Topic: Washers, vertical axis

Question: Use washers to find the volume of the solid generated by revolving the region bounded by $x = \sqrt{y}$, $x = 2$, and $y = 0$ about the y-axis.

Answer choices:

A 2π

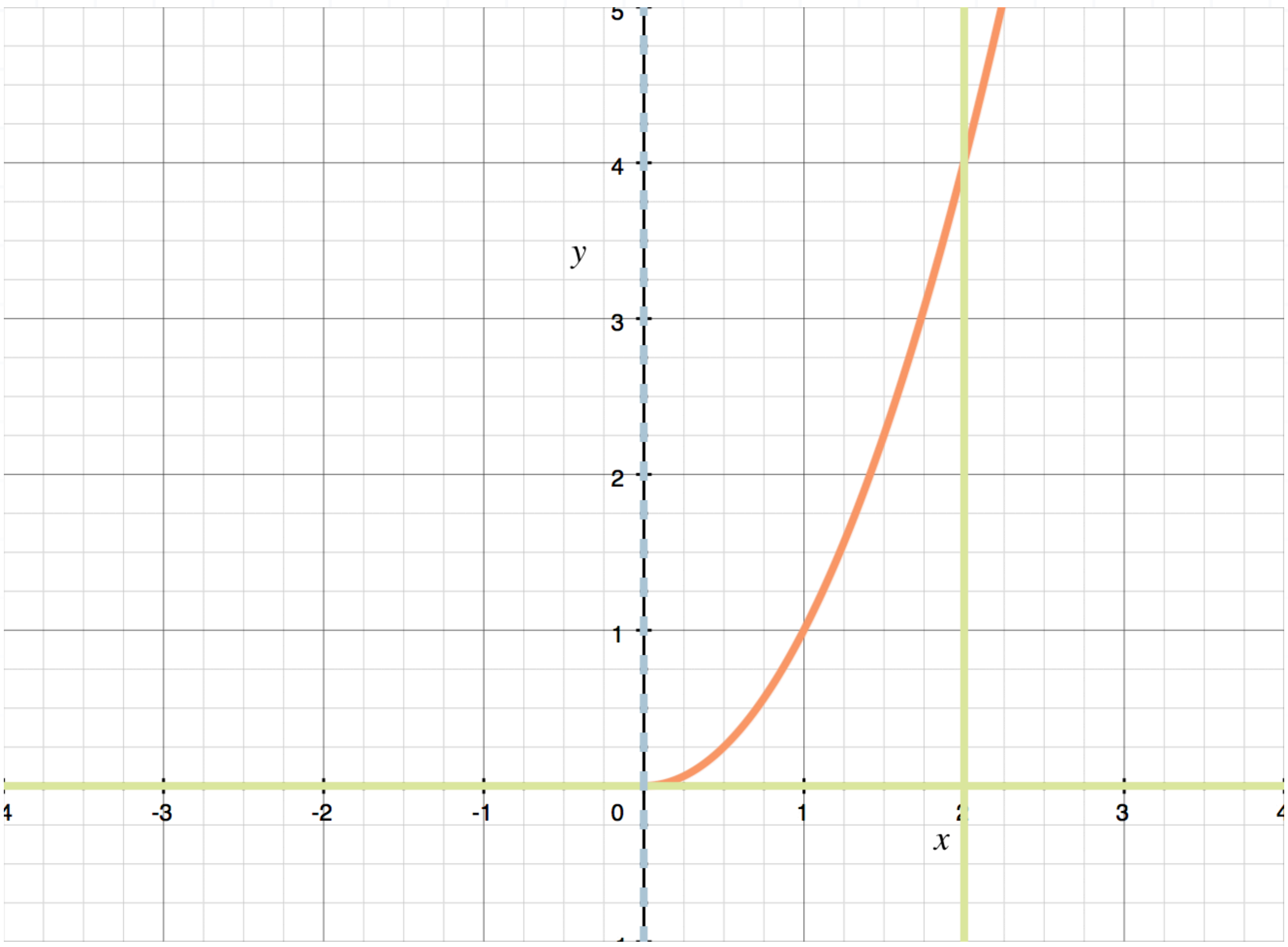
B 4π

C 8π

D 16π

Solution: C

Before doing anything else, we always want to draw a picture of the area. If you don't know how to graph the function, just plug in values for x or y to get individual coordinate points, and plot them until you have a picture of each function.



