Calculus 1
Final Exam Solutions

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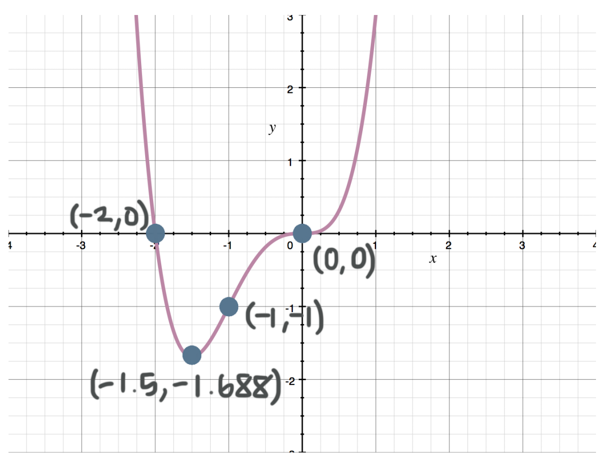
MATH

Calculus 1 Final Exam Answer Key

1. (5 pts)	A	B	■	D	E
2. (5 pts)	A	B	C	■	E
3. (5 pts)	■	B	C	D	E
4. (5 pts)	A	B	■	D	E
5. (5 pts)	A	B	C	■	E
6. (5 pts)	A	■	C	D	E
7. (5 pts)	A	B	C	D	■
8. (5 pts)	A	■	C	D	E

9. (15 pts) $12x + 3$

10. (15 pts)



11. (15 pts) $1/(4\pi)$ cm/s

12. (15 pts) 300 m $\times$ 600 m

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1. C. Find the limit.

$$\lim_{x\to 3} g(x)$$

$$\lim_{x\to 3} x-4$$

$$3-4$$

$$-1$$

Find $f(-1)$.

$$f(-1)=3(-1)^2$$

$$f(-1)=3(1)$$

$$f(-1)=3$$

$$\lim_{x\to 3} f[g(x)]=3$$

2. D. We need to make sure that the piece-wise function is continuous through each domain and at each "break" point (the transition between functions).

$x-3$ is continuous over all of x.

The piecewise function is discontinuous at the transition $x = -1$ since x approaches -4 from the left, and -1 from the right.

$\frac{1}{x}$ is discontinuous at $x = 0$ which is in the interval $-1 < x < 2$

The piecewise function is discontinuous at $x = 2$ since x approaches $1/2$ from the left, and 2 from the right.

$\sqrt{x+2}$ is continuous for all $x > 2$

Therefore the piecewise function is discontinuous at $x = -1, 0, 2$.

3. A. Yes, there's a zero in the interval $[0,1]$. Find the limit at the endpoints by plugging the endpoints into the polynomial.

$$f(0) = 0^3 + 2(0) - 1 = -1$$

$$f(1) = 1^3 + 2(1) - 1 = 2$$

Since $f(0) < 0$ and $f(1) > 0$, the Intermediate Value Theorem states that there must be some x-value in $[0,1]$ where $f(x) = 0$. This means that there is a zero in the interval $[0,1]$.

4. C. Use the chain rule to find the derivative: $f'(g(x))g'(x)$ "derivative of the outside times the derivative of the inside"

$$f(x) = \sqrt{4 - x^2}$$

$$f'(x) = \frac{1}{2}(4 - x^2)^{-1/2} \cdot -2x$$

$$f'(x) = \frac{-x}{\sqrt{4 - x^2}}$$

5. D. Find the first derivative.

$$f(x) = x^4 - 6x^3 + 12x^2$$

$$f'(x) = 4x^3 - 18x^2 + 24x$$

Find the second derivative.

$$f''(x) = 12x^2 - 36x + 24$$

Set the second derivative equal to zero and solve for x.

$$12x^2 - 36x + 24 = 0$$

$$12(x^2 - 3x + 2) = 0$$

$$\frac{12(x^2 - 3x + 2)}{12} = \frac{0}{12}$$

$$x^2 - 3x + 2 = 0$$

$$(x - 1)(x - 2) = 0$$

$$x = 1 \quad \text{or} \quad x = 2$$

Find the y-values of $f(x)$ when $x = 1$ and $x = 2$.

$$f(1) = 1^4 - 6(1)^3 + 12(1)^2$$

$$f(1) = 1 - 6 + 12$$

$$f(1) = 7$$

and

$$f(2) = 2^4 - 6(2)^3 + 12(2)^2$$

$$f(2) = 16 - 6(8) + 12(4)$$

$$f(2) = 16 - 48 + 48$$

$$f(2) = 16$$

The inflection points are at $(1,7)$ and $(2,16)$.

