Sketching polar curves

We'll sketch polar curves by plotting values for r at known values of θ. We can also use the table below to quickly graph polar curves given in these standard forms.

1. Lines

$\theta = \beta$	The line from $(0,0)$ set at the angle β
$r\cos\theta = a$	The vertical line through $x = a$
$r\sin\theta = b$	The horizontal line through $y = b$

2. Circles

$r = a$	The circle centered at $(0,0)$ with radius a
$r = 2a\cos\theta$	The circle centered at $(a,0)$ with radius $\vert a\vert$
$r = 2b\sin\theta$	The circle centered at $(0,b)$ with radius $\vert b\vert$
$r = 2a\cos\theta + 2b\sin\theta$	The circle centered at (a, b) with radius $\sqrt{a^2+b^2}$

3. Cardioids, limaçons and others

$r = a \pm a\cos\theta$	The cardioid through the origin
$r = a \pm a\sin\theta$	

$r = a \pm b\cos\theta,\ a < b$	The limaçon with an inner loop
$r = a \pm b\sin\theta,\ a < b$	
$r = a \pm b\cos\theta,\ a > b$	The limaçon without an inner loop
$r = a \pm b\sin\theta,\ a > b$	

If we can't use the table above to find a standard form for the polar curve we're given, then we can always generate a table of coordinate points (r,θ). In order to do that, we'll take the value inside the trigonometric function that includes θ, set it equal to $\pi/2$, then solve for θ. For example, given the polar curve $r = 6\sin 3\theta$,

