# Average rate of change

So far, we've been calculating the slope of a function at a specific point, by finding the slope of the tangent line there. When we look at the slope of the function at an exact point, we're looking at the rate of change at that exact point, so we call that the **instantaneous rate of change**.

In contrast, we can look at the rate of change over a larger interval, instead of at one specific point. When we look at rate of change over an interval, we call that the **average rate of change**.

## Average rate of change

When we calculate average rate of change of a function over a given interval, we're calculating the average number of units that the function moves up or down along the $y$-axis, per unit along the $x$-axis.

We could also say that we're measuring how much change occurs in the function's value per unit of the $x$-axis.

To find the average rate of change of a function $f(x)$ over an interval $[a, b]$, we'll first calculate the value of the function at both ends of the interval. Then we plug those values and the ends of the interval into the formula for average rate of change,

$$\frac{\Delta f}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Let's do an example where we find the average rate of change.

**Example**

Find the average rate of change of $f(x)$ on the interval $[0,4]$.

$$f(x) = 2x^2 - 2$$

From the interval, we know $x_1 = 0$ and $x_2 = 4$. We'll find $f(x_1)$ and $f(x_2)$ by plugging these values into $f(x) = 2x^2 - 2$. We get

$$f(0) = 2(0)^2 - 2$$

$$f(0) = 0 - 2$$

$$f(0) = -2$$

and

$$f(4) = 2(4)^2 - 2$$

$$f(4) = 2(16) - 2$$

$$f(4) = 30$$

Now we can plug the values we've found into the formula for average rate fo change.

$$\frac{\Delta f}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$\frac{\Delta f}{\Delta x} = \frac{f(4) - f(0)}{4 - 0}$$

$$\frac{\Delta f}{\Delta x}=\frac{30-(-2)}{4}$$

$$\frac{\Delta f}{\Delta x}=\frac{32}{4}$$

$$\frac{\Delta f}{\Delta x}=8$$

The average rate of change of $f(x)$ on the interval $[0,4]$ is $8$.

If we describe what that means, we can say that the function increases by $32$ units, from $f(0)=-2$ up to $f(4)=30$, between $x=0$ and $x=4$. If the function increases by $32$ units over a four-unit span, $[0,4]$, it means the function increases by an average of $8$ units, for each unit we move along the $x$-axis between $x=0$ and $x=4$.