Electric Circuits II: (ELCT 401)

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Lecture 3: AC Analysis

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Sinusoidal Steady-State Analysis

Objectives

To perform circuit analysis using the following techniques:

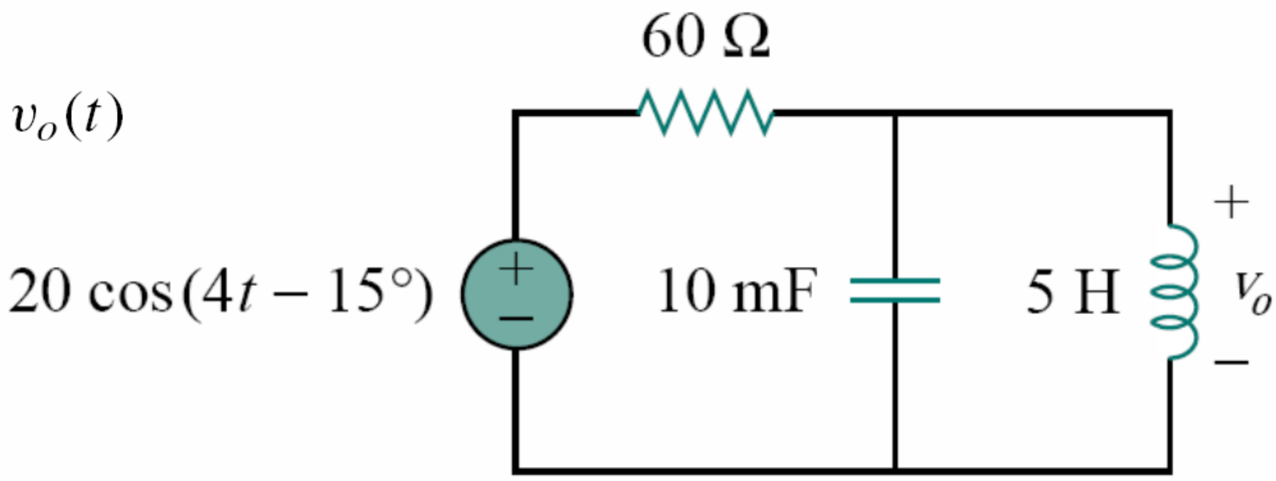
1. Basic Approach (KCL and KVL – Circuit reduction)
2. Nodal Analysis
3. Mesh Analysis
4. Superposition Theorem
5. Source Transformation
6. Thevenin and Norton Equivalent Circuits
7. Bridge Networks

Steps to Analyze AC Circuits:

1. Transform the circuit to the phasor (frequency) domain
2. Solve using an appropriate circuit technique such as nodal analysis, mesh current analysis, superposition, etc…
3. Transform the resulting phasors back into the time domain

Example

Determine $v_o(t)$



Solution

$$v_s = 20\cos(4t - 15^\circ) \quad \Longrightarrow \quad \mathbf{V}_s = 20\angle{-15^\circ}\ \text{V}, \qquad \omega = 4$$

$$10\ \text{mF} \quad \Longrightarrow \quad \frac{1}{j\omega C} = \frac{1}{j4 \times 10 \times 10^{-3}} = -j25\ \Omega$$

$$5\ \text{H} \quad \Longrightarrow \quad j\omega L = j4 \times 5 = j20\ \Omega$$

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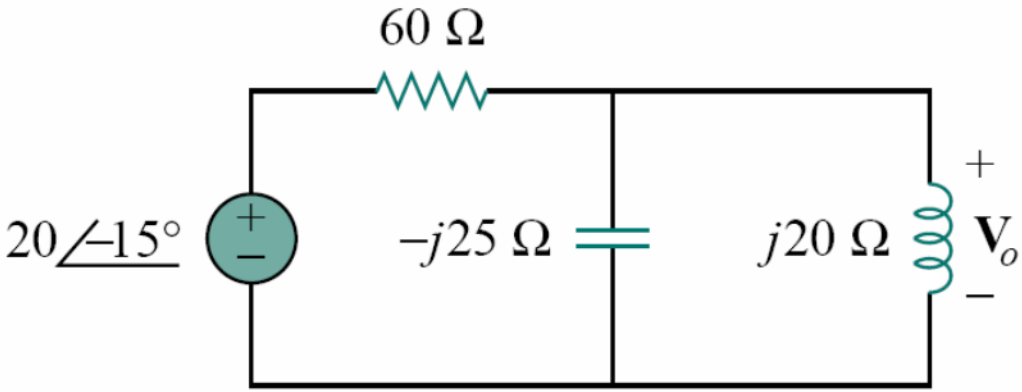

$$\mathbf{Z}_1 = 60\ \Omega$$

$$\mathbf{Z}_2 = -j25 \parallel j20 = \frac{-j25 \times j20}{-j25 + j20} = j100\ \Omega$$

$$\mathbf{V}_o = \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}\mathbf{V}_s = \frac{j100}{60 + j100}(20\angle -15^\circ)$$

$$= (0.8575\angle 30.96^\circ)(20\angle -15^\circ) = 17.15\angle 15.96^\circ \text{ V}.$$

