Calculus 1 Workbook Solutions

Combinations and composites

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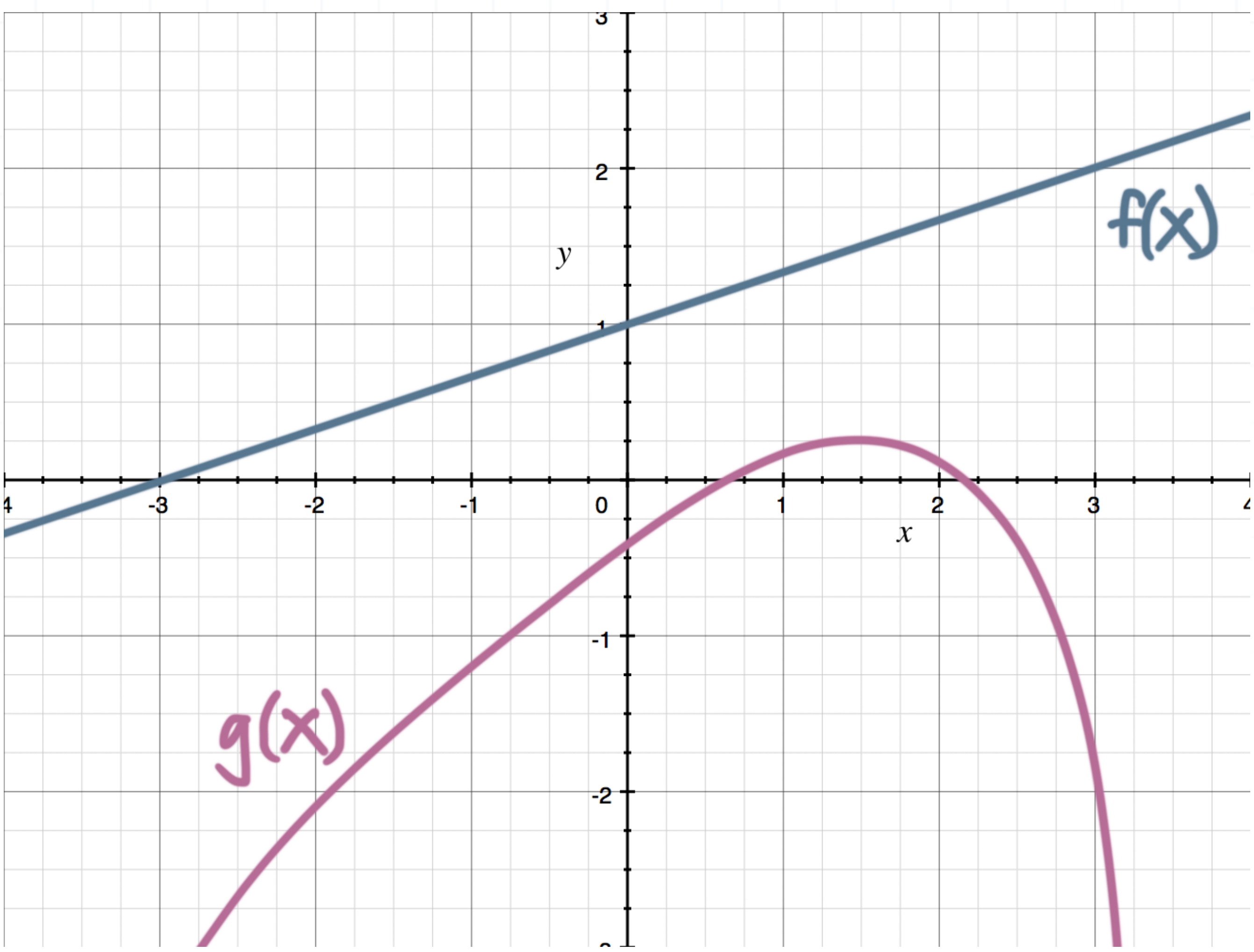
MATH

LIMITS OF COMBINATIONS

■ 1. Use limit laws and the graph below to evaluate the limit.

$$\lim_{x \to 3} \left[4f(x) - 3g(x) \right]$$



Solution:

We can simplify the limit, and then evaluate both functions at $x = 3$.

