STAT230 REVIEW

Basic Concepts and Tips

Contents

1. Probability Basics

A sample space (S) denotes all possible outcomes in a random experiment. A type of outcome is called an event, and is a subset of S.

$$S = \{a_1, a_2, ..., a_{n-1}, a_n\}$$

Probability seeks to determine the likelihood of an event in such experiment.

Three models of probability:
Classic: Number of ways / Number of outcomes
Relative Frequency: Proportion of times an event occurs after many repetitions
Subjective: Educated guess based on no measurable data

Three essential rules of probability:
1) $0 \leq P(a_i) \leq 1$
2) $\Sigma P(a_i) = 1$
3) $P(a) = 1 - \overline{P(a)}$

2. Counting

2.1. Rules for Jobs

1. Job 1 in p ways, job 2 in q ways. Then, $p + q$ ways of doing either job. In other words, the probability of non-nested outcomes is cumulative.

2. Job 1 in p ways, and for each p, job 2 in q ways. Then, pq ways of doing both jobs. In order words, the probability of nested outcomes is multiplicative.

2.2. Combinatorics

1. $n!$ - Arrangements of length n without replacement.
2. $n^{(k)} = \frac{n!}{(n-k)!}$ - Arrangements of length k from n elements, using each element at most once.
3. n^k - Arrangements of length k from n elements with replacement.
4. $\binom{n}{k}$ - Number of subsets of size k from n objects.
5. $\frac{n!}{n_1!n_2!...n_k!}$ - Arrangement of n symbols, where we have n_i symbols of type i.

2.3. Tips for Counting

1. If you're dealing with cases, you may need to apply $P(A) = \frac{O(a_1)+O(a_2)+...+O(a_k)}{O(S)}$, where $O(x)$ is the number of outcomes in x and a_i is a case that yields A.
2. If dealing with an arrangement problem, break the "array" into sections and use appropriate combinatoric formulas for each section given your sample.
3. If dealing with a problem that's without replacement, exploit the multiplication rule for jobs.

3. (In)dependence and Conditions

3.1. (In)dependence

1. Two events are independent iff the occurrence of A does not affect the occurrence of B. This means that both A and B can occur at once. (Example: You flip a coin and get 2 heads in a row)

   Here are some useful formulas you may use if dealing with independent events:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$P(A \cap B) = P(A)P(B)$$
$$P(A) = P(A \cap B) + P(A \cap \overline{B})$$

2. Two events are mutually exclusive iff the occurrence of A does affect the occurrence of b. This means that A and B cannot occur at once. (Example: 2 occurs on first roll, total sum of two rolls is 10)

$$P(A \cup B) = P(A) + P(B)$$

3.2. Conditional Probability

1. The probability of A occurring given that B occurs is given by:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

   Provided $P(B) > 0$. Note that A and B are just place-holders, yet their order matters in context of the formula for conditional probability. Note that in the case of independence, $P(A|B)$ is $P(A)$.

2. The law of Total Probability states that for k ways to split an event B:

$$P(B) = P(A_1 \cap B) + P(A_2 \cap B) + ... + P(A_k \cap B) = \sum_{i=1}^{k} P(B|A_i)P(A_i)$$

   For the case of an experiment with success or failure:

$$P(B) = P(A \cap B) + P(\overline{A} \cap B) = P(B|A)P(A) + P(B|\overline{A})P(\overline{A})$$

3. Bayes' Theorem is simply a merge of the two above theorems:

$$P(A|B) = \frac{P(B|A)P(A)}{\sum_{i=1}^{k} P(B|A_i)P(A_i)}$$

3.3. Tips for (In)dependence and Conditions

1. Draw a Venn-Diagram, fill in given information, and shade what you need to find. Determine if the Venn-Diagram overlaps (If it does, they're independent, and if they don't, they are mutually exclusive).

2. Experiments with replacement or no potential restrictions are more likely to be independent, while experiments without replacement or with potential restrictions are more likely to be mutually exclusive.