$$v_o(t) = 17.15\cos(4t + 15.96^\circ)\text{V}$$

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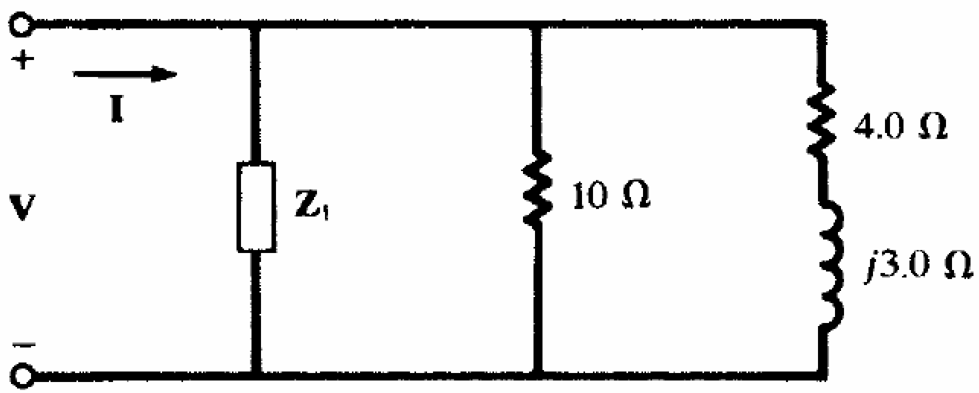
Example

Find $\mathbf{Z}_1$ in the three-branch network if $\mathbf{I} = 31.5\ \underline{/24.0^\circ}$ A for an applied voltage $\mathbf{V} = 50.0\ \underline{/60.0^\circ}$ V.



Solution

$$\mathbf{Y} = \frac{\mathbf{I}}{\mathbf{V}} = 0.630\ \underline{/-36.0^\circ} = 0.510 - j0.370 \quad \mathrm{S}$$

$$0.510 - j0.370 = \mathbf{Y}_1 + \frac{1}{10} + \frac{1}{4.0 + j3.0}$$

$$\mathbf{Y}_1 = 0.354\ \underline{/-45^\circ}\ \mathrm{S} \text{ and } \mathbf{Z}_1 = 2.0 + j2.0\ \Omega.$$

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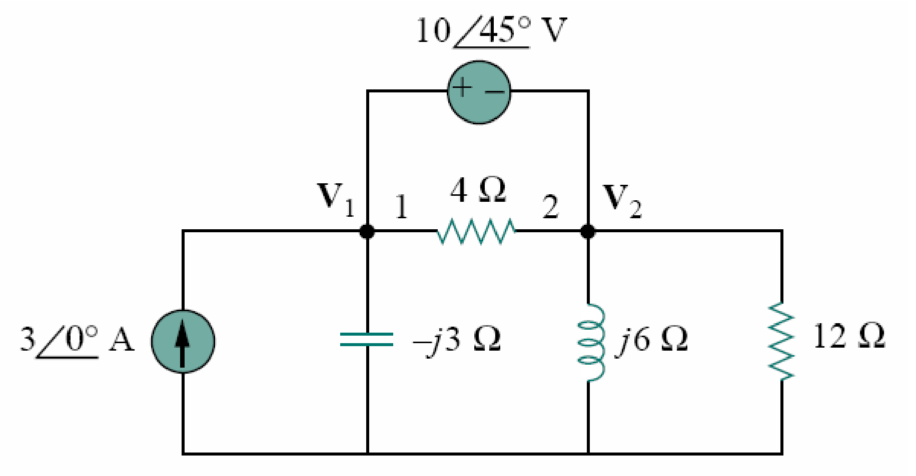
Nodal Analysis

Example:

Using nodal analysis, find V_1 and V_2.



Solution:

$$-3+\frac{V_1-V_2}{4}+\frac{V_1}{-j3}+\frac{V_2-V_1}{4}+\frac{V_2}{j6}+\frac{V_2}{12}=0 \qquad (1)$$

$$V_1-V_2=10\angle 45° \qquad (2)$$

Solve 1& 2:

