



# Calculus 1 Workbook Solutions

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Other derivatives

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MATH

## INVERSE TRIGONOMETRIC DERIVATIVES

■ 1. Find  $f'(t)$ .

$$f(t) = 4 \sin^{-1} \left( \frac{t}{4} \right)$$

*Solution:*

The derivative of inverse sine is given by

$$\frac{d}{dt} a \sin^{-1}(y(t)) = a \cdot \frac{y'(t)}{\sqrt{1 - [y(t)]^2}}$$

If  $a = 4$  and  $y(t) = t/4$ , then  $y'(t) = 1/4$ . Then the derivative is

$$f'(t) = 4 \cdot \frac{\frac{1}{4}}{\sqrt{1 - \left(\frac{t}{4}\right)^2}} = \frac{1}{\sqrt{\frac{16}{16} - \frac{t^2}{16}}} = \frac{1}{\sqrt{\frac{16 - t^2}{16}}} = \frac{1}{\frac{\sqrt{16 - t^2}}{4}} = \frac{4}{\sqrt{16 - t^2}}$$

■ 2. Find  $g'(t)$ .

$$g(t) = -6 \cos^{-1}(2t + 3)$$

*Solution:*



The derivative of inverse cosine is given by

$$\frac{d}{dt} a \cos^{-1}(y(t)) = a \cdot \left( -\frac{y'(t)}{\sqrt{1 - [y(t)]^2}} \right)$$

If  $a = -6$  and  $y(t) = 2t + 3$ , then  $y'(t) = 2$ , and the derivative is

$$g'(t) = (-6) \cdot \left( -\frac{2}{\sqrt{1 - (2t + 3)^2}} \right)$$

$$g'(t) = \frac{12}{\sqrt{1 - 4t^2 - 12t - 9}}$$

$$g'(t) = \frac{12}{\sqrt{-4t^2 - 12t - 8}}$$

$$g'(t) = \frac{12}{2\sqrt{-t^2 - 3t - 2}}$$

$$g'(t) = \frac{6}{\sqrt{-(t + 1)(t + 2)}}$$

■ 3. Find  $h'(t)$ .

$$h(t) = 2 \sec^{-1}(6t^2 + 3) - 8 \cot^{-1}\left(\frac{t^3}{3}\right)$$



*Solution:*

We differentiate one term at a time. The derivative of inverse secant is given by

$$\frac{d}{dt} a \sec^{-1}(y(t)) = a \cdot \frac{y'(t)}{|y(t)| \sqrt{[y(t)]^2 - 1}}$$

If  $a = 2$  and  $y(t) = 6t^2 + 3$ , then  $y'(t) = 12t$ , and the derivative of the first term is

$$2 \cdot \frac{12t}{|6t^2 + 3| \sqrt{(6t^2 + 3)^2 - 1}}$$

$$\frac{24t}{|6t^2 + 3| \sqrt{36t^4 + 36t^2 + 9 - 1}}$$

$$\frac{24t}{|6t^2 + 3| \sqrt{36t^4 + 36t^2 + 8}}$$

$$\frac{12t}{|6t^2 + 3| \sqrt{9t^4 + 9t^2 + 2}}$$

$$\frac{4t}{|2t^2 + 1| \sqrt{9t^4 + 9t^2 + 2}}$$

The derivative of inverse cotangent is given by

$$\frac{d}{dt} a \cot^{-1}(y(t)) = a \cdot \left( -\frac{y'(t)}{1 + [y(t)]^2} \right)$$

If  $a = -8$  and  $y(t) = t^3/3$ , then  $y'(t) = t^2$  and the derivative is



$$-8 \cdot \left( -\frac{t^2}{1 + \left(\frac{t^3}{3}\right)^2} \right) = \frac{8t^2}{1 + \frac{t^6}{9}} = \frac{72t^2}{9 + t^6}$$

Then

$$h'(t) = \frac{4t}{|2t^2 + 1| \sqrt{9t^4 + 9t^2 + 2}} + \frac{72t^2}{9 + t^6}$$

■ 4. Find the derivative.

