Parametric arc length

The arc length of a parametric curve over the interval $\alpha \le t \le \beta$ is given by

$$L = \int_{\alpha}^{\beta} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\ dt$$

where α and β are the limits of the interval

where dx/dt is the derivative of $x(t)$

where dy/dt is the derivative of $y(t)$

Example

Find the length of the parametric curve.

$$x = 5\sin t$$

$$y = 5\cos t$$

for $0 \le t \le 2\pi$

We need to find the derivatives of the parametric equations.

$$x = 5\sin t$$

$$\frac{dx}{dt} = 5\cos t$$

and

$$y = 5\cos t$$

$$\frac{dy}{dt} = -5\sin t$$

Since we were given the limits of integration in the problem, we're ready to plug everything into the arc length formula.

$$L = \int_0^{2\pi} \sqrt{(5\cos t)^2 + (-5\sin t)^2}\ dt$$

$$L = \int_0^{2\pi} \sqrt{25\cos^2 t + 25\sin^2 t}\ dt$$

$$L = \int_0^{2\pi} \sqrt{25\left(\cos^2 t + \sin^2 t\right)}\ dt$$

Since $\cos^2 t + \sin^2 t = 1$, we get

$$L = \int_0^{2\pi} \sqrt{25(1)}\ dt$$

$$L = \int_0^{2\pi} 5\ dt$$

$$L = 5t\Big|_0^{2\pi}$$

$$L = 5(2\pi) - 5(0)$$

$$L = 10\pi$$