3. If you're dealing with conditional probability, you'll most likely be given $P(A|B)$ in the question, which you may substitute in your required formula. Clearly list your givens, as well as indicate events A and B. Then, apply the formulas above.

4. Apply Bayes' theorem if you need to find $P(A|B)$ given $P(B|A)$.

4. Distributions

4.1. Random Variables and the Probability Function

A random variable X is a numerical variable with a range A, the set of possible values for X, that represents the outcome of an event. It assigns a real number to each point to a sample space S. Thus, the probability function of X is the function:

$$f(x) = P(X = x), x \in \mathbb{R}$$

and the points $(x, f(x))$ forms a probability distribution. The cumulative probability function of X is the function:

$$f(x) = P(X \leq x), x \in \mathbb{R}$$

4.2. Discrete Uniform Distribution

Situation: X takes values $a, a+1, ..., b-1, b$ with all values equally likely.
Restrictions: $a \leq x \leq b$, else $f(x) = 0$

$$f(x) = P(X = x) = \frac{1}{b-a+1}$$

4.3. Hypergeometric Distribution

Situation: N objects with r "successes" and $N-r$ "failures", picking n objects **without replacement.** Let X be the number of successes.
Restrictions: $x \leq \min(r, n)$

$$f(x) = P(X = x) = \frac{\binom{r}{n}\binom{N-r}{n-x}}{\binom{N}{n}}$$

4.4. Binomial Distribution

Situation: An experiment is conducted **with replacement** n times, with a p chance of "success" and $1-p$ chance of "failure". Let X be the number of successes.
Name: $\sim \mathit{Binomial}(n, p)$

$$f(x) = P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$$

4.5. Negative Binomial Distribution

Situation: An experiment is conducted **with replacement** until k successes has been obtained, with a p chance of "success" and $1 - p$ chance of "failure".
Let X be the number of trails needed to obtain k successes.
Name: $\sim NB(k, p)$

$$f(x) = P(X = x) = \binom{n+k-1}{x} p^x (1-p)^x$$

4.6. Geometric Distribution

Situation: An experiment is conducted **with replacement** until one success has been obtained, with a p chance of "success" and $1 - p$ chance of "failure".
Let X be the number of trails needed to obtain one success.
Name: $\sim Geometric(p)$

$$f(x) = P(X = x) = p(1-p)^x$$

4.7. Poisson Distribution

Situation: Let X be the number of event occurrences over a time period of length t that have an intensity λ. Let $\mu = \lambda t$.
Name: $\sim Poisson(\mu)$
Restrictions: Units are consistent, $\mu = \lambda t, x \geq 0$

$$f(x) = P(X = x) = e^{-\mu} \frac{\mu^x}{x!}$$

Notes: Poisson may be used to approximate binomial distribution with $\mu = np$, provided n is large and p is small.

4.8. Tips for Distribution Problems

1. Don't forget that probability functions (distributions) are... functions. That means that they may be substituted in other formulas, such as conditional probability.

2. If you struggle to pick a distribution for a situation, think about whether or not it would make sense to model each distribution with it.

3. Apply $P(X \geq x) = 1 - P(X < x)$ or $P(X > x) = 1 - P(X \leq x)$

5. Expected Value and Variance

5.1. Expected Value

Expected Value (E.V., μ) is conceptually the same as the mean (average).

$$E(X) = \sum_{\text{All } x} x f(x) \qquad \text{(E.V. of } X \text{ with p.f. } f(x))$$

$$E(g(X)) = \sum_{\text{All } x} g(x) f(x) \qquad \text{(E.V. of } g(X)\text{, with } X \text{ having p.f. } f(x))$$

E.V. is linear, that is, $E(ag(X) + b) = aE(g(x)) + b$.

