Calculus 2 Workbook Solutions

Biology

krista king MATH

CARDIAC OUTPUT

1. Find the cardiac output, in liters/second, if 8 mg of dye is injected into the heart and the amount of dye remaining in the heart t seconds after the injection is modeled by $C(t) = 14te^{-0.6t}$. Assume $0 \le t \le 20$.

Solution:

From the problem, we know $A = 8$, $C(t) = 14te^{-0.6t}$, and $T = 20$. Substitute these values into the formula for blood flow.

$$F = \frac{A}{\int_0^T C(t)\ dt}$$

$$F = \frac{8}{\int_0^{20} 14te^{-0.6t}\ dt}$$

Work specifically on the integral.

$$\int_0^{20} te^{-0.6t}\ dt$$

Use integration by parts to solve it.

$$u = t$$

$$du = dt$$

$$dv = e^{-0.6t}\ dt$$

$$v = \frac{1}{-0.6}e^{-0.6t}$$

Find the antiderivative by integrating without the limits.

$$\int te^{-0.6t}\ dt$$

$$t\frac{e^{-0.6t}}{-0.6} - \int \frac{e^{-0.6t}}{-0.6}\ dt$$

$$-\frac{10t}{6}e^{-0.6t} + \frac{10}{6}\int e^{-0.6t}\ dt$$

$$-\frac{5t}{3}e^{-0.6t} + \frac{5}{3}\left(\frac{e^{-0.6t}}{-0.6}\right)$$

$$-\frac{5t}{3}e^{-0.6t} - \frac{25}{9}(e^{-0.6t})$$

Evaluating this over the interval gives

$$-\frac{5t}{3}e^{-0.6t} - \frac{25}{9}(e^{-0.6t})\Big|_0^{20}$$

$$\left[-\frac{5(20)}{3}e^{-0.6(20)} - \frac{25}{9}(e^{-0.6(20)})\right] - \left[-\frac{5(0)}{3}e^{-0.6(0)} - \frac{25}{9}(e^{-0.6(0)})\right]$$

$$-0.000221874 + \frac{25}{9}$$

$$2.777555$$

Then blood flow is

$$F = \frac{8}{14(2.777555)} = 0.2057307853 = 0.206 \text{ liters/second}$$

■ 2. Find the cardiac output, in liters/second, if 4 mg of dye is injected into the heart and the amount of dye remaining in the heart t seconds after the injection is modeled by $C(t) = 6te^{-0.2t}$. Assume $0 \le t \le 5$.

Solution:

From the problem, we know $A = 4$, $C(t) = 6te^{-0.2t}$, and $T = 5$. Substitute these values into the formula for blood flow.

$$F = \frac{A}{\int_0^T C(t)\ dt}$$

$$F = \frac{4}{\int_0^5 6te^{-0.2t}\ dt}$$

Work specifically on the integral.

$$\int_0^5 te^{-0.2t}\ dt$$

Use integration by parts to solve it.

$$u = t$$

$$du = dt$$

$$dv = e^{-0.2t}\ dt$$

$$v = \frac{1}{-0.2}e^{-0.2t}$$

Find the antiderivative by integrating without the limits.

$$\int te^{-0.2t}\ dt$$

$$t\frac{e^{-0.2t}}{-0.2} - \int \frac{e^{-0.2t}}{-0.2}\ dt$$

$$-5te^{-0.2t} + 5\int e^{-0.2t}\ dt$$

$$-5te^{-0.2t} + 5\left(\frac{e^{-0.2t}}{-0.2}\right)$$

$$-5te^{-0.2t} - 25(e^{-0.2t})$$

Evaluating this over the interval gives

$$-5te^{-0.2t} - 25(e^{-0.2t})\Big|_0^5$$

$$\left[-5(5)e^{-0.2(5)} - 25(e^{-0.2(5)})\right] - \left[-5(0)e^{-0.2(0)} - 25(e^{-0.2(0)})\right]$$

$$-18.39397206 + 25$$

$$6.6060279$$

Then blood flow is

$$F = \frac{4}{6(6.6060279)} = 0.1009179302 = 0.101 \text{ liters/second}$$

■ 3. Find the cardiac output, in liters/second, if 9 mg of dye is injected into the heart and the amount of dye remaining in the heart t seconds after the injection is modeled by $C(t) = 28te^{-0.85t}$. Assume $0 \le t \le 10$.

