Topic: U-substitution

Question: Use u-substitution to evaluate the integral.

$$\int x^2\sqrt{x^3+2}\ dx$$

Answer choices:

A $\quad \dfrac{3x^4}{2\left(x^3+2\right)^{\frac{1}{2}}}+2x\left(x^3+2\right)^{\frac{1}{2}}+C$

B $\quad \dfrac{2}{9}x^3\left(x^3+2\right)^{\frac{3}{2}}+C$

C $\quad \dfrac{2}{9}x^{\frac{3}{2}}+C$

D $\quad \dfrac{2}{9}\left(x^3+2\right)^{\frac{3}{2}}+C$

Solution: D

We use u-substitution to solve this integral. Letting

$$u = x^3 + 2$$

$$du = 3x^2 \ dx$$

$$x^2 \ dx = \frac{1}{3} \ du$$

Making these substitutions, we have

$$\int x^2 \sqrt{x^3 + 2} \ dx$$

$$\int u^{\frac{1}{2}} \left(\frac{1}{3} \right) \ du$$

$$\frac{1}{3} \int u^{\frac{1}{2}} \ du$$

$$\frac{\frac{1}{3}}{\frac{3}{2}} u^{\frac{3}{2}} + C$$

$$\frac{2}{9} u^{\frac{3}{2}} + C$$

Back-substituting, we'll get

$$\frac{2}{9} \left(x^3 + 2 \right)^{\frac{3}{2}} + C$$

Topic: U-substitution

Question: Use u-substitution to evaluate the integral.

$$\int x^{-2}e^{\frac{1}{x}}\ dx$$

Answer choices:

A $\quad -e^{\frac{1}{x}}+C$

B $\quad -e^{x}+C$

C $\quad e^{\frac{1}{x}}+C$

D $\quad e^{x}+C$

Solution: A

Let

$$u = \frac{1}{x} = x^{-1}$$

$$du = -x^{-2}\ dx$$

By substitution:

$$\int x^{-2} e^{\frac{1}{x}}\ dx = -\int e^{\frac{1}{x}} \left(-x^{-2}\right)\ dx$$

$$-\int e^{u}\ du$$

$$-e^{u} + C$$

$$-e^{\frac{1}{x}} + C$$

Topic: U-substitution

Question: Use u-substitution to evaluate the integral.

$$\int \csc^2 x(1-\cot x)\ dx$$

Answer choices:

A $\quad \frac{1}{2}(1-\cot x)^2+C$

B $\quad -\cot x\left(x+\csc^2 x\right)+C$

C $\quad -\csc^2 x\left(2\cot x+1+2\cot^2 x\right)+C$

D $\quad \csc^2 x\left(\csc^2 x+\cot x-2\cot^2 x\right)+C$

Solution: A

First, we see that

$$\frac{d}{dx}(1 - \cot x) = \csc^2 x$$

and so we'll use u-substitution with

$$u = 1 - \cot x$$

$$du = \csc^2 x \ dx$$

Plugging these in, we get

$$\int \csc^2 x(1 - \cot x) \ dx$$

$$\int u \ du$$

$$\frac{1}{2}u^2 + C$$

$$\frac{1}{2}(1 - \cot x)^2 + C$$