$$3\theta = \frac{\pi}{2}$$

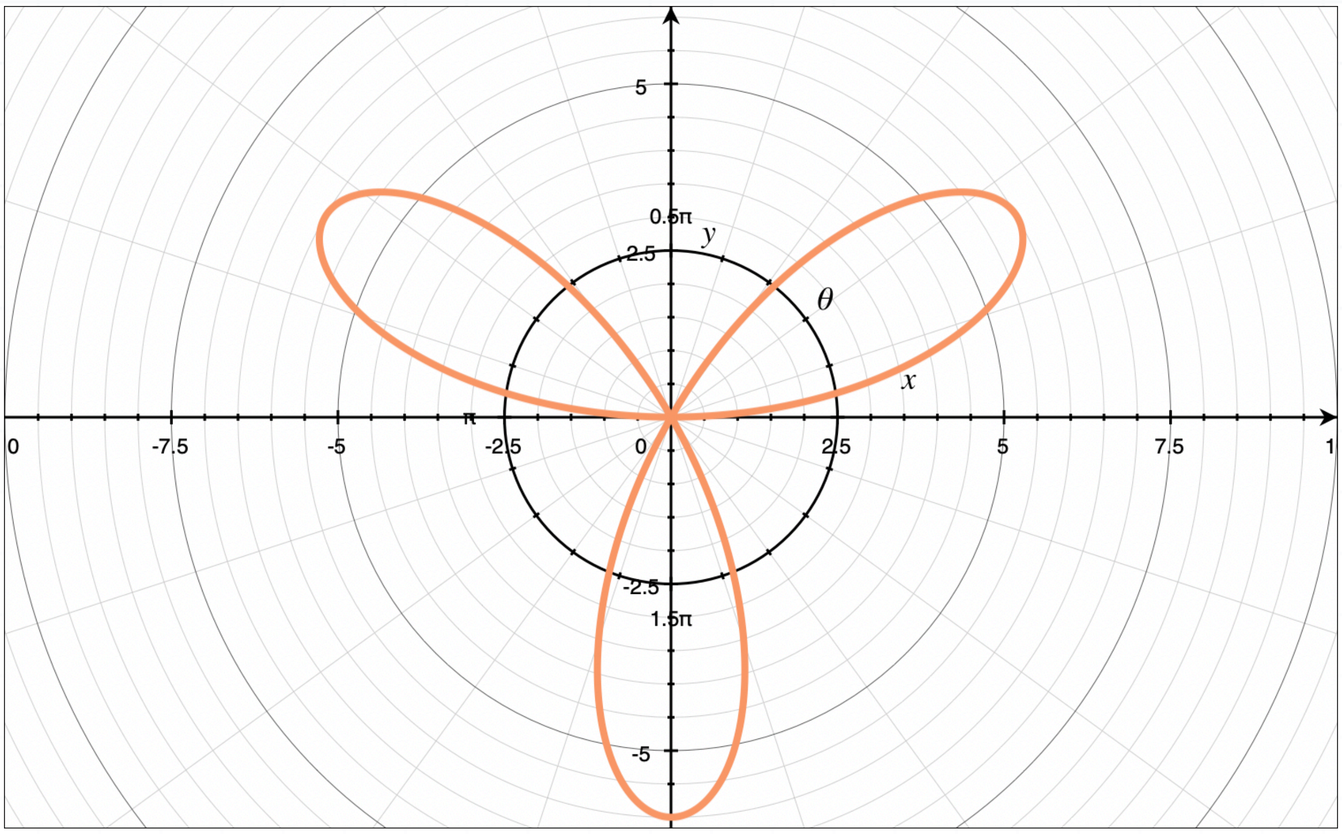
$$\theta = \frac{\pi}{6}$$

Then we'll find r for the increments of $\pi/6$ on the interval $0 \le \theta \le 2\pi$.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$r = 6\sin 3\theta$	0	6	0	-6
θ	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$
$r = 6\sin 3\theta$	0	6	0	-6
θ	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$

$r = 6\sin 3\theta$	0	6	0	-6

Plotting these points on polar axes, we get



Let's try some examples with lines defined in terms of polar coordinates.

Example

Graph the polar curves on the same axes.