Here are some common E.V.'s for different distributions:
Binomial: $\mu = np$ Poisson: $\mu = \lambda t$ Hypergeometric: $\mu = \frac{np}{N}$ NB: $\mu = \frac{k(1-p)}{p}$

5.2. Variance

The variance (Var(X), σ^2) of X is the average square of the distance from the mean, and is given by several formulas:

$$\sigma^2 = E[(X - \mu)^2] = E(X^2) - \mu^2 = E(X(X - 1)) + \mu - \mu^2$$

Here are some common σ^2's for different distributions:
Binomial: $\sigma^2 = np(1 - p)$ Poisson: $\sigma^2 = \mu$

Additionally, here are some properties of the mean and variance:

$$\mu_Y = a\mu_X + b \text{ and } \sigma_Y^2 = a^2\sigma_X^2 \text{ where } Y = aX + b \text{ for constants } a \text{ and } b$$

5.3. Standard Deviation

The standard deviation (sd(X), σ) of X is the average distance from the mean, and is given by:

$$\sigma = \sqrt{\text{Var}(X)}$$

5.4. Tips for Expected Value and Variance

1. X need not consist of consecutive values. You may define X to be any quantity as required, determining the probability of each X occurring when calculating E(X). For example, we might have $X = 0, 5, 50, 100$ for lottery winnings, with points $f(0) = 0.9, f(5) = 0.07, f(50) = 0.02, f(100) = 0.01$.

2. Functions may be substituted, evaluated at a specific point, etc.

3. If the variables confuse you, write out what they are supposed to represent. Calculate them before performing any substitutions.

6. Continuous Random Variables

6.1. General Concepts

1. Continuous Random Variables are RVs that may take any real number, and as such, $P(x = x)$ will be equal to zero.

2. There are two functions that we use to represent Continuous Random Variables. We first have the Cumulative Distribution Function, which is defined as:

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(u)du$$

   Here are some properties of the CDF:

   (a) $F(x)$ is defined for all x
   (b) $F(x)$ is a non-decreasing function of x for all x
   (c) $\lim_{x\to-\infty} F(x) = 0$ and $\lim_{x\to\infty} F(x) = 1$
   (d) $P(a < X \leq b) = F(b) - F(a)$

   Then, we can define the Probability Density Function as the derivative of $F(x)$, which is $f(x)$. Here are some properties of the Probability Density Function:

   (a) $P(a \leq X \leq b) = \int_a^b f(x)dx$
   (b) $f(x) \geq 0$
   (c) $\int_{-\infty}^{\infty} f(x)dx = 1$

3. Note that for each formula, the bound choice has no effect.

6.2. Quantile and Percentile

Suppose X is a continuous random variable with $F(x)$. The pth quantile of X is the value $q(p)$ such that $P(X \leq q(p)) = q(p)$.

Example: If $F(1) = 0.4$, then the 40th percentile is equal to 1.

6.3. E(X) and Var(X) for C.R.V's

The standard formulas for $E(X)$ and $Var(X)$ still hold for continuous random variables, though there is a subtle difference with $E(g(X))$:

$$E(g(X)) = \int_{-\infty}^{\infty} g(x)f(x)dx$$

6.4. Continuous Uniform Distribution

Situation: X takes values in an interval $[a, b]$ with all subintervals equally likely.
Restrictions: $b > a$
Name: $\sim Uniform(a, b)$

$$f(x) = \begin{cases} \frac{1}{b-a} & a \leq x \leq b \\ 0 & \text{Otherwise} \end{cases}$$

$$F(x) = \begin{cases} 0 & x < a \\ \int_a^x \frac{1}{b-a} dx = \frac{x-a}{b-a} & a \leq x \leq b \\ 0 & x > b \end{cases}$$

Mean: $E(X) = \frac{a+b}{2}$
Variance: $Var(X) = \frac{(b-a)^2}{12}$

6.5. Exponential Distribution

Situation: X is the time elapsed until the first event in a Poisson process
Restrictions: $\lambda > 0$
Name: $\sim Exponential(\lambda)$

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x > 0 \\ 0 & x \leq 0 \end{cases}$$

$$F(x) = \begin{cases} 1 - e^{-\lambda x} & x > 0 \\ 0 & x \leq 0 \end{cases}$$

Mean: $E(X) = \frac{1}{\lambda}$
Variance: $Var(X) = \frac{1}{\lambda^2}$
Additional Notes: λ may be replaced with $\theta = \frac{1}{\lambda}$.

