From Additive to Classical Proof Search

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Introduction

Outline:

- Additive Linear Logic (ALL)
- Proof search in ALL through *coalescence*
- Classical Logic (CL)
- Proof search in CL through *coalescence* and *additive stratification*
- Complexity bounds of proof search
- Dimensionality of CL formulae

Additive Linear Logic (ALL)

A formula of ALL is constructed:

$$A, B, C \quad ::= \quad 0 \mid 1 \mid a \mid \overline{a} \mid A + B \mid A \times B$$

where $0, a, +$ are duals of $1, \overline{a}, \times$ respectively.
E.g. $(a + \overline{b}) \times 1$ is the dual of $(\overline{a} \times b) + 0$

Additive Linear Logic (ALL)

A sequent calculus for ALL is given by the following rules:

$$\frac{}{\vdash a, \overline{a}}\ ax \qquad \frac{}{\vdash 1, A}\ 1 \qquad \frac{\vdash A, C}{\vdash A + B, C}\ +_1$$

$$\frac{\vdash B, C}{\vdash A + B, C}\ +_2 \qquad \frac{\vdash A, C \qquad \vdash B, C}{\vdash A \times B, C}\ \times$$

Note that all sequents are comprised of a pair of terms and this is maintained by deduction rules.

Additive Linear Logic (ALL)

Consider the proof of $\vdash a \times b, \overline{a} + \overline{b}$ as follows:

$$\dfrac{\dfrac{\dfrac{}{a, \overline{a}}\,ax}{a, \overline{a} + \overline{b}}\,+ \qquad \dfrac{\dfrac{}{b, \overline{b}}\,ax}{b, \overline{a} + \overline{b}}\,+}{a \times b, \overline{a} + \overline{b}}\,\times$$

Coalescence

First discovered by Galmiche & Marion (1995) and later developed by Heijltjes & Hughes (2015).

Classical Logic (CL)

Additive Stratification

Coalescence

Dimensionality

Conclusion

References

Galmiche, D. & Marion, J.-Y. (1995), Semantic proof search methods for all-a rst approach, *in* '4th Workshop on Theorem Proving with Analytic Tableaux and Related Methods, St Goar am Rhein, Germany'.

Heijltjes, W. & Hughes, D. J. (2015), Complexity bounds for sum-product logic via additive proof nets and petri nets, *in* '2015 30th Annual ACM/IEEE Symposium on Logic in Computer Science', IEEE, pp. 80–91.