$$\theta = \frac{\pi}{3}$$

$$r\cos\theta = 3$$

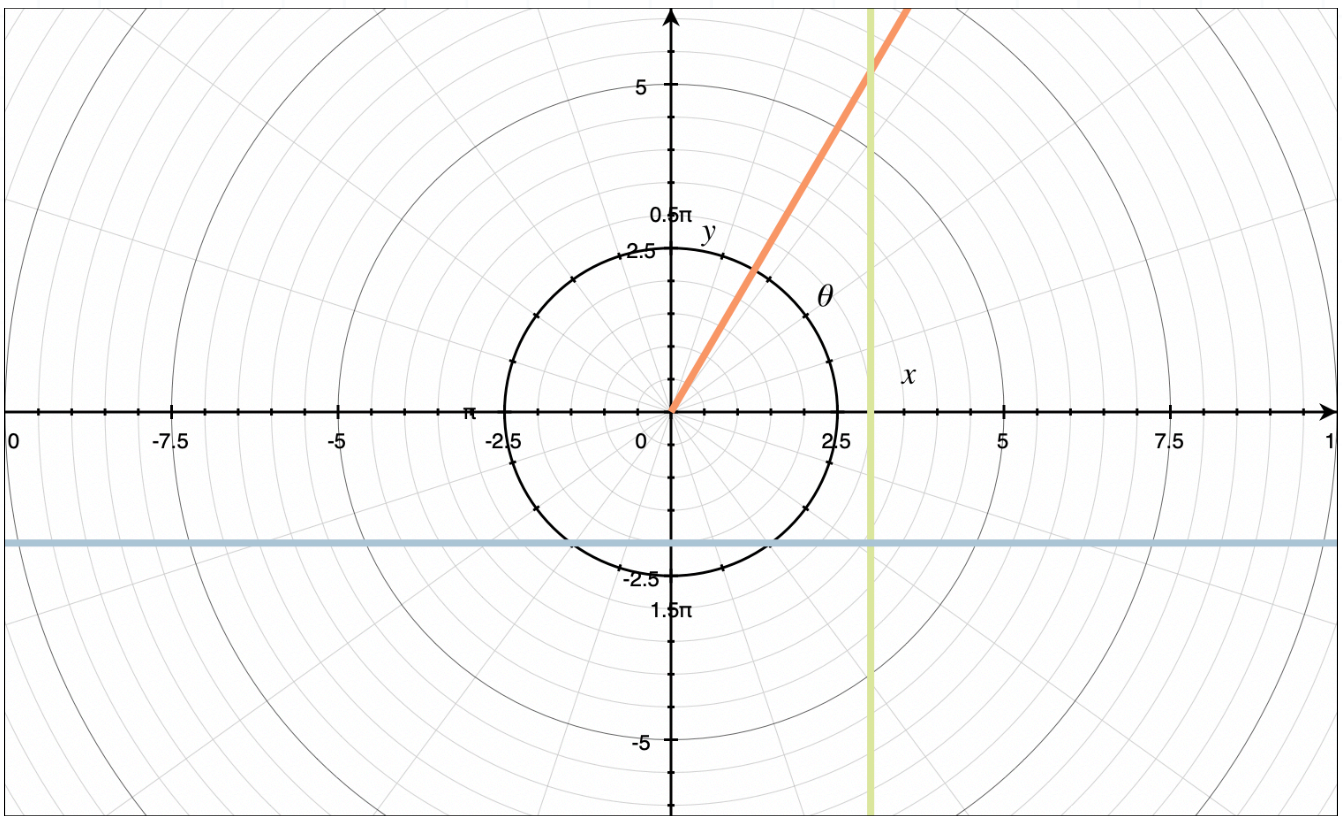
$$r\sin\theta = -2$$

Using the table of standard curves, we can plot all of these on the same axes.

1. $\theta = \pi/3$ is like $\theta = \beta$, so it's a straight line through the origin at the angle $\pi/3$.

2. $r\cos\theta = 3$ is like $r\cos\theta = a$, so it's a vertical line through $x = 3$.

3. $r\sin\theta = -2$ is like $r\sin\theta = b$, so it's a horizontal line through $y = -2$.



Let's try some examples with circles defined in terms of polar coordinates.

Example

Graph the polar curves on the same axes.

