Topic: Rectilinear motion

Question: Find the position function that models the rectilinear motion of a particle moving along the x-axis.

$$a(t) = 16t + 4$$

$$v(0) = -12 \text{ and } x(0) = 3$$

Answer choices:

A $\quad x(t) = \frac{8}{3}t^3 + 2t^2 - 12t$

B $\quad x(t) = 8t^3 + 2t^2 - 12t + 3$

C $\quad x(t) = \frac{8}{3}t^3 + 2t^2 - 12t + 3$

D $\quad x(t) = \frac{4}{3}t^3 + 2t^2 - t + 3$

Solution: C

With particle motion, acceleration, velocity and position have the following relationships.

$$a(t) = v'(t) = x''(t)$$

$$v(t) = x'(t)$$

Which means that the given acceleration function $a(t) = 16t + 4$ is the second derivative of the position function $x(t)$ that we need to find.

Let's begin by integrating $a(t)$ to find $v(t)$.

$$a(t) = 16t + 4$$

$$v(t) = \int a(t)\ dt = \int 16t + 4\ dt$$

$$v(t) = 8t^2 + 4t + C$$

Now we'll use the initial condition for the velocity function $v(0) = -12$ to find the a value for C.

$$-12 = 8(0)^2 + 4(0) + C$$

$$-12 = C$$

So the velocity function is

$$v(t) = 8t^2 + 4t - 12$$

To find $x(t)$, we'll integrate the velocity function we just found.

$$x(t) = \int v(t)\ dt = \int 8t^2 + 4t - 12\ dt$$

$$x(t) = \frac{8}{3}t^3 + 2t^2 - 12t + D$$

Now we'll use the initial condition for the position function $x(0) = 3$ to find the a value for D.

$$3 = \frac{8}{3}(0)^3 + 2(0)^2 - 12(0) + D$$

$$3 = D$$

So the position function is

$$x(t) = \frac{8}{3}t^3 + 2t^2 - 12t + 3$$

Topic: Rectilinear motion

Question: Find the position function that models the rectilinear motion of a particle moving along the x-axis.

$$a(t) = t^2 + 7t + 2$$

$$v(0) = 6 \text{ and } x(0) = 5$$

Answer choices:

A $\quad x(t) = \frac{1}{12}t^4 + \frac{7}{6}t^3 + t^2 + 6t + 5$

B $\quad x(t) = \frac{1}{12}t^4 + \frac{7}{6}t^3 + t^2 + 6t$

C $\quad x(t) = t^4 + t^3 + t^2 + 6t + 5$

D $\quad x(t) = t^4 + 7t^3 + t^2 + 6t + 5$

Solution: A

With particle motion, acceleration, velocity and position have the following relationships.

$$a(t) = v'(t) = x''(t)$$

$$v(t) = x'(t)$$

Which means that the given acceleration function $a(t) = 16t + 4$ is the second derivative of the position function $x(t)$ that we need to find.

Let's begin by integrating $a(t)$ to find $v(t)$.

$$a(t) = t^2 + 7t + 2$$

$$v(t) = \int a(t)\ dt = \int t^2 + 7t + 2\ dt$$

$$v(t) = \frac{1}{3}t^3 + \frac{7}{2}t^2 + 2t + C$$

Now we'll use the initial condition for the velocity function $v(0) = 6$ to find the a value for C.

$$6 = \frac{1}{3}(0)^3 + \frac{7}{2}(0)^2 + 2(0) + C$$

$$6 = C$$

So the velocity function is

$$v(t) = \frac{1}{3}t^3 + \frac{7}{2}t^2 + 2t + 6$$

To find $x(t)$, we'll integrate the velocity function we just found.

$$x(t) = \int v(t)\ dt = \int \frac{1}{3}t^3 + \frac{7}{2}t^2 + 2t + 6\ dt$$

$$x(t) = \frac{1}{12}t^4 + \frac{7}{6}t^3 + t^2 + 6t + D$$

Now we'll use the initial condition for the position function $x(0) = 5$ to find the a value for D.

$$5 = \frac{1}{12}(0)^4 + \frac{7}{6}(0)^3 + (0)^2 + 6(0) + D$$

$$5 = D$$

So the position function is

$$x(t) = \frac{1}{12}t^4 + \frac{7}{6}t^3 + t^2 + 6t + 5$$

Topic: Rectilinear motion

Question: Find the position function that models the rectilinear motion of a particle moving along the x-axis.

$$a(t) = e^t + 3$$

$$v(0) = 9 \text{ and } x(0) = 14$$

Answer choices:

A $\quad x(t) = e^t + \frac{3}{2}t^2 + 8t$

B $\quad x(t) = e^t + t^2 + 8t + 13$

C $\quad x(t) = e^t + \frac{3}{2}t^2 + 9t + 14$

D $\quad x(t) = e^t + \frac{3}{2}t^2 + 8t + 13$

Solution: D

With particle motion, acceleration, velocity and position have the following relationships.

$$a(t) = v'(t) = x''(t)$$

$$v(t) = x'(t)$$

Which means that the given acceleration function $a(t) = 16t + 4$ is the second derivative of the position function $x(t)$ that we need to find.

Let's begin by integrating $a(t)$ to find $v(t)$.

$$a(t) = e^t + 3$$

$$v(t) = \int a(t)\ dt = \int e^t + 3\ dt$$

$$v(t) = e^t + 3t + C$$

Now we'll use the initial condition for the velocity function $v(0) = 9$ to find the a value for C.

$$9 = e^0 + 3(0) + C$$

$$9 = 1 + C$$

$$8 = C$$

So the velocity function is

$$v(t) = e^t + 3t + 8$$

To find $x(t)$, we'll integrate the velocity function we just found.

$$x(t) = \int v(t)\ dt = \int e^t + 3t + 8\ dt$$

$$x(t) = e^t + \frac{3}{2}t^2 + 8t + D$$

Now we'll use the initial condition for the position function $x(0) = 14$ to find the a value for D.

$$14 = e^0 + \frac{3}{2}(0)^2 + 8(0) + D$$

$$14 = 1 + D$$

$$13 = D$$

So the position function is

$$x(t) = e^t + \frac{3}{2}t^2 + 8t + 13$$