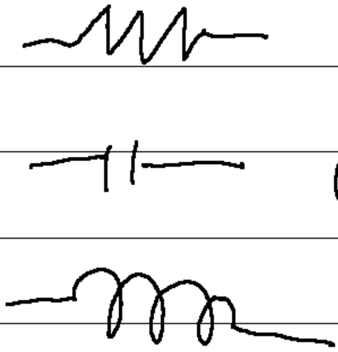
First Day - Syllabus Review



R

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Parts of a circuit

- Buy textbook
- Office hours: 11 AM - Noon 1-2 P.M (every weekday)

MUST PASS FINAL WITH A C!

CI - EEAE I

CII - EEAE II

Electronic Devices - EEAE III

- 4 Regular test, then final (so 5 total tests)
- Schedule on back of syllabus.

Submitting Homework

- One .pdf file (can include all pages)

Ch. 1 Basic Concepts

1.3 Charge and Current

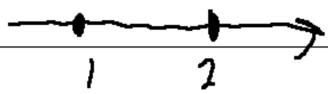
DC   AC/DC

↑   ↑

- Electric current, I or $i(t)$
- Amount of moving electric charges, Q or $q(t)$ per time [Ampere = coulomb/sec]

$$\begin{cases} I = Q/t & \text{or } i(t) = d(q)/dt \\ Q = I \cdot t & \text{or } q = \int_{t_1}^{t_2} i(t)\,dt \end{cases}$$

Direction polarity



I_{12} $I_{12} = -I_{21}$

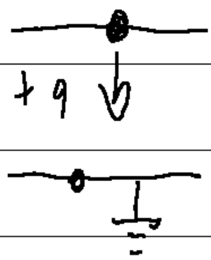
1.4 Polarity

- Voltage, v or $V(t)$
- Amount of energy, W or $w(t)$, per charge required to move charges across two locations

[Volt = Joule/Coulomb]

Kinetic energy

Direction of polarity



$V_1\ (+)$

$+q$

$V_2\ (-) = 0$

$v_{12} = V_1 - V_2 = -V_{21}$

$V = W/Q$ or $v(t) = dw(t)/dq$

$W = V \cdot Q$ or $w = \int_{q(t_1)}^{q(t_2)} V(t)\, dq$

Potential = Voltage

$$\underbrace{W}_{\text{kinetic energy}} = \underbrace{V}_{\text{height}} \cdot \underbrace{Q}_{\text{mass}}$$

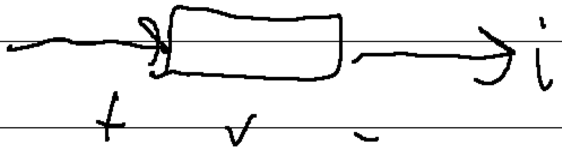
1.5 Power

- Electrical power p or $p(t)$
- Amount of energy per time absorbed (consumed) by a circuit element [watt = Joule/sec]
- $p(t) = \frac{dw(t)}{dt} = \frac{dw(t)}{dq}\frac{dq}{dt} = v(t)\,i(t)$

$$w = \int_{t_1}^{t_2} p(t)\,dt = \int_{t_1}^{t_2} v(t)\,i(t)\,dt$$

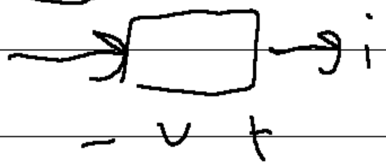
Sign conversion

· Absorption (consumption): passive sign convention



$p = vi$ (absorbed)

· Supply (generation): active sign convention

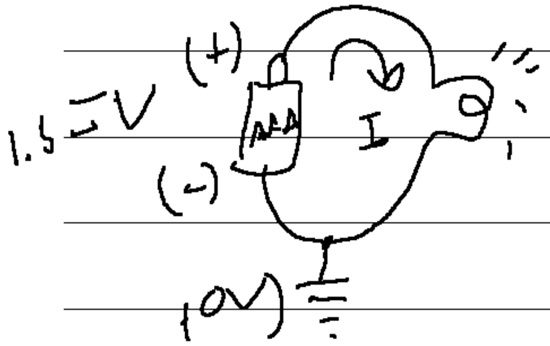


$p = -vi$ (absorbed)

$J/sec = W$

· Power balance

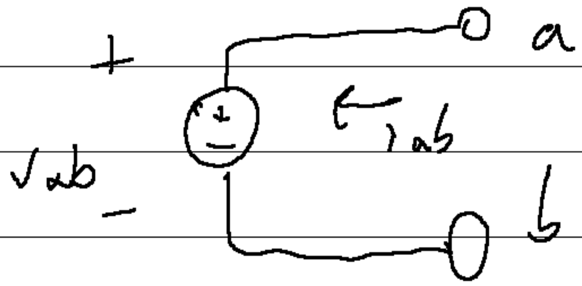
$\sum p(t)$ supplied $= \sum p(t)$ absorbed (in a network)



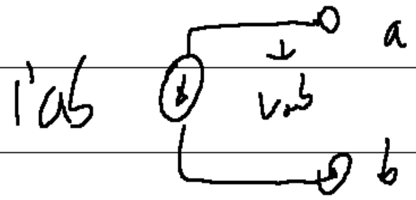
1.6 Circuits Elements (sources)

Independent Sources

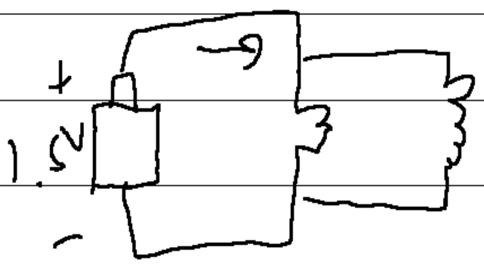
- (Ideal) independent voltage source: V_{ab} is independent of i_{ab}



- (Ideal) independent current source: i_{ab} is independent of V_{ab}



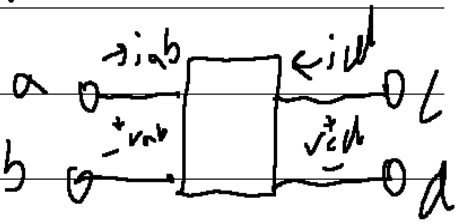
- Dependent Sources
- Output voltage (or current) dependent on input voltage



Dependent Sources

Output voltage (or current) dependent on input voltage (or current):

v_{cd} (or i_{cd}) = f(v_{ab} or i_{ab})



Coeff

Voltage-controlled voltage source: voltage gain [V/V]

—"— current source: transconductance [A/V]

Current-controlled voltage source: transresistance [V/A]

—"— current source: current gain [A/A]

Symbol

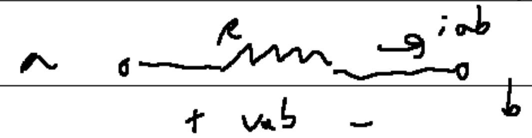
Dep. voltage source

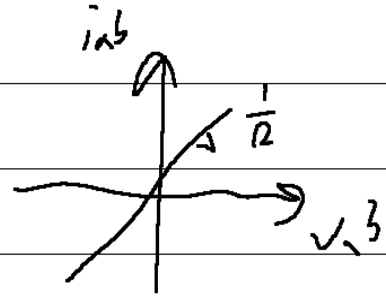
Dependent current source

Ch.2 Basic Laws

2.2 Ohm's Law

- Resistor : a device to dissipate electric energy into other forms of energy
- Resistance / conductance
  - Ohm's Law : $v_{ab} = R \cdot i_{ab}$

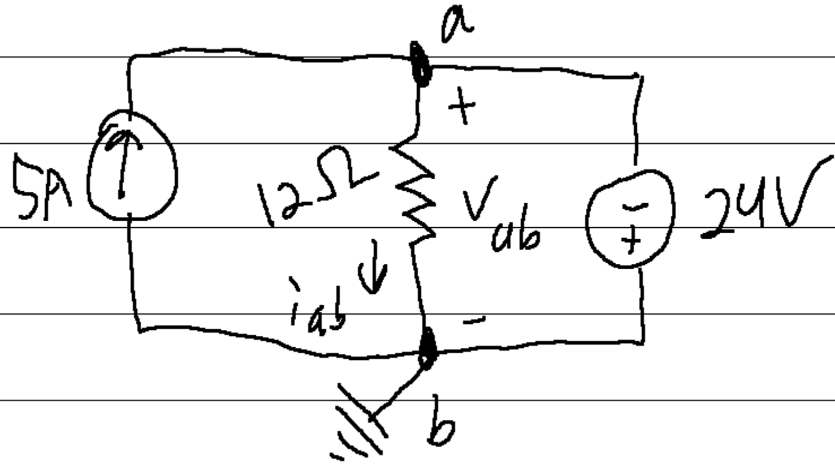




(≠ power sign convention)

- Resistance R [ohm (Ω) = volt/Ampere]

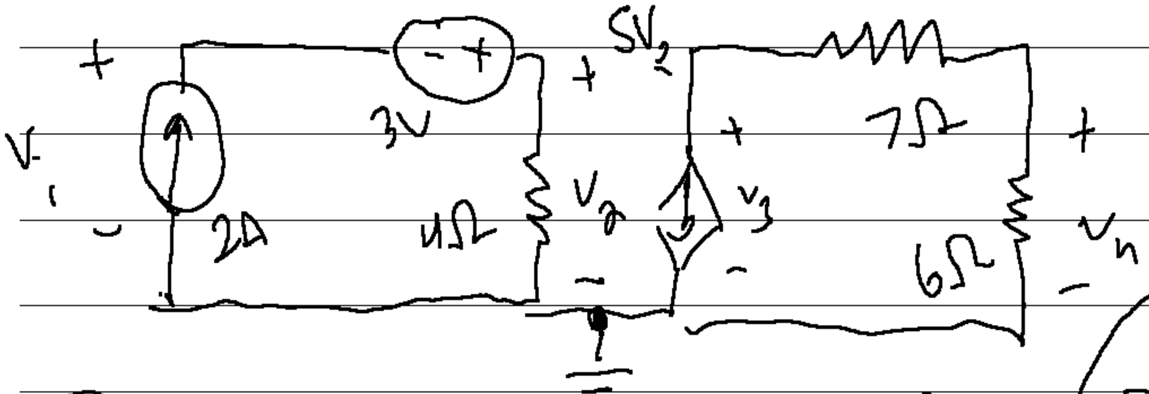
(conductance, G ($= 1/R$) [Siemens (S) or mho (℧)]

