Topic: Definite integrals of even and odd functions

Question: If this is the integral of an even function, rewrite the integral.

$$\int_{-4}^{4} x^4 - 2x^2 \, dx$$

Answer choices:

A The function isn't even or can't be rewritten.

B The function is even and can be rewritten as $\int_{0}^{4} x^4 - 2x^2 \, dx$

C The function is even and can be rewritten as $2\int_{0}^{4} x^4 - 2x^2 \, dx$

D The function is even and can be rewritten as $2\int_{-2}^{2} x^4 - 2x^2 \, dx$

Solution: C

In order for us to be able to rewrite the integral, we need to know that the area under the function to the left of the y-axis is equal to the area under the function to the right of the y-axis. We can say that these two areas are equal if we can show two things:

1. That the function is even, which means it's symmetrical about the y-axis.

2. That the limits of integration are symmetrical about the y-axis.

We can use simple algebra to determine whether or not the function is even. The way we do this is by substituting $-x$ for x in our original function. If we simplify and the result is equal to our original function, then we know that the function is even.

$$f(x) = x^4 - 2x^2$$

$$f(-x) = (-x)^4 - 2(-x)^2$$

$$f(-x) = x^4 - 2x^2$$

$$f(x) = f(-x)$$

Since we've shown that $f(x) = f(-x)$, we know that the function is even. We can also easily see that the limits of integration are symmetrical about the y-axis, because the interval is $[-4,4]$, which is in the form $[-a, a]$.

With these two requirements satisfied, we can rewrite the integral, changing the limits of integration from $[-a, a]$ to $[0,a]$ and multiply the integral by 2. So we get

$$\int_{-a}^{a} f(x)\ dx = 2\int_{0}^{a} f(x)\ dx$$

$$\int_{-4}^{4} x^4 - 2x^2\ dx = 2\int_{0}^{4} x^4 - 2x^2\ dx$$

Topic: Definite integrals of even and odd functions

Question: If this is the integral of an even function, rewrite the integral.

$$\int_{0}^{3} x^2 + 18\ dx$$

Answer choices:

A The function is even and can be rewritten as $\int_{0}^{3} x^2 + 18\ dx$

B The function isn't even or can't be rewritten.

C The function is even and can be rewritten as $2\int_{0}^{\frac{1}{2}} x^2 + 18\ dx$

D The function is even and can be rewritten as $2\int_{-3}^{0} x^2 + 18\ dx$

Solution: B

In order for us to be able to rewrite the integral, we need to know that the area under the function to the left of the y-axis is equal to the area under the function to the right of the y-axis. We can say that these two areas are equal if we can show two things:

1. That the function is even, which means it's symmetrical about the y-axis.

2. That the limits of integration are symmetrical about the y-axis.

We can use simple algebra to determine whether or not the function is even. The way we do this is by substituting $-x$ for x in our original function. If we simplify and the result is equal to our original function, then we know that the function is even.

$$f(x) = x^2 + 18$$

$$f(-x) = (-x)^2 + 18$$

$$f(-x) = x^2 + 18$$

$$f(x) = f(-x)$$

Since we've shown that $f(x) = f(-x)$, we know that the function is even. However, the limits of integration are $[0,3]$. Since that doesn't match the form $[-a, a]$, we know that the limits of integration are not symmetrical about the y-axis.

So even though the function is even, we can't rewrite the integral.

Topic: Definite integrals of even and odd functions

Question: Definite integrals of odd functions evaluated on the interval $[-a, a]$...

Answer choices:

A ... will have different values depending on the function.

B ... will always equal 0.

C ... will never exist.

D ... will always equal ∞.

Solution: B

Odd functions are symmetric about the origin. If a function is symmetric about the origin, it means that any area above the x-axis in the first quadrant will be reflected below the x-axis in the third quadrant. Or that any area above the x-axis in the second quadrant will be reflected below the x-axis in the fourth quadrant.

Therefore, if we take the integral of an odd function on the interval $[-a,a]$, it means that the area above the x-axis will be equal to the area below the x-axis, and therefore that the value of the integral will always be 0.

If the interval is anything other than $[-a,a]$, we know that value of the integral will be non-zero.