**Topic**: Improper integrals, case 5

**Question**: Evaluate the improper integral.

$$\int_0^5 \frac{3\ln x}{x}\,dx$$

**Answer choices**:

A $\quad \frac{3}{2}\ln 5$

B $\quad -\frac{3}{2}\ln 5$

C $\quad \infty$

D $\quad -\infty$

**Solution**: D

The integral in this problem is considered to be an improper integral, case 5, because the integrand is undefined at the lower limit limit of integration. Evaluating this type of improper integral follows this general rule:

$$\int_a^b f(x)\ dx = \lim_{c \to a^+} \int_c^b f(x)\ dx$$

Let's begin by re-writing the integral using the rule.

$$\int_0^5 \frac{3 \ln x}{x}\ dx = \lim_{c \to 0^+} \int_c^5 \frac{3 \ln x}{x}\ dx$$

Use a u-substitution.

$$u = \ln x$$

$$du = \frac{1}{x}\ dx$$

$$dx = x\ du$$

Make substitutions into the integral, then integrate and back-substitute.

$$\lim_{c \to 0^+} \int_{x=c}^{x=5} \frac{3u}{x} \left( x\ du \right)$$

$$\lim_{c \to 0^+} \int_{x=c}^{x=5} 3u\ du$$

$$\frac{3}{2} \lim_{c \to 0^+} u^2 \Big|_{x=c}^{x=5}$$

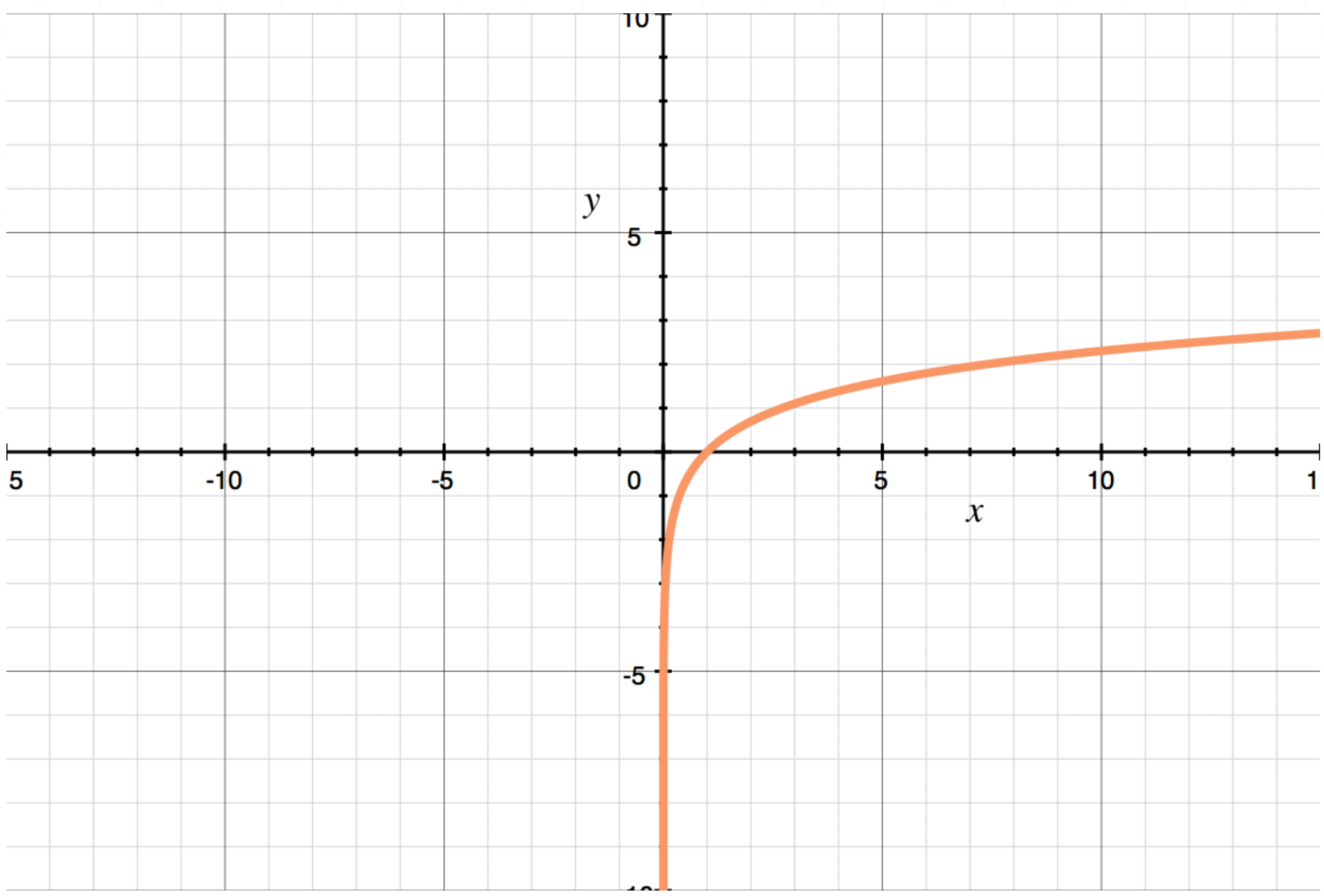
$$\frac{3}{2}\lim_{c\to 0^+}\ln^2 x\Big|_c^5$$