$$\lim_{x \to 3} \left[4f(x) - 3g(x) \right]$$

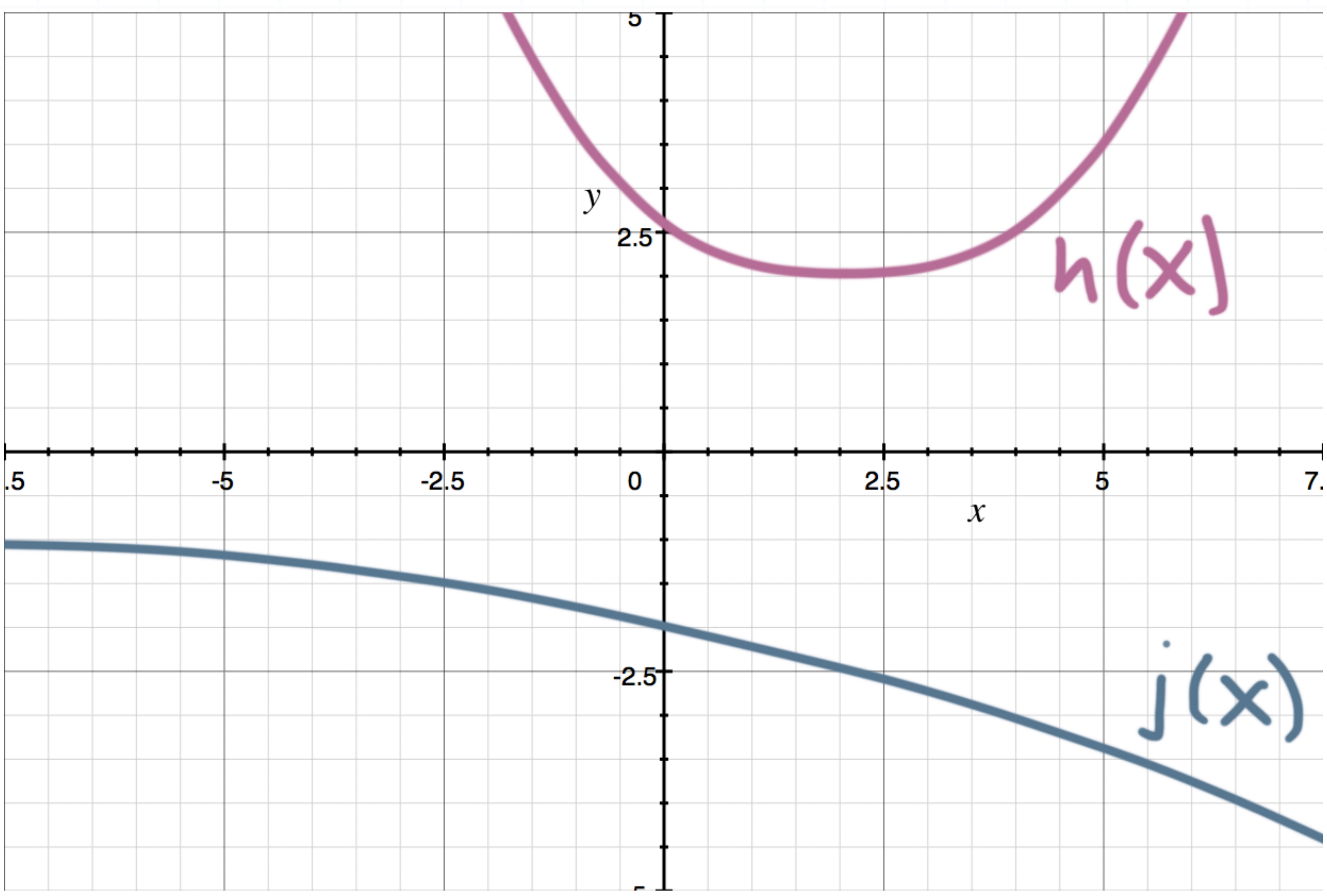
$$4\lim_{x\to3} f(x) - 3\lim_{x\to3} g(x)$$

$$4(2) - 3(-2)$$

$$14$$

■ 2. Use limit laws and the graph below to evaluate the limit.

