Topic: Product rule with two functions

Question: Find the derivative.

$$y = (x^2 + 2)(x^3 + 1)$$

Answer choices:

A $y' = 5x^3 + 6x + 2$

B $y' = x^4 + 12x^2 + 2x$

C $y' = 5x^4 + 6x^2 + 2x$

D $y' = 5x^4 - 6x^2 + 2x$

Solution: C

Let $f(x) = x^2 + 2$ and $g(x) = x^3 + 1$, and then apply product rule.

$$y' = f(x)g'(x) + f'(x)g(x)$$

$$y' = (x^2 + 2)(3x^2) + (2x)(x^3 + 1)$$

Expand the derivative, then collect like terms.

$$y' = 3x^2(x^2) + 3x^2(2) + 2x(x^3) + 2x(1)$$

$$y' = 3x^4 + 6x^2 + 2x^4 + 2x$$

$$y' = 5x^4 + 6x^2 + 2x$$

Topic: Product rule with two functions

Question: Find the derivative.

$$y = (3x^2 + 2x)(x^4 - 3x + 1)$$

Answer choices:

A $y' = 18x^4 + 10x^3 - 27x^2 - 6x + 2$

B $y' = 10x^5 + 27x^4 - 27x^2 - 6x + 2$

C $y' = 18x^5 + 10x^4 + 27x^2 - 6x + 2$

D $y' = 18x^5 + 10x^4 - 27x^2 - 6x + 2$

Solution: D

Let $f(x) = 3x^2 + 2x$ and $g(x) = x^4 - 3x + 1$, and then apply product rule.

$$y' = f(x)g'(x) + f'(x)g(x)$$

$$y' = (3x^2 + 2x)(4x^3 - 3) + (6x + 2)(x^4 - 3x + 1)$$

Expand the derivative, then collect like terms.

$$y' = 3x^2(4x^3 - 3) + 2x(4x^3 - 3) + 6x(x^4 - 3x + 1) + 2(x^4 - 3x + 1)$$

$$y' = 3x^2(4x^3) - 3x^2(3) + 2x(4x^3) - 2x(3)$$

$$+6x(x^4) - 6x(3x) + 6x(1) + 2(x^4) - 2(3x) + 2(1)$$

$$y' = 12x^5 - 9x^2 + 8x^4 - 6x + 6x^5 - 18x^2 + 6x + 2x^4 - 6x + 2$$

$$y' = 12x^5 + 6x^5 + 8x^4 + 2x^4 - 9x^2 - 18x^2 - 6x + 6x - 6x + 2$$

$$y' = 18x^5 + 10x^4 - 27x^2 - 6x + 2$$

Topic: Product rule with two functions

Question: Find the derivative.

$$h(x) = (3x^2 - 7)(x^2 - 4x + 3)$$

Answer choices:

A $h'(x) = 6x^3 - 36x^2 + 4x + 28$

B $h'(x) = 12x^3 - 36x^2 + 32x + 28$

C $h'(x) = 12x^3 - 12x^2 + 4x + 28$

D $h'(x) = 12x^3 - 36x^2 + 4x + 28$

Solution: D

Let $f(x) = 3x^2 - 7$ and $g(x) = x^2 - 4x + 3$, and then apply product rule.

$$h'(x) = f(x)g'(x) + f'(x)g(x)$$

$$h'(x) = (3x^2 - 7)(2x - 4) + (6x)(x^2 - 4x + 3)$$

Expand the derivative, then collect like terms.

$$h'(x) = (3x^2 - 7)(2x) - (3x^2 - 7)(4) + (6x)(x^2) - (6x)(4x) + (6x)(3)$$

$$h'(x) = 2x(3x^2) - 2x(7) - 4(3x^2) - 4(-7) + (6x)(x^2) - (6x)(4x) + (6x)(3)$$

$$h'(x) = 6x^3 - 14x - 12x^2 + 28 + 6x^3 - 24x^2 + 18x$$

$$h'(x) = 6x^3 + 6x^3 - 12x^2 - 24x^2 - 14x + 18x + 28$$

$$h'(x) = 12x^3 - 36x^2 + 4x + 28$$