Topic: Chain rule with quotient rule

Question: Apply quotient rule and chain rule to find the derivative.

$$y = \frac{(2x^2 + 1)^3}{x}$$

Answer choices:

A $\quad y' = 12x(2x^2 + 1)^2$

B $\quad y' = \dfrac{(2x^2 + 1)^2(10x^2 - 1)}{x^2}$

C $\quad y' = \dfrac{1}{12x(2x^2 + 1)^2}$

D $\quad y' = \dfrac{1 - 10x^2}{(2x^2 + 1)^4}$

Solution: B

List out $f(x)$ and $g(x)$ and their derivatives.

$$f(x) = (2x^2 + 1)^3$$

$$f'(x) = 3(2x^2 + 1)^2(4x)$$

$$f'(x) = 12x(2x^2 + 1)^2$$

and

$$g(x) = x$$

$$g'(x) = 1$$

Now we can plug these values directly into the quotient rule formula.

$$y' = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

$$y' = \frac{(12x(2x^2 + 1)^2)(x) - ((2x^2 + 1)^3)(1)}{x^2}$$

$$y' = \frac{12x^2(2x^2 + 1)^2 - (2x^2 + 1)^3}{x^2}$$

Within the numerator, we have a common factor of $(2x^2 + 1)^2$, so factor that out.

$$y' = \frac{(2x^2 + 1)^2(12x^2 - (2x^2 + 1))}{x^2}$$

$$y' = \frac{(2x^2 + 1)^2(12x^2 - 2x^2 - 1)}{x^2}$$

$$y' = \frac{(2x^2+1)^2(10x^2-1)}{x^2}$$

Topic: Chain rule with quotient rule

Question: Apply quotient rule and chain rule to find the derivative.

$$y = \frac{4}{(x^2 - 1)^3}$$

Answer choices:

A $\quad y' = -\frac{24x^2}{(x^2 - 1)^4}$

B $\quad y' = \frac{4}{(x^2 - 1)^3}$

C $\quad y' = -\frac{24x}{(x^2 - 1)^4}$

D $\quad y' = -\frac{12}{(x^2 - 1)^4}$

Solution: C

List out $f(x)$ and $g(x)$ and their derivatives.

$$f(x) = 4$$

$$f'(x) = 0$$

and

$$g(x) = (x^2 - 1)^3$$

$$g'(x) = 3(x^2 - 1)^2(2x) = 6x(x^2 - 1)^2$$

Now we can plug these values directly into the quotient rule formula.

$$y' = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

$$y' = \frac{(0)((x^2 - 1)^3) - (4)(6x(x^2 - 1)^2)}{[(x^2 - 1)^3]^2}$$

$$y' = \frac{0 - 24x(x^2 - 1)^2}{(x^2 - 1)^6}$$

$$y' = -\frac{24x(x^2 - 1)^2}{(x^2 - 1)^6}$$

$$y' = -\frac{24x}{(x^2 - 1)^4}$$

Topic: Chain rule with quotient rule

Question: Apply quotient rule and chain rule to find the derivative.

$$y = \frac{(3x^2 + 4)^2}{(4 - 2x)^4}$$

Answer choices:

A $\quad y' = -\frac{16(3x^2 + 4)}{(4 - 2x)^4}$

B $\quad y' = -\frac{16(3x^2 + 4)(3x + 2)}{(4 - 2x)^5}$

C $\quad y' = \frac{16(3x^2 + 4)}{(4 - 2x)^4}$

D $\quad y' = \frac{16(3x^2 + 4)(3x + 2)}{(4 - 2x)^5}$

Solution: D

List out $f(x)$ and $g(x)$ and their derivatives.

$$f(x) = (3x^2 + 4)^2$$

$$f'(x) = 2(3x^2 + 4)(6x)$$

$$f'(x) = 12x(3x^2 + 4)$$

and

$$g(x) = (4 - 2x)^4$$

$$g'(x) = 4(4 - 2x)^3(-2)$$

$$g'(x) = -8(4 - 2x)^3$$

Now we can plug these values directly into the quotient rule formula.

$$y' = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

$$y' = \frac{(12x(3x^2 + 4))((4 - 2x)^4) - ((3x^2 + 4)^2)(-8(4 - 2x)^3)}{[(4 - 2x)^4]^2}$$

$$y' = \frac{12x(3x^2 + 4)(4 - 2x)^4 + 8(3x^2 + 4)^2(4 - 2x)^3}{(4 - 2x)^8}$$

Within the fraction, we have a common factor of $(4 - 2x)^3$, so cancel that out.

$$y' = \frac{12x(3x^2 + 4)(4 - 2x) + 8(3x^2 + 4)^2}{(4 - 2x)^5}$$

Within the numerator, we have a common factor of $4(3x^2+4)$, so factor that out.

$$y' = \frac{4(3x^2+4)\left[3x(4-2x)+2(3x^2+4)\right]}{(4-2x)^5}$$

$$y' = \frac{4(3x^2+4)\left[(12x-6x^2)+(6x^2+8)\right]}{(4-2x)^5}$$

$$y' = \frac{4(3x^2+4)(12x-6x^2+6x^2+8)}{(4-2x)^5}$$

$$y' = \frac{4(3x^2+4)(12x+8)}{(4-2x)^5}$$

Factor out another 4 from the $12x+8$.

$$y' = \frac{16(3x^2+4)(3x+2)}{(4-2x)^5}$$