Trigonometric derivatives

We've learned about the basic derivative rules, and now we want to shift our attention toward the derivatives of specific kinds of functions. In this lesson we'll be looking at the derivatives of trigonometric functions, and later on we'll look at the derivatives of exponential and logarithmic functions.

Trigonometric derivatives

There are six basic trig functions, and we should know the derivative of each one.

Trigonometric function	Its derivative
$y = \sin x$	$y' = \cos x$
$y = \cos x$	$y' = -\sin x$
$y = \tan x$	$y' = \sec^2 x$
$y = \cot x$	$y' = -\csc^2 x$
$y = \sec x$	$y' = \sec x \tan x$
$y = \csc x$	$y' = -\csc x \cot x$

Remember that, in a trig function like $y = \sin x$, the $\sin$ and x are not multiplied together. So y is not the product of $\sin$ and x. Instead, x is the argument of the sine function, so we read $\sin x$ as "sine of x."

Let's look at an example of what it looks like to take the derivative of a trig function.

Example

Find the derivative.

$$y = 4\cos x$$

The derivative of $y = \cos x$ is $y' = -\sin x$. So the derivative will be

$$y' = 4(-\sin x)$$

$$y' = -4\sin x$$

Let's do another example with a little more going on.

Example

Find the derivative.

$$y = 8x^5 - 9\cot x$$

Dealing with one term at a time, we get

$$y' = 8(5)x^{5-1} - 9(-\csc^2 x)$$

$$y' = 40x^4 + 9\csc^2 x$$

Let's try one more.

Example

Find the derivative.

$$y = \sec x - 7x^5 \sin x + 3\csc x \cos x$$

Let's look at one term at a time. The derivative of $\sec x$ is $\sec x \tan x$.

To find the derivative of $-7x^5 \sin x$, we'll need to use product rule. If $f(x) = -7x^5$ and $f'(x) = -35x^4$, and $g(x) = \sin x$ and $g'(x) = \cos x$, then we can plug directly into the product rule formula.

$$f(x)g'(x) + f'(x)g(x)$$

$$(-7x^5)(\cos x) + (-35x^4)(\sin x)$$

$$-7x^5 \cos x - 35x^4 \sin x$$

The find the derivative of $3\csc x \cos x$, we'll need to use product rule. If $f(x) = 3\csc x$ and $f'(x) = -3\csc x \cot x$, and $g(x) = \cos x$ and $g'(x) = -\sin x$, then we can plug directly into the product rule formula.

$$f(x)g'(x) + f'(x)g(x)$$

$$3\csc x(-\sin x) + (-3\csc x \cot x)\cos x$$

$$-3\csc x\left(\frac{1}{\csc x}\right)-3\left(\frac{1}{\sin x}\right)\left(\frac{\cos x}{\sin x}\right)\cos x$$

$$-3-3\frac{\cos^2 x}{\sin^2 x}$$

$$-3-3\cot^2 x$$

Putting these derivatives together, we get

$$y'=\sec x\tan x-7x^5\cos x-35x^4\sin x-3-3\cot^2 x$$
