Topic: Disks, horizontal axis

Question: Use disks to find the volume of the solid formed by rotating the region enclosed by the curves.

$y = x^2$ and $y = 0$

$x = 0$ and $x = 3$

about the x-axis

Answer choices:

A $\quad V = \frac{243}{5}\pi$ cubic units

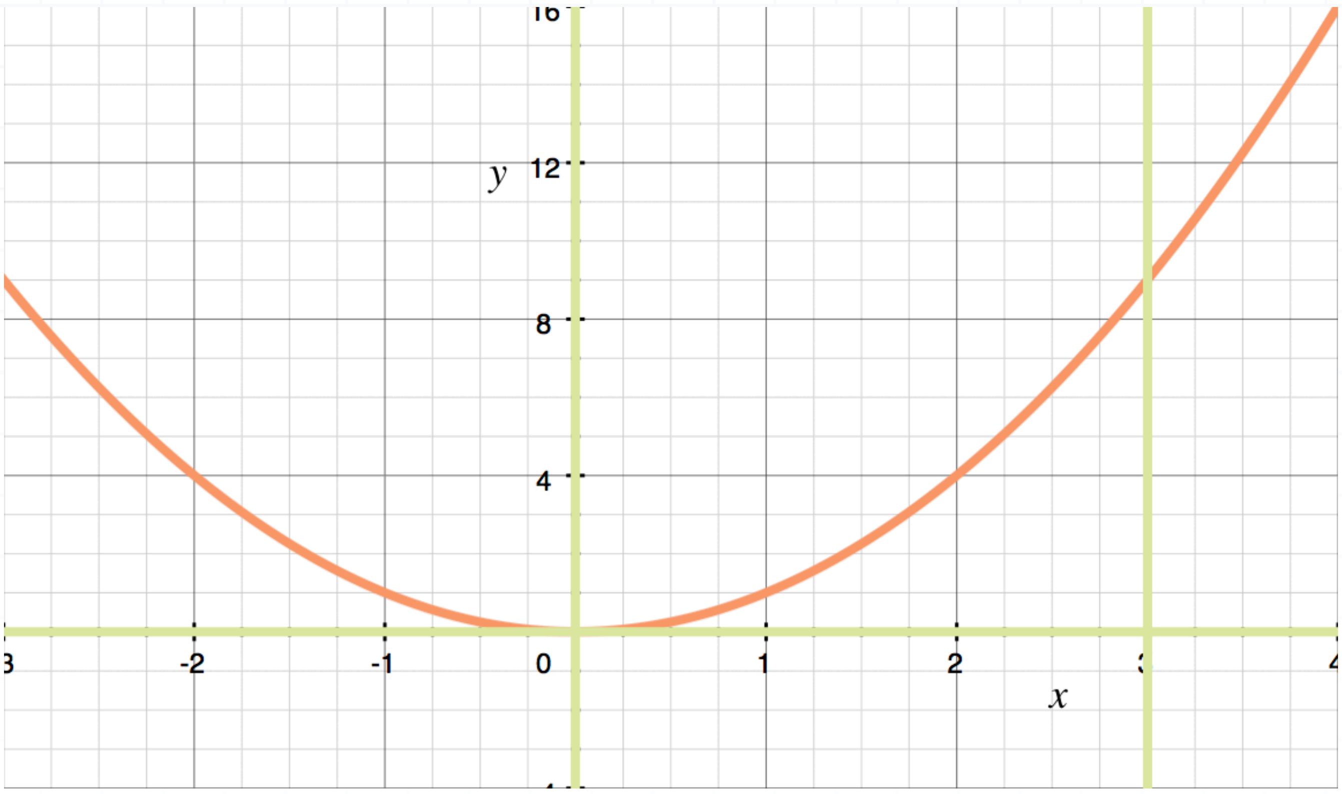
B $\quad V = \frac{243}{5}$ cubic units

C $\quad V = 243\pi$ cubic units

D $\quad V = 81\pi$ cubic units

Solution: A

The region enclosed by $y = x^2$, $y = 0$, $x = 0$ and $x = 3$ is



Because we're rotating about the x-axis, and because our slices of volume must always be perpendicular to the axis of rotation, that means we'll be taking vertical slices of volume. Which means that the width of each infinitely thin slice of volume can be given by dx, which means we'll be integrating with respect to x. Therefore, the limits of integration will be given by $x = [0,3]$. The outer radius will be defined by $y = x^2$. So the volume can be given by

$$V = \int_a^b \pi \left[f(x)\right]^2 \ dx$$

$$V = \int_0^3 \pi \left(x^2\right)^2 \ dx$$

$$V = \int_0^3 \pi x^4 \ dx$$

Integrate, then evaluate over the interval.