$$\lim_{x\to4} \frac{h(x)}{j(x)}$$



Solution:

We can simplify the limit, and then evaluate both functions at $x = 4$.

$$\lim_{x\to 4} \frac{h(x)}{j(x)}$$

$$\frac{\lim_{x\to 4} h(x)}{\lim_{x\to 4} j(x)}$$

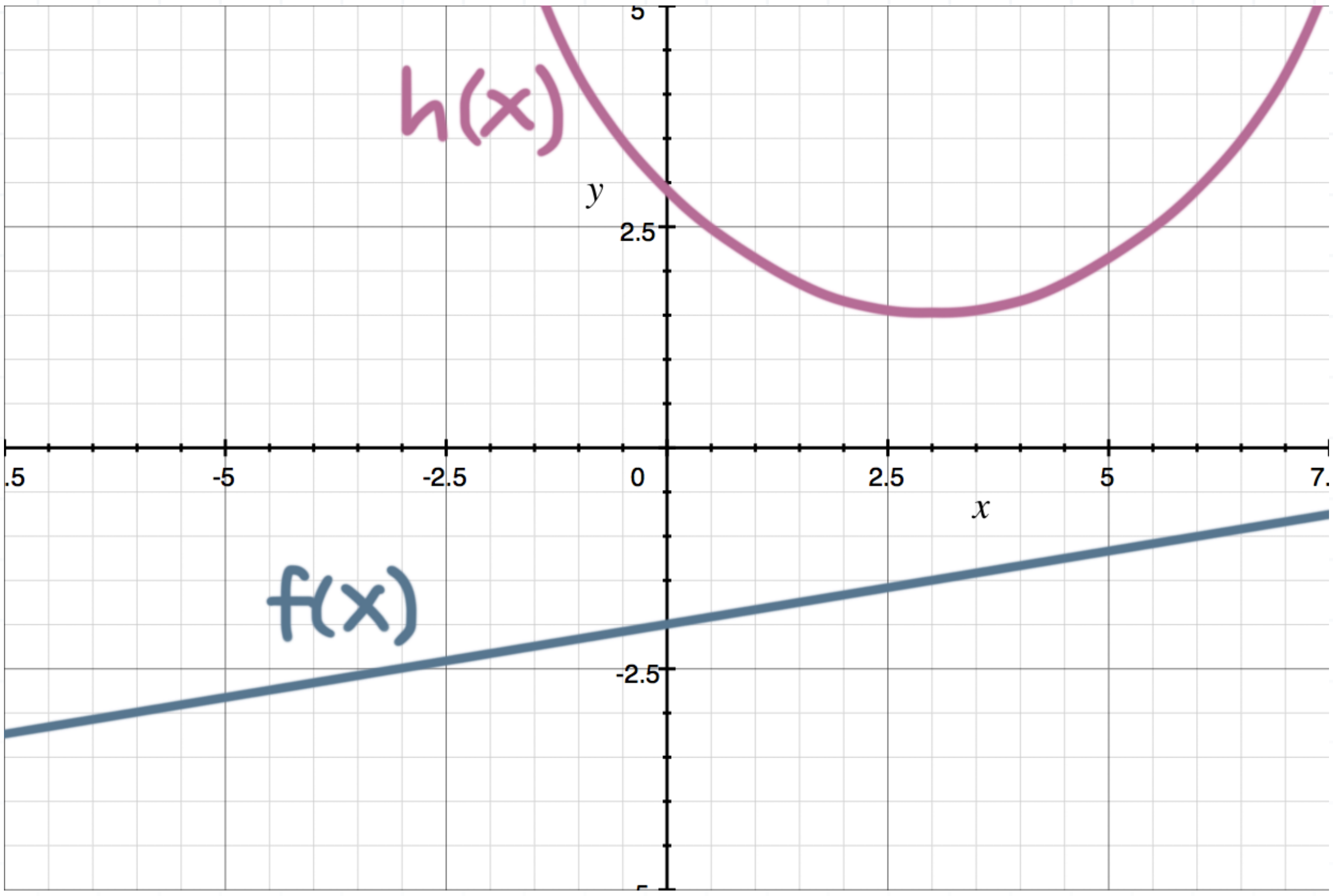
$$\frac{\frac{5}{2}}{-3}$$

$$\frac{5}{2} \cdot \frac{1}{-3}$$

$$-\frac{5}{6}$$

3. Use limit laws and the graph below to evaluate the limit.

$$\lim_{x\to 0} \left[2f(x) \cdot 3h(x)\right]$$



Solution:

We can simplify the limit, and then evaluate both functions at $x = 0$.

$$\lim_{x \to 0} \left[2f(x) \cdot 3h(x) \right]$$

$$2 \lim_{x \to 0} f(x) \cdot 3 \lim_{x \to 0} h(x)$$

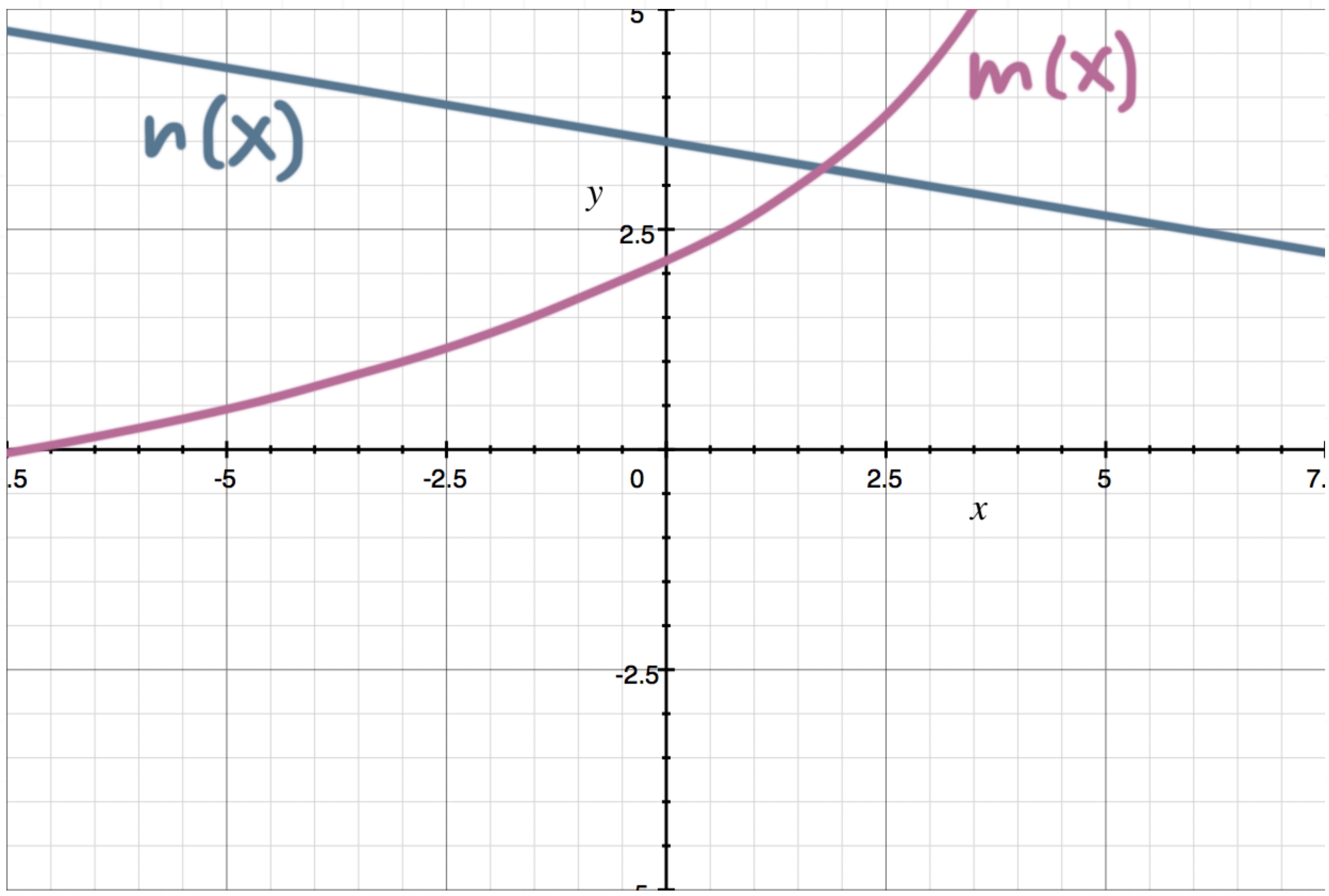
$$6 \lim_{x \to 0} f(x) \cdot \lim_{x \to 0} h(x)$$