$$r = 4$$

$$r = 6\cos\theta$$

$$r = -4\sin\theta$$

$$r = 2\cos\theta + 6\sin\theta$$

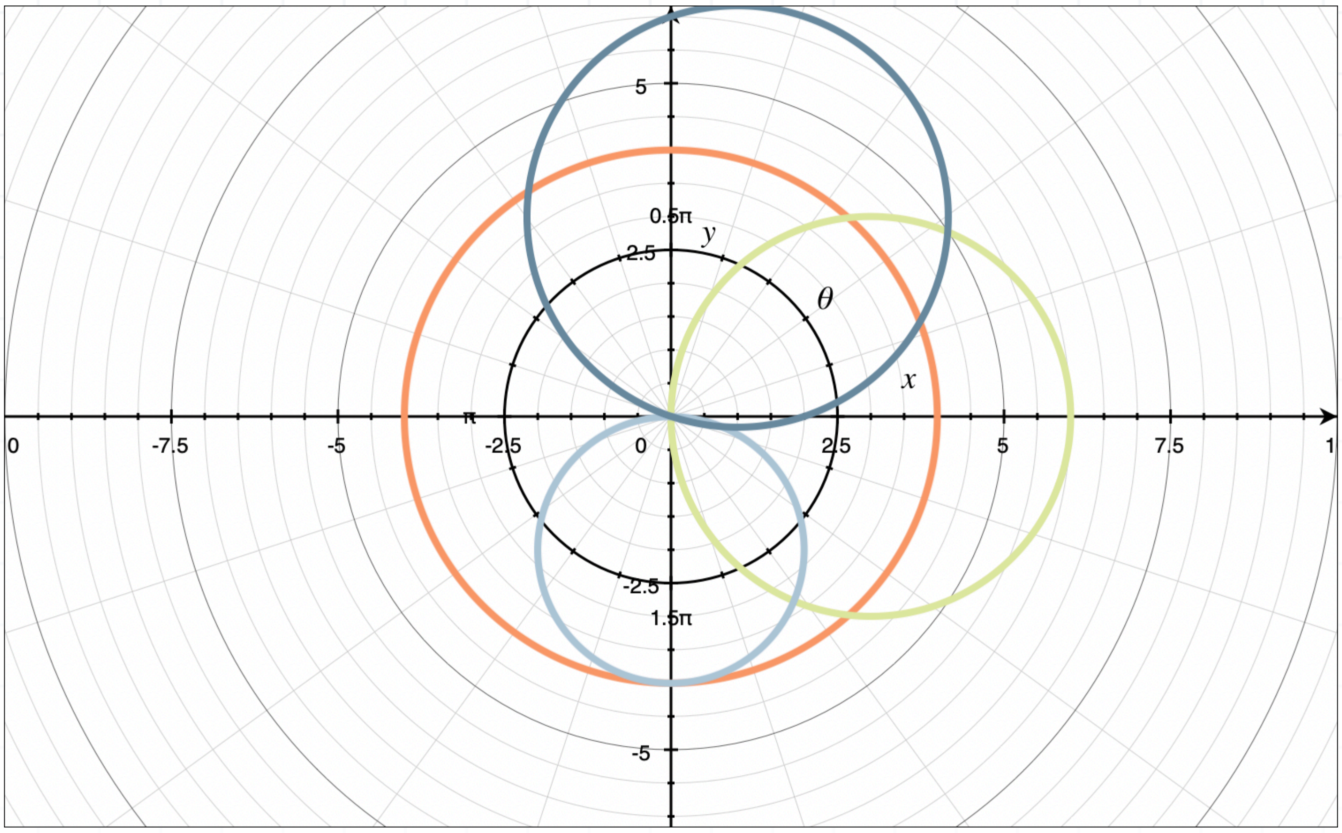
Using the table of standard curves, we can plot all of these on the same axes.

1. $r = 4$ is like $r = a$, so it's a circle centered at $(0,0)$ with radius 4.

2. $r = 6\cos\theta$ is like $r = 2a\cos\theta$, so it's a circle centered at $(3,0)$ with radius $|3|$.

3. $r = -4\sin\theta$ is like $r = 2b\sin\theta$, so it's a circle centered at $(0,-2)$ with radius $|-2|$.

4. $r = 2\cos\theta + 6\sin\theta$ is like $r = 2a\cos\theta + 2b\sin\theta$, so it's a circle centered at $(1,3)$ with radius $\sqrt{a^2+b^2} = \sqrt{10}$.



Let's try some examples with more complex curves defined in terms of polar coordinates.

Example

Graph the polar curves.

