Topic: Part 1 of the FTC

Question: Use Part 1 of the Fundamental Theorem of Calculus to find the derivative.

$$f(x) = \int_3^x 5t^3 + 1 \ dt$$

Answer choices:

A $\quad f'(x) = \frac{5}{4}x^4 - x$

B $\quad f'(x) = 5x^3 - 1$

C $\quad f'(x) = \frac{5}{4}x^4 + x$

D $\quad f'(x) = 5x^3 + 1$

Solution: D

We've been given an integral with a constant lower limit of integration, and the variable x as the upper limit of integration.

Given integral	**How to solve it**
$f(x) = \int_a^x f(t)\ dt$	Plug x in for t.
$f(x) = \int_x^a f(t)\ dt$	Reverse limits of integration and multiply by -1, then plug x in for t.
$f(x) = \int_a^{g(x)} f(t)\ dt$	Plug $g(x)$ in for t, then multiply by dg/dx.
$f(x) = \int_{g(x)}^a f(t)\ dt$	Reverse limits of integration and multiply by -1, then plug $g(x)$ in for t and multiply by dg/dx.
$f(x) = \int_{g(x)}^{h(x)} f(t)\ dt$	Split the limits of integration as $\int_{g(x)}^0 f(t)\ dt + \int_0^{h(x)} f(t)\ dt$. Reverse limits of integration on $\int_{g(x)}^0 f(t)\ dt$ and multiply by -1, then plug $g(x)$ and $h(x)$ in for t, multiplying by dg/dx and dh/dx respectively.

Looking at the chart, we can see that this is the situation described by the first row, which means the steps for solving this using FTC Part 1 are:

1. Plug x in for t.

$$f(x) = \int_3^x 5t^3 + 1 \ dt$$

$$f'(x) = 5x^3 + 1$$

Topic: Part 1 of the FTC

Question: Use Part 1 of the Fundamental Theorem of Calculus to find the derivative.

$$f(x) = \int_{2x^2}^{6} 6t^3 - 6t \; dt$$

Answer choices:

A $\quad f'(x) = -\frac{3}{2}x^4 + 3x^2$

B $\quad f'(x) = 48x^3 - 192x^7$

C $\quad f'(x) = 192x^7 - 48x^3$

D $\quad f'(x) = \frac{3}{2}x^4 - 3x^2$

Solution: B

We've been given an integral with a function as the lower limit of integration, and a constant as the upper limit of integration.

Given integral	**How to solve it**
$f(x)=\int_a^x f(t)\ dt$	Plug x in for t.
$f(x)=\int_x^a f(t)\ dt$	Reverse limits of integration and multiply by -1, then plug x in for t.
$f(x)=\int_a^{g(x)} f(t)\ dt$	Plug $g(x)$ in for t, then multiply by dg/dx.
$f(x)=\int_{g(x)}^a f(t)\ dt$	Reverse limits of integration and multiply by -1, then plug $g(x)$ in for t and multiply by dg/dx.
$f(x)=\int_{g(x)}^{h(x)} f(t)\ dt$	Split the limits of integration as $\int_{g(x)}^0 f(t)\ dt+\int_0^{h(x)} f(t)\ dt$. Reverse limits of integration on $\int_{g(x)}^0 f(t)\ dt$ and multiply by -1, then plug $g(x)$ and $h(x)$ in for t, multiplying by dg/dx and dh/dx respectively.

Looking at the chart, we can see that this is the situation described by the fourth row, which means the steps for solving this using FTC Part 1 are:

1. Reverse the limits of integration and multiply by -1.

$$f(x) = -\int_{6}^{2x^2} 6t^3 - 6t\ dt$$

2. Plug $g(x)$ in for t and multiply by dg/dx.

$$f'(x) = -\left[6\left(2x^2\right)^3 - 6\left(2x^2\right)\right](4x)$$

$$f'(x) = -\left(48x^6 - 12x^2\right)(4x)$$

$$f'(x) = -192x^7 + 48x^3$$

$$f'(x) = 48x^3 - 192x^7$$

Topic: Part 1 of the FTC

Question: Use Part 1 of the Fundamental Theorem of Calculus to find the derivative.

$$f(x) = \int_{6x-1}^{x^3} \cos(2t) - 4t \ dt$$

Answer choices:

A $\quad f'(x) = -12x^5 + 3x^2 \cos\left(2x^3\right) + 144x - 6\cos(12x - 2) - 24$

B $\quad f'(x) = 3x^2 \cos\left(2x^3\right) + 6\cos(12x - 2) - 12x^5 - 144x + 24$

C $\quad f'(x) = x^2 \cos\left(2x^3\right) - 2\cos(12x - 2) - 4x^5 + 48x - 8$

D $\quad f'(x) = x^2 \cos\left(2x^3\right) + 2\cos(12x - 2) - 4x^5 - 48x + 8$

Solution: A

We've been given an integral with functions as the upper and lower limits of integration.

Given integral	**How to solve it**
$f(x)=\int_a^x f(t)\ dt$	Plug x in for t.
$f(x)=\int_x^a f(t)\ dt$	Reverse limits of integration and multiply by -1, then plug x in for t.
$f(x)=\int_a^{g(x)} f(t)\ dt$	Plug $g(x)$ in for t, then multiply by dg/dx.
$f(x)=\int_{g(x)}^a f(t)\ dt$	Reverse limits of integration and multiply by -1, then plug $g(x)$ in for t and multiply by dg/dx.
$f(x)=\int_{g(x)}^{h(x)} f(t)\ dt$	Split the limits of integration as $\int_{g(x)}^0 f(t)\ dt+\int_0^{h(x)} f(t)\ dt$. Reverse limits of integration on $\int_{g(x)}^0 f(t)\ dt$ and multiply by -1, then plug $g(x)$ and $h(x)$ in for t, multiplying by dg/dx and dh/dx respectively.

Looking at the chart, we can see that this is the situation described by the fifth row, which means the steps for solving this using FTC Part 1 are:

1. Split the limits of integration at 0.

$$f(x)=\int_{6x-1}^{0}\cos(2t)-4t\ dt+\int_{0}^{x^3}\cos(2t)-4t\ dt$$

2. For the first integral, reverse the limits of integration and multiply by -1.

$$f(x)=-\int_{0}^{6x-1}\cos(2t)-4t\ dt+\int_{0}^{x^3}\cos(2t)-4t\ dt$$

3. Plug in $g(x)$ and $h(x)$ and multiply by dg/dx and dh/dx.

$$f'(x)=-\left\{\cos\left[2(6x-1)\right]-4(6x-1)\right\}(6)+\left\{\cos\left[2\left(x^3\right)\right]-4\left(x^3\right)\right\}\left(3x^2\right)$$

$$f'(x)=-6\left[\cos\left(12x-2\right)-24x+4\right]+3x^2\left[\cos\left(2x^3\right)-4x^3\right]$$

$$f'(x)=-6\cos\left(12x-2\right)+144x-24+3x^2\cos\left(2x^3\right)-12x^5$$

$$f'(x)=-12x^5+3x^2\cos\left(2x^3\right)+144x-6\cos\left(12x-2\right)-24$$