Topic: Over and underestimation

Question: Use a Riemann sum to estimate the maximum area and minimum area under this curve on $[1,6]$. Use rectangular approximation methods with 5 equal subintervals.

$$f(x) = \frac{1}{8}x^2 - \frac{1}{4}x + \frac{1}{2}$$

Answer choices:

A Minimum of $\frac{48}{8}$ Maximum of $\frac{73}{8}$

B Minimum of $\frac{23}{4}$ Maximum of $\frac{37}{4}$

C Minimum of $\frac{45}{8}$ Maximum of $\frac{35}{4}$

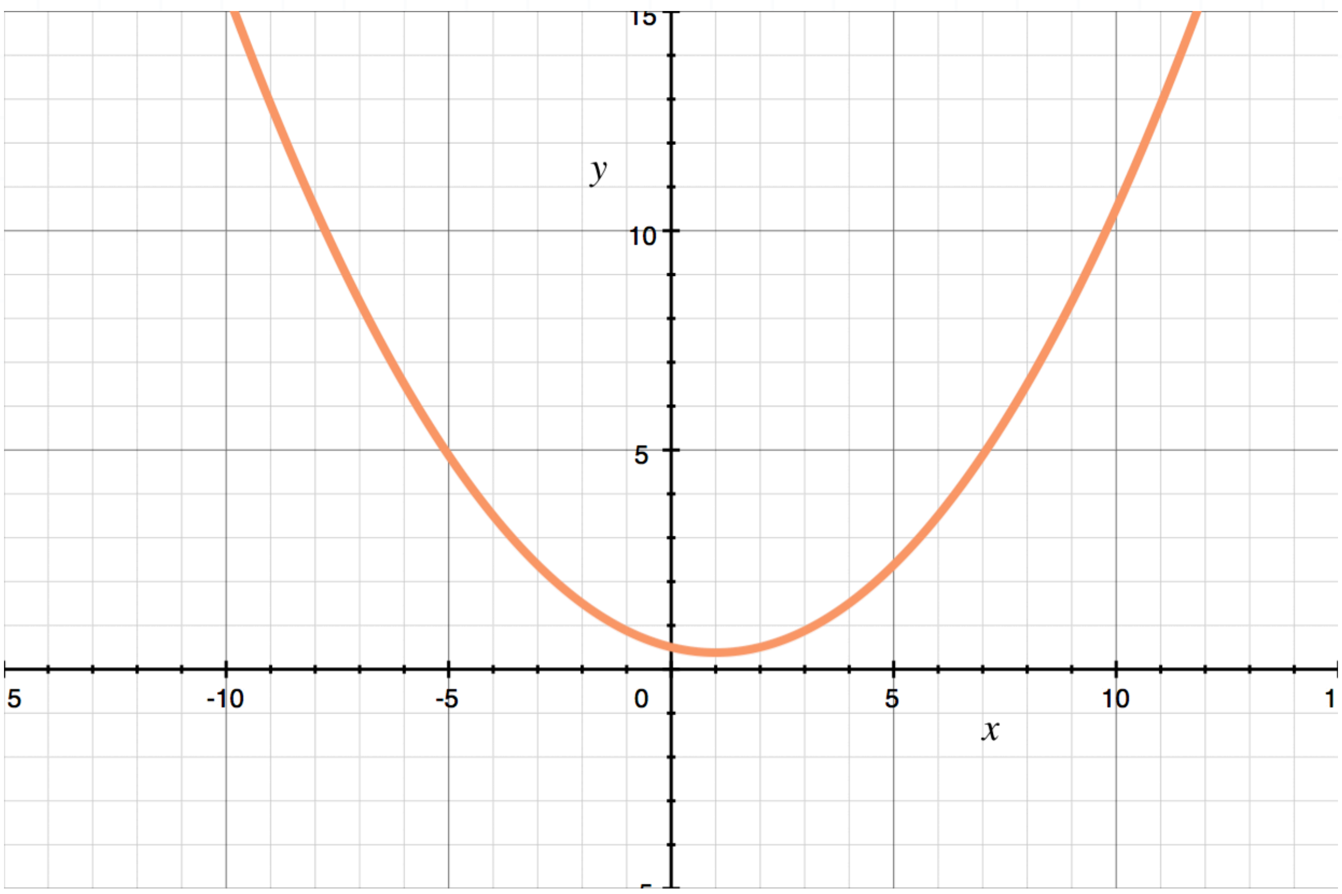
D Minimum of $\frac{35}{4}$ Maximum of $\frac{45}{8}$

