

Applied Mathematics - Assignment 3

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1 Problem 1

(a), (b), (c)

```
1  function euler(f, u0, tspan, h)
2      n = abs(-(tspan...)) / h
3      collect(
4          begin
5              u0 += h * f(u0); u0
6          end
7          for t in tspan[1]:n
8      )
9  end
```

Listing 1: Euler's Method

```
1  function f!(u)
2      du1 = 4*u[1] + 2*u[2]
3      du2 = 3*u[1] + 3*u[2]
4      return [du1, du2]
5  end
```

Listing 2: The differential equation

```
1  euler(f, [10.0; -5.0], (0.0, 0.5) 0.01)
```

Listing 3: Calling the function

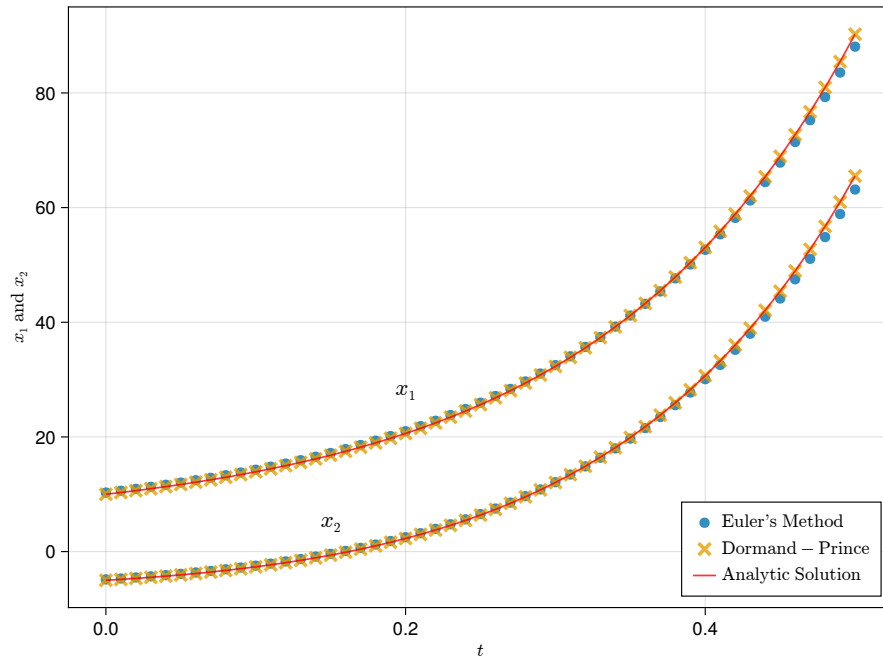


Figure 1: Comparison of numeric to analytic solutions

(d)

The Dormand-Prince method, which I believe is the default used by the ode45 solver, offered substantially improved accuracy over the Euler method, even though its abilities were stunted by using a fixed step-size. However, it is not as performant as the Tsitouras method used in the other problems in this assignment

2 Problem 2

(a)

x represents the prey, and y the predator.

(b)