6.6. Normal Distribution

Name: $\sim N(\mu, \sigma^2)$

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$F(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2} dy$$

Mean: $E(X) = \mu$
Variance: $Var(X) = \sigma^2$
Additional Notes: If $X \sim N(\mu, \sigma^2)$, then we have:

$$P(a \leq x \leq b) = P\left(\frac{a-\mu}{\sigma} \leq Z \leq \frac{b-\mu}{\sigma}\right) = P\left(Z \leq \frac{b-\mu}{\sigma}\right) - P\left(Z \leq \frac{a-\mu}{\sigma}\right)$$

where $Z \sim N(0, 1)$. Working with $\sim N(0, 1)$ allows us to calculate probabilities using a provided $\sim N(0, 1)$ table. This is called standardization.

6.7. Tips for Continuous Random Variables

1. Like for discrete random variables, apply $P(X \geq x) = 1 - P(X < x)$. Note that there's no difference between $\geq$ and $>$ here.

2. As before, don't forget that probability functions are functions. You may find yourself using $(f(x))^n$ a lot if you have n objects independent from each other with distribution $f(x)$, or you may have to solve for μ/σ using a table.

7. Multivariant Distributions

7.1. Joint, Marginal, and Conditional Probability Functions

The joint probability function of X and Y is given by $f(x, y) = P(X = x, Y = y)$. Generalizing this function:

$$f(x_1, x_2, ..., x_n) = P(X_1 = x_1, X_2 = x_2, ..., X_n = x_1)$$

Marginal Probability: $f_1(x) = P(X = x)$ (sum of all y's for which X = x)
Conditional Probability: $f_1(x|y) = \frac{f(x,y)}{f_2(y)}$ or $f_2(y|x) = \frac{f(x,y)}{f_1(x)}$

		Parents		
		Used	Did not use	Total
Student	Uses	125	94	219
	Does not Use	85	141	226
	Total	210	235	445

- Marginal probability
  - Probability based on a single variable

  P(Student = uses)
  = 219/445
- Joint Probability
  - Probability based on two or more variables

  P(Student = uses and Parent = uses)
  = 125/445 = 0.28
- Conditional Probability
  - Probability of one event conditional upon another event

  P(Student = use | parents = used)
  = 125/210 = 0.60

The joint prob. of A_2 and B_1

	A_1	A_2	Total
B_1	a/n	b/n	(a+b)/n
B_2	c/n	d/n	(c+d)/n
Total	(a+c)/n	(b+d)/n	1

The marginal probability of A_1.

3

Similar to events, random variables X and Y are independent if $f(x, y) = f_1(x)f_2(y)$.

7.2. Multinomial Distribution

Situation: An experiment is repeated independently n times with k_m outcomes, with each outcome having probabilities p_1, p_2, ... , p_m. Let X_1 be the number of times k_1 occurs, X_2 be the number of times k_2 occurs, and so forth.

Name: $(X_1, X_2, ..., X_n) \sim Multinomial(n; p_1, p_2, ...p_k)$

$$f(x_1, x_2, ..., x_n) = \frac{n!}{x_1!x_2!...x_n!} p_1^{x_1} p_2^{x_2} ... p_n^{x_n}$$

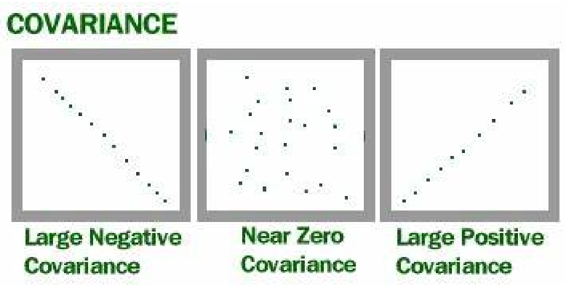
Mean: $E[g(X_1, X_2, ..., X_n)] = \sum g(x_1, x_2, ..., x_n) f(x_1, x_2, ..., x_n)$

7.3. Covariance and Correlation

Covariance and Correlation are ways to measure the strength of a relationship between two random variables.