Using washers means we'll take slices of our area that are perpendicular to the axis of rotation. Therefore, since the axis of rotation is vertical, we'll take horizontal slices of our area and rotate each of them around the axis to form washers.

Using washers around a vertical axis, specifically the y-axis, tells us that we'll use the volume formula

$$V=\int_c^d \pi[f(y)]^2-\pi[g(y)]^2\ dy$$

We can see from the formula that we need our curves and our limits of integration defined in terms of y. The given curves are already defined for x in terms of y, so now we just need to find limits of integration, which will be the smallest and largest y-values for which the area is defined.

We can see from the graph that the smallest y-value for which the area is defined is $y=0$. This was given in the original problem. We can see that the largest value for which it's defined is a point of intersection, so we can set the curves equal to one another and solve for y.

$$\sqrt{y}=2$$

$$y=4$$

Now we know that our limits of integration are $c=0$ and $d=4$.

The curve $f(y)$ is the radius of the curve that's further from the axis of revolution, and $g(y)$ is the radius of the curve that's closer to the axis of revolution.

To figure out which curve is further away and which one is closer, we can look at the graph or we can plug a y-value between the points of intersection (between $y=0$ and $y=4$) into both curves to see which function returns a larger value (this will be the further curve) and which one returns a smaller value (this will be the closer curve). Let's plug in $y=1$ to check.

$$x=\sqrt{y}$$

$$x = \sqrt{1}$$

$$x = 1$$

and

$$x = 2$$

Since $x = 2$ returns a larger value than $x = \sqrt{y}$, we can say

$$g(y) = \sqrt{y}$$

and

$$f(y) = 2$$

Plugging everything we know into the volume formula, we get

$$V = \int_0^4 \pi(2)^2 - \pi(\sqrt{y})^2 \ dy$$

$$V = \int_0^4 4\pi - \pi y \ dy$$

$$V = \left(4\pi y - \frac{\pi}{2}y^2\right)\Big|_0^4$$

$$V = \left[4\pi(4) - \frac{\pi}{2}(4)^2\right] - \left[4\pi(0) - \frac{\pi}{2}(0)^2\right]$$

$$V = 16\pi - 8\pi$$

$$V = 8\pi$$

Topic: Washers, vertical axis

Question: Use washers to find the volume of the solid generated by revolving the region bounded by the curves about $x = -1$.

$$x = \frac{y^4}{4} - \frac{y^2}{2} \text{ and } x = \frac{y^2}{2}$$

Answer choices:

A $\quad V = \dfrac{1{,}984\pi}{315}$ cubic units

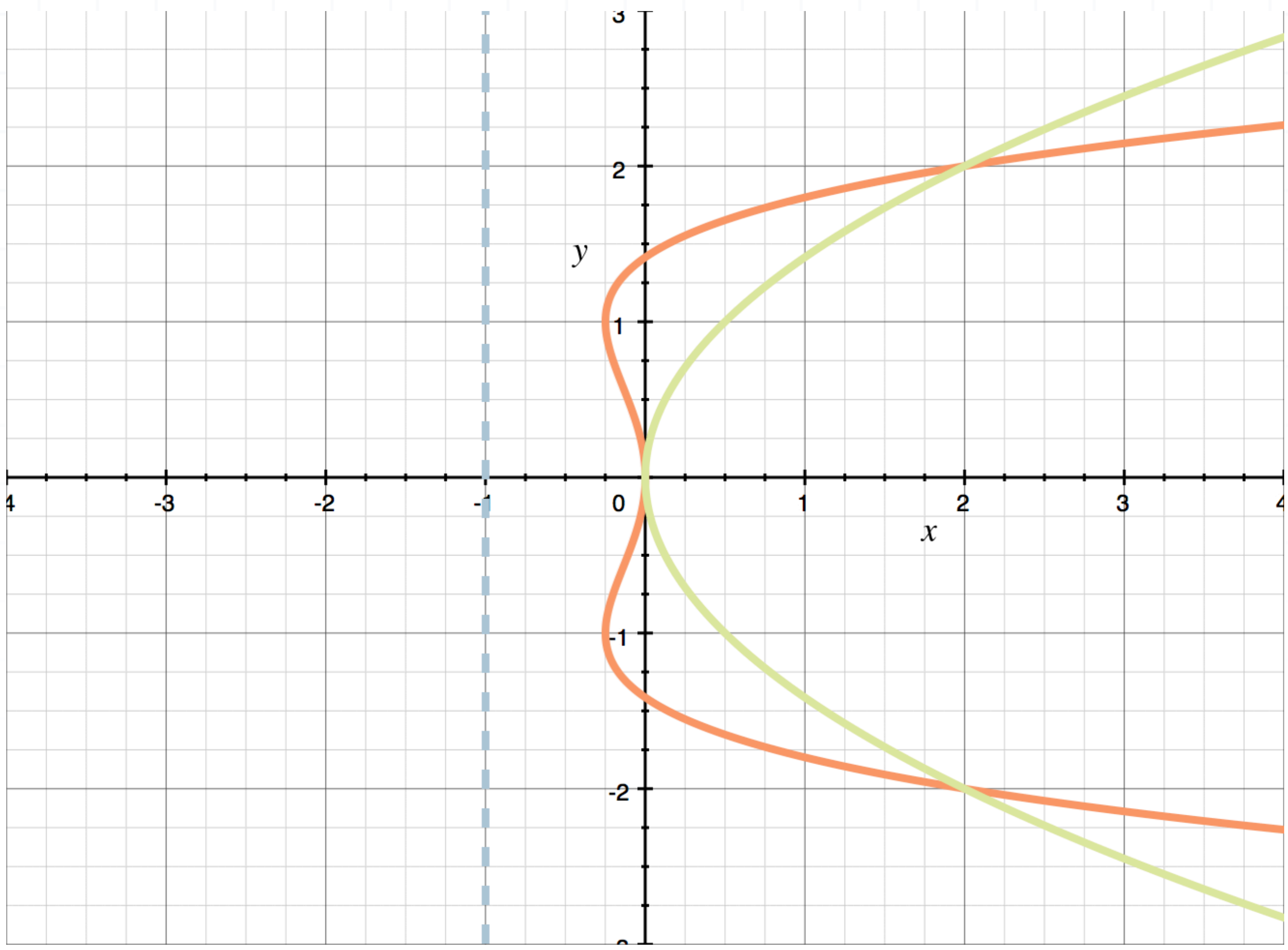
B $\quad V = \dfrac{1{,}984\pi}{315}$ square units

C $\quad V = \dfrac{986\pi}{315}$ cubic units

D $\quad V = \dfrac{986\pi}{315}$ square units

Solution: A

Before doing anything else, we always want to draw a picture of the area. If you don't know how to graph the function, just plug in values for x or y to get individual coordinate points, and plot them until you have a picture of each function.



Using washers means we'll take slices of our area that are perpendicular to the axis of rotation. Therefore, since the axis of rotation is vertical, we'll take horizontal slices of our area and rotate each of them around the axis to form washers.

Using washers around a vertical axis, specifically the line $x = -1$, tells us that we'll use the volume formula

$$V=\int_{c}^{d}\pi\left[k+f(y)\right]^2-\pi\left[k+g(y)\right]^2\ dy$$

We can see from the formula that we need our curves and our limits of integration defined in terms of y. The given curves are already defined for x in terms of y, so now we just need to find limits of integration, which will be the smallest and largest y-values for which the area is defined.

From the graph, it looks like the area is defined between $y=-2$ and $y=2$. To be sure, we'll set the curves equal to each other and solve for y.

$$\frac{y^2}{2}=\frac{y^4}{4}-\frac{y^2}{2}$$

$$2y^2=y^4-2y^2$$

$$4y^2=y^4$$

$$4=y^2$$

$$y=\pm 2$$

Now we know that our limits of integration are $c=-2$ and $d=2$.