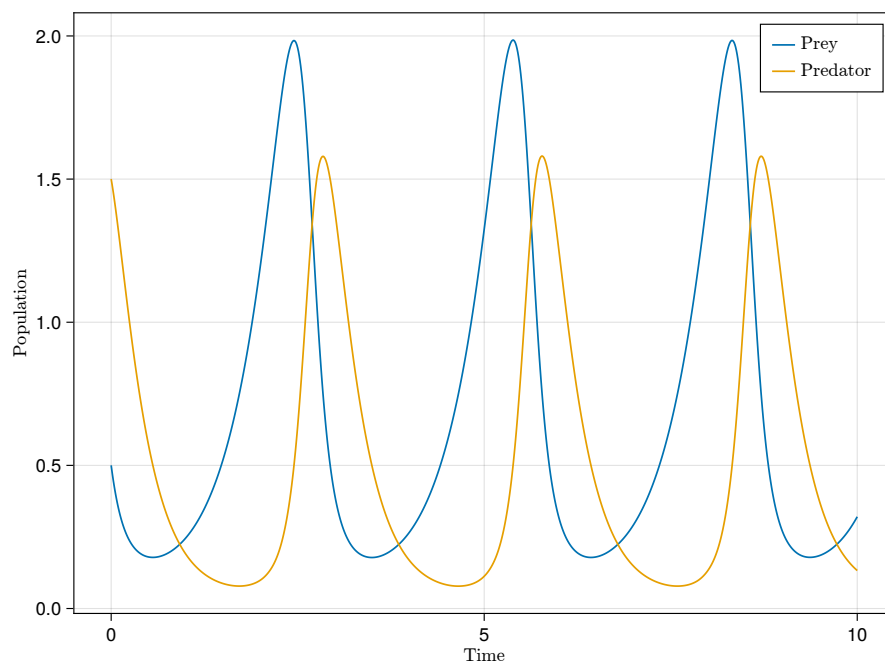


Figure 2: Lotka-Volterra predator-prey model over 10 years

(c)

The populations are first equal at 0.93 days.

The extrema of the prey are 0.178 and 1.99, and the predators 0.0782 and 1.58.

3 Problem 3

(a)

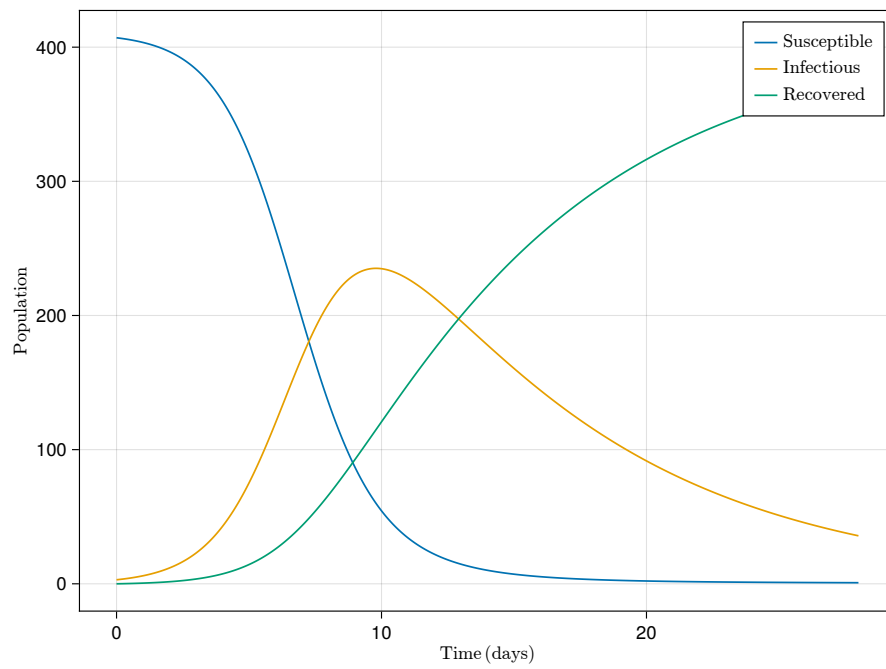


Figure 3: SIR model over 4 weeks

(b)

(i)

235.14

(ii)

35.78

4 Problem 4

(a)

With the substitution $y = x'$, we may describe the Van der Pol oscillator with the first-order system:

$$\begin{aligned}x' &= y \\ y' &= \mu(1 - x^2)y - x\end{aligned}$$

(b)

