Calculus 2 Workbook Solutions

Alternating series test

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ALTERNATING SERIES TEST

1. Use the alternating series test to say whether the series converges or diverges.

$$\sum_{n=1}^{\infty} (-1)^{n-1} \left(\frac{3}{5n+6} \right)$$

Solution:

An alternating series will only converge if the series is decreasing. Check the first few terms of the series.

$$a_n = \frac{3}{5n+6}$$

$$a_1 = \frac{3}{5(1)+6} = \frac{3}{5+6} = \frac{3}{11}$$

$$a_2 = \frac{3}{5(2)+6} = \frac{3}{10+6} = \frac{3}{17}$$

$$a_3 = \frac{3}{5(3)+6} = \frac{3}{15+6} = \frac{3}{21}$$

$$a_4 = \frac{3}{5(4)+6} = \frac{3}{20+6} = \frac{3}{26}$$

We can see that the series is decreasing. The limit as $n \to \infty$ must also be 0 if the alternating series is going to converge.

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{3}{5n+6} = \frac{3}{5(\infty)+6} = \frac{3}{\infty} = 0$$

Because these two conditions are met, the alternating series converges.

2. Use the alternating series test to say whether the series converges or diverges.

$$\sum_{n=1}^{\infty} (-1)^{n-1} n \left(\frac{2}{7}\right)^n$$

Solution:

An alternating series will only converge if the series is decreasing. Check the first few terms of the series.

$$a_n = n\left(\frac{2}{7}\right)^n$$

$$a_1 = 1\left(\frac{2}{7}\right)^1 = \frac{2}{7} \approx 0.2857$$

$$a_2 = 2\left(\frac{2}{7}\right)^2 = 2 \cdot \frac{4}{49} = \frac{8}{49} \approx 0.1633$$

$$a_3 = 3\left(\frac{2}{7}\right)^3 = 3 \cdot \frac{8}{343} = \frac{24}{343} \approx 0.0988$$

$$a_4 = 4\left(\frac{2}{7}\right)^4 = 4 \cdot \frac{16}{2401} = \frac{64}{2401} \approx 0.0267$$

$$a_5 = 5\left(\frac{2}{7}\right)^5 = 5 \cdot \frac{32}{16{,}807} = \frac{160}{16807} \approx 0.0095$$

$$a_6 = 6\left(\frac{2}{7}\right)^6 = 6 \cdot \frac{64}{117{,}649} = \frac{384}{117{,}649} \approx 0.0033$$

$$a_7 = 7\left(\frac{2}{7}\right)^7 = 7 \cdot \frac{128}{823{,}453} = \frac{896}{823{,}453} \approx 0.0011$$

$$a_8 = 8\left(\frac{2}{7}\right)^8 = 8 \cdot \frac{256}{5{,}764{,}801} = \frac{800}{5{,}764{,}801} \approx 0.00039$$

We can see that the series is decreasing. The limit as $n \to \infty$ must also be 0 if the alternating series is going to converge.

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} n\left(\frac{2}{7}\right)^n = \lim_{n\to\infty} \frac{n}{\left(\frac{7}{2}\right)^n}$$

Use L'Hospital's Rule to evaluate the limit.

$$\lim_{n\to\infty} \frac{n}{\left(\frac{7}{2}\right)^n} = \lim_{n\to\infty} \frac{1}{\left(\frac{7}{2}\right)^n \ln\left(\frac{7}{2}\right)} = 0$$

Because these two conditions are met, the alternating series converges.

3. Use the alternating series test to say whether the series converges or diverges.

$$\sum_{n=3}^{\infty} (-1)^{n+1} \frac{n^3}{n!}$$

Solution:

An alternating series will only converge if the series is decreasing. Check the first few terms of the series.

$$a_n = \frac{n^3}{n!}$$

$$a_3 = \frac{3^3}{3!} = \frac{27}{6} \approx 4.5$$

$$a_4 = \frac{4^3}{4!} = \frac{81}{24} \approx 3.375$$

$$a_5 = \frac{5^3}{5!} = \frac{125}{120} \approx 1.042$$

$$a_6 = \frac{6^3}{6!} = \frac{216}{720} \approx 0.3$$

$$a_7 = \frac{7^3}{7!} = \frac{343}{5040} \approx 0.681$$

We can see that the series is decreasing. The limit as $n \to \infty$ must also be 0 if the alternating series is going to converge.