$$V = \frac{1}{5}\pi x^5 \Big|_0^3$$

$$V = \frac{1}{5}\pi(3)^5 - \left(\frac{1}{5}\pi(0)^5\right)$$

$$V = \frac{243\pi}{5}$$

Topic: Disks, horizontal axis

Question: Use disks to find the volume of the solid formed by rotating the region enclosed by the curves.

$y = -x^2 + 5x$ and $y = 0$

about the x-axis

Answer choices:

A $\quad V = \frac{625}{5}\pi$ cubic units

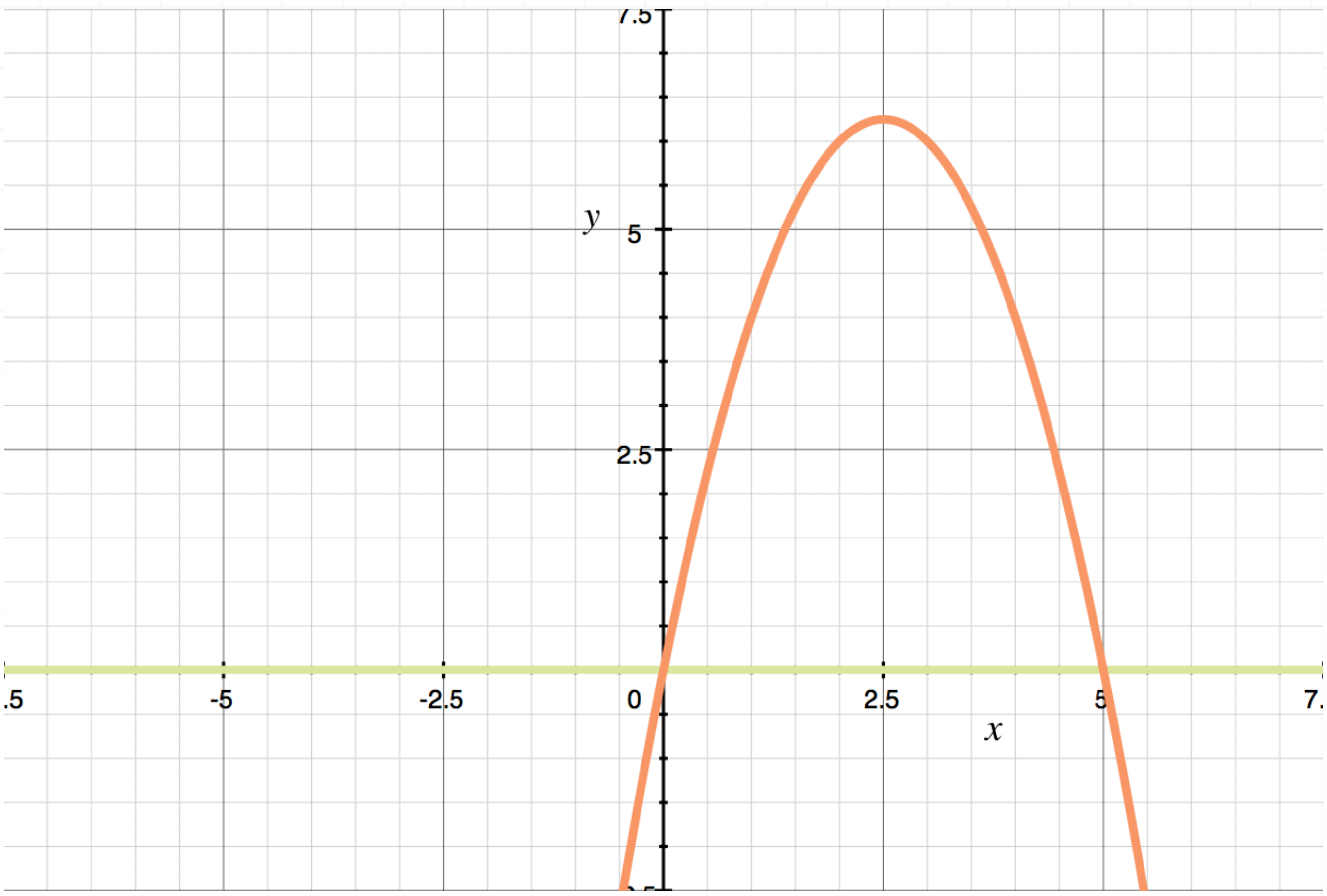
B $\quad V = 625\pi$ cubic units

C $\quad V = \frac{625}{6}\pi$ cubic units

D $\quad V = \frac{626}{5}\pi$ cubic units

Solution: C

The region enclosed by $y = -x^2 + 5x$ and $y = 0$ is



Because we're rotating about the x-axis, and because our slices of volume must always be perpendicular to the axis of rotation, that means we'll be taking vertical slices of volume. Which means that the width of each infinitely thin slice of volume can be given by dx, which means we'll be integrating with respect to x. Therefore, the limits of integration will be given by the points where $y = -x^2 + 5x$ intersects $y = 0$.

$$-x^2 + 5x = 0$$

$$x^2 - 5x = 0$$

$$x(x - 5) = 0$$

$x = 0$ and $x = 5$