Evaluate over the interval.

$$\frac{3}{2}\lim_{c\to 0^+}\ln^2 5-\ln^2 c$$

$$\frac{3}{2}\left(\ln^2 5-\ln^2 0\right)$$

When we look at $\ln^2 0$, we know that $\ln 0$ is undefined. If we look at the graph of the natural logarithm, we can see that the value approaches $-\infty$.




Therefore, we can evaluate the limit using the graph.

$$\frac{3}{2}\left(\ln^2 5-(-\infty)^2\right)$$

$$\frac{3}{2}\left(\ln^2 5 - \infty\right)$$

$$-\infty$$

**Topic**: Improper integrals, case 5

**Question**: Evaluate the improper integral.

$$\int_{\pi}^{\frac{5\pi}{4}} \frac{\cos\theta}{\sin\theta}\ d\theta$$

**Answer choices**:

A $\quad -\infty$

B $\quad \ln\frac{\sqrt{2}}{2}$

C $\quad \infty$

D $\quad -\ln\frac{\sqrt{2}}{2}$

**Solution**: C

The integral in this problem is considered to be an improper integral, case 5, because the integrand is undefined at the lower limit limit of integration. Evaluating this type of improper integral follows this general rule:

$$\int_a^b f(x)\ dx = \lim_{c \to a^+} \int_c^b f(x)\ dx$$

Let's begin by re-writing the integral using the rule.

$$\int_{\pi}^{\frac{5\pi}{4}} \frac{\cos\theta}{\sin\theta}\ dx = \lim_{c \to \pi^+} \int_c^{\frac{5\pi}{4}} \frac{\cos\theta}{\sin\theta}\ dx$$

Use a u-substitution.

$$u = \sin\theta$$

$$du = \cos\theta\ dx$$

$$dx = \frac{du}{\cos\theta}$$

Make substitutions into the integral, then integrate and back-substitute.

$$\lim_{c \to \pi^+} \int_{x=c}^{x=\frac{5\pi}{4}} \frac{\cos\theta}{u} \left(\frac{du}{\cos\theta}\right)$$

