Topic: tan^m sec^n, even n

Question: Evaluate the trigonometric integral.

$$\int \tan^2 x \sec^4 x \ dx$$

Answer choices:

A $\quad \frac{1}{5}\tan^5 x - \frac{1}{3}\tan^3 x + C$

B $\quad \frac{1}{5}\tan^5 x + \frac{1}{3}\tan^3 x + C$

C $\quad \frac{1}{5}\sec^5 x + \frac{1}{3}\sec^3 x + C$

D $\quad \frac{1}{5}\sec^5 x - \frac{1}{3}\sec^3 x + C$

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Solution: B

In the specific case where our function is the product of

an **even** number of **secant** factors and

an **even or odd** number of **tangent** factors,

our plan is to

1. save one $\sec^2 x$ factor and use the identity $\sec^2 x = 1 + \tan^2 x$ to write the other cosine factors in terms of tangent, then

2. use u-substitution with $u = \tan x$.

We'll separate a $\sec^2 x$, and then replace the remaining secant factors using the identity.

$$\int \tan^2 x \sec^4 x \ dx$$

$$\int \tan^2 x \sec^2 x \sec^2 x \ dx$$

$$\int \tan^2 x \sec^2 x \left(1 + \tan^2 x\right) \ dx$$

Using u-substitution with $u = \tan x$, we get

$$u = \tan x$$

$$du = \sec^2 x \ dx$$

Substitute into the integral.

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$$\int u^2 \sec^2 x \left(1 + u^2\right) \ dx$$

$$\int u^2 \left(1 + u^2\right) \left(\sec^2 x \ dx\right)$$

$$\int u^2 \left(1 + u^2\right) \left(du\right)$$

$$\int u^2 \left(1 + u^2\right) \ du$$

$$\int u^2 + u^4 \ du$$

$$\frac{1}{3}u^3 + \frac{1}{5}u^5 + C$$

Back-substituting for u, we get

$$\frac{1}{3}\tan^3 x + \frac{1}{5}\tan^5 x + C$$

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Topic: tan^m sec^n, even n

Question: Evaluate the trigonometric integral.

$$\int \tan^4 x \sec^4 x \ dx$$

Answer choices:

A $\frac{1}{7}\sec^7 x + \frac{1}{5}\sec^5 x + C$

B $\frac{1}{7}\sec^7 x - \frac{1}{5}\sec^5 x + C$

C $\frac{1}{7}\tan^7 x - \frac{1}{5}\tan^5 x + C$

D $\frac{1}{7}\tan^7 x + \frac{1}{5}\tan^5 x + C$

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Solution: D

In the specific case where our function is the product of

an **even** number of **secant** factors and

an **even or odd** number of **tangent** factors,

our plan is to

1. save one $\sec^2 x$ factor and use the identity $\sec^2 x = 1 + \tan^2 x$ to write the other cosine factors in terms of tangent, then

2. use u-substitution with $u = \tan x$.

We'll separate a $\sec^2 x$, and then replace the remaining secant factors using the identity.

$$\int \tan^4 x \sec^4 x \ dx$$

$$\int \tan^4 x \sec^2 x \sec^2 x \ dx$$

$$\int \tan^4 x \sec^2 x \left(1 + \tan^2 x\right) \ dx$$

Using u-substitution with $u = \tan x$, we get

$$u = \tan x$$

$$du = \sec^2 x \ dx$$

Substitute into the integral.

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$$\int u^4 \sec^2 x \left(1 + u^2\right)\ dx$$

$$\int u^4 \left(1 + u^2\right) \left(\sec^2 x\ dx\right)$$

$$\int u^4 \left(1 + u^2\right) \left(du\right)$$

$$\int u^4 \left(1 + u^2\right)\ du$$

$$\int u^4 + u^6\ du$$

$$\frac{1}{5}u^5 + \frac{1}{7}u^7 + C$$

Back-substituting for u, we get

$$\frac{1}{5}\tan^5 x + \frac{1}{7}\tan^7 x + C$$

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Topic: tan^m sec^n, even n

Question: Evaluate the trigonometric integral.

$$\int_0^{\frac{\pi}{4}} \tan^2 x \sec^6 x \ dx$$

Answer choices:

A $-\frac{105}{8}$

B $\frac{105}{8}$

C $-\frac{92}{105}$

D $\frac{92}{105}$

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Solution: D

In the specific case where our function is the product of

an **even** number of **secant** factors and

an **even or odd** number of **tangent** factors,

our plan is to

1. save one $\sec^2 x$ factor and use the identity $\sec^2 x = 1 + \tan^2 x$ to write the other cosine factors in terms of tangent, then

2. use u-substitution with $u = \tan x$.

