Topic: Average value

Question: Find the average value of the function over the given interval.

$$f(x) = 5 + 2x - x^2$$

on the interval $[-2,3]$

Answer choices:

A $\frac{11}{3}$

B -2

C -10

D $\frac{55}{3}$

Solution: A

To find average value of a function over a given interval, we use the integration formula

$$f_{avg} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

where $f(x)$ is the function for which we want the average, and $[a, b]$ is the interval we're interested in.

Plugging the given function and the interval into the formula, we get

$$\frac{1}{3-(-2)}\int_{-2}^{3} 5+2x-x^2\,dx$$

$$\frac{1}{5}\left(5x+x^2-\frac{1}{3}x^3\right)\Bigg|_{-2}^{3}$$

$$\frac{1}{5}\left[\left(5(3)+(3)^2-\frac{1}{3}(3)^3\right)-\left(5(-2)+(-2)^2-\frac{1}{3}(-2)^3\right)\right]$$

$$\frac{1}{5}\left[15+9-9-\left(-10+4+\frac{8}{3}\right)\right]$$

$$\frac{1}{5}\left(15+10-4-\frac{8}{3}\right)$$

$$\frac{1}{5}\left(21-\frac{8}{3}\right)$$

$$\frac{21}{5}-\frac{8}{15}$$

$$\frac{63}{15} - \frac{8}{15}$$

$$\frac{55}{15}$$

$$\frac{11}{3}$$

Topic: Average value

Question: Find the average value of the function over the given interval.

$$f(x) = x^3 + 4x$$

on the interval $[2,5]$

Answer choices:

A 89

B $\frac{259}{4}$

C $\frac{89}{3}$

D $\frac{259}{3}$

Solution: B

To find average value of a function over a given interval, we use the integration formula

$$f_{avg} = \frac{1}{b-a}\int_a^b f(x)\ dx$$

where $f(x)$ is the function for which we want the average, and $[a, b]$ is the interval we're interested in.

Plugging the given function and the interval into the formula, we get

$$f_{avg} = \frac{1}{5-2}\int_2^5 x^3 + 4x\ dx$$

$$f_{avg} = \frac{1}{3}\int_2^5 x^3 + 4x\ dx$$

$$f_{avg} = \frac{1}{3}\left(\frac{x^4}{4} + \frac{4x^2}{2}\right)\Bigg|_2^5$$

$$f_{avg} = \left(\frac{x^4}{12} + \frac{2x^2}{3}\right)\Bigg|_2^5$$

$$f_{avg} = \left[\frac{(5)^4}{12} + \frac{2(5)^2}{3}\right] - \left[\frac{(2)^4}{12} + \frac{2(2)^2}{3}\right]$$

$$f_{avg} = \left(\frac{625}{12} + \frac{50}{3}\right) - \left(\frac{16}{12} + \frac{8}{3}\right)$$

$$f_{avg} = \frac{825}{12} - \frac{48}{12}$$

$$f_{avg} = \frac{777}{12}$$

$$f_{avg} = \frac{259}{4}$$

Topic: Average value

Question: Find the average value of the function over the given interval.

$$f(x) = 4xe^{2x^2}$$

on the interval $[1,3]$

Answer choices:

A $\dfrac{e^{18} - e^2}{2}$

B $8e^{16}$

C $\dfrac{e^{16}}{2}$

D $12e^{18} - 4e^2$

Solution: A

To find average value of a function over a given interval, we use the integration formula

$$f_{avg} = \frac{1}{b-a}\int_a^b f(x)\ dx$$

where $f(x)$ is the function for which we want the average, and $[a, b]$ is the interval we're interested in.

Plugging the given function and the interval into the formula, we get

$$f_{avg} = \frac{1}{3-1}\int_1^3 4xe^{2x^2}\ dx$$

$$f_{avg} = \frac{1}{2}\int_1^3 4xe^{2x^2}\ dx$$

In order to solve this integral, we'll need to use u-substitution, letting

$$u = 2x^2$$

$$du = 4x\ dx$$

Plugging these back into the integral, remembering that our limits of integration still relate to x, and not u, we get

$$f_{avg} = \frac{1}{2}\int_{x=1}^{x=3} 4xe^{u}\ dx$$

$$f_{avg} = \frac{1}{2}\int_{x=1}^{x=3} e^{u}(4x\ dx)$$

$$f_{avg} = \frac{1}{2}\int_{x=1}^{x=3} e^u \, du$$

$$f_{avg} = \frac{1}{2}e^u\Big|_{x=1}^{x=3}$$

$$f_{avg} = \frac{e^u}{2}\Big|_{x=1}^{x=3}$$

Back-substituting before we evaluate over the interval, we get

$$f_{avg} = \frac{e^{2x^2}}{2}\Big|_{1}^{3}$$

$$f_{avg} = \frac{e^{2(3)^2}}{2} - \frac{e^{2(1)^2}}{2}$$

$$f_{avg} = \frac{e^{18}}{2} - \frac{e^2}{2}$$

$$f_{avg} = \frac{e^{18} - e^2}{2}$$