Topic: Work done to lift a weight or mass

Question: Find the work required.

A cable weighing 2 lbs/ft pulls a 200 pound load up a shaft that is 500 feet deep.

Answer choices:

A $35{,}000$ ft-lbs

B $330{,}000$ ft-lbs

C $350{,}000$ ft-lbs

D $340{,}000$ ft-lbs

Solution: C

We have to calculate the work required to lift the load, then calculate the work required to lift the rope itself, and then add them together to get total work required.

To calculate the work required to lift the load, we'll use $W = Fd$, where W is work, F is force, and d is distance. Normally to find force, we would take the product of the mass and the gravitational constant, but since we're given a weight instead of a mass, the gravitational constant is already included and we can just use the load's weight of 200 lbs as the force.

$$W_L = 200 \text{ lbs} \cdot 500 \text{ ft}$$

$$W_L = 100{,}000 \text{ ft-lbs}$$

To calculate the work required to lift the rope, we'll divide the rope into cross sections. We have to multiply the height of the cross section, Δx, by the weight, 2 lbs, by the distance between the cross section and the top of the shaft, x. Putting this into an integral to find the work required to lift the entire rope, we get

$$W_R = \int_0^{500} 2x \ dx$$

$$W_R = x^2 \Big|_0^{500}$$

$$W_R = 500^2 - 0^2$$

$$W_R = 250{,}000 \text{ ft-lbs}$$

Adding them together, we get

$$W = W_L + W_R$$

$$W = 100{,}000 + 250{,}000$$

$$W = 350{,}000 \text{ ft-lbs}$$

Topic: Work done to lift a weight or mass

Question: As a container moves along an assembly line, it's filled with material so the force to keep it moving is given in pounds by $f(x)$. Find the work needed to move the container from $x=0$ to $x=8$ feet.

$$f(x)=6+\frac{1}{4}x^3$$

Answer choices:

A $\quad W=304$ pounds

B $\quad W=320$ feet

C $\quad W=2{,}112$ foot-pounds

D $\quad W=304$ foot-pounds

Solution: D

To find the amount of work required, we use the integral formula for work. We've been told that the container moves from $x = 0$ to $x = 8$, so those become the limits on the integral. We plug the given equation into the integral as the integrand, and integrate with respect to x. So the work required is given as

$$W = \int_0^8 6 + \frac{1}{4}x^3 \ dx$$

Integrate, and then evaluate over the interval.

$$W = 6x + \frac{1}{16}x^4 \Big|_0^8$$

$$W = 6(8) + \frac{1}{16}(8)^4 - \left(6(0) + \frac{1}{16}(0)^4\right)$$

$$W = 48 + 256$$

$$W = 304$$

Recall the force was provided in pounds and the distance was provided in feet. Therefore, the work required to move the container 8 feet is 304 foot-pounds.

Topic: Work done to lift a weight or mass

Question: Two objects move along a path from $x=4$ to $x=9$ feet. The first object needs a constant force of 25 pounds to move and the other object needs a variable force given by $f(x)$ to move. Find the total work required to move the two objects.

$$f(x)=\frac{250}{x}$$

Answer choices:

A $\quad W=750$ foot-pounds

B $\quad W=125+250\ln\frac{9}{4}$ foot-pounds

C $\quad W=125+250\ln\frac{9}{4}$ pounds

D $\quad W=250+\ln\frac{9}{4}$ feet

Solution: B

Since we are moving two objects, we can find total force required to move them by adding together the force required to move each object individually.

The first object needs a constant force of 25 pounds, so $W_1 = 25$. The second object needs a variable force, given in the problem by $f(x)$. Therefore, total force required is

$$F = 25 + \frac{250}{x}$$

The integration limits represent the distance the container is moved along the assembly line, and since we're moving both objects from $x = 4$ to $x = 9$, the work required to move both objects is given by

$$W = \int_4^9 25 + \frac{250}{x}\ dx$$

Integrate, then evaluate over the interval.

$$W = 25x + 250 \ln|x| \Big|_4^9$$

$$W = 25(9) + 250 \ln|9| - \left(25(4) + 250 \ln|4|\right)$$

$$W = 225 + 250 \ln 9 - 100 - 250 \ln 4$$

$$W = 125 + 250 \left(\ln 9 - \ln 4\right)$$

$$W = 125 + 250 \ln \frac{9}{4}$$