Calculus 1 Workbook Solutions

Physics and economics

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POSITION, VELOCITY, AND ACCELERATION

■ 1. Find the velocity $v(t)$, speed, and acceleration $a(t)$ at $t=2$ of the position function.

$$s(t) = -\frac{t^3}{3} + t^2 + 3t - 1$$

Solution:

Velocity is given by the first derivative of the position function.

$$s'(t) = v(t) = -t^2 + 2t + 3$$

$$v(2) = -(2)^2 + 2(2) + 3$$

$$v(2) = 3$$

Acceleration is given by the second derivative of the position function or the first derivative of the velocity function.

$$s''(t) = v'(t) = a(t) = -2t + 2$$

$$a(2) = -2(2) + 2$$

$$a(2) = -2$$

Speed is the absolute value of velocity. So speed is

$$|v(2)| = |3| = 3$$

■ 2.The position of a particle which moves along the x-axis is given by $s(t) = \cos t + \sin t$. What is the acceleration of the particle at the point where the velocity is first equal to zero?

Solution:

Take the derivative of the position function to find the velocity.

$$s(t) = \cos t + \sin t$$

$$v(t) = s'(t) = -\sin t + \cos t$$

We need to find time when velocity is 0.

$$-\sin t + \cos t = 0$$

$$\cos t = \sin t$$

$t = \dfrac{\pi}{4} + \pi k$, where k is any integer

Since we need to find the acceleration of the particle at the point where the velocity is first equal to zero, then $t = \pi/4$.

Take the derivative of the velocity function to find the acceleration.

$$v(t) = -\sin t + \cos t$$

$$a(t) = v'(t) = -\cos t - \sin t$$

$$a\left(\frac{\pi}{4}\right) = -\cos\left(\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{4}\right)$$

$$a\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}$$

$$a\left(\frac{\pi}{4}\right) = -\sqrt{2}$$

■ 3. Find the velocity $v(t)$, speed, and acceleration $a(t)$ at $t = 4$ of the position function.

$$s(t) = \frac{t^2}{2t+4}$$

Solution:

Velocity is given by the first derivative of the position function.

$$s'(t) = v(t) = \frac{(2t)(2t+4) - (t^2)(2)}{(2t+4)^2} = \frac{4t^2 + 8t - 2t^2}{4t^2 + 16t + 16} = \frac{t^2 + 4t}{2t^2 + 8t + 8} = \frac{t(t+4)}{2(t+2)(t+2)}$$

$$v(4) = \frac{4(4+4)}{2(4+2)(4+2)} = \frac{4(8)}{2(6)(6)}$$

$$v(4) = \frac{32}{72} = \frac{4}{9}$$

Acceleration is given by the second derivative of the position function or the first derivative of the velocity function.

$$s''(t) = v'(t) = a(t) = \frac{(2t+4)(2t^2+8t+8) - (t^2+4t)(4t+8)}{(2t^2+8t+8)^2}$$

$$a(t) = \frac{16t+32}{(2t^2+8t+8)^2} = \frac{16(t+2)}{4(t^2+4t+4)^2} = \frac{4(t+2)}{(t+2)^4} = \frac{4}{(t+2)^3}$$

$$a(4) = \frac{4}{(4+2)^3} = \frac{4}{216}$$

$$a(4) = \frac{1}{54}$$

Speed is the absolute value of velocity. So speed is

$$|v(4)| = \left|\frac{4}{9}\right| = \frac{4}{9}$$

■ 4. Let $s(t) = 2t^3 - 12t^2 + 18t + 2$ be the position of a particle. What is the velocity when acceleration is zero? What is the total distance traveled by the particle from $t = 0$ to $t = 2$?

Solution:

Take the derivative of the position function to find the velocity.

$$s(t) = 2t^3 - 12t^2 + 18t + 2$$

$$v(t) = s'(t) = 6t^2 - 24t + 18$$

Take the derivative of the velocity function to find the acceleration.

$$v(t) = 6t^2 - 24t + 18$$

$$a(t) = v'(t) = 12t - 24$$

We need to find time when acceleration is 0.

$$12t - 24 = 0$$

$$12t = 24$$

$$t = 2$$

Then we need to find the velocity when acceleration is zero, $v(2)$.

$$v(2) = 6(2)^2 - 24(2) + 18$$

$$v(2) = 6(4) - 48 + 18$$

$$v(2) = -6$$

To find the total distance traveled by the particle from $t = 0$ to $t = 2$, first we need to find when the particle is at rest.

