

Topic: Repeated linear factors

Question: Rewrite the integral using partial fractions. Do not solve it.

$$\int \frac{x+2}{(x-1)^2} dx$$

Answer choices:

A $\int \frac{1}{(x+1)^2} + \frac{3}{(x-1)} dx$

B $\int \frac{3}{(x-1)^2} + \frac{1}{(x-1)} dx$

C $\int \frac{3}{(x-1)^2} - \frac{1}{(x-1)} dx$

D $\int \frac{1}{(x-1)^2} - \frac{3}{(x-1)} dx$



Solution: B

First, factor the denominator. Since we have a repeated factor, we need to include all factors of a lesser degree.

$$\int \frac{x+2}{(x-1)^2} dx = \int \frac{A}{(x-1)^2} + \frac{B}{(x-1)} dx$$

Using partial fractions decomposition containing a quadratic factor, we have

$$\frac{x+2}{(x-1)^2} = \frac{A}{(x-1)^2} + \frac{B}{(x-1)}$$

Now we'll solve for constants.

$$\frac{(x+2)(x-1)^2}{(x-1)^2} = \frac{A(x-1)^2}{(x-1)^2} + \frac{B(x-1)^2}{(x-1)}$$

$$x+2 = A + B(x-1)$$

$$x+2 = A + Bx - B$$

$$x+2 = Bx + A - B$$

$$x+2 = Bx + (A - B)$$

Equating coefficients on both sides, we get

$$\text{[1]} \quad B = 1$$

$$\text{[2]} \quad A - B = 2$$

We already know the value of B . Plugging [1] into [2] to solve for A , we get



$$A - 1 = 2$$

$$A = 3$$

Plugging the values for both constants back into the partial fractions decomposition, and putting the decomposition back into the integral, we get

$$\int \frac{x+2}{(x-1)^2} dx = \int \frac{3}{(x-1)^2} + \frac{1}{(x-1)} dx$$



Topic: Repeated linear factors

Question: Rewrite the integral using partial fractions. Do not solve it.

$$\int \frac{4x^2 + 10x + 8}{(x + 1)^3} dx$$

Answer choices:

A $\int \frac{2}{(x + 1)} + \frac{2}{(x + 1)^2} + \frac{4}{(x + 1)^3} dx$

B $\int \frac{2}{(x + 1)^3} + \frac{3}{(x + 1)^2} + \frac{4}{(x + 1)} dx$

C $\int \frac{2}{(x + 1)^3} - \frac{2}{(x + 1)^2} + \frac{4}{(x + 1)} dx$

D $\int \frac{2}{(x + 1)^3} + \frac{2}{(x + 1)^2} + \frac{4}{(x + 1)} dx$



Solution: D

First, factor the denominator. Since we have a repeated factor, we need to include all factors of a lesser degree.

$$\int \frac{4x^2 + 10x + 8}{(x + 1)^3} dx = \int \frac{A}{(x + 1)^3} + \frac{B}{(x + 1)^2} + \frac{C}{(x + 1)} dx$$

Using partial fractions decomposition containing a quadratic factor, we have

$$\frac{4x^2 + 10x + 8}{(x + 1)^3} = \frac{A}{(x + 1)^3} + \frac{B}{(x + 1)^2} + \frac{C}{(x + 1)}$$

Now we'll solve for constants.

$$\frac{(4x^2 + 10x + 8)(x + 1)^3}{(x + 1)^3} = \frac{A(x + 1)^3}{(x + 1)^3} + \frac{B(x + 1)^3}{(x + 1)^2} + \frac{C(x + 1)^3}{(x + 1)}$$

$$4x^2 + 10x + 8 = A + B(x + 1) + C(x + 1)^2$$

$$4x^2 + 10x + 8 = A + Bx + B + C(x^2 + 2x + 1)$$

$$4x^2 + 10x + 8 = A + Bx + B + Cx^2 + 2Cx + C$$

$$4x^2 + 10x + 8 = Cx^2 + Bx + 2Cx + A + B + C$$

$$4x^2 + 10x + 8 = Cx^2 + (B + 2C)x + (A + B + C)$$

Equating coefficients on both sides, we get

$$\text{[1]} \quad C = 4$$

$$\text{[2]} \quad B + 2C = 10$$



$$\text{[3]} \quad A + B + C = 8$$

We already know the value of C . Plugging [1] into [2] to solve for B , we get

$$B + 2(4) = 10$$

$$B = 2$$

Plugging the values for B and C into [3] to solve for A , we get

$$A + 2 + 4 = 8$$

$$A = 2$$

Plugging the values for each of the three constants back into the partial fractions decomposition, and putting the decomposition back into the integral, we get

$$\int \frac{4x^2 + 10x + 8}{(x + 1)^3} dx = \int \frac{2}{(x + 1)^3} + \frac{2}{(x + 1)^2} + \frac{4}{(x + 1)} dx$$