6. B. When we try to use direct substitution to find

$$\lim_{x\to\infty} \frac{e^x}{2x^3}$$

we get ∞/∞. This means we can use L'Hospital's Rule to find the limit. To use L'Hospital's Rule keep taking the derivative of the top and bottom until we can get a determinate form.

$$\lim_{x\to\infty} \frac{e^x}{6x^2}$$

$$\lim_{x\to\infty} \frac{e^x}{12x}$$

$$\lim_{x\to\infty} \frac{e^x}{12} = \infty$$

7. E. Use the formula

$$y(t) = -\frac{1}{2}gt^2 + v_0t + y_0$$

where $y(t)$ is the position of the ball at time t, g is the gravitational constant, v_0 is the initial velocity, and y_0 is the initial position.

In this problem $g = 32$ ft/s ^{2}, $v_0 = 48$ ft/s, and $y_0 = 0$ (the ball is thrown from the ground).

$$y(t) = -\frac{1}{2}(32)t^2 + 48t + 0$$

$$y(t) = -16t^2 + 48t$$

Velocity is the derivative of position so $v(t) = y'(t)$.

$$v(t) = -32t + 48$$

Set velocity equal to 0 to find t at the ball's maximum height.

$$0 = -32t + 48$$

$$32t = 48$$

$$t = 1.5$$

Plug 1.5 in for t into the equation $y(t)$ to find the position of the ball at its maximum height.

$$y(1.5) = -16(1.5)^2 + 48(1.5)$$

$$y(1.5) = -36 + 72$$

$$y(1.5) = 36$$

The ball's maximum height is 36 ft.

8. B. To find marginal revenue, take the derivative of the revenue formula $R(x) = -0.2x^2 + 300x$.

$$R'(x) = -0.4x + 300$$

To maximize revenue, set the marginal revenue equal to zero and solve for x.

$$0 = -0.4x + 300$$

$$0.4x = 300$$

$$x = 750$$

The coffee shop needs to sell 750 cups of coffee each week to maximize weekly revenue.

9. Find the derivative of $f(x)$.

$$f(x) = 3x^2$$

$$f'(x) = 6x$$

Find the derivative of $g(x)$.

$$g(x) = \frac{1}{x} + 2$$

$$g'(x) = -x^{-2}$$

Plug these into the product rule formula.

$$f'(x)g(x) + f(x)g'(x)$$

$$6x\left(\frac{1}{x} + 2\right) + 3x^2(-x^{-2})$$

$$6 + 12x - 3$$

$$12x + 3$$

10. Find the first derivative.

$$f'(x) = 4x^3 + 6x^2$$

Find the second derivative.

$$f''(x) = 12x^2 + 12x$$

Find the zeros of the function.

$$x^4 + 2x^3 = 0$$

$$x^3(x + 2) = 0$$

$$x = 0, \ -2$$

Find the zeros of the first derivative.

$$4x^3 + 6x^2 = 0$$

$$2x^2(2x + 3) = 0$$

$$x = 0, \ -\frac{3}{2}$$

Find the zeros of the second derivative.

$$12x^2 + 12x = 0$$

$$12x(x + 1) = 0$$

$$x = 0, \ -1$$

Organize the information into a sign chart.

f(x)	decreasing	local min	increasing		increasing
f'(x)	-	0	+	0	+
x		-3/2		0	

f(x)	concave up	inf. point	concave dn.	inf. point	concave up
f''(x)	+	0	-	0	+
x		-1		0	

