalent polar equation.

$$4x^2 + 4y^2 = 64$$

2. Convert the rectangular equation to an equivalent polar equation.

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

3. Convert the rectangular equation to an equivalent polar equation.

$$(x - 2)^2 + (y + 2)^2 = 8$$

4. Convert the rectangular equation to an equivalent polar equation.

$$\frac{x^2}{9} - \frac{y^2}{8} = 1$$

CONVERTING POLAR EQUATIONS

■ 1. Convert the polar equation to an equivalent rectangular equation.

$$r = 4\cos\theta + 4\sin\theta$$

■ 2. Convert the polar equation to an equivalent rectangular equation.

$$r = 12\cos\theta - 12\sin\theta$$

■ 3. Convert the polar equation to an equivalent rectangular equation.

$$r = 3\sin\left(\theta + \frac{\pi}{4}\right)$$

■ 4. Convert the polar equation to an equivalent rectangular equation.

$$r = 6\cos\theta - 10\sin\theta$$

■ 5. Convert the polar equation to an equivalent rectangular equation.

$$r = 12\sin\theta$$

DISTANCE BETWEEN POLAR POINTS

■ 1. Calculate the distance between the polar coordinate points.

$$\left(2,\frac{\pi}{3}\right) \text{ and } \left(2,\frac{11\pi}{6}\right)$$

■ 2. Calculate the distance between the polar coordinate points.

$$\left(4,\frac{7\pi}{12}\right) \text{ and } \left(2,\frac{\pi}{12}\right)$$

■ 3. Calculate the distance between the polar coordinate points.

$$\left(4,\frac{\pi}{4}\right) \text{ and } \left(9,\frac{3\pi}{4}\right)$$

SKETCHING POLAR CURVES

■ 1. Graph the polar curve. How many petals does the curve have, and what is the length of each petal?

$$r = 5\sin(4\theta)$$

TANGENT LINE TO THE POLAR CURVE

■ 1. Find the tangent line to the polar curve at $\theta = 2\pi/3$.

$$r = 3\cos\theta$$

■ 2. Find the tangent line to the polar curve at $\theta = \pi/3$.

$$r = 5\sin\theta$$

■ 3. Find the tangent line to the polar curve at $\theta = \pi/4$.

$$r = 4 - 2\cos\theta$$

■ 4. Find the tangent line to the polar curve at $\theta = \pi$.

$$r = 8 - 5\sin\theta$$

■ 5. Find the tangent line to the polar curve at $\theta = \pi/2$.

$$r = 7 - 6\cos\theta$$

VERTICAL AND HORIZONTAL TANGENT LINES TO THE POLAR CURVE

■ 1. At which points does the polar curve have horizontal tangent lines?

$$r = 4 - 4\sin\theta$$

■ 2. At which points does the polar curve have vertical tangent lines?

$$r = 6 - 6\cos\theta$$

■ 3. At which points does the polar curve have horizontal tangent lines?

$$r = 8 - 2\sin\theta$$

INTERSECTION OF THE POLAR CURVES

1. Find the rectangular points of intersection of the polar curves.

$$r = 3\cos\theta$$

$$r = 3\sin\theta$$

2. Find the polar points of intersection of the polar curves.

$$r = 4$$

$$r = -8\sin\theta$$

3. Find the rectangular points of intersection of the polar curves.

$$r = 6 - 4\cos\theta$$

$$r = 5\cos\theta$$

AREA INSIDE A POLAR CURVE

■ 1. Find the area bounded by the polar curve over the interval.

$$r = 2 + 2\cos\theta$$

$$0 \le \theta \le 2\pi$$

■ 2. Find the area bounded by the polar curve over the interval.

$$r = 2\sin 2\theta$$

$$0 \le \theta \le 2\pi$$

■ 3. Find the area bounded by the polar curve over the interval.

$$r = 4 + 2\sin\theta$$

$$0 \le \theta \le 2\pi$$

■ 4. Find the area bounded by the polar curve over the interval.

$$r^2 = \sin\theta$$

$$0 \le \theta \le \pi$$

5. Find the area bounded by the polar curve over the interval.

$$r = 2\cos\theta$$

$$-\frac{\pi}{4} \le \theta \le \frac{\pi}{4}$$

AREA BOUNDED BY ONE LOOP OF A POLAR CURVE

■ 1. Find the area of one loop of the polar curve.

