Chapter I

Sinusoidal Steady State Analysis

“Circuit Elements in the Phasor Domain”

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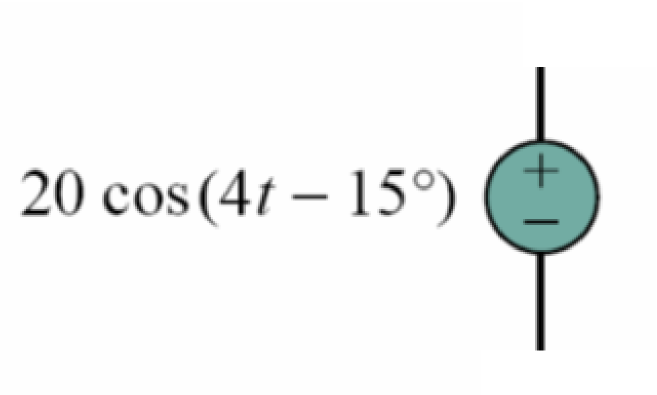
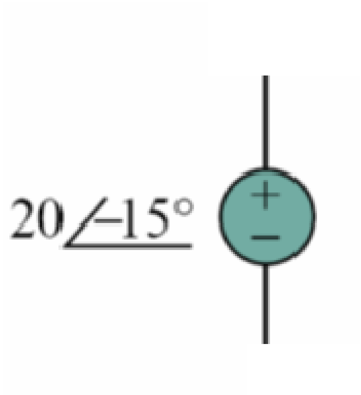
Objectives

- To introduce phasors and convert the time domain sinusoidal waveform into phasors
- To develop the phasor relationships for the basic circuit elements
- To solve electric circuits in the phasor domain

