BDA Asymptotics and BiBa Model - 2023/4

List 4

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III. Random Walks and PageRank

1. Let $G = (\{1,2,3,4,5,7,8,9,10\}, \{1 \to 2, 2 \to 1, 2 \to 3, 2 \to 2, 3 \to 4, 3 \to 2, 4 \to 2, 4 \to 4, 4 \to 5, 5 \to 4, 4 \to 1, 1 \to 5, 9 \to 10, 8 \to 7, 7 \to 8, 4 \to 9, 10 \to 7, 8 \to 1\})$. An agent is placed at in a vertex 1. In each step the agent chooses randomly out-going edge and goes to the vertex pointed by this edge. Find the distribution of the position of the agent after $2, 3, 4, 50$ and 100 steps. HINT: Do this numerically. Remember that $p^{(t+1)} = p^{(t)}\mathbf{P}$.

2. Let $G = (\{1,2,3,4,5,6\}, \{1 \to 2, 2 \to 1, 2 \to 2, 2 \to 6, 2 \to 3, 3 \to 4, 4 \to 5, 5 \to 6, 6 \to 1, 6 \to 6\})$. Consider a random walk on G and find numerically its ergodic probabilities.

3. $G = (\{1,2,3,4,5,6,7,8,9\}, \{1 \to 2, 2 \to 1, 2 \to 2, 2 \to 6, 2 \to 3, 3 \to 4, 4 \to 5, 5 \to 6, 6 \to 1, 6 \to 6, 6 \to 7, 5 \to 8, 7 \to 9, 8 \to 9\})$. Find PageRank value for each vertex representing a web-site. Beware that there are some some dead-ends. Assume taxation parameters $\beta = 0$ and 0.1.

4. Generate a directed graph G with n nodes in a following way. For each of n nodes choose l unique out-neighbours. For such randomly generated structure start a random walk of an agent in a randomly chosen vertex. Find the distribution after $1, 2$, $10, 50, 100$ and n steps for all configurations of parameters:

   - $n = 2, 10, 20, 1000$
   - $l = 1, 2, 5, 10, 20, 50$
   - $\beta = 0, 0.01, 0.05, 0.1, 0.2$