Solution:

From the problem, we know $A = 9$, $C(t) = 28te^{-0.85t}$, and $T = 10$. Substitute these values into the formula for blood flow.

$$F = \frac{A}{\int_0^T C(t)\ dt}$$

$$F = \frac{9}{\int_0^{10} 28te^{-0.85t}\ dt}$$

Work specifically on the integral.

$$\int_0^{10} te^{-0.85t}\ dt$$

Use integration by parts to solve it.

$$u = t$$

$$du = dt$$

$$dv = e^{-0.85t}\ dt$$

$$v = \frac{1}{-0.85}e^{-0.85t}$$

Find the antiderivative by integrating without the limits.

$$\int te^{-0.85t}\ dt$$

$$t\frac{e^{-0.85t}}{-0.85} - \int \frac{e^{-0.85t}}{-0.85}\ dt$$

$$-\frac{20t}{17}e^{-0.85t} + \frac{20}{17}\int e^{-0.85t}\ dt$$

$$-\frac{20t}{17}e^{-0.85t} + \frac{20}{17}\left(\frac{e^{-0.85t}}{-0.85}\right)$$

$$-\frac{20t}{17}e^{-0.85t} - \frac{400}{289}(e^{-0.85t})$$

Evaluating this over the interval gives

$$-\frac{20t}{17}e^{-0.85t} - \frac{400}{289}(e^{-0.85t})\Big|_0^{10}$$

$$\left[-\frac{20(10)}{17}e^{-0.85(10)} - \frac{400}{289}(e^{-0.85(10)})\right] - \left[-\frac{20(0)}{17}e^{-0.85(10)} - \frac{400}{289}(e^{-0.85(0)})\right]$$

$$-0.0026753626 + \frac{400}{289}$$

$$1.3814077$$

Then blood flow is

$$F = \frac{9}{28(1.3814077)} = 0.2326819023 = 0.233 \text{ liters/second}$$

POISEUILLE'S LAW

1. Use Poiseuille's law to find the flow of blood in the human artery in which $n = 0.031$, $R = 0.008$ cm, $L = 6$ cm, and $P = 3{,}900$ dynes/cm^2. Express the answer using scientific notation.

Solution:

The blood flow is

$$F = \frac{\pi P R^4}{8nL}$$

$$F = \frac{\pi(3{,}900)(0.008)^4}{8(0.031)(6)}$$

$$F = 3.37 \times 10^{-5} \text{ cm}^3/\text{sec}$$

2. Use Poiseuille's law to find the flow of blood in the human artery in which $n = 0.028$, $R = 0.007$ cm, $L = 3.5$ cm, and $P = 3{,}600$ dynes/cm^2. Express the answer using scientific notation.

Solution:

The blood flow is

$$F = \frac{\pi P R^4}{8nL}$$

$$F = \frac{\pi(3{,}600)(0.007)^4}{8(0.028)(3.5)}$$

$$F = 3.46 \times 10^{-5} \text{ cm}^3/\text{sec}$$

3. Use Poiseuille's law to find the flow of blood in the human artery in which $n = 0.027$, $R = 0.006$ cm, $L = 2.5$ cm, and $P = 3{,}800$ dynes/cm^2. Express the answer using scientific notation.