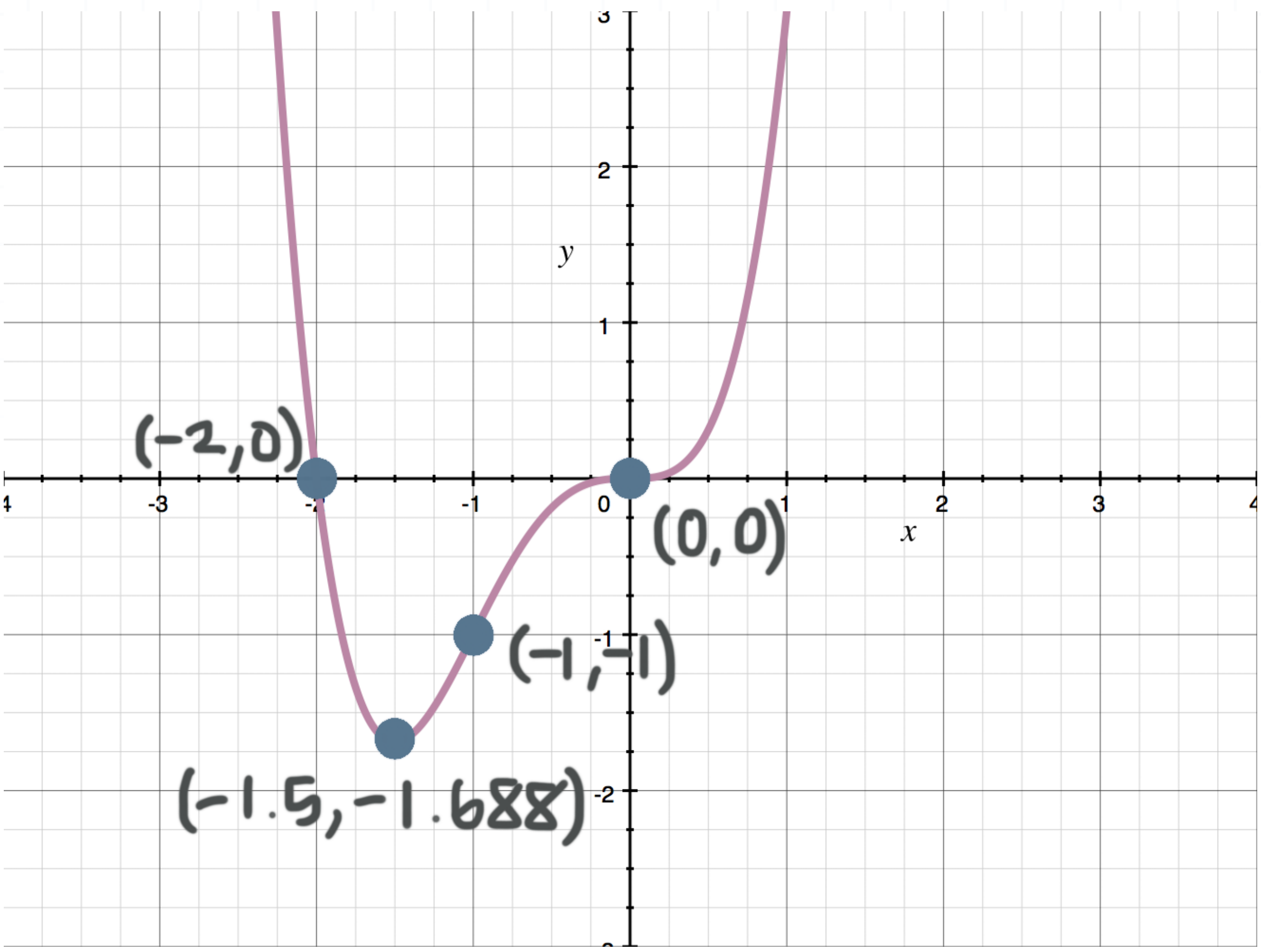
To summarize, we can say

$f(x)$ is decreasing from $(-\infty,-3/2)$ and increasing from $(-3/2,\infty)$.

$f(x)$ has a local minimum at $(-3/2,-27/16)$.

$f(x)$ is concave up from $(-\infty,-1)$ and $(0,\infty)$ and concave down from $(-1,0)$.

$f(x)$ has inflection points at $(-1,-1)$ and $(0,0)$.



11. Find the derivative of the volume of a sphere with respect to time t.

$$\frac{dV}{dt}=4\pi r^2\frac{dr}{dt}$$

Plug in $r = 12$ and $dV/dt = 144$ cm^3/s.

$$144\frac{\text{cm}^3}{\text{s}} = 4\pi(12\text{ cm})^2\frac{dr}{dt}$$

$$144\frac{\text{cm}^3}{\text{s}} = 576\pi\text{ cm}^2\frac{dr}{dt}$$

$$\frac{144}{576\pi\text{ cm}^2}\cdot\frac{\text{cm}^3}{\text{s}} = \frac{dr}{dt}$$

$$\frac{1}{4\pi}\text{ cm/s} = \frac{dr}{dt}$$

The radius is increasing at a rate of $1/(4\pi)$ cm/s when the radius is 12 cm.

12. Since one side of the rectangle does not need any fencing, the perimeter of required fencing is given by $F = 2x + y$.

$$A = xy$$

Plug in $180{,}000$ for area and solve for y.

$$180{,}000 = xy$$

$$y = \frac{180{,}000}{x}$$

Plug in $180{,}000/x$ for y into equation F.

$$F = 2x + \frac{180{,}000}{x}$$

Find the first derivative to get critical numbers, then use it to find the minimum.

$$F' = 2 - \frac{180{,}000}{x^2}$$

$$2 - \frac{180{,}000}{x^2} = 0$$

$$2 = \frac{180{,}000}{x^2}$$

$$2x^2 = 180{,}000$$

$$x^2 = 90{,}000$$

$$x = \pm\sqrt{90{,}000}$$

$$x = \pm 300$$

Since the problem is about length of fencing, negative values for x don't make sense and must be $x = 300$. We'll also need to check the endpoints to make sure we have the minimum. The interval of possible x-values is $[1, \ 180{,}000]$ because the rectangular area must be at least one meter wide for cows. Plug in the endpoints and $x = 300$ to find the minimum.

$$F(1) = 2(1) + \frac{180{,}000}{(1)} = 180{,}002$$

$$F(300) = 2(300) + \frac{180{,}000}{300} = 1{,}200$$

$$F(180{,}000) = 2(180{,}000) + \frac{180{,}000}{180{,}000} = 360{,}001$$

$x = 300$ results in the least amount of fencing. Plug 300 in for x into the equation that we isolated y.

$$y = \frac{180{,}000}{300} = 600$$

The dimensions for the least amount of fencing are 300×600 m.

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