$$r = 6\cos(4\theta)$$

■ 2. Find the area of one loop of the polar curve.

$$r = 4\sin(5\theta)$$

■ 3. Find the area of one loop of the polar curve.

$$r = 7\sin(6\theta)$$

■ 4. Find the area of one loop of the polar curve.

$$r = 5\sin(3\theta)$$

AREA BETWEEN POLAR CURVES

■ 1. Find the area of the region that inside both polar curves.

$$r = 4\cos\theta$$

$$r = 2$$

■ 2. Find the area of the region inside $r = 1 - \cos\theta$ but outside $r = 1$.

■ 3. Find the area of the region inside $r = 1 + \cos\theta$ but outside the circle $r = \cos\theta$.

■ 4. Find the area of the region inside $r = 2 + \cos\theta$ but outside the circle $r = 5\cos\theta$.

AREA INSIDE BOTH POLAR CURVES

1. Find the area of the region that's inside both polar curves.

$$r = 2\cos\theta$$

$$r = 2\sin\theta$$

2. Find the area of the region that's inside both polar curves.

$$r = 2\sin\theta$$

$$r = 1$$

3. Find the area of the region that's inside both polar curves.

$$r = 2(1 - \cos\theta)$$

$$r = 2$$

4. Find the area of the region that's inside both polar curves.

$$r = 2(1 + \cos\theta)$$

$$r = 2(1 - \cos\theta)$$

5. Find the area of the region that's inside both polar curves.

$$r = 3 + 2\sin\theta$$

$$r = 2$$

SURFACE AREA OF REVOLUTION OF A POLAR CURVE

■ 1. Find the surface area generated by revolving the polar curve about the y-axis over the interval $0 \leq \theta \leq \pi$.

$$r = 2\cos\theta$$

■ 2. Find the surface area generated by revolving the polar curve about the x-axis over the interval $0 \leq \theta \leq \pi/2$.

$$r = 4\cos\theta$$

■ 3. Find the surface area generated by revolving the polar curve about the y-axis over the interval $0 \leq \theta \leq \pi/2$.

$$r = 8\sin\theta$$

■ 4. Find the surface area generated by revolving the polar curve about the x-axis over the interval $0 \leq \theta \leq \pi$.

$$r = 7\sin\theta$$

SEQUENCES VS. SERIES

1. Determine whether the expression is a sequence or a series.

$$5, 10, 15, 20\ , 25, 30$$

2. Determine whether the expression is a sequence or a series.

$$\sum_{n=1}^{15} 5n - 2$$

3. Determine whether the expression is a sequence or a series.

$$3 + 6 + 9 + 12 + 15 + 18 + 21$$

LISTING THE FIRST TERMS

1. Write the first five terms of the sequence.

$$a_{n+1} = 3a_n + 4$$

$$a_1 = 4$$

2. Write the first five terms of the sequence.

$$a_{n+1} = 4a_n - 5$$

$$a_1 = 3$$

3. Write the first five terms of the sequence.

$$a_{n+1} = a_n + 9$$

$$a_1 = 24$$

CALCULATING THE FIRST TERMS

■ 1. Write the first five terms of the sequence and find the limit of the sequence a_n as $n \to \infty$.

$$a_n = \frac{5n^2 - 2}{n^2 + 3n - 2}$$

■ 2. Write the first five terms of the sequence and find the limit of the sequence a_n as $n \to \infty$.

$$a_n = \frac{6n}{e^{2n}}$$

■ 3. Write the first five terms of the sequence and find the limit of the sequence a_n as $n \to \infty$.

$$a_n = \frac{n^2 + 1}{n^2 + 8n}$$

FORMULA FOR THE GENERAL TERM

■ 1. What is a formula for the general term of the sequence?

$$\frac{3}{4}, \frac{5}{8}, \frac{7}{12}, \frac{9}{16}, \frac{11}{20}$$

■ 2. What is a formula for the general term of the sequence?

$$5, 8, 13, 20, 29, 40$$

■ 3. What is a formula for the general term of the sequence?

$$-\frac{1}{6}, \frac{2}{7}, -\frac{3}{8}, \frac{4}{9}, -\frac{1}{2}, \frac{6}{11}$$

