Topic: Second derivative of a parametric curve

Question: Find the second derivative of the parametric curve.

$$x = 3t$$

$$y = t^2$$

Answer choices:

A $\frac{9}{2}$

B $\frac{2}{9}$

C $\frac{2}{9}t$

D $\frac{9}{2}t$

Solution: B

Before we can find the second derivative of a parametric curve, we have to find the first derivative using the formula

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

We'll find

the derivative of y with respect to t, dy/dt, and

the derivative of x with respect to t, dx/dt

and then plug both of them into the formula above. The derivatives of our separate equations are

$$\frac{dy}{dt} = 2t$$

and

$$\frac{dx}{dt} = 3$$

Plugging these into the formula for the derivative of a parametric curve, we get

$$\frac{dy}{dx} = \frac{2t}{3}$$

With this first derivative in hand, we'll use the formula

$$\frac{d^2y}{dx^2}=\frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$$

to find the second derivative, plugging in the values we already found for dy/dx and dx/dt.

$$\frac{d^2y}{dx^2}=\frac{\frac{d}{dt}\left(\frac{2t}{3}\right)}{3}$$

The d/dt in the numerator tells us to take the derivative of our first derivative with respect to t.

$$\frac{d^2y}{dx^2}=\frac{\frac{2}{3}}{3}$$

$$\frac{d^2y}{dx^2}=\frac{2}{3}\left(\frac{1}{3}\right)$$

$$\frac{d^2y}{dx^2}=\frac{2}{9}$$

Topic: Second derivative of a parametric curve

Question: Find the second derivative of the parametric curve.

$$x = 4t^2$$

$$y = \sin t$$

Answer choices:

A $\dfrac{t\sin t - \cos t}{64t^3}$

B $\dfrac{t\sin t + \cos t}{64t^3}$

C $\dfrac{-t\sin t + \cos t}{64t^3}$

D $-\dfrac{t\sin t + \cos t}{64t^3}$

Solution: D

Before we can find the second derivative of a parametric curve, we have to find the first derivative using the formula

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

We'll find

the derivative of y with respect to t, dy/dt, and

the derivative of x with respect to t, dx/dt

and then plug both of them into the formula above. The derivatives of our separate equations are

$$\frac{dy}{dt} = \cos t$$

and

$$\frac{dx}{dt} = 8t$$

Plugging these into the formula for the derivative of a parametric curve, we get

$$\frac{dy}{dx} = \frac{\cos t}{8t}$$

With this first derivative in hand, we'll use the formula

$$\frac{d^2y}{dx^2}=\frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$$

to find the second derivative, plugging in the values we already found for dy/dx and dx/dt.

$$\frac{d^2y}{dx^2}=\frac{\frac{d}{dt}\left(\frac{\cos t}{8t}\right)}{8t}$$

The d/dt in the numerator tells us to take the derivative of our first derivative with respect to t. We'll use quotient rule to find the derivative of just the value in the parentheses.

$$\frac{d^2y}{dx^2}=\frac{\frac{(-\sin t)(8t)-(\cos t)(8)}{(8t)^2}}{8t}$$

$$\frac{d^2y}{dx^2}=\frac{\frac{-8t\sin t-8\cos t}{64t^2}}{8t}$$

$$\frac{d^2y}{dx^2}=\frac{-8t\sin t-8\cos t}{64t^2}\left(\frac{1}{8t}\right)$$

$$\frac{d^2y}{dx^2}=\frac{-t\sin t-\cos t}{64t^3}$$

$$\frac{d^2y}{dx^2}=-\frac{t\sin t+\cos t}{64t^3}$$

Topic: Second derivative of a parametric curve

Question: Find the second derivative of the parametric curve.

$$x = \cos 2t$$

$$y = 3\sin t - t^2$$

Answer choices:

A $$\frac{3\sin t \sin 2t + 2\sin 2t + 6\cos t \cos 2t - 4t\cos 2t}{4\sin^3 2t}$$

B $$\frac{3\sin t \sin 2t + 2\sin 2t + 6\cos t \cos 2t - 4t\cos 2t}{8\sin^3 2t}$$

C $$-\frac{3\sin t \sin 2t + 2\sin 2t + 6\cos t \cos 2t - 4t\cos 2t}{4\sin^3 2t}$$

D $$-\frac{3\sin t \sin 2t + 2\sin 2t + 6\cos t \cos 2t - 4t\cos 2t}{8\sin^3 2t}$$

Solution: C

Before we can find the second derivative of a parametric curve, we have to find the first derivative using the formula

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

We'll find

the derivative of y with respect to t, dy/dt, and

the derivative of x with respect to t, dx/dt

and then plug both of them into the formula above. The derivatives of our separate equations are

$$\frac{dy}{dt} = 3\cos t - 2t$$

and

$$\frac{dx}{dt} = -2\sin 2t$$

Plugging these into the formula for the derivative of a parametric curve, we get

$$\frac{dy}{dx} = \frac{3\cos t - 2t}{-2\sin 2t}$$

With this first derivative in hand, we'll use the formula

$$\frac{d^2y}{dx^2}=\frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$$

to find the second derivative, plugging in the values we already found for dy/dx and dx/dt.

$$\frac{d^2y}{dx^2}=\frac{\frac{d}{dt}\left(\frac{3\cos t-2t}{-2\sin 2t}\right)}{-2\sin 2t}$$

The d/dt in the numerator tells us to take the derivative of our first derivative with respect to t. We'll use quotient rule to find the derivative of just the value in the parentheses.

$$\frac{d^2y}{dx^2}=\frac{\frac{(-3\sin t-2)(-2\sin 2t)-(3\cos t-2t)(-4\cos 2t)}{(-2\sin 2t)^2}}{-2\sin 2t}$$

$$\frac{d^2y}{dx^2}=\frac{(-3\sin t-2)(-2\sin 2t)-(3\cos t-2t)(-4\cos 2t)}{(-2\sin 2t)^2}\cdot\frac{1}{-2\sin 2t}$$

$$\frac{d^2y}{dx^2}=\frac{(-3\sin t-2)(-2\sin 2t)-(3\cos t-2t)(-4\cos 2t)}{(-2\sin 2t)^3}$$

$$\frac{d^2y}{dx^2}=\frac{6\sin t\sin 2t+4\sin 2t-(-12\cos t\cos 2t+8t\cos 2t)}{-8\sin^3 2t}$$

$$\frac{d^2y}{dx^2}=\frac{6\sin t\sin 2t+4\sin 2t+12\cos t\cos 2t-8t\cos 2t}{-8\sin^3 2t}$$

$$\frac{d^2y}{dx^2}=-\frac{3\sin t\sin 2t+2\sin 2t+6\cos t\cos 2t-4t\cos 2t}{4\sin^3 2t}$$