Phasor Domain Sources

- **Convert time domain elements and sources into phasors**



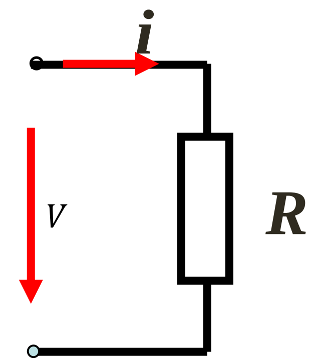


Time Domain

Phasor Domain

Circuit Elements in Frequency Domain

1. Resistance

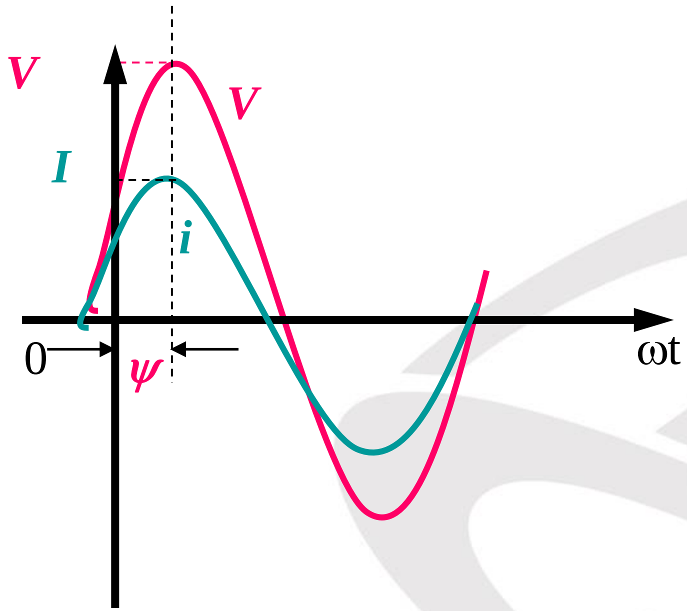


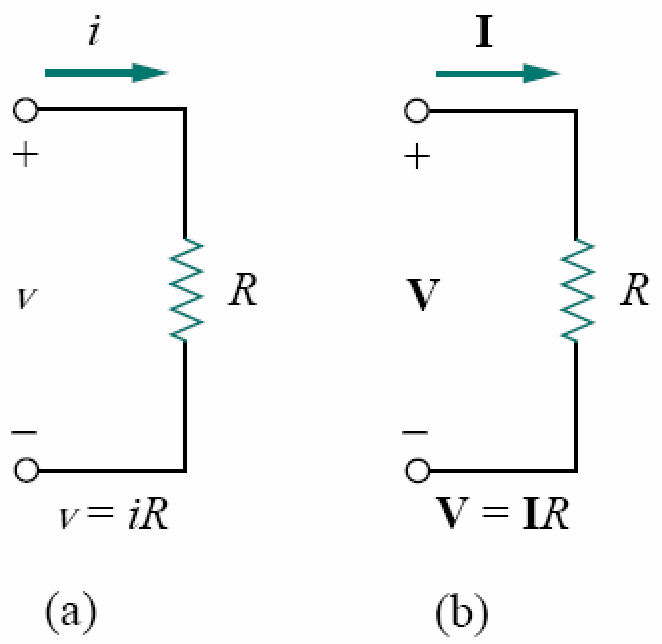
$$i(t) = I_m \cos(\omega t - \psi)$$

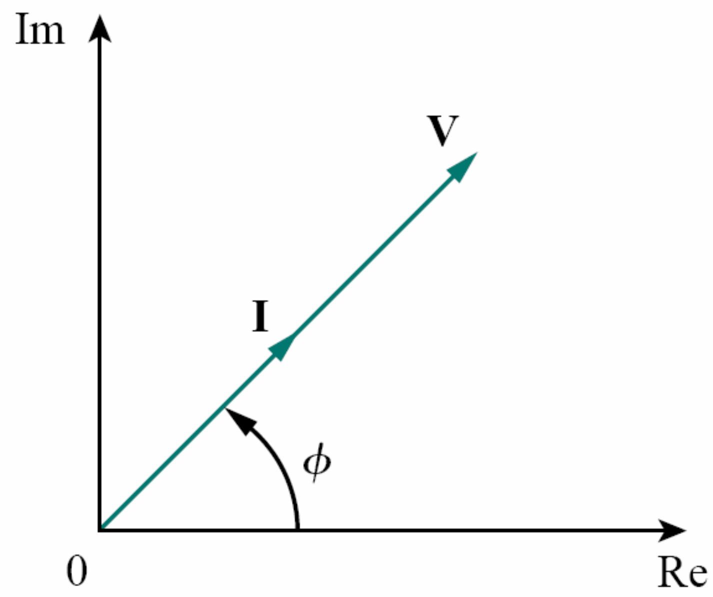
$$v(t) = Ri = RI_m \cos(\omega t - \psi)$$
$$= V_m \cos(\omega t - \psi)$$

Then:

$$R = \frac{V_m}{I_m}$$




i
+
v
R
−
v = iR
(a)
I
+
V
R
−
V = IR
(b)


