Topic: Limit process to find area on [-a,a]

Question: Use the limit process to find the area of the region between $f(x) = 9 - x^2$ and the x-axis on the interval $[-3,3]$.

Answer choices:

A 92

B 18

C 54

D 36

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Solution: D

The function is even because $f(x) = f(-x)$. So instead of calculating area under the curve over the interval $[-3,3]$, we'll calculate area over $[0,3]$ and then double the result.

$$\Delta x = \frac{b-a}{n} = \frac{3-0}{n} = \frac{3}{n}$$

Find x_i by adding $i\Delta x$ to the left edge of the interval, 0.

$$x_i = 0 + i\Delta x = 0 + \frac{3i}{n} = \frac{3i}{n}$$

Set up the limit expression,

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} f\left(\frac{3i}{n}\right)\frac{3}{n}$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left[9 - \left(\frac{3i}{n}\right)^2\right]\frac{3}{n}$$

then simplify.

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{27}{n} - \frac{3}{n}\left(\frac{3i}{n}\right)^2$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{27}{n} - \frac{3}{n}\left(\frac{9i^2}{n^2}\right)$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{27}{n} - \frac{27i^2}{n^3}$$

$$\lim_{n\to\infty} \left[\sum_{i=1}^{n} \frac{27}{n} - \sum_{i=1}^{n} \frac{27i^2}{n^3} \right]$$

$$\lim_{n\to\infty} \left[\frac{27}{n} \sum_{i=1}^{n} 1 - \frac{27}{n^3} \sum_{i=1}^{n} i^2 \right]$$

For values of i, we know

$$\sum_{i=1}^{n} a = an$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

Use these equations to make substitutions.

$$\lim_{n\to\infty} \left[\frac{27}{n}(n) - \frac{27}{n^3} \left(\frac{n(n+1)(2n+1)}{6} \right) \right]$$

$$\lim_{n\to\infty} 27 - \frac{9(n+1)(2n+1)}{2n^2}$$

$$\lim_{n\to\infty} 27 - \frac{9(2n^2+3n+1)}{2n^2}$$

$$\lim_{n\to\infty} 27 - \frac{18n^2 + 27n + 9}{2n^2}$$

Split up the fraction,

$$\lim_{n\to\infty} 27 - \frac{18n^2}{2n^2} - \frac{27n}{2n^2} - \frac{9}{2n^2}$$

$$\lim_{n\to\infty} 27 - 9 - \frac{27}{2n} - \frac{9}{2n^2}$$

$$\lim_{n\to\infty} 18 - \frac{27}{2n} - \frac{9}{2n^2}$$

then evaluate the limit.

$$18 - 0 - 0$$

$$18$$

This is the area over the interval $[0,4]$, so the area over the interval $[-4,4]$ is double this value, which is 36.

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Topic: Limit process to find area on [-a,a]

Question: Use the limit process to find the area of the region between $f(x) = x^2 + 2$ and the x-axis on the interval $[-1,1]$.

Answer choices:

A $\frac{14}{3}$

B $\frac{2}{3}$

C $\frac{28}{3}$

D $\frac{8}{3}$

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Solution: A

The function is even because $f(x) = f(-x)$. So instead of calculating area under the curve over the interval $[-1,1]$, we'll calculate area over $[0,1]$ and then double the result.

$$\Delta x = \frac{b-a}{n} = \frac{1-0}{n} = \frac{1}{n}$$

Find x_i by adding $i\Delta x$ to the left edge of the interval, 0.

$$x_i = 0 + i\Delta x = 0 + \frac{1i}{n} = \frac{i}{n}$$

Set up the limit expression,

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} f\left(\frac{i}{n}\right)\frac{1}{n}$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(\frac{i}{n}\right)^2 + 2\right]\frac{1}{n}$$

then simplify.

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{i^2}{n^2}\right)\frac{1}{n} + \frac{2}{n}$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{i^2}{n^3} + \frac{2}{n}$$

$$\lim_{n\to\infty}\left[\sum_{i=1}^{n}\frac{i^2}{n^3}+\sum_{i=1}^{n}\frac{2}{n}\right]$$

$$\lim_{n\to\infty}\left[\frac{1}{n^3}\sum_{i=1}^{n}i^2+\frac{2}{n}\sum_{i=1}^{n}1\right]$$

For values of i, we know

$$\sum_{i=1}^{n}a=an$$

$$\sum_{i=1}^{n}i=\frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n}i^2=\frac{n(n+1)(2n+1)}{6}$$

Use these equations to make substitutions.

$$\lim_{n\to\infty}\frac{1}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)+\frac{2}{n}(n)$$

$$\lim_{n\to\infty}\frac{(n+1)(2n+1)}{6n^2}+2$$

$$\lim_{n\to\infty}\frac{2n^2+3n+1}{6n^2}+2$$

Split up the fraction,

$$\lim_{n\to\infty}\frac{2n^2}{6n^2}+\frac{3n}{6n^2}+\frac{1}{6n^2}+2$$

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$$\lim_{n\to\infty} \frac{1}{3} + \frac{1}{2n} + \frac{1}{6n^2} + 2$$

$$\lim_{n\to\infty} \frac{7}{3} + \frac{1}{2n} + \frac{1}{6n^2}$$

then evaluate the limit.

$$\frac{7}{3} + 0 + 0$$

$$\frac{7}{3}$$

This is the area over the interval $[0,1]$, so the area over the interval $[-1,1]$ is double this value, which is $14/3$.

