Topic: Power series multiplication

Question: Use power series multiplication to find the first four non-zero terms of the Maclaurin series.

$$y = \sin(2x)e^{2x}$$

Answer choices:

A $\quad \sin(2x)e^{2x} = 2x - 4x^2 + \frac{8x^3}{3} - \frac{16x^5}{5}$

B $\quad \sin(2x)e^{2x} = 2x + 4x^2 - \frac{8x^3}{3} - \frac{16x^5}{5}$

C $\quad \sin(2x)e^{2x} = 2x + 4x^2 + \frac{8x^3}{3} + \frac{16x^5}{5}$

D $\quad \sin(2x)e^{2x} = 2x + 4x^2 + \frac{8}{3}x^3 - \frac{16}{15}x^5$

Solution: D

When we multiply two power series together, we want to find the expansion of the sum of each series, so that we essentially have polynomial representations. Then finding the product of the series will be like multiplying polynomials.

We need to recognize that the given series is the product of two other series

$$y = \sin(2x)$$

$$y = e^{2x}$$

There are common Maclaurin series that are similar to each of these.

$$\sin x = x - \frac{1}{6}x^3 + \frac{1}{120}x^5 - \frac{1}{5,040}x^7 + \frac{1}{362,880}x^9 - \ldots$$

$$e^x = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5 + \ldots$$

We want to modify each of these common series to match the given series.

For $y = \sin(2x)$, we'll substitute $2x$ for x:

$$\sin(2x) = 2x - \frac{1}{6}(2x)^3 + \frac{1}{120}(2x)^5 - \frac{1}{5,040}(2x)^7 + \frac{1}{362,880}(2x)^9 - \ldots$$

$$\sin(2x) = 2x - \frac{1}{6}(8x^3) + \frac{1}{120}(32x^5) - \frac{1}{5,040}(128x^7) + \frac{1}{362,880}(512x^9) - \ldots$$

$$\sin(2x) = 2x - \frac{4}{3}x^3 + \frac{4}{15}x^5 - \frac{8}{315}x^7 + \frac{4}{2,835}x^9 - \ldots$$

For $y = e^{2x}$, we'll substitute $2x$ for x:

$$e^{2x} = 1 + 2x + \frac{1}{2}(2x)^2 + \frac{1}{6}(2x)^3 + \frac{1}{24}(2x)^4 + \frac{1}{120}(2x)^5 + \dots$$

$$e^{2x} = 1 + 2x + \frac{1}{2}(4x^2) + \frac{1}{6}(8x^3) + \frac{1}{24}(16x^4) + \frac{1}{120}(32x^5) + \dots$$

$$e^{2x} = 1 + 2x + 2x^2 + \frac{4}{3}x^3 + \frac{2}{3}x^4 + \frac{4}{15}x^5 + \dots$$

Multiplying the modified series together, we get

$$\sin(2x)e^{2x} = \left(2x - \frac{4}{3}x^3 + \frac{4}{15}x^5 - \frac{8}{315}x^7 + \frac{4}{2{,}835}x^9 - \dots\right)$$

$$\left(1 + 2x + 2x^2 + \frac{4}{3}x^3 + \frac{2}{3}x^4 + \frac{4}{15}x^5 + \dots\right)$$

We need to multiply every term in the first series by every term in the second series.

$$\sin(2x)e^{2x} = 2x\left(1 + 2x + 2x^2 + \frac{4}{3}x^3 + \frac{2}{3}x^4 + \frac{4}{15}x^5 + \dots\right)$$

$$-\frac{4}{3}x^3\left(1 + 2x + 2x^2 + \frac{4}{3}x^3 + \frac{2}{3}x^4 + \frac{4}{15}x^5 + \dots\right)$$

$$+\frac{4}{15}x^5\left(1 + 2x + 2x^2 + \frac{4}{3}x^3 + \frac{2}{3}x^4 + \frac{4}{15}x^5 + \dots\right)$$

