Quantum Cohomology and the Gromov-Witten Theory of $\mathbb{P}^2$

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1. Introduction

2. Quantum Cohomology

3. Gromov-Witten Theory for $\mathbb{P}^2$

The Gromov-Witten potential for $\mathbb{P}^2$ is given by

$$\Phi_0^{\mathbb{P}^2} := \frac{1}{2}(y_0^2 y_2 + y_0 y_1^2) + \sum_{d=1}^{\infty} N_d \frac{e^{dy_1} y_2^{3d-3}}{(3d-1)!} q^d. \tag{1}$$

For convenience, we define

$$\Phi_{ijk} := \frac{\partial^3 \Phi_0^{\mathbb{P}^2}}{\partial y_i \partial y_j \partial y_k}, \tag{2}$$

then the WDVV equation for $\mathbb{P}^2$ is given by

$$\Phi_{222} = \Phi_{112}^2 - \Phi_{111}\Phi_{122}. \tag{3}$$

Evaluating both sides we obtain the expression

$$\sum_{d=1}^{\infty} N_d \frac{e^{dy_1} y_2^{3d-4}}{(3d-4)!} q^d = \left(\sum_{d=1}^{\infty} d^2 N_d \frac{e^{dy_1} y_2^{3d-2}}{(3d-2)!} q^d \right)^2 \tag{4}$$

4. Axiomatization of Gromov-Witten-invariants

References