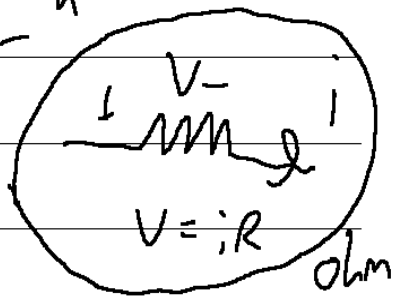


① $V_{ab} = -24(V)$

② $V_{ab} = R \cdot i_{ab} \rightarrow i_{ab} = V_{ab}/R = -24/12 = -2(A)$

③ $P_{12\Omega} = i_{ab} \cdot V_{ab} = -2 \times -24 = 48(W)$ (absorbed)





① (KVL) $-V_1 - 3 + V_2 = 0 \rightarrow V_1 = 5$ (V)

② $V_2 = 2 \times 4 = 8$ (V)

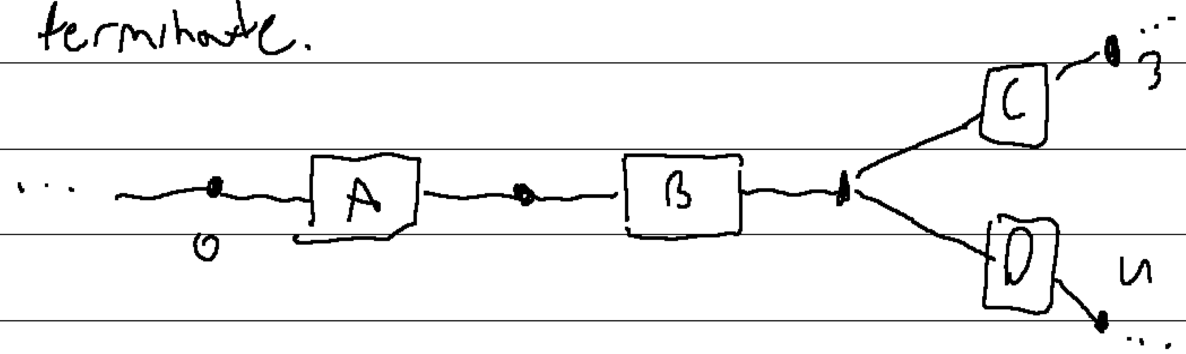
③ (KVL) $-V_3 + V_4 + (-5V_2 \times 7) = 0 \rightarrow V_3 = -$ (V)

④ $V_4 = 5 \times V_2 = -(5 \times 8) \times 6 = -240$ (V)

2.3 Nodes, Branches, Loops

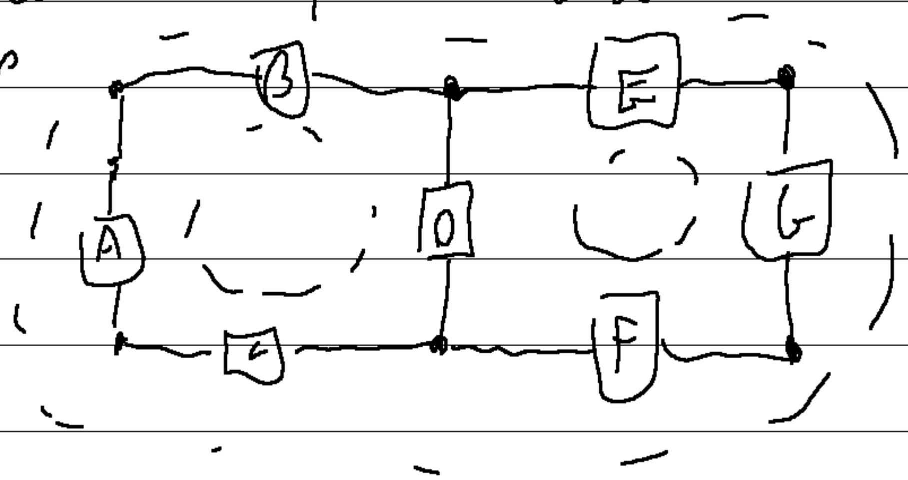
Nodes

1. Branch: a circuit element and attached line segment
2. Node: termination of one or more branches
3. Junction: a node where 3 or more branches terminate.



Loops

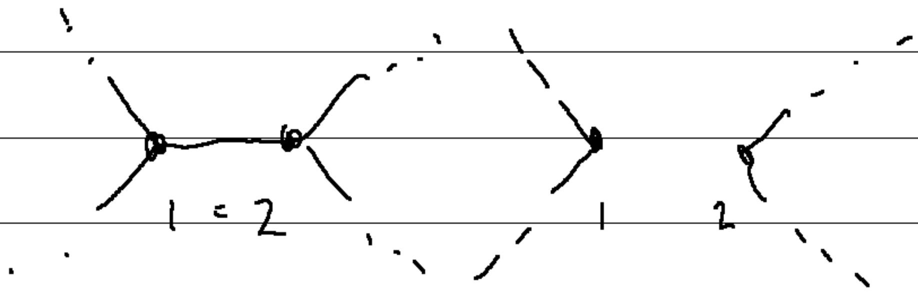
1. Loop: a closed path of branches
2. mesh: a loop that does not enclose another loop



Short / Open

1. Short circuit: multiple nodes are connected (resistance between nodes are zero)
2. open circuit: multiple nodes are isolated

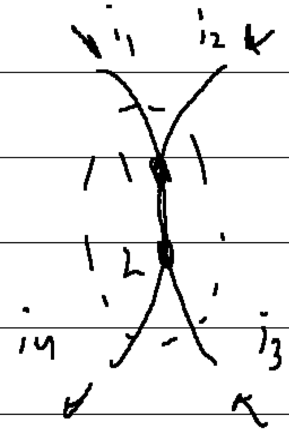
(resistance) between nodes are ∞)



2.4 Kirchoff's Law

Kirchoff's Current Law (KCL)

① sum of entering currents and sum of leaving currents over a closed surface is same

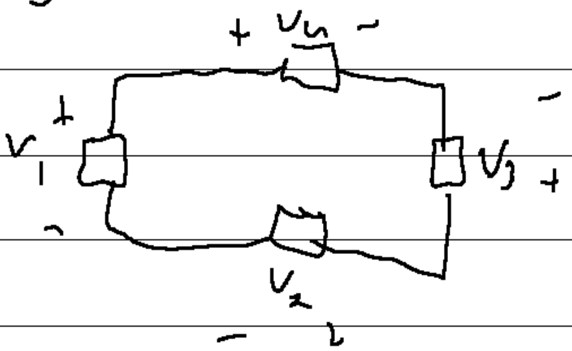


② $i_1 + i_2 + i_3 = i_4$

$$\rightarrow \begin{cases} i_1 + i_2 + i_3 - i_4 = 0 \\ -i_1 - i_2 - i_3 + i_4 = 0 \end{cases}$$

Kirchoff's Voltage Law (KVL)

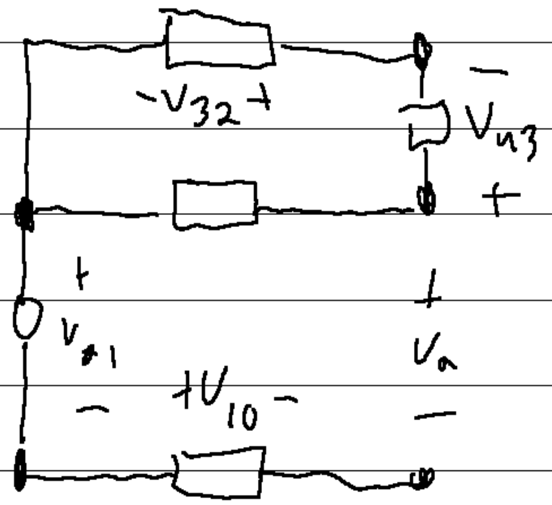
Sum of all voltage drops along a closed loop is zero.



$$\begin{cases} V_1 - V_2 + V_3 - V_4 = 0 \\ -V_1 + V_2 - V_3 + V_4 = 0 \end{cases}$$

Attendance Worksheet (continuing from Wednesday)

② KVL



$V_{10} = -2(V)$ $(V_1 - V_0 = -2)$

$V_{20} = 1$

$V_{30} = 18$

$V_{40} = 6$ $(V_4 - V_0 = 6)$

$V_a = ?$

① $V_a (= V_{40}) = 6(V)$

② (KVL) $-V_{10} - V_{21} - V_{32} - V_{43} + V_a = 0$

$V_{21} = V_2 - V_1 = V_{20} - V_{10} = 1 - (-2) = 3(V)$

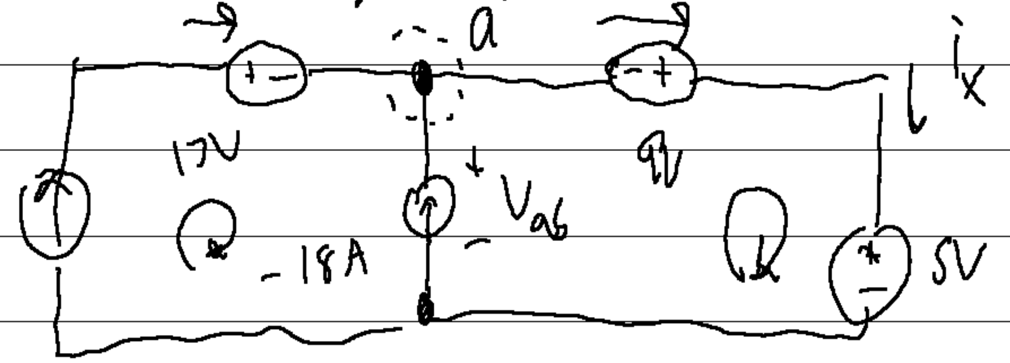
$V_{32} = V_{30} - V_{20} = 17$

$V_{43} = V_{40} - V_{30} = -12$

$\longrightarrow V_a = 6(V)$

Attn WO (Friday)
(Sources, Ohm, KVL/KCL)

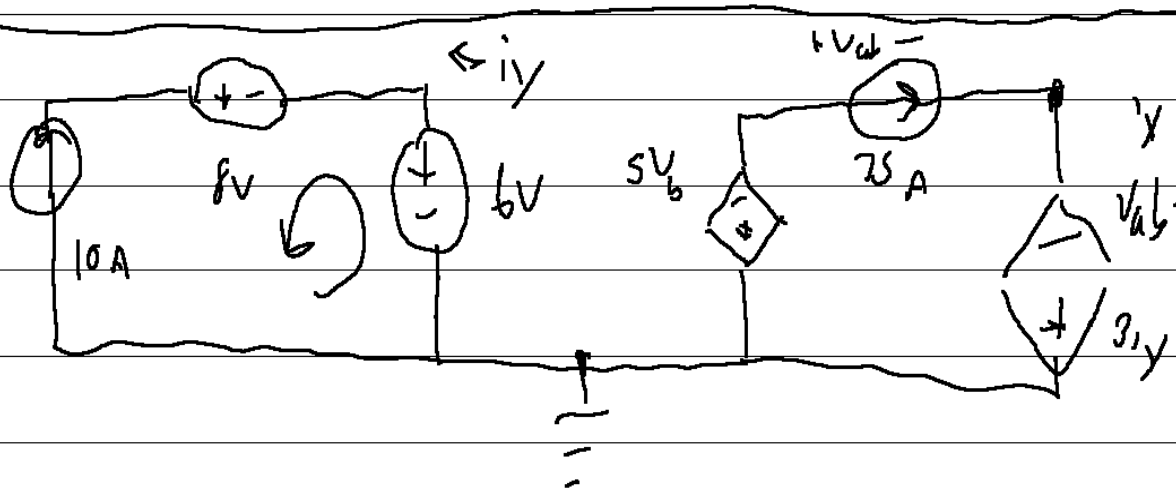
(1)



$i_y = ?$
$V_{ab} = ?$
P_{5V} (absorbed) = ?