$$-\frac{8}{315}x^7\left(1 + 2x + 2x^2 + \frac{4}{3}x^3 + \frac{2}{3}x^4 + \frac{4}{15}x^5 + \dots\right)$$

$$+\frac{4}{2{,}835}x^9\left(1+2x+2x^2+\frac{4}{3}x^3+\frac{2}{3}x^4+\frac{4}{15}x^5+\dots\right)$$

$$\sin(2x)e^{2x}=2x+4x^2+4x^3+\frac{8}{3}x^4+\frac{4}{3}x^5+\frac{8}{15}x^6$$

$$-\frac{4}{3}x^3-\frac{8}{3}x^4-\frac{8}{3}x^5-\frac{16}{9}x^6-\frac{8}{9}x^7-\frac{16}{45}x^8$$

$$+\frac{4}{15}x^5+\frac{8}{15}x^6+\frac{8}{15}x^7+\frac{16}{45}x^8+\frac{8}{45}x^9+\frac{16}{225}x^{10}$$

$$-\frac{8}{315}x^7-\frac{16}{315}x^8-\frac{16}{315}x^9-\frac{32}{945}x^{10}-\frac{16}{945}x^{11}-\frac{32}{4{,}725}x^{12}$$

$$+\frac{4}{2{,}835}x^9+\frac{8}{2{,}835}x^{10}+\frac{8}{2{,}835}x^{11}+\frac{16}{8{,}505}x^{12}+\frac{8}{8{,}505}x^{13}+\frac{16}{42{,}525}x^{14}$$

Group like terms together.

$$\sin(2x)e^{2x}=2x+4x^2+\left(4x^3-\frac{4}{3}x^3\right)+\left(\frac{8}{3}x^4-\frac{8}{3}x^4\right)$$

$$+\left(\frac{4}{3}x^5-\frac{8}{3}x^5+\frac{4}{15}x^5\right)+\left(\frac{8}{15}x^6-\frac{16}{9}x^6+\frac{8}{15}x^6\right)$$

$$+\left(-\frac{8}{9}x^7+\frac{8}{15}x^7-\frac{8}{315}x^7\right)+\left(-\frac{16}{45}x^8+\frac{16}{45}x^8-\frac{16}{315}x^8\right)$$

$$+\left(\frac{8}{45}x^9-\frac{16}{315}x^9+\frac{4}{2{,}835}x^9\right)+\left(\frac{16}{225}x^{10}-\frac{32}{945}x^{10}+\frac{8}{2{,}835}x^{10}\right)$$

$$+\left(-\frac{16}{945}x^{11}+\frac{8}{2{,}835}x^{11}\right)+\left(-\frac{32}{4{,}725}x^{12}+\frac{16}{8{,}505}x^{12}\right)$$

$$+\frac{8}{8,505}x^{13}+\frac{16}{42,525}x^{14}$$

Since we only need the first four non-zero terms, we can at least simplify to

$$\sin(2x)e^{2x}=2x+4x^2+\left(\frac{12}{3}x^3-\frac{4}{3}x^3\right)+\left(\frac{8}{3}x^4-\frac{8}{3}x^4\right)$$

$$+\left(\frac{20}{15}x^5-\frac{40}{15}x^5+\frac{4}{15}x^5\right)+\left(\frac{24}{45}x^6-\frac{80}{45}x^6+\frac{24}{45}x^6\right)$$

$$\sin(2x)e^{2x}=2x+4x^2+\frac{8}{3}x^3+0-\frac{16}{15}x^5-\frac{32}{45}x^6$$

Take just the first four non-zero terms, and then the answer is

$$\sin(2x)e^{2x}=2x+4x^2+\frac{8}{3}x^3-\frac{16}{15}x^5$$

Topic: Power series multiplication

Question: Use power series multiplication to find the first four non-zero terms of the Maclaurin series.

$$y = 3\tan(3x)\ln(1 + 2x)$$

Answer choices:

A $3\tan(3x)\ln(1 + 2x) = 6x^2 + 6x^3 + 26x^4 + 30x^5$

B $3\tan(3x)\ln(1 + 2x) = 18x^2 + 18x^3 + 78x^4 + 90x^5$

C $3\tan(3x)\ln(1 + 2x) = 18x^2 - 18x^3 + 78x^4 - 90x^5$

D $3\tan(3x)\ln(1 + 2x) = 6x^2 - 6x^3 + 26x^4 - 30x^5$

Solution: C

When we multiply two power series together, we want to find the expansion of the sum of each series, so that we essentially have polynomial representations. Then finding the product of the series will be like multiplying polynomials.

