Rectilinear motion

Rectilinear motion problems deal with an object that moves laterally, or horizontally. The object can be moving along the ground or at any other height, as long as it's moving horizontally. We call this type of motion "rectilinear" motion.

Problems like these require you to know the relationship between position $x(t)$, velocity $v(t)$, and acceleration $a(t)$. The important thing to know is that the derivative of position is velocity, and the derivative of velocity is acceleration.

$$x(t)$$

$$x'(t) = v(t)$$

$$x''(t) = v'(t) = a(t)$$

We can also describe the above relationship using integrals instead of derivatives, and we see that the integral of acceleration is velocity, and the integral of velocity is position.

$$a(t)$$

$$\int a(t)\ dt = v(t)$$

$$\int \int a(t)\ dt = \int v(t) = x(t)$$

Example

An object is moving along the ground. Its acceleration is $a(t) = 3t + 5$, its velocity at time $t = 4$ is $v(4) = 6$, and its position at $t = 5$ is $x(5) = 25$. Find the equation for position that describes this object's motion.

We can integrate the acceleration function to get a velocity function, $v(t)$.

$$v(t) = \int a(t)\ dt$$

$$v(t) = \int 3t + 5\ dt$$

$$v(t) = \int 3t\ dt + \int 5\ dt$$

$$v(t) = 3\int t\ dt + 5\int 1\ dt$$

$$v(t) = \frac{3t^2}{2} + 5t + C$$

Now we need to solve for C using $v(4) = 6$.

$$6 = \frac{3(4)^2}{2} + 5(4) + C$$

$$C = -38$$

So the equation for velocity, $v(t)$, is

$$v(t) = \frac{3t^2}{2} + 5t - 38$$

Now we can integrate the velocity function to get the position function.

$$x(t)=\int v(t)\ dt$$

$$x(t)=\int \frac{3t^2}{2}+5t-38\ dt$$

$$x(t)=\int \frac{3t^2}{2}\ dt+\int 5t\ dt+\int -38\ dt$$

$$x(t)=\frac{3}{2}\int t^2\ dt+5\int t\ dt-38\int 1\ dt$$

$$x(t)=\frac{3}{2}\left(\frac{t^3}{3}\right)+5\left(\frac{t^2}{2}\right)-38t+C$$

$$x(t)=\frac{t^3}{2}+\frac{5t^2}{2}-38t+C$$

Now we need to solve for C using $x(5)=25$.

$$25=\frac{(5)^3}{2}+\frac{5(5)^2}{2}-38(5)+C$$

$$C=90$$

So the equation for position, $x(t)$, is

$$x(t)=\frac{t^3}{2}+\frac{5t^2}{2}-38t+90$$