Topic: Rolle's Theorem

Question: Which of these is not part of Rolle’s Theorem?

Answer choices:

A The function $f(x)$ must be continuous on the closed interval $[a, b]$.

B That the function $f(x)$ meets the condition $f(a) = f(b)$ for the closed interval $[a, b]$.

C The function $f(x)$ can be integrated over the open interval (a, b).

D The function $f(x)$ is differentiable on the open interval (a, b).

Solution: C

Rolle's Theorem requires three conditions be met in order for its conclusion to be true:

- The function $f(x)$ must be continuous on the closed interval $[a, b]$
- The function $f(x)$ must be differentiable on the open interval (a, b)
- The function $f(x)$ meets the condition $f(a) = f(b)$ for the interval $[a, b]$

If these three conditions are met, Rolle's Theorem states that there must exist a point c within the interval (a, b) where $f'(c) = 0$.

Topic: Rolle's Theorem

Question: Does the function meet the criteria of Rolle's Theorem on the interval $[0,1]$?

$$f(x) = x^2 - x + 6$$

Answer choices:

A Yes, it's continuous and differentiable over the interval, and $f(0) = f(1)$.

B Yes, it's continuous and $f(0) \neq f(1)$.

C No, it's not differentiable over the interval.

D No, it's discontinuous, and $f(0) \neq f(1)$.

Solution: A

Since this is a polynomial function, and we know that polynomial functions are continuous for all real numbers, we know that the function is continuous and differentiable on the interval $[0,1]$.

Confirm that $f(0)=f(1)$.

$$f(0)=0^2-0+6$$

$$f(0)=6$$

and

$$f(1)=1^2-1+6$$

$$f(1)=6$$

Since $f(0)=6=f(1)$, we've confirmed that this function over the given interval meets all three conditions of Rolle's Theorem.

Topic: Rolle's Theorem

Question: Use Rolle's Theorem to find the point in the interval $[0,4]$ where the function has a horizontal tangent line.

$$f(x) = -x^2 + 4x + 16$$

Answer choices:

A $(0,4)$

B $(-2,20)$

C $(0,-4)$

D $(2,20)$

Solution: D

Since this is a polynomial function, and we know that polynomial functions are continuous for all real numbers, we know that the function is continuous and differentiable on the interval $[0,4]$. Evaluating the function at the endpoints of the interval, we get

$$f(0) = -0^2 + 4(0) + 16$$

$$f(0) = 16$$

and

$$f(4) = -4^2 + 4(4) + 16$$

$$f(4) = 16$$

Since $f(0) = 16 = f(4)$, we've confirmed that the function over the given interval meets all three conditions of Rolle's Theorem.

Now we can find the point c by solving the equation $f'(c) = 0$.

$$f'(x) = -2x + 4$$

$$-2c + 4 = 0$$

$$2c = 4$$

$$c = 2$$

To find the coordinate point associated with $c = 2$, we'll plug it back into the original function.

$$f(2) = -2^2 + 4(2) + 16$$

$$f(2) = -4 + 8 + 16$$

$$f(2) = 20$$

The conclusion of Rolle's Theorem tells us that the function has a horizontal tangent line at $(2,20)$ inside the interval $[0,4]$.