The curve $f(y)$ is the radius of the curve that's further from the axis of revolution, and $g(y)$ is the radius of the curve that's closer to the axis of revolution.

To figure out which curve is further away and which one is closer, we can look at the graph or we can plug a y-value between the points of intersection (between $y=-2$ and $y=2$) into both curves to see which function returns a larger value (this will be the further curve) and which one returns a smaller value (this will be the closer curve). Let's plug in $y = 1$ to check.

$$x = \frac{y^4}{4} - \frac{y^2}{2}$$

$$x = \frac{1^4}{4} - \frac{1^2}{2}$$

$$x = -\frac{1}{4}$$

and

$$x = \frac{y^2}{2}$$

$$x = \frac{1^2}{2}$$

$$x = \frac{1}{2}$$

Since $x = y^2/2$ returns a larger value than $x = y^4/4 - y^2/2$, we can say

$$g(y) = \frac{y^4}{4} - \frac{y^2}{2}$$

and

$$f(y) = \frac{y^2}{2}$$

Plugging everything we know into the volume formula, we get

$$V=\int_{-2}^{2}\pi\left(1+\frac{y^2}{2}\right)^2-\pi\left(1+\frac{y^4}{4}-\frac{y^2}{2}\right)^2\ dy$$

$$V=\int_{-2}^{2}\pi\left(1+y^2+\frac{y^4}{4}\right)-\pi\left(1+\frac{y^4}{4}-\frac{y^2}{2}+\frac{y^4}{4}+\frac{y^8}{16}-\frac{y^6}{8}-\frac{y^2}{2}-\frac{y^6}{8}+\frac{y^4}{4}\right)\ dy$$

$$V=\int_{-2}^{2}\pi\left(1+y^2+\frac{y^4}{4}\right)-\pi\left(1-y^2+\frac{3y^4}{4}-\frac{y^6}{4}+\frac{y^8}{16}\right)\ dy$$

$$V=\int_{-2}^{2}\pi+\pi y^2+\frac{\pi}{4}y^4-\left(\pi-\pi y^2+\frac{3\pi}{4}y^4-\frac{\pi}{4}y^6+\frac{\pi}{16}y^8\right)\ dy$$

$$V=\int_{-2}^{2}\pi+\pi y^2+\frac{\pi}{4}y^4-\pi+\pi y^2-\frac{3\pi}{4}y^4+\frac{\pi}{4}y^6-\frac{\pi}{16}y^8\ dy$$

$$V=\int_{-2}^{2}2\pi y^2-\frac{\pi}{2}y^4+\frac{\pi}{4}y^6-\frac{\pi}{16}y^8\ dy$$

Integrate, then evaluate over the interval.

$$V=\frac{2\pi}{3}y^3-\frac{\pi}{10}y^5+\frac{\pi}{28}y^7-\frac{\pi}{144}y^9\Big|_{-2}^{2}$$

$$V=\frac{2\pi}{3}(2)^3-\frac{\pi}{10}(2)^5+\frac{\pi}{28}(2)^7-\frac{\pi}{144}(2)^9$$

$$-\left(\frac{2\pi}{3}(-2)^3-\frac{\pi}{10}(-2)^5+\frac{\pi}{28}(-2)^7-\frac{\pi}{144}(-2)^9\right)$$

$$V=\frac{16\pi}{3}-\frac{16\pi}{5}+\frac{32\pi}{7}-\frac{32\pi}{9}-\left(-\frac{16\pi}{3}+\frac{16\pi}{5}-\frac{32\pi}{7}+\frac{32\pi}{9}\right)$$

$$V=\frac{16\pi}{3}-\frac{16\pi}{5}+\frac{32\pi}{7}-\frac{32\pi}{9}+\frac{16\pi}{3}-\frac{16\pi}{5}+\frac{32\pi}{7}-\frac{32\pi}{9}$$

$$V=\frac{32\pi}{3}-\frac{32\pi}{5}+\frac{64\pi}{7}-\frac{64\pi}{9}$$

$$V=\frac{3{,}360\pi}{315}-\frac{2{,}016\pi}{315}+\frac{2{,}880\pi}{315}-\frac{2{,}240\pi}{315}$$

$$V=\frac{1{,}984\pi}{315}$$

Topic: Washers, vertical axis

Question: Use washers to find the volume of the solid generated by revolving the region bounded by the curves about $x = 4$.

