Root test

The root test for convergence lets us determine the convergence or divergence of a series a_n using the limit

$$L = \lim_{n \to \infty} \sqrt[n]{|a_n|}$$

The convergence or divergence of the series depends on the value of L.

- the series converges absolutely if $L < 1$.
- the series diverges if $L > 1$ or if L is infinite.
- the test is inconclusive if $L = 1$.

The root test is used most often when our series includes something raised to the nth power.

Example

Use the root test to say whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{6^n}{(n+2)^n}$$

To use the root test, we need to solve for the limit

$$L = \lim_{n \to \infty} \sqrt[n]{|a_n|}$$

and then evaluate the value of L.

$$L = \lim_{n\to\infty} \sqrt[n]{\left| \frac{6^n}{(n+2)^n} \right|}$$

We can drop the absolute value bars since all of our terms will be positive.

$$L = \lim_{n\to\infty} \sqrt[n]{\frac{6^n}{(n+2)^n}}$$

$$L = \lim_{n\to\infty} \left[\frac{6^n}{(n+2)^n} \right]^{\frac{1}{n}}$$

$$L = \lim_{n\to\infty} \left[\left(\frac{6}{n+2} \right)^n \right]^{\frac{1}{n}}$$

$$L = \lim_{n\to\infty} \left(\frac{6}{n+2} \right)^{\frac{n}{n}}$$

$$L = \lim_{n\to\infty} \frac{6}{n+2}$$

$$L = \frac{6}{\infty + 2}$$

$$L = \frac{6}{\infty}$$

$$L = 0$$

Since $L < 1$, we can say that the original series a_n converges absolutely.