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} (-1)^{n-1}\frac{n^3}{n!}$$

Use the ratio test to find the limit.

$$\lim_{n\to\infty} (-1)^{n-1}\frac{n^3}{n!}$$

$$\lim_{n\to\infty} \left| \frac{(-1)^{n+1-1}\frac{(n+1)^3}{(n+1)!}}{(-1)^{n-1}\frac{n^3}{n!}} \right|$$

$$\lim_{n\to\infty} \left| \frac{\frac{(n+1)^3}{(n+1)!}}{\frac{n^3}{n!}} \right|$$

$$\lim_{n\to\infty} \left| \frac{(n+1)^3}{(n+1)!} \cdot \frac{n!}{n^3} \right|$$

$$\lim_{n\to\infty} \left| \frac{(n+1)^3}{(n+1)n!} \cdot \frac{n!}{n^3} \right|$$

$$\lim_{n\to\infty} \left| \frac{(n+1)^3}{(n+1)} \cdot \frac{1}{n^3} \right|$$

$$\lim_{n\to\infty} \left| \frac{(n+1)^3}{n^3(n+1)} \right|$$

$$\lim_{n\to\infty}\left|\frac{n+3n^2+3n+1}{n^4+n^3}\right|$$

$$\lim_{n\to\infty}\left|\frac{\frac{n}{n^4}+\frac{3n^2}{n^4}+\frac{3n}{n^4}+\frac{1}{n^4}}{\frac{n^4}{n^4}+\frac{n^3}{n^4}}\right|$$

$$\lim_{n\to\infty}\left|\frac{\frac{1}{n^3}+\frac{3}{n^2}+\frac{3}{n^3}+\frac{1}{n^4}}{1+\frac{1}{n}}\right|$$

$$\left|\frac{0+0+0+0}{1+0}\right|$$

$$0$$

Because these two conditions are met, the alternating series converges.

ALTERNATING SERIES ESTIMATION THEOREM

1. Approximate the sum of the alternating series to three decimal places, using the first 5 terms. Then find the remainder of the approximation, to the nearest six decimal places.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1} n^3}{12^n}$$

Solution:

The first several terms of the series are

$$a_n = \frac{(-1)^{n-1} n^3}{12^n}$$

$$a_1 = \frac{(-1)^{1-1}(1)^3}{12^1} = \frac{1 \cdot 1}{12} = \frac{1}{12} \approx 0.083333$$

$$a_2 = \frac{(-1)^{2-1}(2)^3}{12^2} = \frac{-1 \cdot 8}{144} = \frac{-8}{144} = -\frac{1}{18} \approx -0.055556$$

$$a_3 = \frac{(-1)^{3-1}(3)^3}{12^3} = \frac{1 \cdot 27}{1728} = \frac{27}{1728} = \frac{1}{64} \approx 0.015625$$

$$a_4 = \frac{(-1)^{4-1}(4)^3}{12^4} = \frac{-1 \cdot 81}{20736} = -\frac{81}{20736} = -\frac{1}{256} \approx -0.003086$$

$$a_5 = \frac{(-1)^{5-1}(5)^3}{12^5} = \frac{1 \cdot 125}{248{,}832} = \frac{125}{248{,}832} \approx 0.0005023$$

$$a_6 = \frac{(-1)^{6-1}(6)^3}{12^6} = \frac{-1 \cdot 216}{2,985,984} = -\frac{216}{2,985,984} = -\frac{27}{373,248} \approx -0.0000723$$

Then the first five partial sums are

$$s_1 = a_1 = 0.08333$$

$$s_2 = a_1 + a_2 = 0.08333 - 0.05556 = 0.02777$$

$$s_3 = a_1 + a_2 + a_3 = 0.08333 - 0.05556 + 0.015625 = 0.043395$$

$$s_4 = a_1 + a_2 + a_3 + a_4$$

$$= 0.08333 - 0.05556 + 0.015625 - 0.003086 = 0.04031$$

$$s_5 = a_1 + a_2 + a_3 + a_4 + a_5$$

$$= 0.08333 - 0.05556 + 0.015625 - 0.003086 + 0.000502 = 0.040811$$

$$s_6 = a_1 + a_2 + a_3 + a_4 + a_5 + a_6$$

$$= 0.08333 - 0.05556 + 0.015625 - 0.003086 + 0.000502 - 0.0000723$$

$$= 0.040739$$

The approximation to three decimal places is $s = 0.041$. Verify that the series is decreasing, $b_{n+1} \leq b_n$.