Covariance is defined as the mean of the product of the deviations of two variates from their respective means.

$$Cov(X, Y) = E[(X - \mu_x)(Y - \mu_y)] = E(XY) - E(X)E(Y)$$



Properties:

1. $-1 \leq Cov(X, Y) \leq 1$
2. $Cov(X, X) = Var(X)$
3. $Cov(aX + b, Y) = a(Cov(X, Y))$
4. $|Cov(X, Y)| \leq \sigma_X \sigma_Y$
5. $Cov(X, Y) = 0$ if X, Y are independent

The correlation coefficient of X and Y is given by:

$$\rho_{XY} = \frac{Cov(X, Y)}{\sigma_X \sigma_Y} = \frac{E(XY) - E(X)E(Y)}{\sqrt{E(X^2) - [E(X)]^2} \cdot \sqrt{E(Y^2) - [E(Y)]^2}}$$

7.4. Tips for Multivariant Distributions

1. Follow these steps to solve multivariant distributions problems:
   (a) Break down the multivariant distribution into several univariate distribution problems, and determine their distributions
   (b) Apply the necessary formulas for each univariate distribution
   (c) With each univariate distribution figured out, apply the necessary multivariate distribution formulas to solve the problem
2. Apply the toolkit strategy (list information, find the appropriate formulas for the problem and/or the information given, calculate)
3. Depending on the problem, you may have to apply formulas in (8.1) and (8.2).

8. Additional Topics

8.1. Transformations of Random Variables

Note that if the Random Variables X and Y are independent:

$$Cov(X,Y) = 0$$

We thus have the groundwork for a variety of useful formulas for μ, σ^2, and $Cov(X,Y)$:

$$\begin{aligned}
E(aX + bY) &= aE(X) + bE(Y) \\
E\left(\sum_{i=1}^{n} X_i\right) &= \sum_{i=1}^{n} E(X_i) \\
Cov(aX + bY, cU + dV) &= acCov(X,U) + adCov(X,V) + bcCov(Y,U) + bdCov(Y,V) \\
Var(aX + bY) &= a^2Var(X) + b^2Var(Y) + 2abCov(X,Y) \\
Var\left(\sum_{i=1}^{n} a_iX_i\right) &= \sum_{i=1}^{n} a_i^2\sigma_i^2 + 2\sum_{i=1}^{n}\sum_{j=i+1}^{n} a_ia_jCov(X_i,X_j)
\end{aligned}$$

Additionally, we have a variety of useful formulas for the Normal distribution:

1. If $X \sim N(\mu, \sigma^2)$ and $Y = aX + b$, then $Y \sim N(a\mu + b, a^2\sigma^2)$
2. If $X \sim N(\mu_1, \sigma_1^2)$ and $Y \sim N(\mu_2, \sigma_2^2)$, then $aX + bY \sim N(a\mu_1 + b\mu_2, a^2\sigma_1^2 + b^2\sigma_2^2)$
3. If $X_i \sim N(\mu_i, \sigma_i^2)$, then $\sum_{i=1}^{n} \sim N\left(\sum_{i=1}^{n} a_i\mu_i, \sum_{i=1}^{n} a_i^2, \sigma_i^2\right)$
4. If $X_1, X_2, ..., X_n$ are independent $N(\mu, \sigma^2)$ random variables, then $\sum_{i=1}^{n} X_i \sim N(n\mu, n\sigma^2)$ and $\overline{X} \sim N(\mu, \sigma^2/n)$.