CONVERGENCE OF A SEQUENCE

1. If the sequence converges, find its limit.

$$a_n = \frac{5n}{n^2 + 2n - 1}$$

2. If the sequence converges, find its limit.

$$a_n = \frac{9n^3 - 27n^2 + 5n}{3n^3 + 12n^2 - n}$$

3. If the sequence converges, find its limit.

$$a_n = \left(\frac{n^2 + 3}{n^3}\right)^2$$

LIMIT OF A CONVERGENT SEQUENCE

■ 1. Find the limit of the convergent sequence.

$$a_n = \frac{3n^2 - 6}{9n^2 + 3n - 12}$$

■ 2. Find the limit of the convergent sequence.

$$a_n = \frac{n^3}{3^n}$$

■ 3. Find the limit of the convergent sequence.

$$a_n = n^5 e^{-2n}$$

INCREASING, DECREASING, AND NOT MONOTONIC

■ 1. State whether the sequence is increasing, decreasing, and monotonic or not monotonic.

$$a_n = \frac{17}{4n^2 + 6n + 3}$$

■ 2. State whether the sequence is increasing, decreasing, and monotonic or not monotonic.

$$a_n = \frac{3n^2 - 5}{4n + 2}$$

■ 3. State whether the sequence is increasing, decreasing, and monotonic or not monotonic.

$$a_n = n^5 + 1$$

BOUNDED SEQUENCES

1. Describe how the sequence is bounded by indicating the upper and lower bounds, or say whether there is no upper bound or now lower bound.

$$a_n = \frac{2n+5}{n^2}$$

2. Describe how the sequence is bounded by indicating the upper and lower bounds, or say whether there is no upper bound or now lower bound.

$$a_n = \frac{3n^3+2}{n^4}$$

3. Describe how the sequence is bounded by indicating the upper and lower bounds, or say whether there is no upper bound or now lower bound.

$$a_n = \frac{7n^3+15}{2n^3}$$

■ 4. Describe how the sequence is bounded by indicating the upper and lower bounds, or say whether there is no upper bound or now lower bound.

$$a_n = \frac{3n^4 + 9}{4n^3}$$

CALCULATING THE FIRST TERMS OF A SERIES OF PARTIAL SUMS

■ 1. Approximate the first four terms of the series of partial sums.

$$\sum_{n=1}^{\infty} \frac{7n}{3n^2+2}$$

■ 2. Approximate the first four terms of the series of partial sums.

$$\sum_{n=1}^{\infty} \frac{5n^2}{7n+4}$$

■ 3. Approximate the first four terms of the series of partial sums.

$$\sum_{n=1}^{\infty} \frac{9n^3}{8n^2+13}$$

SUM OF THE SERIES OF PARTIAL SUMS

■ 1. Use the partial sums equation to find the sum of the series.

$$s_n = 12 + \frac{9}{n}$$

■ 2. Use the partial sums equation to find the sum of the series.

$$s_n = \frac{7n^2 + 9n}{n^2 - 6}$$

■ 3. Use the partial sums equation to find the sum of the series.

$$s_n = \frac{9n^3 + 7n + 9}{8n^3 + 2n^2 + 5}$$

■ 4. Use the partial sums equation to find the sum of the series.

$$s_n = \frac{13}{15n^3} + \frac{12}{n} + 5$$

■ 5. Use the partial sums equation to find the sum of the series.

$$s_n = \frac{14n^2}{15n^3} - \frac{n}{16n^2} - \frac{1}{4n} + \frac{1}{3}$$

GEOMETRIC SERIES TEST

■ 1. Use the geometric series test to say whether the geometric series converges or diverges, then give the value of the common ratio r.

$$\sum_{n=1}^{\infty} 6\left(\frac{2}{3}\right)^{n-1}$$

■ 2. Use the geometric series test to say whether the geometric series converges or diverges, then give the value of the common ratio r.

$$\sum_{n=1}^{\infty} \left(\frac{3}{7}\right)^{n-1}$$

■ 3. Use the geometric series test to say whether the geometric series converges or diverges, then give the value of the common ratio r.

$$\frac{\pi}{2} + \frac{\pi^2}{6} + \frac{\pi^3}{18} + \frac{\pi^4}{54} + \cdots$$