Solution:

The blood flow is

$$F = \frac{\pi P R^4}{8nL}$$

$$F = \frac{\pi(3{,}800)(0.006)^4}{8(0.027)(2.5)}$$

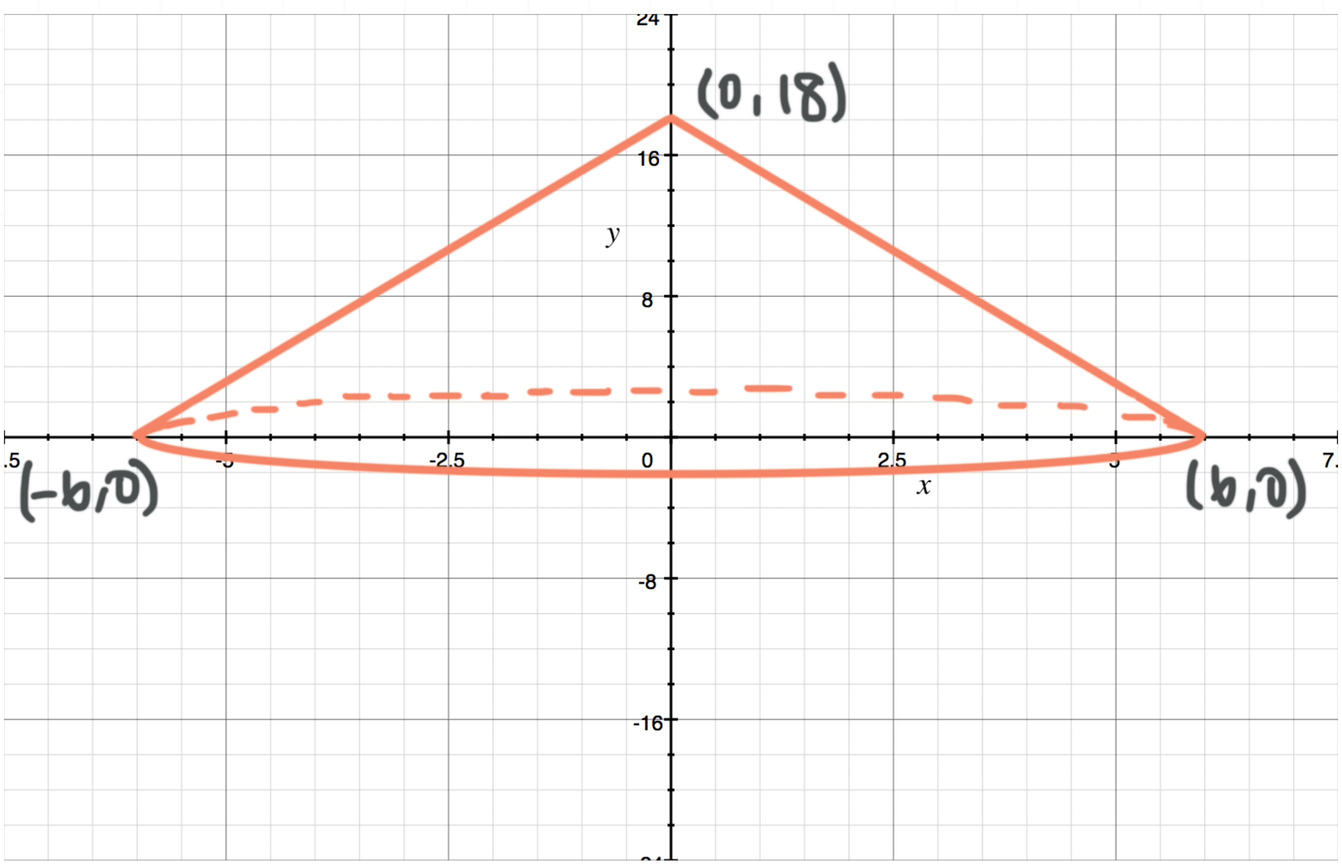
$$F = 2.87 \times 10^{-5} \text{ cm}^3/\text{sec}$$

THEOREM OF PAPPUS

1. Use the Theorem of Pappus to find the exact volume of a right circular cone with radius 6 feet and height 18 feet.

Solution:

The right circular cone drawn with the center of the base at the origin is



The cross section that the Theorem of Pappus uses is the area of a triangle drawn from the vertex of the cone to the center of the base, and then to the edge of the cone. The area of this cross section is

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(6)(18)$$

$$A = 54$$

Two points on the cone are $(0,18)$ and $(6,0)$. Use these points to calculate the slope of the slant height.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 18}{6 - 0} = \frac{-18}{6} = -3$$

