Topic: Maclaurin series to estimate an indefinite integral

Question: Use a Maclaurin series to estimate the indefinite integral.

$$\int \frac{e^x - 1}{2x}\ dx$$

Answer choices:

A $\quad C + \sum_{n=1}^{\infty} \frac{x^n}{(2n)!}$

B $\quad C + \sum_{n=1}^{\infty} \frac{x^n}{n(n!)}$

C $\quad C + \sum_{n=1}^{\infty} \frac{x^n}{2n(n!)}$

D $\quad C + \sum_{n=1}^{\infty} \frac{x^n}{2n!}$

Solution: C

When we're asked to use Maclaurin series to estimate an indefinite integral, it means we're supposed to substitute the Maclaurin series expansion for part of the function we've been asked to integrate, simplify the polynomial expression, and then integrate that polynomial instead of the original function.

We know that the Maclaurin series expansion of e^x is

$$e^x = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \dots$$

Plugging the expansion into the given function in place of e^x, we get

$$\int \frac{e^x - 1}{2x}\ dx = \int \frac{1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \dots - 1}{2x}\ dx$$

$$\int \frac{e^x - 1}{2x}\ dx = \int \frac{x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \dots}{2x}\ dx$$

$$\int \frac{e^x - 1}{2x}\ dx = \int \frac{x}{2x} + \frac{\frac{1}{2}x^2}{2x} + \frac{\frac{1}{6}x^3}{2x} + \dots\ dx$$

$$\int \frac{e^x - 1}{2x}\ dx = \int \frac{1}{2} + \frac{1}{4}x + \frac{1}{12}x^2 + \dots\ dx$$

Now we can integrate the polynomial expansion instead of the original function.

$$\int \frac{e^x - 1}{2x}\ dx = \frac{1}{2}x + \frac{1}{8}x^2 + \frac{1}{36}x^3 + \dots + C$$

Since the right side is still an infinite series, we need to give our answer in terms of an infinite sum. We'll rewrite the right side as

$$\frac{1}{2}x+\frac{1}{8}x^2+\frac{1}{36}x^3+\ldots+C$$

$$\frac{1}{2}\left(x+\frac{1}{4}x^2+\frac{1}{18}x^3+\ldots\right)+C$$

$$\frac{1}{2}\left[\frac{1}{1(1)}x^1+\frac{1}{2(2)}x^2+\frac{1}{3(6)}x^3+\ldots\right]+C$$

$$\frac{1}{2}\left[\frac{1}{1(1!)}x^1+\frac{1}{2(2!)}x^2+\frac{1}{3(3!)}x^3+\ldots\right]+C$$

Therefore, the integral can be represented as

$$\int\frac{e^x-1}{2x}\ dx=C+\frac{1}{2}\sum_{n=1}^{\infty}\frac{x^n}{n(n!)}$$

$$\int\frac{e^x-1}{2x}\ dx=C+\sum_{n=1}^{\infty}\frac{x^n}{2n(n!)}$$

Topic: Maclaurin series to estimate an indefinite integral

Question: Use a Maclaurin series to estimate the indefinite integral.

$$\int \sin x - x \ dx$$

Answer choices:

A $\quad C + \sum_{n=1}^{\infty} \frac{x^{2n+2}}{2^{n+2}(n+1)}$

B $\quad C + \sum_{n=1}^{\infty} \frac{(-1)^n x^{2n+2}}{(2n+2)!}$

C $\quad C + \sum_{n=1}^{\infty} \frac{x^{2n+1}}{2^{n+2}(2n+1)}$

D $\quad C + \sum_{n=1}^{\infty} \frac{x^{2n+2}}{2^{n+1}(2n+1)}$

Solution: B

When we're asked to use Maclaurin series to estimate an indefinite integral, it means we're supposed to substitute the Maclaurin series expansion for part of the function we've been asked to integrate, simplify the polynomial expression, and then integrate that polynomial instead of the original function.