$$y = (x^4 + x^2) \csc^{-1} x + \sin(5x^3)$$

*Solution:*

We'll need to use product rule for the first term,  $(x^4 + x^2) \csc^{-1} x$ , as well as the formula for the derivative of inverse cosecant.

$$\frac{d}{dt} a \csc^{-1}(y(t)) = a \cdot \left( -\frac{y'(t)}{|y(t)| \sqrt{[y(t)]^2 - 1}} \right)$$

Then the derivative is

$$y' = (4x^3 + 2x)(\csc^{-1} x) + (x^4 + x^2) \left( -\frac{1}{|x| \sqrt{x^2 - 1}} \right) + \cos(5x^3)(15x^2)$$



$$y' = (4x^3 + 2x)(\csc^{-1} x) - \frac{x^3 + x}{\sqrt{x^2 - 1}} + 15x^2 \cos(5x^3)$$

■ 5. Find the derivative.

$$y = \frac{\sin^{-1} \left( x + \frac{x^2}{2} \right)}{1 + x}$$

*Solution:*

We'll need to use quotient rule and the formula for the derivative of the inverse sine function.

$$\frac{d}{dt} a \sin^{-1}(y(t)) = a \cdot \frac{y'(t)}{\sqrt{1 - [y(t)]^2}}$$

With

$$y(t) = x + \frac{x^2}{2}$$

$$y'(t) = 1 + \frac{2x}{2} = 1 + x$$

the derivative is

$$y' = \frac{\frac{d}{dx} \sin^{-1} \left( x + \frac{x^2}{2} \right) \cdot (1 + x) - \frac{d}{dx}(1 + x) \cdot \sin^{-1} \left( x + \frac{x^2}{2} \right)}{(1 + x)^2}$$



$$y' = \frac{\frac{1+x}{\sqrt{1-\left(x+\frac{x^2}{2}\right)^2}} \cdot (1+x) - 1 \cdot \sin^{-1}\left(x+\frac{x^2}{2}\right)}{(1+x)^2}$$

$$y' = \frac{\frac{(1+x)^2}{\sqrt{1-\left(x+\frac{x^2}{2}\right)^2}} - \sin^{-1}\left(x+\frac{x^2}{2}\right)}{(1+x)^2}$$

$$y' = \frac{\frac{(1+x)^2}{\sqrt{1-\left(x+\frac{x^2}{2}\right)^2}}}{(1+x)^2} - \frac{\sin^{-1}\left(x+\frac{x^2}{2}\right)}{(1+x)^2}$$

$$y' = \frac{1}{\sqrt{1-\left(x+\frac{x^2}{2}\right)^2}} - \frac{\sin^{-1}\left(x+\frac{x^2}{2}\right)}{(1+x)^2}$$

■ 6. Find the derivative.

$$y = \frac{1 - \sin^{-1}(2x)}{1 + \cos^{-1}(2x)}$$

*Solution:*

We'll need to use quotient rule and the formulas for the derivatives of the inverse sine and cosine functions.



$$\frac{d}{dt} a \sin^{-1}(y(t)) = a \cdot \frac{y'(t)}{\sqrt{1 - [y(t)]^2}}$$

$$\frac{d}{dt} a \cos^{-1}(y(t)) = a \cdot \left( -\frac{y'(t)}{\sqrt{1 - [y(t)]^2}} \right)$$

If  $y(t) = 2x$ , then  $y'(t) = 2$ . Then the derivative is

$$y' = \frac{\frac{d}{dx}(1 - \sin^{-1}(2x)) \cdot (1 + \cos^{-1}(2x)) - \frac{d}{dx}(1 + \cos^{-1}(2x)) \cdot (1 - \sin^{-1}(2x))}{(1 + \cos^{-1}(2x))^2}$$

$$y' = \frac{\left( -\frac{2}{\sqrt{1 - (2x)^2}} \right) \cdot (1 + \cos^{-1}(2x)) - \left( -\frac{2}{\sqrt{1 - (2x)^2}} \right) \cdot (1 - \sin^{-1}(2x))}{(1 + \cos^{-1}(2x))^2}$$

$$y' = \frac{\left( -\frac{2}{\sqrt{1 - (2x)^2}} \right) [(1 + \cos^{-1}(2x)) - (1 - \sin^{-1}(2x))]}{(1 + \cos^{-1}(2x))^2}$$

$$y' = -\frac{2(1 + \cos^{-1}(2x) - 1 + \sin^{-1}(2x))}{\sqrt{1 - (2x)^2}(1 + \cos^{-1}(2x))^2}$$

$$y' = -\frac{2(\cos^{-1}(2x) + \sin^{-1}(2x))}{\sqrt{1 - 4x^2}(1 + \cos^{-1}(2x))^2}$$