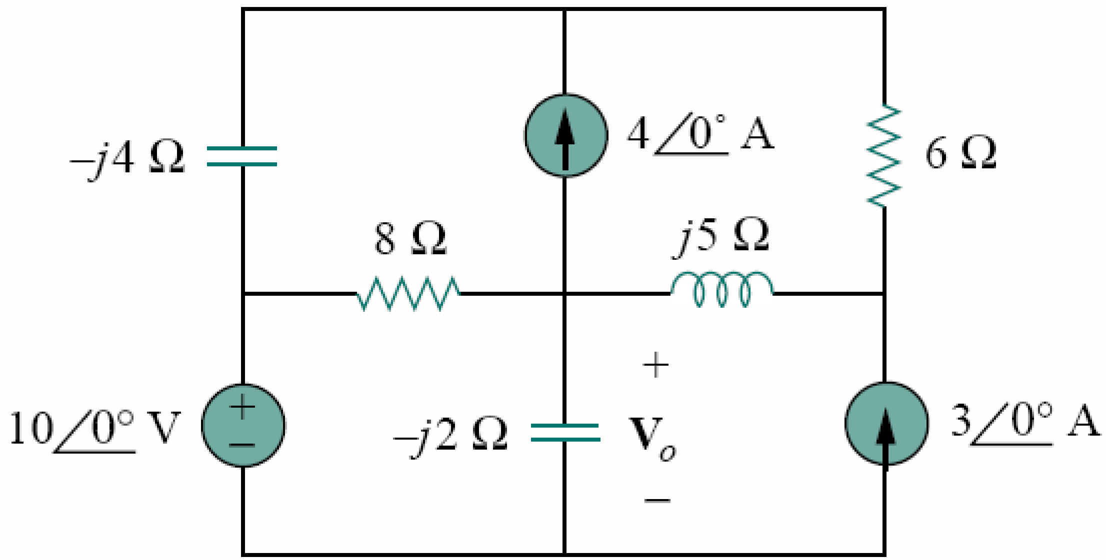
$$\mathbf{V}_2 = 31.41\angle -87.18° \text{ V}$$

$$\mathbf{V}_1 = \mathbf{V}_2 + 10\angle 45° = 25.78\angle -70.48° \text{ V}$$

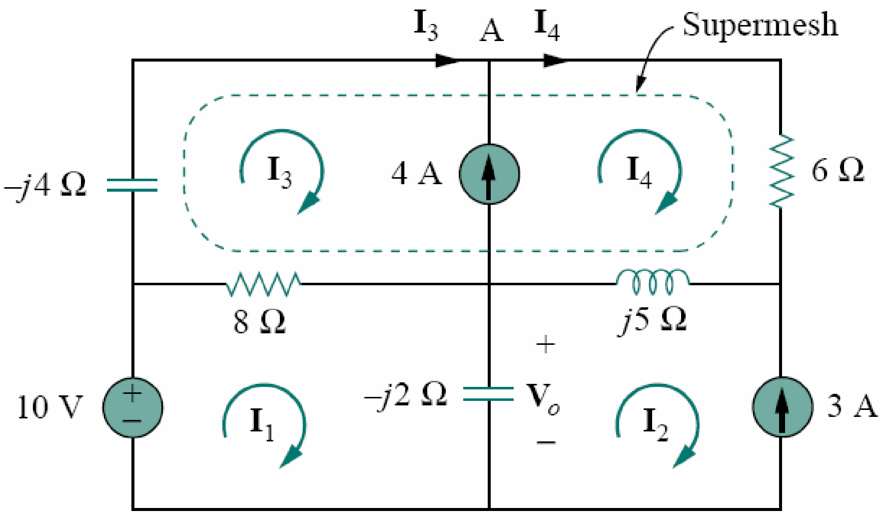
Mesh Analysis

Example

Find V_o in the following figure using mesh analysis.



Solution



- Mesh 1: $-10 + 8(I_1 - I_3) - j2(I_1 - I_2) = 0$
- Mesh 2: $I_2 = -3$
- Super mesh:

  $-j4I_3 + 6I_4 + j5(I_4 - I_2) + 8(I_3 - I_1) = 0$
- Constraint: $I_4 - I_3 = 4$

Solving the 4 equations:

$$I_1 = \frac{41}{145} - j\frac{523}{145}\ A,\quad I_2 = -3\ A,\quad I_3 = -\frac{271}{145} - j\frac{642}{145}\ A, I_4 = \frac{309}{145} - j\frac{642}{145}\ A$$

$$V_0 = -2j(I_1 - I_2) = -7.2134 - j6.568 = 9.756\angle 222.32°\ V$$

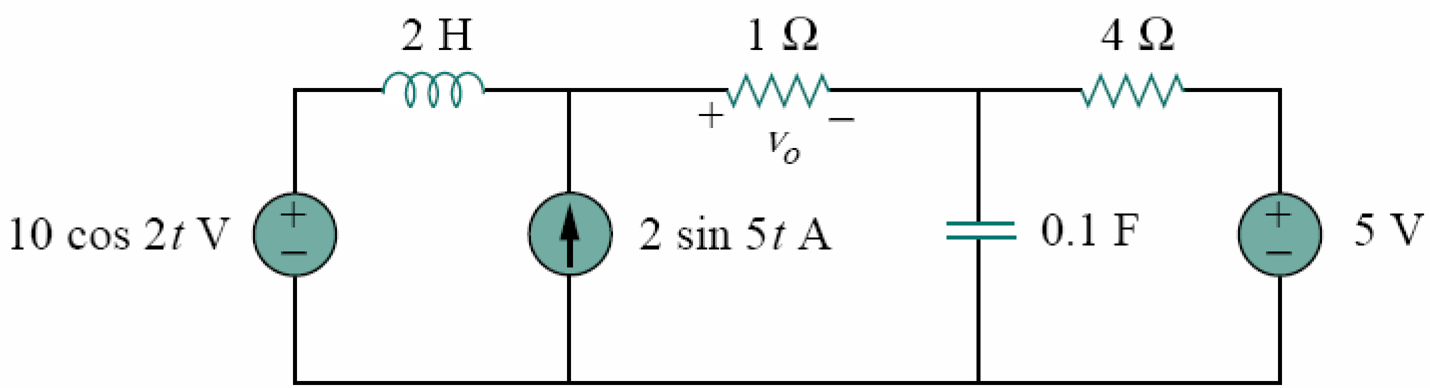
Superposition Theorem

If a circuit has sources operating at different frequencies, then:

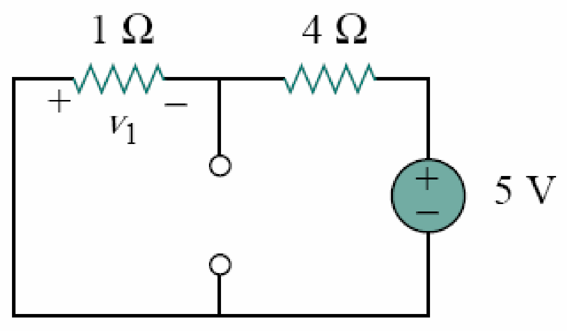
- The separate phasor circuit for each frequency must be solved independently, and
- The total response is the sum of the time-domain responses of all the individual phasor circuits

Example 5

Calculate $v_o(t)$ in the circuit shown below using the superposition theorem

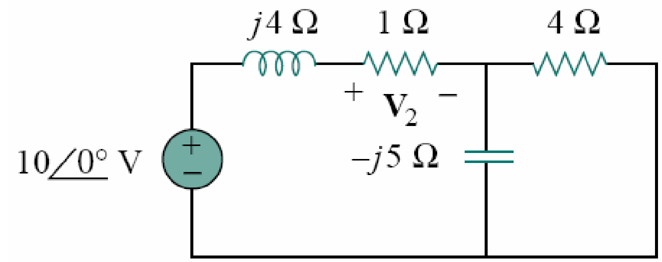


$$\boxed{v_o = v_1 + v_2 + v_3}$$



(a)

$$-v_1 = \frac{1}{1+4}(5) = 1 \text{ V}$$



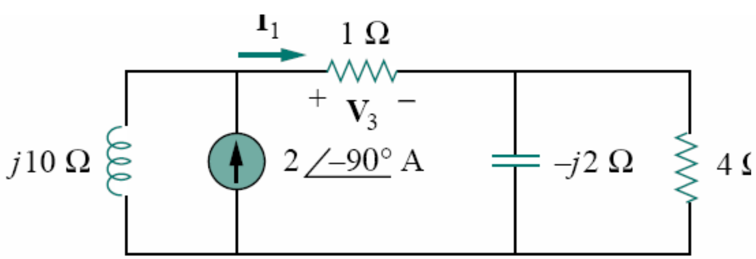
(b)

$\omega = 2\,rad/s$

Voltage divider

$$\mathbf{V}_2 = \frac{1}{1+j4+\mathbf{Z}}(10\underline{/0^\circ}) = \frac{10}{3.439+j2.049} = 2.498\underline{/-30.79^\circ}$$

$$v_2 = 2.498\cos(2t - 30.79^\circ)$$



$\omega = 5\,rad/s$

(c)

Current divider

$$\mathbf{V}_3 = \mathbf{I}_1 \times 1 = \frac{j10}{1.8+j8.4}(-j2) = 2.328\underline{/-77.91^\circ} \text{ V}$$

$$v_3 = 2.33\cos(5t - 80^\circ) = 2.33\sin(5t + 10^\circ) \text{ V}$$

$$\boxed{v_o = v_1 + v_2 + v_3}$$

$$v_o(t) = -1 + 2.498\cos(2t - 30.79^\circ) + 2.33\sin(5t + 10^\circ) \text{ V}$$

