Topic: Precise definition of the limit

Question: Which of these is the precise definition of the limit?

Answer choices:

A Let f be a function defined on a closed interval containing c (except possibly at c itself) and let L be a real number. The statement $\lim_{x \to c} f(x) = L$ means that for each $\epsilon > 0$ there exists a $\delta > 0$ such that if $0 < x - c < \delta$, then $f(x) - L < \epsilon$.

B Let f be a function defined on an open interval containing c (except possibly at c itself) and let L be a real number. The statement $\lim_{x \to c} f(x) = L$ means that for each $\epsilon > 0$ there exists a $\delta > 0$ such that if $0 < |x - c| < \delta$, then $|f(x) - L| < \epsilon$.

C Let f be a function defined on an open interval containing c (except possibly at c itself) and let L be a real number. The statement $\lim_{x \to c} f(x) = L$ means that for each $\epsilon > 0$ there exists a $\delta > 0$ such that if $|f(x) - L| < \epsilon$, then $0 < |x - c| < \delta$.

Solution: B

The correct statement of the precise definition of the limit is:

Let f be a function defined on an open interval containing c (except possibly at c itself) and let L be a real number. The statement $\lim_{x \to c} f(x) = L$ means that for each $\epsilon > 0$ there exists a $\delta > 0$ such that if $0 < |x - c| < \delta$, then $|f(x) - L| < \epsilon$.

Topic: Precise definition of the limit

Question: Use the the precise definition of the limit to prove the value of the limit by finding a relationship between ϵ and δ that guarantees the limit exists.

$$\lim_{x \to 2}(x-1)=1$$

Answer choices:

A $\quad \delta = \epsilon^2$

B $\quad \delta = \sqrt{\epsilon}$

C $\quad \delta = \epsilon$

D $\quad \delta = \dfrac{\epsilon}{2}$

Solution: C

To prove the limit equation,

$$\lim_{x\to 2}(x-1)=1$$

we need to show that, on some open interval surrounding $x=2$, for every $\epsilon>0$ there exists a $\delta>0$ such that

$|(x-1)-1|<\epsilon$ whenever $0<|x-2|<\delta$

Let $\epsilon>0$ and $0<|x-2|<\delta$. We need to find a δ (which will be in terms of ϵ) that will give $|(x-1)-1|<\epsilon$. So,

$$|(x-1)-1|<\epsilon$$

$$|x-2|<\epsilon$$

Now if $|x-2|<\epsilon$ and $0<|x-2|<\delta$, then if $\epsilon>0$, then $\delta=\epsilon$. Therefore, the limit equation is true.

Topic: Precise definition of the limit

Question: True or false? The precise definition of the limit implies that picking a value of x inside the δ interval will return a resulting value in the ϵ interval.

Answer choices:

A True

B False

Solution: A

According to the epsilon-delta definition of the limit, choosing a value for x between $x - \delta$ and $x + \delta$ will return a function value between $L - \epsilon$ and $L + \epsilon$.