Solution: C

The question asks us to estimate the minimum area and maximum area under this curve on the interval $[1,6]$, using rectangular approximation methods with 5 equal subintervals. We will use the left rectangular approximation method (LRAM) and the right rectangular approximation method (RRAM) to accomplish this task.

$$f(x) = \frac{1}{8}x^2 - \frac{1}{4}x + \frac{1}{2}$$

A graph of $f(x)$ is shown below.



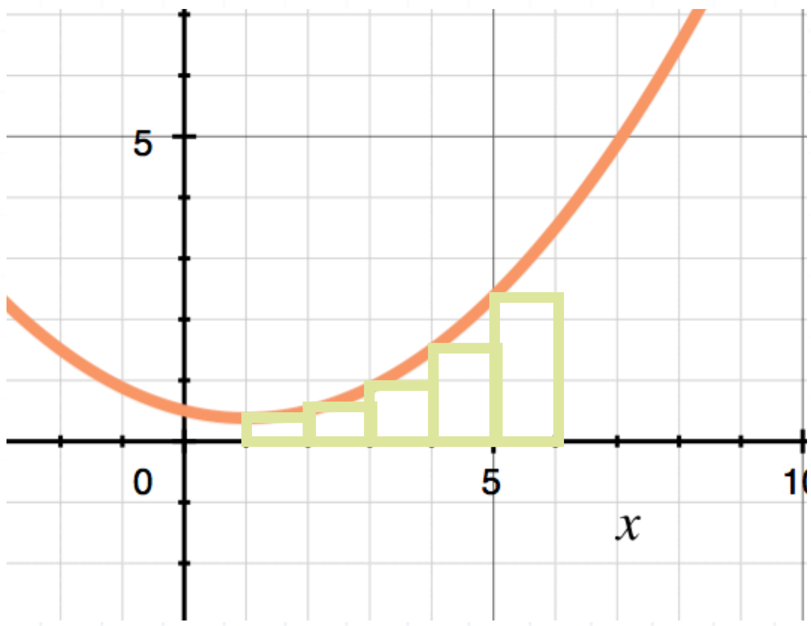
A quick observation of the graph shows that $f(x)$ is an increasing function everywhere on the interval $[1,6]$.

The term rectangular approximation method means we will approximate the area under the curve using rectangles. We calculate the area of each

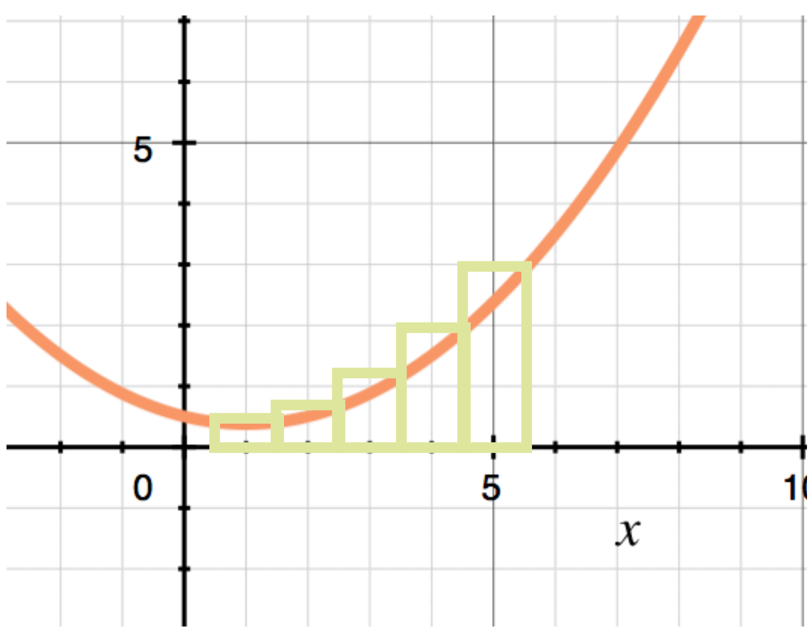
rectangle by multiplying the height of the rectangle (the function value) times the width of the rectangle (the length of the subinterval).

