Topic: Definition of the derivative

Question: Use the definition of the derivative to find the simplified form of the limit.

$$f(x) = x^3 - 2x$$

Answer choices:

$$A f'(x) = \lim_{\Delta x \to 0} \frac{(x + \Delta x)^3 - 2(x + \Delta x)}{\Delta x}$$

B
$$f'(x) = \lim_{\Delta x \to 0} (3x^2 + 3x\Delta x + \Delta x^2 - 2)$$

C
$$f'(x) = \lim_{\Delta x \to 0} (\Delta x^2 - 2)$$

D
$$f'(x) = \lim_{\Delta x \to 0} \frac{(x + \Delta x)^3 - (x^3 - 2x)}{\Delta x}$$



Solution: B

After replacing x with $(x + \Delta x)$ in f(x),

$$f(x) = (x + \Delta x)^3 - 2(x + \Delta x)$$

we'll substitute for $f(x + \Delta x)$.

$$f'(x) = \lim_{\Delta x \to 0} \frac{(x + \Delta x)^3 - 2(x + \Delta x) - f(x)}{\Delta x}$$

Then plug f(x) into the definition.

$$f'(x) = \lim_{\Delta x \to 0} \frac{(x + \Delta x)^3 - 2(x + \Delta x) - (x^3 - 2x)}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{x^3 + x^2 \Delta x + 2x^2 \Delta x + 2x \Delta x^2 + x \Delta x^2 + \Delta x^3 - 2x - 2\Delta x - x^3 + 2x}{\Delta x}$$

Collect like terms,

$$f'(x) = \lim_{\Delta x \to 0} \frac{x^2 \Delta x + 2x^2 \Delta x + 2x \Delta x^2 + x \Delta x^2 + \Delta x^3 - 2x - 2\Delta x + 2x}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{x^2 \Delta x + 2x^2 \Delta x + 2x \Delta x^2 + x \Delta x^2 + \Delta x^3 - 2\Delta x}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{3x^2 \Delta x + 3x \Delta x^2 + \Delta x^3 - 2\Delta x}{\Delta x}$$

then factor Δx out of the numerator and cancel out that common factor from the numerator and denominator.

$$f'(x) = \lim_{\Delta x \to 0} \frac{\Delta x (3x^2 + 3x\Delta x + \Delta x^2 - 2)}{\Delta x}$$



$$f'(x) = \lim_{\Delta x \to 0} (3x^2 + 3x\Delta x + \Delta x^2 - 2)$$



Topic: Definition of the derivative

Question: Use the definition of the derivative to find the derivative of the function.

$$f(x) = x^2$$

Answer choices:

$$\mathbf{A} \qquad f'(x) = 0$$

$$\mathsf{B} \qquad f'(x) = 2$$

$$C f'(x) = 2x$$

$$D f'(x) = x^2 + 2x$$

Solution: C

After replacing x with $(x + \Delta x)$ in f(x),

$$f(x) = (x + \Delta x)^2$$

we'll substitute for $f(x + \Delta x)$.

$$f'(x) = \lim_{\Delta x \to 0} \frac{(x + \Delta x)^2 - f(x)}{\Delta x}$$

Then plug f(x) into the definition.

$$f'(x) = \lim_{\Delta x \to 0} \frac{(x + \Delta x)^2 - x^2}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{x^2 + x\Delta x + x\Delta x + \Delta x^2 - x^2}{\Delta x}$$

Collect like terms,

$$f'(x) = \lim_{\Delta x \to 0} \frac{x\Delta x + x\Delta x + \Delta x^2}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{2x\Delta x + \Delta x^2}{\Delta x}$$

then factor Δx out of the numerator and cancel out that common factor from the numerator and denominator.

$$f'(x) = \lim_{\Delta x \to 0} \frac{\Delta x (2x + \Delta x)}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \to 0} (2x + \Delta x)$$



Now we evaluate the limit using substitution, which means we'll substitute

$$\Delta x = 0$$
.

$$f'(x) = 2x + 0$$

$$f'(x) = 2x$$

Topic: Definition of the derivative

Question: Use the definition of the derivative to find the derivative of the function.

$$f(x) = 2 - x^2 + x$$

Answer choices:

$$\mathbf{A} \qquad f'(x) = 2$$

$$\mathsf{B} \qquad f'(x) = 2x$$

$$C f'(x) = -2x$$

$$D f'(x) = -2x + 1$$

Solution: D

After replacing x with $(x + \Delta x)$ in f(x),

$$f(x) = 2 - (x + \Delta x)^2 + (x + \Delta x)$$

we'll substitute for $f(x + \Delta x)$.

$$f'(x) = \lim_{\Delta x \to 0} \frac{2 - (x + \Delta x)^2 + (x + \Delta x) - f(x)}{\Delta x}$$

Then plug f(x) into the definition.

$$f'(x) = \lim_{\Delta x \to 0} \frac{2 - (x + \Delta x)^2 + (x + \Delta x) - (2 - x^2 + x)}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{2 - (x^2 + x\Delta x + x\Delta x + \Delta x^2) + x + \Delta x - 2 + x^2 - x}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{2 - x^2 - x\Delta x - x\Delta x - \Delta x^2 + x + \Delta x - 2 + x^2 - x}{\Delta x}$$

Collect like terms,

$$f'(x) = \lim_{\Delta x \to 0} \frac{2 - x\Delta x - x\Delta x - \Delta x^2 + x + \Delta x - 2 - x}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{-x\Delta x - x\Delta x - \Delta x^2 + x + \Delta x - x}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{-x\Delta x - x\Delta x - \Delta x^2 + \Delta x}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{-2x\Delta x - \Delta x^2 + \Delta x}{\Delta x}$$



then factor Δx out of the numerator and cancel out that common factor from the numerator and denominator.

$$f'(x) = \lim_{\Delta x \to 0} \frac{\Delta x (-2x - \Delta x + 1)}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \to 0} (-2x - \Delta x + 1)$$

Now we evaluate the limit using substitution, which means we'll substitute $\Delta x = 0$.

$$f'(x) = -2x - 0 + 1$$

$$f'(x) = -2x + 1$$