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Topic: Limit process to find area on [-a,a]

Question: Use the limit process to find the area of the region between $f(x) = x^3 + 4$ and the x-axis on the interval $[-2,2]$.

Answer choices:

A 12

B 16

C 20

D 24

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Solution: B

First, find Δx.

$$\Delta x = \frac{b-a}{n} = \frac{2-(-2)}{n} = \frac{4}{n}$$

Find x_i by adding $i\Delta x$ to the left edge of the interval, 0.

$$x_i = -2 + i\Delta x = -2 + \frac{4i}{n}$$

Set up the limit expression,

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} f\left(-2+\frac{4i}{n}\right)\frac{4}{n}$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(\left(-2+\frac{4i}{n}\right)^3 + 4\right)\frac{4}{n}$$

then simplify.

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(-2+\frac{4i}{n}\right)^3 \frac{4}{n} + \frac{16}{n}$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(4-\frac{16i}{n}+\frac{16i^2}{n^2}\right)\left(-2+\frac{4i}{n}\right)\frac{4}{n} + \frac{16}{n}$$

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$$\lim_{n\to\infty}\sum_{i=1}^{n}\left(-8+\frac{16i}{n}+\frac{32i}{n}-\frac{64i^2}{n^2}-\frac{32i^2}{n^2}+\frac{64i^3}{n^3}\right)\frac{4}{n}+\frac{16}{n}$$

$$\lim_{n\to\infty}\sum_{i=1}^{n}-\frac{32}{n}+\frac{64i}{n^2}+\frac{128i}{n^2}-\frac{256i^2}{n^3}-\frac{128i^2}{n^3}+\frac{256i^3}{n^4}+\frac{16}{n}$$

$$\lim_{n\to\infty}\sum_{i=1}^{n}-\frac{16}{n}+\frac{192i}{n^2}-\frac{384i^2}{n^3}+\frac{256i^3}{n^4}$$

$$\lim_{n\to\infty}\left[\sum_{i=1}^{n}-\frac{16}{n}+\sum_{i=1}^{n}\frac{192i}{n^2}-\sum_{i=1}^{n}\frac{384i^2}{n^3}+\sum_{i=1}^{n}\frac{256i^3}{n^4}\right]$$

$$\lim_{n\to\infty}\left[-\frac{16}{n}\sum_{i=1}^{n}1+\frac{192}{n^2}\sum_{i=1}^{n}i-\frac{384}{n^3}\sum_{i=1}^{n}i^2+\frac{256}{n^4}\sum_{i=1}^{n}i^3\right]$$

For values of i, we know

$$\sum_{i=1}^{n}a=an$$

$$\sum_{i=1}^{n}i=\frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n}i^2=\frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n}i^3=\frac{n^2(n+1)^2}{4}$$

Use these equations to make substitutions.

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$$\lim_{n\to\infty}\left[-\frac{16}{n}(n)+\frac{192}{n^2}\left(\frac{n(n+1)}{2}\right)-\frac{384}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)\right.$$

$$\left.+\frac{256}{n^4}\left(\frac{n^2(n+1)^2}{4}\right)\right]$$

$$\lim_{n\to\infty}-16+\frac{96(n+1)}{n}-\frac{64(n+1)(2n+1)}{n^2}+\frac{64(n+1)^2}{n^2}$$

$$\lim_{n\to\infty}-16+\frac{96n+96}{n}-\frac{64(2n^2+3n+1)}{n^2}+\frac{64(n^2+2n+1)}{n^2}$$

$$\lim_{n\to\infty}-16+\frac{96n+96}{n}-\frac{128n^2+192n+64}{n^2}+\frac{64n^2+128n+64}{n^2}$$

Split up the fraction,

$$\lim_{n\to\infty}-16+\frac{96n}{n}+\frac{96}{n}-\frac{128n^2}{n^2}-\frac{192n}{n^2}-\frac{64}{n^2}+\frac{64n^2}{n^2}+\frac{128n}{n^2}+\frac{64}{n^2}$$

$$\lim_{n\to\infty}-16+96+\frac{96}{n}-128-\frac{192}{n}-\frac{64}{n^2}+64+\frac{128}{n}+\frac{64}{n^2}$$

then evaluate the limit.

$$-16+96+0-128-0-0+64+0+0$$

$$-16+96-128+64$$

$$16$$

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