In the LRAM, the height of each rectangle is the function value at the x-value at the left end of each subinterval. In the RRAM, the height of each rectangle is the function value at the x-value at the right end of each subinterval. Since the function is consistently increasing, the function values will be lower in the LRAM than in the RRAM.

Therefore, the LRAM will underestimate the area under the curve,



and the RRAM will overestimate the area under the curve.



Now let's calculate the heights of the endpoints of each subinterval. We begin with the interval $[1,6]$ and make 5 equal subintervals. Thus, we will calculate the value of $f(x)$ at $x = 1,2,3,4,5,$ and 6. The values are in the table below, followed by the work.

x	1	2	3	4	5	6
$f(x)$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{7}{8}$	$\frac{3}{2}$	$\frac{19}{8}$	$\frac{7}{2}$

$$f(1) = \frac{1}{8}(1)^2 - \frac{1}{4}(1) + \frac{1}{2} = \frac{1}{8} - \frac{1}{4} + \frac{1}{2} = \frac{3}{8}$$

$$f(2) = \frac{1}{8}(2)^2 - \frac{1}{4}(2) + \frac{1}{2} = \frac{4}{8} - \frac{2}{4} + \frac{1}{2} = \frac{1}{2}$$

$$f(3) = \frac{1}{8}(3)^2 - \frac{1}{4}(3) + \frac{1}{2} = \frac{9}{8} - \frac{3}{4} + \frac{1}{2} = \frac{7}{8}$$

$$f(4) = \frac{1}{8}(4)^2 - \frac{1}{4}(4) + \frac{1}{2} = \frac{16}{8} - \frac{4}{4} + \frac{1}{2} = \frac{3}{2}$$

$$f(5) = \frac{1}{8}(5)^2 - \frac{1}{4}(5) + \frac{1}{2} = \frac{25}{8} - \frac{5}{4} + \frac{1}{2} = \frac{19}{8}$$

$$f(6) = \frac{1}{8}(6)^2 - \frac{1}{4}(6) + \frac{1}{2} = \frac{36}{8} - \frac{6}{4} + \frac{1}{2} = \frac{7}{2}$$

The LRAM uses the function values at the left endpoints of each subinterval, at $x = 1, 2, 3, 4,$ and 5. The width of each subinterval is 1, so the area of each rectangle is the function value. The area under the curve is the sum of the areas of the five rectangles.

$$LRAM = \frac{3}{8} + \frac{1}{2} + \frac{7}{8} + \frac{3}{2} + \frac{19}{8} = \frac{45}{8}$$

The RRAM uses the function values at the right endpoints of each subinterval, at $x = 2, 3, 4, 5,$ and 6. The width of each subinterval is still 1, so the area of each rectangle is the function value. The area under the curve is the sum of the areas of the five rectangles.

$$RRAM = \frac{1}{2} + \frac{7}{8} + \frac{3}{2} + \frac{19}{8} + \frac{7}{2} = \frac{35}{4}$$

Now that we know the minimum area under the curve is $45/8$ and the maximum area under the curve is $35/4$.