$$r = 3 + 3\sin\theta$$

$$r = 2 + 4\cos\theta$$

$$r = 7 + 6\cos\theta$$

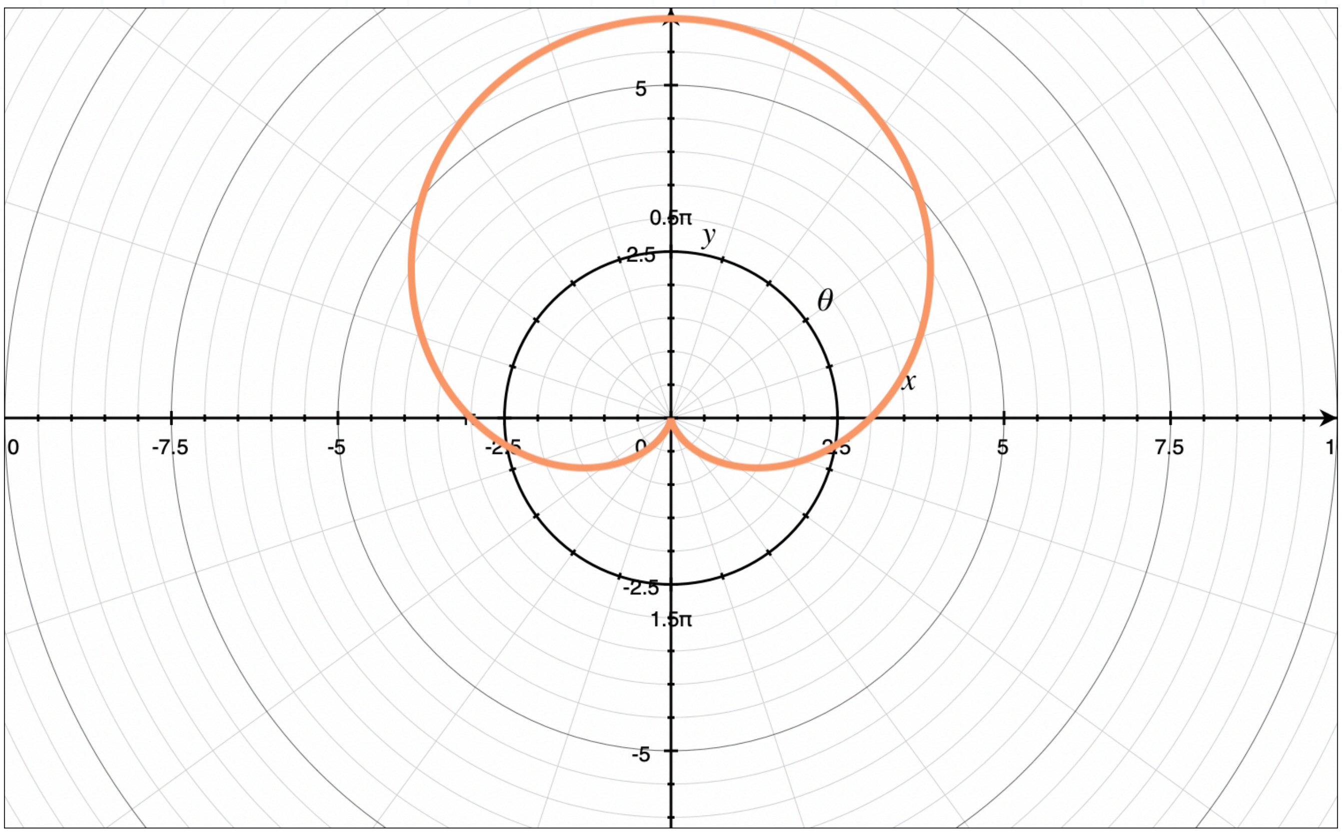
$$r = 6\sin 2\theta$$

For $r = 3 + 3\sin\theta$:

$r = 3 + 3\sin\theta$ is like $r = a \pm a\sin\theta$, so it's a cardioid through the origin. We'll generate a table of values over the interval $0 \le \theta \le 2\pi$.

θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$r = 3 + 3\sin\theta$	3	6	3	0	3

With these points and knowing the shape of our polar curve, we can sketch the graph.