$$6(-2)(3)$$

$$-36$$

4. Use limit laws and the graph below to evaluate the limit.

$$\lim_{x \to -3} \left[\frac{5m(x)}{n(x)} - \frac{4m(x)}{n(x)} \right]$$



Solution:

We can simplify the limit, and then evaluate both functions at $x = -3$.

$$\lim_{x \to -3} \left[\frac{5m(x)}{n(x)} - \frac{4m(x)}{n(x)} \right]$$

$$\frac{5 \lim_{x \to -3} m(x)}{\lim_{x \to -3} n(x)} - \frac{4 \lim_{x \to -3} m(x)}{\lim_{x \to -3} n(x)}$$

$$\frac{5(1)}{4} - \frac{4(1)}{4}$$

$$\frac{1}{4}$$

■ 5. Evaluate the limit.

$$\lim_{x \to 6} \left(\sqrt{x-2} + \frac{e^x}{2x+3} - x^2 - 12 \right)$$

Solution:

We'll start by distributing the limit across the combination.

$$\lim_{x \to 6} \sqrt{x-2} + \lim_{x \to 6} \frac{e^x}{2x+3} - \lim_{x \to 6} (x^2 + 12)$$

$$\sqrt{\lim_{x \to 6} (x-2)} + \frac{\lim_{x \to 6} e^x}{\lim_{x \to 6} (2x+3)} - \lim_{x \to 6} (x^2 + 12)$$

Now we'll substitute the value we're approaching into each function.

$$\sqrt{4} + \frac{e^6}{2(6)+3} - (6^2 + 12)$$

$$2 + \frac{e^6}{15} - 48$$

$$\frac{e^6}{15} - 46$$

■ 6. If $f(x) = x^2 + 4$, $g(x) = x - 5$, and $h(x) = -5x$, evaluate the limit.

$$\lim_{x \to 1} \sqrt{\frac{f(x)g(x)}{h(x)}}$$

Solution:

We'll start by distributing the limit across the combination.

$$\sqrt{\lim_{x \to 1} \frac{f(x)g(x)}{h(x)}}$$

$$\sqrt{\frac{\lim_{x \to 1} f(x) \lim_{x \to 1} g(x)}{\lim_{x \to 1} h(x)}}$$

$$\sqrt{\frac{\lim_{x \to 1}(x^2 + 4) \lim_{x \to 1}(x - 5)}{\lim_{x \to 1}(-5x)}}$$

Now we'll substitute the value we're approaching into each function.

$$\sqrt{\frac{(1^2 + 4)(1 - 5)}{(-5(1))}}$$

$$\sqrt{\frac{5(-4)}{-5}}$$

$$\sqrt{\frac{-20}{-5}}$$

$$\sqrt{4}$$

$$2$$

LIMITS OF COMPOSITES

■ 1. What is $\lim_{x\to 3} f(g(x))$ if $f(x) = 4x$ and $g(x) = 6x - 9$?

Solution:

If f is continuous at $x = 3$, then

$$\lim_{x\to 3} f(g(x)) = f\left(\lim_{x\to 3} g(x)\right)$$

$$\lim_{x\to 3} f(g(x)) = f\left(\lim_{x\to 3}(6x - 9)\right)$$

$$\lim_{x\to 3} f(g(x)) = f(6(3) - 9) = f(9) = 4(9) = 36$$

■ 2. What is $\lim_{x\to -4} f(g(x))$ if $f(x) = 2x^2$ and $g(x) = 2x - 1$?