## HYPERBOLIC DERIVATIVES

■ 1. Find  $f'(\theta)$  if  $f(\theta) = 3 \sinh(2\theta^2 - 5\theta + 2)$ .

*Solution:*

The derivative of hyperbolic sine is given by

$$\frac{d}{d\theta} a \sinh(y(\theta)) = a \cdot \cosh(y(\theta)) \cdot y'(\theta)$$

If  $a = 3$  and  $y(\theta) = 2\theta^2 - 5\theta + 2$ , then  $y'(\theta) = 4\theta - 5$ . Then the derivative is

$$f'(\theta) = 3 \cosh(2\theta^2 - 5\theta + 2)(4\theta - 5)$$

$$f'(\theta) = 3(4\theta - 5)\cosh(2\theta^2 - 5\theta + 2)$$

■ 2. Find  $g'(\theta)$  if  $g(\theta) = 2 \cosh(5\theta^{\frac{3}{2}} + 6\theta)$ .

*Solution:*

The derivative of hyperbolic cosine is given by

$$\frac{d}{d\theta} a \cosh(y(\theta)) = a \cdot \sinh(y(\theta)) \cdot y'(\theta)$$

If  $a = 2$  and  $y(\theta) = 5\theta^{\frac{3}{2}} + 6\theta$ , then  $y'(\theta) = 5(3/2)\theta^{\frac{1}{2}} + 6$ . Then the derivative is



$$g'(\theta) = 2 \sinh(5\theta^{\frac{3}{2}} + 6\theta) \left( \frac{15}{2}\theta^{\frac{1}{2}} + 6 \right)$$

$$g'(\theta) = (15\theta^{\frac{1}{2}} + 12)\sinh(5\theta^{\frac{3}{2}} + 6\theta)$$

$$g'(\theta) = 3(5\theta^{\frac{1}{2}} + 4)\sinh(5\theta^{\frac{3}{2}} + 6\theta)$$

■ 3. Find  $h'(\theta)$  if  $h(\theta) = 9 \tanh(3\theta^2 - \theta\sqrt{3})$ .

*Solution:*

The derivative of hyperbolic tangent is given by

$$\frac{d}{d\theta} a \tanh(y(\theta)) = a \cdot \text{sech}^2(y(\theta)) \cdot y'(\theta)$$

If  $a = 9$  and  $y(\theta) = 3\theta^2 - \theta\sqrt{3}$ , then  $y'(\theta) = 6\theta - \sqrt{3} \cdot \theta^{\sqrt{3}-1}$ . Then the derivative is

$$h'(\theta) = 9(6\theta - \sqrt{3} \cdot \theta^{\sqrt{3}-1})\text{sech}^2(3\theta^2 - \theta\sqrt{3})$$

■ 4. Find the derivative of the hyperbolic function.

$$y = \coth(x^2 + 3x) - x^4 \text{csch}(x^2)$$

*Solution:*



Let's work on one term at a time. The derivative of the first term can be found by applying the formula for the derivative of hyperbolic cotangent with  $g(x) = x^2 + 3x$ . The derivative of that first term will be given by

$$-\operatorname{csch}^2[g(x)][g'(x)]$$

$$-\operatorname{csch}^2(x^2 + 3x)(2x + 3)$$

$$-(2x + 3)\operatorname{csch}^2(x^2 + 3x)$$

To find the derivative of the second term, we'll use product rule and the formula for the derivative of hyperbolic cosecant.

