Topic: Work done on elastic springs

Question: Find the work required to stretch the spring by 2 feet from its natural length.

The force required to stretch a spring s feet is given by $F(s) = 11s$ lbs.

Answer choices:

A 11 ft-lbs

B 12 ft-lbs

C 21 ft-lbs

D 22 ft-lbs

Solution: D

Using Hooke's Law, $F(s) = ks$, we get

$$11s = ks$$

$$k = 11$$

The work done in stretching the spring 2 feet can be calculated using

$$W = \int_a^b F(s)\ ds$$

When the spring is at its natural length, then $a = 0$, and when it's stretched by 2 feet, $b = 2$.

$$W = \int_0^2 11s\ ds$$

$$W = 11\left(\frac{1}{2}s^2\right)\Big|_0^2$$

$$W = \left[\frac{11}{2}(2)^2\right] - \left[\frac{11}{2}(0)^2\right]$$

$$W = \frac{44}{2}$$

$$W = 22 \text{ ft-lbs}$$

Topic: Work done on elastic springs

Question: Find the work required to compress the spring from its natural length to 6 feet.

A force of 200 lbs is required to keep a spring compressed to 8 feet, when its natural length is 10 feet.

Answer choices:

A 200 ft-lbs

B 400 ft-lbs

C 600 ft-lbs

D 800 ft-lbs

Solution: D

We'll first determine the spring constant using Hooke's Law.

$$k = \frac{F(x)}{x}$$

$$k = \frac{200}{10 - 8}$$

$$k = 100$$

The work done to compress the spring can be calculated using

$$W = \int_a^b F(x)\ dx$$

When the spring is at its natural length, $a = 0$, and when it's compressed to 6 feet, or by 4 feet from its natural length, $b = 4$.

$$W = \int_0^4 100x\ dx$$

$$W = \frac{100}{2}x^2\Big|_0^4$$

$$W = 50x^2\Big|_0^4$$

$$W = 50(4)^2 - 50(0)^2$$

$$W = 800 \text{ ft-lbs}$$

The work done in compressing the spring from its natural length of 10 feet to a length of 6 feet is 800 ft-lbs.

Topic: Work done on elastic springs

Question: Find the value of the spring constant and the work done to stretch the spring 1/2 foot beyond its natural length.

A force of 8 lbs is required to keep a spring stretched $1/2$ foot beyond its natural length.

Answer choices:

A 16 lbs/ft and 2 ft-lbs

B 12 lbs/ft and 4 ft-lbs

C 8 lbs/ft and 6 ft-lbs

D 20 lbs/ft and 8 ft-lbs

Solution: A

To determine the spring constant k, we'll use Hooke's Law, which states that the force $F(x)$ necessary to keep a spring stretched (or compressed) by x units beyond (or short of) its natural length, is given by $F(x) = kx$. For the given spring where $F(x) = 8$ lbs and $x = 0.5$ ft, we'll solve for k by substituting these values into Hooke's Law.

$$k = \frac{F(x)}{x}$$

$$k = \frac{8}{0.5}$$

$$k = 16$$

The spring constant is 16 lbs/ft.

The work done to stretch the spring $1/2$ foot beyond its natural length can be calculated using

$$W = \int_a^b F(x)\ dx$$

When the spring is at its natural length, $a = 0$ and when it's stretched $1/2$ foot beyond its natural length, $b = 1/2$.

$$W = \int_0^{\frac{1}{2}} 16x\ dx$$

$$W = \frac{16}{2}x^2 \Big|_0^{\frac{1}{2}}$$

$$W = 8\left(\frac{1}{2}\right)^2 - 8(0)^2$$

$$W = 2$$

The work done in stretching the spring $1/2$ foot beyond its natural length is 2 ft-lbs.