① $-10 - (-18) + i_x = 0 \rightarrow i_x = -8\,(A)$ (KCL)
② $-V_{ab} - 9 + 5 = 0 \rightarrow V_{ab} = -4\,(V)$ (KVL)
③ $P_{5V} = 5 \cdot i_x = -40\,(W)$ (absorbed) (pass)

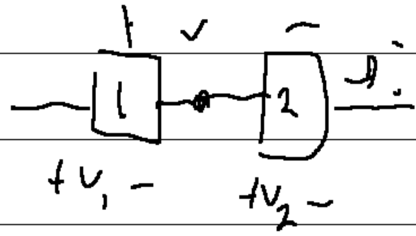
(2)



$i_y = ?$
$V_{ab} = ?$

① $-10 - i_y = 0 \rightarrow i_y = \underline{-10\,(A)}$ (KCL)
② $V_{ab} - 3i_y + 5V_y = 0 \rightarrow V_{ab} =$
$-V_y + 8 + 6 = 0 \rightarrow V_y = 14$ (KVL)

2.5 Series resistors and Voltage Division



(1) "single-loop" (series ckt)

① Series connection:

Same current through each component

Different voltage drop across each element

② $V = V_1 + V_2 = iR_1 + iR_2 = iR_s$ ($R_s = R_1 + R_2$; equiv resistance)

(2) Series elements

① Resistance: $R_S = R_1 + R_2 + \dots + R_N$

② Conductance: $G_S = \dfrac{1}{1/G_1 + \frac{1}{G_2} + \dots + 1/G_N}$

③ Inductance: $L_S = L_1 + L_2 + \dots + L_N$

④ Capacitance: $C_S = \dfrac{1}{1/C_1 + \frac{1}{C_2} + \dots + 1/C_N}$

(3) Voltage Division $\rightarrow V_s \dfrac{R_2}{R_1 + R_2}$

voltage is divided proportionally to each resistance

$$V_K = \left(\frac{R_K}{R_1 + R_2 + \dots + R_K + \dots R_N}\right) V = \frac{R_K}{R_S} V$$

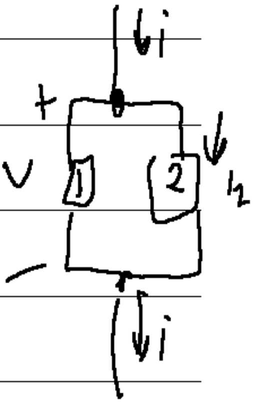
2.6 Parallel Resistors and Current Division

(1) "Two-node" parallel (16)

① Parallel connection:

Same voltage across each element,

Different current through each element



② $i = i_1 + i_2 = \frac{v}{R_1} + \frac{v}{R_2} = \frac{v}{R_p}$

$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2}$; equiv resistance)

(2) Parallel elements

① Resistance: $R_p = \frac{1}{1/R_1 + 1/R_2 + \cdots 1/R_N}$

② Conductance: $G_p = G_1 + G_2 + \dots + G_N$

③ Inductance: $L_p \quad \frac{1}{1/L_1 + 1/L_2 + \cdots 1/L_N}$

④ Capacitance $= C_p + C_1 + C_2 + \dots + C_N$

(3) Current Division

③ Current is divided inverse-proportionally to each resistance.

Test #1 Practice Samples

1

(1) $i_1 = 90/10 = 9A$ [Ohm's Law]

(2) $-90 + 25 \cdot i_2 + 30 + 15 i_2 = 0$

$i_2 = 1.5\ A$

(3) $V_{ab} = 25 i_2 + 30$

$\to V_{ab} = 67.5\ (V)$

2

(1) $V_{ab} = V_{oc} = 60\ (V)$

(2) $i_b = 60/10 = 1\ (A)$

(3) $-i_a + i_b + i_c = 0$ $\quad (i_c = 4 i_b)$

$\to i_a = 5\ (A)$

(4) $i_c = 4 \cdot i_b = 4\ (A)$

(5) $P_{60V} = -60 \cdot i_a = -300$ [W] absorbed (active)

3

(1) $-24 + 16 i_1 + 4 v_1 = 0$

(2) $v_1 = -0.5 i_1 + 4$

$\to v_1 = -6\ (V)$

$i_1 = 3\ A$

(1) [Total KVL over the whole loop]

① $-100 + 2i_1 + 8i_1 + 5i_1 + 9i_1 - 20 = 0$

$\rightarrow i_1 = 5(A)$

② $-V_{ab} + 10 \times 0 - 30 + 5i_1 - 20 + 9i_1 = 0$

$\rightarrow V_{ab} = 20(V)$

④

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① $V_{ab} = 6 \cdot 10 = 60(V)$

② $V_{bc} = 4 \cdot V_{ab} = 240(V)$

③ $V_{ac} = V_{ab} + V_{bc} = 300(V)$

④ $V_{oc} = V_{bc} = 240(V)$

⑤ $P_{6A} = -6 \times 300 = 1,800(W)$ absorber

(origin)

⑤

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① $i: -12 + 5i + 10i + 5i + 20i + 2 - 15 = 0$

$i = 0.5(A)$

② $V_1 = 5i = 2.5(V)$

$V_2 = 20i = 10(V)$

(or $V_2 = \left(\frac{20}{5+10+5+20}\right) 20 = 10$

⑨

①' $-12 - 5i_a - 10i_a - 15i_a - 20i_a + 2 - 15 = 0$

$\rightarrow i_a = -0.5(A)$

(6)

(1) $54+18=72$

$36 // 72 = 24$

$(16+24 = 40)$

$V_2 = \left(\frac{24}{16+24}\right) 60 = \underline{48(V)}$

(2) $V_1 = \left(\frac{18}{54+18}\right) V_2 = 12(V)$

option #2

$$\begin{cases} -80+16i+36i_2=0 & (V_2 = 36i_2) \\ -36i_2+54i_3+18i_3 & (V_1 = 18i_3) \\ -60+16i_1+54i_3+18i_3 = 0 \end{cases}$$

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(7)

(1) $i_x = \left(\frac{1/9}{1/9+1/18}\right) 12 = 8(A)$

(2) $-V_x = 3i_x = +24V$

(3) $-V + 12\cdot 4 - V_x + 6i_x - 5V_x = 0$

$\rightarrow V = 240\ (V)$

(4) $P_{12A} = -(12\cdot 240) = -2{,}880\ (W)$ absorbed

(active)

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(8)

(1) $V = 36\ (V)$

(2) $6//3 = 2$, $R_{ab} = 4+2 = 6$

$i_y = 36/6 = 6(A)$

(3) $i_x = \left(\frac{1/6}{1/6+1/3}\right) i_y = 2(A)$

(4) $-i + i_y - 2i_x = 0$

$\rightarrow i = 2(A)$

Ch.3 Method of analysis

3.2 Nodal analysis

(1) Node Voltage (nV)

① Ref node: ref for all other nodes (node #0 : $v_0 = 0$)

② Node Voltage: $V_{k0} = V_k - V_0 = V_k$ (Voltage of node k)

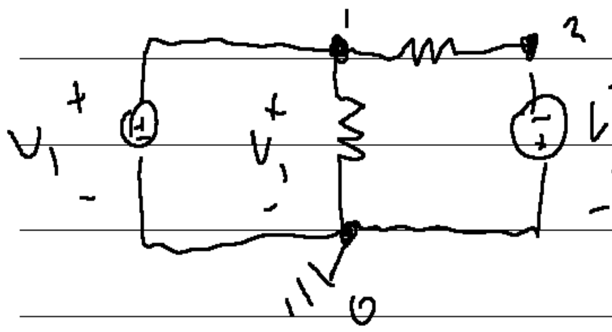
③ Node Voltage eqs: a set of eqs with NVs as variables

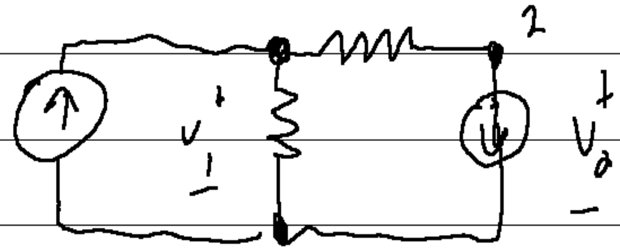
$$(V_1, V_2, \dots V_k)$$

(2) NV eqs

① One KVL eq: for each node connected to ref node by a path of voltage sources

② One KCL eq: for each node NOT connected to voltage source





Test #1 Solutions

1. a) 0.02 (A)
   b) −8 V
   c) −0.04 W (absorbed)
   d) −16 W (absorbed)

2. a) 25 A
   b) 250 V?
   c) 150 V
   d) 240 V

3. a) 2A
   b) 24V
   c) 20 V
   d) − 24 W absorbed

4. a) 3A
   b) −30V
   c) −7V
   d) −2A

3.3 Node analysis with Voltage Sources

(1) Supernode

① A group of nodes connected to each other by voltage source(s), but NOT directly to ref node by voltage sources(s).

② One KCL eq: for a surface enclosing a supernode

③ KVL eq(s): for other node(s) within supernode in terms of "principal node".

(2) Network analysis by NV

(1) procedure

① Identify ref node and supernode(s).

② # of NV eqs = # of total nodes −1.

③ Build a set of NV eqs.

