Making the function continuous

In this lesson, we want to build on what we already know about discontinuities and one-sided limits.

Point discontinuities

Previously, we learned that a point discontinuity was a single pinpoint of discontinuity in the graph. We saw that we could take a function like

$$f(x)=\frac{x^2+11x+28}{x+4}$$

and simplify it as

$$f(x)=\frac{(x+4)(x+7)}{x+4}$$

$$f(x)=x+7$$

Because we canceled the $x+4$, we know the function has a point discontinuity at $x=-4$, and we know we can make the function continuous if we "plug the hole" be redefining it as

$$f(x)=\begin{cases}\frac{x^2+11x+28}{x+4} & x\neq-4\\ 3 & x=-4\end{cases}$$

Remember that this kind of function is a piecewise function or piecewise-defined function, because it's defined in "pieces." At this point, we want to spend more time with piecewise functions. Specifically, we want to investigate how to find the value of an unknown in order to force the continuity of the piecewise function.

Piecewise-defined functions

Sometimes we'll be given a piecewise-defined function, and asked to find the value of an unknown constant that will make the function continuous. For instance, consider this function:

$$f(x) = \begin{cases} k\sqrt{x+1} & 0 \le x \le 3 \\ 5 - x & 3 < x \le 5 \end{cases}$$

It's a piecewise-defined function, where the first piece defines the function from $x = 0$ to $x = 3$ (including at $x = 3$), and the second piece defines the function for values greater than $x = 3$, all the way up to $x = 5$.

When $x = 3$, the first piece stops defining the function and the second piece takes over, so we can think of $x = 3$ as the "break point" between the pieces. If we can make the two pieces meet at the break point, then the function will be continuous.

So for a problem like this one, we need to find the value of k that makes $f(x)$ continuous, which is the value of k that ensures that both pieces of the graph meet at the same value when $x = 3$.

To visualize this, here's what the graph of $f(x)$ looks like with some different values of k.