We need to recognize that the given series is the product of two other series

$$y = 3\tan(3x)$$

$$y = \ln(1 + 2x)$$

There are common Maclaurin series that are similar to each of these.

$$\tan x = x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + \dots$$

$$\ln(1 + x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \dots$$

We want to modify each of these common series to match the given series.

For $y = 3\tan(3x)$, we'll substitute $3x$ for x and then multiply by 3:

$$\tan(3x) = 3x + \frac{1}{3}(3x)^3 + \frac{2}{15}(3x)^5 + \dots$$

$$\tan(3x) = 3x + \frac{27x^3}{3} + \frac{486x^5}{15} + \dots$$

$$3\tan(3x) = 3\left(3x + \frac{27x^3}{3} + \frac{486x^5}{15} + \dots\right)$$

$$3\tan(3x) = 9x + 27x^3 + \frac{486x^5}{5} + \dots$$

For $y = \ln(1 + 2x)$, we'll substitute $2x$ for x:

$$\ln(1 + 2x) = 2x - \frac{1}{2}(2x)^2 + \frac{1}{3}(2x)^3 - \frac{1}{4}(2x)^4 + \dots$$

$$\ln(1 + 2x) = 2x - 2x^2 + \frac{8x^3}{3} - 4x^4 + \dots$$

Multiplying the modified series together, we get

$$3\tan(3x)\ln(1 + 2x) = \left(9x + 27x^3 + \frac{486x^5}{5} + \dots\right)\left(2x - 2x^2 + \frac{8x^3}{3} - 4x^4 + \dots\right)$$

We need to multiply every term in the first series by every term in the second series.

$$3\tan(3x)\ln(1 + 2x) = 9x\left(2x - 2x^2 + \frac{8x^3}{3} - 4x^4 + \dots\right)$$

$$+27x^3\left(2x - 2x^2 + \frac{8x^3}{3} - 4x^4 + \dots\right)$$

$$+\frac{486x^5}{5}\left(2x - 2x^2 + \frac{8x^3}{3} - 4x^4 + \dots\right)$$

$$3\tan(3x)\ln(1 + 2x) = 18x^2 - 18x^3 + 24x^4 - 36x^5 + 54x^4 - 54x^5 + 72x^6 - 108x^7$$

$$+\frac{972x^6}{5} - \frac{972x^7}{5} + \frac{3{,}888x^8}{15} - \frac{1{,}944x^9}{5}$$

Group like terms together.

$$3\tan(3x)\ln(1+2x) = 18x^2 - 18x^3 + \left(24x^4 + 54x^4\right) + \left(-36x^5 - 54x^5\right)$$

$$+\left(72x^6 + \frac{972x^6}{5}\right) + \left(-108x^7 - \frac{972x^7}{5}\right) + \frac{3{,}888x^8}{15} - \frac{1{,}944x^9}{5}$$

$$3\tan(3x)\ln(1+2x) = 18x^2 - 18x^3 + 78x^4 - 90x^5$$

$$+\left(72x^6 + \frac{972x^6}{5}\right) + \left(-108x^7 - \frac{972x^7}{5}\right) + \frac{3{,}888x^8}{15} - \frac{1{,}944x^9}{5}$$

Since we only need the first four non-zero terms, our answer will be

$$3\tan(3x)\ln(1+2x) = 18x^2 - 18x^3 + 78x^4 - 90x^5$$

Topic: Power series multiplication

Question: Use power series multiplication to find the first four non-zero terms of the Maclaurin series.

$$y = \sin^{-1}(3x)e^{3x}$$

Answer choices:

A $\sin^{-1}(3x)e^{3x} = 3x + 9x^2 + 18x^3 + 27x^4$

B $\sin^{-1}(3x)e^{3x} = x - 3x^2 + 9x^3 - 18x^4$

C $\sin^{-1}(3x)e^{3x} = -x + 3x^2 - 9x^3 + 18x^4$

D $\sin^{-1}(3x)e^{3x} = x - 3x^2 + 9x^3 - 9x^4$

Solution: A

When we multiply two power series together, we want to find the expansion of the sum of each series, so that we essentially have polynomial representations. Then finding the product of the series will be like multiplying polynomials.

We need to recognize that the given series is the product of two other series

$$y = \sin^{-1}(3x)$$

$$y = e^{3x}$$

There are common Maclaurin series that are similar to each of these.

$$\sin^{-1} x = x + \frac{1}{6}x^3 + \frac{3}{40}x^5 + \dots$$

$$e^x = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \dots$$

We want to modify each of these common series to match the given series.

For $y = \sin^{-1}(3x)$, we'll substitute $3x$ for x:

$$\sin^{-1}(3x) = 3x + \frac{1}{6}(3x)^3 + \frac{3}{40}(3x)^5 + \dots$$

$$\sin^{-1}(3x) = 3x + \frac{9x^3}{2} + \frac{729x^5}{40} + \dots$$

For $y = e^{3x}$, we'll substitute $3x$ for x:

$$e^{3x} = 1 + 3x + \frac{1}{2}(3x)^2 + \frac{1}{6}(3x)^3 + \dots$$

$$e^{3x} = 1 + 3x + \frac{9x^2}{2} + \frac{9x^3}{2} + \dots$$

Multiplying the modified series together, we get

$$\sin^{-1}(3x)e^{3x} = \left(3x + \frac{9x^3}{2} + \frac{729x^5}{40} + \dots\right)\left(1 + 3x + \frac{9x^2}{2} + \frac{9x^3}{2} + \dots\right)$$

We need to multiply every term in the first series by every term in the second series.

$$\sin^{-1}(3x)e^{3x} = 3x\left(1 + 3x + \frac{9x^2}{2} + \frac{9x^3}{2} + \dots\right) + \frac{9}{2}x^3\left(1 + 3x + \frac{9x^2}{2} + \frac{9x^3}{2} + \dots\right)$$

$$+\frac{729x^5}{40}\left(1 + 3x + \frac{9x^2}{2} + \frac{9x^3}{2} + \dots\right)$$

$$\sin^{-1}(3x)e^{3x} = 3x + 9x^2 + \frac{27x^3}{2} + \frac{27x^4}{2} + \frac{9x^3}{2} + \frac{27x^4}{2} + \frac{81x^5}{4} + \frac{81x^6}{4}$$

$$+\frac{729x^5}{40} + \frac{2{,}187x^6}{40} + \frac{6{,}561x^7}{80} + \frac{6{,}561x^8}{80}$$

Group like terms together.

$$\sin^{-1}(3x)e^{3x} = 3x + 9x^2 + \left(\frac{27x^3}{2} + \frac{9x^3}{2}\right) + \left(\frac{27x^4}{2} + \frac{27x^4}{2}\right)$$

$$+\left(\frac{81x^5}{4}+\frac{729x^5}{40}\right)+\left(\frac{81x^6}{4}+\frac{2{,}187x^6}{40}\right)+\frac{6{,}561x^7}{80}+\frac{6{,}561x^8}{80}$$

$$\sin^{-1}(3x)e^{3x}=3x+9x^2+18x^3+27x^4$$

$$+\left(\frac{81x^5}{4}+\frac{729x^5}{40}\right)+\left(\frac{81x^6}{4}+\frac{2{,}187x^6}{40}\right)+\frac{6{,}561x^7}{80}+\frac{6{,}561x^8}{80}$$

Since we only need the first four non-zero terms, our answer will be

$$\sin^{-1}(3x)e^{3x}=3x+9x^2+18x^3+27x^4$$