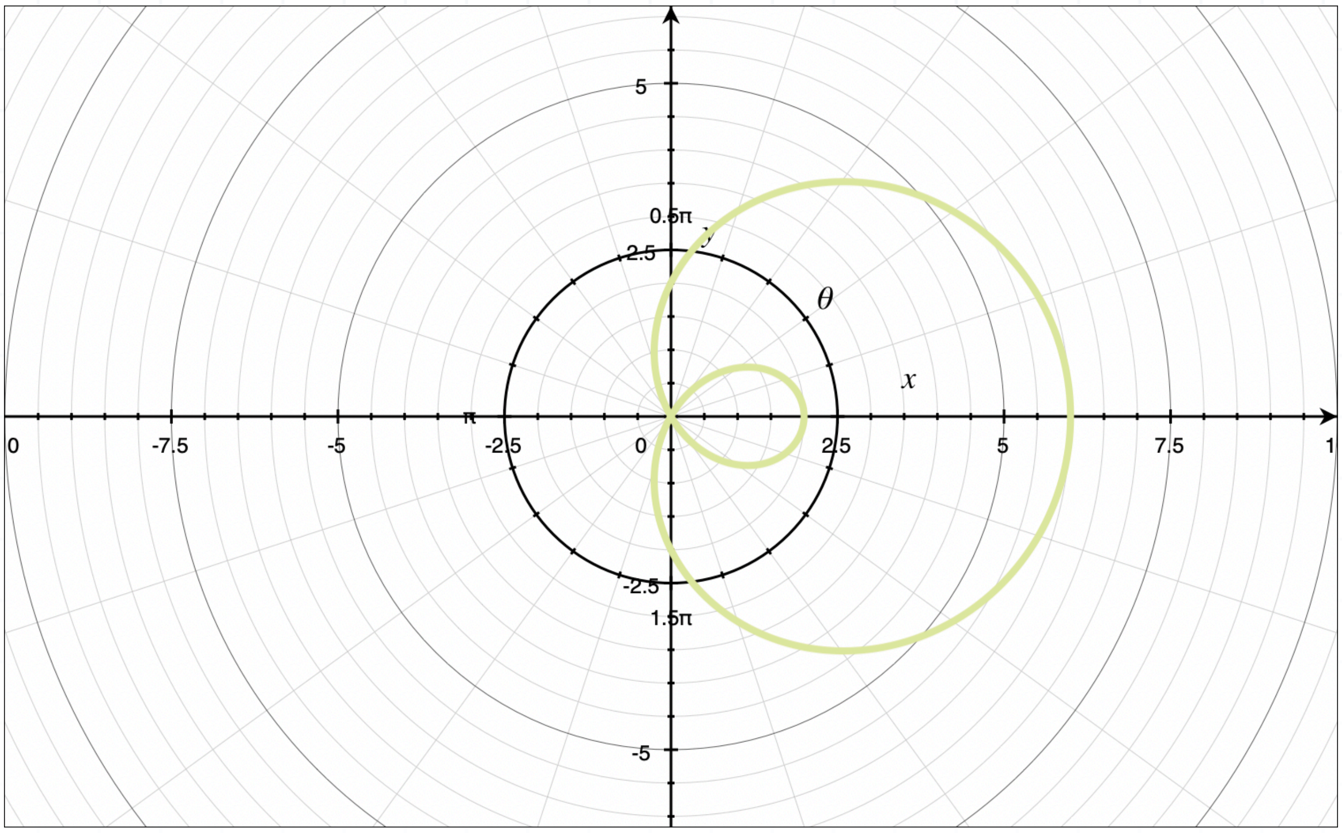


For $r = 2 + 4\cos\theta$:

$r = 2 + 4\cos\theta$ is like $r = a \pm b\cos\theta$ with $a < b$, so it's a limaçon with an inner loop. We'll generate a table of values over the interval $0 \le \theta \le 2\pi$.

θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$r = 2 + 4\cos\theta$	6	2	-2	2	6

With these points and knowing the shape of our polar curve, we can sketch the graph.