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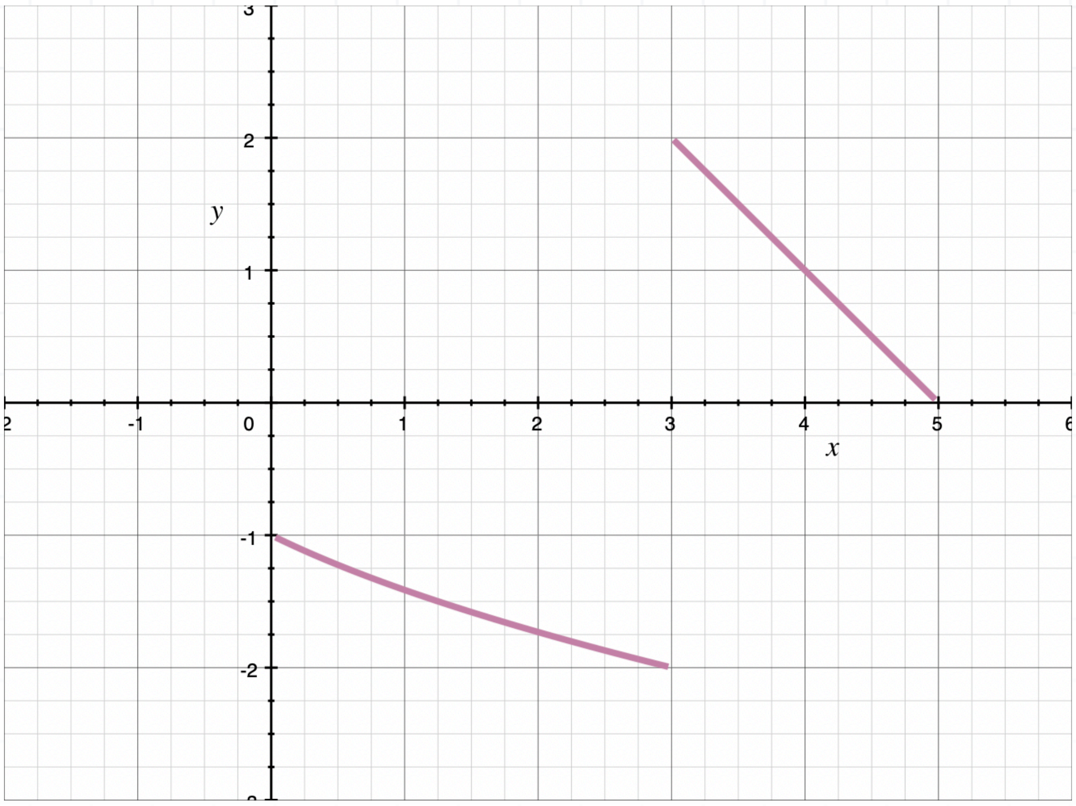
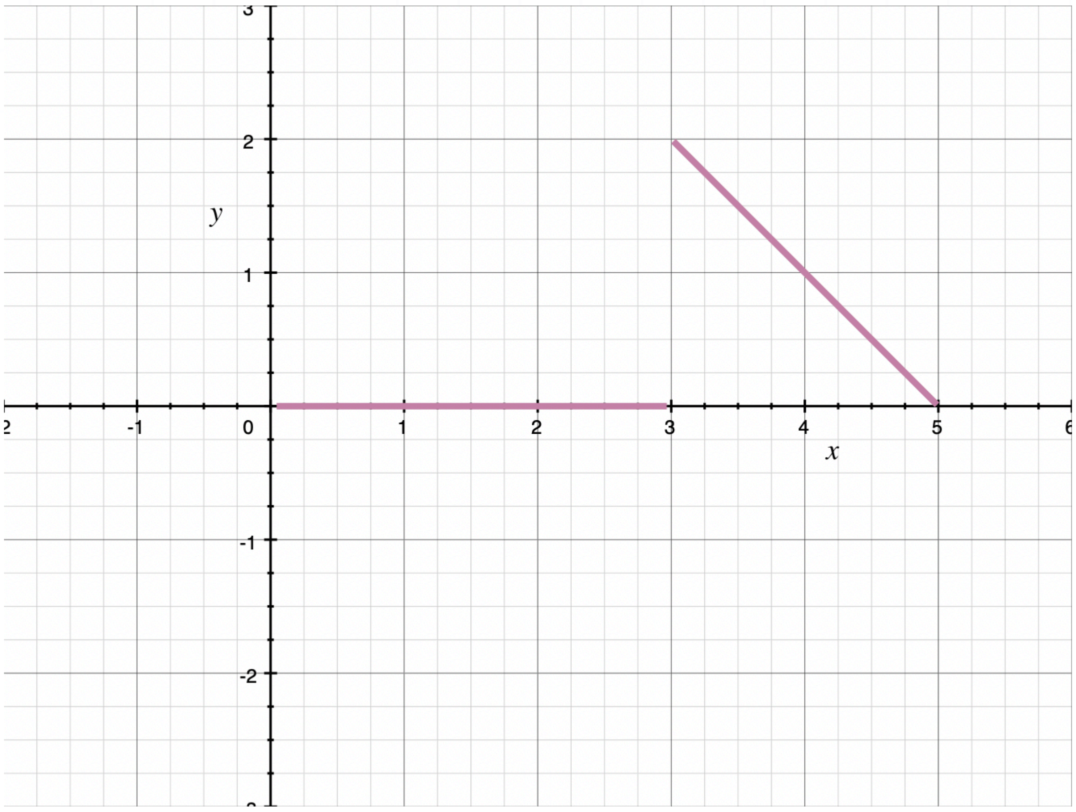
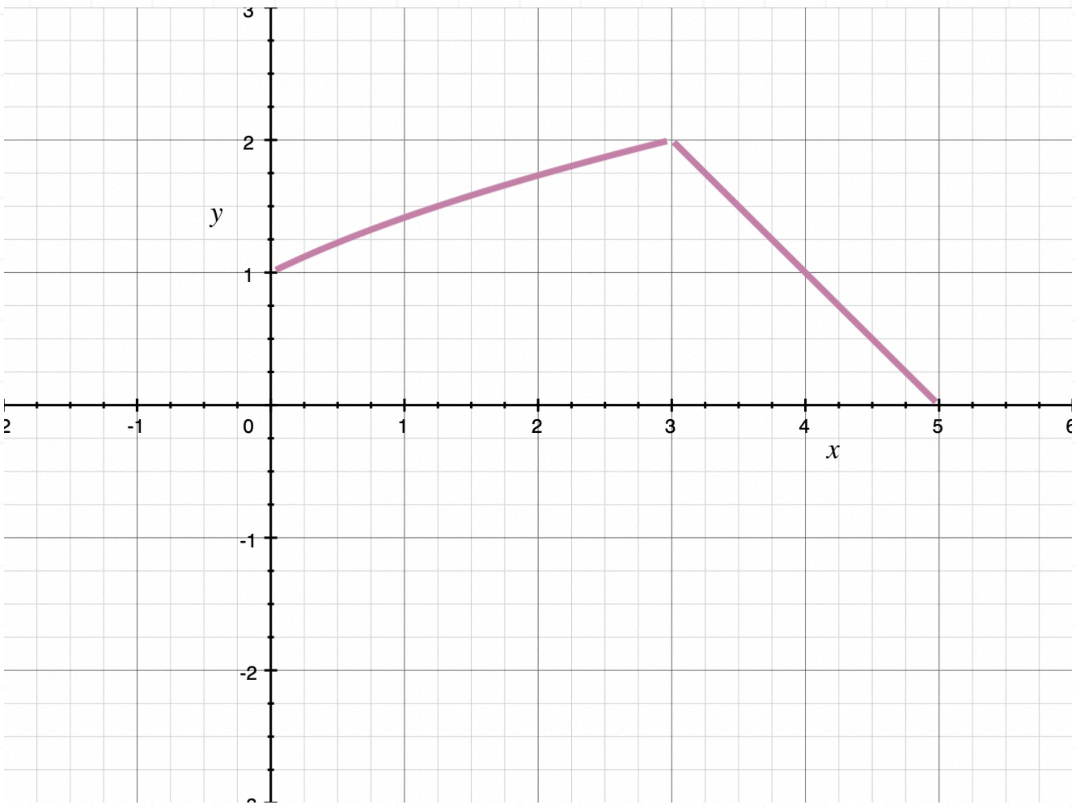
If $k = -1$, the graph of $f(x)$ is



If $k = 0$, the graph of $f(x)$ is



If $k = 1$, the graph of $f(x)$ is



We can see from the graphs that $k=1$ will be the value that makes the function continuous. But how do we solve for the value of k algebraically, so that we can avoid picking random values of the unknown constant and graphing the function with that value?

Well, we always want to start at the "break point" that we talked about earlier. For this function, $x=3$ is the break point between the two pieces, so we need the pieces to have equal value at that point. Therefore, we'll set the pieces equal to one another, plug in $x=3$, and then solve the equation for the unknown k.

$$k\sqrt{x+1}=5-x$$

$$k\sqrt{3+1}=5-3$$

$$k\sqrt{4}=2$$

$$2k=2$$

$$k = 1$$

So $k = 1$ is the value that forces the continuity of the function. For any other value of k, we'll get a jump discontinuity at the break point $x = 3$, but $k = 1$ makes the two pieces meet at the same point, thereby making the function continuous there.

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