**Topic**: sin^m cos^n, odd n

**Question**: Evaluate the trigonometric integral.

$$\int_0^{\frac{\pi}{2}} \sin^2 x \cos x \ dx$$

**Answer choices**:

A $\frac{1}{4}$

B $\frac{1}{2}$

C $\frac{1}{3}$

D $\frac{2}{3}$

**Solution**: C

In the specific case where our function is the product of

an **odd** number of **cosine** factors and

an **even or odd** number of **sine** factors,

our plan is to

1. save one cosine factor and use the identity $\cos^2 x = 1 - \sin^2 x$ to write the other cosine factors in terms of sine, then

2. use u-substitution with $u = \sin x$.

Since we only have one cosine factor to begin with, we don't need to separate factors and use the identity. Instead, we'll go straight to the u-substitution.

Using u-substitution with $u = \sin x$, we get

$$u = \sin x$$

$$du = \cos x \ dx$$

Because we're dealing with a definite integral, we have to either change the limits of integration when we make our substitution, or we have to indicate that the limits of integration are in terms of $x$ until we back-substitute. Substitute into the integral.

$$\int_0^{\frac{\pi}{2}} \sin^2 x \cos x \ dx$$

$$\int_{x=0}^{x=\frac{\pi}{2}} u^2(\cos x \ dx)$$

$$\int_{x=0}^{x=\frac{\pi}{2}} u^2(du)$$

$$\int_{x=0}^{x=\frac{\pi}{2}} u^2 \ du$$

$$\frac{1}{3}u^3 \Big|_{x=0}^{x=\frac{\pi}{2}}$$

Back-substituting for $u$, we get

$$\frac{1}{3}\sin^3 x \Big|_0^{\frac{\pi}{2}}$$

$$\frac{1}{3}\sin^3\left(\frac{\pi}{2}\right) - \sin^3 0$$

$$\frac{1}{3}(1)^3 - (0)^3$$

$$\frac{1}{3}$$

**Topic**: sin^m cos^n, odd n

**Question**: Evaluate the trigonometric integral.

$$\int \sin^2\theta\cos^3\theta\ d\theta$$

**Answer choices**:

A $\quad \frac{1}{3}\sin^3\theta - \frac{1}{5}\sin^5\theta + C$

B $\quad \frac{1}{3}\cos^3\theta + \frac{1}{5}\cos^5\theta + C$

C $\quad \frac{1}{3}\cos^3\theta - \frac{1}{5}\cos^5\theta + C$

D $\quad \frac{1}{3}\sin^3\theta + \frac{1}{5}\sin^5\theta + C$

**Solution**: A

In the specific case where our function is the product of

an **odd** number of **cosine** factors and

an **even or odd** number of **sine** factors,

our plan is to

1. save one cosine factor and use the identity $\cos^2 x = 1 - \sin^2 x$ to write the other cosine factors in terms of sine, then

2. use u-substitution with $u = \sin x$.

We'll separate a single cosine factor and then replace the remaining cosine factors using the identity.

$$\int \sin^2 \theta \cos^3 \theta \ d\theta$$

$$\int \sin^2 \theta \cos^2 \theta \cos \theta \ d\theta$$

$$\int \sin^2 \theta \left(1 - \sin^2 \theta\right) \cos \theta \ d\theta$$

Using u-substitution with $u = \sin \theta$, we get

$$u = \sin \theta$$

$$du = \cos \theta \ d\theta$$

Substitute into the integral.

$$\int u^2\left(1-u^2\right)\cos\theta\ d\theta$$

$$\int u^2\left(1-u^2\right)\left(\cos\theta\ d\theta\right)$$

$$\int u^2\left(1-u^2\right)\left(du\right)$$

$$\int u^2\left(1-u^2\right)\ du$$

$$\int u^2-u^4\ du$$

$$\frac{1}{3}u^3-\frac{1}{5}u^5+C$$

Back-substituting for $u$, we get

$$\frac{1}{3}\sin^3\theta-\frac{1}{5}\sin^5\theta+C$$

**Topic**: sin^m cos^n, odd n

**Question**: Evaluate the trigonometric integral.

$$\int \sin^4 \pi x \cos^3 \pi x \ dx$$

**Answer choices**:

A $\quad \frac{1}{5\pi}\cos^5 \pi x - \frac{1}{7\pi}\cos^7 \pi x + C$

B $\quad \frac{1}{5\pi}\sin^5 \pi x + \frac{1}{7\pi}\sin^7 \pi x + C$

C $\quad \frac{1}{5\pi}\sin^5 \pi x - \frac{1}{7\pi}\sin^7 \pi x + C$

D $\quad \frac{1}{5\pi}\cos^5 \pi x + \frac{1}{7\pi}\cos^7 \pi x + C$

**Solution**: C

In the specific case where our function is the product of

an **odd** number of **cosine** factors and

an **even or odd** number of **sine** factors,

our plan is to

1. save one cosine factor and use the identity $\cos^2 x = 1 - \sin^2 x$ to write the other cosine factors in terms of sine, then

2. use u-substitution with $u = \sin x$.

We'll separate a single cosine factor and then replace the remaining cosine factors using the identity.

$$\int \sin^4 \pi x \cos^3 \pi x \ dx$$

$$\int \sin^4 \pi x \cos^2 \pi x \cos \pi x \ dx$$

$$\int \sin^4 \pi x \left(1 - \sin^2 \pi x\right) \cos \pi x \ dx$$

Using u-substitution with $u = \sin \pi x$, we get

$$u = \sin \pi x$$

$$du = \pi \cos \pi x \ dx$$

$$\frac{du}{\pi} = \cos \pi x \ dx$$

Substitute into the integral.

$$\int u^4\left(1-u^2\right)\cos\pi x\ dx$$

$$\int u^4\left(1-u^2\right)\left(\cos\pi x\ dx\right)$$

$$\int u^4\left(1-u^2\right)\left(\frac{du}{\pi}\right)$$

$$\frac{1}{\pi}\int u^4\left(1-u^2\right)\ du$$

$$\frac{1}{\pi}\int u^4-u^6\ du$$

$$\frac{1}{\pi}\left(\frac{1}{5}u^5-\frac{1}{7}u^7\right)+C$$

$$\frac{1}{5\pi}u^5-\frac{1}{7\pi}u^7+C$$

Back-substituting for $u$, we get

$$\frac{1}{5\pi}\sin^5\pi x-\frac{1}{7\pi}\sin^7\pi x+C$$