Calculus 2 Workbook Solutions

Group

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SIN^M COS^N, ODD M

1. Evaluate the trigonometric integral.

$$\int \sin^5(3x^2+2x+1)\cos(3x^2+2x+1)(6x+2)\ dx$$

Solution:

Use u-substitution.

$$u=\sin(3x^2+2x+1)$$

$$\frac{du}{dx}=\cos(3x^2+2x+1)(6x+2)$$

$$du=\cos(3x^2+2x+1)(6x+2)\ dx$$

$$dx=\frac{du}{\cos(3x^2+2x+1)(6x+2)}$$

Substitute.

$$\int u^5\cos(3x^2+2x+1)(6x+2)\cdot\frac{du}{\cos(3x^2+2x+1)(6x+2)}$$

$$\int u^5\ du$$

Integrate and back-substitute.

$$\frac{1}{6}u^6 + C$$

$$\frac{1}{6}(\sin(3x^2 + 2x + 1))^6 + C$$

$$\frac{1}{6}\sin^6(3x^2 + 2x + 1) + C$$

SIN^M COS^N, ODD N

1. Evaluate the trigonometric integral.

$$\int_{-\frac{\pi}{6}}^{\frac{\pi}{3}} (4+\cos x)\sin x \ dx$$

Solution:

Use u-substitution.

$$u = 4 + \cos x$$

$$\frac{du}{dx} = -\sin x, \text{ so } du = -\sin x \ dx, \text{ so } dx = -\frac{du}{\sin x}$$

Change the limits of integration.

$$u\left(\frac{\pi}{3}\right) = 4 + \cos\left(\frac{\pi}{3}\right) = 4 + \frac{1}{2} = \frac{9}{2}$$

$$u\left(-\frac{\pi}{6}\right) = 4 + \cos\left(-\frac{\pi}{6}\right) = 4 + \frac{\sqrt{3}}{2}$$

Substitute into the integral.

$$\int_{4+\frac{\sqrt{3}}{2}}^{\frac{9}{2}} u \sin x \left(-\frac{du}{\sin x}\right)$$

$$-\int_{4+\frac{\sqrt{3}}{2}}^{\frac{9}{2}} u\ du$$

Integrate and evaluate over the interval.

$$-\frac{1}{2}u^2\Big|_{4+\frac{\sqrt{3}}{2}}^{\frac{9}{2}}$$

$$-\frac{1}{2}\left(\frac{9}{2}\right)^2+\frac{1}{2}\left(4+\frac{\sqrt{3}}{2}\right)^2$$

$$-\frac{1}{2}\left(\frac{81}{4}\right)+\frac{1}{2}\left(16+4\sqrt{3}+\frac{3}{4}\right)$$

$$-\frac{81}{8}+8+2\sqrt{3}+\frac{3}{8}$$

$$-\frac{7}{4}+2\sqrt{3}$$

2. Evaluate the trigonometric integral.

$$\int \sin(2x)\cos^3(2x)\ dx$$

Solution:

Use u-substitution.

$$u = \cos(2x)$$

$$\frac{du}{dx} = -2\sin(2x), \text{ so } du = -2\sin(2x)\ dx, \text{ so } dx = -\frac{du}{2\sin(2x)}$$

Substitute into the integral.

$$\int \sin(2x) \cdot u^3 \cdot -\frac{du}{2\sin(2x)}$$

$$-\frac{1}{2}\int u^3\ du$$

Integrate, then back-substitute.

$$-\frac{1}{2}\left(\frac{1}{4}u^4\right) + C$$

$$-\frac{1}{8}u^4 + C$$

$$-\frac{1}{8}\cos^4(2x) + C$$

SIN^M COS^N, M AND N EVEN

■ 1. Evaluate the trigonometric integral.

$$\int \sin^2(2x+3)\cos^2(2x+3)\ dx$$

Solution:

Use the trig identity $\sin\theta\cos\theta = (1/2)\sin(2\theta)$ to rewrite the integrand.