$$v(t) = 6t^2 - 24t + 18$$

$$6t^2 - 24t + 18 = 0$$

$$6(t - 1)(t - 3) = 0$$

$t = 1$ and $t = 3$

So to find the total distance we need to find the distance from $t = 0$ to $t = 1$ and the distance from $t = 1$ to $t = 2$, then add them.

$$s(0) = 2(0)^3 - 12(0)^2 + 18(0) + 2$$

$$s(0) = 2$$

$$s(1) = 2(1)^3 - 12(1)^2 + 18(1) + 2$$

$$s(1) = 10$$

The distance from $t = 0$ to $t = 1$ is $|10 - 2| = 8$.

$$s(2) = 2(2)^3 - 12(2)^2 + 18(2) + 2$$

$$s(2) = 6$$

The distance from $t = 1$ to $t = 2$ is $|6 - 10| = |-4| = 4$. Therefore the total distance is $8 + 4 = 12$.

■ 5. The position of a particle moving along a line is given. For what values of t is the speed of the particle decreasing?

$$s(t) = \frac{4}{3}t^3 - 12t^2 + 32t - 12 \text{ for } t \geq 0$$

Solution:

Speed is decreasing when velocity and acceleration have opposite signs, such that $v(t) > 0$ with $a(t) < 0$, or $v(t) < 0$ with $a(t) > 0$.

Take the derivative of the position function to find velocity.

$$s(t) = \frac{4}{3}t^3 - 12t^2 + 32t - 12$$

$$v(t) = s'(t) = 4t^2 - 24t + 32$$

We need to find time when acceleration is 0.

$$4t^2 - 24t + 32 = 0$$

$$4(t - 4)(t - 2) = 0$$

$t = 2$ and $t = 4$

Using the first derivative test with test values of $t = 1$, $t = 3$, and $t = 5$, we can determine that velocity is positive to the left of $t = 2$, negative between $t = 2$ and $t = 4$, and positive again to the right of $t = 4$.

$$v(1) = 4(1)^2 - 24(1) + 32 = 12 > 0$$

$$v(3) = 4(3)^2 - 24(3) + 32 = -4 < 0$$

$$v(5) = 4(5)^2 - 24(5) + 32 = 12 > 0$$

Take the derivative of the velocity function to find the acceleration.

$$v(t) = 4t^2 - 24t + 32$$

$$a(t) = v'(t) = 8t - 24$$

We need to find time when acceleration is 0.

$$8t - 24 = 0$$

$$8t = 24$$

$$t = 3$$

Now we need to determine where acceleration is positive and negative, so we'll use test values of $t = 1$ and $t = 4$.

$$a(1) = 8(1) - 24 = -16 < 0$$

$$a(4) = 8(4) - 24 = 8 > 0$$

Therefore the speed is decreasing on $[0,2]$ and $[3,4]$.

■ 6. A particle moves along the x-axis with its position at time t given by $s(t) = a(t + a)(t - b)$, where a and b are constants and $a \neq b$. Find the values of t when the particle is at rest.

Solution:

To find when the particle is at rest, we first find velocity.

$$s(t) = a(t + a)(t - b)$$

$$s(t) = a(t^2 - bt + at - ab)$$

$$s(t) = at^2 - abt + a^2t - a^2b$$

$$v(t) = 2at - ab + a^2$$

Set velocity equal to zero and solve for t.

$$2at - ab + a^2 = 0$$

$$a(2t - b + a) = 0$$

$$2t - b + a = 0$$

$$2t = b - a$$

$$t = \frac{b - a}{2}$$

So the particle is at rest whenever time satisfies $t = (b - a)/2$.

BALL THROWN UP FROM THE GROUND

1. A ball is thrown straight upward from the ground with an initial velocity of $v_0 = 86$ ft/sec. Assuming constant gravity, find the maximum height, in feet, that the ball attains, the time, in seconds, that it's in the air, as well as the ball's velocity, in ft/sec, when it hits the ground.

