Topic: Surface of revolution equation

Question: Find an equation for the surface generated by revolving the curve around the given axis.

$$x^2 + 4y^2 = 5$$

about the x-axis

Answer choices:

A $x^2 + 4y^2 + z^2 = 5$

B $x^2 + 4y^2 + 4z^2 = 5$

C $x^2 + y^2 + 4z^2 = 5$

D $x^2 + y^2 + z^2 = 5$

Solution: B

If we select a generic point $P(x, y, z)$ on the surface of revolution, and then another point on the curve with the same x-coordinate as P, like $Q(x, y_1, 0)$, then for Q we get

$$x^2 + 4y_1^2 = 5$$

Remember that both P and Q are on the surface of revolution and both have the same x-coordinate. Therefore, their distances from the x-axis are the same and the squares of these distances are equal. Therefore, the squared distance from P to the x-axis is $y^2 + z^2$, and the squared distance from Q to the x-axis is $y_1^2 + 0^2$ or just y_1^2.

Since the two distances are equal, we get

$$y_1^2 = y^2 + z^2$$

Substituting this into $x^2 + 4y_1^2 = 5$, the surface of revolution is

$$x^2 + 4\left(y^2 + z^2\right) = 5$$

$$x^2 + 4y^2 + 4z^2 = 5$$

Topic: Surface of revolution equation

Question: Find an equation for the surface generated by revolving the curve around the given axis.

$$x^2 = 4y^2$$

about the y-axis

Answer choices:

A $x^2 + 4z^2 = 4y^2$

B $x^2 + z^2 = 4y^2$

C $4x^2 + 4z^2 = 4y^2$

D $x^2 + z^2 = y^2$

Solution: B

If we select a generic point $P(x, y, z)$ on the surface of revolution, and then another point on the curve with the same y-coordinate as P, like $Q(x_1, y, 0)$, then for Q we get

$$x_1^2 = 4y^2$$

Remember that both P and Q are on the surface of revolution and both have the same y-coordinate. Therefore, their distances from the y-axis are the same and the squares of these distances are equal. Therefore, the squared distance from P to the y-axis is $x^2 + z^2$ and the squared distance from Q to the y-axis is $x_1^2 + 0^2$ or just x_1^2.

Since the two distances are equal, we get

$$x_1^2 = x^2 + z^2$$

Substituting this into $x_1^2 = 4y^2$, the surface of revolution equation is

$$x^2 + z^2 = 4y^2$$

Topic: Surface of revolution equation

Question: Find an equation for the surface generated by revolving the curve around the given axis.

$$3z^2 = 4 - y^2$$

about the z-axis

Answer choices:

A $\quad z^2 = 4 - x^2 - y^2$

B $\quad 3z^2 = x^2 + y^2$

C $\quad 3z^2 = 4 + x^2 - y^2$

D $\quad 3z^2 = 4 - x^2 - y^2$

Solution: D

If we select a generic point $P(x, y, z)$ on the surface of revolution, and then another point on the curve with the same z-coordinate as P, like $Q(0, y_1, z)$, then for Q we get

$$3z^2 = 4 - y_1^2$$

Remember that both P and Q are on the surface of revolution and both have the same z-coordinate. Therefore, their distances from the z-axis are the same and the squares of these distances are equal. Therefore, the squared distance from P to the z-axis is $x^2 + y^2$ and the squared distance from Q to the z-axis is $y_1^2 + 0^2$ or just y_1^2.

Since the two distances are equal, we get

$$y_1^2 = x^2 + y^2$$

Substituting this into $3z^2 = 4 - y_1^2$, the surface of revolution is

$$3z^2 = 4 - \left(x^2 + y^2\right)$$

$$3z^2 = 4 - x^2 - y^2$$