$$\sin^2(2x+3)\cos^2(2x+3)$$

$$\left[\sin(2x+3)\cos(2x+3)\right]\left[\sin(2x+3)\cos(2x+3)\right]$$

$$\left[\frac{1}{2}\sin(2(2x+3))\right]^2$$

$$\frac{1}{4}\sin^2(4x+6)$$

Use the trig identity $\sin^2\theta = (1/2)(1-\cos(2\theta))$ to rewrite the integrand.

$$\frac{1}{4}\left[\frac{1}{2}(1-\cos(2(4x+6)))\right]$$

$$\frac{1}{8}(1-\cos(8x+12))$$

$$\frac{1}{8}-\frac{1}{8}\cos(8x+12)$$

Integrate.

$$\int \frac{1}{8} - \frac{1}{8}\cos(8x + 12)\ dx$$

$$\frac{1}{8}x - \frac{1}{64}\sin(8x + 12) + C$$

■ 2. Evaluate the trigonometric integral.

$$\int \sin^4(2x)\cos^2(2x)\ dx$$

Solution:

Use the trig identity $\sin\theta\cos\theta = (1/2)\sin(2\theta)$ to rewrite the integrand.

$$\sin^4(2x)\cos^2(2x)$$

$$\sin^2(2x)\sin^2(2x)\cos^2(2x)$$

$$\sin^2(2x)(\sin(2x)\cos(2x))(\sin(2x)\cos(2x))$$

$$\sin^2(2x)\left[\frac{1}{2}\sin(2(2x))\right]^2$$

$$\frac{1}{4}\sin^2(2x)\sin^2(4x)$$

Use the trig identity $\sin^2\theta = (1/2)(1 - \cos(2\theta))$ to rewrite the integrand.

$$\frac{1}{4}\left(\frac{1}{2}(1-\cos(2(2x)))\right)\left(\frac{1}{2}(1-\cos(2(4x)))\right)$$

$$\frac{1}{16}(1-\cos(4x))(1-\cos(8x))$$

$$\frac{1}{16}\left[1-\cos(8x)-\cos(4x)+\cos(8x)\cos(4x)\right]$$

Use the trig identity $\cos a \cos b = (1/2)\left[\cos(a-b)+\cos(a+b)\right]$ to rewrite the integrand.

$$\frac{1}{16}\left[1-\cos(8x)-\cos(4x)+\frac{1}{2}\left[\cos(8x-4x)+\cos(8x+4x)\right]\right]$$

$$\frac{1}{16}\left[1-\cos(8x)-\cos(4x)+\frac{1}{2}\cos(4x)+\frac{1}{2}\cos(12x)\right]$$

$$\frac{1}{16}\left[1-\cos(8x)-\frac{1}{2}\cos(4x)+\frac{1}{2}\cos(12x)\right]$$

Integrate.

$$\frac{1}{16}\int 1-\cos(8x)-\frac{1}{2}\cos(4x)+\frac{1}{2}\cos(12x)\ dx$$

$$\frac{1}{16}\left(x-\frac{1}{8}\sin(8x)-\frac{1}{8}\sin(4x)+\frac{1}{24}\sin(12x)\right)+C$$

$$\frac{1}{16}x-\frac{1}{128}\sin(8x)-\frac{1}{128}\sin(4x)+\frac{1}{384}\sin(12x)+C$$

3. Evaluate the trigonometric integral.

$$\int \sin^6(3x)\cos^4(3x)\ dx$$

Solution:

Use the trig identity $\sin\theta\cos\theta = (1/2)\sin(2\theta)$ to rewrite the integrand.

$$\int \sin^2(3x)\sin^4(3x)\cos^4(3x)\ dx$$

$$\int \sin^2(3x)\left[\sin(3x)\cos(3x)\right]^4\ dx$$

$$\int \sin^2(3x)\left[\frac{1}{2}\sin(2(3x))\right]^4\ dx$$

$$\frac{1}{16}\int \sin^2(3x)\sin^4(6x)\ dx$$

Use the trig identity $\sin^2\theta = (1/2)(1-\cos(2\theta))$ to rewrite the integrand.

$$\frac{1}{16}\int \frac{1}{2}(1-\cos(2(3x)))\sin^4(6x)\ dx$$

$$\frac{1}{32}\int (1-\cos(6x))\sin^4(6x)\ dx$$

Use the trig identity $\sin^2\theta = (1/2)(1-\cos(2\theta))$ to rewrite the integrand.