$$b_n = \frac{n^3}{12^n}$$

$$b_1 = \frac{(1)^3}{12^1} = \frac{1}{12} \approx 0.08333$$

$$b_2 = \frac{(2)^3}{12^2} = \frac{8}{144} = \frac{8}{144} = \frac{1}{18} \approx 0.05556$$

$$b_3 = \frac{(3)^3}{12^3} = \frac{27}{1728} = \frac{1}{64} \approx 0.015625$$

$$b_4 = \frac{(4)^3}{12^4} = \frac{64}{20{,}736} = \frac{1}{324} \approx 0.003086$$

$$b_5 = \frac{(5)^3}{12^5} = \frac{125}{248{,}832} \approx 0.000502$$

$$b_6 = \frac{(6)^3}{12^6} = \frac{216}{2{,}985{,}984} \approx 0.0000723$$

Verify that the limit as $n \to \infty$ is 0. Use L'Hospital's rule.

$$\lim_{n\to\infty} \frac{n^3}{12^n} = \lim_{n\to\infty} \frac{3n^2}{12^n \ln 12} = \lim_{n\to\infty} \frac{6n}{12^n (\ln 12)^2} = \lim_{n\to\infty} \frac{6}{12^n (\ln 12)^3} = \frac{6}{\infty} = 0$$

Find the remainder.

$$\left|R_n\right| = \left|S - S_n\right| \le b_{n+1}$$

$$\left|R_5\right| = \left|S - S_5\right| \le b_{5+1}$$

$$\left|R_5\right| \le b_6$$

$$\left|R_5\right| \le 0.0000723$$

So the approximation of the sum of the alternating series is $S_5 \approx 0.041$, with an error of $\left|R_5\right| \le 0.0000723$.

2. Approximate the sum of the alternating series to three decimal places, using the first 12 terms. Then find the remainder of the approximation, to the nearest six decimal places.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{3n^3}$$

Solution:

The first several terms of the series are

$$a_n = \frac{(-1)^{n+1}}{3n^3}$$

$$a_1 = \frac{(-1)^{1+1}}{3(1)^3} = \frac{1}{3} \approx 0.333333$$

$$a_2 = \frac{(-1)^{2+1}}{3(2)^3} = \frac{-1}{24} \approx -0.041667$$

$$a_3 = \frac{(-1)^{3+1}}{3(3)^3} = \frac{1}{81} \approx 0.012346$$

$$a_4 = \frac{(-1)^{4+1}}{3(4)^3} = \frac{-1}{192} \approx -0.005208$$

$$a_5 = \frac{(-1)^{5+1}}{3(5)^3} = \frac{1}{729} \approx 0.002667$$

$$a_6 = \frac{(-1)^{6+1}}{3(6)^3} = \frac{-1}{648} \approx -0.001543$$

$$a_7 = \frac{(-1)^{7+1}}{3(7)^3} = \frac{1}{1029} \approx 0.000972$$

$$a_8 = \frac{(-1)^{8+1}}{3(8)^3} = \frac{-1}{1536} \approx -0.000651$$

$$a_9 = \frac{(-1)^{9+1}}{3(9)^3} = \frac{1}{2187} \approx 0.000457$$

$$a_{10} = \frac{(-1)^{10+1}}{3(10)^3} = \frac{-1}{3000} \approx -0.000333$$

$$a_{11} = \frac{(-1)^{11+1}}{3(11)^3} = \frac{1}{3993} \approx 0.000250$$

$$a_{12} = \frac{(-1)^{12+1}}{3(12)^3} = \frac{-1}{5184} \approx -0.000193$$

$$a_{13} = \frac{(-1)^{13+1}}{3(13)^3} = \frac{1}{6591} \approx 0.000152$$

Then the first five partial sums are

$$s_1 = a_1 = 0.333333$$

$$s_2 = a_1 + a_2 = 0.333333 - 0.041667 = 0.291666$$

$$s_3 = a_1 + a_2 + a_3 = 0.333333 - 0.041667 + 0.012346 = 0.304012$$

$$s_4 = a_1 + a_2 + a_3 + a_4$$

$$= 0.333333 - 0.041667 + 0.012346 - 0.005208 = 0.298804$$

$$s_5 = a_1 + a_2 + a_3 + a_4 + a_5$$

$$= 0.333333 - 0.041667 + 0.012346 - 0.005208 + 0.002667 = 0.301471$$

$$s_6 = a_1 + a_2 + a_3 + a_4 + a_5 + a_6$$

$$= 0.333333 - 0.041667 + 0.012346 - 0.005208 + 0.002667 - 0.001543$$

$$= 0.299928$$

$$s_7 = a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7$$

$$= 0.333333 - 0.041667 + 0.012346 - 0.005208$$

$$+0.002667 - 0.001543 + 0.000972$$

$$= 0.300900$$

$$s_8 = a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8$$

$$= 0.333333 - 0.041667 + 0.012346 - 0.005208$$

$$+0.002667 - 0.001543 + 0.000972 - 0.000651$$

$$= 0.300249$$

$$s_9 = a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8 + a_9$$

$$= 0.333333 - 0.041667 + 0.012346 - 0.005208 + 0.002667$$

$$-0.001543+0.000972-0.000651+0.000457$$

$$=0.300706$$

$$s_{10}=a_1+a_2+a_3+a_4+a_5+a_6+a_7+a_8+a_9+a_{10}$$

$$=0.333333-0.041667+0.012346-0.005208+0.002667$$

$$-0.001543+0.000972-0.000651+0.000457-0.000333$$

$$=0.300373$$

$$s_{11}=a_1+a_2+a_3+a_4+a_5+a_6+a_7+a_8+a_9+a_{10}+a_{11}$$

$$=0.333333-0.041667+0.012346-0.005208+0.002667-0.001543$$

$$+0.000972-0.000651+0.000457-0.000333+0.000250$$

$$=0.300623$$

$$s_{12}=a_1+a_2+a_3+a_4+a_5+a_6+a_7+a_8+a_9+a_{10}+a_{11}+a_{12}$$

$$=0.333333-0.041667+0.012346-0.005208$$

$$+0.002667-0.001543+0.000972-0.000651$$

$$+0.000457-0.000333+0.000250-0.000193$$

$$=0.300430$$

$$s_{13}=a_1+a_2+a_3+a_4+a_5+a_6+a_7+a_8+a_9+a_{10}+a_{11}+a_{12}+a_{13}$$

$$=0.333333-0.041667+0.012346-0.005208$$

$$+0.002667 - 0.001543 + 0.000972 - 0.000651$$

$$+0.000457 - 0.000333 + 0.000250 - 0.000193 + 0.000152$$

$$= 0.300195$$

The approximation to three decimal places is $s = 0.300$. Verify that the series is decreasing, $b_{n+1} \leq b_n$.

$$b_n = \frac{1}{3n^3}$$

$$b_1 = \frac{1}{3(1)^3} = \frac{1}{3} \approx 0.333333$$

$$b_2 = \frac{1}{3(2)^3} = \frac{1}{24} \approx 0.041667$$

$$b_3 = \frac{1}{3(3)^3} = \frac{1}{81} \approx 0.012346$$

$$b_4 = \frac{1}{3(4)^3} = \frac{1}{192} \approx 0.005208$$

$$b_5 = \frac{1}{3(5)^3} = \frac{1}{729} \approx 0.002667$$

$$b_6 = \frac{1}{3(6)^3} = \frac{1}{648} \approx 0.001543$$

$$b_7 = \frac{1}{3(7)^3} = \frac{1}{1029} \approx 0.000972$$

$$b_8 = \frac{1}{3(8)^3} = \frac{1}{1536} \approx 0.000651$$

$$b_9 = \frac{1}{3(9)^3} = \frac{1}{2187} \approx 0.000457$$

$$b_{10} = \frac{1}{3(10)^3} = \frac{1}{3000} \approx 0.000333$$

$$b_{11} = \frac{1}{3(11)^3} = \frac{1}{3993} \approx 0.000250$$

$$b_{12} = \frac{1}{3(12)^3} = \frac{1}{5184} \approx 0.000193$$

$$b_{13} = \frac{1}{3(13)^3} = \frac{1}{6591} \approx 0.000152$$

Verify that the limit as $n \to \infty$ is 0.