Topic: Over and underestimation

Question: Use a Riemann sum to estimate the maximum area and minimum area under the curve on $[0,10]$, using 5 equal subintervals.

$$g(x) = 3(0.85)^x$$

Answer choices:

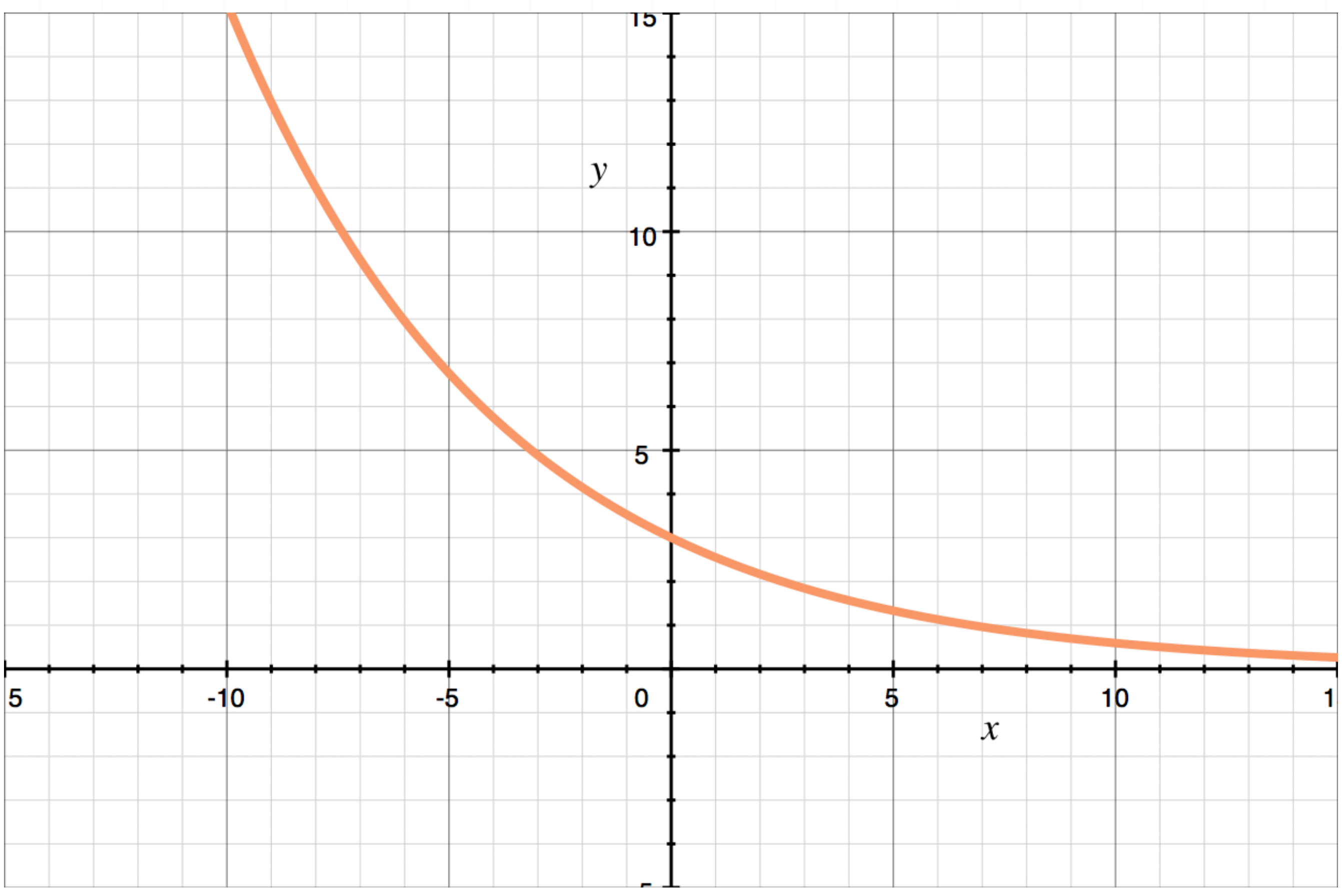
A	Minimum of 17.365	Maximum of 18.546
B	Minimum of 12.546	Maximum of 17.365
C	Minimum of 8.683	Maximum of 9.273
D	Minimum of 6.273	Maximum of 12.273

Solution: B

The question asks us to estimate the minimum area and maximum area under this curve on the interval $[0,10]$, using rectangular approximation methods with 5 equal subintervals. We will use the left rectangular approximation method (LRAM) and the right rectangular approximation method (RRAM) to accomplish this task.

$$g(x) = 3(0.85)^x$$

A graph of $g(x)$ is shown below.