Im
V
I
ϕ
0
Re

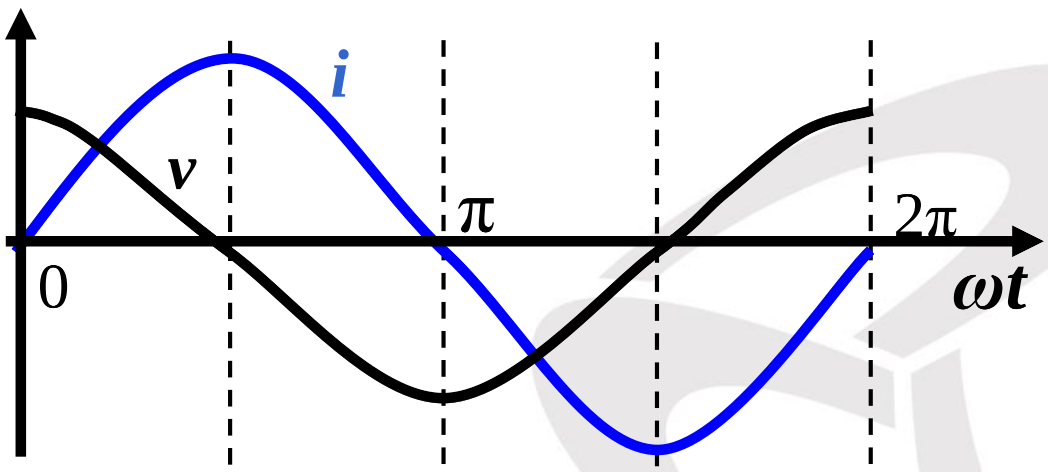
2. Inductance

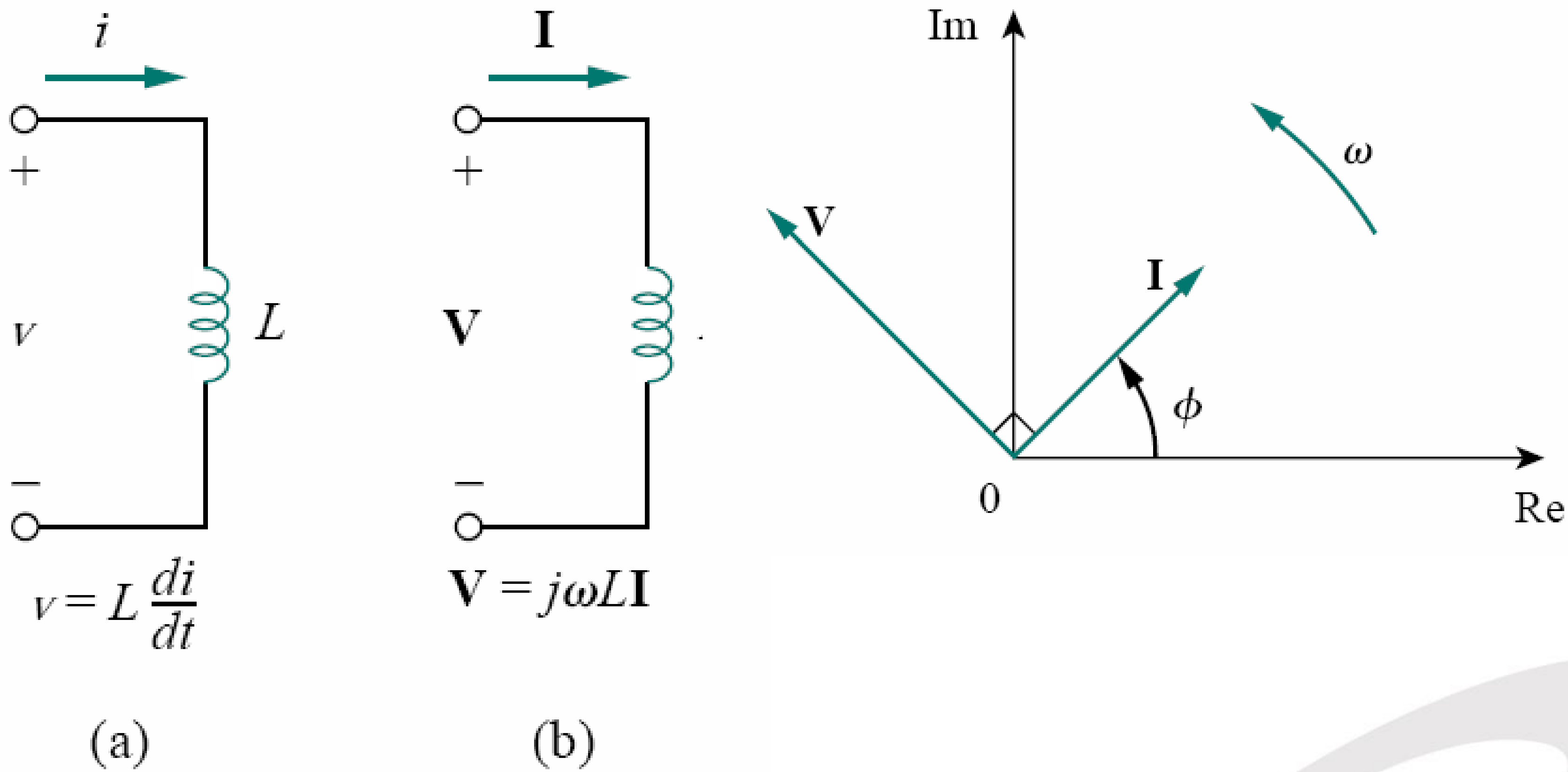
Relationship between voltage and current

$$i(t) = I_m \cos(\omega t - 90) = I_m \sin(\omega t)$$

$$v(t) = L\,\frac{\mathrm{d}i}{\mathrm{d}t} = \omega L I_m \cos(\omega t) \qquad V_m = \omega L I_m$$

v and *i* have the same frequency, *i* lags *v* by 90^o





$$X_L = \frac{V_m}{I_m} = \omega L = 2\pi f L$$

X_L is the inductive reactance (Ω)