Solution:

Plugging everything we know into the formula for standard projectile motion, we get

$$y(t) = -\frac{1}{2}gt^2 + v_0t + y_0$$

$$h(t) = -\frac{1}{2}(32)t^2 + 86t + 0$$

$$h(t) = -16t^2 + 86t$$

When the ball is at its maximum height, velocity is 0, so find $h'(t)$ and set it equal to 0.

$$v(t) = h'(t) = -32t + 86$$

$$-32t + 86 = 0$$

$$32t = 86$$

$$t = \frac{43}{16} \approx 2.69 \text{ seconds}$$

Next, find the maximum height.

$$h(t) = -16\left(\frac{43}{16}\right)^2 + 86\left(\frac{43}{16}\right) = \frac{1,849}{16} \approx 115.56 \text{ feet}$$

To find the time the ball stays in the air, set the height equal to 0 and solve for t.

$$h(t) = -16t^2 + 86t$$

$$-16t^2 + 86t = 0$$

$$t(43 - 8t) = 0$$

$$t = 0, \frac{43}{8} \approx 5.38 \text{ seconds}$$

Now, find the final velocity of the ball when it hits the ground. Substitute the time the ball lands into the velocity function.

$$v\left(\frac{43}{8}\right) = -32\left(\frac{43}{8}\right) + 86 = -86 \text{ ft/sec}$$

■ 2. A ball is thrown straight upward from the top of a building, which is 56 feet above the ground, with an initial velocity of $v_0 = 48$ ft/sec. Assuming constant gravity, find the maximum height, in feet, that the ball attains, the time, in seconds, that it's in the air, as well as the ball's velocity, in ft/sec, when it hits the ground.

Solution:

Plugging everything we know into the formula for standard projectile motion, we get

$$y(t) = -\frac{1}{2}gt^2 + v_0t + y_0$$

$$h(t) = -\frac{1}{2}(32)t^2 + 48t + 56$$

$$h(t) = -16t^2 + 48t + 56$$

When the ball is at its maximum height, velocity is 0, so find $h'(t)$ and set it equal to 0.

$$h'(t) = v(t) = -32t + 48$$

$$-32t + 48 = 0$$

$$32t = 48$$

$$t = \frac{3}{2} = 1.5 \text{ seconds}$$

Next, find the maximum height.

$$h(t) = -16\left(\frac{3}{2}\right)^2 + 48\left(\frac{3}{2}\right) + 56 = 92 \text{ feet}$$

To find the time the ball stays in the air, set the height equal to 0 and solve for t.

$$-16t^2 + 48t + 56 = 0$$

$$2t^2 - 6t - 7 = 0$$

$$t = \frac{3 + \sqrt{23}}{2} \approx 3.90 \text{ seconds}$$

Now, find the final velocity of the ball when it hits the ground. Substitute the time the ball lands into the velocity function.

$$v\left(\frac{3 + \sqrt{23}}{2}\right) = -32\left(\frac{3 + \sqrt{23}}{2}\right) + 48 \approx -76.73 \text{ ft/sec}$$

■ 3. A ball is thrown straight upward from a bridge, which is 24 meters above the water, with an initial velocity of $v_0 = 20$ m/sec. Assuming constant gravity, find the maximum height, in meters, that the ball attains, the time, in seconds, that it's in the air, as well as the ball's velocity, in m/sec, when it hits the water below.

Solution:

Plugging everything we know into the formula for standard projectile motion, we get

$$y(t) = -\frac{1}{2}gt^2 + v_0t + y_0$$

$$h(t) = -\frac{1}{2}(9.8)t^2 + 20t + 24$$

$$h(t) = -4.9t^2 + 20t + 24$$

When the ball is at its maximum height, velocity is 0, so find $h'(t)$ and set it equal to 0.

$$h'(t) = v(t) = -9.8t + 20$$

$$-9.8t + 20 = 0$$

$$9.8t = 20$$

$$t = \frac{20}{9.8} \approx 2.041 \text{ seconds}$$

Next, find the maximum height.

$$h(t) = -4.9\left(\frac{100}{49}\right)^2 + 20\left(\frac{100}{49}\right) + 24 \approx 44.41 \text{ meters}$$

To find the time the ball stays in the air, set the height equal to 0 and solve for t.

$$h(t) = -4.9t^2 + 20t + 24$$

$$-4.9t^2 + 20t + 24 = 0$$

$$t \approx 5.05 \text{ seconds}$$

Now, find the final velocity of the ball when it hits the water. Substitute the time the ball lands into the velocity function.

$$v(5.05) = -9.8(5.05) + 20 \approx -29.5 \text{ m/sec}$$

4. A boy needs to jump 2.8 ft in the air in order to dunk a basketball. The height that the boy's feet are above the ground is given by the function $h(t) = -16t^2 + 10t$. What is the maximum height the boy's feet will ever be above the ground, and will he be able to dunk the basketball?

