Topic: Income stream, compounded continuously, present value

Question: If $\$25{,}000$ is deposited into an account every year for 7 years and the account pays $6.25\,\%$, compounded continuously, find the present value of the account.

Answer choices:

A $\$217{,}483.98$

B $\$219{,}532.12$

C $\$142{,}897.16$

D $\$141{,}740.59$

Solution: D

Plugging what we know into the present value formula for a continuous income stream, we get

$$PV = \int_0^T S(t)e^{-rt}\ dt$$

$$PV = \int_0^7 25{,}000e^{-0.0625t}\ dt$$

Integrate, then evaluate over the interval.

$$PV = -\frac{25{,}000}{0.0625}e^{-0.0625t}\Big|_0^7$$

$$PV = -\frac{25{,}000}{0.0625}e^{-0.0625(7)} - \left(-\frac{25{,}000}{0.0625}e^{-0.0625(0)}\right)$$

$$PV = -\frac{25{,}000}{0.0625}e^{-0.4375} + \frac{25{,}000}{0.0625}(1)$$

$$PV \approx \$141{,}740.59$$

Topic: Income stream, compounded continuously, present value

Question: Suppose that money is deposited steadily into an account at a constant rate of $\$1{,}250$ per year for 5 years. Find the present value of this income stream if the account pays $8.35\ \%$, compounded continuously.

Answer choices:

A $\$7{,}684.76$

B $\$5{,}109.41$

C $\$7{,}756.91$

D $\$5{,}151.25$

Solution: B

Plugging what we know into the present value formula for a continuous income stream, we get

$$PV = \int_0^T S(t)e^{-rt}\ dt$$

$$PV = \int_0^5 1{,}250e^{-0.0835t}\ dt$$

Integrate, then evaluate over the interval.

$$PV = -\frac{1{,}250}{0.0835}e^{-0.0835t}\Big|_0^5$$

$$PV = -\frac{1{,}250}{0.0835}e^{-0.0835(5)} - \left(-\frac{1{,}250}{0.0835}e^{-0.0835(0)}\right)$$

$$PV = -\frac{1{,}250}{0.0835}e^{-0.4175} + \frac{1{,}250}{0.0835}$$

$$PV \approx \$5{,}109.41$$

Topic: Income stream, compounded continuously, present value

Question: Suppose that money is deposited steadily into an account at a constant rate of $\$5{,}600$ per year for 12 years. Find the present value of this income stream if the account pays 4.9%, compounded continuously.

Answer choices:

A $\$91{,}472.46$

B $\$50{,}533.10$

C $\$50{,}807.19$

D $\$92{,}077.31$

Solution: C

Plugging what we know into the present value formula for a continuous income stream, we get

$$PV = \int_0^T S(t)e^{-rt}\ dt$$

$$PV = \int_0^{12} 5{,}600e^{-0.049t}\ dt$$

Integrate, then evaluate over the interval.

$$PV = -\frac{5{,}600}{0.049}e^{-0.049t}\Big|_0^{12}$$

$$PV = -\frac{5{,}600}{0.049}e^{-0.049(12)} - \left(-\frac{5{,}600}{0.049}e^{-0.049(0)}\right)$$

$$PV = -\frac{5{,}600}{0.049}e^{-0.588} + \frac{5{,}600}{0.049}$$

$$PV \approx \$50{,}807.19$$