$$x = \frac{y^2}{2} \text{ and } x = 2$$

Answer choices:

A $\quad V = \dfrac{448}{15}$ cubic units

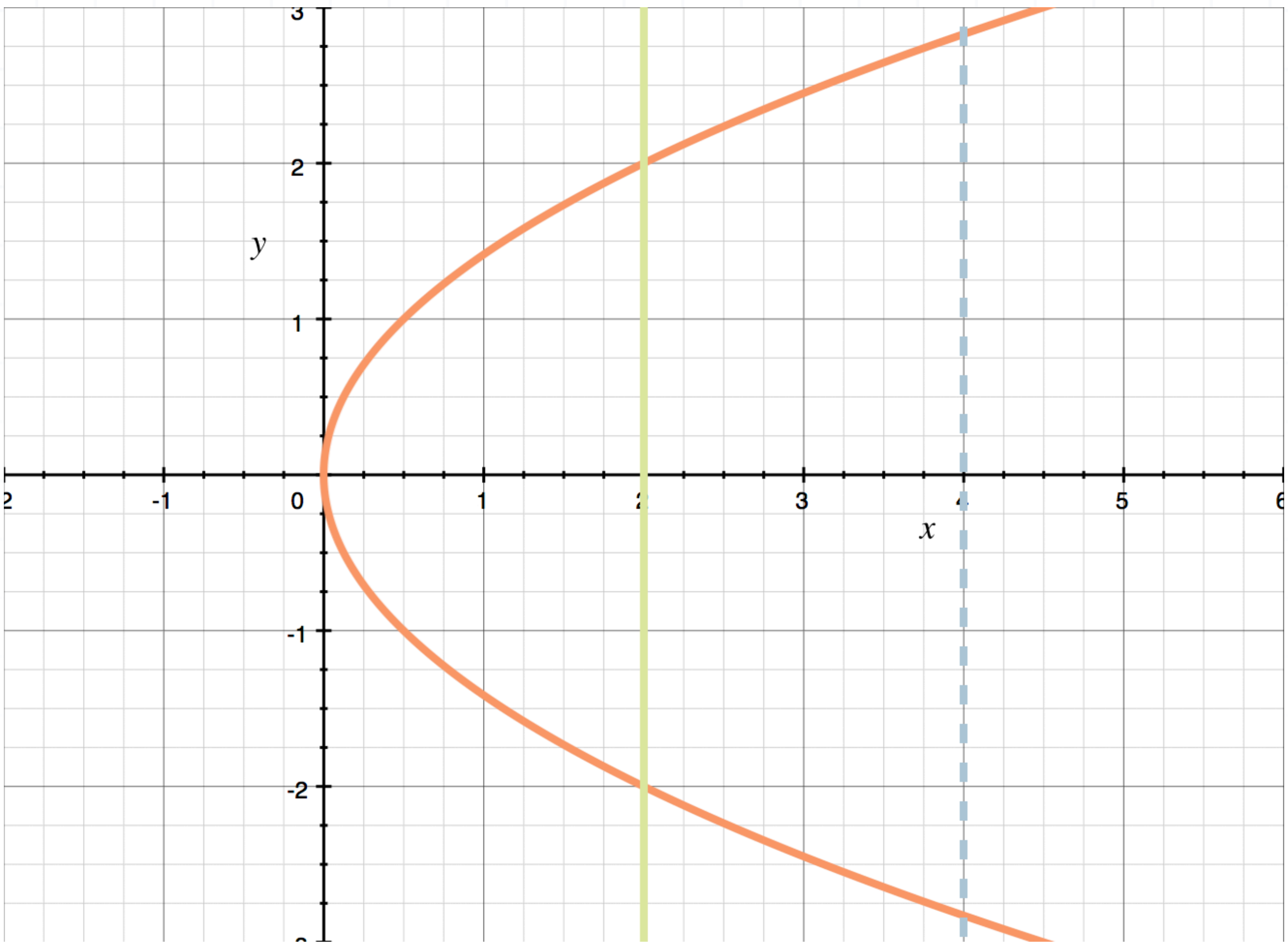
B $\quad V = \dfrac{1{,}552}{51}$ cubic units