■ 4. Use the geometric series test to say whether the geometric series converges or diverges, then give the value of the common ratio r.

$$1-\frac{1}{3}+\frac{1}{9}-\frac{1}{27}+\cdots+\left(-\frac{1}{3}\right)^{n-1}+\cdots$$

■ 5. Use the geometric series test to say whether the geometric series converges or diverges, then give the value of the common ratio r.

$$\sum_{n=1}^{\infty}\left(\frac{e}{\pi}\right)^{n}$$

SUM OF THE GEOMETRIC SERIES

■ 1. Find the sum of the geometric series.

$$\sum_{n=1}^{\infty} 7\left(\frac{3}{8}\right)^{n-1}$$

■ 2. Find the sum of the geometric series.

$$\sum_{n=1}^{\infty} 9\left(\frac{5}{14}\right)^{n-1}$$

■ 3. Find the sum of the geometric series.

$$\frac{1}{3}-\frac{2}{9}+\frac{4}{27}-\frac{8}{81}+\cdots$$

■ 4. Find the sum of the geometric series.

$$\sum_{n=1}^{\infty}\left(\frac{e}{\pi}\right)^{n}$$

VALUES FOR WHICH THE SERIES CONVERGES

1. Find the values of x for which the geometric series converges.

$$\sum_{n=1}^{\infty} \frac{17}{3} x^{n-1}$$

2. Find the values of x for which the geometric series converges.

$$\sum_{n=1}^{\infty} 5 \left(\frac{x-2}{3} \right)^{n-1}$$

3. Find the values of x for which the geometric series converges.

$$\sum_{n=0}^{\infty} 4^n x^n$$

GEOMETRIC SERIES FOR REPEATING DECIMALS

■ 1. Express the repeating decimal $0.\overline{17}$ as a geometric series.

■ 2. Express the repeating decimal $23.\overline{23}$ as a geometric series.

■ 3. Express the repeating decimal $6.7\overline{2}$ as a geometric series.

■ 4. Express the repeating decimal $9.15\overline{65}$ as a geometric series.

CONVERGENCE OF A TELESCOPING SERIES

■ 1. Say whether the telescoping series converges or diverges.

$$\sum_{n=1}^{\infty} \left(5^n - 5^{n-1}\right)$$

■ 2. Say whether the telescoping series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{1}{n(n+2)}$$

■ 3. Say whether the telescoping series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{2}{n^2+n}$$

■ 4. Say whether the telescoping series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{4}{n^2+3n+2}$$

■ 5. Say whether the telescoping series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{5}{n+1} - \frac{5}{n+2}$$

SUM OF A TELESCOPING SERIES

1. Calculate the sum of the telescoping series.

$$\sum_{n=1}^{\infty} \frac{3}{n^2+n}$$

2. Calculate the sum of the telescoping series.

$$\sum_{n=1}^{\infty} \frac{4}{n^2+3n+2}$$

3. Calculate the sum of the telescoping series.

$$\sum_{n=1}^{\infty} \frac{6}{n+2} - \frac{6}{n+3}$$

LIMIT VS. SUM OF THE SERIES

1. Find the limit of the series, and if it converges, find its sum.

$$\sum_{n=1}^{\infty} 3e^{-n} + 2^{-n}$$

2. Find the limit of the series, and if it converges, find its sum.

$$\sum_{n=1}^{\infty} \frac{3^n + 2^n}{6^n}$$

3. Find the limit of the series, and if it converges, find its sum.

$$\sum_{n=1}^{\infty} \frac{3}{5^n} + \frac{2}{n}$$

INTEGRAL TEST

1. Use the integral test to say whether the series converges or diverges. If it converges, give the value to which it converges.

$$\sum_{n=1}^{\infty} \frac{7}{n^{\frac{3}{2}}}$$

2. Use the integral test to say whether the series converges or diverges. If it converges, give the value to which it converges.

$$\sum_{n=1}^{\infty} \frac{9}{n+1}$$

3. Use the integral test to say whether the series converges or diverges. If it converges, give the value to which it converges.

