Topic: sin^m cos^n, odd m

Question: Evaluate the trigonometric integral.

$$\int \sin^7 3x \cos^2 3x \ dx$$

Answer choices:

A $\quad -\frac{1}{9}\cos^3 3x + \frac{1}{5}\cos^5 3x - \frac{1}{7}\cos^7 3x + \frac{1}{27}\cos^9 3x + C$

B $\quad \frac{1}{9}\cos^3 3x - \frac{1}{5}\cos^5 3x + \frac{1}{7}\cos^7 3x - \frac{1}{27}\cos^9 3x + C$

C $\quad -\frac{1}{9}\cos^9 3x + \frac{1}{5}\cos^7 3x - \frac{1}{7}\cos^5 3x + \frac{1}{27}\cos^3 3x + C$

D $\quad \frac{1}{9}\cos^9 3x - \frac{1}{5}\cos^7 3x + \frac{1}{7}\cos^5 3x - \frac{1}{27}\cos^3 3x + C$

Solution: A

In the specific case where our function is the product of

an **odd** number of **sine** factors and

an **even or odd** number of **cosine** factors,

our plan is to

1. save one sine factor and use the identity $\sin^2 x = 1 - \cos^2 x$ to write the other sine factors in terms of cosine, then

2. use u-substitution with $u = \cos x$.

We'll separate a single sine factor and then replace the remaining sine factors using the identity.

$$\int \sin^7 3x \cos^2 3x \ dx$$

$$\int \sin 3x \sin^6 3x \cos^2 3x \ dx$$

$$\int \sin 3x \left(\sin^2 3x\right)^3 \cos^2 3x \ dx$$

$$\int \sin 3x \left(1 - \cos^2 3x\right)^3 \cos^2 3x \ dx$$

Using u-substitution with $u = \cos 3x$, we get

$$u = \cos 3x$$

$$du = -3\sin 3x\ dx$$

$$\sin 3x\ dx = \frac{du}{-3}$$

Substitute into the integral.

$$\int \sin 3x\left(1-u^2\right)^3 u^2\ dx$$

$$\int \left(1-u^2\right)^3 u^2\left(\sin 3x\ dx\right)$$

$$\int \left(1-u^2\right)^3 u^2\left(\frac{du}{-3}\right)$$

$$-\frac{1}{3}\int \left(1-u^2-2u^2+2u^4+u^4-u^6\right)u^2\ du$$

$$-\frac{1}{3}\int \left(1-3u^2+3u^4-u^6\right)u^2\ du$$

$$-\frac{1}{3}\int u^2-3u^4+3u^6-u^8\ du$$

$$-\frac{1}{3}\left(\frac{1}{3}u^3-\frac{3}{5}u^5+\frac{3}{7}u^7-\frac{1}{9}u^9\right)+C$$

Back-substituting for u, we get

$$-\frac{1}{3}\left(\frac{1}{3}\cos^3 3x-\frac{3}{5}\cos^5 3x+\frac{3}{7}\cos^7 3x-\frac{1}{9}\cos^9 3x\right)+C$$

$$-\frac{1}{9}\cos^3 3x+\frac{3}{15}\cos^5 3x-\frac{3}{21}\cos^7 3x+\frac{1}{27}\cos^9 3x+C$$

$$-\frac{1}{9}\cos^3 3x + \frac{1}{5}\cos^5 3x - \frac{1}{7}\cos^7 3x + \frac{1}{27}\cos^9 3x + C$$

Topic: sin^m cos^n, odd m

Question: Evaluate the trigonometric integral.

$$\int \sin^5\theta\cos^4\theta\ d\theta$$

Answer choices:

A $\quad \frac{1}{5}\sin^5\theta-\frac{2}{7}\sin^7\theta+\frac{1}{9}\sin^9\theta+C$

B $\quad -\frac{1}{5}\sin^5\theta+\frac{2}{7}\sin^7\theta-\frac{1}{9}\sin^9\theta+C$

C $\quad -\frac{1}{5}\cos^5\theta+\frac{2}{7}\cos^7\theta-\frac{1}{9}\cos^9\theta+C$

D $\quad \frac{1}{5}\cos^5\theta-\frac{2}{7}\cos^7\theta+\frac{1}{9}\cos^9\theta+C$

Solution: C

In the specific case where our function is the product of

an **odd** number of **sine** factors and

an **even or odd** number of **cosine** factors,

our plan is to

1. save one sine factor and use the identity $\sin^2 x = 1 - \cos^2 x$ to write the other sine factors in terms of cosine, then

2. use u-substitution with $u = \cos x$.