Use $(0,18)$ and the slope $m = -3$ to write the equation.

$$y = mx + b$$

$$y = -3x + 18$$

$$f(x) = -3x + 18$$

Find the x-value of the centroid of the cross section, $\bar{x}$.

$$\bar{x} = \frac{1}{A}\int_a^b xf(x)\ dx$$

$$\bar{x} = \frac{1}{54}\int_0^6 x(-3x + 18)\ dx$$

$$\bar{x} = \frac{1}{18}\int_0^6 -x^2 + 6x\ dx$$

Integrate, then evaluate over the interval.

$$\bar{x} = \frac{1}{18}\left(-\frac{1}{3}x^3 + 3x^2\right)\Big|_0^6$$

$$\bar{x} = \frac{1}{18}\left(-\frac{1}{3}(6)^3 + 3(6)^2\right) - \frac{1}{18}\left(-\frac{1}{3}(0)^3 + 3(0)^2\right)$$

$$\bar{x} = \frac{1}{18}(-72 + 108)$$

$$\bar{x} = 2$$

Find the distance traveled by the x-value of the centroid.

$$d = 2\pi\bar{x}$$

$$d = 2\pi(2)$$

$$d = 4\pi$$

Then the volume is

$$V = Ad$$

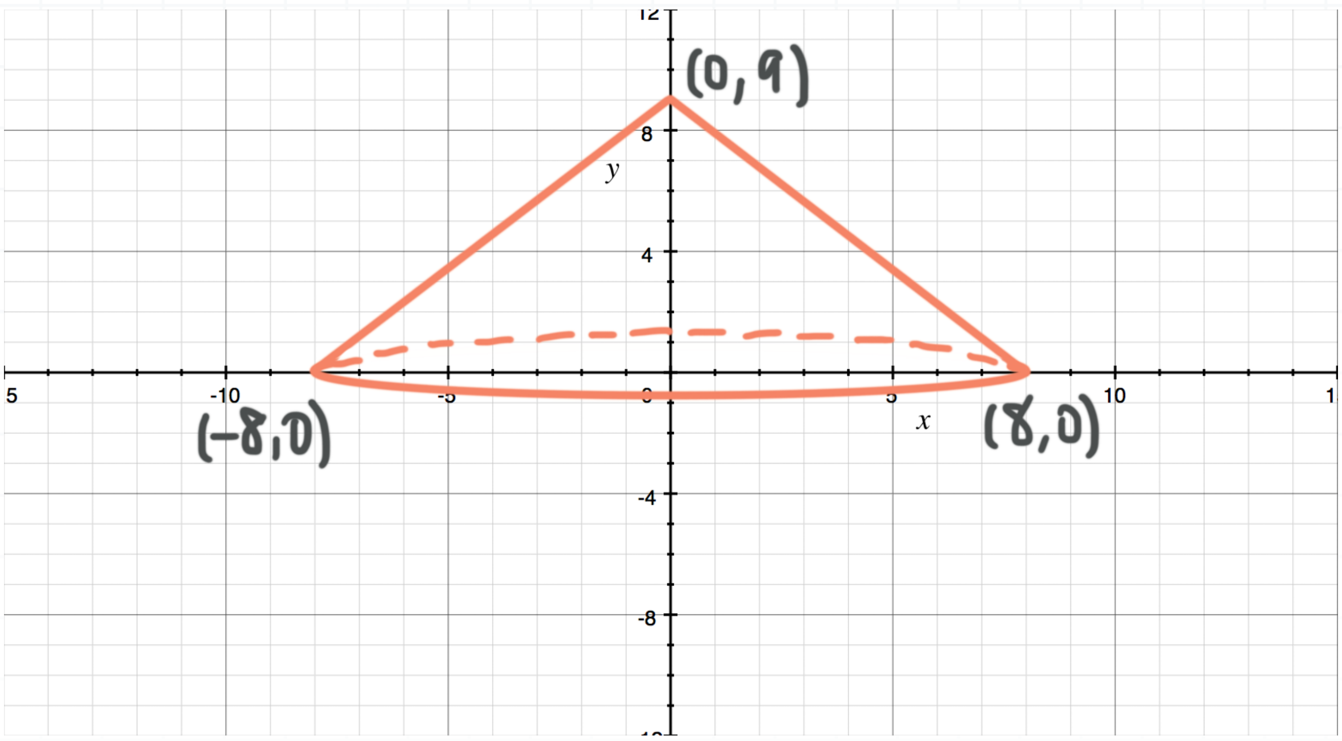
$$V = 54(4\pi)$$

$$V = 216\pi \text{ ft}^3$$

■ 2. Use the Theorem of Pappus to find the exact volume of a right circular cone with radius 8 inches and height 9 inches.

Solution:

The right circular cone drawn with the center of the base at the origin is



The cross section that the Theorem of Pappus uses is the area of a triangle drawn from the vertex of the cone to the center of the base, and then to the edge of the cone. The area of this cross section is