$$\sum_{n=1}^{\infty} \frac{9}{7n-2}$$

P-SERIES TEST

1. Use the p-series test to say whether the series converges of diverges.

$$\sum_{n=1}^{\infty} \frac{23}{4\sqrt[3]{n}}$$

2. Use the p-series test to say whether the series converges of diverges.

$$\sum_{n=1}^{\infty} \frac{7}{5n^3}$$

3. Use the p-series test to say whether the series converges of diverges.

$$\sum_{n=1}^{\infty} \frac{6n^2 + 2n}{9n^4}$$

NTH TERM TEST

1. Use the nth term test to say whether the series diverges, or whether the nth term test is inconclusive.

$$\sum_{n=1}^{\infty} \frac{1}{2n-1}$$

2. Use the nth term test to say whether the series diverges, or whether the nth term test is inconclusive.

$$\sum_{n=1}^{\infty} a_n = 8 + 2 + \frac{1}{2} + \frac{1}{8} + \frac{1}{32} + \frac{1}{128} + \dots$$

3. Use the nth term test to say whether the series diverges, or whether the nth term test is inconclusive.

$$\sum_{n=1}^{\infty} \frac{11^n}{10^n}$$

4. Use the nth term test to say whether the series diverges, or whether the nth term test is inconclusive.

$$\sum_{n=1}^{\infty} \frac{n}{n+1}$$

COMPARISON TEST

1. Use the comparison test to say whether or not the series converges.

$$\sum_{n=0}^{\infty} \frac{4}{3^n + n}$$

2. Use the comparison test to say whether or not the series converges.

$$\sum_{n=1}^{\infty} \frac{n}{n^4 + 7}$$

3. Use the comparison test to say whether or not the series converges.

$$\sum_{n=2}^{\infty} \frac{5}{\ln n}$$

LIMIT COMPARISON TEST

1. Use the limit comparison test to say whether or not the series converges.

$$\sum_{n=1}^{\infty} \frac{3n+2}{(2n-1)^4}$$

2. Use the limit comparison test to say whether or not the series converges.

$$\sum_{n=1}^{\infty} \frac{12n^2+5}{n^3-7}$$

3. Use the limit comparison test to say whether or not the series converges.

$$\sum_{n=1}^{\infty} \frac{n^4+3n^2}{7n^6+3n^4}$$

ERROR OR REMAINDER OF A SERIES

■ 1. Estimate the remainder of the series using the first three terms.

$$\sum_{n=1}^{\infty} \frac{3}{7n^3 + 2n^2 + 3}$$

■ 2. Estimate the remainder of the series using the first three terms.

$$\sum_{n=1}^{\infty} \frac{5}{\sqrt{n^4 + 6}}$$

■ 3. Estimate the remainder of the series using the first three terms.

$$\sum_{n=1}^{\infty} \frac{4n^2}{n^5 + n^2 + 3}$$

RATIO TEST

■ 1. Use the ratio test to determine the convergence of the series.

$$\sum_{n=1}^{\infty} \frac{7^n}{n^3}$$

■ 2. Use the ratio test to determine the convergence of the series.

$$\sum_{n=1}^{\infty} \frac{9(n+3)}{n^2}$$

■ 3. Use the ratio test to determine the convergence of the series.

$$\sum_{n=1}^{\infty} \frac{10^n}{5^{3n+1}(n+2)}$$

■ 4. Use the ratio test to determine the convergence of the series.

$$\sum_{n=1}^{\infty} \frac{6n+17}{3^{2n+1}}$$

■ 5. Use the ratio test to determine the convergence of the series.

$$\sum_{n=1}^{\infty} \frac{(-1)^n \cdot 5^{n+3}}{6^{n+1}}$$

RATIO TEST WITH FACTORIALS

1. Use the ratio test to determine the convergence of the series.

$$\sum_{n=1}^{\infty} \frac{n^3}{(2n-1)!}$$

2. Use the ratio test to determine the convergence of the series.

$$\sum_{n=1}^{\infty} \frac{8^n}{2^{n+1} \cdot n!}$$

3. Use the ratio test to determine the convergence of the series.

$$\sum_{n=1}^{\infty} \frac{(-1)^n n!}{n^3+1}$$

4. Use the ratio test to determine the convergence of the series.

$$\sum_{n=0}^{\infty} \frac{(n+2)!}{(3n)^2+7}$$

5. Use the ratio test to determine the convergence of the series.

$$\sum_{n=0}^{\infty} \frac{4^n(n+1)}{n!}$$

ROOT TEST

1. Use the root test to determine the convergence of the series.

$$\sum_{n=3}^{\infty} \left(\frac{5n^3 + 3n^2 - 6}{\sqrt{6n^6 + 7n^4 - 8}} \right)^n$$