We'll separate a single sine factor and then replace the remaining sine factors using the identity.

$$\int \sin^5 \theta \cos^4 \theta \ d\theta$$

$$\int \sin \theta \sin^4 \theta \cos^4 \theta \ d\theta$$

$$\int \sin \theta \left(\sin^2 \theta\right)^2 \cos^4 \theta \ d\theta$$

$$\int \sin \theta \left(1 - \cos^2 \theta\right)^2 \cos^4 \theta \ d\theta$$

Using u-substitution with $u = \cos \theta$, we get

$$u = \cos \theta$$

$$du = -\sin\theta \ d\theta$$

$$-du = \sin\theta \ d\theta$$

Substitute into the integral.

$$\int \sin\theta \left(1 - u^2\right)^2 u^4 \ d\theta$$

$$\int \left(1 - u^2\right)^2 u^4 \left(\sin\theta \ d\theta\right)$$

$$\int \left(1 - u^2\right)^2 u^4 \left(- \ du\right)$$

$$-\int \left(1 - u^2\right)^2 u^4 \ du$$

$$-\int \left(1 - 2u^2 + u^4\right) u^4 \ du$$

$$-\int u^4 - 2u^6 + u^8 \ du$$

$$-\left(\frac{1}{5}u^5 - \frac{2}{7}u^7 + \frac{1}{9}u^9\right) + C$$

$$-\frac{1}{5}u^5 + \frac{2}{7}u^7 - \frac{1}{9}u^9 + C$$

Back-substituting for u, we get

$$-\frac{1}{5}\cos^5\theta + \frac{2}{7}\cos^7\theta - \frac{1}{9}\cos^9\theta + C$$

Topic: sin^m cos^n, odd m

Question: Evaluate the trigonometric integral.

$$\int \sin^3\theta\cos^2\theta\ d\theta$$

Answer choices:

A $\quad \frac{1}{3}\sin^3\theta - \frac{1}{5}\sin^5\theta + C$

B $\quad -\frac{1}{3}\cos^3\theta + \frac{1}{5}\cos^5\theta + C$

C $\quad -\frac{1}{3}\sin^3\theta + \frac{1}{5}\sin^5\theta + C$

D $\quad \frac{1}{3}\cos^3\theta - \frac{1}{5}\cos^5\theta + C$

Solution: B

In the specific case where our function is the product of

an **odd** number of **sine** factors and

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our plan is to

1. save one sine factor and use the identity $\sin^2 x = 1 - \cos^2 x$ to write the other sine factors in terms of cosine, then

2. use u-substitution with $u = \cos x$.

We'll separate a single sine factor and then replace the remaining sine factors using the identity.

$$\int \sin^3 \theta \cos^2 \theta \ d\theta$$

$$\int \sin \theta \sin^2 \theta \cos^2 \theta \ d\theta$$

$$\int \sin \theta \left(1 - \cos^2 \theta\right) \cos^2 \theta \ d\theta$$

Using u-substitution with $u = \cos \theta$, we get

$$u = \cos \theta$$

$$du = -\sin \theta \ d\theta$$

$$-du = \sin \theta \ d\theta$$

Substitute into the integral.

$$\int \sin\theta\left(1-u^2\right)u^2\ d\theta$$

$$\int \left(1-u^2\right)u^2\left(\sin\theta\ d\theta\right)$$

$$\int \left(1-u^2\right)u^2\left(-\ du\right)$$

$$-\int \left(1-u^2\right)u^2\ du$$

$$-\int u^2-u^4\ du$$

$$-\left(\frac{1}{3}u^3-\frac{1}{5}u^5\right)+C$$

$$-\frac{1}{3}u^3+\frac{1}{5}u^5+C$$

Back-substituting for u, we get

$$-\frac{1}{3}\cos^3\theta+\frac{1}{5}\cos^5\theta+C$$