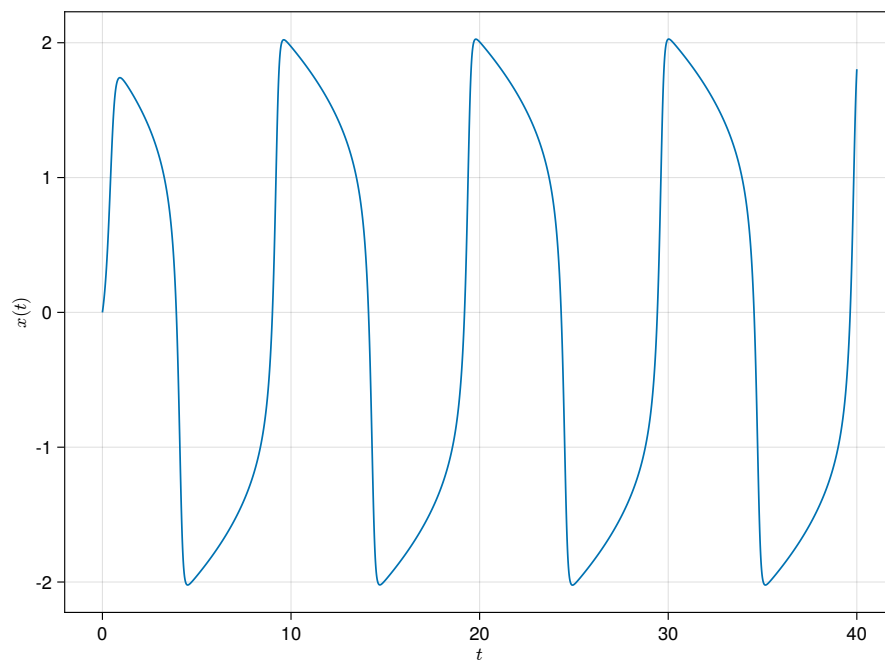


Figure 4: Solution to the Van der Pol oscillator

(c)

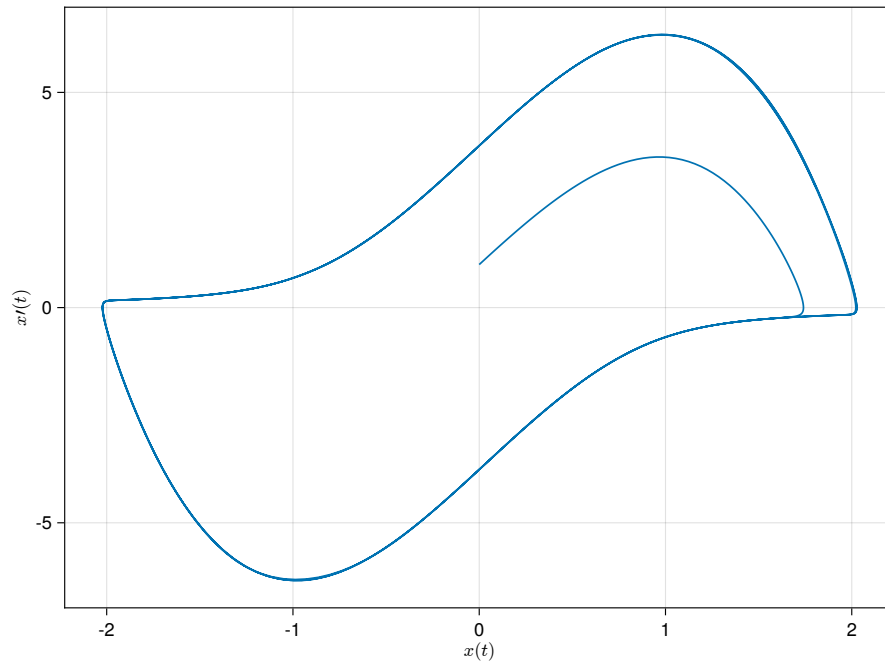


Figure 5: Phase plot of the Van der Pol oscillator

A static export of the notebook containing all analysis and figures is available at https://adammenne.github.io/applied_mathematics_244/assignment_3/notebook.html.

With full source code available at https://github.com/AdamMenne/applied_mathematics_244/tree/master/assignment_3