Solution:

The boy's feet will reach their maximum height when velocity is 0, so find $h'(t)$ and set it equal to 0.

$$h(t) = -16t^2 + 10t$$

$$h'(t) = v(t) = -32t + 10$$

$$-32t + 10 = 0$$

$$32t = 10$$

$$t = \frac{10}{32} \approx 0.31 \text{ seconds}$$

Next, find the maximum height.

$$h(t) = -16(0.31)^2 + 10(0.31) = 1.56 \text{ ft}$$

The boy won't be able to dunk the ball because he'll never reach the 2.8 ft required.

5. A diver jumps up from a platform and then falls down into a pool. His height as a function of time can be modeled by $h(t) = -16t^2 + 12t + 60$, where t is the time in seconds and h is the height in feet. How long did it take for the diver to reach his maximum height? What was the highest point that he reached? In how many seconds does he hit the water?

Solution:

When the diver reaches his maximum height, velocity is 0, so find $h'(t)$ and set it equal to 0.

$$h(t) = -16t^2 + 12t + 60$$

$$h'(t) = v(t) = -32t + 12$$

$$-32t + 12 = 0$$

$$32t = 12$$

$$t = \frac{12}{32} \approx 0.375 \text{ seconds}$$

Next, find the maximum height.

$$h(t) = -16(0.375)^2 + 12(0.375) + 60 \approx 62.25 \text{ ft}$$

To find the time at which the diver hits the water, set the height equal to 0 and solve for t.

$$h(t) = -16t^2 + 12t + 60$$

$$-16t^2 + 12t + 60 = 0$$

$t \approx 2.35$ seconds

■ 6. An amateur rocketry club is holding a competition. There is cloud cover at 890 ft. If they launch a rocket with an initial velocity of 365 ft/s, determine the amount of time that the rocket is out of site in the cloud cover.

Solution:

Plugging everything we know into the formula for standard projectile motion, we get

$$y(t) = -\frac{1}{2}gt^2 + v_0t + y_0$$

$$h(t) = -\frac{1}{2}(32)t^2 + 365t + 0$$

$$h(t) = -16t^2 + 365t$$

We need to find out when the rocket will reach the height of 890 ft. Substitute $h(t) = 890$ into the formula and solve for t.

$$890 = -16t^2 + 365t$$

$$-16t^2 + 365t - 890 = 0$$

$t \approx 2.78$ seconds and $t \approx 20.04$ seconds

Therefore, the time that the rocket is out of sight is $20.04 - 2.78 = 17.26$ seconds.

COIN DROPPED FROM THE ROOF

■ 1. A rock is dropped from the top of an 800 foot tall cliff, with an initial velocity of $v_0 = 0$ ft/sec. Assuming constant gravity, when does the rock hit the ground, and what is its velocity when it hits the ground?

Solution:

Plugging everything we know into the formula for standard projectile motion, we get

$$y(t) = -\frac{1}{2}gt^2 + v_0t + y_0$$

$$y(t) = -\frac{1}{2}(32)t^2 + 0t + 800$$

$$y(t) = -16t^2 + 800$$

The rock hits the ground when its height is 0.

$$-16t^2 + 800 = 0$$

$$16t^2 = 800$$

$$t^2 = 50$$

$$t = \sqrt{50} \approx 7 \text{ seconds}$$

To find the velocity of the rock when it hits the ground, find $y'(t)$ and evaluate it at the time the rock hits the ground.

$$y'(t) = -32t$$

$$y'(7.071) = -32(7) \approx -224 \text{ ft/sec}$$

■ 2. A rock is tossed from the top of a 300 foot tall cliff, with an initial velocity of $v_0 = 15$ ft/sec. Assuming constant gravity, when does the rock hit the ground, and what is its velocity when it hits the ground?

