

1.a)

$$I^+ = IDGF$$

$$H^+ = HCEA$$

$$BI^+ = ABCDEFGHIJK$$

$$B^+ = BHCEA$$

$$CI^+ = CIKDGF$$

$I \rightarrow DGF$, $H \rightarrow CEA$, $B \rightarrow H$, $CI \rightarrow K$ violate BCNF since the left sides are not super keys.

b)

For the first split of the relation I will be using the closure of CI in part (a):

$$R(ABCDEFGHIJK) \rightarrow R_1(CDFGIK), R_2(ABCEHIJ)$$

$$R_1(CDFGIK):$$

The closures of C,D,F,G,I are trivial as the original FDs don't have a single C,D,F,G,I on the left side, however the closure of I, $I^+ = IDGF$, is not trivial and is not a super key of R_1 . I will split R_1 using the closure of I:

$$R_1(CDFGIK) \rightarrow R_3(CKI), R_4(DFGI)$$

$$R_3(CKI):$$

All FDs are trivial here except for CI which is a super key. So this relation is done.

$$R_4(DFGI):$$

I is the super key and any closure including I is also a super key. Closure of D,F,G are trivial. $DF^+ = DF$, $FG^+ = FG$ as you can see the other attributes are all trivial with the exception of those that include I which is a super key. So this relation is done.

$$R_2(ABCEHIJ):$$

The original FD $B^+ = BHCEA$ is not a super key on the left side so I will split using this FD.

$$R_2(ABCEHIJ) \rightarrow R_5(BIJ), R_6(ABCEH)$$

$$R_5(BIJ):$$

Closures of B,I,J are trivial. BI is a super key of R, BJ and IJ is trivial, mainly because J does not appear at all on the left sides of any FD. So this relation is done.

$R_6(ABCEH)$:

The original FD $H^+ = HCEA$ is not a super key on the left side so I will split on this FD:

$R_6(ABCEH) \rightarrow R_7(BH), R_8(ACEH)$

$R_7(BH)$:

This relation only has two attributes it is impossible to split further.

$R_8(ACEH)$:

All FDs including H is a super key of this relation. Since there are no A,C,E on the left sides of the original FDs, any and all FD combinations of these will be trivial. So this relation is done.

FINAL RELATIONS:

$R_3(CKI)$

$R_4(DFGI)$

$R_5(BIJ)$

$R_7(BH)$

$R_8(ACEH)$

2.a)

Enumerating FD with singleton RHS:

1. $ACDE \rightarrow B$

2. $BF \rightarrow A$

3. $BF \rightarrow D$

4. $B \rightarrow C$

5. $B \rightarrow F$

6. $CD \rightarrow A$

7. $CD \rightarrow F$

8. $ABF \rightarrow C$

9. $ABF \rightarrow D$

10. $ABF \rightarrow H$

FD	Exclude	Closure	Decision
1	1	Can't get B without this FD	Keep
2	2	$BF^+ = BFCDA$	Discard
3	2,3	$BF^+ = BFC$	Keep
4	2,4	$B^+ = BFD$	Keep
5	2,5	$B^+ = BC$	Keep
6	2,6	Can't get A without this FD	Keep
7	2,7	$CD^+ = CDA$ Need E to get B	Keep
8	2,8	$ABF^+ = ABFC$	Discard
9	2,8,9	$ABF^+ = ABFD$	Discard
10	2,8,9,10	Can't get H without this FD	Keep

KEEP FDs:

1. $ACDE \rightarrow B$

3. $BF \rightarrow D$

4. $B \rightarrow C$

5. $B \rightarrow F$

6. $CD \rightarrow A$

7. $CD \rightarrow F$

10. $ABF \rightarrow H$

1. All single LHS closures are trivial because the no remaining FDs have a single LHS except for B. E is present in only this FD so it is included automatically.

$CD^+ = CDA \rightarrow \text{Remove } A$

Final FD: $CDE \rightarrow B$

3. $B^+ = BFD \rightarrow \text{Remove } F$

Final FD: $B \rightarrow D$

4. Singleton LHS no possible modification
5. Singleton LHS no possible modification
6. $C^+ = C, D^+ = D$ This FD remains the same
7. Same reasoning as 6.

10. $B^+ = BFCD \rightarrow \text{Remove } F$ Final FD: $AB \rightarrow H$

FD	Exclude	Closure	Decision
$AB \rightarrow H$	$AB \rightarrow H$	Can't get H without this FD	Keep
$B \rightarrow C$	$B \rightarrow C$	Can't get C without this FD	Keep
$B \rightarrow D$	$B \rightarrow D$	Can't get D without this FD	Keep
$B \rightarrow F$	$B \rightarrow F$	$B^+ = BCDF$	Discard
$CD \rightarrow A$	$B \rightarrow F, CD \rightarrow A$	Can't get A without this FD	Keep
$CD \rightarrow F$	$B \rightarrow F, CD \rightarrow F$	Can't get F without this FD	Keep
$CDE \rightarrow B$	$B \rightarrow F, CDE \rightarrow B$	Can't get B without this FD	Keep

FINAL MINIMAL BASIS:

$AB \rightarrow H$

$B \rightarrow C$

$B \rightarrow D$

$CD \rightarrow A$

$CD \rightarrow F$

$CDE \rightarrow B$

b) Combining minimal basis FDs:

$AB \rightarrow H$

$B \rightarrow CD$

$CD \rightarrow AF$

$CDE \rightarrow B$

The keys are AB, B, CD, CDE

c) The relations are:

$R_1(ABH), R_2(ACDF), R_3(BCD), R_4(BCDE)$

Since no relation is a super key (G was never present) then we add the new relation:

$R_5(CDEG), CDEG^+ = CDEGABFH$

Final relations:

$R_1(ABH), R_2(ACDF), R_3(BCD), R_4(BCDE), R_5(CDEG)$

d) AB is most certainly NOT a super key despite it being on the left side of an FD. This shows that the relation R_1 allows for redundancy. Thus, the entire schema allows for redundancy.