Topic: Sum of the Maclaurin series

Question: Find the sum of the Maclaurin series.

$$\sum_{n=0}^{\infty} \frac{5(x+2)^n}{n!}$$

Answer choices:

A $5e^{-x}$

B $5e^{x-2}$

C $5e^{x}$

D $5e^{x+2}$

Solution: D

The easiest way to find the sum of the series of a Maclaurin series is to identify a similar Maclaurin series with a known sum, and then manipulate the given series until it matches the known series.

In this case, the given series is similar to the known series

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

We'll manipulate this series until it matches the given series. We'll start by replacing x with $x + 2$.

$$e^{x+2} = \sum_{n=0}^{\infty} \frac{(x+2)^n}{n!}$$

Then we'll multiply both sides by 5.

$$5e^{x+2} = \sum_{n=0}^{\infty} \frac{5(x+2)^n}{n!}$$

Since the right side of this manipulation now matches the given series, we can say that the sum of the given series is $5e^{x+2}$.

Topic: Sum of the Maclaurin series

Question: Find the sum of the Maclaurin series.

$$\sum_{n=0}^{\infty} 3(2)^n$$

Answer choices:

A 3

B 1

C -3

D ∞

Solution: D

The first thing we notice is that the 2^n term will only get larger and larger as n increases. Multiplying the result of 2^n by 3 only makes each term bigger. Which means that, when we add up larger and larger terms, the sum of the series will diverge to ∞.

n	$3(2)^n$	sum
0	3	3
1	6	9
2	12	21
3	24	45
4	48	93
5	96	189
...	...	...

Topic: Sum of the Maclaurin series

Question: Find the sum of the Maclaurin series.

$$\sum_{n=0}^{\infty} \frac{(-1)^n 9^n \pi^{2n}}{(2n)!}$$

Answer choices:

A -1

B 0

C 1

D ∞

Solution: A

The easiest way to find the sum of the series of a Maclaurin series is to identify a similar Maclaurin series with a known sum, and then manipulate the given series until it matches the known series.

In this case, the given series is similar to the known series

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

We'll manipulate this series until it matches the given series. But first we need to work on the given series.

$$\sum_{n=0}^{\infty} \frac{(-1)^n 9^n \pi^{2n}}{(2n)!}$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n \left(3^2\right)^n \pi^{2n}}{(2n)!}$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n 3^{2n} \pi^{2n}}{(2n)!}$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n (3\pi)^{2n}}{(2n)!}$$

Now we'll just replace x with 3π, and then simplify.

$$\cos(3\pi) = \sum_{n=0}^{\infty} \frac{(-1)^n (3\pi)^{2n}}{(2n)!}$$

$$-1 = \sum_{n=0}^{\infty} \frac{(-1)^n (3\pi)^{2n}}{(2n)!}$$

Since the right side of this manipulation now matches the given series, we can say that the sum of the given series is -1.