Topic: Price of the product

Question: An item is currently selling for $\$50$/unit. The quantity supplied is decreasing by 10 units/week. At what rate is the price of the item changing?

$$q = 4{,}000e^{-0.01p}$$

Answer choices:

A Increasing by $\$2.43$ per week

B Decreasing by $\$1.86$ per week

C Decreasing by $\$6.59$ per week

D Increasing by $\$0.41$ per week

Solution: D

Use implicit differentiation to take the derivative of both sides of the quantity equation.

$$q = 4{,}000e^{-0.01p}$$

$$\frac{dq}{dt} = -40e^{-0.01p}\frac{dp}{dt}$$

From the question, we know that $p = 50$ and $dq/dt = -10$, so we'll plug those in.

$$-10 = -40e^{-0.01(50)}\frac{dp}{dt}$$

$$-10 = -40e^{-0.50}\frac{dp}{dt}$$

Solve for dp/dt, which is the rate we were asked to find.

$$\frac{dp}{dt} = \frac{-10}{-40e^{-0.50}}$$

$$\frac{dp}{dt} = \frac{1}{4e^{-0.50}}$$

$$\frac{dp}{dt} = \frac{e^{0.50}}{4}$$

$$\frac{dp}{dt} \approx \$0.41$$

Topic: Price of the product

Question: A company has determined that the demand curve for their product is given by q, where p is the price in dollars, and q is the quantity in millions. If the company is increasing the price of the product by $\$1.25$ per week, find the rate at which demand is changing when the price is $\$25$.

$$q = \sqrt{2{,}000 - p^2}$$

Answer choices:

A Increasing by 0.85 million items per week

B Decreasing by 1.85 million items per week

C Decreasing by 0.85 million items per week

D Increasing by 1.85 million items per week

Solution: C

Use implicit differentiation to take the derivative of both sides of the quantity equation.

$$q = \sqrt{2{,}000 - p^2}$$

$$\frac{dq}{dt} = \frac{1}{2}(2{,}000 - p^2)^{-\frac{1}{2}}\left(-2p\frac{dp}{dt}\right)$$

$$\frac{dq}{dt} = -p(2{,}000 - p^2)^{-\frac{1}{2}}\frac{dp}{dt}$$

From the question, we know that $p = 25$ and $dp/dt = 1.25$, so we'll plug those in.

$$\frac{dq}{dt} = -25(2{,}000 - 25^2)^{-\frac{1}{2}}(1.25)$$

$$\frac{dq}{dt} \approx -0.85$$

Demand is decreasing by 0.85 million items per week.

Topic: Price of the product

Question: Suppose that the price p of a product is given by the demand function, where q represents the demand quantity. If the daily demand is decreasing at a rate of 8 units per day, at what rate is the price changing when the demand is 30 units?

$$p = \frac{900 - 10q}{250 - 2q}$$

Answer choices:

A The price is changing at a rate of $\$6.45$ per day

B The price is changing at a rate of $\$0.155$ per day

C The price is changing at a rate of $\$6.45$ per week

D The price is changing at a rate of $\$0.155$ per week

Solution: B

Use implicit differentiation to take the derivative of both sides of the price equation.

$$p = \frac{900 - 10q}{250 - 2q}$$

$$\frac{dp}{dt} = \frac{-10\frac{dq}{dt}(250 - 2q) - \left(-2\frac{dq}{dt}\right)(900 - 10q)}{(250 - 2q)^2}$$

$$\frac{dp}{dt} = \frac{\frac{dq}{dt}(-2{,}500 + 20q) + \frac{dq}{dt}(1{,}800 - 20q)}{(250 - 2q)^2}$$

$$\frac{dp}{dt} = \frac{\frac{dq}{dt}(-2{,}500 + 20q + 1{,}800 - 20q)}{(250 - 2q)^2}$$

$$\frac{dp}{dt} = \frac{-700\frac{dq}{dt}}{(250 - 2q)^2}$$

From the question, we know that $q = 30$ and $dq/dt = -8$, so we'll plug those in.

$$\frac{dp}{dt} = \frac{-700(-8)}{(250 - 2(30))^2}$$

$$\frac{dp}{dt} = \frac{5{,}600}{36{,}100}$$

$$\frac{dp}{dt} = 0.155$$