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(8)(9)$$

$$A = 36$$

Two points on the cone are $(0,9)$ and $(8,0)$. Use these points to calculate the slope of the slant height.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-9}{8-0} = -\frac{9}{8}$$

Use $(0,9)$ and the slope $m = -9/8$ to write the equation.

$$y = mx + b$$

$$y = -\frac{9}{8}x + 9$$

$$f(x) = -\frac{9}{8}x + 9$$

Find the x-value of the centroid of the cross section, $\bar{x}$.

$$\bar{x} = \frac{1}{A}\int_a^b xf(x)\ dx$$

$$\bar{x} = \frac{1}{36}\int_0^8 x\left(-\frac{9}{8}x + 9\right)\ dx$$

$$\bar{x} = \frac{1}{4}\int_0^8 -\frac{1}{8}x^2 + x\ dx$$

Integrate, then evaluate over the interval.

$$\bar{x} = \frac{1}{4}\left(-\frac{1}{24}x^3 + \frac{1}{2}x^2\right)\Big|_0^8$$

$$\bar{x} = \frac{1}{4}\left(-\frac{1}{24}(8)^3 + \frac{1}{2}(8)^2\right) - \frac{1}{4}\left(-\frac{1}{24}(0)^3 + \frac{1}{2}(0)^2\right)$$

$$\bar{x} = \frac{1}{4}\left(-\frac{64}{3} + 32\right)$$

$$\bar{x} = -\frac{16}{3} + 8$$

$$\bar{x} = \frac{8}{3}$$

Find the distance traveled by the x-value of the centroid.