We'll separate a $\sec^2 x$, and then replace the remaining secant factors using the identity.

$$\int_0^{\frac{\pi}{4}} \tan^2 x \sec^6 x \ dx$$

$$\int_0^{\frac{\pi}{4}} \tan^2 x \sec^2 x \sec^4 x \ dx$$

$$\int_0^{\frac{\pi}{4}} \tan^2 x \sec^2 x \left(\sec^2 x\right)^2 \ dx$$

$$\int_0^{\frac{\pi}{4}} \tan^2 x \sec^2 x \left(1 + \tan^2 x\right)^2 \ dx$$

Using u-substitution with $u = \tan x$, we get

$$u = \tan x$$

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$$du = \sec^2 x \ dx$$

Because we're dealing with a definite integral, we have to either change the limits of integration when we make our substitution, or we have to indicate that the limits of integration are in terms of x until we back-substitute. Substitute into the integral.

$$\int_{x=0}^{x=\frac{\pi}{4}} u^2 \sec^2 x \left(1 + u^2\right)^2 \ dx$$

$$\int_{x=0}^{x=\frac{\pi}{4}} u^2 \left(1 + u^2\right)^2 \left(\sec^2 x \ dx\right)$$

$$\int_{x=0}^{x=\frac{\pi}{4}} u^2 \left(1 + u^2\right)^2 \left(du\right)$$

$$\int_{x=0}^{x=\frac{\pi}{4}} u^2 \left(1 + u^2\right)^2 \ du$$

$$\int_{x=0}^{x=\frac{\pi}{4}} u^2 \left(1 + 2u^2 + u^4\right) \ du$$

$$\int_{x=0}^{x=\frac{\pi}{4}} u^2 + 2u^4 + u^6 \ du$$

$$\left(\frac{1}{3}u^3 + \frac{2}{5}u^5 + \frac{1}{7}u^7\right)\Bigg|_{x=0}^{x=\frac{\pi}{4}}$$

Back-substituting for u, we get

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$$\left(\frac{1}{3}\tan^3 x+\frac{2}{5}\tan^5 x+\frac{1}{7}\tan^7 x\right)\Bigg|_0^{\frac{\pi}{4}}$$

$$\left(\frac{1}{3}\tan^3\frac{\pi}{4}+\frac{2}{5}\tan^5\frac{\pi}{4}+\frac{1}{7}\tan^7\frac{\pi}{4}\right)-\left(\frac{1}{3}\tan^3 0+\frac{2}{5}\tan^5 0+\frac{1}{7}\tan^7 0\right)$$

$$\left(\frac{1}{3}\frac{\sin^3\frac{\pi}{4}}{\cos^3\frac{\pi}{4}}+\frac{2}{5}\frac{\sin^5\frac{\pi}{4}}{\cos^5\frac{\pi}{4}}+\frac{1}{7}\frac{\sin^7\frac{\pi}{4}}{\cos^7\frac{\pi}{4}}\right)-\left(\frac{1}{3}\frac{\sin^3 0}{\cos^3 0}+\frac{2}{5}\frac{\sin^5 0}{\cos^5 0}+\frac{1}{7}\frac{\sin^7 0}{\cos^7 0}\right)$$

$$\left[\frac{1}{3}\frac{\left(\frac{\sqrt{2}}{2}\right)^3}{\left(\frac{\sqrt{2}}{2}\right)^3}+\frac{2}{5}\frac{\left(\frac{\sqrt{2}}{2}\right)^5}{\left(\frac{\sqrt{2}}{2}\right)^5}+\frac{1}{7}\frac{\left(\frac{\sqrt{2}}{2}\right)^7}{\left(\frac{\sqrt{2}}{2}\right)^7}\right]-\left(\frac{1}{3}\frac{0^3}{1^3}+\frac{2}{5}\frac{0^5}{1^5}+\frac{1}{7}\frac{0^7}{1^7}\right)$$

$$\left[\frac{1}{3}(1)+\frac{2}{5}(1)+\frac{1}{7}(1)\right]-\left[\frac{1}{3}(0)+\frac{2}{5}(0)+\frac{1}{7}(0)\right]$$

$$\frac{1}{3}+\frac{2}{5}+\frac{1}{7}$$

$$\frac{35}{105}+\frac{42}{105}+\frac{15}{105}$$

$$\frac{92}{105}$$

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