8.2. Indicator Random Variables

Situation: Suppose we have a random variable X, of which we can define an event with probability p that occurs or does not occur with X.

$$X_{\text{event}} = \begin{cases} 1 & \text{If the event occurs} \\ 0 & \text{If the event does not occur} \end{cases}$$

$$E(X_{\text{event}}) = p$$

How to apply Indicator Random Variables

1. For independent events, exploit the property of independent random variables. In particular, note that you can have a "chain of probabilities" by simply multiplying. Additionally, note that the following holds for independent events:

$$E(X_{event}) = E(X_{event}^2) = ... \quad \text{and} \quad E(X_1 + X_2 + ...) = E(X_1) + E(X_2) + ...$$

2. For dependent events, you will have to determine every bit of potentially information individually and apply previously discussed formulas.

3. Before calculating, think about what the problem actually means and get any bit of information possible from the problem. Common sense helps here.

4. If you have a problem that can be thought of a "chain", define the event clearly and use a subscript i that represents each object in the event for $i = 1, 2, 3, ...$

8.3. Central Limit Theorem

Theorem

If $X_1, X_2, ..., X_n$ are independent random variables all having the same distribution with mean μ and variance σ^2, then as $n \to \infty$, the cumulative distribution functions

$$\frac{\sum_{i=1}^n X_i - n\mu}{\sigma\sqrt{n}} = \frac{S_n - n\mu}{\sigma\sqrt{n}} \quad \text{and} \quad \frac{\overline{X} - \mu}{\sigma/\sqrt{n}}$$

approaches the $N(0, 1)$ cumulative distribution function.

Usage

This theorem may be used to approximate properties for linear combinations of random variables having a non-normal distribution. μ and σ^2 must exist.
A "continuity correction" may be used to have a higher accuracy approximation.

Approximation Formulas

1. $S_n = \sum_{i=1}^n X_i$ has approximately a $N(n\mu, n\sigma^2)$ distribution for large n

2. $\overline{X} = \frac{1}{n}\sum_{i=1}^n X_i$ has approximately a $N\left(\mu, \frac{\sigma^2}{n}\right)$ distribution for large n

Using these approximation formulas, we can derive the normal approximation from $X \sim Binom(n, p)$ using the random variable W, defined as:

$$W = \frac{X - np}{\sqrt{np(1-p)}}$$

W will approximately have a $N(0, 1)$ distribution.

8.4. Moment Generating Functions

Moment Generating Functions provide an alternative way of expressing distributions. They express distributions in terms of *moments*, which are the expectations of X^k at $k = 1, 2, 3, ...$ Thus, *moments* are $E(X), E(X^2), E(X^3), ...$ where

$$E(X^k) = M^{(k)}(0) = \frac{d^k}{dt^k} M(t) \quad \text{at } t = 0$$

Univariate Discrete Distributions

For some $a > 0$ and $t \in [-a, a]$, we have:

$$M(t) = E(e^{tX}) = \sum_{\text{all } x} e^{tx} f(x)$$

Univariate Continuous Distributions

For some $a > 0$ and $t \in [-a, a]$, we have:

$$M(t) = E(e^{tX}) = \int_{-\infty}^{\infty} e^{tx} f(x) dx$$

Multivariate Distributions

For some s and t, we have:

$$M(s, t) = E(e^{sT+tY}) = M_X(s) M_Y(t)$$

8.5. Additional Tips

1. Do not be afraid of defining new transformed random variables, and you may be forced to depending on the problem. Simply look at the formulas in (8.1) and determine which formulas suit your problem.
2. Look at how the problem is structured and identify "buzzwords" or phrases that indicate independence, "chains", repetition of trials, a specific type of distribution, conditional probability, cases, etc.
3. If you do not know how to solve a specific problem, extract as many part marks as possible. You can do this by listing the aforementioned "buzzwords" and phrases, listing the needed formulas, and extracting as much information as possible.
4. If exp is an expression, then don't be afraid to algebraically modify exp in $P(exp)$. In particular, $P(X > Y) = P(X - Y > 0)$.