$$(x^4)(-\operatorname{csch}(x^2)\coth(x^2))(2x) + (4x^3)(\operatorname{csch}(x^2))$$

$$-2x^5\operatorname{csch}(x^2)\coth(x^2) + 4x^3\operatorname{csch}(x^2)$$

Putting these derivatives together gives the derivative for the original function.

$$y' = -(2x + 3)\operatorname{csch}^2(x^2 + 3x) - (-2x^5\operatorname{csch}(x^2)\coth(x^2) + 4x^3\operatorname{csch}(x^2))$$

$$y' = -(2x + 3)\operatorname{csch}^2(x^2 + 3x) + 2x^5\operatorname{csch}(x^2)\coth(x^2) - 4x^3\operatorname{csch}(x^2)$$

■ 5. Find the derivative of the hyperbolic function.

$$y = \frac{2x + 3e^x}{\cosh(x^{-5})}$$

*Solution:*



To find the derivative, we'll use quotient rule, and apply the formula for the derivative of hyperbolic cosine.

$$y' = \frac{(2 + 3e^x)(\cosh(x^{-5})) - (2x + 3e^x)(\sinh(x^{-5}))(-5x^{-6})}{(\cosh(x^{-5}))^2}$$

$$y' = \frac{(2 + 3e^x)\cosh(x^{-5}) + 5x^{-6}(2x + 3e^x)\sinh(x^{-5})}{(\cosh(x^{-5}))^2}$$

■ 6. Find the derivative of the hyperbolic function.

$$y = \tanh(x^2)\tan(x^2)$$

*Solution:*

To find the derivative, we'll use product rule and the formula for the derivative of hyperbolic tangent. Following the format of product rule,

$$y' = f(x)g'(x) + f'(x)g(x)$$

we'll identify values for the product rule formula.

$$f(x) = \tanh(x^2)$$

$$f'(x) = \operatorname{sech}^2(x^2)(2x) = 2x\operatorname{sech}^2(x^2)$$

and

$$g(x) = \tan(x^2)$$



$$g'(x) = \sec^2(x^2)(2x) = 2x \sec^2(x^2)$$

Then the derivative of the original function is

$$y' = (\tanh(x^2))(2x \sec^2(x^2)) + (2x \operatorname{sech}^2(x^2))(\tan(x^2))$$

$$y' = 2x \tanh(x^2) \sec^2(x^2) + 2x \tan(x^2) \operatorname{sech}^2(x^2)$$



## INVERSE HYPERBOLIC DERIVATIVES

■ 1. Find  $f'(t)$  if  $f(t) = 7 \sinh^{-1}(5t^4)$ .

*Solution:*

The derivative of inverse hyperbolic sine is given by

$$\frac{d}{dt} a \sinh^{-1}(y(t)) = a \cdot \frac{y'(t)}{\sqrt{[y(t)]^2 + 1}}$$

If  $a = 7$  and  $y(t) = 5t^4$ , then  $y'(t) = 20t^3$ . Then the derivative is

$$f'(t) = 7 \cdot \frac{20t^3}{\sqrt{(5t^4)^2 + 1}} = \frac{140t^3}{\sqrt{25t^8 + 1}}$$

■ 2. Find  $g'(t)$  if  $g(t) = 4 \cosh^{-1}(2t - 3)$ .

*Solution:*

The derivative of inverse hyperbolic cosine is given by

$$\frac{d}{dt} a \cosh^{-1}(y(t)) = a \cdot \frac{y'(t)}{\sqrt{[y(t)]^2 - 1}}$$



If  $a = 4$  and  $y(t) = 2t - 3$ , then  $y'(t) = 2$ . Then the derivative is

$$g'(t) = 4 \cdot \frac{2}{\sqrt{(2t-3)^2 - 1}}$$

$$g'(t) = \frac{8}{\sqrt{4t^2 - 12t + 9 - 1}}$$

$$g'(t) = \frac{8}{\sqrt{4t^2 - 12t + 8}}$$

$$g'(t) = \frac{8}{\sqrt{4(t-1)(t-2)}}$$

$$g'(t) = \frac{4}{\sqrt{(t-1)(t-2)}}$$

■ 3. Find  $h'(t)$  if  $h(t) = 9 \tanh^{-1}(-7t + 2)$ .