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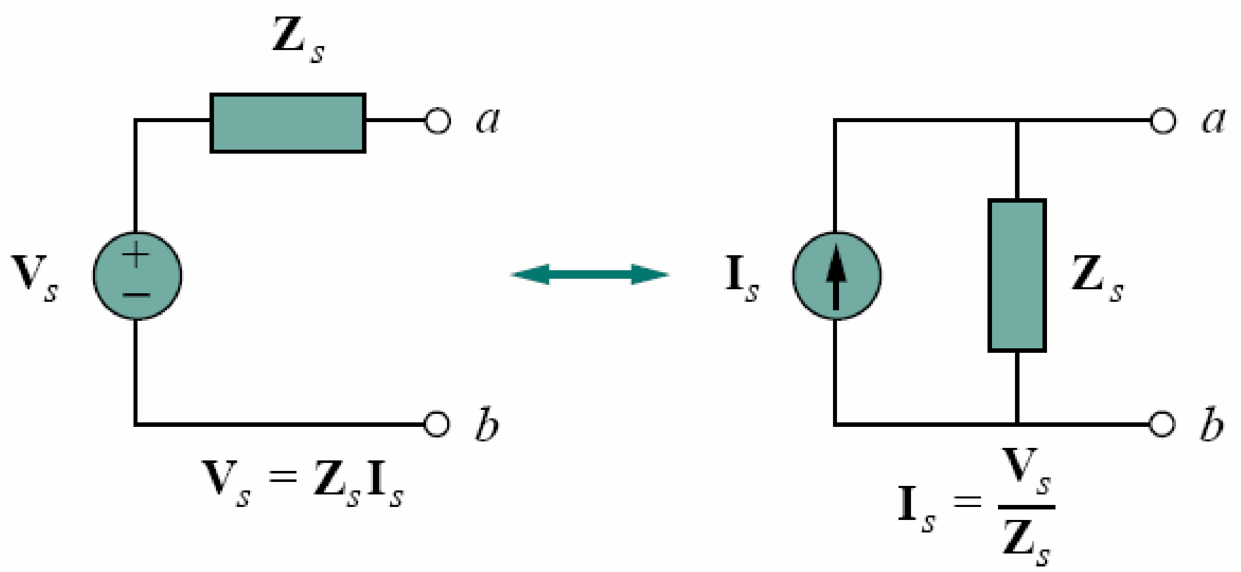
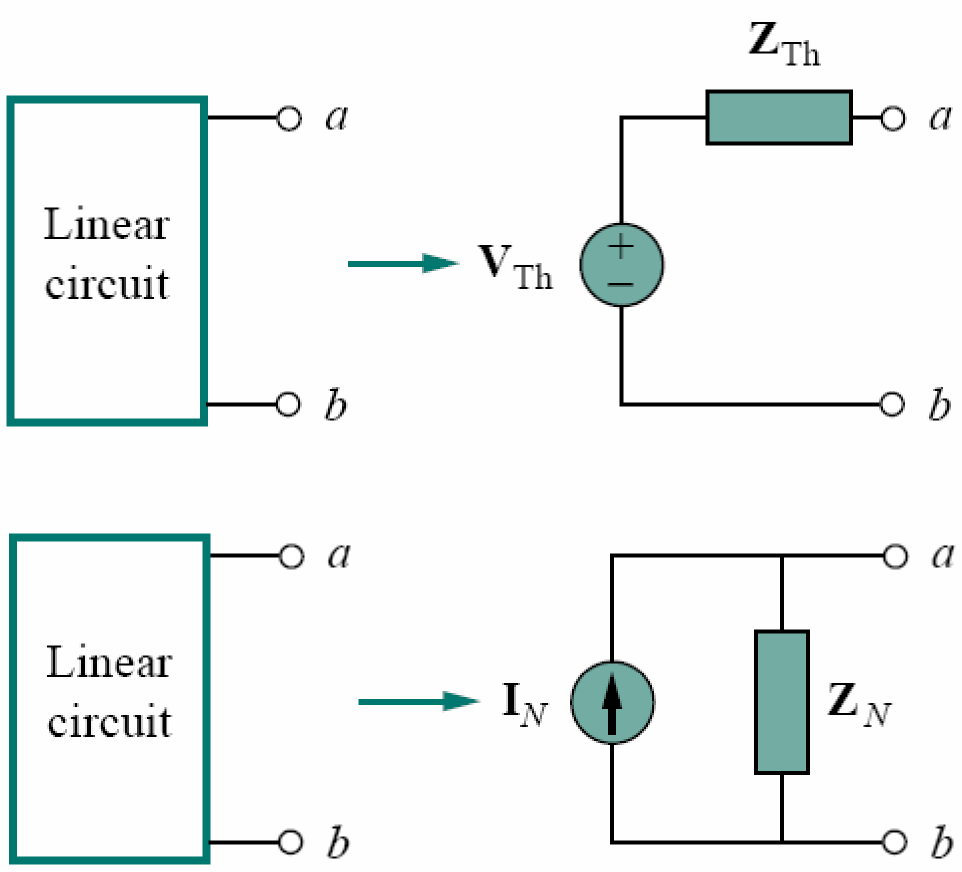
Source Transformation



Thevenin and Norton Equivalent Circuits



Thevenin transform

Norton transform

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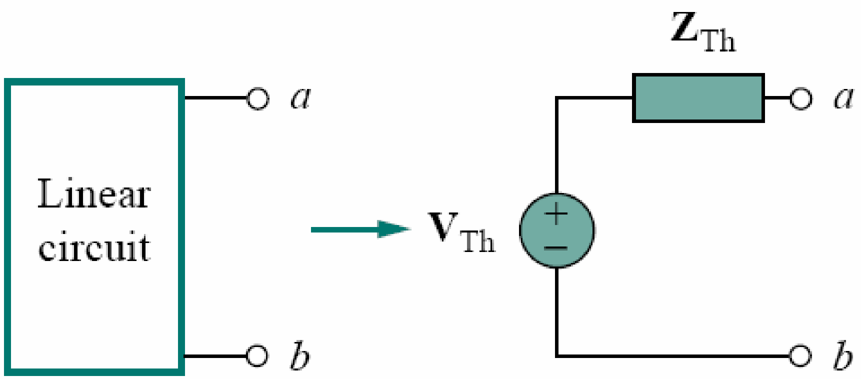
Thevenin and Norton Equivalent Circuits