Topic: Repeated linear factors

Question: Rewrite the integral using partial fractions. Do not solve it.

$$\int \frac{x^3 - 2x^2 + 2x - 3}{(x - 2)^4} dx$$

Answer choices:

A $\int \frac{1}{(x - 2)^4} + \frac{4}{(x - 2)^3} + \frac{4}{(x - 2)^2} + \frac{1}{(x - 2)} dx$

B $\int \frac{1}{(x - 2)^4} - \frac{6}{(x - 2)^3} + \frac{4}{(x - 2)^2} - \frac{1}{(x - 2)} dx$

C $\int \frac{1}{(x - 2)^4} + \frac{6}{(x - 2)^3} + \frac{4}{(x - 2)^2} + \frac{1}{(x - 2)} dx$

D $\int \frac{1}{(x - 2)} + \frac{6}{(x - 2)^2} + \frac{4}{(x - 2)^3} + \frac{1}{(x - 2)^4} dx$



Solution: C

First, factor the denominator. Since we have a repeated factor, we need to include all factors of a lesser degree.

$$\int \frac{x^3 - 2x^2 + 2x - 3}{(x - 2)^4} dx = \int \frac{A}{(x - 2)^4} + \frac{B}{(x - 2)^3} + \frac{C}{(x - 2)^2} + \frac{D}{(x - 2)} dx$$

Using partial fractions decomposition containing a quadratic factor, we have

$$\frac{x^3 - 2x^2 + 2x - 3}{(x - 2)^4} = \frac{A}{(x - 2)^4} + \frac{B}{(x - 2)^3} + \frac{C}{(x - 2)^2} + \frac{D}{(x - 2)}$$

Now we'll solve for constants.

$$\frac{(x^3 - 2x^2 + 2x - 3)(x - 2)^4}{(x - 2)^4} = \frac{A(x - 2)^4}{(x - 2)^4} + \frac{B(x - 2)^4}{(x - 2)^3} + \frac{C(x - 2)^4}{(x - 2)^2} + \frac{D(x - 2)^4}{(x - 2)}$$

$$x^3 - 2x^2 + 2x - 3 = A + B(x - 2) + C(x - 2)^2 + D(x - 2)^3$$

$$x^3 - 2x^2 + 2x - 3 = A + Bx - 2B + C(x^2 - 4x + 4) + D(x^3 - 6x^2 + 12x - 8)$$

$$x^3 - 2x^2 + 2x - 3 = A + Bx - 2B + Cx^2 - 4Cx + 4C + Dx^3 - 6Dx^2 + 12Dx - 8D$$

$$x^3 - 2x^2 + 2x - 3 = Dx^3 + Cx^2 - 6Dx^2 + Bx - 4Cx + 12Dx + A - 2B + 4C - 8D$$

$$x^3 - 2x^2 + 2x - 3 = Dx^3 + (C - 6D)x^2 + (B - 4C + 12D)x + (A - 2B + 4C - 8D)$$

Equating coefficients on both sides, we get

[1] $D = 1$

[2] $C - 6D = -2$



$$\text{[3]} \quad B - 4C + 12D = 2$$

$$\text{[4]} \quad A - 2B + 4C - 8D = -3$$

We already know the value of D . Plugging [1] into [2] to solve for C , we get

$$C - 6(1) = -2$$

$$C = 4$$

Plugging the values for C and D into [3] to solve for B , we get

$$B - 4(4) + 12(1) = 2$$

$$B = 6$$

Plugging the values for B , C and D into [4] to solve for A , we get

$$A - 2(6) + 4(4) - 8(1) = -3$$

$$A = 1$$

Plugging the values for each of the four constants back into the partial fractions decomposition, and putting the decomposition back into the integral, we get

$$\int \frac{x^3 - 2x^2 + 2x - 3}{(x - 2)^4} dx = \int \frac{1}{(x - 2)^4} + \frac{6}{(x - 2)^3} + \frac{4}{(x - 2)^2} + \frac{1}{(x - 2)} dx$$