*Solution:*

The derivative of inverse hyperbolic tangent is given by

$$\frac{d}{dt} a \tanh^{-1}(y(t)) = a \cdot \frac{y'(t)}{1 - [y(t)]^2}$$

If  $a = 9$  and  $y(t) = -7t + 2$ , then  $y'(t) = -7$ . Then the derivative is



$$h'(t) = 9 \cdot \frac{-7}{1 - (-7t + 2)^2}$$

$$h'(t) = -\frac{63}{1 - (49t^2 - 28t + 4)}$$

$$h'(t) = -\frac{63}{1 - 49t^2 + 28t - 4}$$

$$h'(t) = -\frac{63}{-49t^2 + 28t - 3}$$

$$h'(t) = \frac{63}{49t^2 - 28t + 3}$$

■ 4. Find the derivative of the inverse hyperbolic function.

$$y = \cosh^{-1}(3x^3 + 4x^2) - x^2 \sinh^{-1}(e^x)$$

*Solution:*

Apply the formula for the derivative of inverse hyperbolic cosine with  $g(x) = 3x^3 + 4x^2$  and  $g'(x) = 9x^2 + 8x$ , and the formula for the derivative of inverse hyperbolic sine with  $g(x) = e^x$  and  $g'(x) = e^x$ . We'll also need to use product rule for the second term.

$$y' = \left( \frac{1}{\sqrt{(3x^3 + 4x^2)^2 - 1}} \right) (9x^2 + 8x)$$





$$- \left[ (2x)(\sinh^{-1}(e^x)) + (x^2) \left( \frac{1}{\sqrt{(e^x)^2 + 1}} \right) (e^x) \right]$$

$$y' = \frac{9x^2 + 8x}{\sqrt{(3x^3 + 4x^2)^2 - 1}} - 2x \sinh^{-1}(e^x) - \frac{x^2 e^x}{\sqrt{e^{2x} + 1}}$$

■ 5. Find the derivative of the inverse hyperbolic function.

$$y = \left( \operatorname{csch}^{-1} \left( \frac{x^2}{3x^4 + 1} \right) \right)^5$$

*Solution:*

Use a substitution with

$$u = \operatorname{csch}^{-1} \left( \frac{x^2}{3x^4 + 1} \right)$$

and apply the formula for the derivative of inverse hyperbolic cosecant.

With

$$g(x) = \frac{x^2}{3x^4 + 1}$$

$$g'(x) = \frac{2x - 6x^5}{(3x^4 + 1)^2}$$



we get

$$u' = - \frac{1}{\left| \frac{x^2}{3x^4 + 1} \right| \sqrt{\left( \frac{x^2}{3x^4 + 1} \right)^2 + 1}} \cdot \frac{2x - 6x^5}{(3x^4 + 1)^2}$$

$$u' = - \frac{\left| \frac{3x^4 + 1}{x^2} \right| \cdot \frac{2x - 6x^5}{(3x^4 + 1)^2}}{\sqrt{\left( \frac{x^2}{3x^4 + 1} \right)^2 + 1}}$$

$$u' = - \frac{\frac{2 - 6x^4}{x(3x^4 + 1)}}{\sqrt{\left( \frac{x^2}{3x^4 + 1} \right)^2 + 1}}$$

$$u' = - \frac{2 - 6x^4}{x(3x^4 + 1) \sqrt{\left( \frac{x^2}{3x^4 + 1} \right)^2 + 1}}$$

$$u' = - \frac{2 - 6x^4}{x(3x^4 + 1) \sqrt{\frac{x^4}{(3x^4 + 1)^2} + 1}}$$

$$u' = - \frac{2 - 6x^4}{x(3x^4 + 1) \sqrt{\frac{x^4 + (3x^4 + 1)^2}{(3x^4 + 1)^2}}}$$

$$u' = - \frac{2 - 6x^4}{x(3x^4 + 1) \frac{\sqrt{x^4 + (3x^4 + 1)^2}}{3x^4 + 1}}$$



$$u' = -\frac{2 - 6x^4}{x\sqrt{x^4 + (3x^4 + 1)^2}}$$

Then the function is

$$y = u^5$$

and its derivative is

$$y'(x) = 5u^4 \cdot u'$$

$$y'(x) = 5 \left( \operatorname{csch}^{-1} \left( \frac{x^2}{3x^4 + 1} \right) \right)^4 \cdot \left( -\frac{2 - 6x^4}{x\sqrt{x^4 + (3x^4 + 1)^2}} \right)$$

$$y'(x) = -\frac{10 - 30x^4}{x\sqrt{x^4 + (3x^4 + 1)^2}} \left( \operatorname{csch}^{-1} \left( \frac{x^2}{3x^4 + 1} \right) \right)^4$$