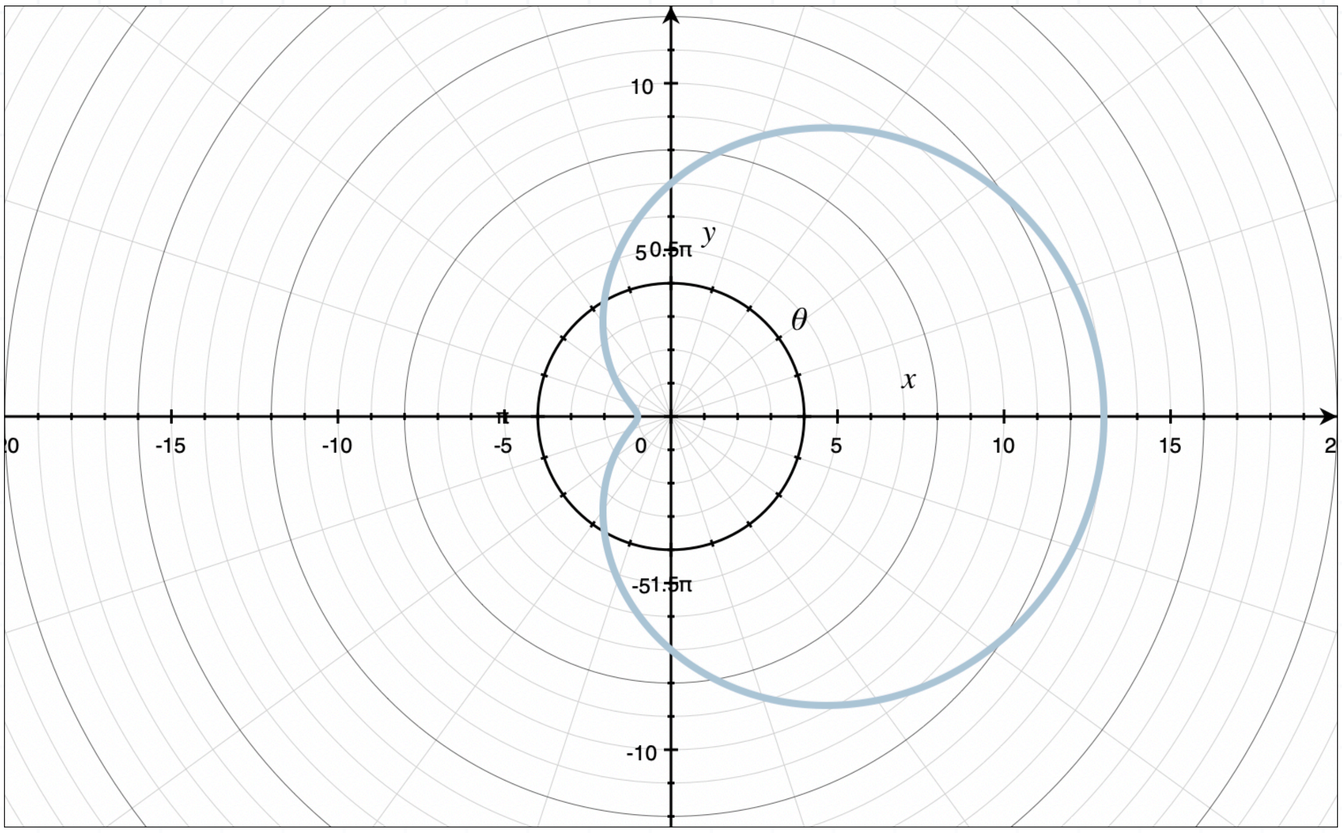


For $r = 7 + 6\cos\theta$:

$r = 7 + 6\cos\theta$ is like $r = a \pm b\cos\theta$ with $a > b$, so it's a limaçon without an inner loop. We'll generate a table of values over the interval $0 \leq \theta \leq 2\pi$.

θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$r = 7 + 6\cos\theta$	13	7	1	7	13

With these points and knowing the shape of our polar curve we can sketch the graph.



For $r = 6\sin 2\theta$:

$r = 6\sin 2\theta$ doesn't match any of the standard forms in our table. In this case, we'll set the value inside our trigonometric function equal to $\pi/2$ and then solve for θ.