2. Use the root test to determine the convergence of the series.

$$\sum_{n=1}^{\infty} \frac{7n^3}{e^{2n^2}}$$

3. Use the root test to determine the convergence of the series.

$$\sum_{n=0}^{\infty} \left(\frac{7n - 6n^4}{9n^4 + 3} \right)^n$$

ABSOLUTE AND CONDITIONAL CONVERGENCE

1. Use the root test to determine the absolute or conditional convergence of the series.

$$\sum_{n=1}^{\infty}\left(\frac{6n}{8n+5}\right)^n$$

2. Use the ratio test to determine the absolute or conditional convergence of the series, or say if the series diverges of if the ratio test is inconclusive.

$$\sum_{n=1}^{\infty}\frac{(-1)^n}{25n}$$

3. Use the root test to determine the absolute or conditional convergence of the series.

$$\sum_{n=1}^{\infty}\left(\frac{8n-9n^5}{14n^5+7}\right)^n$$

4. Use the ratio test to determine the absolute or conditional convergence of the series, or say if the series diverges of if the ratio test is inconclusive.

$$\sum_{n=1}^{\infty} \frac{n!}{9^n}$$

ALTERNATING SERIES TEST

■ 1. Use the alternating series test to say whether the series converges or diverges.

$$\sum_{n=1}^{\infty} (-1)^{n-1} \left(\frac{3}{5n+6} \right)$$

■ 2. Use the alternating series test to say whether the series converges or diverges.

$$\sum_{n=1}^{\infty} (-1)^{n-1} n \left(\frac{2}{7} \right)^n$$

■ 3. Use the alternating series test to say whether the series converges or diverges.

$$\sum_{n=3}^{\infty} (-1)^{n+1} \frac{n^3}{n!}$$

ALTERNATING SERIES ESTIMATION THEOREM

■ 1. Approximate the sum of the alternating series to three decimal places, using the first 5 terms. Then find the remainder of the approximation, to the nearest six decimal places.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}n^3}{12^n}$$

■ 2. Approximate the sum of the alternating series to three decimal places, using the first 12 terms. Then find the remainder of the approximation, to the nearest six decimal places.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{3n^3}$$

■ 3. Approximate the sum of the alternating series to three decimal places, using the first 10 terms. Then find the remainder of the approximation.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1} \cdot 3}{12n^3 + 4n^2}$$

POWER SERIES REPRESENTATION

1. Find the power series representation of the function.

$$f(x) = \frac{3x}{7 + x^2}$$

2. Find the power series representation of the function.

$$f(x) = \frac{5}{4 - 6x}$$

3. Find the power series representation of the function.

$$f(x) = \frac{4}{x^2 - x^3}$$

4. Find the power series representation of the function.

$$f(x) = \frac{5x^2}{1 + x^3}$$

5. Find the power series representation of the function.

$$f(x) = \frac{x}{8 - x}$$

POWER SERIES MULTIPLICATION

■ 1. Use power series multiplication to find the first four non-zero terms of the Maclaurin series.

$$y = \cos(3x)e^{3x}$$

■ 2. Use power series multiplication to find the first four non-zero terms of the Maclaurin series.

$$y = \arctan(2x)\sin x$$

■ 3. Use power series multiplication to find the first four non-zero terms of the Maclaurin series.

$$y = e^{-2x}\cos(2x)$$

■ 4. Use power series multiplication to find the first four non-zero terms of the Maclaurin series.

$$y = e^{5x}\ln(1 + 3x)$$

■ 5. Use power series multiplication to find the first four non-zero terms of the Maclaurin series.

$$y = e^{3x} \cdot \frac{3}{1-x}$$

POWER SERIES DIVISION

■ 1. Use power series division to find the first four non-zero terms of the Maclaurin series.

$$y = \frac{e^{3x}}{x^2}$$

■ 2. Use power series division to find the first four non-zero terms of the Maclaurin series.

$$y = \frac{6x}{\ln(1 + 6x)}$$

■ 3. Use power series division to find the first four non-zero terms of the Maclaurin series.