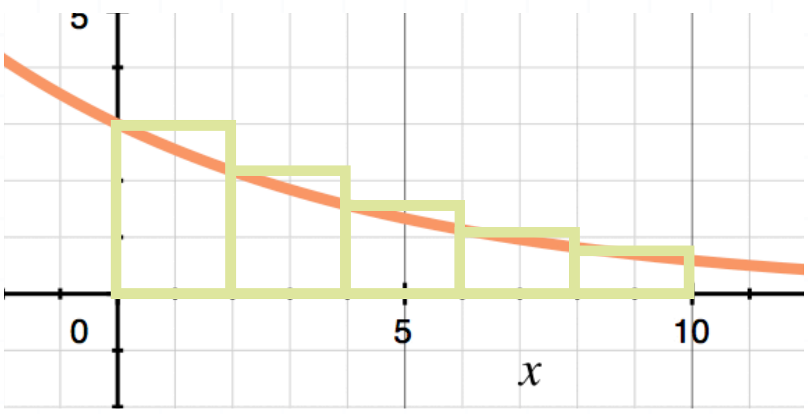
A quick observation of the graph shows that $g(x)$ is a decreasing function everywhere on the interval $[0,10]$.

The term rectangular approximation method means we will approximate the area under the curve using rectangles. We calculate the area of each

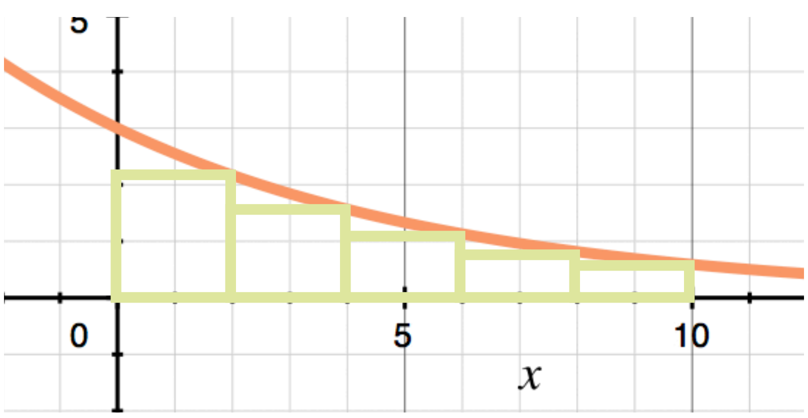
rectangle by multiplying the height of the rectangle (the function value) times the width of the rectangle (the length of the subinterval).

In the LRAM, the height of each rectangle is the function value at the x-value at the left end of each subinterval. In the RRAM, the height of each rectangle is the function value at the x-value at the right end of each subinterval. Since the function is consistently decreasing, the function values will be higher in the LRAM than in the RRAM.

Therefore, the LRAM will overestimate the area under the curve,



and the RRAM will underestimate the area under the curve.



Now we'll calculate the height of the function at the endpoints of each subinterval. When we divide $[0,10]$ into 5 equal subintervals, the endpoints of the subintervals are at $x=0,\ 2,\ 4,\ 6,\ 8,$ and 10, and the height of the function at each endpoint is

$$g(0) = 3(0.85)^0 = 3(1) = 3$$

$$g(2) = 3(0.85)^2 = 3(0.7225) = 2.1675$$

$$g(4) = 3(0.85)^4 = 3(0.5220) = 1.5660$$

$$g(6) = 3(0.85)^6 = 3(0.37715) = 1.1314$$

$$g(8) = 3(0.85)^8 = 3(0.27249) = 0.8175$$

$$g(10) = 3(0.85)^{10} = 3(0.19687) = 0.5906$$

The LRAM uses the function values at the left endpoints of each subinterval, at $x = 0, 2, 4, 6,$ and 8. The width of each subinterval is 2, so the area of each rectangle is two times of the function's value. The area under the curve is the sum of the areas of the five rectangles.

$$LRAM = 2(3) + 2(2.1675) + 2(1.5660) + 2(1.1314) + 2(0.8175) = 17.365$$

The RRAM uses the function values at the right endpoints of each subinterval, at $x = 2, 4, 6, 8,$ and 10. The width of each subinterval is still 2, so the area of each rectangle is two times of the function's value. The area under the curve is the sum of the areas of the five rectangles.

$$RRAM = 2(2.1675) + 2(1.5660) + 2(1.1314) + 2(0.8175) + 2(0.5906) = 12.546$$

Now that we know the minimum area under the curve is 12.546 and the maximum area under the curve is 17.365.

