Topic: Cardiac output

Question: Find the cardiac output if 5 mg of dye is injected into the heart and the concentration of dye remaining in the heart t seconds after the injection is modeled by $C(t) = 18te^{-0.7t}$. Assume $0 \leq t \leq 15$.

Answer choices:

A 0.136 liters/second

B 8.169 liters/second

C 0.136 liters/minute

D 2.040 liters/second

Solution: A

Cardiac output is given by

$$F = \frac{A}{\int_0^T C(t)\ dt}$$

where F is blood flow, A is the amount of dye that was injected into the heart, $C(t)$ is the concentration of dye injected t seconds after the injection, and T is the time at which all of the dye concentration is out of the heart.

Plugging everything we've been given into the formula gives

$$F = \frac{5}{\int_0^{15} 18te^{-0.7t}\ dt}$$

Use integration by parts to evaluate the integral.

$$u = 18t$$

$$du = 18\ dt$$

$$dv = e^{-0.7t}\ dt$$

$$v = -\frac{1}{0.7}e^{-0.7t}$$

The integration by parts formula is

$$\int u\ dv = uv - \int v\ du$$

So the cardiac output formula is now

$$F = \frac{5}{(18t)\left(-\frac{1}{0.7}e^{-0.7t}\right)\Big|_0^{15} - \int_0^{15} -\frac{1}{0.7}e^{-0.7t}(18)\,dt}$$

$$F = \frac{5}{-\frac{18t}{0.7}e^{-0.7t}\Big|_0^{15} + \int_0^{15}\frac{18}{0.7}e^{-0.7t}\,dt}$$

$$F = \frac{5}{-\frac{18t}{0.7}e^{-0.7t} - \frac{18}{0.49}e^{-0.7t}\Big|_0^{15}}$$

$$F = \frac{5}{-\frac{18(15)}{0.7}e^{-0.7(15)} - \frac{18}{0.49}e^{-0.7(15)} - \left(-\frac{18(0)}{0.7}e^{-0.7(0)} - \frac{18}{0.49}e^{-0.7(0)}\right)}$$

$$F = \frac{5}{-\frac{270}{0.7}e^{-10.5} - \frac{18}{0.49}e^{-10.5} + (0)(1) + \frac{18}{0.49}(1)}$$

$$F = \frac{5}{-\frac{270}{0.7}e^{-10.5} - \frac{18}{0.49}e^{-10.5} + \frac{18}{0.49}}$$

$$F \approx 0.136$$

Topic: Cardiac output

Question: Find the cardiac output if 7 mg of dye is injected into the heart and the concentration of dye remaining in the heart t seconds after the injection is modeled by $C(t) = 20te^{-0.5t}$. Assume $0 \leq t \leq 12$.

Answer choices:

A 5.343 liters/second

B 0.089 liters/minute

C 3.931 liters/second

D 0.089 liters/second

Solution: D

Cardiac output is given by

$$F = \frac{A}{\int_0^T C(t)\ dt}$$

where F is blood flow, A is the amount of dye that was injected into the heart, $C(t)$ is the concentration of dye injected t seconds after the injection, and T is the time at which all of the dye concentration is out of the heart.

Plugging everything we've been given into the formula gives

$$F = \frac{7}{\int_0^{12} 20te^{-0.5t}\ dt}$$

Use integration by parts to evaluate the integral.

$$u = 20t$$

$$du = 20\ dt$$

$$dv = e^{-0.5t}\ dt$$

$$v = -\frac{1}{0.5}e^{-0.5t}$$

The integration by parts formula is

$$\int u\ dv = uv - \int v\ du$$

So the cardiac output formula is now

$$F = \frac{7}{(20t)\left(-\frac{1}{0.5}e^{-0.5t}\right)\Big|_0^{12} - \int_0^{12} -\frac{1}{0.5}e^{-0.5t}(20)\ dt}$$

$$F = \frac{7}{-\frac{20t}{0.5}e^{-0.5t}\Big|_0^{12} + \int_0^{12} \frac{20}{0.5}e^{-0.5t}\ dt}$$

$$F = \frac{7}{-\frac{20t}{0.5}e^{-0.5t} - \frac{20}{0.25}e^{-0.5t}\Big|_0^{12}}$$

$$F = \frac{7}{-\frac{20(12)}{0.5}e^{-0.5(12)} - \frac{20}{0.25}e^{-0.5(12)} - \left(-\frac{20(0)}{0.5}e^{-0.5(0)} - \frac{20}{0.25}e^{-0.5(0)}\right)}$$

$$F = \frac{7}{-960e^{-6} - 80e^{-6} - \left(-(0)(1) - (80)(1)\right)}$$

$$F = \frac{7}{-960e^{-6} - 80e^{-6} + 80}$$

$$F \approx 0.089$$

Topic: Cardiac output

Question: Find the cardiac output if 10 mg of dye is injected into the heart and the concentration of dye remaining in the heart t seconds after the injection is modeled by $C(t) = 56te^{-0.8t}$. Assume $0 \leq t \leq 20$.

Answer choices:

A 0.114 liters/minute

B 1.562 liters/minute

C 0.114 liters/second

D 1.562 liters/second

Solution: C

Cardiac output is given by

$$F = \frac{A}{\int_0^T C(t)\ dt}$$

where F is blood flow, A is the amount of dye that was injected into the heart, $C(t)$ is the concentration of dye injected t seconds after the injection, and T is the time at which all of the dye concentration is out of the heart.

Plugging everything we've been given into the formula gives

$$F = \frac{10}{\int_0^{20} 56te^{-0.8t}\ dt}$$

Use integration by parts to evaluate the integral.

$$u = 56t$$

$$du = 56\ dt$$

$$dv = e^{-0.8t}\ dt$$

$$v = -\frac{1}{0.8}e^{-0.8t}$$

The integration by parts formula is

$$\int u\ dv = uv - \int v\ du$$

So the cardiac output formula is now

$$F = \frac{10}{(56t)\left(-\frac{1}{0.8}e^{-0.8t}\right)\Big|_0^{20} - \int_0^{20} -\frac{1}{0.8}e^{-0.8t}(56)\,dt}$$

$$F = \frac{10}{-\frac{56t}{0.8}e^{-0.8t} - \frac{56}{0.64}e^{-0.8t}\Big|_0^{20}}$$

$$F = \frac{10}{-\frac{56(20)}{0.8}e^{-0.8(20)} - \frac{56}{0.64}e^{-0.8(20)} - \left(-\frac{56(0)}{0.8}e^{-0.8(0)} - \frac{56}{0.64}e^{-0.8(0)}\right)}$$

$$F = \frac{10}{-1{,}400e^{-16} - 87.5e^{-16} - \left(-(0)(1) - 87.5(1)\right)}$$

$$F = \frac{10}{-1{,}400e^{-16} - 87.5e^{-16} + 87.5}$$

$$F \approx 0.114$$