3. Capacitors

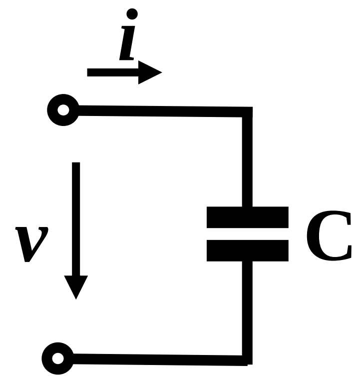
Relationship between voltage and current

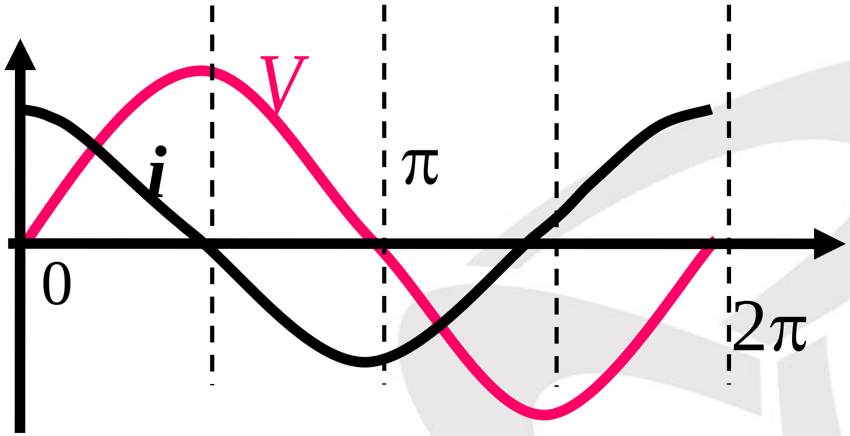
$$v = V_m \sin \omega t$$

$$i = C\frac{dv}{dt} = C\omega U_m \sin(\omega t + 90^0)$$

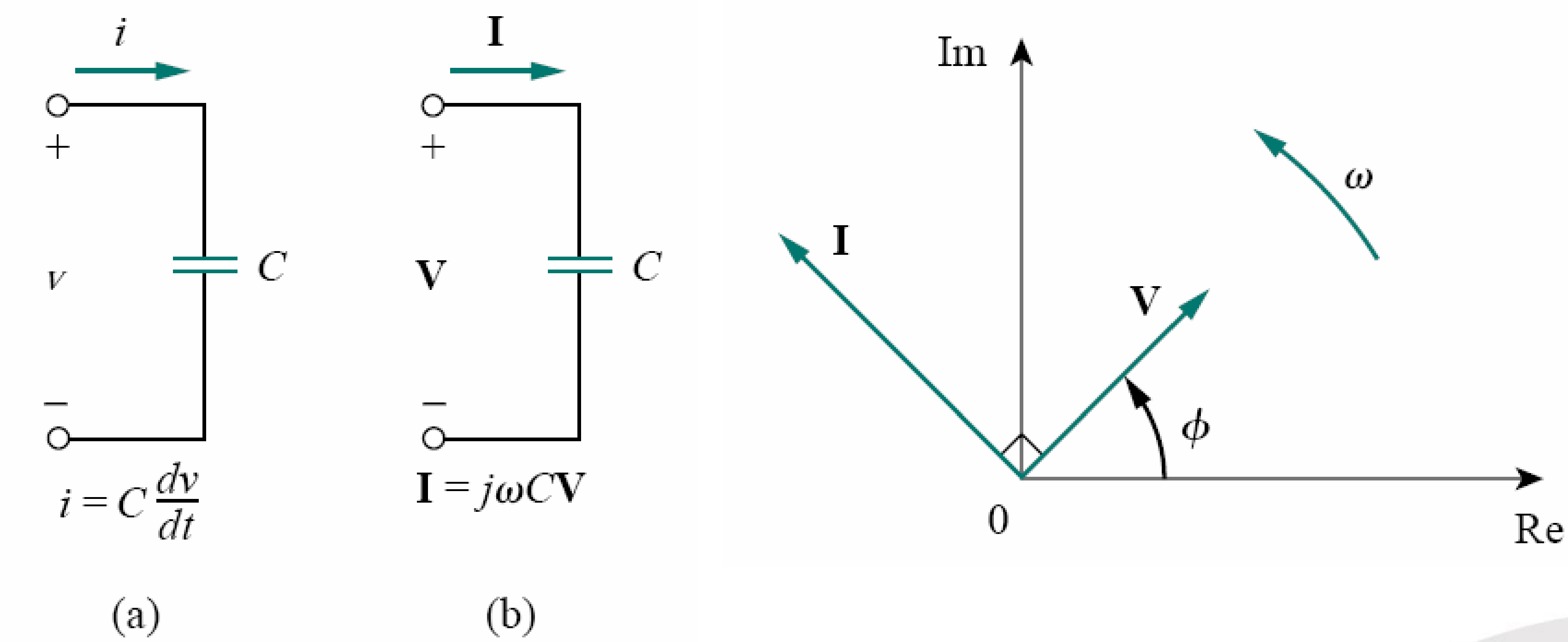
$$= I_m \sin(\omega t + 90^0)$$

$$I_m = \omega C V_m = 2\pi f C V_m$$





Current leads voltage by 90°



$$X_C = \frac{V_m}{I_m} = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

X_C is the capacitive reactance (Ω)

Summary of voltage-current relationship		
Element	Time domain	Frequency domain
R	$v = Ri$	$V = RI$
L	$v = L\frac{di}{dt}$	$V = j\omega LI$
C	$i = C\frac{dv}{dt}$	$V = \frac{I}{j\omega C}$

Example

The voltage $v = 12\cos(60t + 45^\circ)$ is applied to a 0.1-H inductor. Find the steady-state current through the inductor.

Solution:

For the inductor, $\mathbf{V} = j\omega L\mathbf{I}$, where $\omega = 60$ rad/s and $\mathbf{V} = 12\underline{/45^\circ}$ V. Hence

$$\mathbf{I} = \frac{\mathbf{V}}{j\omega L} = \frac{12\underline{/45^\circ}}{j60 \times 0.1} = \frac{12\underline{/45^\circ}}{6\underline{/90^\circ}} = 2\underline{/-45^\circ}\ \text{A}$$