Topic: Over and underestimation

Question: Use a Riemann sum to estimate the maximum area and minimum area under this curve on $[0,64]$. Use rectangular approximation methods with 8 unequal subintervals, beginning and ending at x-values that are perfect square numbers.

$$h(x) = 2\sqrt{x}$$

Answer choices:

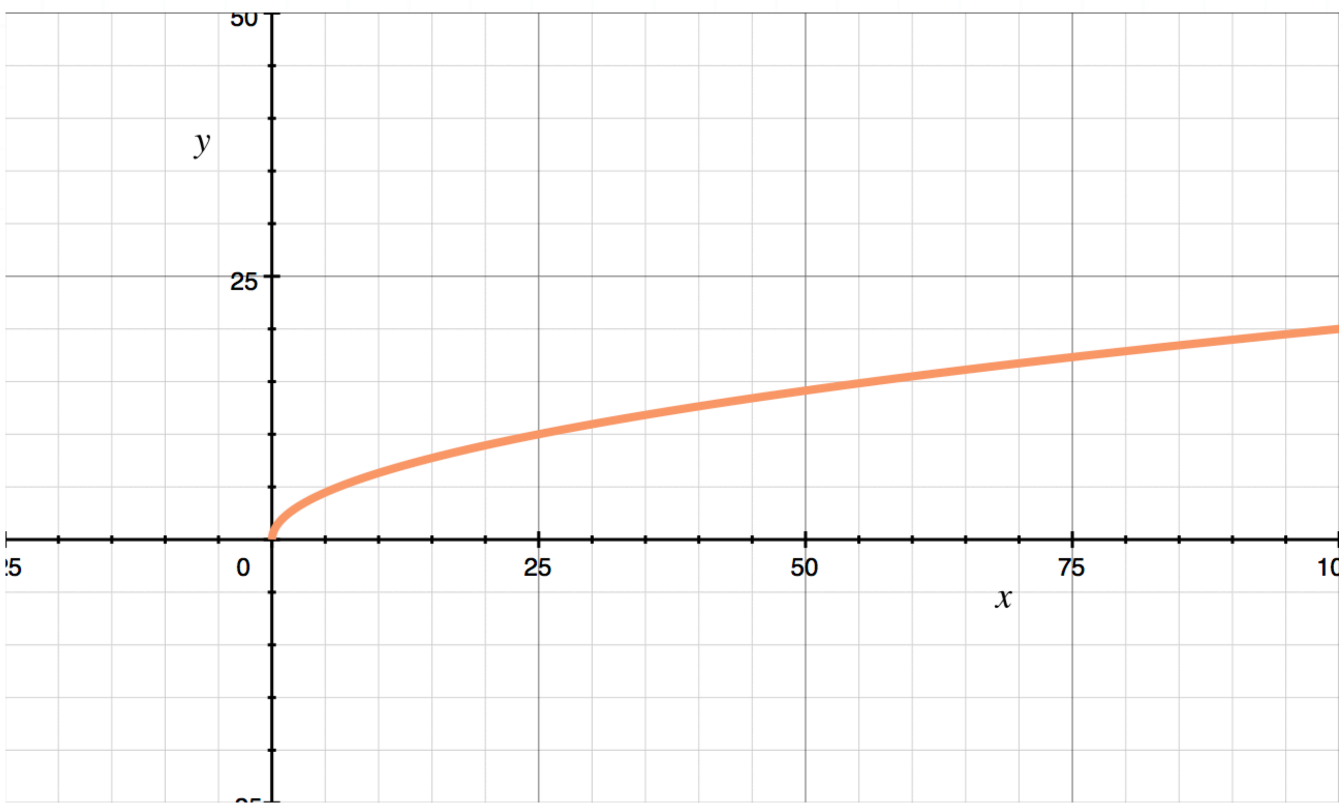
A	Minimum of 299	Maximum of 373
B	Minimum of 305	Maximum of 373
C	Minimum of 599	Maximum of 745
D	Minimum of 616	Maximum of 744

