Calculus 2 Workbook Solutions

Average value

krista king
MATH

AVERAGE VALUE

■ 1. Find the average value of $f(x)$ over the interval $[-3,5]$.

$$f(x) = -3x^3 - 5x^2 + x + 4$$

Solution:

Plug the interval and the function into the average value integral formula.

$$f_{avg} = \frac{1}{b-a}\int_a^b f(x)\ dx$$

$$f_{avg} = \frac{1}{5-(-3)}\int_{-3}^{5} -3x^3 - 5x^2 + x + 4\ dx$$

$$f_{avg} = \frac{1}{8}\left(-\frac{3}{4}x^4 - \frac{5}{3}x^3 + \frac{1}{2}x^2 + 4x\right)\Bigg|_{-3}^{5}$$

$$f_{avg} = \frac{1}{8}\left(-\frac{3}{4}(5)^4 - \frac{5}{3}(5)^3 + \frac{1}{2}(5)^2 + 4(5)\right)$$

$$-\frac{1}{8}\left(-\frac{3}{4}(-3)^4 - \frac{5}{3}(-3)^3 + \frac{1}{2}(-3)^2 + 4(-3)\right)$$

$$f_{avg} = \frac{1}{8}\left(-\frac{1{,}875}{4} - \frac{625}{3} + \frac{25}{2} + 20\right) - \frac{1}{8}\left(-\frac{243}{4} + \frac{135}{3} + \frac{9}{2} - 12\right)$$

$$f_{avg} = -\frac{1{,}875}{32} - \frac{625}{24} + \frac{25}{16} + \frac{20}{8} + \frac{243}{32} - \frac{135}{24} - \frac{9}{16} + \frac{12}{8}$$

$$f_{avg} = -\frac{1{,}632}{32} - \frac{760}{24} + \frac{16}{16} + \frac{32}{8}$$

$$f_{avg} = -51 - \frac{95}{3} + 1 + 4$$

$$f_{avg} = -46 - \frac{95}{3}$$

$$f_{avg} = -\frac{138}{3} - \frac{95}{3}$$

$$f_{avg} = -\frac{233}{3}$$

■ 2. Find the average value of $g(x)$ over the interval $[-4,3]$.

$$g(x) = \frac{1}{3}x^3 + \frac{3}{2}x^2 + \frac{2}{5}x - 2$$

Solution:

Plug the interval and the function into the average value integral formula.

$$g_{avg} = \frac{1}{b-a}\int_a^b g(x)\ dx$$

$$g_{avg} = \frac{1}{3-(-4)}\int_{-4}^{3} \frac{1}{3}x^3 + \frac{3}{2}x^2 + \frac{2}{5}x - 2\ dx$$

$$g_{avg} = \frac{1}{7}\left(\frac{1}{12}x^4 + \frac{1}{2}x^3 + \frac{1}{5}x^2 - 2x\right)\Bigg|_{-4}^{3}$$

$$g_{avg} = \frac{1}{7}\left(\frac{1}{12}(3)^4 + \frac{1}{2}(3)^3 + \frac{1}{5}(3)^2 - 2(3)\right) - \frac{1}{7}\left(\frac{1}{12}(-4)^4 + \frac{1}{2}(-4)^3 + \frac{1}{5}(-4)^2 - 2(-4)\right)$$

$$g_{avg} = \frac{27}{28} + \frac{27}{14} + \frac{9}{35} - \frac{6}{7} - \frac{64}{21} + \frac{24}{7} - \frac{16}{35}$$

$$g_{avg} = -\frac{7}{35} + \frac{27}{28} - \frac{64}{21} + \frac{27}{14} + \frac{18}{7}$$

$$g_{avg} = -\frac{84}{420} + \frac{405}{420} - \frac{1,280}{420} + \frac{810}{420} + \frac{1,080}{420}$$

$$g_{avg} = \frac{931}{420}$$

$$g_{avg} = \frac{133}{60}$$

3. Find the average value of $h(x)$ over the interval $[-2,3]$.

$$h(x) = 3(2x-5)^2$$

Solution:

Plug the interval and the function into the average value integral formula.

$$h_{avg} = \frac{1}{b-a}\int_a^b h(x)\ dx$$

$$h_{avg} = \frac{1}{3-(-2)} \int_{-2}^{3} 3(2x-5)^2\ dx$$

$$h_{avg} = \frac{3}{5} \int_{-2}^{3} 4x^2 - 20x + 25\ dx$$

$$h_{avg} = \frac{3}{5} \left(\frac{4}{3}x^3 - 10x^2 + 25x \right) \Big|_{-2}^{3}$$

$$h_{avg} = \frac{4}{5}x^3 - 6x^2 + 15x \Big|_{-2}^{3}$$

$$h_{avg} = \frac{4}{5}(3)^3 - 6(3)^2 + 15(3) - \left(\frac{4}{5}(-2)^3 - 6(-2)^2 + 15(-2) \right)$$

$$h_{avg} = \frac{108}{5} - 54 + 45 + \frac{32}{5} + 24 + 30$$

$$h_{avg} = \frac{140}{5} + 45$$

$$h_{avg} = 28 + 45$$

$$h_{avg} = 73$$

■ 4. Set up the average value formula for $f(x)$ over the interval $[-4,4]$. Do not evaluate the integral.

$$f(x) = \sqrt{16 - x^2}$$

Solution:

Plug the interval and the function into the average value integral formula.

$$f_{avg} = \frac{1}{b-a}\int_a^b f(x)\ dx$$

$$f_{avg} = \frac{1}{4-(-4)}\int_{-4}^4 \sqrt{16-x^2}\ dx$$

$$f_{avg} = \frac{1}{8}\int_{-4}^4 \sqrt{16-x^2}\ dx$$

MEAN VALUE THEOREM FOR INTEGRALS

1. Use the Mean Value Theorem for integrals to find a value for $f(c)$.

$$\int_{4}^{20} f(x)\ dx = 26$$

Solution:

Comparing the integral to the Mean Value Theorem formula,

$$\int_{a}^{b} f(x)\ dx = f(c)(b-a)$$

we have $a = 4$ and $b = 20$. So we can set up the equation for $f(c)$.

$$f(c)(20-4) = 26$$

$$16f(c) = 26$$

$$f(c) = \frac{26}{16} = \frac{13}{8}$$

2. Use the Mean Value Theorem for integrals to find a value for $g(c)$.

$$\int_{-15}^{35} g(x)\ dx = -20$$

Solution:

Comparing the integral to the Mean Value Theorem formula,

$$\int_a^b g(x)\ dx = g(c)(b-a)$$

we have $a = -15$ and $b = 35$. So we can set up the equation for $g(c)$.

$$g(c)(35-(-15)) = -20$$

$$50g(c) = -20$$

$$g(c) = -\frac{20}{50} = -\frac{2}{5}$$

3. Use the Mean Value Theorem for integrals to find a value for $h(c)$.

$$\int_{-1}^{5} h(x)\ dx = 48$$

Solution:

Comparing the integral to the Mean Value Theorem formula,

$$\int_a^b h(x)\ dx = h(c)(b-a)$$

we have $a = -1$ and $b = 5$. So we can set up the equation for $h(c)$.

$$h(c)(5-(-1)) = 48$$

$$6h(c) = 48$$

$$h(c) = \frac{48}{6} = 8$$

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