HW Notes

#1) No need for V_x

#2) Supernode 2, 3

* Just complete matrix

3.4 Mesh Currents

(1) Mesh current (MC)

① Mesh current: i_x at the outer path of xth mesh (i_j, i_k)

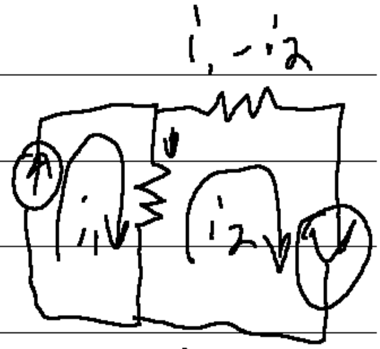
② Branch current: current at a shared branch ($i_j - i_k$)

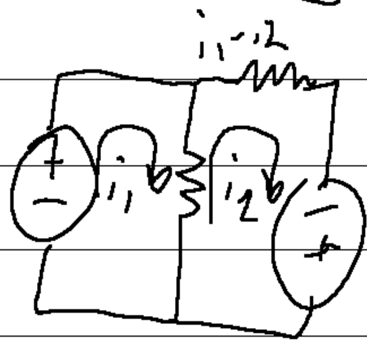
③ MC eqs: a set of eqs with MCs as variables
($i_1, i_2, \cdots, i_k$)

(2) MC eqs

① One KCL eq: for each branches with current source(s)

② One KVL eq: for each loop without current source(s)





3.5 Mesh analysis with Current Sources

(1) Supermesh

① A group of meshes showing a current source with other meshes: none of which contains current source(s) in outer loop

② One KVL eq: for each mesh

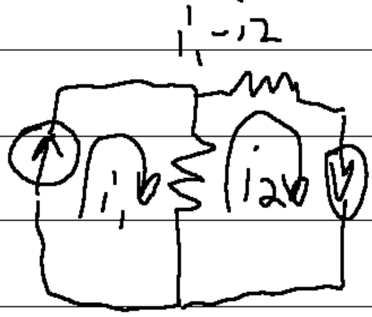
③ KCL eq(s): for branches within supermesh in terms of "principal mesh".

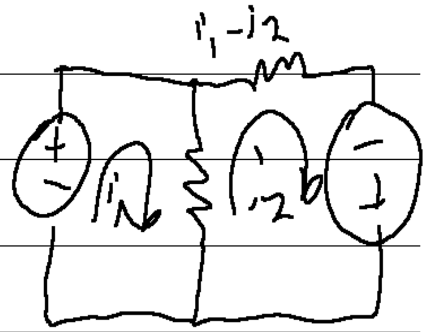
(2) Network analysis procedure by MC

① Identify each mesh and supermesh(es)

② # of MC eqs = # of mesh.

③ Build MC eqs.





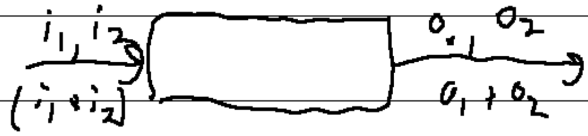
Ch. 4 Circuit theorems

4.2 Linearity

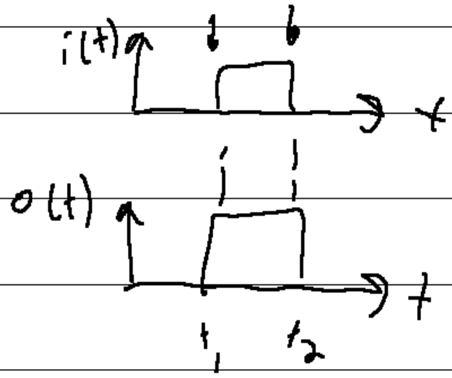
(1) Linear network

① Linearity:

② Superposition:



③ Time invariance:



4.3 Superposition

① Indep. Voltage Source → short-ckt (V=0)
Indep. current → open-ckt (i=0)
Dep. source → no change

② Response to each indep. source at a time
→ total response = sum of each response

4.4 Source Transformation

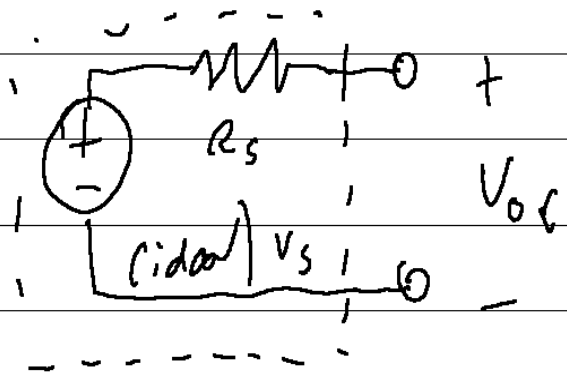
(1) Sources

① Practical Voltage Source:

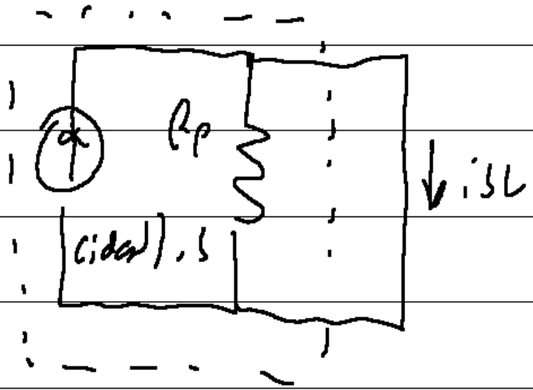
an ideal voltage source with a series resistance

② Practical current source:

an ideal current source with a parallel resistance



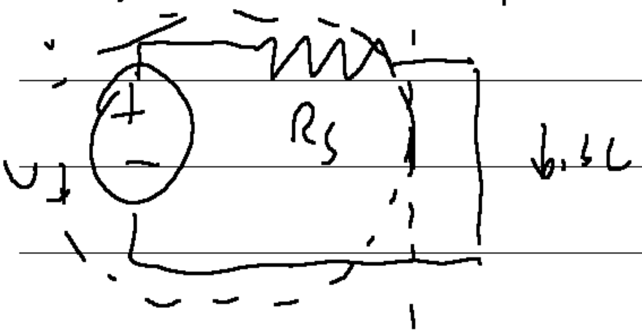
Practical



Practical

(V_{oc}: open-ckt voltage, i_{sc} is short-ckt current)

(2) ideal vs. practical (with extreme loads)

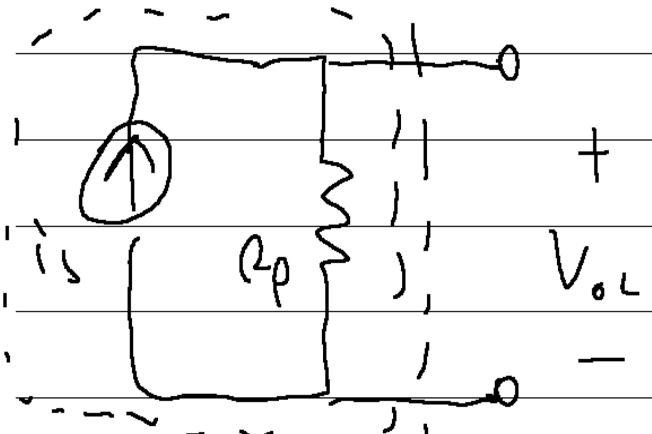


$v_s = i_{sc} \times R_s$

(finite = finite × finite (practical)

(finite = $\infty \times 0$ (ideal)

(loop resistance = 0)



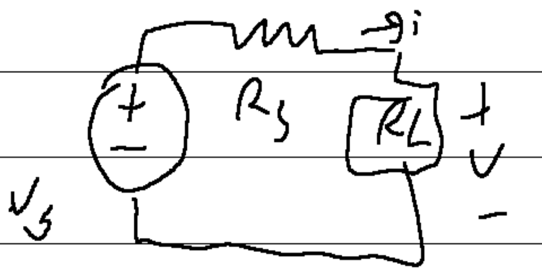
(load resistance = ∞)

$i_s = V_{oc}/R_p$

finite = finite/finite (practical)

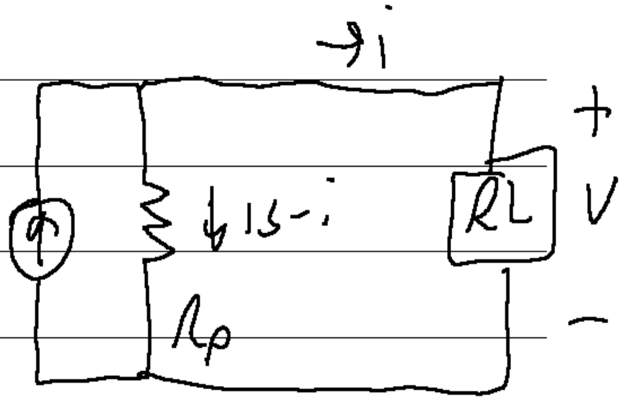
finite = ∞/∞ (ideal)

(3) Source Transformation



$(V_s \neq V)$

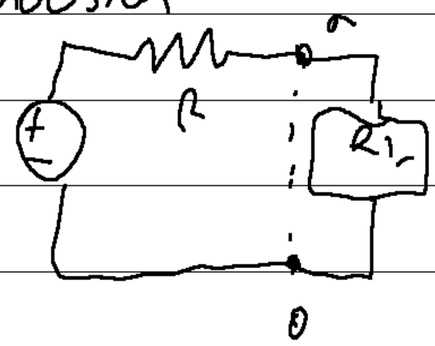
⟺



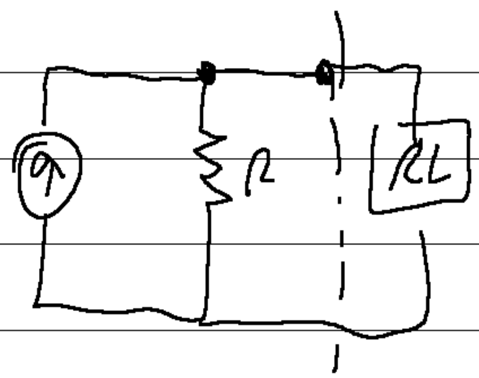
$(i_s \neq i)$

① Voltage source: $V = V_s - R_s i$
Current source: $V = R_p(i_s - i) = R_p i_s - R_p i$
$\rightarrow V_s = i_s R$ (if $R_s = R_p = R$)

② Conversion:



