Topic: Proving that the limit does not exist

Question: How do we know that the limit does not exist?

$$\lim_{x \to 0} \frac{1}{x}$$

Answer choices:

A   The left-hand limit is 0.

B   The right-hand limit does not equal the left-hand limit.

C   The right-hand limit is equal to the left-hand limit.

D   The right-hand limit exists.

Solution: B

We know that the general limit only exists if both the left- and right-hand limits exist. We can't find the limit using substitution, so we'll pick values close to $x = 0$, but on either side of it, to get an idea of what the one-sided limits are doing.

$$f(-0.0001) = \frac{1}{-0.0001} = -10{,}000$$

$$f(0.0001) = \frac{1}{0.0001} = 10{,}000$$

These values tell us that the left-hand limit is $-\infty$, and the right-hand limit is ∞. Because the one-sided limits aren't equal, the general limit does not exist.

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Question: How do we know that the limit does not exist?

$$\lim_{x \to 0} \frac{\sqrt{x^2}}{x}$$

Answer choices:

A   The right-hand limit is 1.

B   The right-hand limit is equal to the left-hand limit.

C   The right-hand limit does not equal the left-hand limit.

D   The left-hand limit does exist.

Solution: C

We know that the general limit only exists if both the left- and right-hand limits exist. We can't find the limit using substitution, so we'll pick values close to $x = 0$, but on either side of it, to get an idea of what the one-sided limits are doing.

$$f(-0.0001) = \frac{\sqrt{(-0.0001)^2}}{-0.0001} = \frac{0.0001}{-0.0001} = -1$$

$$f(0.0001) = \frac{\sqrt{(0.0001)^2}}{0.0001} = \frac{0.0001}{0.0001} = 1$$

These values tell us that the left-hand limit is -1, and the right-hand limit is 1. Because the one-sided limits aren't equal, the general limit does not exist.

Topic: Proving that the limit does not exist

Question: How do we know that the limit does not exist?

$$\lim_{x \to 1} \ln(x - 1)$$

Answer choices:

A   The right-hand limit does not exist.

B   The right-hand limit is equal to the left-hand limit.

C   The left-hand limit is approaching $-\infty$.

D   The left-hand limit does not exist.

Solution: D

We know that the general limit only exists if both the left- and right-hand limits exist. We can't find the limit using substitution (because the natural log function is undefined when the argument is 0, and substituting $x = 1$ makes the argument 0), so we'll pick values close to $x = 1$, but on either side of it, to get an idea of what the one-sided limits are doing.

$$f(0.9999) = \ln(0.9999 - 1) = \ln(-0.0001) = \text{DNE}$$

$$f(1.0001) = \ln(1.0001 - 1) = \ln(0.0001) = -\infty$$

The natural log function isn't defined in real numbers for negative arguments. So, because we end up with $\ln(-0.0001)$ when we investigate the left-hand limit, we can say that the left-hand limit does not exist (DNE).

This fact alone tells us that the general limit does not exist, since both of the one-sided limits must exist in order for the general limit to be defined.