Topic: Jump discontinuities

Question: Which of the following statements is true?

Answer choices:

A A jump discontinuity occurs when the right- and left-hand limits both exist, but aren't equal.

B A jump discontinuity occurs when the right- and left-hand limits are not equal, and only one exists.

C A jump discontinuity occurs when the right- and left-hand limits do not exist.

D A jump discontinuity occurs when the right- and left-hand limits both exist, and they're equal.

Solution: A

For a jump discontinuity to occur, the left- and right-hand limits must both exist, but they must not be equal. In other words, the left-hand limit will exist, and the right-hand limit will exist, but the left- and right-hand limits will have different values.

Topic: Jump discontinuities

Question: Choose the correct description of the jump discontinuity that exists in the function.

$$f(x) = \frac{x}{|x|}$$

Answer choices:

A The function has a jump discontinuity at $x = 1$.

B The function has a jump discontinuity at $x = -1$.

C The function has a jump discontinuity at $x = \infty$.

D The function has a jump discontinuity at $x = 0$.

Solution: D

To answer this question, we could investigate the limit at each of the values given in the answer choices.

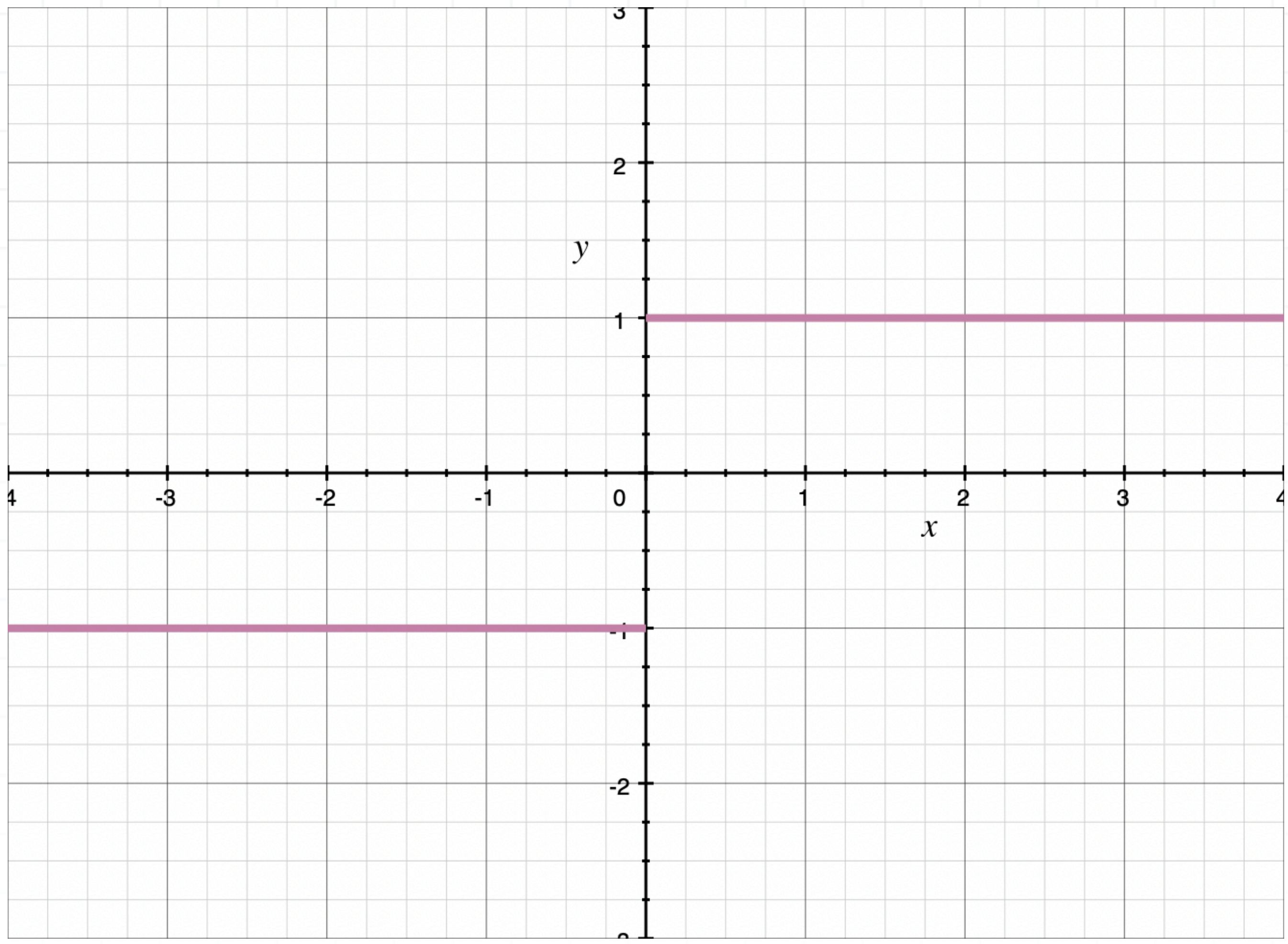
However, if we consider the answer choices briefly, or if we investigate answer choice D, we see that the function is undefined at $x = 0$. So $x = 0$ is an interesting place to start.

We'll look at what the function is doing on either side of $x = 0$.

$$f(-0.0001) = \frac{-0.0001}{|-0.0001|} = \frac{-0.0001}{0.0001} = -1$$

$$f(0.0001) = \frac{0.0001}{|0.0001|} = \frac{0.0001}{0.0001} = 1$$

What we see when we substitute values on either side of $x = 0$ is that, no matter which values we pick, any value to the left of $x = 0$ will return a value of -1, and any value to the right of $x = 0$ will return a value of 1.



