Calculus 1 Final Exam

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This exam is comprehensive over the entire course and includes 12 questions. You have 60 minutes to complete the exam.

The exam is worth 100 points. The 8 multiple choice questions are worth 5 points each (40 points total) and the 4 free response questions are worth 15 points each (60 points total).

Mark your multiple choice answers on this cover page. For the free response questions, show your work and make sure to circle your final answer.

1. (5 pts) A B C D E

2. (5 pts) A B C D E

3. (5 pts) A B C D E

4. (5 pts) A B C D E

5. (5 pts) A B C D E

6. (5 pts) A B C D E

7. (5 pts) A B C D E

8. (5 pts) A B C D E

1. (5 pts) Evaluate the limit of the composite function when $f(x) = 3x^2$ and $g(x) = x - 4$.

$$\lim_{x \to 3} f[g(x)]$$

A	-3		C	3		E	-4
B	2		D	4			

2. (5 pts) Find the points of discontinuity of the function.

$$f(x) = \begin{cases} x - 3 & x \leq -1 \\ \dfrac{1}{x} & -1 < x < 2 \\ \sqrt{x+2} & x > 2 \end{cases}$$

A	$x = 2$		C	$x = -1$		E	$x = -1, 2$
B	$x = 0, 2$		D	$x = -1, 0, 2$			

3. (5 pts) Does the polynomial function $x^3 + 2x - 1$ have a zero in the interval $[0,1]$?

A   Yes, there is a zero in the interval $[0,1]$

B   Yes, there are 3 zeros in the interval $[0,1]$

C   No, there is not a zero in the interval $[0,1]$

D   No, the function does not have any zeros

E   Cannot be determined

4. (5 pts) Find the derivative of $f(x) = \sqrt{4 - x^2}$.

A $\dfrac{1}{2\sqrt{4 - x^2}}$ C $\dfrac{-x}{\sqrt{4 - x^2}}$ E $\dfrac{-1}{2\sqrt{4 - x^2}}$

B $2 - x$ D $\dfrac{x}{\sqrt{4 - x^2}}$

5. (5 pts) Find all inflection points for the polynomial $f(x) = x^4 - 6x^3 + 12x^2$.

A	$(1,7)$ and $(2, -16)$		D	$(1,7)$ and $(2,16)$

B	$(2, -16)$		E	There are no inflection points

C	$(1,7)$

6. (5 pts) Find the limit.

$$\lim_{x \to \infty} \frac{e^x}{2x^3}$$

A	The limit does not exist		D	0

B	∞		E	1

C	$-\infty$

7. (5 pts) A ball is thrown straight up from the ground with initial velocity of $v_0 = 48$ ft/s. What is the ball's maximum height?

A	16 ft		C	48 ft		E	36 ft
B	72 ft		D	32 ft			

8. (5 pts) A coffee shop has a weekly revenue described by $R(x) = -0.2x^2 + 300x$, where x represents a single cup of coffee. How many cups need to be sold to maximize weekly revenue?

A	500		C	150		E	6,000
B	750		D	600			

9. (15 pts) Find the derivative of the function.

$$f(x) = 3x^2 \left(\frac{1}{x} + 2 \right)$$

10. (15 pts) Graph $f(x) = x^4 + 2x^3$. Your work must include the intervals on which $f(x)$ is increasing and decreasing, any local extrema, the intervals on which $f(x)$ is concave up and concave down, and any points of inflection. Your sketch should clearly reflect this information.

11. (15 pts) Air is being pumped into a spherical balloon so that its volume is increasing by 144 cm^3/s. How fast is the radius increasing when the radius is 12 cm?

12. (15 pts) A rancher plans to build a rectangular fenced area adjacent to an existing stone wall. The rancher wants the fence to enclose an area of $180{,}000$ square meters for his cows, but is low on fencing materials. What dimensions require the least amount of fencing?