Solution:

Plugging everything we know into the formula for standard projectile motion, we get

$$y(t) = -\frac{1}{2}gt^2 + v_0t + y_0$$

$$y(t) = -\frac{1}{2}(32)t^2 + 15t + 300$$

$$y(t) = -16t^2 + 15t + 300$$

The rock hits the ground when its height is 0.

$$-16t^2 + 15t + 300 = 0$$

$$t = \frac{-15 \pm \sqrt{(-15)^2 - 4(-16)(300)}}{2(-16)} = \frac{15 \pm 5\sqrt{777}}{32} \approx 4.8242 \text{ seconds}$$

To find the velocity of the rock when it hits the ground, find $y'(t)$ and evaluate it at the time the rock hits the ground.

$$y'(t) = -32t + 15$$

$$y'(4.8242) = -32(4.8242) + 15 \approx -139.37 \text{ ft/sec}$$

■ 3. A coin is tossed downward from the top of a 36 meter tall building, with an initial velocity of $v_0 = 6$ m/sec. Assuming constant gravity, when does the rock hit the ground, and what is its velocity when it hits the ground?

Solution:

Plugging everything we know into the formula for standard projectile motion, we get

$$y(t) = -\frac{1}{2}gt^2 + v_0t + y_0$$

$$y(t) = -\frac{1}{2}(9.8)t^2 + 6t + 36$$

$$y(t) = -4.9t^2 + 6t + 36$$

The rock hits the ground when its height is 0.

$$-4.9t^2 + 6t + 36 = 0$$

$$t = \frac{-6 \pm \sqrt{(-6)^2 - 4(-4.9)(36)}}{2(-4.9)} = \frac{6 \pm \sqrt{741.6}}{9.8} \approx 3.391 \text{ seconds}$$

To find the velocity of the rock when it hits the ground, find $y'(t)$ and evaluate it at the time the rock hits the ground.

$$y'(t) = -9.8t + 6$$

$$y'(3.391) = -9.8(3.391) + 6 \approx -27.23 \text{ m/sec}$$

■ 4. A raindrop falls from the sky and takes 25 seconds to reach the ground. Assuming constant gravity, what is the raindrop's velocity at impact? How far did it fall? What is its acceleration when $t = 5$ seconds?

Solution:

Given the formula for standard projectile motion,

$$y(t) = -\frac{1}{2}gt^2 + v_0t + y_0$$

and the formula for velocity,

$$v(t) = x'(t) = -gt + v_0$$

we can find instantaneous velocity at $t = 25$ by substituting into the velocity function.

$$v(25) = -9.8(25) + 0$$

$$v(25) = -245 \text{ m/s}$$

Substitute what we know into the position function to determine how far the raindrop fell.

$$0 = -\frac{1}{2}(9.8)(25)^2 + (0)(15) + y_0$$

$$\frac{1}{2}(9.8)(25)^2 = y_0$$

$$y_0 = 3{,}062.5 \text{ m}$$

Acceleration is relatively constant, $a = g = 9.8$ m/s^2.

■ 5. You throw a rock into the Grand Canyon and it takes 7.55 seconds to hit the ground. Calculate the velocity of the rock at impact in m/s and then find the distance the rock fell in feet.

Solution:

Given the formula for standard projectile motion,

$$y(t) = -\frac{1}{2}gt^2 + v_0t + y_0$$

and the formula for velocity,

$$v(t) = x'(t) = -gt + v_0$$

we can find instantaneous velocity at $t = 7.55$ by substituting into the velocity function.

$$v(7.55) = -9.8(7.55) + 0$$

$$v(7.55) = -73.99 \text{ m/s}$$

To find the distance the rock fell in feet, substitute what we know into the position formula, using $g = 32$ ft/s^2.

$$0 = -\frac{1}{2}(32)(7.55)^2 + (0)(7.55) + y_0$$

$$\frac{1}{2}(32)(7.55)^2 = y_0$$

$$y_0 = 912.04 \text{ ft}$$

■ 6. A coin is dropped into a very deep wishing well. It hits the water 4.5 s later. How far is it from the top of the well to the water at the bottom? At what velocity does the coin hit the water? How far had the coin fallen when it reached -20m/s?

Solution:

Substitute what we know into the position formula.

$$y(t) = -\frac{1}{2}gt^2 + v_0t + y_0$$

$$0 = -\frac{1}{2}(9.8)(4.5)^2 + (0)(4.5) + y_0$$

$$\frac{1}{2}(9.8)(4.5)^2 = y_0$$

$$y_0 = 99.225 \text{ m}$$

Find the velocity function by differentiating the position function.

$$v(t) = x'(t) = -gt + v_0$$

To find instantaneous velocity at $t = 4.5$, substitute $t = 4.5$ into the velocity function.

$$v(4.5) = -9.8(4.5) + 0$$

$$v(4.5) = -44.1 \text{ m/s}$$

Now we need to find the time at which the velocity of the coin is -20 m/s.