If a load impedance Z_L is connected between the terminals *a* and *b*, the maximum power is transferred to the load when:

$$Z_L = Z_{th}^{*}$$



In this case, the maximum power transferred to the load is given by:

$$P_{max} = \frac{|V_{th-max}|^2}{8R_{th}}$$

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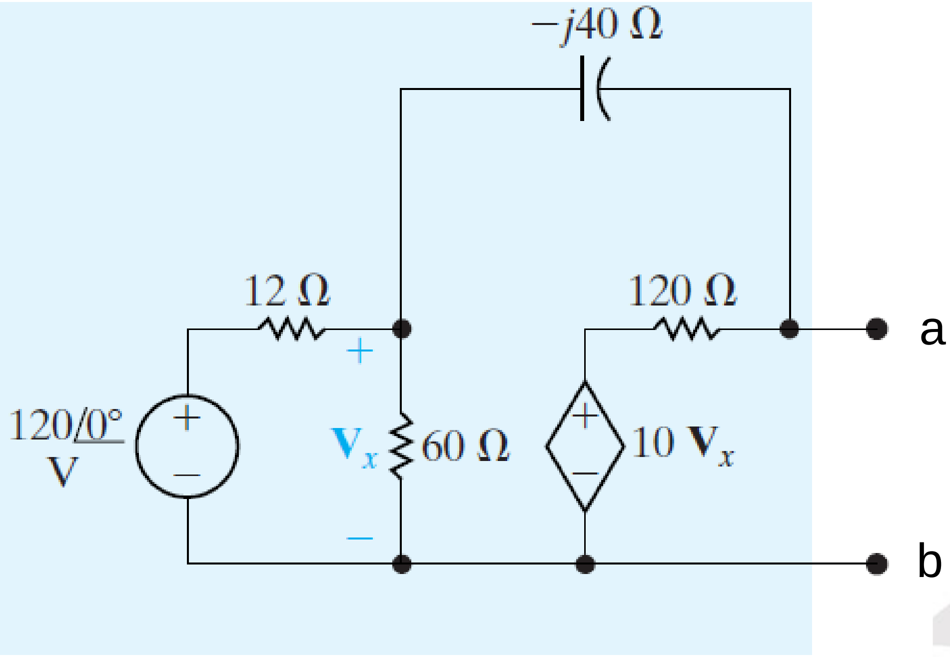
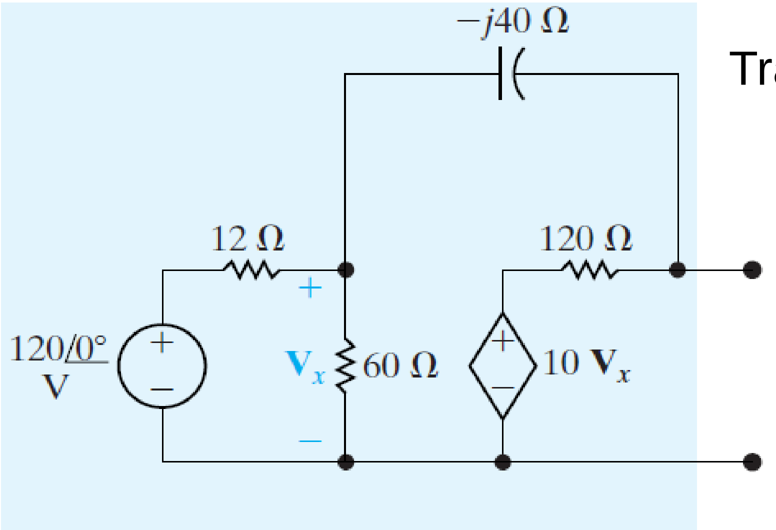
Example

Find the Thevenin equivalent circuit between terminals a & b

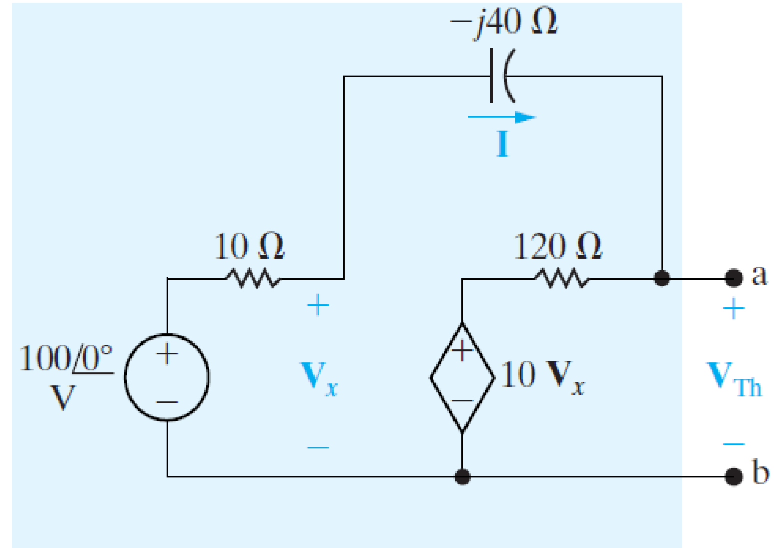


This means that we need to find V_{Th} and Z_{Th}

Solution:



Source Transformation



Finding V_{TH}

$$\mathbf{V}_x = 100 - 10\mathbf{I}.$$

$$\mathbf{I} = \frac{-900}{30 - j40} = 18 \angle -126.87^\circ \text{ A}.$$

$$\mathbf{V}_x = 100 - 180 \angle -126.87^\circ = 208 + j144 \text{ V}.$$

$$\mathbf{V}_{\text{Th}} = 10\mathbf{V}_x + 120\mathbf{I} = 2080 + j1440 + 120(18) \angle -126.87^\circ$$

$$= 784 - j288 = 835.22 \angle -20.17^\circ \text{ V}.$$

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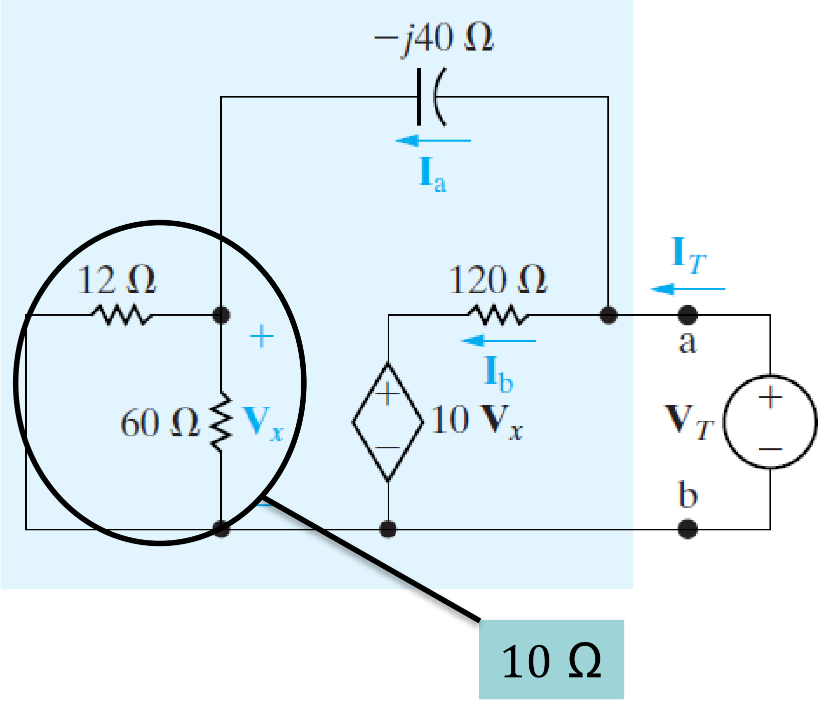
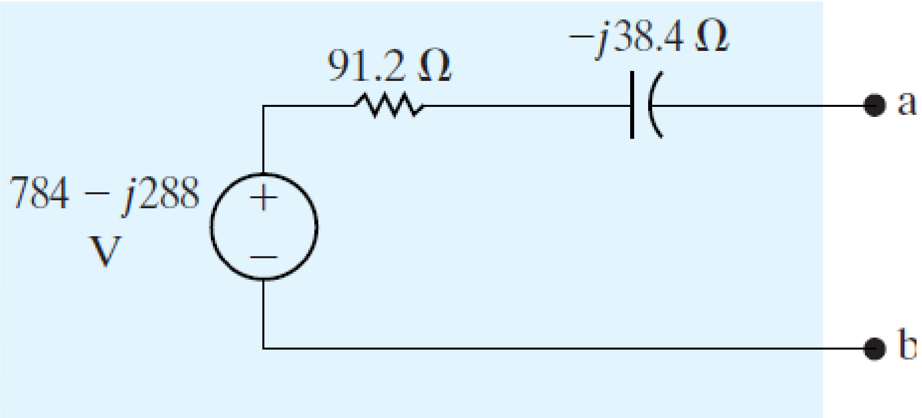
Finding Z_{TH}



$$\mathbf{I}_a = \frac{\mathbf{V}_T}{10 - j40}, \quad \mathbf{V}_x = 10\mathbf{I}_a,$$

$$\mathbf{I}_b = \frac{\mathbf{V}_T - 10\mathbf{V}_x}{120}$$

$$\begin{aligned}\mathbf{I}_T &= \mathbf{I}_a + \mathbf{I}_b \\ &= \frac{\mathbf{V}_T}{10 - j40}\left(1 - \frac{9 + j4}{12}\right) \\ &= \frac{\mathbf{V}_T(3 - j4)}{12(10 - j40)},\end{aligned}$$



$$Z_{Th} = \frac{\mathbf{V}_T}{\mathbf{I}_T} = 91.2 - j38.4\ \Omega.$$

Bridge Networks

- A bridge is a 4-arm circuit, connected as shown below
- It is used to measure unknown impedances
- We place the unknown impedance as Z_4, a known impedance as Z_1, then we keep varying Z_2 and Z_3 until V and I are equal to zero
- At balance, the unknown quantities can be calculated

