Average value of a function

In the same way that we can find the average of set of numbers, we can also find the average value of a function over a specific interval.

The formula we use to find the average value of a function $f(x)$ over the interval $[a,b]$ is

$$f_{avg} = \frac{1}{b-a}\int_a^b f(x)\ dx$$

Think about the average value of a function as the average height the function attains above the x-axis. If the function were $y = 3$, then the height of the function is always 3 everywhere, so the average height of the function would also be 3. When the function gets more complicated, we can use the average value formula to find its average height on $[a,b]$.

Example

Calculate the average value of the function over the interval.

$$f(x) = x^3 - 2x^2 + e^{2x}$$

on $[3,7]$

We'll use the formula for average value

$$f_{avg} = \frac{1}{b-a}\int_a^b f(x)\ dx$$

and get

$$f_{avg} = \frac{1}{7-3}\int_{3}^{7} x^3 - 2x^2 + e^{2x}\ dx$$

$$f_{avg} = \frac{1}{4}\int_{3}^{7} x^3 - 2x^2 + e^{2x}\ dx$$

Next we can break the integral apart by term.

$$f_{avg} = \frac{1}{4}\int_{3}^{7} x^3\ dx + \frac{1}{4}\int_{3}^{7} -2x^2\ dx + \frac{1}{4}\int_{3}^{7} e^{2x}\ dx$$

$$f_{avg} = \frac{1}{4}\int_{3}^{7} x^3\ dx - \frac{2}{4}\int_{3}^{7} x^2\ dx + \frac{1}{4}\int_{3}^{7} e^{2x}\ dx$$

Integrate.

$$f_{avg} = \frac{1}{4}\left(\frac{x^4}{4}\right)\Bigg|_{3}^{7} - \frac{2}{4}\left(\frac{x^3}{3}\right)\Bigg|_{3}^{7} + \frac{1}{4}\left(\frac{e^{2x}}{2}\right)\Bigg|_{3}^{7}$$

$$f_{avg} = \frac{x^4}{16} - \frac{x^3}{6} + \frac{e^{2x}}{8}\Bigg|_{3}^{7}$$

Now we can evaluate on the interval.

$$f_{avg} = \left[\frac{(7)^4}{16} - \frac{(7)^3}{6} + \frac{e^{2(7)}}{8}\right] - \left[\frac{(3)^4}{16} - \frac{(3)^3}{6} + \frac{e^{2(3)}}{8}\right]$$

$$f_{avg} = 150{,}367$$

The average value of the function $f(x) = x^3 - 2x^2 + e^{2x}$ over the interval $[3,7]$ is $150,367$.