■ 6. Find the derivative of the inverse hyperbolic function.

$$y = -\frac{\coth^{-1} x}{\tanh^{-1}(2x^4)}$$

*Solution:*

Apply the formulas for the derivative of inverse hyperbolic cotangent and inverse hyperbolic tangent. We'll also need to use quotient rule.



$$y' = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

First, let's list out  $f(x)$  and  $g(x)$  and their derivatives.

$$f(x) = \coth^{-1} x$$

$$f'(x) = \frac{1}{1-x^2}$$

and

$$g(x) = \tanh^{-1}(2x^4)$$

$$g'(x) = \frac{1}{1-(2x^4)^2} \cdot (8x^3) = \frac{8x^3}{1-4x^8}$$

Now we can plug these values directly into the quotient rule formula.

$$y' = - \frac{\left(\frac{1}{1-x^2}\right)(\tanh^{-1}(2x^4)) - (\coth^{-1} x)\left(\frac{8x^3}{1-4x^8}\right)}{(\tanh^{-1}(2x^4))^2}$$

$$y' = - \frac{\left(\frac{1}{1-x^2}\right)(\tanh^{-1}(2x^4))}{(\tanh^{-1}(2x^4))^2} + \frac{(\coth^{-1} x)\left(\frac{8x^3}{1-4x^8}\right)}{(\tanh^{-1}(2x^4))^2}$$

$$y' = - \frac{\frac{1}{1-x^2}}{\tanh^{-1}(2x^4)} + \frac{\coth^{-1} x \left(\frac{8x^3}{1-4x^8}\right)}{(\tanh^{-1}(2x^4))^2}$$

$$y' = - \frac{1}{(1-x^2)\tanh^{-1}(2x^4)} + \frac{8x^3 \coth^{-1} x}{(1-4x^8)(\tanh^{-1}(2x^4))^2}$$



## LOGARITHMIC DIFFERENTIATION

- 1. Use logarithmic differentiation to find  $dy/dx$ .

$$y = (\ln x)^{\ln(x^2)}$$

*Solution:*

Take the natural log of both sides.

$$\ln y = \ln((\ln x)^{\ln(x^2)})$$

Use properties of logarithms to rewrite the equation.

$$\ln y = \ln(x^2)\ln(\ln x)$$

Differentiate, remembering to apply product and chain rule, then solve for  $dy/dx$ .

$$\frac{1}{y} \cdot y' = \frac{2x}{x^2} \cdot \ln(\ln x) + \ln x^2 \cdot \frac{1}{\ln x} \cdot \frac{1}{x}$$

$$\frac{1}{y} \cdot y' = \frac{2 \ln(\ln x)}{x} + \frac{\ln x^2}{x \ln x}$$

$$y' = y \left( \frac{2 \ln(\ln x)}{x} + \frac{\ln x^2}{x \ln x} \right)$$

Substitute for  $y$ .



$$y' = (\ln x)^{\ln(x^2)} \left( \frac{2 \ln(\ln x)}{x} + \frac{\ln x^2}{x \ln x} \right)$$

■ 2. Use logarithmic differentiation to find  $dy/dx$ .

$$y = 5x^4 e^{3x} \sqrt[4]{x}$$

*Solution:*

Take the natural log of both sides.

$$\ln y = \ln \left( 5x^4 e^{3x} \sqrt[4]{x} \right)$$

Use properties of logarithms to rewrite the equation.

