Topic: Observer and the airplane

Question: An airplane is flying horizontally at 720 miles/hr, 3 miles above the ground, when it passes an observer on the ground. How fast is the distance between the person and the plane increasing 10 seconds later?

Answer choices:

A About 400 miles/hr

B About 500 miles/hr

C About 600 miles/hr

D About 700 miles/hr

Solution: A

The horizontal path of the plane, the vertical distance between the observer and that path, and the diagonal connecting the observer to the plane's location, together form a right triangle.

So we'll use the Pythagorean theorem, which relates the three side lengths of a right triangle.

$$a^2 + b^2 = c^2$$

Use implicit differentiation to take the derivative of both sides.

$$2a\frac{da}{dt} + 2b\frac{db}{dt} = 2c\frac{dc}{dt}$$

We'll call a the vertical distance, b the horizontal distance, and c the diagonal distance. We know from the question that $a = 3$. And because a stays constant, $da/dt = 0$.

$$2(3)(0) + 2b\frac{db}{dt} = 2c\frac{dc}{dt}$$

$$2b\frac{db}{dt} = 2c\frac{dc}{dt}$$

$$b\frac{db}{dt} = c\frac{dc}{dt}$$

Convert $t = 10$ seconds to hours,

$$x \text{ hours} = 10 \text{ seconds} \times \frac{1 \text{ minute}}{60 \text{ seconds}} \times \frac{1 \text{ hour}}{60 \text{ minutes}}$$

$$x \text{ hours} = \frac{1}{360} \text{ hours}$$

then use it to find the horizontal distance b.

$$b \text{ miles} = \frac{1}{360} \text{ hours} \times \frac{720 \text{ miles}}{\text{hour}}$$

$$b \text{ miles} = 2 \text{ miles}$$

With $a = 3$ and $b = 2$, we can find c.

$$a^2 + b^2 = c^2$$

$$3^2 + 2^2 = c^2$$

$$9 + 4 = c^2$$

$$13 = c^2$$

$$c \approx 3.61$$

Substitute $b = 2$ and $c = 3.61$, along with $db/dt = 720$, into the derivative. This will give us the rate at which the distance between the observer and the airplane is increasing.

$$b\frac{db}{dt} = c\frac{dc}{dt}$$

$$2(720) = (3.61)\ \frac{dc}{dt}$$

$$\frac{dc}{dt} \approx 399$$

Topic: Observer and the airplane

Question: Car A starts 50 miles directly west of a car B and begins moving east at 75 mph. At the same moment, car B begins moving south at 90 mph. At what rate is the distance between the cars changing after 20 minutes?

Answer choices:

A About 39 miles/hr

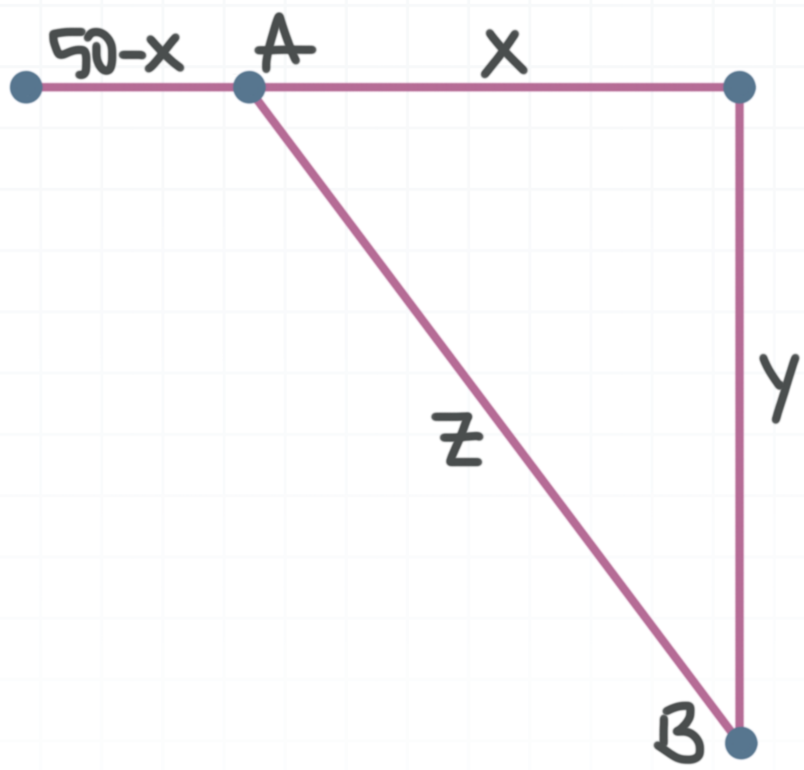
B About 21 miles/hr

C About 18 miles/hr

D About 58 miles/hr

Solution: B

Draw a diagram.



We'll use the Pythagorean Theorem, which relates the three side lengths of a right triangle.

$$x^2 + y^2 = z^2$$

Use implicit differentiation to take the derivative of both sides.