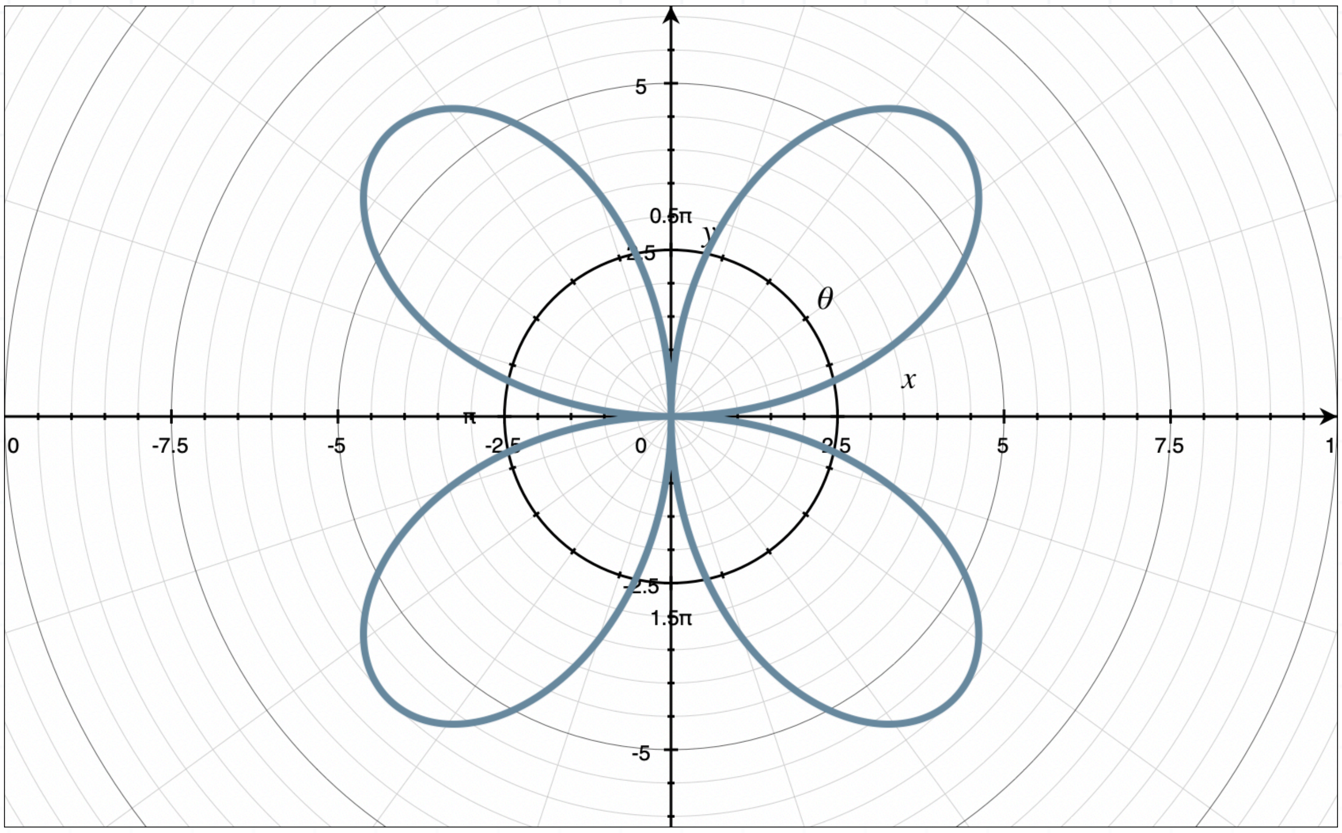
$$2\theta = \frac{\pi}{2}$$

$$\theta = \frac{\pi}{4}$$

Then we'll find r for the increments of $\pi/4$ on the interval $0 \le \theta \le 2\pi$.

θ	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
$r = 6\sin 2\theta$	0	6	0	-6	0	6	0	-6	0

Plotting these points on polar axes, we get


5
0.5π
y
2.5
θ
x
π
-7.5
-5
-2.5
0
2.5
5
7.5
-2.5
1.5π
-5