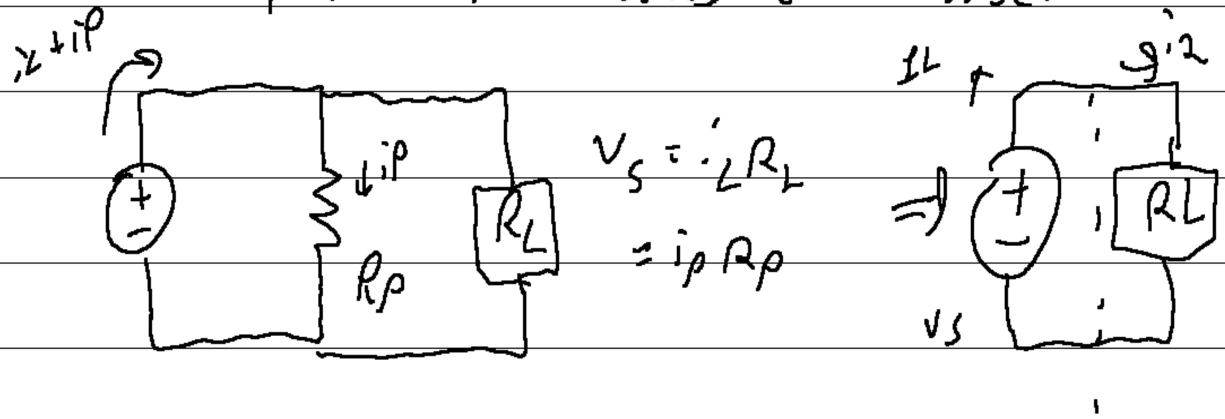
⟺



(4) Effect of R_S, R_P on ideal sources

(1) Ideal voltage source with R_p:

equiv to ideal voltage source itself

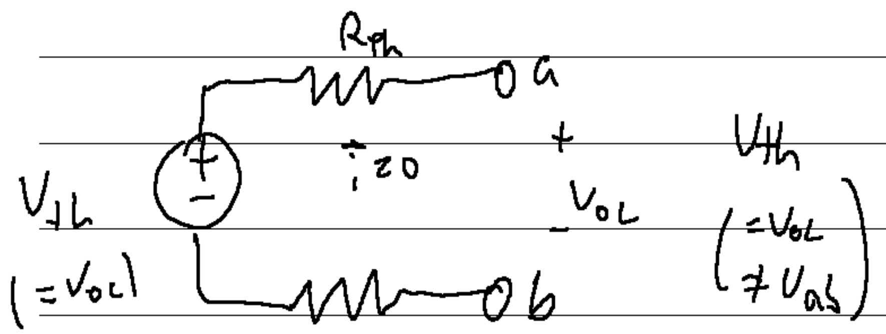


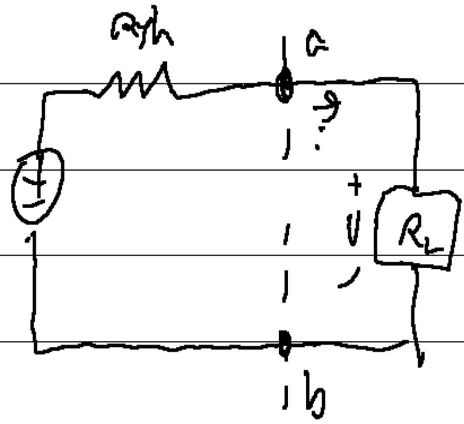
$v_S = i_L R_L$
$= i_P R_P$

$\Rightarrow$

$V_S = i_L R_L$

4.5 Thévenin Theorem





(1) Components

① Thévenin voltage source, V_{th}:
open-ckt ($i=0$), equiv voltage source

② Thevenin resistor, R_{th}:
equiv. resistor with all indep. sources are deactivated; can be found by replacing load with an imaginary source.

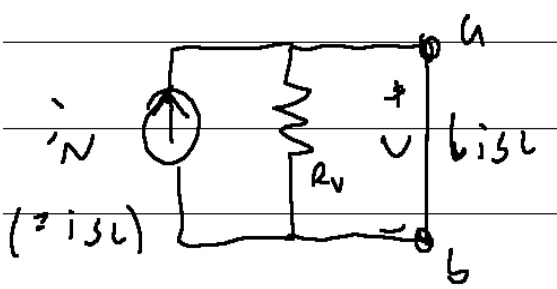
(2) Thévenin equiv. ckt

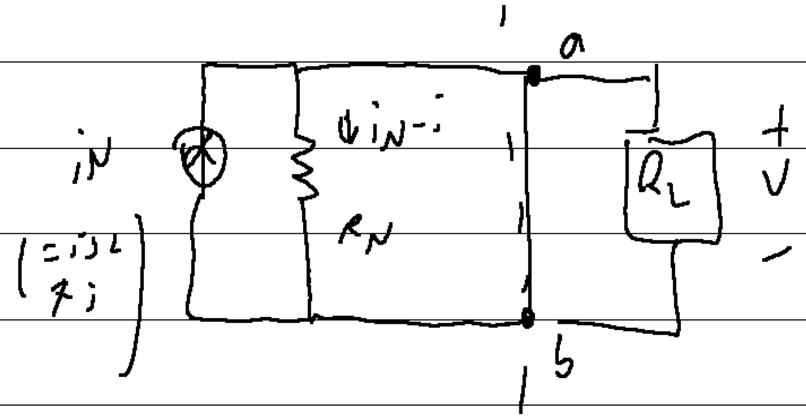
① Equiv practical voltage source seen from open terminals

② terminal characteristic (with R_L): $V_{th} = V + R_{th} i$

③ V_{th} ($=V_{oc}$) and R_{th} are indep. from R_L:
once R_L is attached, not open-ckt anymore
($i \neq 0$, $V_{th} \neq V$)

4.6 Norton Theorem





(1) Components

① Norton current source, i_N:

closed-ckt ($v=0$), equiv current source

② Norton resistor, R_N:

equiv. resistor with all indep. sources deactivated; can be found by replacing load with a imaginary source.

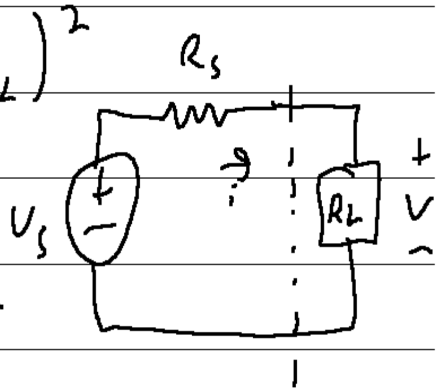
(3) Source transformation (when $R_{Th} = R_N = R$)

$V_{th} = i_N \cdot R$ ($V_{oc} = i_{sc} \cdot R$)

4.8 Max Power Transfer

(1) Theorem: $P_L = R_L \cdot i^2 = R_L\left(\frac{V_S}{R_S+R_L}\right)^2$

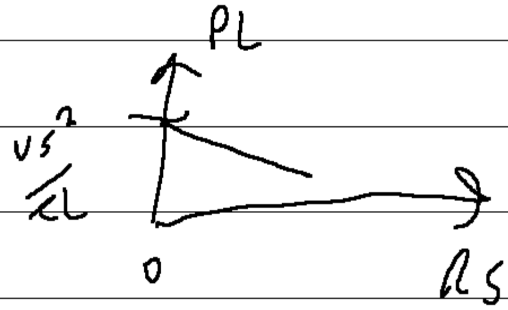
$$\left(P = Vi = i^2R = V^2/R\right)$$

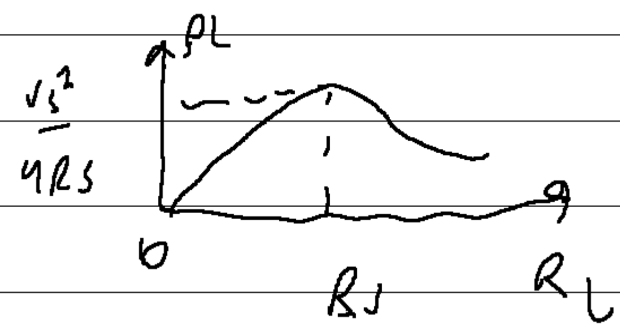


(2) Max power delivery to load R_L:

$R_S = 0$: $P_L = \frac{V_S^2}{R_L}$ (for ideal source)

$R_L = R_S$: $P_L = V_S^2/4R_S$ (for practical source)





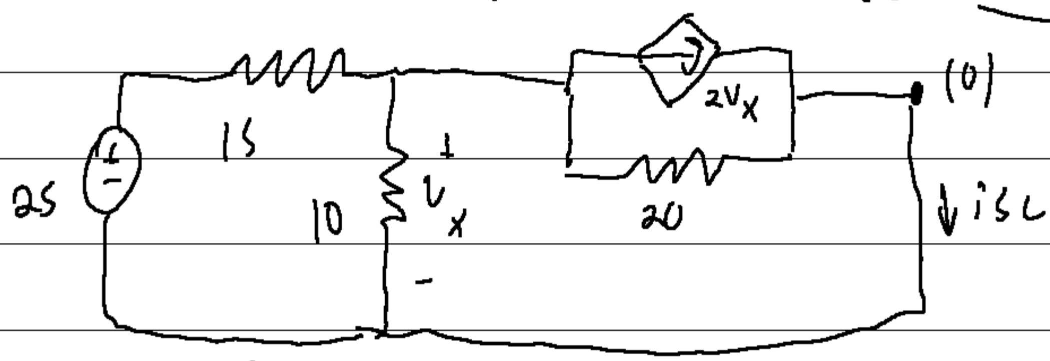
Test #2 Sample Problems

9) ① V_{th}: KCL

$$\frac{(V_x - 25)}{15} + \frac{V_x}{10} + 0 = 0 \rightarrow V_x = 10$$

(KCL) $-2V_x + (V_{OC} - V_x)/20 = 0 \rightarrow V_{OC} = \underline{410\,(V)}$

② i_N:



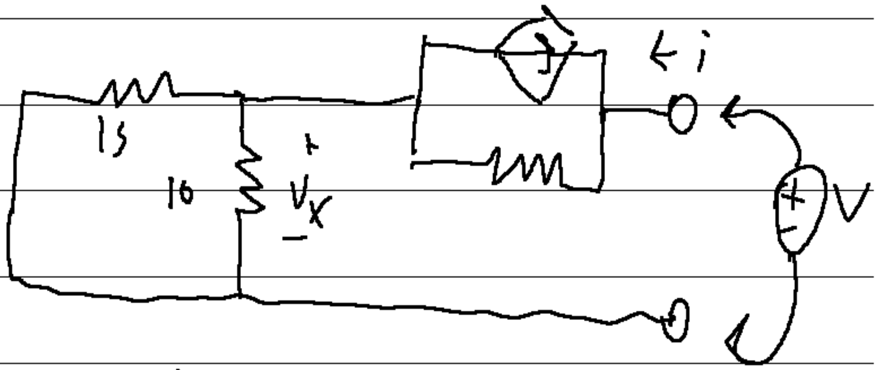
(KCL) $\frac{(v_x - 25)}{15} + \frac{v_x}{10} + i_{SC} = 0$