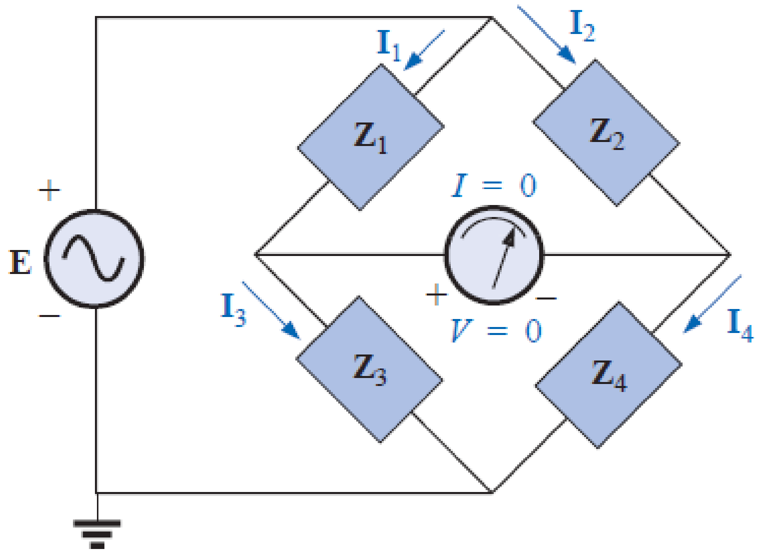
For balance, $\quad \mathbf{I}$ and $\mathbf{V} = 0.$

Since $\mathbf{I} = 0,$ $\quad \boxed{\mathbf{I}_1 = \mathbf{I}_3}$

$\boxed{\mathbf{I}_2 = \mathbf{I}_4}$

for $\mathbf{V} = 0,$ $\quad \boxed{\mathbf{I}_1\mathbf{Z}_1 = \mathbf{I}_2\mathbf{Z}_2} \quad (1)$

$\boxed{\mathbf{I}_3\mathbf{Z}_3 = \mathbf{I}_4\mathbf{Z}_4} \quad (2)$



$$\xrightarrow{(1)/(2)} \quad \boxed{\frac{\mathbf{Z}_1}{\mathbf{Z}_3} = \frac{\mathbf{Z}_2}{\mathbf{Z}_4}}$$

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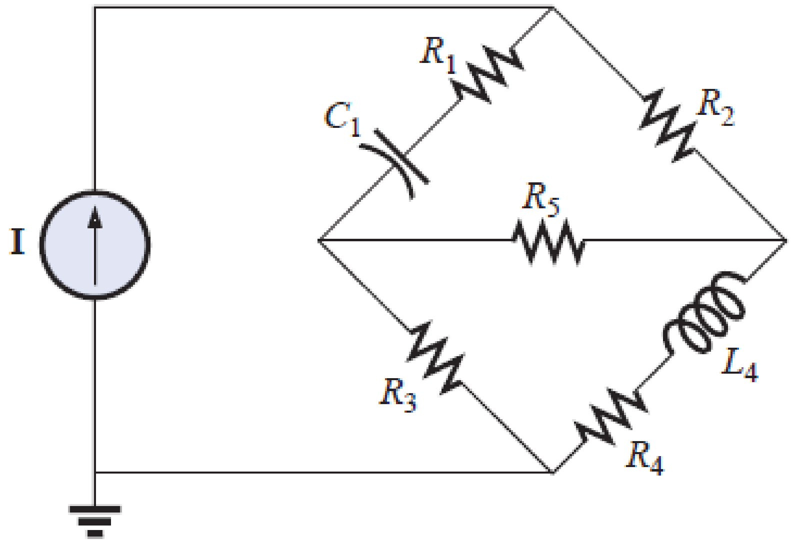
Example

Hay's Bridge

L_4 & R_4 are the unknowns



$$\mathbf{Z}_1 = R_1 - j\,X_C$$
$$\mathbf{Z}_2 = R_2$$
$$\mathbf{Z}_3 = R_3$$
$$\mathbf{Z}_4 = R_4 + j\,X_L$$

At Balance:

$$\mathbf{Z}_2\mathbf{Z}_3 = \mathbf{Z}_4\mathbf{Z}_1$$

$$R_2R_3 = (R_4 + j\,X_L)(R_1 - j\,X_C)$$

$$R_2R_3 = R_1R_4 + j\,(R_1X_L - R_4X_C) + X_CX_L$$

Therefore

$$\boxed{R_2R_3 = R_1R_4 + X_CX_L}$$

$$\boxed{0 = R_1X_L - R_4X_C}$$

Since, $X_L = \omega L$ and $X_C = \dfrac{1}{\omega C}$

$$R_2R_3 = R_1R_4 + \frac{L}{C}$$

$$R_1\omega L = \frac{R_4}{\omega C}$$

$$\boxed{L = \frac{CR_2R_3}{1 + \omega^2C^2R_1^2}}$$

$$\boxed{R_4 = \frac{\omega^2C^2R_1R_2R_3}{1 + \omega^2C^2R_1^2}}$$

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