p-series test

If we have a series a_n in the form

$$a_n = \sum_{n=1}^{\infty} \frac{1}{n^p}$$

then we can use the p-series test for convergence to say whether or not a_n will converge. The p-series test says that

a_n will converge when $p > 1$

a_n will diverge when $p \leq 1$

The key is to make sure that the given series matches the format above for a p-series, and then to look at the value of p to determine convergence.

Example

Use the p-series test to say whether or not the series converges.

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$$

In order to use the p-series test, we need to make sure the format of the given series matches the format above for a p-series, so we'll rewrite the given series as

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} = \sum_{n=1}^{\infty} \frac{1}{n^{\frac{1}{2}}}$$

In this format, we can see that $p = 1/2$. The p-series test tells us that a_n diverges when $p \leq 1$, so we can say that this series diverges.

Let's try a second example.

Example

Use the p-series test to say whether or not the series converges.

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^4}}$$

In order to use the p-series test, we need to make sure the format of the given series matches the format above for a p-series, so we'll rewrite the given series as

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^4}} = \sum_{n=1}^{\infty} \frac{1}{\left(n^4\right)^{\frac{1}{3}}}$$

$$\sum_{n=1}^{\infty} \frac{1}{n^{\frac{4}{3}}}$$

In this format, we can see that $p = 4/3$. The p-series test tells us that a_n converges when $p > 1$, so we can say that this series converges.