Calculus 2 Workbook Solutions

Hyperbolic integrals

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INVERSE HYPERBOLIC INTEGRALS

1. Evaluate the hyperbolic integral.

$$\int x \sinh(3x^2+7)\ dx$$

Solution:

Use u-substitution.

$$u = 3x^2+7$$

$$\frac{du}{dx} = 6x, \text{ so } du = 6x\ dx, \text{ so } dx = \frac{du}{6x}$$

Substitute.

$$\int x \sinh u \cdot \frac{du}{6x}$$

$$\frac{1}{6}\int \sinh u\ du$$

Integrate and back-substitute.

$$\frac{1}{6}\cosh u + C$$

$$\frac{1}{6}\cosh(3x^2+7) + C$$

■ 2. Evaluate the hyperbolic integral using the substitution $x = \sqrt{3}\cosh u$.

$$\int \frac{\sqrt{x^2 - 3}}{x^2}\ dx$$

Solution:

Starting with the substitution $x = \sqrt{3}\cosh u$, we get

$$x = \sqrt{3}\cosh u$$

$$x^2 = 3\cosh^2 u$$

$$x^2 - 3 = 3\cosh^2 u - 3$$

$$\sqrt{x^2 - 3} = \sqrt{3\cosh^2 u - 3}$$

$$\sqrt{x^2 - 3} = \sqrt{3(\cosh^2 u - 1)}$$

$$\sqrt{x^2 - 3} = \sqrt{3}\sqrt{\cosh^2 u - 1}$$

Using the substitution $\sinh^2 u = \cosh^2 u - 1$ gives

$$\sqrt{x^2 - 3} = \sqrt{3}\sqrt{\sinh^2 u}$$

$$\sqrt{x^2 - 3} = \sqrt{3}\sinh u$$

And

$$dx = \sqrt{3}\sinh u \ du$$

Substituting into the integral gives

$$\int \frac{\sqrt{3}\sinh u}{\left(\sqrt{3}\cosh u\right)^2} \cdot \sqrt{3}\sinh u \ du$$

$$\int \frac{3\sinh u}{3\cosh^2 u} \cdot \sinh u \ du$$

$$\int \frac{\sinh^2 u}{\cosh^2 u} \ du$$

Substitute using $\sinh^2 u = \cosh^2 u - 1$.

$$\int \frac{\cosh^2 u - 1}{\cosh^2 u} \ du$$

$$\int \frac{\cosh^2 u}{\cosh^2 u} - \frac{1}{\cosh^2 u} \ du$$

$$\int 1 - \text{sech}^2 u \ du$$

Integrate, then back-substitute using

$$u = \cosh^{-1}\left(\frac{x}{\sqrt{3}}\right)$$

You get

$$u - \tanh u + C$$

$$\cosh^{-1}\left(\frac{x}{\sqrt{3}}\right) - \tanh\left[\cosh^{-1}\left(\frac{x}{\sqrt{3}}\right)\right] + C$$

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