9. Tables and Summaries

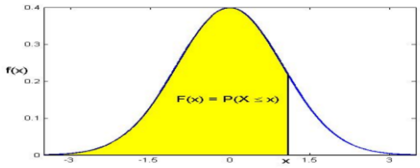
Summary of Discrete Distributions

Notation and Parameters	**Probability Function** $f(x)$	**Mean** $E(X)$	**Variance** $Var(X)$	**Moment Generating Function** $M(t)$
Discrete Uniform(a,b) $b \geq a$ a,b integers	$\frac{1}{b-a+1}$ $x = a, a+1, \ldots, b$	$\frac{a+b}{2}$	$\frac{(b-a+1)^2-1}{12}$	$\frac{1}{b-a+1}\sum_{x=a}^{b} e^{tx}$ $t \in \Re$
Hypergeometric(N,r,n) $N = 1,2,\ldots$ $n = 0,1,\ldots,N$ $r = 0,1,\ldots,N$	$\frac{\binom{r}{x}\binom{N-r}{n-x}}{\binom{N}{n}}$ $x = \max(0, n-N+r),$ $\ldots, \min(r,n)$	$\frac{nr}{N}$	$\frac{nr}{N}(1-\frac{r}{N})\frac{N-n}{N-1}$	Not tractable
Binomial(n,p) $0 \leq p \leq 1,\ q = 1-p$ $n = 1,2,\ldots$	$\binom{n}{x}p^x q^{n-x}$ $x = 0,1,\ldots,n$	np	npq	$(pe^t+q)^n$ $t \in \Re$
Bernoulli(p) $0 \leq p \leq 1,\ q = 1-p$	$p^x q^{1-x}$ $x = 0,1$	p	pq	pe^t+q $t \in \Re$
Negative Binomial(k,p) $0 < p \leq 1,\ q = 1-p$ $k = 1,2,\ldots$	$\binom{x+k-1}{x}p^k q^x$ $= \binom{-k}{x}p^k(-q)^x$ $x = 0,1,\ldots$	$\frac{kq}{p}$	$\frac{kq}{p^2}$	$\left(\frac{p}{1-qe^t}\right)^k$ $t < -\ln q$
Geometric(p) $0 < p \leq 1,\ q = 1-p$	pq^x $x = 0,1,\ldots$	$\frac{q}{p}$	$\frac{q}{p^2}$	$\frac{p}{1-qe^t}$ $t < -\ln q$
Poisson(λ) $\lambda \geq 0$	$\frac{e^{-\lambda}\lambda^x}{x!}$ $x = 0,1,\ldots$	λ	λ	$e^{\lambda(e^t-1)}$ $t \in \Re$
Multinomial$(n; p_1, p_2, \ldots, p_k)$ $0 \leq p_i \leq 1$ $i = 1,2,\ldots,k$ and $\sum_{i=1}^{k} p_i = 1$	$f(x_1,x_2,\ldots,x_k) =$ $\frac{n!}{x_1!x_2!\cdots x_k!}p_1^{x_1}p_2^{x_2}\cdots p_k^{x_k}$ $x_i = 0,1,\ldots,n$ $i = 1,2,\ldots,k$ and $\sum_{i=1}^{k} x_i = n$	$E(X_i) = np_i$ $i = 1,2,\ldots,k$	$Var(X_i)$ $= np_i(1-p_i)$ $i = 1,2,\ldots,k$	$M(t_1,t_2,\ldots,t_k)$ $=(p_1e^{t_1}+p_2e^{t_2}+\cdots$ $+p_{k-1}e^{t_{k-1}}+p_k)^n$ $t_i \in \Re$ $i = 1,2,\ldots,k-1$

Summary of Continuous Distributions

Notation and Parameters	**Probability Density Function** $f(x)$	**Mean** $E(X)$	**Variance** $Var(X)$	**Moment Generating Function** $M(t)$
Uniform(a,b) $b > a$	$\frac{1}{b-a}$ $a \leq x \leq b$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$	$\frac{e^{bt}-e^{at}}{(b-a)t}\quad t \neq 0$ $1\quad t = 0$
Exponential(θ) $\theta > 0$	$\frac{1}{\theta}e^{-x/\theta}$ $x \geq 0$	θ	θ^2	$\frac{1}{1-\theta t}$ $t < \frac{1}{\theta}$
$N(\mu,\sigma^2) = G(\mu,\sigma)$ $\mu \in \Re,\ \sigma^2 > 0$	$\frac{1}{\sqrt{2\pi}\,\sigma}e^{-(x-\mu)^2/(2\sigma^2)}$ $x \in \Re$	μ	σ^2	$e^{\mu t+\sigma^2 t^2/2}$ $t \in \Re$

