Exponential derivatives

Exponential and logarithmic functions have their own set of derivative rules.

The exponential function we'll work with most often is $y = e^x$. The number e is a special constant with a value equal to $e \approx 2.718281828459045...$ It's similar to how we use π to represent the constant $\pi \approx 3.141592653589793...$

These kinds of exponential functions look a lot like the power functions we've seen before (like x^3), but there's one critical difference.

In power functions, the base is a variable, and the exponent is a constant. But in exponential functions, the base is the constant, and the exponent is the variable. So they're opposite situations.

Exponential derivative rules

Here are the derivatives of simple exponential functions:

Exponential functions	Their derivatives
$y = e^x$	$y' = e^x$
$y = a^x$	$y' = a^x(\ln a)$

There are two important things to say about these derivative formulas.

First, we have $\ln a$ as part of the formula for the derivative of $y = a^x$. That $\ln$ value represents the **natural log function**. Similar to the way we used $\sin x$

to represent the sine function, we use $\ln x$ to represent the natural log function. So whenever we see $\ln$, we know that we're dealing with the natural log. Whatever value of x we plug into $\ln x$, the natural log function will output a corresponding value. We'll talk more about logs and natural logs and their derivatives in the next lesson.

Second, there's actually no difference between the derivative formulas for $y = e^x$ and $y = a^x$. When we take the derivative of any $y = a^x$, we always need to multiply by $\ln a$. When we do that for the derivative of $y = e^x$, we get

$$y' = e^x(\ln e)$$

But $\ln e = 1$, so the derivative simplifies to

$$y' = e^x(1)$$

$$y' = e^x$$

So when the base of the exponential function is e, that $\ln a$ essentially disappears. But for any base other than e, that $\ln a$ will remain as part of the derivative.

Let's try an example where we differentiate an exponential function with base a.

Example

Find the derivative of the exponential function.

$$y = 42^x$$

In this function, $a = 42$ and the exponent is x. We'll differentiate by applying the formula for exponential derivatives.

$$y' = a^x(\ln a)$$

$$y' = 42^x(\ln(42))$$

Now let's try an example with base e.

Example

Find the derivative of the exponential function.

$$y = 3e^x$$

The base is e and the exponent is x, so the derivative is

$$y' = 3e^x$$

Let's try one final, more complex example.

Example

Find the derivative of the exponential function.

$$y = 8x^3 - 4^x e^x + 6^x$$

We have to take the derivative one term at a time, remembering to apply product rule when we get to $4^x e^x$.

$$y' = 24x^2 - [4^x(\ln 4)(e^x) + (4^x)(e^x)] + 6^x(\ln 6)$$

$$y' = 24x^2 - 4^x e^x \ln 4 - 4^x e^x + 6^x \ln 6$$