C $\quad V = \dfrac{1{,}552}{51}\pi$ cubic units

D $\quad V = \dfrac{448}{15}\pi$ cubic units

Solution: D

Before doing anything else, we always want to draw a picture of the area. If you don't know how to graph the function, just plug in values for x or y to get individual coordinate points, and plot them until you have a picture of each function.



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Using washers around a vertical axis, specifically the line $x=4$, tells us that we'll use the volume formula

$$V=\int_c^d \pi\left[k-f(y)\right]^2-\pi\left[k-g(y)\right]^2\ dy$$

We can see from the formula that we need our curves and our limits of integration defined in terms of y. The given curves are already defined for x in terms of y, so now we just need to find limits of integration, which will be the smallest and largest y-values for which the area is defined.

From the graph, it looks like the area is defined between $y=-2$ and $y=2$. To be sure, we'll set the curves equal to each other and solve for y.

$$\frac{y^2}{2}=2$$

$$y^2=4$$

$$y=\pm 2$$

Now we know that our limits of integration are $c=-2$ and $d=2$.

The curve $f(y)$ is the radius of the curve that's further from the axis of revolution, and $g(y)$ is the radius of the curve that's closer to the axis of revolution.

To figure out which curve is further away and which one is closer, we can look at the graph or we can plug a y-value between the points of intersection (between $y=-2$ and $y=2$) into both curves to see which function returns a larger value (this will be the further curve) and which one returns a smaller value (this will be the closer curve). Let's plug in $y=1$ to check.

$$x = \frac{y^2}{2}$$

$$x = \frac{1^2}{2}$$

$$x = \frac{1}{2}$$

and

$$x = 2$$

Since $x = y^2/2$ returns a smaller value than $x = 2$, we can say

$$f(y) = \frac{y^2}{2}$$

and

$$g(y) = 2$$

Plugging everything we know into the volume formula, we get

$$V = \int_{-2}^{2} \pi \left(4 - \frac{y^2}{2}\right)^2 - \pi (4 - 2)^2 \ dy$$

$$V = \int_{-2}^{2} \pi \left(16 - 4y^2 + \frac{y^4}{4}\right) - 4\pi \ dy$$

$$V = \int_{-2}^{2} 16\pi - 4\pi y^2 + \frac{\pi y^4}{4} - 4\pi \ dy$$

$$V = \int_{-2}^{2} 12\pi - 4\pi y^2 + \frac{\pi y^4}{4} \ dy$$

Integrate, then evaluate over the interval.

$$V = 12\pi y - \frac{4\pi}{3} y^3 + \frac{\pi}{20} y^5 \Big|_{-2}^{2}$$

$$V = 12\pi(2) - \frac{4\pi}{3}(2)^3 + \frac{\pi}{20}(2)^5 - \left(12\pi(-2) - \frac{4\pi}{3}(-2)^3 + \frac{\pi}{20}(-2)^5\right)$$

$$V = 24\pi - \frac{32\pi}{3} + \frac{8\pi}{5} - \left(-24\pi + \frac{32\pi}{3} - \frac{8\pi}{5}\right)$$

$$V = 24\pi - \frac{32\pi}{3} + \frac{8\pi}{5} + 24\pi - \frac{32\pi}{3} + \frac{8\pi}{5}$$

$$V = 48\pi - \frac{64\pi}{3} + \frac{16\pi}{5}$$

$$V = \frac{720}{15}\pi - \frac{320\pi}{15} + \frac{48\pi}{15}$$

$$V = \frac{448}{15}\pi$$