(KVL) $v_x = (i_{SC} - 2V_x)\,20$

ohm's Law

$\rightarrow i_{SC} = \frac{410}{266}\,(A)$

③ R_N ($= R_{th}$):



(KVL) $-V_x + V - 20(2V_x - i) = 0$

$V_x = (6 /\!/ 15)\, i$

$\rightarrow R_N \,(= R_{th}) = V/i = \underline{266\,(\Omega)}$

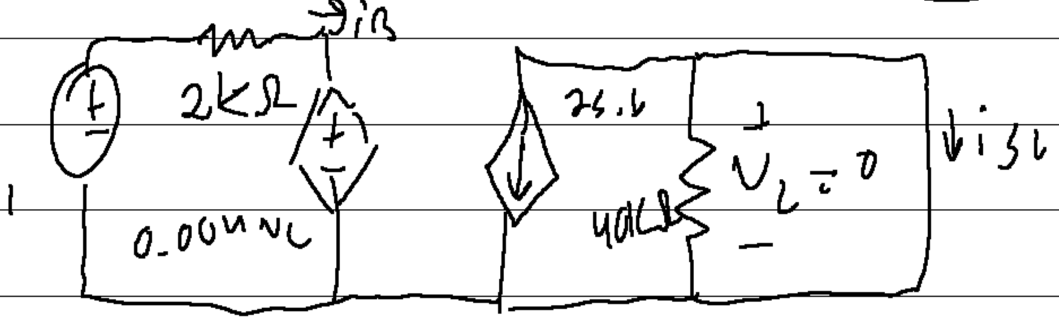
④ $P_L = V_{th}^2 / 4R_{th}$ (when $R_L = R_{Th}$)

$= \frac{(416)^2}{4 \times 266}$ [W]

10) ① V_{th}: $-1 + 2k \cdot i_B + 0.004 V_L = 0$

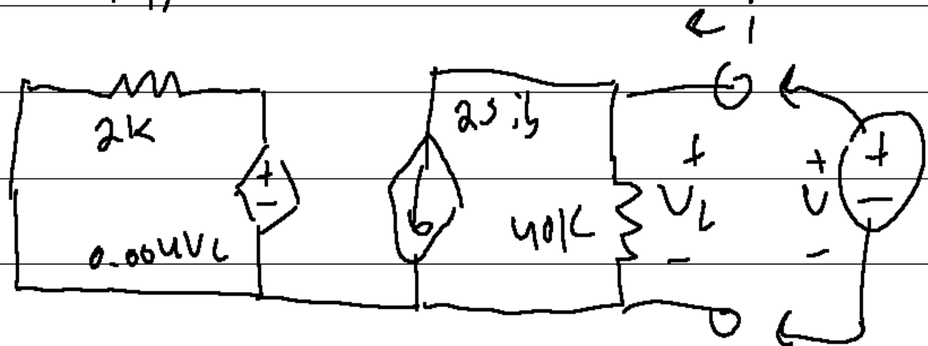
$V_L = -25 i_b \times 40K = -625 = V_{OL}$

② i_N:



$i_b = \frac{1}{2K} = 0.5$(mA), $i_{SC} = -25 i_b = -12.5$(mA)

③ $R_N (= R_{th})$:



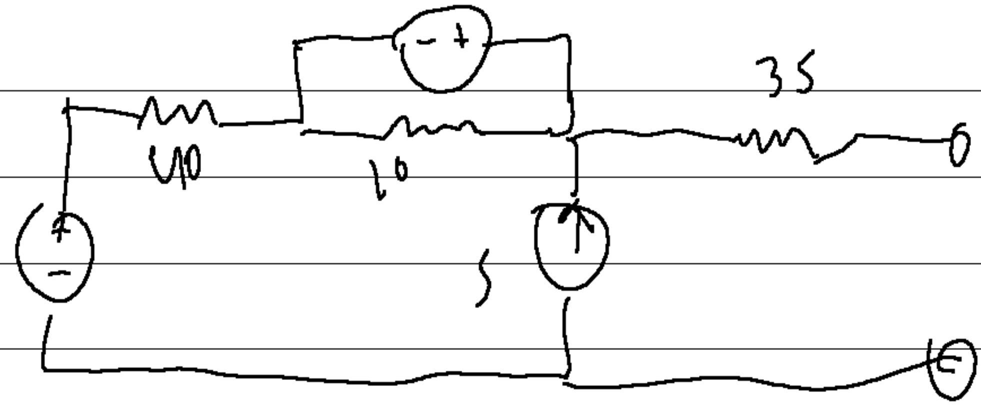
$25 i_b + \frac{v}{40K} - i = 0$ ($v = v_L$)

$0.004 V_L + 2k \cdot i_B = 0$ → $R_N = v/i = 50$ [kΩ]

④ $P_L = \frac{V_{th}^2}{4R_{th}} = \frac{(-625)^2}{4 \times 50k} =$ ___ [W]

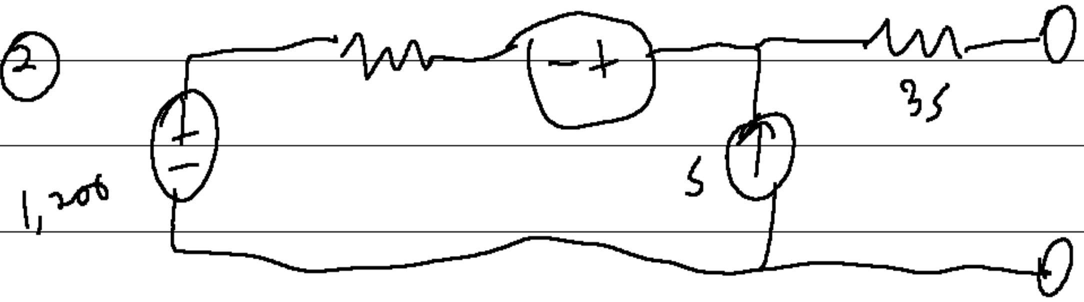
(8)

(1)

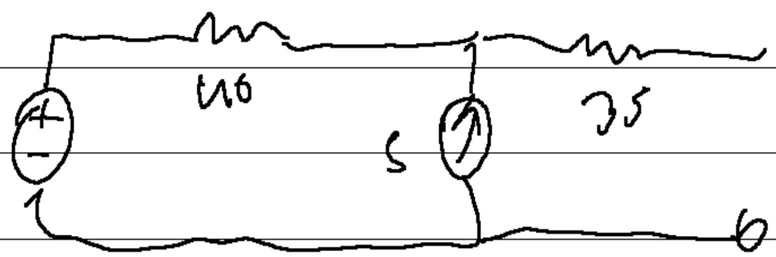


$V_s = 30 \times 40 = 1,200$

(2)

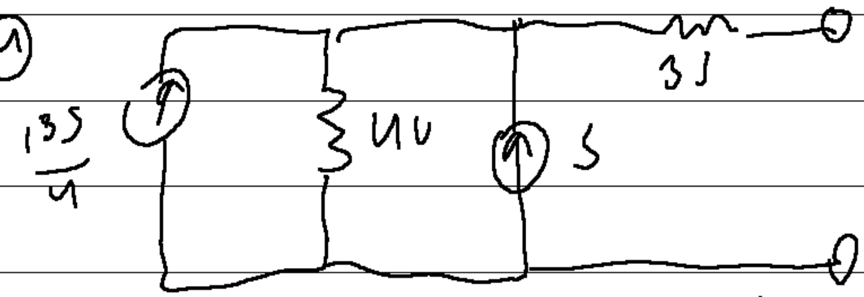


(3)



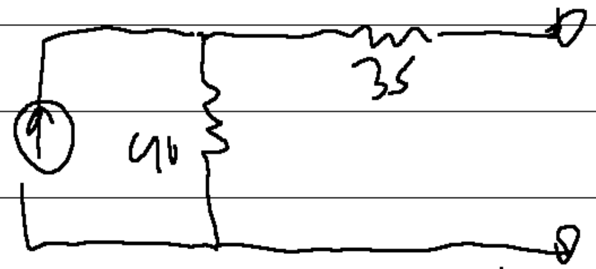
$V_s = 1,200 + 150 = 1,350$

(4)



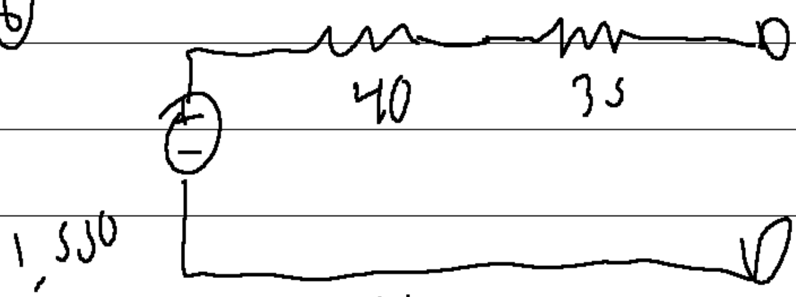
$i_s = 1,350/40 = 135/4$

(5)



$i_s = \frac{135}{4} + 5 = \frac{155}{4}$

(6)

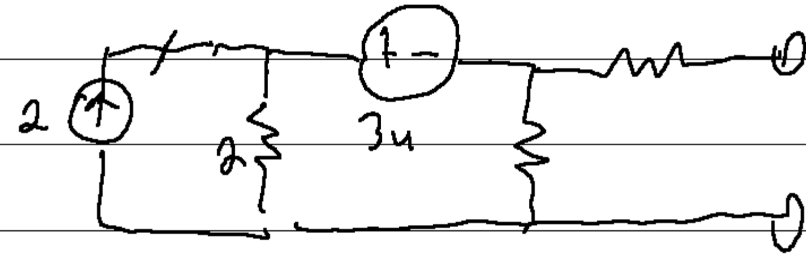


$$v_s = \frac{155}{4} \times 40 = 1,550$$

$$R_s = 40 + 35 = 75$$

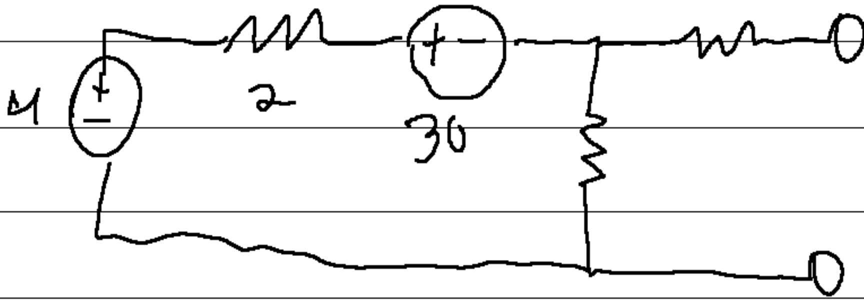
Test

(7) (1)