$$y = \frac{\cos(2x)}{2x^3}$$

■ 4. Use power series division to find the first four non-zero terms of the Maclaurin series.

$$y = \frac{\sin(3x)}{3x^2}$$

5. Use power series division to find the first four non-zero terms of the Maclaurin series.

$$y = \frac{\arctan(4x)}{4x^2}$$

POWER SERIES DIFFERENTIATION

■ 1. Differentiate to find the power series representation of the function.

$$f(x) = \frac{5}{(3-x)^2}$$

■ 2. Differentiate to find the power series representation of the function.

$$f(x) = \frac{3}{(4+x)^2}$$

■ 3. Differentiate to find the power series representation of the function.

$$f(x) = \frac{1}{(-5-x)^2}$$

■ 4. Differentiate to find the power series representation of the function.

$$f(x) = \frac{3}{(6-3x)^2}$$

■ 5. Differentiate to find the power series representation of the function.

$$f(x) = \frac{2}{(1-2x)^2}$$

RADIUS OF CONVERGENCE

1. Find the radius of convergence of the series.

$$\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{4 \cdot 2^{2n}}$$

2. Find the radius of convergence of the series.

$$\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

3. Find the radius of convergence of the series.

$$\sum_{n=0}^{\infty} \frac{x^n}{n+4}$$

4. Find the radius of convergence of the series.

$$\sum_{n=0}^{\infty} \frac{3^n (x+2)^n}{n!}$$

5. Find the radius of convergence of the series.

$$\sum_{n=0}^{\infty} \frac{3^n (x+2)^n}{n+1}$$

INTERVAL OF CONVERGENCE

1. Find the interval of convergence of the series.

$$\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}$$

2. Find the interval of convergence of the series.

$$\sum_{n=0}^{\infty} \frac{(-1)^n (x-3)^n}{n+1}$$

ESTIMATING DEFINITE INTEGRALS

■ 1. Evaluate the definite integral as a power series, using the first four terms.

$$\int_0^2 \frac{24}{x^2+4}\, dx$$

■ 2. Evaluate the definite integral as a power series, using the first four terms.

$$\int_0^1 3x\cos(x^3)\, dx$$

■ 3. Evaluate the definite integral as a power series, using the first four terms.

$$\int_0^1 4e^{x^2}\, dx$$

ESTIMATING INDEFINITE INTEGRALS

1. Evaluate the indefinite integral as a power series.

$$\int x^2 \sin(x^2)\ dx$$

2. Evaluate the indefinite integral as a power series.

$$\int \ln(1+2x)\ dx$$

3. Evaluate the indefinite integral as a power series.

$$\int x^2 \cos(x^3)\ dx$$

BINOMIAL SERIES

1. Use a binomial series to expand the function as a power series.

$$f(x) = (3 + x)^5$$

2. Use a binomial series to expand the function as a power series.

$$f(x) = (6 - x)^4$$

3. Use a binomial series to expand the function as a power series.

$$f(x) = (-4 + x)^5$$

4. Use a binomial series to expand the function as a power series.

$$f(x) = (7 - x)^6$$

5. Use a binomial series to expand the function as a power series.

$$f(x) = (8 + x)^7$$

TAYLOR SERIES

■ 1. Find the third-degree Taylor polynomial and use it to approximate $f(5)$.

$$f(x) = 3\sqrt{x+1}$$

$n = 3$ and $a = 3$

■ 2. Find the third-degree Taylor polynomial and use it to approximate $f(4)$.

$$f(x) = e^{2x} + 9$$

$n = 3$ and $a = 2$

■ 3. Find the fourth-degree Taylor polynomial and use it to approximate $f(\pi/24)$.

$$f(x) = \sin(6x) + 5$$

$n = 4$ and $a = \dfrac{\pi}{12}$

RADIUS AND INTERVAL OF CONVERGENCE OF A TAYLOR SERIES

1. Find the radius of convergence of the Taylor polynomial.

$$P_{(3)}(x) = 1 + 2(x-3) + 4(x-3)^2 + 8(x-3)^3$$

2. Find the radius of convergence of the Taylor polynomial.

$$P_{(3)}(x) = 4 - 4(x-5) + 16(x-5)^2 - 64(x-5)^3$$

3. Find the radius of convergence of the Taylor polynomial.

$$P_{(3)}(x) = \frac{1}{4} - \frac{1}{4}(x-4) + \frac{1}{8}(x-4)^2 - \frac{1}{24}(x-4)^3$$