$$\frac{1}{32}\int (1-\cos(6x))(\sin^2(6x))^2\ dx$$

$$\frac{1}{32}\int(1-\cos(6x))\left[\frac{1}{2}(1-\cos(2(6x)))\right]^2\ dx$$

$$\frac{1}{32}\int(1-\cos(6x))\frac{1}{4}(1-\cos(12x))^2\ dx$$

$$\frac{1}{128}\int(1-\cos(6x))(1-\cos(12x))^2\ dx$$

$$\frac{1}{128}\int(1-\cos(6x))(1-2\cos(12x)+\cos^2(12x))\ dx$$

$$\frac{1}{128}\int 1-2\cos(12x)+\cos^2(12x)-\cos(6x)$$

$$+2\cos(6x)\cos(12x)-\cos(6x)\cos^2(12x)\ dx$$

Use the trig identity $\cos^2\theta=(1/2)(1+\cos(2\theta))$ to rewrite the integrand.

$$\frac{1}{128}\int 1-2\cos(12x)+\frac{1}{2}(1+\cos(2(12x)))-\cos(6x)$$

$$+2\cos(6x)\cos(12x)-\cos(6x)\cos^2(12x)\ dx$$

$$\frac{1}{128}\int 1-2\cos(12x)+\frac{1}{2}+\frac{1}{2}\cos(24x)-\cos(6x)$$

$$+2\cos(6x)\cos(12x)-\cos(6x)\cos^2(12x)\ dx$$

$$\frac{1}{128}\int \frac{3}{2}-\cos(6x)-2\cos(12x)+\frac{1}{2}\cos(24x)$$

$$+2\cos(6x)\cos(12x)-\cos(6x)\cos^2(12x)\ dx$$

Use the trig identity $\cos a \cos b = (1/2)\left[\cos(a-b) + \cos(a+b)\right]$ to rewrite the integrand.

$$\frac{1}{128}\int \frac{3}{2} - \cos(6x) - 2\cos(12x) + \frac{1}{2}\cos(24x)$$

$$+2\left[\frac{1}{2}\left[\cos(12x-6x) + \cos(12x+6x)\right]\right] - \cos(6x)\cos^2(12x)\ dx$$

$$\frac{1}{128}\int \frac{3}{2} - \cos(6x) - 2\cos(12x) + \frac{1}{2}\cos(24x)$$

$$+\cos(6x) + \cos(18x) - \cos(6x)\cos^2(12x)\ dx$$

Use the trig identity $\cos a \cos b = (1/2)\left[\cos(a-b) + \cos(a+b)\right]$ to rewrite the integrand.

$$\frac{1}{128}\int \frac{3}{2} - \cos(6x) - 2\cos(12x) + \frac{1}{2}\cos(24x)$$

$$+\cos(6x) + \cos(18x) - \cos(6x)\cos(12x)\cos(12x)\ dx$$

$$\frac{1}{128}\int \frac{3}{2} - \cos(6x) - 2\cos(12x) + \frac{1}{2}\cos(24x)$$

$$+\cos(6x) + \cos(18x) - \frac{1}{2}\left[\cos(12x-6x) + \cos(12x+6x)\right]\cos(12x)\ dx$$

$$\frac{1}{128}\int \frac{3}{2} - \cos(6x) - 2\cos(12x) + \frac{1}{2}\cos(24x)$$

$$+\cos(6x) + \cos(18x) + \left[-\frac{1}{2}\cos(6x) - \frac{1}{2}\cos(18x)\right]\cos(12x)\ dx$$

$$\frac{1}{128}\int\frac{3}{2}-\cos(6x)-2\cos(12x)+\frac{1}{2}\cos(24x)+\cos(6x)+\cos(18x)$$

$$-\frac{1}{2}\left[\cos(6x)\cos(12x)+\cos(18x)\cos(12x)\right]\ dx$$

$$\frac{1}{128}\int\frac{3}{2}-\cos(6x)-2\cos(12x)+\frac{1}{2}\cos(24x)+\cos(6x)+\cos(18x)$$