Solution:

If f is continuous at $x = -4$, then

$$\lim_{x\to -4} f(g(x)) = f\left(\lim_{x\to -4} g(x)\right)$$

$$\lim_{x \to -4} f(g(x)) = f\left(\lim_{x \to -4} (2x - 1)\right)$$

$$\lim_{x \to -4} f(g(x)) = f(2(-4) - 1) = f(-9) = 2(-9)^2 = 162$$

■ 3. What is $\lim_{x \to \frac{\pi}{2}} f(g(x))$ if $f(x) = \sin x$ and $g(x) = x/2$?

Solution:

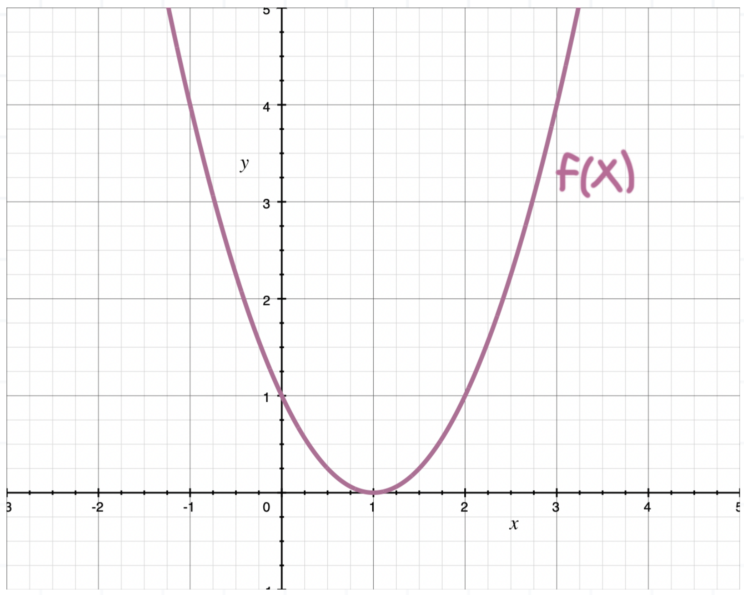
If f is continuous at $x = \pi/2$, then

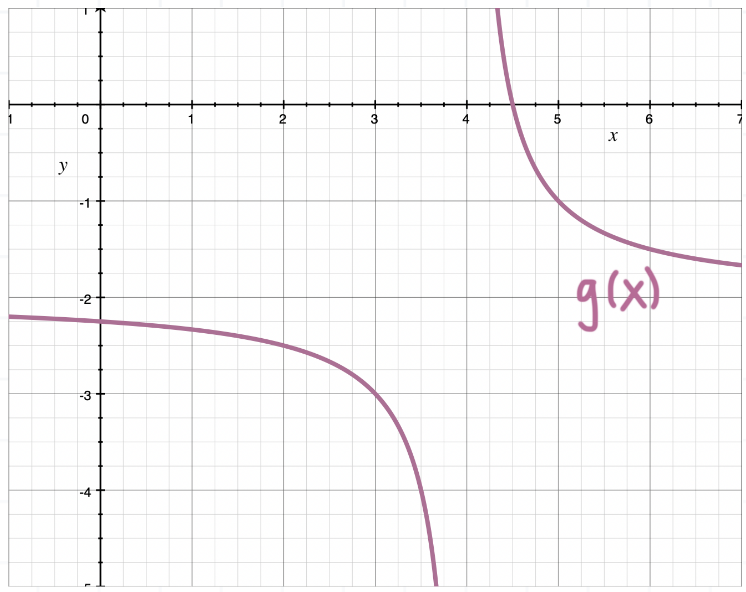
$$\lim_{x \to \frac{\pi}{2}} f(g(x)) = f\left(\lim_{x \to \frac{\pi}{2}} g(x)\right)$$

$$\lim_{x \to \frac{\pi}{2}} f(g(x)) = f\left(\lim_{x \to \frac{\pi}{2}} \frac{x}{2}\right)$$

$$\lim_{x \to \frac{\pi}{2}} f(g(x)) = f\left(\frac{\frac{\pi}{2}}{2}\right) = f\left(\frac{\pi}{4}\right) = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

■ 4. If $f(x)$ and $g(x)$ are graphed below, find $\lim_{x \to 3} g(f(x))$.





