Topic: Horizontal and slant asymptotes

Question: Find the function's horizontal asymptote(s).

$$f(x) = \frac{4x^3 - 2x^2 + 1}{2x^3 - 3x}$$

Answer choices:

A $y = 0$

B $y = -3$

C $y = 2$

D $y = \pm 2$

Solution: C

The degree of the numerator is 3 and the degree of the denominator is 3, so the degree of the numerator is equal to the degree of the denominator

Therefore, the equation of the horizontal asymptote is given as the ratio of the coefficients on the highest-degree terms, and the function has the horizontal asymptote at $y = 2$.

$$y = \frac{4}{2} = 2$$

Topic: Horizontal and slant asymptotes

Question: Find the function's horizontal asymptote(s).

$$y = \frac{x^5 - x + 6}{x^7 - x^4 + 3x^2 - 1}$$

Answer choices:

A The function has a horizontal asymptote at $y = 1$

B The function has a horizontal asymptote at $y = 5/7$

C The function has a horizontal asymptote at $y = 0$

D The function has no horizontal asymptote

Solution: C

The x^5 term is the highest-degree term in the numerator, and the x^7 term is the highest-degree term in the denominator.

Because the degree of the numerator is less than the degree of the denominator, the function has a horizontal asymptote at $y = 0$.

Topic: Horizontal and slant asymptotes

Question: Find the function’s slant asymptote(s).

$$f(x) = \frac{x^2 - x + 3}{x + 1}$$

Answer choices:

A The function has a slant asymptote at $y = x + 2 + \frac{5}{x + 1}$

B The function has a slant asymptote at $y = x - 2 + \frac{5}{x + 1}$

C The function has a slant asymptote at $y = x - 2$

D The function has a slant asymptote at $y = x + 2$

Solution: C

The degree of the numerator is exactly one greater than the degree of the denominator, $2 > 1$, so the function has a slant asymptote.

We want to do polynomial long division with the function, which we set up as

$$x+1\,\overline{\big)\,x^2-x+3}$$

If we work through this division, we end up with

$$f(x) = x - 2 + \frac{5}{x+1}$$

The slant asymptote is what we get when we remove the remainder from this rewritten function. If we remove the remainder, we get

$$f(x) = x - 2$$

So the equation of the slant asymptote is

$$y = x - 2$$