$$\lim_{n\to\infty} \frac{1}{3n^3} = \frac{1}{3} \lim_{n\to\infty} \frac{1}{n^3} = \frac{1}{3} \cdot 0 = 0$$

Find the remainder.

$$\left|R_n\right| = \left|S - S_n\right| \le b_{n+1}$$

$$\left|R_{12}\right| = \left|S - S_{10}\right| \le b_{12+1}$$

$$\left|R_{12}\right| \le b_{13}$$

$$\left|R_{12}\right| \le 0.000152$$

So the approximation of the sum of the alternating series is $S_{10} \approx 0.300$, with an error of $\left|R_{10}\right| \le 0.000152$.

3. Approximate the sum of the alternating series to three decimal places, using the first 10 terms. Then find the remainder of the approximation.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1} \cdot 3}{12n^3 + 4n^2}$$

Solution:

The first several terms of the series are

$$a_n = \frac{(-1)^{n-1} \cdot 3}{12n^3 + 4n^2}$$

$$a_1 = \frac{(-1)^{1-1} \cdot 3}{12(1)^3 + 4(1)^2} = \frac{1 \cdot 3}{12 + 4} = \frac{3}{16} = 0.187500$$

$$a_2 = \frac{(-1)^{2-1} \cdot 3}{12(2)^3 + 4(2)^2} = \frac{-1 \cdot 3}{96 + 16} = -\frac{3}{112} \approx -0.026786$$

$$a_3 = \frac{(-1)^{3-1} \cdot 3}{12(3)^3 + 4(3)^2} = \frac{1 \cdot 3}{324 + 36} = \frac{3}{360} \approx 0.008333$$

$$a_4 = \frac{(-1)^{4-1} \cdot 3}{12(4)^3 + 4(4)^2} = \frac{-1 \cdot 3}{768 + 64} = -\frac{3}{832} \approx -0.003606$$

$$a_5 = \frac{(-1)^{5-1} \cdot 3}{12(5)^3 + 4(5)^2} = \frac{1 \cdot 3}{1500 + 100} = \frac{3}{1600} \approx 0.001875$$

$$a_6 = \frac{(-1)^{6-1} \cdot 3}{12(6)^3 + 4(6)^2} = \frac{-1 \cdot 3}{2592 + 144} = -\frac{3}{2736} \approx -0.001096$$

$$a_7 = \frac{(-1)^{7-1} \cdot 3}{12(7)^3 + 4(7)^2} = \frac{1 \cdot 3}{4116 + 196} = \frac{3}{4312} \approx 0.000696$$

$$a_8 = \frac{(-1)^{8-1} \cdot 3}{12(8)^3 + 4(8)^2} = \frac{-1 \cdot 3}{6144 + 256} = -\frac{3}{6400} \approx -0.000469$$

$$a_9 = \frac{(-1)^{9-1} \cdot 3}{12(9)^3 + 4(9)^2} = \frac{1 \cdot 3}{8748 + 324} = \frac{3}{9072} \approx 0.000331$$

$$a_{10} = \frac{(-1)^{10-1} \cdot 3}{12(10)^3 + 4(10)^2} = \frac{-1 \cdot 3}{12000 + 400} = -\frac{3}{12400} \approx -0.000242$$

$$a_{11} = \frac{(-1)^{11-1} \cdot 3}{12(11)^3 + 4(11)^2} = \frac{1 \cdot 3}{15972 + 484} = \frac{3}{16456} \approx 0.000182$$

