Applied Mathematics - Assignment 2

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1 Problem 1

(a), (b), (c)

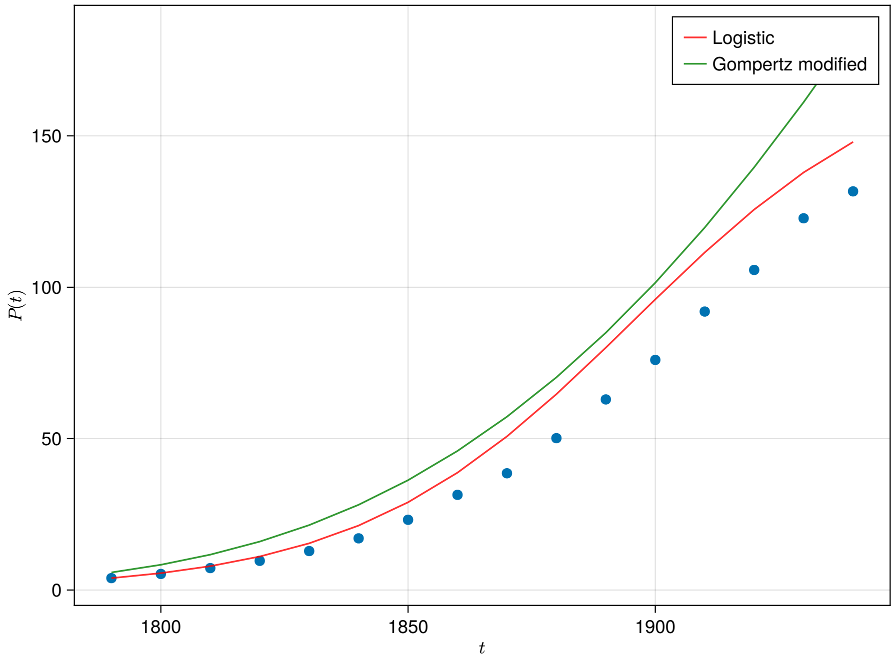

Figure 1:

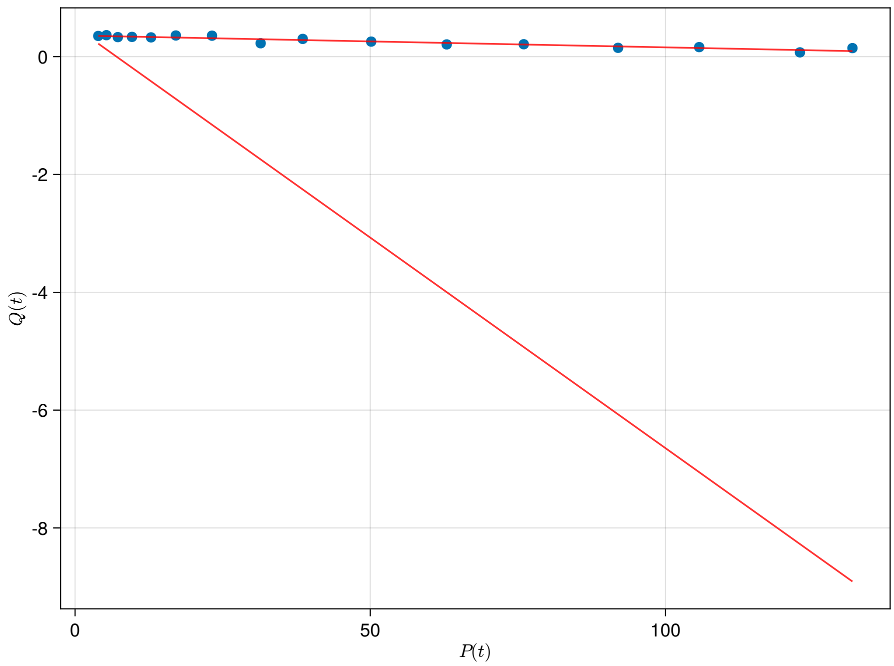

Figure 2:

(d)

176.2 million with $K = 178.1$ million. In 2020 the actual US population was 329.5

2 Problem 2

(a)

Solving the differential equation

$$\frac{dP}{dt} = P(a - b\ln P)$$

$$\int \frac{1}{P(a - b\ln P)}\, dP = \int dt$$

$$u = \ln P \quad , \; du = \frac{1}{p}\; dP$$

$$\int \frac{1}{a - bu}\; du = t + c$$

$$s = a - bu \quad , \; ds = -b\; du$$

$$-\frac{1}{b}\; \int \frac{1}{s}\; ds = t + c$$

$$-\frac{\ln(s)}{b} = t + c$$

$$-\frac{\ln(a - bu)}{b} = t + c$$

$$-\frac{\ln(a - b\ln(P))}{b} = t + c$$

$$\ln(a - b\ln(P)) = -bt + c$$

$$a - b\ln(P) = ce^{-bt}$$

$$\ln P = \frac{ce^{-bt} - a}{-b}$$

$$P = e^{\frac{a}{b} + ce^{-bt}}$$

Solving the IVP

$$P_0 = e^{\frac{a}{b}} e^{ce^{-b(0)}}$$

$$P_0 = e^{\frac{a}{b}+c}$$

$$\ln P_0 = \frac{a}{b} + c$$

$$c = \ln P - \frac{a}{b}$$

(b)

See figure 1

(c)

See figure 2

(e)

2089.5

Full source code for this assignment is availble at `https://github.com/AdamMenne/applied_mathematics_244/tree/master/assignment_2`