$$d = 2\pi\bar{x}$$

$$d = 2\pi\left(\frac{8}{3}\right)$$

$$d = \frac{16}{3}\pi$$

Then the volume is

$$V = Ad$$

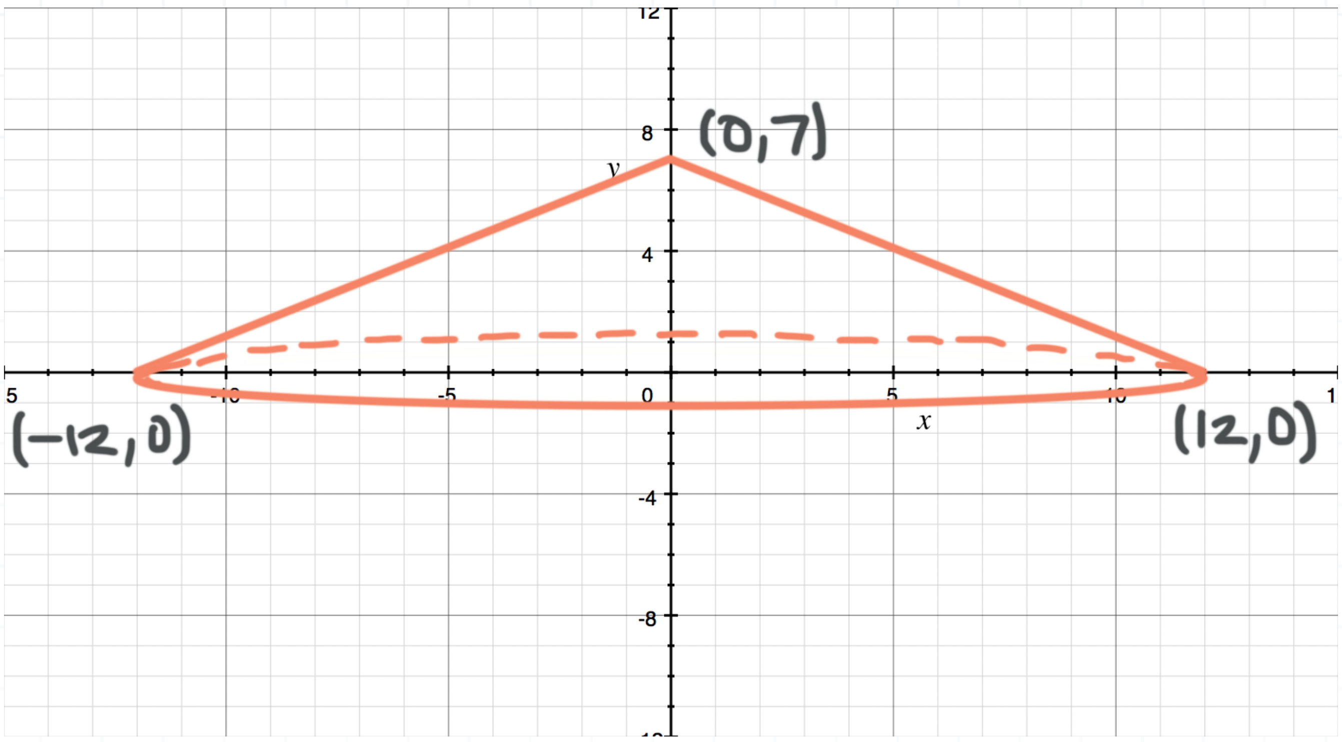
$$V = 36\left(\frac{16}{3}\pi\right)$$

$$V = 192\pi \text{ in}^3$$

■ 3. Use the Theorem of Pappus to find the exact volume of a right circular cone with radius 12 centimeters and height 7 centimeters.

Solution:

The right circular cone drawn with the center of the base at the origin is



The cross section that the Theorem of Pappus uses is the area of a triangle drawn from the vertex of the cone to the center of the base, and then to the edge of the cone. The area of this cross section is

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(12)(7)$$

$$A = 42$$

Two points on the cone are $(0,7)$ and $(12,0)$. Use these points to calculate the slope of the slant height.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 7}{12 - 0} = -\frac{7}{12}$$

Use $(0,7)$ and the slope $m = -7/12$ to write the equation.

$$y = mx + b$$

$$y = -\frac{7}{12}x + 7$$

$$f(x) = -\frac{7}{12}x + 7$$

Find the x-value of the centroid of the cross section, $\bar{x}$.

$$\bar{x} = \frac{1}{A}\int_a^b xf(x)\ dx$$

$$\bar{x} = \frac{1}{42}\int_0^{12} x\left(-\frac{7}{12}x + 7\right)\ dx$$

$$\bar{x} = \frac{1}{6}\int_0^{12} -\frac{1}{12}x^2 + x\ dx$$

Integrate, then evaluate over the interval.

$$\bar{x} = \frac{1}{6}\left(-\frac{1}{36}x^3 + \frac{1}{2}x^2\right)\Bigg|_0^{12}$$

$$\bar{x} = \frac{1}{6}\left(-\frac{1}{36}(12)^3 + \frac{1}{2}(12)^2\right) - \frac{1}{6}\left(-\frac{1}{36}(0)^3 + \frac{1}{2}(0)^2\right)$$

$$\bar{x} = \frac{1}{6}(-48 + 72)$$

$$\bar{x} = 4$$

Find the distance traveled by the x-value of the centroid.

$$d = 2\pi\bar{x}$$

$$d = 2\pi(4)$$

$$d = 8\pi$$

Then the volume is

$$V = Ad$$

$$V = 42(8\pi)$$

$$V = 336\pi \text{ cm}^3$$

W W W . K R I S T A K I N G M A T H . C O M