Then the first five partial sums are

$$s_1 = a_1 = 0.187500$$

$$s_2 = a_1 + a_2 = 0.1875 - 0.026786 = 0.160714$$

$$s_3 = a_1 + a_2 + a_3 = 0.1875 - 0.026786 + 0.008333 = 0.169047$$

$$s_4 = a_1 + a_2 + a_3 + a_4$$

$$= 0.1875 - 0.026786 + 0.008333 - 0.003606 = 0.165441$$

$$s_5 = a_1 + a_2 + a_3 + a_4 + a_5$$

$$= 0.1875 - 0.026786 + 0.008333 - 0.003606 + 0.001875 = 0.167316$$

$$s_6 = a_1 + a_2 + a_3 + a_4 + a_5 + a_6$$

$$= 0.1875 - 0.026786 + 0.008333 - 0.003606 + 0.001875 - 0.001096$$

$$= 0.166220$$

$$s_7 = a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7$$

$$= 0.1875 - 0.026786 + 0.008333 - 0.003606$$

$$+0.001875 - 0.001096 + 0.000696$$

$$= 0.166916$$

$$s_8 = a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8$$

$$= 0.1875 - 0.026786 + 0.008333 - 0.003606$$

$$+0.001875 - 0.001096 + 0.000696 - 0.000469$$

$$= 0.166447$$

$$s_9 = a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8 + a_9$$

$$= 0.1875 - 0.026786 + 0.008333 - 0.003606 + 0.001875$$

$$-0.001096 + 0.000696 - 0.000469 + 0.000331$$

$$= 0.166778$$

$$s_{10} = a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8 + a_9 + a_{10}$$

$$= 0.1875 - 0.026786 + 0.008333 - 0.003606 + 0.001875$$

$$-0.001096 + 0.000696 - 0.000469 + 0.000331 - 0.000242$$

$$= 0.166536'$$

$$s_{11} = a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8 + a_9 + a_{10} + a_{11}$$

$$= 0.1875 - 0.026786 + 0.008333 - 0.003606$$

$$+0.001875 - 0.001096 + 0.000696 - 0.000469$$

$$+0.000331 - 0.000242 + 0.000182$$

$$= 0.166718$$

The approximation to three decimal places is $s = 0.167$. Verify that the series is decreasing, $b_{n+1} \leq b_n$.

$$b_n = \frac{3}{12n^3 + 4n^2}$$

$$b_1 = \frac{3}{12(1)^3 + 4(1)^2} = \frac{3}{12 + 4} = \frac{3}{16} = 0.187500$$

$$b_2 = \frac{3}{12(2)^3 + 4(2)^2} = \frac{3}{96 + 16} = \frac{3}{112} \approx 0.026786$$

$$b_3 = \frac{3}{12(3)^3 + 4(3)^2} = \frac{3}{324 + 36} = \frac{3}{360} \approx 0.008333$$

$$b_4 = \frac{3}{12(4)^3 + 4(4)^2} = \frac{3}{768 + 64} = \frac{3}{832} \approx 0.003606$$

$$b_5 = \frac{3}{12(5)^3 + 4(5)^2} = \frac{3}{1500 + 100} = \frac{3}{1600} \approx 0.001875$$

$$b_6 = \frac{3}{12(6)^3 + 4(6)^2} = \frac{3}{2592 + 144} = \frac{3}{2736} \approx 0.001096$$

$$b_7 = \frac{3}{12(7)^3 + 4(7)^2} = \frac{3}{4116 + 196} = \frac{3}{4312} \approx 0.000696$$

$$b_8 = \frac{3}{12(8)^3 + 4(8)^2} = \frac{3}{6144 + 256} = \frac{3}{6400} \approx 0.000469$$

$$b_9 = \frac{3}{12(9)^3 + 4(9)^2} = \frac{3}{8748 + 324} = \frac{3}{9072} \approx 0.000331$$

$$b_{10} = \frac{3}{12(10)^3 + 4(10)^2} = \frac{3}{12000 + 400} = \frac{3}{12400} \approx 0.000242$$

$$b_{11} = \frac{3}{12(11)^3 + 4(11)^2} = \frac{3}{15972 + 484} = \frac{3}{16456} \approx 0.000182$$

Verify that the limit as $n \to \infty$ is 0. Use L'Hospital's rule.

$$\lim_{n\to\infty} \frac{3}{12n^3 + 4n^2} = \lim_{n\to\infty} \frac{0}{6n^2 + 8n} = 0$$

Find the remainder.

$$\left| R_n \right| = \left| S - S_n \right| \le b_{n+1}$$

$$|R_{10}| = |S - S_{10}| \le b_{10+1}$$

$$|R_{10}| \le b_{11}$$

$$|R_{10}| \le 0.000182$$

So the approximation of the sum of the alternating series is $S_{10} \approx 0.167$, with an error of $|R_{10}| \le 0.000182$.

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