$$-\frac{1}{2}\left[\frac{1}{2}\left[\cos(12x-6x)+\cos(12x+6x)\right]+\cos(18x)\cos(12x)\right]\ dx$$

$$\frac{1}{128}\int\frac{3}{2}-\cos(6x)-2\cos(12x)+\frac{1}{2}\cos(24x)+\cos(6x)+\cos(18x)$$

$$-\frac{1}{2}\left[\frac{1}{2}\cos(6x)+\frac{1}{2}\cos(18x)+\cos(18x)\cos(12x)\right]\ dx$$

$$\frac{1}{128}\int\frac{3}{2}-\cos(6x)-2\cos(12x)+\frac{1}{2}\cos(24x)+\cos(6x)+\cos(18x)$$

$$-\frac{1}{4}\cos(6x)-\frac{1}{4}\cos(18x)-\frac{1}{2}\cos(18x)\cos(12x)\ dx$$

$$\frac{1}{128}\int\frac{3}{2}-\cos(6x)-2\cos(12x)+\frac{1}{2}\cos(24x)+\cos(6x)+\cos(18x)$$

$$-\frac{1}{4}\cos(6x)-\frac{1}{4}\cos(18x)-\frac{1}{2}\left[\frac{1}{2}\left[\cos(18x-12x)+\cos(18x+12x)\right]\right]\ dx$$

$$\frac{1}{128}\int\frac{3}{2}-\cos(6x)-2\cos(12x)+\frac{1}{2}\cos(24x)+\cos(6x)+\cos(18x)$$

$$-\frac{1}{4}\cos(6x)-\frac{1}{4}\cos(18x)-\frac{1}{4}\left[\cos(6x)+\cos(30x)\right]\ dx$$

$$\frac{1}{128}\int \frac{3}{2}-\cos(6x)-2\cos(12x)+\frac{1}{2}\cos(24x)+\cos(6x)+\cos(18x)$$

$$-\frac{1}{4}\cos(6x)-\frac{1}{4}\cos(18x)-\frac{1}{4}\cos(6x)-\frac{1}{4}\cos(30x)\ dx$$

$$\frac{1}{128}\int \frac{3}{2}-\cos(6x)+\cos(6x)-\frac{1}{4}\cos(6x)-\frac{1}{4}\cos(6x)-2\cos(12x)$$

$$+\cos(18x)-\frac{1}{4}\cos(18x)+\frac{1}{2}\cos(24x)-\frac{1}{4}\cos(30x)\ dx$$

$$\frac{1}{128}\int \frac{3}{2}-\frac{1}{2}\cos(6x)-2\cos(12x)$$

$$+\frac{3}{4}\cos(18x)+\frac{1}{2}\cos(24x)-\frac{1}{4}\cos(30x)\ dx$$

Integrate.

$$\frac{1}{128}\left(\frac{3}{2}x-\frac{1}{12}\sin(6x)-\frac{1}{6}\sin(12x)+\frac{1}{24}\sin(18x)+\frac{1}{48}\sin(24x)-\frac{1}{120}\sin(30x)\right)+C$$

$$\frac{1}{256}\left(3x-\frac{1}{6}\sin(6x)-\frac{1}{3}\sin(12x)+\frac{1}{12}\sin(18x)+\frac{1}{24}\sin(24x)-\frac{1}{60}\sin(30x)\right)+C$$

TAN^M SEC^N, ODD M

1. Evaluate the trigonometric integral.

$$\int \tan^3(2x)\sec(2x)\ dx$$

Solution:

Use the trig identity $\tan^2\theta = \sec^2\theta - 1$ to rewrite the integrand.

$$\int \tan^2(2x)\tan(2x)\sec(2x)\ dx$$

$$\int (\sec^2(2x) - 1)\tan(2x)\sec(2x)\ dx$$

$$\int \sec^2(2x)\tan(2x)\sec(2x) - \tan(2x)\sec(2x)\ dx$$

$$\int \sec^2(2x)\tan(2x)\sec(2x)\ dx - \int \tan(2x)\sec(2x)\ dx$$

Use u-substitution.