$$\ln y = \ln 5x^4 + \ln e^{3x} + \ln \sqrt[4]{x}$$

$$\ln y = \ln 5 + \ln x^4 + 3x + \ln x^{\frac{1}{4}}$$

$$\ln y = \ln 5 + 4 \ln x + 3x + \frac{1}{4} \ln x$$

Differentiate, remembering to apply chain rule, then solve for  $dy/dx$ .

$$\frac{1}{y} \cdot \frac{dy}{dx} = 0 + \frac{4}{x} + 3 + \frac{1}{4x}$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{4}{x} + 3 + \frac{1}{4x}$$



$$\frac{dy}{dx} = y \left( \frac{4}{x} + 3 + \frac{1}{4x} \right)$$

Substitute for  $y$ .

$$\frac{dy}{dx} = 5x^4 e^{3x} \sqrt[4]{x} \left( \frac{4}{x} + 3 + \frac{1}{4x} \right)$$

You could leave the answer like this, or try to simplify.

$$\frac{dy}{dx} = 5x^4 e^{3x} \sqrt[4]{x} \left( \frac{16}{4x} + \frac{12x}{4x} + \frac{1}{4x} \right)$$

$$\frac{dy}{dx} = 5x^4 e^{3x} \sqrt[4]{x} \left( \frac{12x + 17}{4x} \right)$$

$$\frac{dy}{dx} = \frac{5x^3 e^{3x} \sqrt[4]{x} (12x + 17)}{4}$$

■ 3. Use logarithmic differentiation to find  $dy/dx$ .

$$y = (7 - 4x^3)^{x^2+9} \sqrt[3]{1 - \cos(3x)}$$

*Solution:*

Take the natural log of both sides.

$$\ln y = \ln[(7 - 4x^3)^{x^2+9} \sqrt[3]{1 - \cos(3x)}]$$

Use properties of logarithms to rewrite the equation.



$$\ln y = \ln(7 - 4x^3)^{x^2+9} + \ln(\sqrt[3]{1 - \cos(3x)})$$

$$\ln y = (x^2 + 9)\ln(7 - 4x^3) + \ln(1 - \cos(3x))^{\frac{1}{3}}$$

$$\ln y = (x^2 + 9)\ln(7 - 4x^3) + \frac{1}{3} \ln(1 - \cos(3x))$$

Differentiate, remembering to apply product and chain rule, then solve for  $dy/dx$ .

$$\frac{1}{y} \cdot \frac{dy}{dx} = 2x \ln(7 - 4x^3) + (x^2 + 9) \left( \frac{-12x^2}{7 - 4x^3} \right) + \frac{1}{3(1 - \cos(3x))} (3 \sin(3x))$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = 2x \ln(7 - 4x^3) - \frac{12x^2(x^2 + 9)}{7 - 4x^3} + \frac{\sin(3x)}{1 - \cos(3x)}$$

$$\frac{dy}{dx} = y \left( 2x \ln(7 - 4x^3) - \frac{12x^2(x^2 + 9)}{7 - 4x^3} + \frac{\sin(3x)}{1 - \cos(3x)} \right)$$

Substitute for  $y$ .

$$\frac{dy}{dx} = (7 - 4x^3)^{x^2+9} \sqrt[3]{1 - \cos(3x)} \left( 2x \ln(7 - 4x^3) - \frac{12x^2(x^2 + 9)}{7 - 4x^3} + \frac{\sin(3x)}{1 - \cos(3x)} \right)$$

■ 4. Use logarithmic differentiation to find  $dy/dx$ .

$$y = \frac{(2e)^{\cos x}}{(3e)^{\sin x}}$$





*Solution:*

Take the natural log of both sides.

$$\ln y = \ln \left( \frac{(2e)^{\cos x}}{(3e)^{\sin x}} \right)$$

Use properties of logarithms to rewrite the equation.

$$\ln y = \ln(2e)^{\cos x} - \ln(3e)^{\sin x}$$

$$\ln y = \cos x \ln(2e) - \sin x \ln(3e)$$

$$\ln y = (\cos x)(\ln 2 + \ln e) - (\sin x)(\ln 3 + \ln e)$$

$$\ln y = (\cos x)(\ln 2 + 1) - (\sin x)(\ln 3 + 1)$$

Differentiate, remembering to apply chain rule, then solve for  $dy/dx$ .

