Topic: sin(mx) sin(nx)

Question: Evaluate the trigonometric integral.

$$\int \sin 3x \sin 2x \ dx$$

Answer choices:

A $\quad -\frac{1}{2}\sin x - \frac{1}{10}\sin 5x + C$

B $\quad \frac{1}{2}\sin x - \frac{1}{10}\sin 5x + C$

C $\quad \frac{1}{2}\cos x + \frac{1}{10}\cos 5x + C$

D $\quad \frac{1}{2}\sin x + \frac{1}{10}\sin 5x + C$

Solution: B

In the specific case where our function is the product of

two **sine** factors,

our plan is to

1. use the identity $\sin A \sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$

We'll use the identity to simplify the integral.

$$\int \sin 3x \sin 2x \ dx$$

$$\int \frac{1}{2}\left[\cos(3x-2x) - \cos(3x+2x)\right] \ dx$$

$$\frac{1}{2}\int \cos x - \cos 5x \ dx$$

$$\frac{1}{2}\left(\sin x - \frac{1}{5}\sin 5x\right) + C$$

$$\frac{1}{2}\sin x - \frac{1}{10}\sin 5x + C$$

Topic: sin(mx) sin(nx)

Question: Evaluate the trigonometric integral.

$$\int \sin 5x \sin 2x \; dx$$

Answer choices:

A $\quad \frac{1}{6}\sin 3x - \frac{1}{14}\sin 7x + C$

B $\quad \frac{1}{6}\sin 3x + \frac{1}{14}\sin 7x + C$

C $\quad \frac{1}{6}\cos 3x - \frac{1}{14}\cos 7x + C$

D $\quad \frac{1}{6}\cos 3x + \frac{1}{14}\cos 7x + C$

Solution: A

In the specific case where our function is the product of

two **sine** factors,

our plan is to

1. use the identity $\sin A \sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$

We'll use the identity to simplify the integral.

$$\int \sin 5x \sin 2x \ dx$$

$$\int \frac{1}{2}\left[\cos(5x-2x) - \cos(5x+2x)\right] \ dx$$

$$\frac{1}{2}\int \cos 3x - \cos 7x \ dx$$

$$\frac{1}{2}\left(\frac{1}{3}\sin 3x - \frac{1}{7}\sin 7x\right) + C$$

$$\frac{1}{6}\sin 3x - \frac{1}{14}\sin 7x + C$$

Topic: sin(mx) sin(nx)

Question: Evaluate the trigonometric integral.

$$\int_0^{\frac{\pi}{2}} \sin 4x \sin 3x \ dx$$

Answer choices:

A $-\frac{4}{7}$

B $\frac{3}{7}$

C $-\frac{3}{7}$

D $\frac{4}{7}$

Solution: D

In the specific case where our function is the product of

two **sine** factors,

our plan is to

1. use the identity $\sin A \sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$

We'll use the identity to simplify the integral.

$$\int_0^{\frac{\pi}{2}} \sin 4x \sin 3x \ dx$$

$$\int_0^{\frac{\pi}{2}} \frac{1}{2}\left[\cos(4x-3x) - \cos(4x+3x)\right] \ dx$$

$$\frac{1}{2}\int_0^{\frac{\pi}{2}} \cos x - \cos 7x \ dx$$

$$\frac{1}{2}\left(\sin x - \frac{1}{7}\sin 7x\right)\Bigg|_0^{\frac{\pi}{2}}$$

$$\left(\frac{1}{2}\sin x - \frac{1}{14}\sin 7x\right)\Bigg|_0^{\frac{\pi}{2}}$$

$$\left[\frac{1}{2}\sin\frac{\pi}{2} - \frac{1}{14}\sin 7\left(\frac{\pi}{2}\right)\right] - \left[\frac{1}{2}\sin(0) - \frac{1}{14}\sin 7(0)\right]$$

$$\left(\frac{1}{2}\sin\frac{\pi}{2}-\frac{1}{14}\sin\frac{7\pi}{2}\right)-\left[\frac{1}{2}(0)-\frac{1}{14}(0)\right]$$

$$\frac{1}{2}(1)-\frac{1}{14}(-1)$$

$$\frac{1}{2}+\frac{1}{14}$$

$$\frac{7}{14}+\frac{1}{14}$$

$$\frac{8}{14}$$

$$\frac{4}{7}$$