$$u = \sec(2x)$$

$\dfrac{du}{dx} = 2\sec(2x)\tan(2x)$, so $du = 2\sec(2x)\tan(2x)\ dx$, so $dx = \dfrac{du}{2\sec(2x)\tan(2x)}$

Substitute and integrate the first integral.

$$\int u^2 \tan(2x)\sec(2x) \cdot \frac{du}{2\sec(2x)\tan(2x)} - \int \tan(2x)\sec(2x)\ dx$$

$$\frac{1}{2}\int u^2\ du - \int \tan(2x)\sec(2x)\ dx$$

$$\frac{1}{2}\left(\frac{1}{3}u^3\right) - \int \tan(2x)\sec(2x)\ dx$$

$$\frac{1}{6}\sec^3(2x) - \int \tan(2x)\sec(2x)\ dx$$

Integrate the second integral.

$$\frac{1}{6}\sec^3(2x) - \frac{1}{2}\sec(2x) + C$$

■ 2. Evaluate the trigonometric integral.

$$\int \tan^5(3x)\sec(3x)\ dx$$

Solution:

Use the trig identity $\tan^2\theta = \sec^2\theta - 1$ to rewrite the integrand.

$$\int \tan^4(3x)\tan(3x)\sec(3x)\ dx$$

$$\int (\tan^2(3x))^2\tan(3x)\sec(3x)\ dx$$

$$\int (\sec^2(3x) - 1)^2 \tan(3x)\sec(3x)\ dx$$

Use u-substitution.

$$u = \sec(3x)$$

$$\frac{du}{dx} = 3\sec(3x)\tan(3x), \text{ so } du = 3\sec(3x)\tan(3x)\ dx, \text{ so } dx = \frac{du}{3\sec(3x)\tan(3x)}$$

Substitute.

$$\int (u^2 - 1)^2 \tan(3x)\sec(3x) \cdot \frac{du}{3\sec(3x)\tan(3x)}$$

$$\frac{1}{3}\int (u^2 - 1)^2\ du$$

$$\frac{1}{3}\int u^4 - 2u^2 + 1\ du$$

Integrate and back-substitute.

$$\frac{1}{3}\left(\frac{1}{5}u^5 - \frac{2}{3}u^3 + u\right) + C$$

$$\frac{1}{3}\left(\frac{1}{5}\sec^5(3x) - \frac{2}{3}\sec^3(3x) + \sec(3x)\right) + C$$

TAN^M SEC^N, EVEN N

1. Evaluate the trigonometric integral.

$$\int \tan^2(4x)\sec^4(4x)\ dx$$

Solution:

Use the trig identity $\sec^2\theta = 1 + \tan^2\theta$ to rewrite the integrand.

$$\int \tan^2(4x)\sec^2(4x)\sec^2(4x)\ dx$$

$$\int \tan^2(4x)\sec^2(4x)(1+\tan^2(4x))\ dx$$

Use u-substitution.

$$u = \tan(4x)$$

$$\frac{du}{dx} = 4\sec^2(4x)\text{, so } du = 4\sec^2(4x)\ dx\text{, so } dx = \frac{du}{4\sec^2(4x)}$$

Substitute.

$$\int u^2\sec^2(4x)(1+u^2)\left(\frac{du}{4\sec^2(4x)}\right)$$

$$\frac{1}{4}\int u^2(1+u^2)\ du$$

$$\frac{1}{4}\int u^2+u^4\ du$$

Integrate and back-substitute.

$$\frac{1}{4}\left(\frac{1}{3}u^3+\frac{1}{5}u^5\right)+C$$

$$\frac{1}{4}\left(\frac{1}{3}\tan^3(4x)+\frac{1}{5}\tan^5(4x)\right)+C$$

■ 2. Evaluate the trigonometric integral.

$$\int \tan^4(2x)\sec^4(2x)\ dx$$

Solution:

Use the trig identity $\sec^2\theta=1+\tan^2\theta$ to rewrite the integrand.

$$\int \tan^4(2x)\sec^2(2x)\sec^2(2x)\ dx$$

$$\int \tan^4(2x)\sec^2(2x)(1+\tan^2\theta)\ dx$$

Use u-substitution.