$$\frac{1}{y} \cdot \frac{dy}{dx} = (-\sin x)(\ln 2 + 1) + (\cos x)(0) - [(\cos x)(\ln 3 + 1) + (\sin x)(0)]$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = -(\ln 2 + 1)\sin x - (\ln 3 + 1)\cos x$$

$$\frac{dy}{dx} = -y [(\ln 2 + 1)\sin x + (\ln 3 + 1)\cos x]$$

Substitute for  $y$ .

$$\frac{dy}{dx} = -\frac{(2e)^{\cos x}}{(3e)^{\sin x}} [(\ln 2 + 1)\sin x + (\ln 3 + 1)\cos x]$$



■ 5. Use logarithmic differentiation to find  $dy/dx$ .

$$y = e^x(2e)^{\sin x}(3e)^{\cos x}$$

*Solution:*

Take the natural log of both sides.

$$\ln y = \ln(e^x(2e)^{\sin x}(3e)^{\cos x})$$

Use properties of logarithms to rewrite the equation.

$$\ln y = \ln e^x + \ln(2e)^{\sin x} + \ln(3e)^{\cos x}$$

$$\ln y = x + \sin x \ln(2e) + \cos x \ln(3e)$$

$$\ln y = x + \sin x(\ln 2 + \ln e) + \cos x(\ln 3 + \ln e)$$

$$\ln y = x + \sin x(\ln 2 + 1) + \cos x(\ln 3 + 1)$$

Differentiate, remembering to apply chain rule, then solve for  $dy/dx$ .

$$\frac{1}{y} \cdot \frac{dy}{dx} = 1 + (\ln 2 + 1)\cos x - (\ln 3 + 1)\sin x$$

$$\frac{dy}{dx} = y [1 + (\ln 2 + 1)\cos x - (\ln 3 + 1)\sin x]$$

Substitute for  $y$ .

$$\frac{dy}{dx} = e^x(2e)^{\sin x}(3e)^{\cos x} [1 + (\ln 2 + 1)\cos x - (\ln 3 + 1)\sin x]$$



■ 6. Use logarithmic differentiation to find  $dy/dx$ .

$$y = \frac{(1 - 2x)^{\sin x}}{(x^3 - 2x)^{5x+7}}$$

*Solution:*

Take the natural log of both sides.

$$\ln y = \ln \left( \frac{(1 - 2x)^{\sin x}}{(x^3 - 2x)^{5x+7}} \right)$$

Use properties of logarithms to rewrite the equation.

$$\ln y = \ln(1 - 2x)^{\sin x} - \ln(x^3 - 2x)^{5x+7}$$

$$\ln y = \sin x \ln(1 - 2x) - (5x + 7)\ln(x^3 - 2x)$$

Differentiate, remembering to apply product and chain rule, then solve for  $dy/dx$ .

$$\frac{1}{y} \cdot \frac{dy}{dx} = \cos x \ln(1 - 2x) + \sin x \left( \frac{-2}{1 - 2x} \right) - \left[ 5 \ln(x^3 - 2x) + (5x + 7) \left( \frac{3x^2 - 2}{x^3 - 2x} \right) \right]$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \cos x \ln(1 - 2x) - \frac{2 \sin x}{1 - 2x} - 5 \ln(x^3 - 2x) - \frac{(3x^2 - 2)(5x + 7)}{x^3 - 2x}$$

Substitute for  $y$ .



$$\frac{dy}{dx} = y \left( \cos x \ln(1 - 2x) - \frac{2 \sin x}{1 - 2x} - 5 \ln(x^3 - 2x) - \frac{(3x^2 - 2)(5x + 7)}{x^3 - 2x} \right)$$

$$\frac{dy}{dx} = \frac{(1 - 2x)^{\sin x}}{(x^3 - 2x)^{5x+7}} \left( \cos x \ln(1 - 2x) - \frac{2 \sin x}{1 - 2x} - 5 \ln(x^3 - 2x) - \frac{(3x^2 - 2)(5x + 7)}{x^3 - 2x} \right)$$