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$$

$$x\frac{dx}{dt} + y\frac{dy}{dt} = z\frac{dz}{dt}$$

Convert $t = 20$ minutes to hours.

$$x \text{ hours} = 20 \text{ minutes} \times \frac{1 \text{ minute}}{60 \text{ seconds}}$$

$$x \text{ hours} = \frac{1}{3} \text{ hours}$$

If $t = 1/3$ hr, then car A has traveled east,

$$\frac{1}{3} \text{ hours} \times \frac{75 \text{ miles}}{\text{hour}} = 25 \text{ miles}$$

$$x = 50 - 25 = 25 \text{ miles}$$

and car B has traveled south

$$\frac{1}{3} \text{ hours} \times \frac{90 \text{ miles}}{\text{hour}} = 30 \text{ miles}$$

With $x = 25$ and $y = 30$, we can find z.

$$x^2 + y^2 = z^2$$

$$30^2 + 25^2 = z^2$$

$$1{,}525 = z^2$$

$$z \approx 39$$

Substitute what we know into the derivative, then solve for dz/dt.

$$25(-75) + 30(90) = 39\frac{dz}{dt}$$

$$\frac{dz}{dt} = \frac{25(-75) + 30(90)}{39}$$

$$\frac{dz}{dt} \approx 21$$

Topic: Observer and the airplane

Question: A truck is 40 miles north of an intersection, traveling toward the intersection at 35 mph. At the same time, another car is 30 miles west of the intersection, traveling away from the intersection at 45 mph. Is the distance between the vehicles increasing or decreasing at that moment and at what rate?

Answer choices:

A Decreasing at a rate of 55 mph

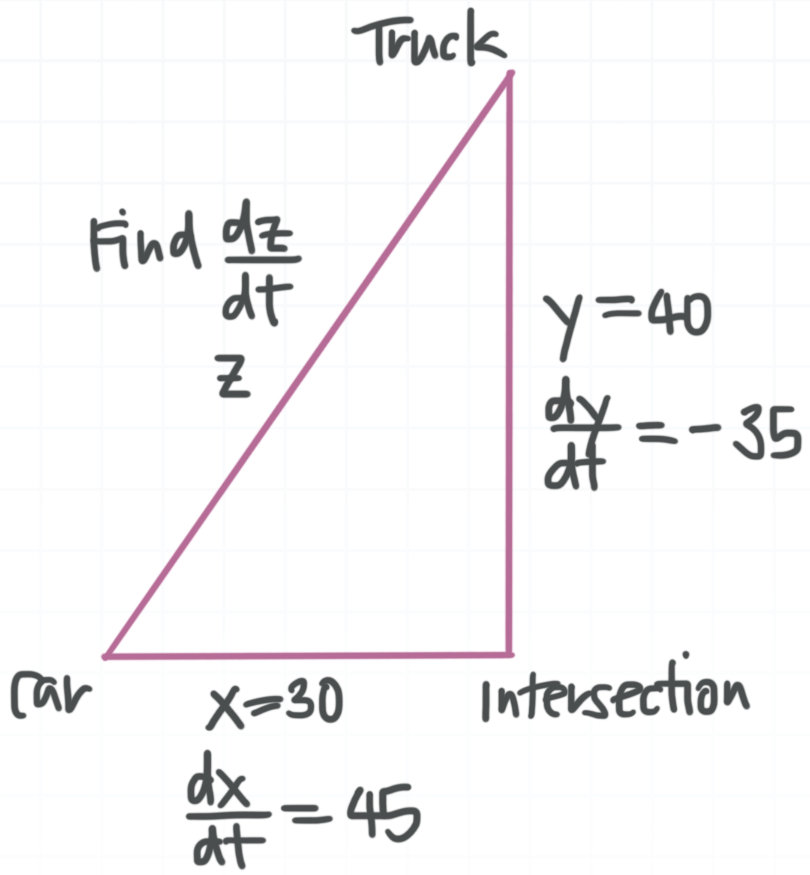
B Decreasing at a rate of 1 mph

C Increasing at a rate of 55 mph

D Increasing at a rate of 1 mph

Solution: B

Draw a diagram.



Use the Pythagorean Theorem $x^2+y^2=z^2$, then differentiate with respect to time.

$$2x\frac{dx}{dt}+2y\frac{dy}{dt}=2z\frac{dz}{dt}$$

$$x\frac{dx}{dt}+y\frac{dy}{dt}=z\frac{dz}{dt}$$

We can use the Pythagorean Theorem to find the distance between the truck and the car, z, when $x=30$ and $y=40$.

$$x^2+y^2=z^2$$

$$30^2+40^2=z^2$$

$$900 + 1{,}600 = z^2$$

$$2{,}500 = z^2$$

$$z = 50$$

Substitute what we know into the derivative, then solve for dz/dt.

$$30(45) + 40(-35) = 50\frac{dz}{dt}$$

$$\frac{dz}{dt} = \frac{30(45) + 40(-35)}{50}$$

$$\frac{dz}{dt} = -\frac{50}{50} = -1$$

The distance between the two vehicles is decreasing at a rate of 1 mph.