$$i(t) = 2\cos(60t - 45^\circ)\ \text{A}$$

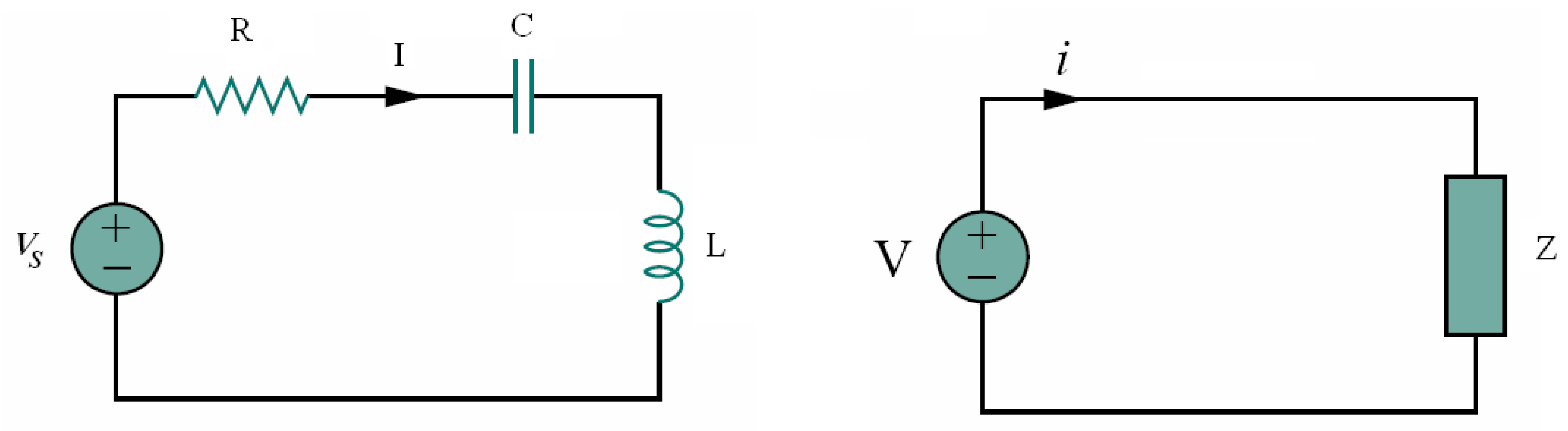
Example

If voltage $v(t) = 6\cos(100t - 30^o)$ is applied to a 50 μF capacitor, calculate the current, $i(t)$, through the capacitor.

Answer: $i(t) = 30 \cos(100t + 60^o)$ mA

The Impedance

- Any combination of series and/or parallel resistors, inductors and capacitors form an impedance.
- Z is the impedance where $Z = R + j\omega L - j/\omega c$



The admittance

The admittance **Y** is the reciprocal of impedance, measured in siemens (S).

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} \quad \text{or} \quad \mathbf{V} = \mathbf{Z}\mathbf{I}$$

$$\mathbf{Y} = \frac{1}{\mathbf{Z}} = \frac{\mathbf{I}}{\mathbf{V}}$$

$$\mathbf{Z} = R \pm jX = |\mathbf{Z}|\underline{/\theta}$$

$$\mathbf{Y} = G + jB$$

$$G + jB = \frac{1}{R + jX}$$

$$G + jB = \frac{1}{R + jX} \cdot \frac{R - jX}{R - jX} = \frac{R - jX}{R^2 + X^2}$$

Equating the real and imaginary parts gives

$$G = \frac{R}{R^2 + X^2}, \qquad B = -\frac{X}{R^2 + X^2}$$

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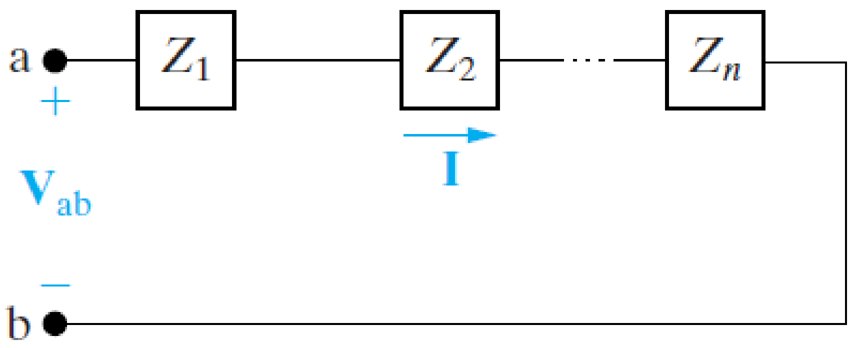
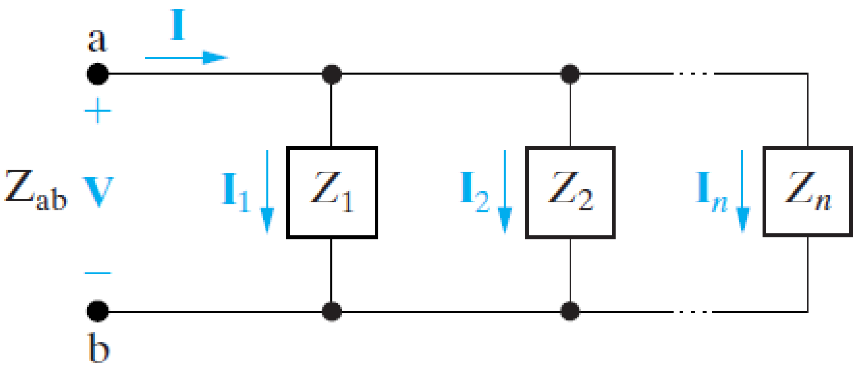
Series and parallel Impedances

