

Topic: Water level in the tank

Question: Water is being pumped into a rectangular tank at a rate of 0.8 cubic feet per minute. How fast is the water level rising if the base of the tank is a rectangle with dimensions 4×5 feet?

Answer choices:

- A 25 ft/m
- B 0.4 ft/m
- C 0.04 ft/m
- D 0.8 ft/m



Solution: C

The formula for the volume of a rectangular prism is

$$V = lwh$$

From the question, we know that $w = 4$ and $l = 5$, so we'll plug that in.

$$V = 5(4)h$$

$$V = 20h$$

Use implicit differentiation to take the derivative of both sides.

$$(1)\frac{dV}{dt} = 20(1)\frac{dh}{dt}$$

$$\frac{dV}{dt} = 20\frac{dh}{dt}$$

From the question, we know that $dV/dt = 0.8$, so make that substitution.

$$0.8 = 20\frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{0.8}{20}$$

$$\frac{dh}{dt} = 0.04$$



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Question: Water is being pumped from a cylindrical tank with a radius of 3 ft at a rate of 18 cubic feet per minute. How fast is the water level falling when the water is 2 ft deep?

Answer choices:

A $-\frac{2}{\pi}$ ft/min

B -2 ft/min

C $-\pi$ ft/min

D $-\frac{\pi}{2}$ ft/min



Solution: A

The formula for the volume of a cylinder is

$$V = \pi r^2 h$$

From the question, we know that $r = 3$, so plug that in.

$$V = \pi(3)^2 h$$

$$V = 9\pi h$$

Use implicit differentiation to take the derivative of both sides.

$$(1) \frac{dV}{dt} = 9\pi(1) \frac{dh}{dt}$$

$$\frac{dV}{dt} = 9\pi \frac{dh}{dt}$$

From the question, we know that $dV/dt = -18$, so make that substitution.

$$-18 = 9\pi \frac{dh}{dt}$$

$$\frac{dh}{dt} = -\frac{18}{9\pi}$$

$$\frac{dh}{dt} = -\frac{2}{\pi}$$



Topic: Water level in the tank

Question: An inverted pyramid is standing on its tip. The base of the pyramid is 4×4 meters, and the depth is 9 meters. If oil is flowing into the vat at $8 \text{ m}^3/\text{min}$, how fast is the oil level rising when the depth of the oil is 7 meters?

Answer choices:

- A $\frac{3}{2} \text{ m/min}$
- B $\frac{81}{98} \text{ m/min}$
- C $\frac{16}{81} \text{ m/min}$
- D $\frac{81}{16} \text{ m/min}$

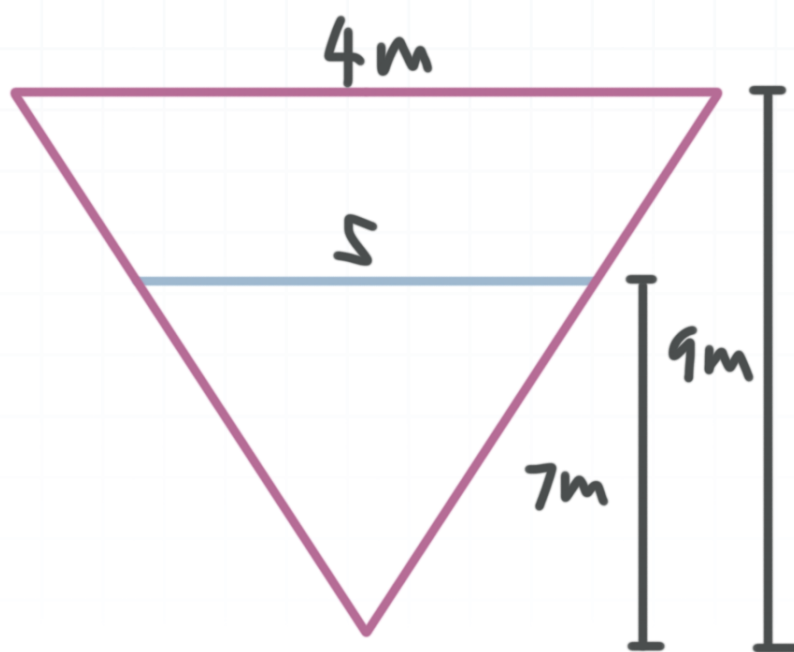


Solution: B

The formula for the volume of a pyramid is

$$V = \frac{1}{3}s^2h$$

We want to express volume as a function of h only. Using the diagram of a cross-section of the pyramid,



and similar triangles, we see that

$$\frac{h}{s} = \frac{9}{4}$$

$$s = \frac{4}{9}h$$

Then the volume of the water is given by

$$V = \frac{1}{3} \left(\frac{4}{9}h \right)^2 h$$



$$V = \frac{1}{3} \left(\frac{16}{81} \right) h^3$$

$$V = \frac{16}{243} h^3$$

Use implicit differentiation to take the derivative of both sides.

$$(1) \frac{dV}{dt} = \frac{16}{243} (3h^2) \frac{dh}{dt}$$

$$\frac{dV}{dt} = \frac{16}{81} h^2 \frac{dh}{dt}$$

From the question, we know that $dV/dt = 8$ and $h = 7$, so make those substitutions.

$$8 = \frac{16}{81} (7)^2 \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{8(81)}{16(49)}$$

$$\frac{dh}{dt} = \frac{81}{98}$$