N(0,1) Cumulative Distribution Function



This table gives values of $F(x) = P(X \leq x)$ for $X \sim N(0,1)$ and $x \geq 0$

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586
0.1	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962	0.56356	0.56749	0.57142	0.57535
0.2	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409
0.3	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173
0.4	0.65542	0.65910	0.66276	0.66640	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793
0.5	0.69146	0.69497	0.69847	0.70194	0.70540	0.70884	0.71226	0.71566	0.71904	0.72240
0.6	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.75490
0.7	0.75804	0.76115	0.76424	0.76730	0.77035	0.77337	0.77637	0.77935	0.78230	0.78524
0.8	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327
0.9	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891
1.0	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214
1.1	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147
1.3	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91309	0.91466	0.91621	0.91774
1.4	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189
1.5	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408
1.6	0.94520	0.94630	0.94738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449
1.7	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.96080	0.96164	0.96246	0.96327
1.8	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062
1.9	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670
2.0	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030	0.98077	0.98124	0.98169
2.1	0.98214	0.98257	0.98300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574
2.2	0.98610	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899
2.3	0.98928	0.98956	0.98983	0.99010	0.99036	0.99061	0.99086	0.99111	0.99134	0.99158
2.4	0.99180	0.99202	0.99224	0.99245	0.99266	0.99286	0.99305	0.99324	0.99343	0.99361
2.5	0.99379	0.99396	0.99413	0.99430	0.99446	0.99461	0.99477	0.99492	0.99506	0.99520
2.6	0.99534	0.99547	0.99560	0.99573	0.99585	0.99598	0.99609	0.99621	0.99632	0.99643
2.7	0.99653	0.99664	0.99674	0.99683	0.99693	0.99702	0.99711	0.99720	0.99728	0.99736
2.8	0.99744	0.99752	0.99760	0.99767	0.99774	0.99781	0.99788	0.99795	0.99801	0.99807
2.9	0.99813	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99851	0.99856	0.99861
3.0	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99896	0.99900
3.1	0.99903	0.99906	0.99910	0.99913	0.99916	0.99918	0.99921	0.99924	0.99926	0.99929
3.2	0.99931	0.99934	0.99936	0.99938	0.99940	0.99942	0.99944	0.99946	0.99948	0.99950
3.3	0.99952	0.99953	0.99955	0.99957	0.99958	0.99960	0.99961	0.99962	0.99964	0.99965
3.4	0.99966	0.99968	0.99969	0.99970	0.99971	0.99972	0.99973	0.99974	0.99975	0.99976
3.5	0.99977	0.99978	0.99978	0.99979	0.99980	0.99981	0.99981	0.99982	0.99983	0.99983

N(0,1) Quantiles: This table gives values of $F^{-1}(p)$ for $p \geq 0.5$

p	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.075	0.08	0.09	0.095
0.5	0.0000	0.0251	0.0502	0.0753	0.1004	0.1257	0.1510	0.1764	0.1891	0.2019	0.2275	0.2404
0.6	0.2533	0.2793	0.3055	0.3319	0.3585	0.3853	0.4125	0.4399	0.4538	0.4677	0.4959	0.5101
0.7	0.5244	0.5534	0.5828	0.6128	0.6433	0.6745	0.7063	0.7388	0.7554	0.7722	0.8064	0.8239
0.8	0.8416	0.8779	0.9154	0.9542	0.9945	1.0364	1.0803	1.1264	1.1503	1.1750	1.2265	1.2536
0.9	1.2816	1.3408	1.4051	1.4758	1.5548	1.6449	1.7507	1.8808	1.9600	2.0537	2.3263	2.5758