Topic: Ladder sliding down the wall

Question: A gardener's shovel is 1 m long and leaning against a fence, sliding down the fence at a rate of 0.25 m/s. When the top of the shovel is 0.5 m off the ground, at what rate is the bottom of the shovel sliding along the ground away from the fence?

Answer choices:

A $\frac{3\sqrt{3}}{4}$ m/s

B $\frac{4\sqrt{3}}{3}$ m/s

C $\frac{4}{3}$ m/s

D $\frac{\sqrt{3}}{12}$ m/s

Solution: D

The ground, the fence, and the shovel form a right triangle, so we'll use the Pythagorean Theorem as the equation that relates the side lengths.

$$a^2 + b^2 = c^2$$

Use implicit differentiation to take the derivative of both sides.

$$2a\frac{da}{dt} + 2b\frac{db}{dt} = 2c\frac{dc}{dt}$$

From the question we know that the length of the shovel is $c = 1$, and that the length of the shovel doesn't change, so $dc/dt = 0$.

$$2a\frac{da}{dt} + 2b\frac{db}{dt} = 2(1)(0)$$

$$2a\frac{da}{dt} + 2b\frac{db}{dt} = 0$$

$$a\frac{da}{dt} + b\frac{db}{dt} = 0$$

If we say that the vertical fence is side b, and that the horizontal ground is side a, then the question tells us that $b = 1/2$ and that $db/dt = -1/4$.

$$a\frac{da}{dt} + \frac{1}{2}\left(-\frac{1}{4}\right) = 0$$

$$a\frac{da}{dt} - \frac{1}{8} = 0$$

Find the value of a when $b = 1/2$ and $c = 1$.

$$a^2 + b^2 = c^2$$

$$a^2 + \left(\frac{1}{2}\right)^2 = 1^2$$

$$a^2 + \frac{1}{4} = 1$$

$$a^2 = \frac{3}{4}$$

$$a = \frac{\sqrt{3}}{2}$$

We're asked to solve for da/dt, so we'll plug in this value of a that we've found and then solve the equation for da/dt.

$$\left(\frac{\sqrt{3}}{2}\right)\frac{da}{dt} - \frac{1}{8} = 0$$

$$\left(\frac{\sqrt{3}}{2}\right)\frac{da}{dt} = \frac{1}{8}$$

$$\frac{da}{dt} = \frac{2}{8\sqrt{3}}$$

$$\frac{da}{dt} = \frac{1}{4\sqrt{3}}$$

Rationalize the denominator.

$$\frac{1}{4\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right)$$

$$\frac{\sqrt{3}}{4(3)}$$

$$\frac{\sqrt{3}}{12}$$

Topic: Ladder sliding down the wall

Question: A 5 foot ladder is sliding down a vertical wall while its bottom slides away from the wall at 3 ft/s. How fast is the top moving when the top is 4 feet off the ground?

Answer choices:

A $-\frac{9}{4}$ ft/s

B $-\frac{4}{9}$ ft/s

C $-\frac{3}{2}$ ft/s

D $-\frac{2}{3}$ ft/s

Solution: A

The ground, the wall, and the ladder form a right triangle, so we'll use the Pythagorean Theorem as the equation that relates the side lengths.

$$a^2 + b^2 = c^2$$

Use implicit differentiation to take the derivative of both sides.

$$2a\frac{da}{dt} + 2b\frac{db}{dt} = 2c\frac{dc}{dt}$$

From the question we know that the length of the ladder is $c = 5$, and that the length of the ladder doesn't change, so $dc/dt = 0$.

$$2a\frac{da}{dt} + 2b\frac{db}{dt} = 2(5)(0)$$

$$2a\frac{da}{dt} + 2b\frac{db}{dt} = 0$$

$$a\frac{da}{dt} + b\frac{db}{dt} = 0$$

If we say that the vertical wall is side b, and that the horizontal ground is side a, then the question tells us that $b = 4$ and that $da/dt = 3$.

$$a(3) + 4\frac{db}{dt} = 0$$

$$3a + 4\frac{db}{dt} = 0$$

Find the value of a when $b = 4$ and $c = 5$.

$$a^2 + b^2 = c^2$$

$$a^2 + 4^2 = 5^2$$

$$a^2 + 16 = 25$$

$$a^2 = 9$$

$$a = 3$$

We're asked to solve for db/dt, so we'll plug in this value of a that we've found and then solve the equation for db/dt.

$$3(3) + 4\frac{db}{dt} = 0$$

$$9 + 4\frac{db}{dt} = 0$$

$$4\frac{db}{dt} = -9$$

$$\frac{db}{dt} = -\frac{9}{4}$$

Topic: Ladder sliding down the wall

Question: A 13-foot ladder is leaning against a wall. The base of the ladder is pushed toward the wall at the rate of 5 ft/s. When the base of the ladder is 12 feet from the wall, at what rate is the top of the ladder moving up the wall?

Answer choices:

A $-\frac{25}{12}$ ft/s

B 12 ft/s

C $\frac{25}{12}$ ft/s

D -12 ft/s

Solution: B

The ground, the wall, and the ladder form a right triangle, so we'll use the Pythagorean Theorem as the equation that relates the side lengths.

$$a^2 + b^2 = c^2$$

Use implicit differentiation to take the derivative of both sides.

$$2a\frac{da}{dt} + 2b\frac{db}{dt} = 2c\frac{dc}{dt}$$

From the question we know that the length of the ladder is $c = 13$, and that the length of the ladder doesn't change, so $dc/dt = 0$.

$$2a\frac{da}{dt} + 2b\frac{db}{dt} = 2(13)(0)$$

$$2a\frac{da}{dt} + 2b\frac{db}{dt} = 0$$

$$a\frac{da}{dt} + b\frac{db}{dt} = 0$$

If we say that the vertical wall is side b, and that the horizontal ground is side a, then the question tells us that $a = 12$ and that $da/dt = -5$.

$$12(-5) + b\frac{db}{dt} = 0$$

$$-60 + b\frac{db}{dt} = 0$$

Find the value of b when $a = 12$ and $c = 13$.

$$a^2 + b^2 = c^2$$

$$12^2 + b^2 = 13^2$$

$$144 + b^2 = 169$$

$$b^2 = 25$$

$$b = 5$$

We're asked to solve for db/dt, so we'll plug in this value of a that we've found and then solve the equation for db/dt.

$$-60 + 5\frac{db}{dt} = 0$$

$$5\frac{db}{dt} = 60$$

$$\frac{db}{dt} = 12$$