Series



$$Z_{ab} = \frac{\mathbf{V}_{ab}}{\mathbf{I}} = Z_1 + Z_2 + \cdots + Z_n.$$

parallel



$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 + \cdots + \mathbf{I}_n,$$

$$\frac{\mathbf{V}}{Z_{ab}} = \frac{\mathbf{V}}{Z_1} + \frac{\mathbf{V}}{Z_2} + \cdots + \frac{\mathbf{V}}{Z_n}.$$

$$Y = \frac{1}{Z} = G + jB$$

$$Y_{ab} = Y_1 + Y_2 + \cdots + Y_n.$$

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Kirchhoff’s Laws in the Frequency Domain

- Both KVL and KCL are hold in the phasor domain or more commonly called frequency domain.
- Moreover, the variables to be handled are phasors, which are complex numbers.
- Series and parallel combinations are the same as in DC circuits analysis.
- All the mathematical operations involved are now in complex domain.

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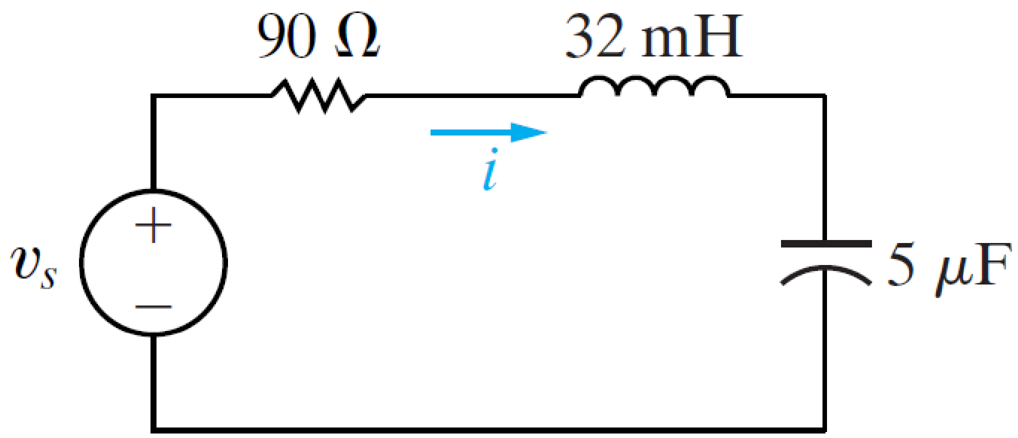
Example

Refer to Figure below, determine the time domain expression for the steady state current.

v_s is $750 \cos(5000t + 30^\circ)$ V



Answer

$$\mathbf{I} = \frac{750 \angle 30^\circ}{150 \angle 53.13^\circ} = 5 \angle -23.13^\circ \text{ A}. \qquad i = 5 \cos(5000t - 23.13^\circ) \text{ A}.$$

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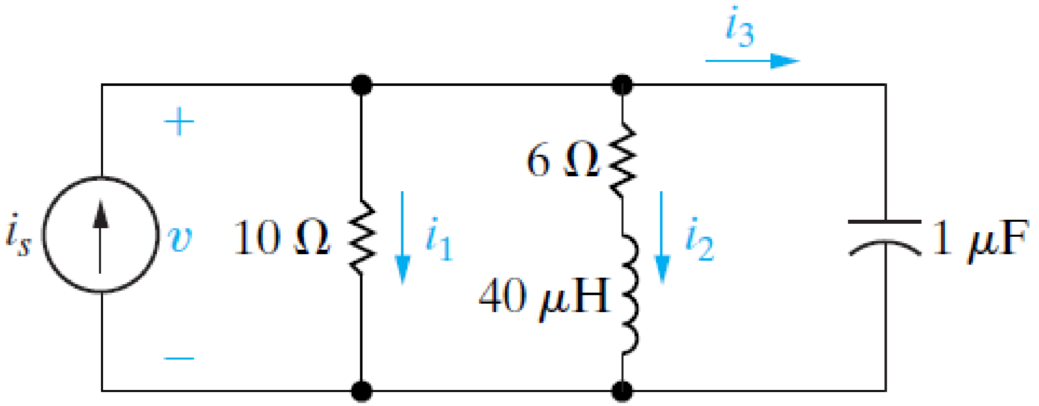
Example