Therefore, the jump discontinuity occurs at $x = 0$.

Topic: Jump discontinuities

Question: Choose the correct description of the jump discontinuity.

$$f(x) = \frac{x - 1}{|x - 1|}$$

Answer choices:

A The function has a jump discontinuity at $x = -1$.

B The function has a jump discontinuity at $x = 1$.

C The function has a jump discontinuity at $x = \infty$.

D The function has a jump discontinuity at $x = 0$.

Solution: B

To answer this question, we could investigate the limit at each of the values given in the answer choices.

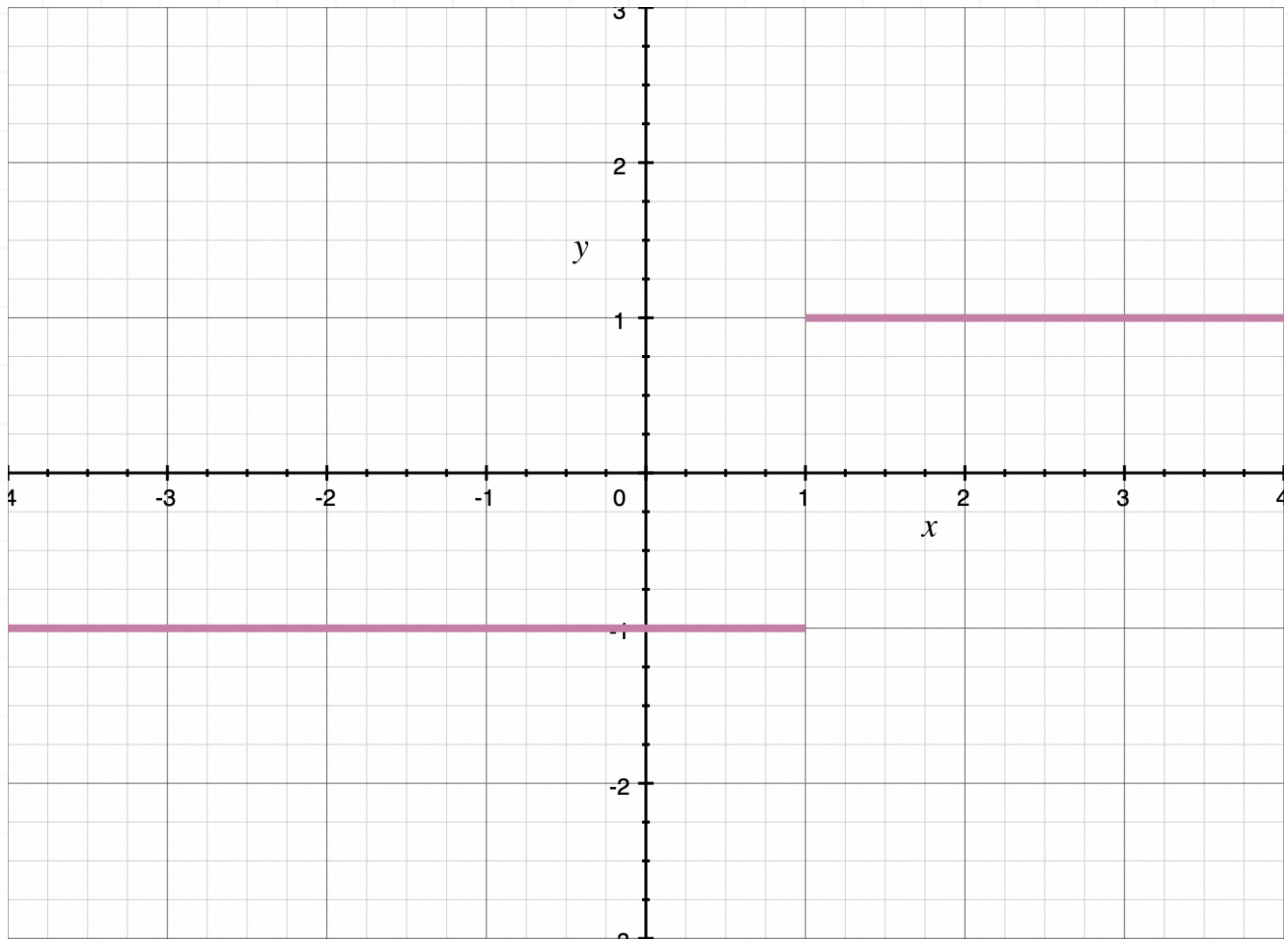
However, if we consider the answer choices briefly, or if we investigate answer choice B, we see that the function is undefined at $x=1$. So $x=1$ is an interesting place to start.

We'll look at what the function is doing on either side of $x=1$.

$$f(0.9999)=\frac{0.9999-1}{|0.9999-1|}=\frac{-0.0001}{|-0.0001|}=\frac{-0.0001}{0.0001}=-1$$

$$f(1.0001)=\frac{1.0001-1}{|1.0001-1|}=\frac{0.0001}{|0.0001|}=\frac{0.0001}{0.0001}=1$$

What we see when we substitute values on either side of $x=1$ is that, no matter which values we pick, any value to the left of $x=1$ will return a value of -1, and any value to the right of $x=1$ will return a value of 1.



Therefore, the jump discontinuity occurs at $x = 1$.