Topic: Hyperbolic derivatives

Question: Find the derivative of the hyperbolic function.

$$y = \cosh(x^2 - 1)$$

Answer choices:

A $y' = -2x \sinh(x^2 - 1)$

B $y' = -2x \cosh(x^2 - 1)$

C $y' = 2x \sinh(x^2 - 1)$

D $y' = 2x \cosh(x^2 - 1)$

Solution: C

Make a substitution, letting $u = x^2 - 1$ and $u' = 2x$. Then $y = \cosh u$, and the derivative is

$$y' = \sinh u \cdot u'$$

$$y' = \sinh(x^2 - 1) \cdot (2x)$$

$$y' = 2x \sinh(x^2 - 1)$$

Topic: Hyperbolic derivatives

Question: Find the derivative of the hyperbolic function.

$$y = x^2 \sinh x$$

Answer choices:

A $\quad y' = x \cosh x + 2 \sinh x$

B $\quad y' = x^2 \cosh x + 2x \sinh x$

C $\quad y' = x \cosh x - 2 \sinh x$

D $\quad y' = x^2 \cosh x - 2x \sinh x$

Solution: B

Use the product rule with

$$f(x) = x^2$$

$$f'(x) = 2x$$

and

$$g(x) = \sinh x$$

$$g'(x) = \cosh x$$

Then the derivative is

$$y' = f(x)g'(x) + f'(x)g(x)$$

$$y' = (x^2)(\cosh x) + (2x)(\sinh x)$$

$$y' = x^2 \cosh x + 2x \sinh x$$

Topic: Hyperbolic derivatives

Question: Find the derivative of the hyperbolic function.

$$f(x) = \sinh(\cosh(2x))$$

Answer choices:

A $\quad f'(x) = 2\sinh(2x)\sinh(\sinh(2x))$

B $\quad f'(x) = 2\sinh(2x)\cosh(\sinh(2x))$

C $\quad f'(x) = 2\sinh(2x)\cosh(\cosh(2x))$

D $\quad f'(x) = 2\cosh(2x)\cosh(\cosh(2x))$

Solution: C

Use a substitution with $u = \cosh(2x)$ and $u' = 2\sinh(2x)$. Then the function can be rewritten as

$$f(x) = \sinh u$$

and the derivative is

$$f'(x) = \cosh u \cdot u'$$

$$f'(x) = \cosh(\cosh(2x)) \cdot (2\sinh(2x))$$

$$f'(x) = 2\sinh(2x)\cosh(\cosh(2x))$$