Solution: D

The question asks us to estimate the minimum area and maximum area under this curve on the interval $[0,64]$, using rectangular approximation methods with 8 unequal subintervals. The intervals are to begin and end with perfect square numbers. We will use the Left Rectangular Approximation Method (LRAM) and the Right Rectangular Approximation Method (RRAM) to accomplish this task.

$$h(x) = 2\sqrt{x}$$

A graph of $h(x)$ is shown below.



A quick observation of the graph shows that $h(x)$ is an increasing function everywhere on the interval $[0,64]$.

The term rectangular approximation method means we will approximate the area under the curve using rectangles. We calculate the area of each

rectangle by multiplying the height of the rectangle (the function value) times the width of the rectangle (the length of the subinterval).

In the LRAM, the height of each rectangle is the function value at the x -value at the left end of each subinterval. In the RRAM, the height of each rectangle is the function value at the x-value at the right end of each subinterval. Since the function is consistently increasing, the function values will be lower in the LRAM than in the RRAM.

Therefore, the LRAM will underestimate the area under the curve, and the RRAM will overestimate the area under the curve.

Now let's calculate the heights of the endpoints of each subinterval. We begin with the interval $[0,64]$ and make 8 unequal subintervals that begin and end with perfect square numbers. Thus, we will calculate the value of $h(x)$ at $x=0$, 1, 4, 9, 16, 25, 36, 49 and 64. The values are in the table below.

x	0	1	4	9	16	25	36	49	64	
$\sqrt{x}$	0	1	2	3	4	5	6	7	8	
$h(x)$	0	2	4	6	8		10	12	14	16

Next, we'll calculate the area of the rectangle in each subdivision, using the left endpoint of each subdivision (LRAM). Each subinterval is identified as $[a,b]$. The height of the rectangle is $h(a)$ at the appropriate endpoint of the subinterval. The width of the subinterval is $b-a$, the upper endpoint minus the lower endpoint. The area of the rectangle is the product of the height and the width, $h(a)(b-a)$, in each subinterval. The figures are shown in the table below.

$[a, b]$	$[0,1]$	$[1,4]$	$[4,9]$	$[9,16]$	$[16,25]$	$[25,36]$	$[36,49]$	$[49,64]$
$h(a)$	0	2	4	6	8	10	12	14
$b-a$	1	3	5	7	9	11	13	15
$h(a)(b-a)$	0	6	20	42	72	110	156	210

Since $h(x)$ is strictly increasing on $[0,64]$, the minimum area under the curve is the sum of the areas of the 8 rectangles.

$$LRAM = 0 + 6 + 20 + 42 + 72 + 110 + 156 + 210 = 616$$

Next, we'll calculate the area of the rectangle in each subdivision, using the right endpoint of each subdivision (RRAM). Each subinterval is identified as $[a, b]$. The height of the rectangle is $h(b)$ at the appropriate endpoint of the subinterval. The width of the subinterval is $b - a$, the upper endpoint minus the lower endpoint. The area of the rectangle is the product of the height and the width, $h(b)(b - a)$, in each subinterval. The figures are shown in the table below.

$[a, b]$	$[0,1]$	$[1,4]$	$[4,9]$	$[9,16]$	$[16,25]$	$[25,36]$	$[36,49]$	$[49,64]$
$h(b)$	2	4	6	8	10	12	14	16
$b-a$	1	3	5	7	9	11	13	15
$h(b)(b-a)$	2	12	30	56	90	132	182	240

Since $h(x)$ is strictly increasing on $[0,64]$, the maximum area under the curve is the sum of the areas of the 8 rectangles.

$$LRAM = 2 + 12 + 30 + 56 + 90 + 132 + 182 + 240 = 744$$

Now we that know the minimum area under the curve is 616 and the maximum area under the curve is 744.