$$v(t) = -gt + v_0$$

$$-20 = -9.8t + 0$$

$$t = \frac{20}{9.8}$$

$$t \approx 2.04 \text{ s}$$

Therefore,

$$y(t) = -\frac{1}{2}gt^2 + v_0t + y_0$$

$$y(t) = -\frac{1}{2}(9.8)(2.04)^2 + (0)(2.04) + 99.225$$

$$y(t) = -\frac{1}{2}(9.8)(2.04)^2 + 99.225$$

$$y(t) = 78.83 \text{ m}$$

The coin had fallen $99.225 - 78.83 = 20.39$ m when it reached -20 m/s.

MARGINAL COST, REVENUE, AND PROFIT

■ 1. A company manufactures and sells basketballs for $\$9.50$ each. The company has a fixed cost of $\$395$ per week and a variable cost of $\$2.75$ per basketball. The company can make up to 300 basketballs per week. Find the marginal cost, marginal revenue, and marginal profit, if the company makes 150 basketballs.

Solution:

The cost function is $C(x) = 395 + 2.75x$, where x is the number of basketballs, so marginal cost is $C'(x) = 2.75$, and $C'(150) = \$2.75$.

The revenue function is $R(x) = 9.50x$, where x is the number of basketballs, so marginal revenue is $R'(x) = 9.50$, and $R'(150) = \$9.50$.

The profit function is

$$P(x) = R(x) - C(x)$$

$$P(x) = 9.50x - (395 + 2.75x)$$

$$P(x) = 6.75x - 395$$

Marginal profit is $P'(x) = 6.75$, and $P'(150) = \$6.75$.

2. A company manufactures and sells high end folding tables for $\$250$ each. The company has a fixed cost of $\$3{,}000$ per week and variable costs of $85x + 150\sqrt{x}$, where x is the number of tables manufactured. The company can make up to 200 tables per week. Find the marginal cost, marginal revenue, and marginal profit, if the company makes 64 tables.

Solution:

The cost function is $C(x) = 3{,}000 + 85x + 150\sqrt{x}$, where x is the number of folding tables, so marginal cost is $C'(x) = 85 + 75/\sqrt{x}$, and $C'(64) = 85 + 75/\sqrt{64} = 85 + 9.375 = \$94.375$.

The revenue function is $R(x) = 250x$, where x is the number of folding tables, so marginal revenue is $R'(x) = 250$, and $R'(64) = \$250$.

The profit function is

$$P(x) = R(x) - C(x)$$

$$P(x) = 250x - (3{,}000 + 85x + 150\sqrt{x})$$

$$P(x) = 165x - 150\sqrt{x} - 3{,}000$$

Marginal profit is $P'(x) = 165 - 75/\sqrt{x}$, and

$$P'(64) = 165 - \frac{75}{\sqrt{64}}$$

$$P'(64) = 165 - 9.375$$

$$P'(64) = \$155.63$$

■ 3. A company manufactures and sells electric food mixers for $\$150$ each. The company has a fixed cost of $\$7,800$ per week and variable costs of $24x + 0.04x^2$, where x is the number of mixers manufactured. The company can make up to 200 mixers per week. Find the marginal cost, marginal revenue, and marginal profit, if the company makes 75 mixers.

Solution:

The cost function is $C(x) = 7,800 + 24x + 0.04x^2$, where x is the number of food mixers, so marginal cost is $C'(x) = 24 + 0.08x$, and $C'(75) = 24 + 0.08(75) = \$30$.

The revenue function is $R(x) = 150x$, where x is the number of food mixers, so marginal revenue is $R'(x) = 150$, and $R'(75) = \$150$.

The profit function is

$$P(x) = R(x) - C(x)$$

$$P(x) = 150x - (7,800 + 24x + 0.04x^2)$$

$$P(x) = 126x - 0.04x^2 - 7,800$$

Marginal profit is $P'(x) = 126 - 0.08x$, and

$$P'(75) = 126 - 0.08(75)$$

$$P'(75) = 126 - 6$$

$$P'(75) = \$120$$

■ 4. A coffee machine manufacturer determines that the demand function for their coffee machines is given by p, while the cost of producing x coffee machines is given by $C(x) = 25x + 10\sqrt{x^3} + 1{,}250$. What is the marginal cost, marginal revenue, and marginal profit at $x = 25$?

$$p = \frac{750}{\sqrt{x^3}}$$

Solution:

Find marginal cost, then evaluate the marginal cost function at $x = 25$.

$$C'(x) = 25 + \frac{15\sqrt{x^3}}{x}$$

$$C'(25) = 25 + \frac{15\sqrt{25^3}}{25} = \$100$$

Revenue is given by the product of demand and the number of units sold.

$$R(x) = x \cdot p$$

$$R(x) = \frac{750x}{\sqrt{x^3}}$$

Find marginal revenue, then evaluate the marginal revenue function at $x = 25$.

$$R'(x) = -\frac{375}{\sqrt{x^3}}$$

$$R'(25) = -\frac{375}{\sqrt{25^3}} = -\$3$$

Now find profit and marginal profit,

$$P(x) = R(x) - C(x)$$

$$P(x) = \frac{750x}{\sqrt{x^3}} - (25x + 10\sqrt{x^3} + 1{,}250)$$

$$P(x) = \frac{750x}{\sqrt{x^3}} - 10\sqrt{x^3} - 25x - 1{,}250$$

$$P'(x) = \frac{-25\sqrt{x^3} - 15x^2 - 375}{\sqrt{x^3}}$$

then evaluate the marginal profit function at $x = 25$.

$$P'(25) = -\$103$$

The coffee machine manufacturer's marginal cost, revenue, and profit at 25 units are $C'(25) = 100$, $R'(25) = -3$, and $P'(25) = -\$103$, respectively. Therefore, their marginal profit from selling the 26th coffee machine is $-\$103$, meaning that the company should not increase production if their goal is to maximize profit.

■ 5. For the given cost and demand functions, find the number of units the company needs to produce in order to maximize profit.

$$C(x) = 15x + 300$$

$$p = 2x - 250$$

Solution:

Profit will be maximized when marginal profit is 0, or when

$$P'(x) = R'(x) - C'(x)$$

$$0 = R'(x) - C'(x)$$

$$R'(x) = C'(x)$$

Find revenue.

$$R(x) = x \cdot p$$

$$R(x) = x \cdot (2x - 250)$$

$$R(x) = 2x^2 - 250x$$

Then marginal revenue is $R'(x) = 4x - 250$. Using the cost function we've been given, find marginal cost.

$$C'(x) = 15$$

Then profit is maximized when

$$R'(x) = C'(x)$$

$$4x - 250 = 15$$

$$4x = 265$$

$$x = 66.25$$

The profit function is

$$P(x) = R(x) - C(x)$$

$$P(x) = 2x^2 - 250x - (15x + 300)$$

$$P(x) = 2x^2 - 250x - 15x - 300$$

$$P(x) = 2x^2 - 265x - 300$$

Let's determine whether profit is maximized at 66 or 67 units.

$$P(66) = 2(66)^2 - 265(66) - 300$$

$$P(66) = -8,478$$

or

$$P(67) = 2(67)^2 - 265(67) - 300$$

$$P(67) = -8,477$$

Even though profit is negative for both values (the company is currently losing money), profit is maximized (they lose the least amount of money), when they produce 67 units.

■ 6. A company manufactures and sells kids' toys. The total cost of producing x toys is $C(x) = -0.3x^2 + 25x + 975$, and demand is given by $p(x) = 12 + 3x$. Calculate the marginal profit from selling the 10th toy.

Solution:

Start by finding marginal cost. Since we need to find marginal profit from selling the 10th toy, we'll use $x = 9$ throughout.

$$C(x) = -0.3x^2 + 25x + 975$$

$$C'(x) = -0.6x + 25$$

$$C'(9) = -0.6(9) + 25$$

$$C'(9) = \$19.60$$

The marginal cost of producing the 10th toy is $\$19.60$.

Now find marginal revenue. Since $R(x) = x \cdot p$ and $p = 12 + 3x$, then $R(x) = 12x + 3x^2$. Therefore, $R'(x) = 12 + 6x$.

$$R'(9) = 12 + 6(9)$$

$$R'(9) = 12 + 54$$

$$R'(9) = \$66$$

The marginal revenue from selling the 10th toy is $\$66$.

Now find the marginal profit as the difference of these.

$$P'(x) = R'(x) - C'(x)$$

$$P'(9) = R'(9) - C'(9)$$

$$P'(9) = \$46.40$$

The marginal profit from selling the 10th toy is $\$46.40$.

W W W . K R I S T A K I N G M A T H . C O M