**Topic**: Values that make the function differentiable

**Question**: Which values of a and b would make the function differentiable?

$$f(x) = \begin{cases} 3x^2 & x > 1\\ bx^2 - a & x \le 1 \end{cases}$$

# **Answer choices:**

$$A \qquad a = 3$$

and 
$$b = 0$$

B 
$$a=0$$
 and  $b=0$ 

$$b = 0$$

C 
$$a = 0$$
 and  $b = 3$ 

$$b = 3$$

D 
$$a=3$$
 and  $b=3$ 

$$b = 3$$

# Solution: C

The break point of the function is at x = 1, because that's where the first piece of the function ends and the second piece takes over.

We'll work on continuity first by setting the one-sided limits at the break point x = 1 equal to one another.

$$\lim_{x \to 1^+} 3x^2 = \lim_{x \to 1^-} (bx^2 - a)$$

$$3(1)^2 = b(1)^2 - a$$

$$3 = b - a$$

$$b - a = 3$$

Now we'll work on smoothness by setting the one-sided limits of the derivatives of each piece at the break point x = 1 equal to one another.

$$\lim_{x \to 1^+} 6x = \lim_{x \to 1^-} 2bx$$

$$6(1) = 2b(1)$$

$$6 = 2b$$

$$b = 3$$

Pull together these two equations into a system of equations.

$$b = 3$$

$$b - a = 3$$

We need to solve the system, which we can do by substituting the first equation b=3 into the second equation.

$$3 - a = 3$$

$$-a = 0$$

$$a = 0$$

Therefore, the values of the constants a and b that make f(x) differentiable are a=0 and b=3.



**Topic**: Values that make the function differentiable

**Question**: Which values of a and b would make the function differentiable?

$$f(x) = \begin{cases} x^2 - 5 & x > 3\\ 4x^2 - 2ax + b & x \le 3 \end{cases}$$

# **Answer choices:**

$$A \qquad a = 22$$

and 
$$b = 9$$

$$b = 9$$

B 
$$a = 22$$
 and  $b = 22$ 

$$b = 22$$

C 
$$a = 9$$
 and  $b = 9$ 

$$b = 9$$

D 
$$a = 9$$
 and  $b = 22$ 

$$b = 22$$

# Solution: D

The break point of the function is at x = 3, because that's where the first piece of the function ends and the second piece takes over.

We'll work on continuity first by setting the one-sided limits at the break point x = 3 equal to one another.

$$\lim_{x \to 3^{+}} (x^{2} - 5) = \lim_{x \to 3^{-}} (4x^{2} - 2ax + b)$$

$$3^2 - 5 = 4(3)^2 - 2a(3) + b$$

$$9 - 5 = 4(9) - 6a + b$$

$$4 = 36 - 6a + b$$

$$-6a + b = -32$$

$$6a - b = 32$$

Now we'll work on smoothness by setting the one-sided limits of the derivatives of each piece at the break point x=3 equal to one another.

$$\lim_{x \to 3^+} 2x = \lim_{x \to 3^-} (8x - 2a)$$

$$2(3) = 8(3) - 2a$$

$$6 = 24 - 2a$$

$$-18 = -2a$$

$$a = 9$$

Pull together these two equations into a system of equations.

$$a = 9$$

$$6a - b = 32$$

We need to solve the system, which we can do by substituting the first equation a=9 into the second equation.

$$6(9) - b = 32$$

$$54 - b = 32$$

$$-b = -22$$

$$b = 22$$

Therefore, the values of the constants a and b that make f(x) differentiable are a=9 and b=22.



Topic: Values that make the function differentiable

**Question**: Which values of a and b would make the function differentiable?

$$f(x) = \begin{cases} ax^2 + 10 & x \le 2\\ x^2 - 6x + b & x > 2 \end{cases}$$

**Answer choices:** 

$$A a = \frac{1}{2} and b = 16$$

B 
$$a = -\frac{1}{2}$$
 and  $b = -16$ 

C 
$$a = \frac{1}{2}$$
 and  $b = -16$ 

D 
$$a = -\frac{1}{2}$$
 and  $b = 16$ 

# Solution: D

The break point of the function is at x = 2, because that's where the first piece of the function ends and the second piece takes over.

We'll work on continuity first by setting the one-sided limits at the break point x = 2 equal to one another.

$$\lim_{x \to 2^{-}} (ax^{2} + 10) = \lim_{x \to 2^{+}} (x^{2} - 6x + b)$$

$$a(2)^2 + 10 = 2^2 - 6(2) + b$$

$$4a + 10 = 4 - 12 + b$$

$$4a - b = -18$$

Now we'll work on smoothness by setting the one-sided limits of the derivatives of each piece at the break point x = 2 equal to one another.

$$\lim_{x \to 2^{-}} 2ax = \lim_{x \to 2^{+}} (2x - 6)$$

$$2a(2) = 2(2) - 6$$

$$4a = 4 - 6$$

$$4a = -2$$

$$a = -\frac{1}{2}$$

Pull together these two equations into a system of equations.

$$a = -\frac{1}{2}$$

$$4a - b = -18$$

We need to solve the system, which we can do by substituting the first equation a=-1/2 into the second equation.

$$4\left(-\frac{1}{2}\right) - b = -18$$

$$-2 - b = -18$$

$$-b = -16$$

$$b = 16$$

Therefore, the values of the constants a and b that make f(x) differentiable are a = -1/2 and b = 16.