$$u=\tan(2x)$$

$$\frac{du}{dx} = 2\sec^2(2x), \text{ so } du = 2\sec^2(2x)\ dx, \text{ so } dx = \frac{du}{2\sec^2(2x)}$$

Substitute.

$$\int u^4 \sec^2(2x)(1+u^2)\left(\frac{du}{2\sec^2(2x)}\right)$$

$$\frac{1}{2}\int u^4(1+u^2)\ du$$

$$\frac{1}{2}\int u^4 + u^6\ du$$

Integrate and back-substitute.

$$\frac{1}{2}\left(\frac{1}{5}u^5 + \frac{1}{7}u^7\right) + C$$

$$\frac{1}{2}\left(\frac{1}{5}\tan^5(2x) + \frac{1}{7}\tan^7(2x)\right) + C$$

■ 3. Evaluate the trigonometric integral.

$$\int \tan^4(3x-1)\sec^4(3x-1)\ dx$$

Solution:

Use the trig identity $\sec^2\theta = 1 + \tan^2\theta$ to rewrite the integrand.

$$\int \tan^4(3x-1)\sec^2(3x-1)(1+\tan^2\theta)\ dx$$

Use u-substitution.

$$u = \tan(3x-1)$$

$\dfrac{du}{dx} = 3\sec^2(3x-1)$, so $du = 3\sec^2(3x-1)\ dx$, so $dx = \dfrac{du}{3\sec^2(3x-1)}$

Substitute.

$$\int u^4\sec^2(3x-1)(1+u^2)\cdot\frac{du}{3\sec^2(3x-1)}$$

$$\frac{1}{3}\int u^4(1+u^2)\ du$$

$$\frac{1}{3}\int u^4+u^6\ du$$

Integrate and back-substitute.

$$\frac{1}{3}\left(\frac{1}{5}u^5+\frac{1}{7}u^7\right)+C$$

$$\frac{1}{3}\left(\frac{1}{5}\tan^5(3x-1)+\frac{1}{7}\tan^7(3x-1)\right)+C$$

SIN(MX) COS(NX)

1. Evaluate the trigonometric integral.

$$\int 5\sin(6x)\cos(3x)\ dx$$

Solution:

Use the trig identity $\sin a \cos b = (1/2)(\sin(a-b)+\sin(a+b))$ to rewrite the integrand.

$$\int 5 \cdot \frac{1}{2}(\sin(6x-3x)+\sin(6x+3x))\ dx$$

$$\frac{5}{2}\int \sin(3x)+\sin(9x)\ dx$$

Integrate.

$$\frac{5}{2}\left(-\frac{1}{3}\cos(3x)-\frac{1}{9}\cos(9x)\right)+C$$

$$-\frac{5}{6}\left(\cos(3x)+\frac{1}{3}\cos(9x)\right)+C$$

2. Evaluate the trigonometric integral.

$$\int 2\sin(9x)\cos(4x)\ dx$$

Solution:

Use the trig identity $\sin a \cos b = (1/2)(\sin(a-b) + \sin(a+b))$ to rewrite the integrand.

$$\int 2 \cdot \frac{1}{2}(\sin(9x-4x) + \sin(9x+4x))\ dx$$

$$\int \sin(5x) + \sin(13x)\ dx$$

Integrate.

$$-\frac{1}{5}\cos(5x) - \frac{1}{13}\cos(13x) + C$$

■ 3. Evaluate the trigonometric integral.

$$\int \frac{1}{3}\sin(12x)\cos(7x)\ dx$$

Solution:

Use the trig identity $\sin a \cos b = (1/2)(\sin(a-b) + \sin(a+b))$ to rewrite the integrand.

$$\int \frac{1}{3} \cdot \frac{1}{2}(\sin(12x - 7x) + \sin(12x + 7x))\ dx$$

$$\frac{1}{6}\int \sin(5x) + \sin(19x)\ dx$$

Integrate.

$$\frac{1}{6}\left(-\frac{1}{5}\cos(5x) - \frac{1}{19}\cos(19x)\right) + C$$

SIN(MX) SIN(NX)

■ 1. Evaluate the trigonometric integral.

$$\int 6\sin(9x)\sin(2x)\ dx$$

Solution:

Use the trig identity $\sin a \sin b = (1/2)(\cos(a-b) - \cos(a+b))$ to rewrite the integrand.

$$\int 6 \cdot \frac{1}{2}(\cos(9x-2x) - \cos(9x+2x))\ dx$$

$$3\int \cos(7x) - \cos(11x)\ dx$$

Integrate.

$$3\left(\frac{1}{7}\sin(7x) - \frac{1}{11}\sin(11x)\right) + C$$

■ 2. Evaluate the trigonometric integral.

$$\int \frac{1}{2}\sin(8x)\sin(4x)\ dx$$

Solution:

Use the trig identity $\sin a \sin b = (1/2)(\cos(a-b) - \cos(a+b))$ to rewrite the integrand.

$$\int \frac{1}{2} \cdot \frac{1}{2}(\cos(8x-4x) - \cos(8x+4x))\ dx$$

$$\frac{1}{4}\int \cos(4x) - \cos(12x)\ dx$$

Integrate.

$$\frac{1}{4}\left(\frac{1}{4}\sin(4x) - \frac{1}{12}\sin(12x)\right) + C$$

$$\frac{1}{16}\left(\sin(4x) - \frac{1}{3}\sin(12x)\right) + C$$

■ 3. Evaluate the trigonometric integral.

$$\int 8\sin(14x)\sin(7x)\ dx$$

Solution:

Use the trig identity $\sin a \sin b = (1/2)(\cos(a-b) - \cos(a+b))$ to rewrite the integrand.

$$\int 8 \cdot \frac{1}{2}(\cos(14x - 7x) - \cos(14x + 7x)) \ dx$$

$$4 \int \cos(7x) - \cos(21x) \ dx$$

Integrate.

$$4\left(\frac{1}{7}\sin(7x) - \frac{1}{21}\sin(21x)\right) + C$$

$$\frac{4}{7}\left(\sin(7x) - \frac{1}{3}\sin(21x)\right) + C$$

COS(MX) COS(NX)

1. Evaluate the trigonometric integral.

$$\int 7\cos(8x)\cos(3x)\ dx$$

Solution:

Use the trig identity $\cos a \cos b = (1/2)(\cos(a-b)+\cos(a+b))$ to rewrite the integrand.

$$\int 7 \cdot \frac{1}{2}(\cos(8x-3x)+\cos(8x+3x))\ dx$$

$$\frac{7}{2}\int \cos(5x)+\cos(11x)\ dx$$

Integrate.

$$\frac{7}{2}\left(\frac{1}{5}\sin(5x)+\frac{1}{11}\sin(11x)\right)+C$$

2. Evaluate the trigonometric integral.

$$\int 5\cos(15x)\cos(5x)\ dx$$

Solution:

Use the trig identity $\cos a \cos b = (1/2)(\cos(a-b) + \cos(a+b))$ to rewrite the integrand.

$$\int 5 \cdot \frac{1}{2}(\cos(15x - 5x) + \cos(15x + 5x)) \ dx$$

$$\frac{5}{2}\int \cos(10x) + \cos(20x) \ dx$$

Integrate.

$$\frac{5}{2}\left(\frac{1}{10}\sin(10x) + \frac{1}{20}\sin(20x)\right) + C$$

$$\frac{1}{4}\left(\sin(10x) + \frac{1}{2}\sin(20x)\right) + C$$

■ 3. Evaluate the trigonometric integral.

$$\int 49\cos(21x)\cos(14x) \ dx$$

Solution:

Use the trig identity $\cos a \cos b = (1/2)(\cos(a-b) + \cos(a+b))$ to rewrite the integrand.

$$\int 49 \cdot \frac{1}{2}(\cos(21x - 14x) + \cos(21x + 14x))\ dx$$

$$\frac{49}{2}\int \cos(7x) + \cos(35x)\ dx$$

Integrate.

$$\frac{49}{2}\left(\frac{1}{7}\sin(7x) + \frac{1}{35}\sin(35x)\right) + C$$

$$\frac{7}{2}\left(\sin(7x) + \frac{1}{5}\sin(35x)\right) + C$$

W W W . K R I S T A K I N G M A T H . C O M