Hyperbolic integrals

Hyperbolic functions follow standard rules for integration. The general rules for the six hyperbolic functions are

$$\int \sinh(ax)\ dx = \frac{\cosh(ax)}{a} + C$$

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$$\int \tanh(ax)\ dx = \frac{\ln\left|\cosh(ax)\right|}{a} + C$$

$$\int \coth(ax)\ dx = \frac{\ln\left|\sinh(ax)\right|}{a} + C$$

$$\int \text{sech}(ax)\ dx = \frac{\arctan[\sinh(ax)]}{a} + C$$

$$\int \text{csch}(ax)\ dx = \frac{\ln\left|\tanh(\frac{ax}{2})\right|}{a} + C$$

We also have a few other standard hyperbolic integrals that are based on the standard hyperbolic derivatives.

$$\int \text{sech}^2(ax)\ dx = \frac{\tanh(ax)}{a} + C$$

$$\int \text{csch}^2(ax)\ dx = \frac{-\coth(ax)}{a} + C$$

$$\int \text{sech}(ax)\tanh(ax)\ dx = \frac{-\text{sech}(ax)}{a} + C$$

$$\int \text{csch}(ax)\coth(ax)\ dx = \frac{-\text{csch}(ax)}{a} + C$$

Example

Evaluate the integral.

$$\int 5\text{sech}(3x)\ dx$$

First, we simplify the integral by factoring out the 5.

$$5\int \text{sech}(3x)\ dx$$

Remembering that $\int \text{sech}(ax)\ dx = \frac{\arctan[\sinh(ax)]}{a} + C$, we integrate and get

$$\int 5\text{sech}(3x)\ dx = \frac{5\arctan[\sinh(3x)]}{3} + C$$

Now let's try a more complex example.

Example

Evaluate the integral.

$$\int 16x^3 - 2\sinh(3x) + 4\text{csch}^2(6x)\ dx$$

First, we will break the integral into parts to simplify it.

$$\int 16x^3\ dx + \int -2\sinh(3x)\ dx + \int 4\text{csch}^2(6x)\ dx$$

$$16\int x^3\ dx - 2\int \sinh(3x)\ dx + 4\int \text{csch}^2(6x)\ dx$$

We'll use our hyperbolic integration formulas to integrate, and we'll get

$$16\left(\frac{1}{4}x^4\right) - 2\left[\frac{\cosh(3x)}{3}\right] + 4\left[\frac{-\coth(6x)}{6}\right] + C$$

$$4x^4 - \frac{2\cosh(3x)}{3} - \frac{2\coth(6x)}{3} + C$$