From Additive to Classical Proof Search

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Introduction

Outline:

- ▶ Additive Linear Logic (ALL)
- ▶ Proof search in ALL through *coalescence*
- ▶ Classical Logic (CL)
- ▶ Proof search in CL through *coalescence* and *additive stratification*
- ▶ Complexity bounds of proof search
- ▶ Dimensionality of CL formulae

Additive Linear Logic (ALL)

A formula of ALL is constructed:

$$A, B, C \quad ::= \quad 0 \mid 1 \mid a \mid \overline{a} \mid A + B \mid A \times B$$

where $0, a, +$ are duals of $1, \overline{a}, \times$ respectively.

Additive Linear Logic (ALL)

Some examples:

- Conceptually, $a + \overline{b}$ describes "Is a true *or* is b false"
- In a looser definition, we could say $a \equiv \overline{\overline{a}}$ and define duality up to non-atomic terms
- Then $(a + \overline{b}) \times 1$ is the dual of $(\overline{a} \times b) + 0$ and vice versa.

Additive Linear Logic (ALL)

A sequent calculus for ALL is given by the following rules:

$$\frac{}{\vdash a, \overline{a}} \; ax \qquad\qquad \frac{}{\vdash 1, A} \; 1 \qquad\qquad \frac{\vdash A, C}{\vdash A + B, C} \; +_1$$

$$\frac{\vdash B, C}{\vdash A + B, C} \; +_2 \qquad\qquad \frac{\vdash A, C \quad \vdash B, C}{\vdash A \times B, C} \; \times$$

Note that all sequents are comprised of a pair of terms and this is maintained by deduction rules.

Additive Linear Logic (ALL)

This sequent calculus exhibits proofs of ALL formulae:

- We start with some grounding — there is a proof that a is either true or false, or there is a proof of 1.

- Given proofs of A, B, C, we can construct further provable formulae.

- The proof tree is grown downwards building up to some conclusion.

Additive Linear Logic (ALL)

Consider the proof of $\vdash a \times b, \overline{a} + \overline{b}$ — either a and b are both true or at least one of them is false — as follows:

$$\cfrac{\cfrac{\overline{\phantom{a,\overline{a}}}}{a, \overline{a}} \; ax}{\cfrac{a, \overline{a} + \overline{b}}{} +} \qquad \cfrac{\cfrac{\overline{\phantom{b, \overline{b}}}}{b, \overline{b}} \; ax}{b, \overline{a} + \overline{b}} + \bigg/ \; \cfrac{}{a \times b, \overline{a} + \overline{b}} \; \times$$

Coalescence

First discovered by Galmiche & Marion (1995) and later developed by Heijltjes & Hughes (2015).

Classical Logic (CL)

Additive Stratification

Coalescence

Dimensionality

Conclusion

References

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