TAYLOR'S INEQUALITY

1. Find Taylor's inequality for the function.

$$f(x) = 5\cos x$$

2. Find Taylor's inequality for the function.

$$f(x) = 3\sin x$$

3. Find Taylor's inequality for the function.

$$f(x) = 7\sin x + 5$$

4. Find Taylor's inequality for the function.

$$f(x) = \pi\cos x$$

5. Find Taylor's inequality for the function.

$$f(x) = e\sin x$$

MACLAURIN SERIES

1. Write the first four non-zero terms of the Maclaurin series and use it to estimate $f(\pi/9)$.

$$f(x) = \cos(3x)$$

2. Write the first three non-zero terms of the Maclaurin series and use it to estimate $f(2\pi/3)$.

$$f(x) = \cos^2 x$$

3. Write the first four non-zero terms of the Maclaurin series and use it to estimate $f(2)$.

$$f(x) = (x+4)^{\frac{3}{2}}$$

SUM OF THE MACLAURIN SERIES

1. Find the sum of the Maclaurin series.

$$\sum_{n=0}^{\infty} \frac{7(x+4)^n}{n!}$$

2. Find the sum of the Maclaurin series.

$$\sum_{n=0}^{\infty} \frac{6(-1)^n(x-\pi)^{2n+1}}{7(2n+1)!}$$

3. Find the sum of the Maclaurin series.

$$4 + \sum_{n=0}^{\infty} \frac{e(-1)^n(x+\pi)^{2n}}{3(2n)!}$$

RADIUS AND INTERVAL OF CONVERGENCE OF A MACLAURIN SERIES

■ 1. Find the radius of convergence of the Maclaurin series.

$$f(x) = \frac{5}{1 - x^3}$$

■ 2. Find the radius of convergence of the Maclaurin series.

$$f(x) = 4\cos(x^2)$$

■ 3. Find the radius of convergence of the Maclaurin series.

$$\sum_{n=1}^{\infty} \frac{x^n \cdot 3^n}{n}$$

INDEFINITE INTEGRAL AS AN INFINITE SERIES

1. Use an infinite series to evaluate the indefinite integral.

$$\int x^2 \cos(x^3)\ dx$$

2. Use an infinite series to evaluate the indefinite integral.

$$\int 4x^3 \sin(x^4)\ dx$$

3. Use an infinite series to evaluate the indefinite integral.

$$\int 2x \ln(1+x^2)\ dx$$

MACLAURIN SERIES TO ESTIMATE AN INDEFINITE INTEGRAL

■ 1. Use a Maclaurin series to estimate the indefinite integral.

$$\int \frac{\sin(2x)}{4x} \, dx$$

■ 2. Use a Maclaurin series to estimate the indefinite integral.

$$\int \frac{\cos x}{x^2} \, dx$$

■ 3. Use a Maclaurin series to estimate the indefinite integral.

$$\int \frac{\arctan x}{x^2} \, dx$$

MACLAURIN SERIES TO ESTIMATE A DEFINITE INTEGRAL

■ 1. Use a Maclaurin series to estimate the value of the definite integral.

$$\int_0^3 3xe^{\frac{1}{2}x^2}\ dx$$

■ 2. Use a Maclaurin series to estimate the value of the definite integral.

$$\int_0^{\sqrt{\pi/2}} 12\cos(x^2)\ dx$$

■ 3. Use a Maclaurin series to estimate the value of the definite integral.

$$\int_0^{\sqrt[3]{\pi}} 15\sin(x^3)\ dx$$

MACLAURIN SERIES TO EVALUATE A LIMIT

■ 1. Use a Maclaurin series to evaluate the limit.

$$\lim_{x \to 0} \frac{e^{2x} - 1 - 2x}{x^2}$$

■ 2. Use a Maclaurin series to evaluate the limit.

$$\lim_{x \to 0} \frac{\arctan x - x}{x^3}$$

■ 3. Use a Maclaurin series to evaluate the limit.

$$\lim_{x \to 0} \frac{\cos(3x) + \frac{9}{2}x^2 - 1}{x^4}$$

W W W . K R I S T A K I N G M A T H . C O M