$$\lim_{c \to \pi^+} \int_{x=c}^{x=\frac{5\pi}{4}} \frac{1}{u}\ du$$

$$\lim_{c \to \pi^+} \ln|u| \Big|_{x=c}^{x=\frac{5\pi}{4}}$$

$$\lim_{c \to \pi^+} \ln |\sin \theta| \Big|_c^{\frac{5\pi}{4}}$$

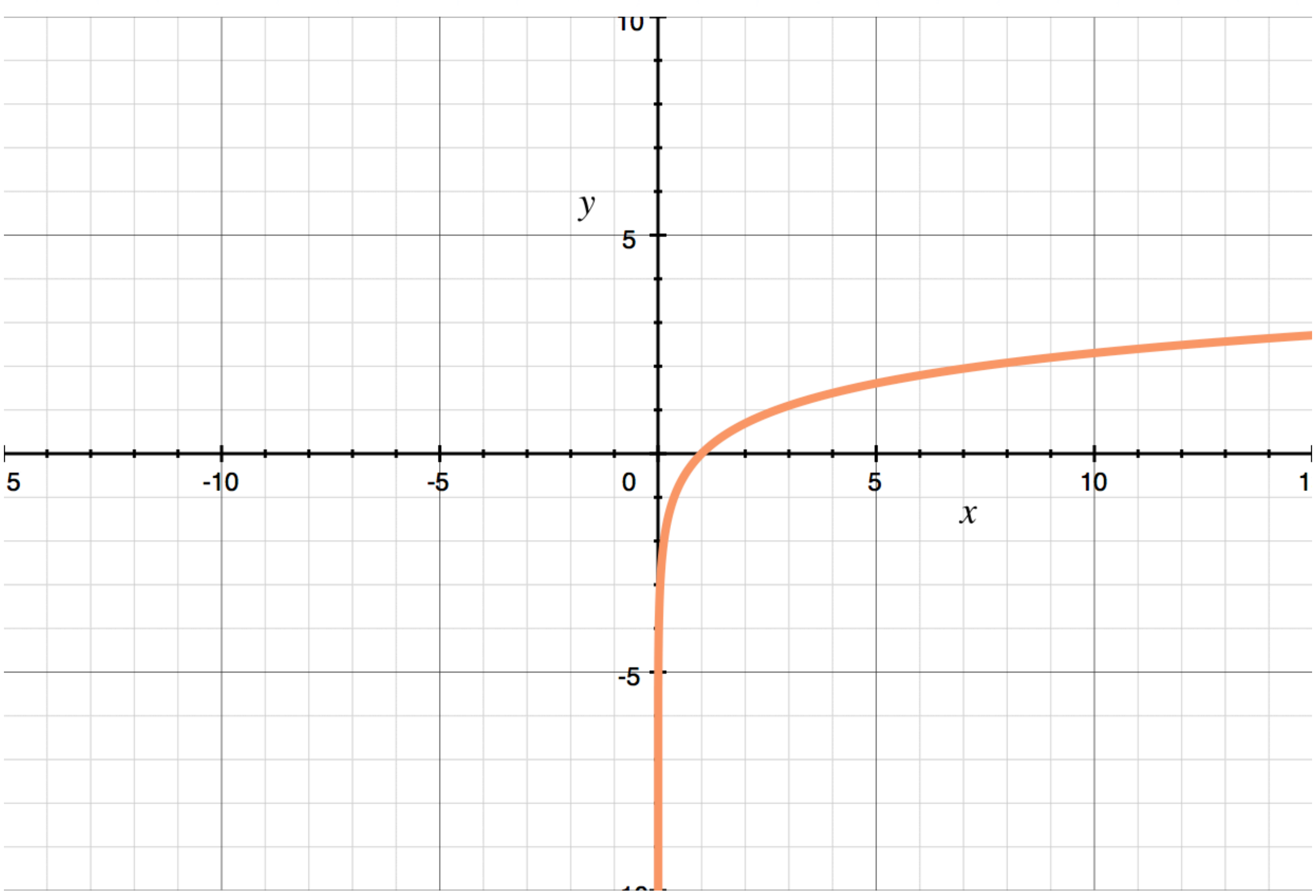
Evaluate over the interval.

$$\lim_{c \to \pi^+} \left( \ln \left| \sin \frac{5\pi}{4} \right| - \ln |\sin c| \right)$$

$$\lim_{c \to \pi^+} \left( \ln \frac{\sqrt{2}}{2} - \ln |\sin c| \right)$$

$$\ln \frac{\sqrt{2}}{2} - \ln |\sin \pi|$$

When we look at $\ln \sin \pi = \ln 0$, we know that $\ln 0$ is undefined. If we look at the graph of the natural logarithm, we can see that the value approaches $-\infty$.



Therefore, we can evaluate the limit using the graph.

$$\ln\frac{\sqrt{2}}{2} - (-\infty)$$

$$\infty$$

**Topic**: Improper integrals, case 5

**Question**: Evaluate the improper integral.

$$\int_6^{15} \frac{8}{\sqrt{x-6}}\, dx$$

**Answer choices**:

A $48$

B $-\infty$

C $-48$

D $\infty$

**Solution**: A

The integral in this problem is considered to be an improper integral, case 5, because the integrand is undefined at the lower limit limit of integration. Evaluating this type of improper integral follows this general rule:

$$\int_a^b f(x)\ dx = \lim_{c \to a^+} \int_c^b f(x)\ dx$$

Let's begin by re-writing the integral using the rule.

$$\int_6^{15} \frac{8}{\sqrt{x-6}}\ dx = \lim_{c \to 6^+} \int_c^{15} \frac{8}{\sqrt{x-6}}\ dx$$

$$8 \lim_{c \to 6^+} \int_c^{15} (x-6)^{-\frac{1}{2}}\ dx$$

Integrate.

$$8 \lim_{c \to 6^+} 2(x-6)^{\frac{1}{2}} \Big|_c^{15}$$

$$16 \lim_{c \to 6^+} \sqrt{x-6} \Big|_c^{15}$$

Evaluate over the interval.

$$16 \lim_{c \to 6^+} \left(\sqrt{15-6} - \sqrt{c-6}\right)$$

$$16 \lim_{c \to 6^+} \left(3 - \sqrt{c-6}\right)$$

$$16\left(3-\sqrt{6-6}\right)$$

$$16(3-0)$$

$$48$$