Topic: Summation notation, collapsing

Question: Use summation notation to rewrite the sum.

$$\frac{(x-1)}{1-2}+\frac{(x-1)^2}{2-4}+\frac{(x-1)^3}{3-8}+\frac{(x-1)^4}{4-16}+\frac{(x-1)^5}{5-32}$$

Answer choices:

A $\displaystyle\sum_{n=1}^{5}\frac{(x-1)^n}{2^n-n}$

B $\displaystyle\sum_{n=0}^{4}\frac{(x-1)^n}{n-2^n}$

C $\displaystyle\sum_{n=1}^{5}\frac{(x-1)^n}{n-2^n}$

D $\displaystyle\sum_{n=1}^{5}\frac{(x-1)^{n+1}}{n-2^{n+1}}$

Solution: C

To rewrite the summation

$$\frac{(x-1)}{1-2}+\frac{(x-1)^2}{2-4}+\frac{(x-1)^3}{3-8}+\frac{(x-1)^4}{4-16}+\frac{(x-1)^5}{5-32}$$

we notice that in the terms of the sum there are three subsequences, one in the numerator and two in the denominator. We can find the summation rule for each of these subsequences separately and then put them together in one summation at the end.

In the numerator, we have

n (term #)	1	2	3	4	5
expression	$(x-1)^1$	$(x-1)^2$	$(x-1)^3$	$(x-1)^4$	$(x-1)^5$

We can see that the numerator is the expression $x-1$ raised to the power of n.

We see a changing difference in the denominator. Associating these differences with the term number we have

n (term #)	1	2	3	4	5
expression	$1-2$	$2-4$	$3-8$	$4-16$	$5-32$

We can see that the first number in each difference is the term number, n. Now let's look at the second number. That number appears to be a power of 2.

n (term #)	1	2	3	4	5

expression	$1-2$	$2-4$	$3-8$	$4-16$	$5-32$
2nd number	2	4	8	16	32
As a power of 2		2^1	2^2	2^3	2^4 2^5

Again, after careful observation, we can see that the denominator contains the difference of n and 2 raised to the power of n.

The sum contains five terms, starting with $n=1$, and ending with $n=5$. Therefore, we can rewrite the sum as a summation.

$$\sum_{n=1}^{5} \frac{(x-1)^n}{n-2^n} = \frac{(x-1)}{1-2} + \frac{(x-1)^2}{2-4} + \frac{(x-1)^3}{3-8} + \frac{(x-1)^4}{4-16} + \frac{(x-1)^5}{5-32}$$

Topic: Summation notation, collapsing

Question: Use summation notation to rewrite the sum.

$$3x+\frac{9x^2}{4}+3x^3+\frac{81x^4}{16}+\frac{243x^5}{25}$$

Answer choices:

A $\quad \sum_{n=1}^{5}\frac{3^n x^n}{2n+2}$

B $\quad \sum_{n=0}^{4}\frac{3^n x^n}{n^2}$

C $\quad \sum_{n=1}^{5}\frac{3^{n+1} x^{n+1}}{n^2}$

D $\quad \sum_{n=1}^{5}\frac{3^n x^n}{n^2}$

Solution: D

To rewrite the summation

$$3x+\frac{9x^2}{4}+3x^3+\frac{81x^4}{16}+\frac{243x^5}{25}$$

it may be helpful to make every term appear to be a fraction to find a pattern of the denominators.

We can see that the denominators are all squared numbers

$$1^2, 2^2, 1^2, 4^2, 5^2$$

The third denominator is either out of place or should be 9. To form a pattern of denominators, we will change the denominator of that term, as well as the numerator of that term so the value of the term remains the same as before. The new sum is

$$\frac{3x}{1}+\frac{9x^2}{4}+\frac{27x^3}{9}+\frac{81x^4}{16}+\frac{243x^5}{25}$$

Now we can see three patterns in the sum; the coefficients of the numerator, the exponents in the numerator, and the denominator.