Solution:

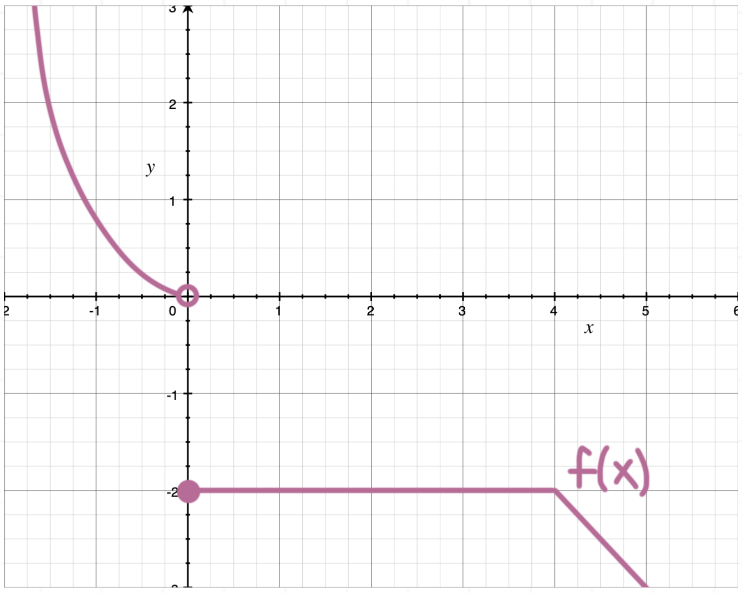
If f is continuous at $x = 3$, then

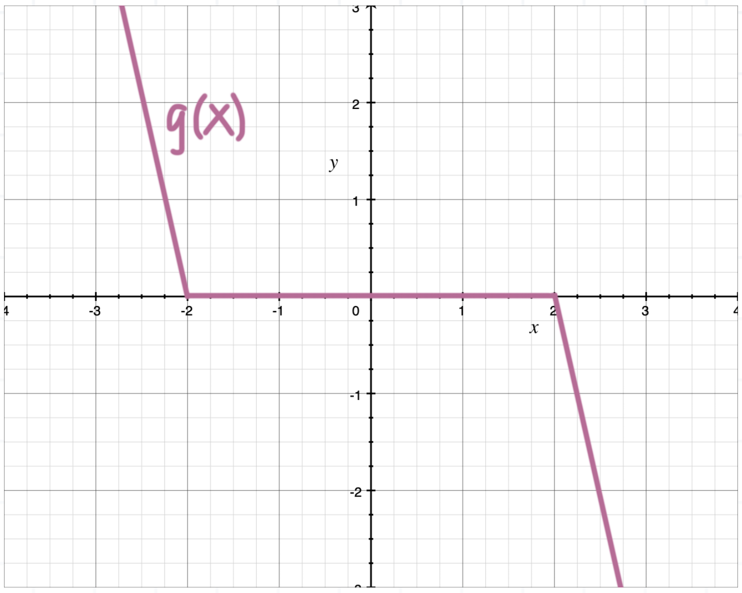
$$\lim_{x \to 3} g(f(x)) = g\left(\lim_{x \to 3} f(x)\right)$$

Because $\lim_{x \to 3} f(x) = 4$,

$$\lim_{x \to 3} g(f(x)) = g(4) = \text{DNE}$$

■ 5. If $f(x)$ and $g(x)$ are graphed below, find $\lim_{x \to 2} g(f(x))$.





Solution:

If f is continuous at $x=2$, then

$$\lim_{x\to 2} g(f(x)) = g\left(\lim_{x\to 2} f(x)\right)$$

Because $\lim_{x\to 2} f(x) = -2$,

$$\lim_{x\to 2} g(f(x)) = g(-2) = 0$$

■ 6. If $f(x)=2x+1$ and $\lim_{x\to 3} h(x) = -2$, find $\lim_{x\to 3} f(h(x))$.

Solution:

If f is continuous at $x=3$, then

$$\lim_{x \to 3} f(h(x)) = f\left(\lim_{x \to 3} h(x)\right)$$

$$\lim_{x \to 3} f(h(x)) = f(-2) = 2(-2) + 1 = -4 + 1 = -3$$

W W W . K R I S T A K I N G M A T H . C O M