The limits of integration are therefore $x = [0,5]$. The outer radius will be defined by $y = -x^2 + 5x$. So the volume can be given by

$$V = \int_a^b \pi \left[f(x)\right]^2 \ dx$$

$$V = \int_0^5 \pi \left(-x^2 + 5x\right)^2 \ dx$$

$$V = \int_0^5 \pi \left(x^4 - 10x^3 + 25x^2\right) \ dx$$

$$V = \pi \int_0^5 x^4 - 10x^3 + 25x^2 \ dx$$

Integrate, then evaluate over the interval.

$$V = \pi \left[\frac{1}{5}x^5 - \frac{5}{2}x^4 + \frac{25}{3}x^3\right]\Big|_0^5$$

$$V = \pi \left[\frac{1}{5}(5)^5 - \frac{5}{2}(5)^4 + \frac{25}{3}(5)^3\right] - \pi \left[\frac{1}{5}(0)^5 - \frac{5}{2}(0)^4 + \frac{25}{3}(0)^3\right]$$

$$V = \pi \left(5^4 - \frac{5^5}{2} + \frac{5^5}{3}\right)$$

$$V = \pi \left(\frac{6(5^4)}{6} - \frac{3(5^5)}{6} + \frac{2(5^5)}{6}\right)$$

$$V = \frac{6(5^4) - 5^5}{6}\pi$$

$$V = \frac{625}{6}\pi$$

Topic: Disks, horizontal axis

Question: Use disks to find the volume of the solid formed by rotating the region enclosed by the curves.

$$y = \sqrt{9 - x^2} \text{ and } y = 0$$

$$x = -\frac{5}{2} \text{ and } x = \frac{5}{2}$$

about the x-axis

Answer choices:

A $\quad V = \frac{415}{6}\pi$ cubic units

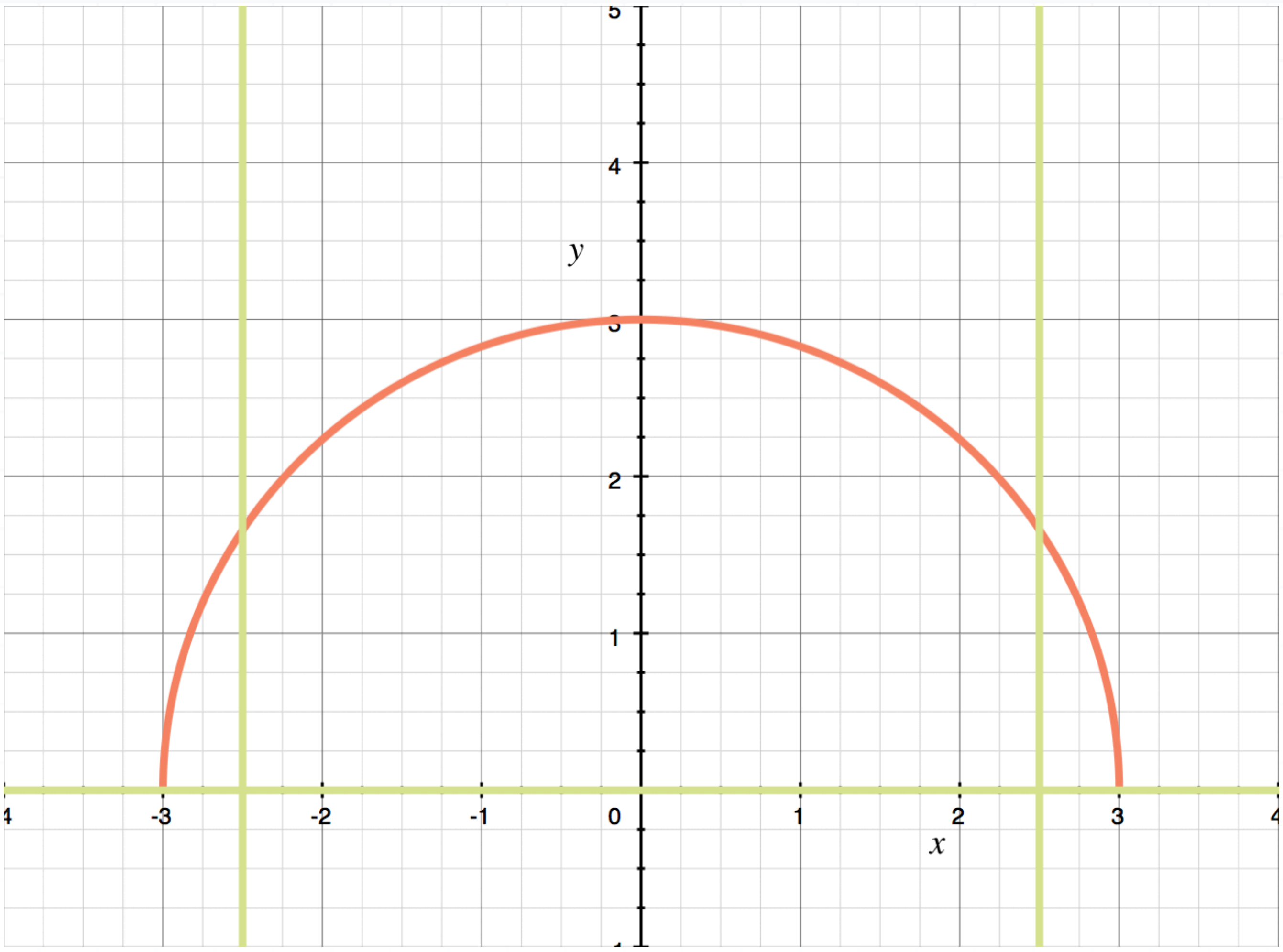
B $\quad V = \frac{415}{24}\pi$ cubic units

C $\quad V = \frac{415}{12}\pi$ cubic units

D $\quad V = 415\pi$ cubic units

Solution: C

The region enclosed by $y = \sqrt{9 - x^2}$, $y = 0$, $x = -5/2$ and $x = 5/2$ is



Because we're rotating about the x-axis, and because our slices of volume must always be perpendicular to the axis of rotation, that means we'll be taking vertical slices of volume. Which means that the width of each infinitely thin slice of volume can be given by dx, which means we'll be integrating with respect to x. Therefore, the limits of integration will be given by $x = [-5/2, 5/2]$. The outer radius will be defined by $y = \sqrt{9 - x^2}$. So the volume can be given by

$$V = \int_a^b \pi \left[f(x)\right]^2 \ dx$$

$$V=\int_{-\frac{5}{2}}^{\frac{5}{2}}\pi\left(\sqrt{9-x^2}\right)^2\ dx$$

$$V=\int_{-\frac{5}{2}}^{\frac{5}{2}}\pi\left(9-x^2\right)\ dx$$

$$V=\int_{-\frac{5}{2}}^{\frac{5}{2}}9\pi-\pi x^2\ dx$$

Integrate, then evaluate over the interval.

$$V=9\pi x-\frac{1}{3}\pi x^3\Big|_{-\frac{5}{2}}^{\frac{5}{2}}$$

$$V=9\pi\left(\frac{5}{2}\right)-\frac{1}{3}\pi\left(\frac{5}{2}\right)^3-\left[9\pi\left(-\frac{5}{2}\right)-\frac{1}{3}\pi\left(-\frac{5}{2}\right)^3\right]$$

$$V=\frac{45}{2}\pi-\frac{125}{24}\pi+\frac{45}{2}\pi-\frac{125}{24}\pi$$

$$V=45\pi-\frac{250}{24}\pi$$

$$V=\frac{415}{12}\pi$$