The coefficient in the numerator is a power of 3.

n (term #)	1	2	3	4	5
coefficient	3	9	27	81	243
as a power of 3	3^1	3^2	3^3	3^4	3^5

We can see that the coefficient in the numerator is the number 3 raised to the power of n, the term number. Now let's look at the power of x in the numerator.

n (term #)	1	2	3	4	5
x term	x^1	x^2	x^3	x^4	x^5

We can see that the exponent of x in the numerator is the same as the value of n, the term number. Next, let's look at the denominator of each term in the sum.

n (term #)	1	2	3	4	5
denominator	1	4	9	16	25
square of n	$1^2 = 1$	$2^2 = 4$	$3^2 = 9$	$4^2 = 16$	$5^2 = 25$

We can see that each denominator is the square of the term number. Now we can rewrite the sum as a summation.

$$\sum_{n=1}^{5} \frac{3^n x^n}{n^2} = \frac{3x}{1} + \frac{9x^2}{4} + \frac{27x^3}{9} + \frac{81x^4}{16} + \frac{243x^5}{25}$$

Topic: Summation notation, collapsing

Question: Use summation notation to rewrite the sum.

$$\frac{1}{3}-\frac{x}{4}+\frac{x^2}{5}-\frac{x^3}{6}+\frac{x^4}{7}-\frac{x^5}{8}$$

Answer choices:

A $\displaystyle\sum_{n=0}^{5}\frac{(-1)^n x^n}{n+3}$

B $\displaystyle\sum_{n=1}^{6}\frac{(-1)^n x^n}{n+2}$

C $\displaystyle\sum_{n=0}^{5}\frac{x^n}{n+3}$

D $\displaystyle\sum_{n=0}^{5}\frac{-x^n}{n+3}$

Solution: A

To rewrite the summation

$$\frac{1}{3}-\frac{x}{4}+\frac{x^2}{5}-\frac{x^3}{6}+\frac{x^4}{7}-\frac{x^5}{8}$$

we need to notice that there's a pattern in the denominators, the sign of the terms alternate between positive and negative, and there is a pattern of the exponents, if we change the first term. We will begin by changing the first term so it contains a power of x, and we will place an exponent of 1 in the second term.

$$\frac{x^0}{3}-\frac{x^1}{4}+\frac{x^2}{5}-\frac{x^3}{6}+\frac{x^4}{7}-\frac{x^5}{8}$$

Now we see three definite patterns. The changing signs, the exponent of x, and the denominator. Let's look at each one. Recall that multiplying by (-1) changes the sign of terms. So the summation must contain this multiplication in each term. If we consider the value of n to be the term number, the signs of each term will be incorrect, unless we begin n with 0.

n	0	1	2	3	4	5
Sign	$+$	$-$	$+$	$-$	$+$	$-$
Power of -1	$(-1)^0$	$(-1)^1$	$(-1)^2$	$(-1)^3$	$(-1)^4$	$(-1)^5$
Sign	$+$	$-$	$+$	$-$	$+$	$-$

We can see that if n begins with a value of 0, and we raise -1 to the power of n, we have the correct sign of each term. Next let's look at the power of x in the numerators.

n	0	1	2	3	4	5
x term	x^0	x^1	x^2	x^3	x^4	x^5
Power of x	0	1	2	3	4	5

We can see that the power of x is the same as the value of n. Now, let's look at the denominator of each term.

n	0	1	2	3	4	5
Denominator	3	4	5	6	7	8
$n+3$	$0+3$	$1+3$	$2+3$	$3+3$	$4+3$	$5+3$
Answer	3	4	5	6	7	8

After careful study, we can see that the denominator of each term in the sum is three more than the value of n.

Now we have the pattern of all of the parts of the sum. Let's write the summation.

$$\sum_{n=0}^{5} \frac{(-1)^n x^n}{n+3} = \frac{1}{3} - \frac{x}{4} + \frac{x^2}{5} - \frac{x^3}{6} + \frac{x^4}{7} - \frac{x^5}{8}$$