Economics

When we study present and future value in calculus, usually we're trying to calculate the amount a sum of money will be worth in the *future* after it's had time to grow and earn interest, or we're trying to calculate how much money we had in the *past* given the sum of money in the account today.

The present and future value formulas we use will vary depending on the rate at which interest is compounded, and whether we're calculating the value of a single deposit, or a continuous income stream. Use the table below to determine which formula to use.

Compounding	Future value	Present value
n times annually, single deposit	$FV = PV\left(1 + \dfrac{r}{n}\right)^{nt}$	$PV = \dfrac{FV}{\left(1 + \dfrac{r}{n}\right)^{nt}}$

FV is the future value

PV is the present value

r is the yearly rate

n is the number of times compounded annually

t is the number of years

Continuously, single deposit	$FV = PVe^{rt}$	$PV = \dfrac{FV}{e^{rt}}$

FV is the future value

PV is the present value

r is the yearly rate

t is the number of years

Continuously, continuous stream $FV = \int_0^T S(t)e^{r(T-t)}\,dt \qquad PV = \int_0^T S(t)e^{-rt}\,dt$

FV is the future value

$S(t)$ is the continuous income stream

r is the yearly rate in decimal form

T is the number of years elapsed

t is the variable we'll solve for

Let's do an example in which interest is compounded n times annually for a single deposit.

Example

Find the value of a $3,000 investment after 3 years, if the interest rate is 3% and interest is compounded every 3 months (4 times per year).

Here's what we know.

$$PV = 3{,}000$$

$$r = 0.03$$

$$n = 4$$

$$t = 3$$

Plugging these into the future value formula for interest compounded n times per year for a single deposit, we get

$$FV = 3{,}000 \left(1 + \frac{0.03}{4} \right)^{(4)(3)}$$

$$FV = 3{,}281.42$$

The value of the account after 3 years is $\$3{,}281.42$.

Let's try an example in which interest is compounded continuously for a single deposit.

Example

Find the value after 5 years of an investment that's worth $\$1{,}500$ right now, if the interest rate is 6% compounded continuously.

Here's what we know.

$$PV = 1{,}500$$

$$r = 0.06$$

$$t = 5$$

Plugging these into the future value equation for interest compounded continuously for a single deposit, we get

$$FV = 1,500e^{(0.06)(5)}$$

$$FV = 2,024.79$$

The value of the account after 5 years is $\$2,024.79$.

Now let's do an example where interest is compounded continuously for a continuous income stream.

Example

Find the future value after 3 years of an account that has $\$2,000$ added to it annually, if the interest rate is 10% compounded continuously. Assume that no money is withdrawn from the account during these 3 years, and that no money is added to the account other than the $\$2,000$ annual deposit.

Here's what we know.

$$S(t) = 2,000$$

$$r = 0.10$$

$$T = 3$$

Plugging these into the future value equation for interest compounded continuously for a continuous income stream, we get

$$FV = \int_0^3 2{,}000e^{0.10(3-t)} \, dt$$

$$FV = \int_0^3 2{,}000e^{0.30-0.10t} \, dt$$

$$FV = \int_0^3 2{,}000e^{0.30}e^{-0.10t} \, dt$$

$$FV = 2{,}000e^{0.30} \int_0^3 e^{-0.10t} \, dt$$

$$FV = \left(2{,}000e^{0.30}\right) \left(\frac{e^{-0.10t}}{-0.10}\right) \Bigg|_0^3$$

$$FV = \left(-20{,}000e^{0.30}\right) \left(e^{-0.10t}\right) \Bigg|_0^3$$

$$FV = \left(-20{,}000e^{0.30}\right) \left[e^{-0.10(3)} - e^{-0.10(0)}\right]$$

$$FV = 7{,}020.00$$

The value of the account after 3 years is $\$7{,}020.00$.

We'll do one last example for compounding interest n times annually for a continuous income stream.

Example

You deposit $\$10{,}000$ every year for 5 years into a new bank account. Interest on the account is compounded continuously at 8%. What is the present value?

Here's what we know.

$$S(t) = 10{,}000$$

$$r = 0.08$$

$$T = 5$$

Plugging these into the present value equation for interest compounded n times annually for a continuous income stream, we get

$$PV = \int_0^5 10{,}000e^{-0.08t}\ dt$$

$$PV = 10{,}000 \int_0^5 e^{-0.08t}\ dt$$

$$PV = 10{,}000 \left(\frac{e^{-0.08t}}{-0.08} \right) \Bigg|_0^5$$

$$PV = -\,125{,}000\left(e^{-0.08t}\right)\Big|_{0}^{5}$$

$$PV = -\,125{,}000\left[e^{-0.08(5)} - e^{-0.08(0)}\right]$$

$$PV = 41{,}209.99$$

The value of the account today, assuming you make the $10,000 annual deposits for 5 years and get the interest rate you've been promised, is $41,209.99.