We know that the Maclaurin series expansion of $\sin x$ is

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$

Plugging the expansion into the given function in place of $\sin x$, we get

$$\int \sin x - x \ dx = \int x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots - x \ dx$$

$$\int \sin x - x \ dx = \int -\frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots \ dx$$

Now we can integrate the polynomial expansion instead of the original function.

$$\int \sin x - x \ dx = -\frac{x^4}{4(3!)} + \frac{x^6}{6(5!)} - \frac{x^8}{8(7!)} + \dots + C$$

Since the right side is still an infinite series, we need to give our answer in terms of an infinite sum. We'll rewrite the right side as

$$-\frac{x^4}{4(3!)} + \frac{x^6}{6(5!)} - \frac{x^8}{8(7!)} + \dots + C$$

$$-\frac{x^4}{4!}+\frac{x^6}{6!}-\frac{x^8}{8!}+\ldots+C$$

$$-\frac{x^4}{4!}+\frac{x^6}{6!}-\frac{x^8}{8!}+\ldots+C$$

$$-\frac{x^{2(1)+2}}{(2(1)+2)!}+\frac{x^{2(2)+2}}{(2(2)+2)!}-\frac{x^{2(3)+2}}{(2(3)+2)!}+\ldots+C$$

Therefore, the integral can be represented as

$$\int \sin x - x\ dx = C + \sum_{n=1}^{\infty} \frac{(-1)^n x^{2n+2}}{(2n+2)!}$$

Topic: Maclaurin series to estimate an indefinite integral

Question: Use a Maclaurin series to estimate the indefinite integral.

$$\int x \ln(1+2x)\ dx$$

Answer choices:

A $\quad C + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}x^{n+2}}{n(n+1)}$

B $\quad C + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}x^{n+1}}{n(n+2)}$

C $\quad C + \sum_{n=1}^{\infty} \frac{(-1)^{n}x^{n+2}}{n(n+2)}$

D $\quad C + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}2^n x^{n+2}}{n(n+2)}$

Solution: D

When we're asked to use Maclaurin series to estimate an indefinite integral, it means we're supposed to substitute the Maclaurin series expansion for part of the function we've been asked to integrate, simplify the polynomial expression, and then integrate that polynomial instead of the original function.

We know that the Maclaurin series expansion of $\ln(1+x)$ is

$$\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \dots$$

In the case of our function though, we have $\ln(1+2x)$ instead of $\ln(1+x)$, which means we need to replace x with $2x$ everywhere.

$$\ln(1+2x) = 2x - \frac{1}{2}(2x)^2 + \frac{1}{3}(2x)^3 - \dots$$

$$\ln(1+2x) = 2x - \frac{1}{2}\left(4x^2\right) + \frac{1}{3}\left(8x^3\right) - \dots$$

$$\ln(1+2x) = 2x - 2x^2 + \frac{8}{3}x^3 - \dots$$

Plugging the expansion into the given function in place of $\ln(1+2x)$, we get

$$\int x\ln(1+2x)\ dx = \int x\left(2x - 2x^2 + \frac{8}{3}x^3 - \dots\right)\ dx$$

$$\int x\ln(1+2x)\ dx = \int 2x^2 - 2x^3 + \frac{8}{3}x^4 - \dots\ dx$$

Now we can integrate the polynomial expansion instead of the original function.

$$\int x\ln(1+2x)\ dx = \frac{2}{3}x^3 - \frac{2}{4}x^4 + \frac{8}{15}x^5 - \ldots + C$$

Since the right side is still an infinite series, we need to give our answer in terms of an infinite sum. We'll rewrite the right side as

$$\frac{2}{3}x^3 - \frac{2}{4}x^4 + \frac{8}{15}x^5 - \ldots + C$$

$$\frac{2}{3}x^{1+2} - \frac{2}{4}x^{2+2} + \frac{8}{15}x^{3+2} - \ldots + C$$

$$(-1)^{1+1}\frac{2}{3}x^{1+2} + (-1)^{2+1}\frac{2}{4}x^{2+2} + (-1)^{3+1}\frac{8}{15}x^{3+2} + \ldots + C$$

$$(-1)^{1+1}\frac{2}{3}x^{1+2} + (-1)^{2+1}\frac{4}{8}x^{2+2} + (-1)^{3+1}\frac{8}{15}x^{3+2} + \ldots + C$$

$$(-1)^{1+1}\frac{2^1}{3}x^{1+2} + (-1)^{2+1}\frac{2^2}{8}x^{2+2} + (-1)^{3+1}\frac{2^3}{15}x^{3+2} + \ldots + C$$

$$(-1)^{1+1}\frac{2^1}{1(1+2)}x^{1+2} + (-1)^{2+1}\frac{2^2}{2(2+2)}x^{2+2} + (-1)^{3+1}\frac{2^3}{3(3+2)}x^{3+2} + \ldots + C$$

Therefore, the integral can be represented as

$$\int x\ln(1+2x)\ dx = C + \sum_{n=1}^{\infty}\frac{(-1)^{n+1}2^n x^{n+2}}{n(n+2)}$$