Part 1 of the FTC

Part 1 of the Fundamental Theorem of Calculus (FTC) is the formula that relates the derivative to the integral. In other words, it connects the first big idea of calculus, the derivative, to the second big idea of calculus, the integral. It states that

If $r(x)$ is continuous on $[a,b]$ then

$$f(x)=\int_a^x r(t)\ dt,$$

is continuous on $[a,b]$, it's differentiable on (a,b), and

$$f'(x)=r(x)$$

This means that if you take the integral of the function $r(t)$ over the interval $[a,x]$, the answer you get, $f(x)$, can be differentiated to get back to $r(x)$.

Therefore, the FTC is something you can use to double check your integration for mistakes.

When it comes to solving a problem using Part 1 of the Fundamental Theorem, we can use the chart below to help us figure out how to do it. The chart tells us how to solve for $f'(x)$, depending on the kinds of bounds we find on the integral.

Given integral	**How to solve it**
$f(x)=\int_a^x r(t)\ dt$	Plug x in for t.

$f(x) = \int_x^a r(t)\ dt$

Reverse limits of integration and multiply by -1, then plug x in for t.

$f(x) = \int_a^{g(x)} r(t)\ dt$

Plug $g(x)$ in for t, then multiply by dg/dx.

$f(x) = \int_{g(x)}^a r(t)\ dt$

Reverse limits of integration and multiply by -1, then plug $g(x)$ in for t and multiply by dg/dx.

$f(x) = \int_{g(x)}^{h(x)} r(t)\ dt$

Split the limits of integration as $\int_{g(x)}^0 r(t)\ dt + \int_0^{h(x)} r(t)\ dt$. Reverse limits of integration on $\int_{g(x)}^0 r(t)\ dt$ and multiply by -1, then plug $g(x)$ and $h(x)$ in for t, multiplying by dg/dx and dh/dx respectively.

Example

Use Part 1 of the Fundamental Theorem of Calculus to find $f'(x)$.

$$f(x) = \int_0^{x^2} t^2 - 1\ dt$$

Since the given lower bound is a constant and the upper bound is a function in terms of x, the integral we're given follows the pattern of the third integral in the table,

$$f(x) = \int_{a}^{g(x)} r(t)\ dt$$

So we know we need to plug $g(x)$ (the upper bound) in for t, then multiply by dg/dx (the derivative of the upper bound). Therefore, we can say that $f'(x)$ is

$$f'(x) = \left((x^2)^2 - 1\right)(2x)$$

$$f'(x) = (x^4 - 1)(2x)$$

$$f'(x) = 2x^5 - 2x$$