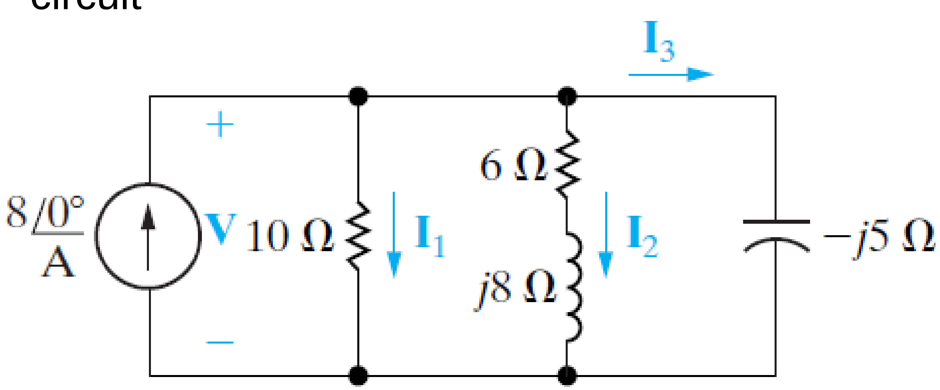
$$i_s = 8 \cos 200{,}000t \text{ A}$$



Find the steady-state expressions for v, i_1, i_2

Solution

Step 1: Construct the phasor domain circuit



Step 2: Apply circuit analysis technique

$$Y = Y_1 + Y_2 + Y_3$$

$$= 0.16 + j0.12$$

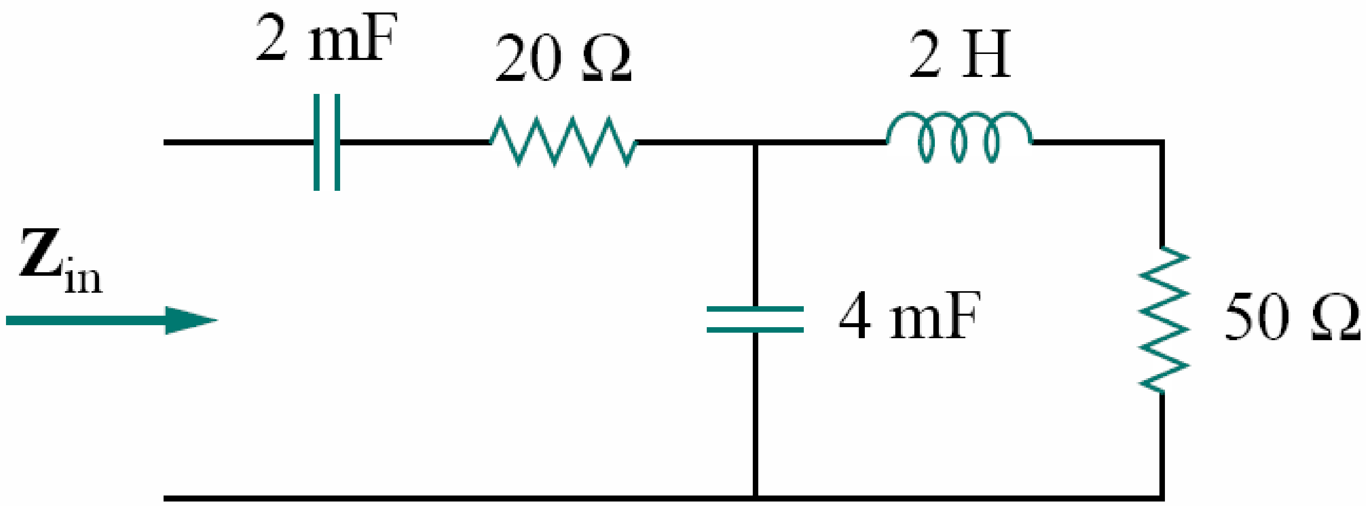
$$= 0.2\angle 36.87° \text{ S.}$$

$$Z = \frac{1}{Y} = 5\angle -36.87° \ \Omega$$

$$\mathbf{V} = Z\mathbf{I} = 40\angle -36.87° \text{ V.}$$

Impedance Combinations

Determine the input impedance of the circuit in figure below at ω =10 rad/s.



Answer: $\boldsymbol{Z_{in}}$ = 32.38 – j73.76

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