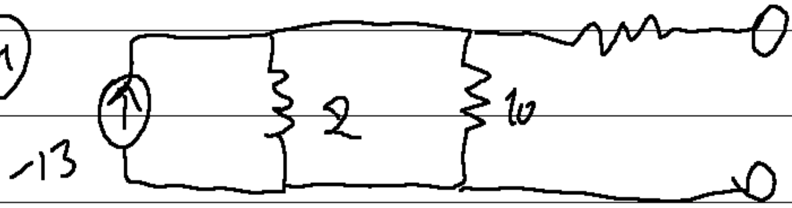
(ignore R_s)

(2)

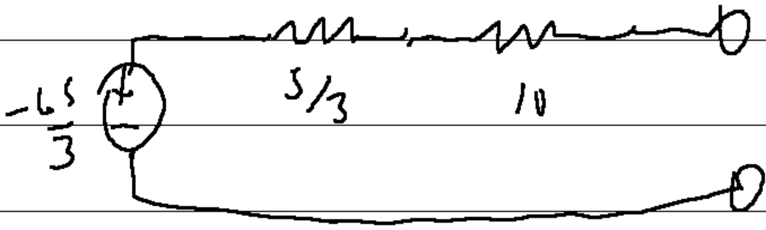


(3)

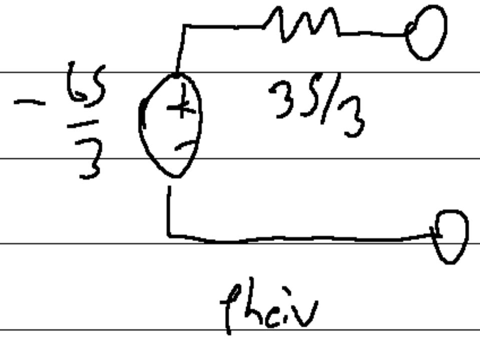
(4)



(5)



5)



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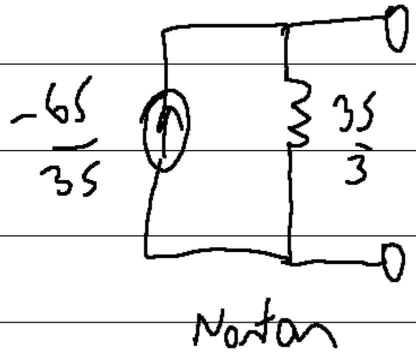
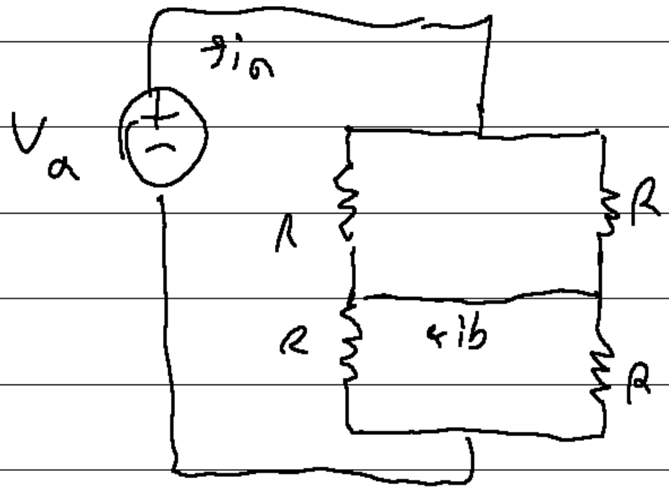
5)'



6)

(1)

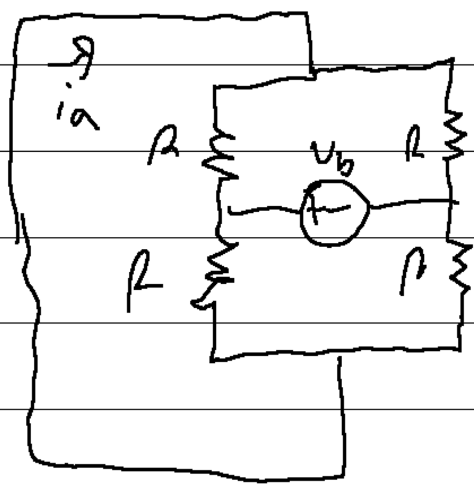


$i_a = \frac{V_a}{(2R // 2R)} = \frac{V_a}{R}$

$i_b = 0$

$i_c = -\frac{V_a}{2R}$

(2)



(3) $i_a = V_a/R_1$ $\quad i_b = V_b/R_1$ $\quad i_c = \frac{V_b - V_a}{R}$

⑤ ① V_a: KVL

$$\begin{cases} -V_a + 40 i_a + 1(i_a + 4 i_a) = 0 \rightarrow i_a = \dfrac{V_a}{50} \\ \text{KVL} \\ 40 i_b + 1(i_a + 9 i_a) = 0 \rightarrow i_b = 0 \end{cases}$$

$$\rightarrow V_0 = 20(9 i_a + 9 i_b) = -\frac{18}{5} V_a$$

② V_b:

③ $V_0 = -\frac{18}{5} V_a - \frac{18}{5} V_b$

④ ① Supermesh 1+2+3 (KVL)

$$-27 - V_x + 2 i_y + i_3 \left(\frac{1}{6}\right) = 0$$

$$\left(i_y = i_1,\ V_x = -i_1/5\right)$$

② Supermesh 1+2 (KCL)

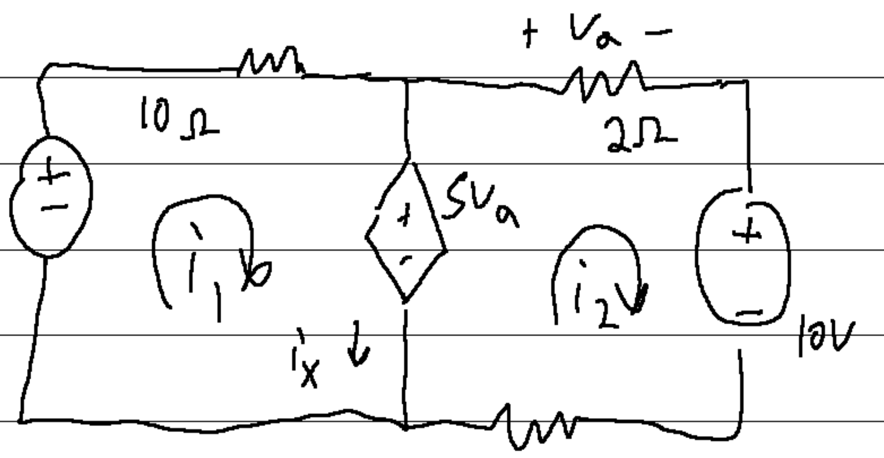
$$i_1 - i_2 = -10$$

③ Supermesh 2+3 (KCL)

$$i_2 - i_3 = -2V_x$$

$$\begin{bmatrix} 11/5 & 0 & 1/6 \\ -1 & 1 & 0 \\ -2/5 & 1 & -1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 27 \\ 10 \\ 0 \end{bmatrix}$$

③



① mesh 1 (KVL)

$-20 + 10i_1 + 5V_a = 0 \quad (V_a = 2i_2)$

② mesh 2 (KVL)

$10 + 10i_2 - 5V_a + V_a = 0$

$i_1 = 7, \quad i_2 = -5\,(A)$

$i_x = i_1 - i_2 = 12\,(A)$

①

① node 1 (KCL)

$20 + \underbrace{V_1 \cdot 10} + 5i_a = 0 \quad (i_a = 2V_2)$

② node 2 (KCL)

$(-5i_a + i_a) + 2V_2$

$\rightarrow V_1 = -7, \quad V_2 = 5\,(V)$

③ $i_x = 5i_a - i_a = 40\,(A)$

②

① Supernode 1+2+3 (KCL):

$-27 + i_x + 2V_y + V_3/6$

$(V_y = V_1,\ i_x = V_1/5)$

② Supernode 1+2 (KVL):

$V_1 - V_2 = -10$

③ Supernode 2+3 (KVL)

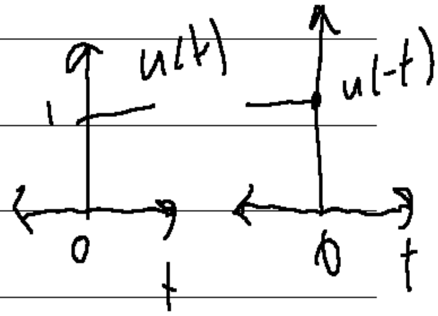
$V_2 - V_3 = -2i_x$

$$\rightarrow \begin{bmatrix} 11/5 & 0 & 1/6 \\ -1 & 1 & 0 \\ -2/5 & -1 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 27 \\ 10 \\ 0 \end{bmatrix}$$

7.4 Singularity functions

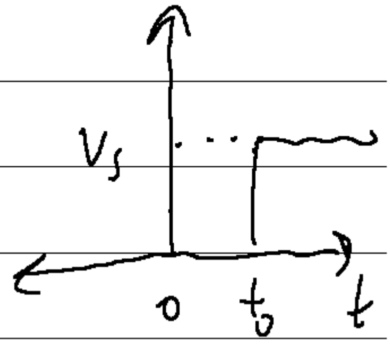
(1) Unit step ft. $u(t)$

① Definition $u(t) \begin{cases} = 0 & (t<0) \\ = 1 & (t>1) \end{cases}$



② Shifted/multiplied $u(t)$:

$V_s \; u(t-t_0)$



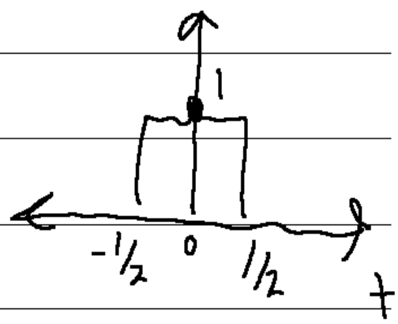
③ Unit pulse ft, $\pi(t)$:

$\pi(t) = u(t+\frac{1}{2}) - u(t-\frac{1}{2})$

(2) Unit ramp ft, $r(t)$

① Definition:

$r(t) \begin{cases} = 0 & (t \leq 0) \\ = t & (t \geq 0) \end{cases}$

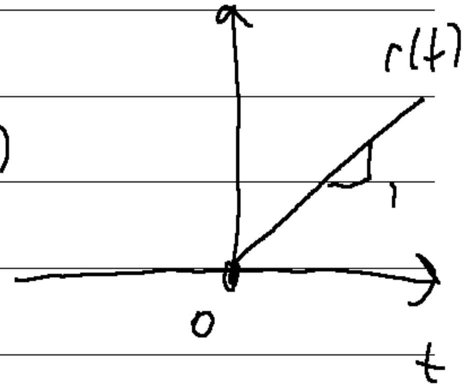


$= t \cdot u(t) = t \; (t \geq 0)$

② Shifted Multiplied $r(t)$

$V_s \cdot r(t-t_0) = V_s (t-t_0) \cdot u(t-t_0)$

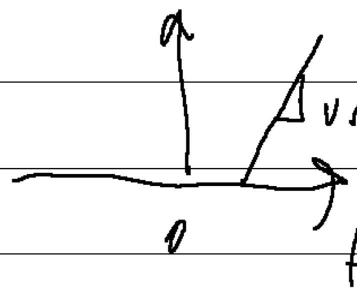
$= V_s (t-t_0) \; (t \geq 0)$



③ integ/diff

$\frac{dr(t)}{dt} = u(t)$

$\int_{-\infty}^{t} u(\lambda) d\lambda = t \cdot u(t) = r(t)$



Circuits Test #2 Recap

1a) Supernode 1+2

① KCL: $\frac{V_1}{10} + \frac{(V_1 - V_4)}{2} + \frac{V_2}{20} + \frac{-V_x}{2} = 0$ $(-V_x = V_2 - V_3)$

$\rightarrow 12V_1 + 11V_2 - 10V_3 - 10V_4 = 0$

② KVL: $V_1 - V_2 = 10$

$\left(\frac{12}{20} \quad \frac{11}{20} \quad \frac{-10}{20} \quad \frac{-10}{20}\right)$

$$\begin{bmatrix} 12 & 11 & -10 & -10 \\ 1 & -1 & 0 & 0 \\ n/a & n/a & n/a & n/a \\ n/a & n/a & n/a & n/a \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ n/a \\ n/a \end{bmatrix}$$

1b)

① KVL: $20(i_2 - i_1) - V_x + 40 + 100 = 0$, $-V_x = 2(i_2 - i_4)$

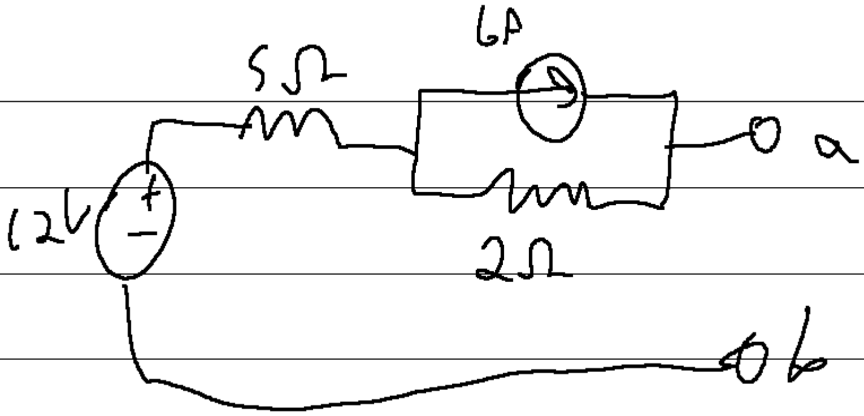
$\rightarrow -20i_1 + 22i_2 - 2i_4 = -140$

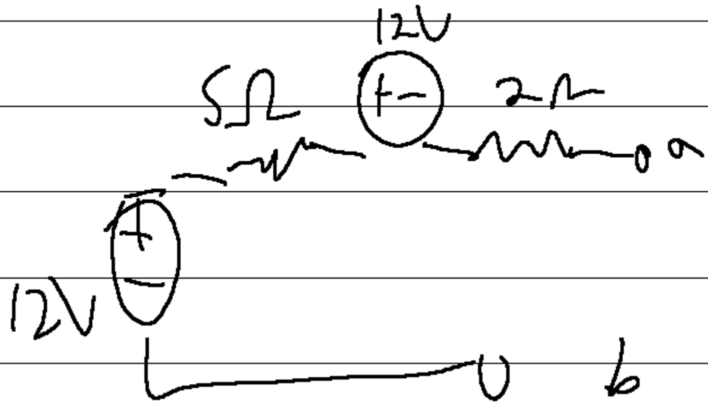
② KCL: $i_2 - i_3 = -2V_x$ $V_x = 2(i_4 - i_2)$

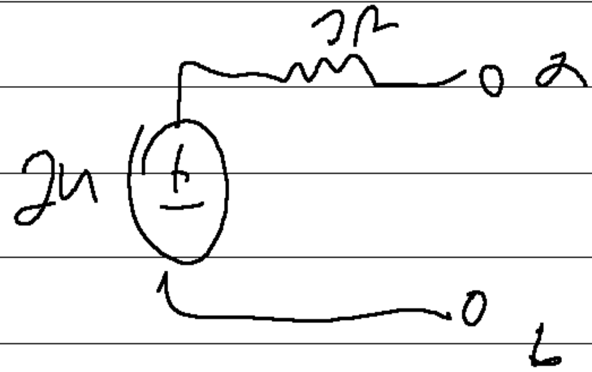
$\rightarrow -3i_2 - i_3 + 4i_4 = 0$

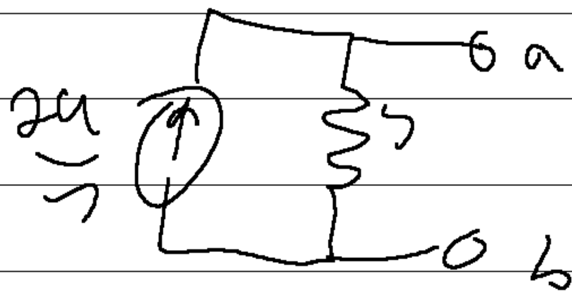
$$\begin{bmatrix} -20 & 22 & 0 & -2 \\ 0 & -3 & -1 & 4 \\ n/a & n/a & n/a & n/a \\ n/a & n/a & n/a & n/a \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} -140 \\ 0 \\ n/a \\ n/a \end{bmatrix}$$

2)

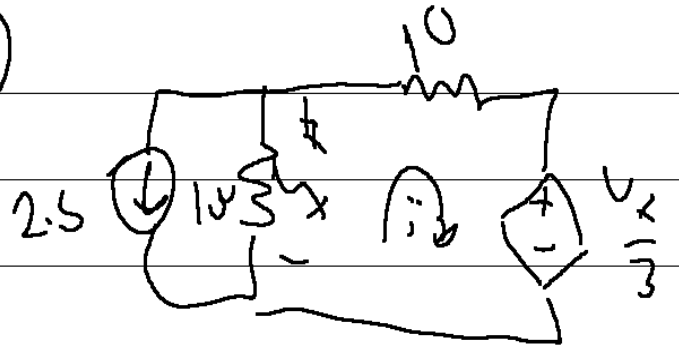








3a)



(KCL) $2.5 + \frac{V_x}{10} + \frac{\left(V_x - \frac{V_x}{3}\right)}{10} = 0$

$\rightarrow V_x = -15\,(V)$

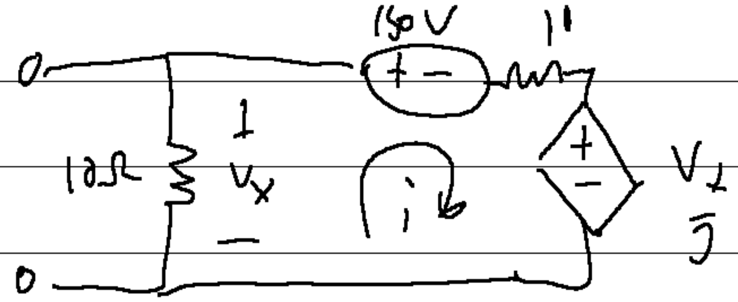
or

(KVL) $-V_x + 10i + \frac{V_x}{3} = 0$

$V_x = -10(i + 2.5)$

$\rightarrow V_x = -15$

3b)



(KCL) $\frac{V_x}{10} + \frac{(V_x - 150) - \frac{V_x}{3}}{10} = 0$

$\rightarrow V_x = 90\,(V)$

or

(KCL) $\frac{V_x}{10} + \frac{(V_x - 150) - \frac{V_x}{3}}{10} = 0$

$\rightarrow V_x = \underline{90\,(V)}$

(KVL) $-V_x + 150 + 10i + \frac{V_x}{3} = 0$

$V_x = -10i \rightarrow V_x = 90$

3c) $V_x = 90 - 15 = \underline{75\,(V)}$

4a)

(KVL) $-80 - 80 \times 2 + 40 \times 0 + V_{oc} = 0$

$\rightarrow V_{oc} = 240 (V)$

or

(KCL) $\frac{(V_{oc} - 80)}{80} - 2 + 0 = 0$

$\rightarrow V_{oc} = 240 (V)$

4b) (KVL) $-80 + 80 (i_{sc} - 2) + 40 \cdot i_{sc} = 0$

$\rightarrow i_{sc} = \underline{2 (A)}$

or

(KCL) $\frac{(V - 80)}{80} - 2 + i_{sc} = 0, \quad V = 40 \cdot i_{sc}$

$\rightarrow i_{sc} = 2 (A)$

4c) 

$\leftarrow$ $R_{th} (= R_N) = 80 + 40 = \underline{120 \Omega}$

4d) $P_L = \frac{v_{th}^2}{4 R_{th}} = \frac{240^2}{4 \cdot 120} = \underline{120 (W)}$

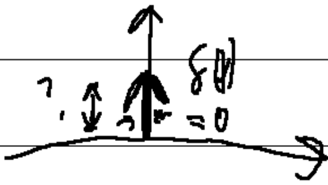
$\left(\frac{v_s^2}{4R_s} \right)$

Final:

1) General Concepts
2) Mesh Current
3) Node Voltage Matrix
4) Superposition
5) Source Transformation
6) Thévinin Norton

$\left.\begin{array}{l}\cdot\\\cdot\\\cdot\\\cdot\end{array}\right\}$ Upcoming

13) Unit impulse ft, $\delta(t)$



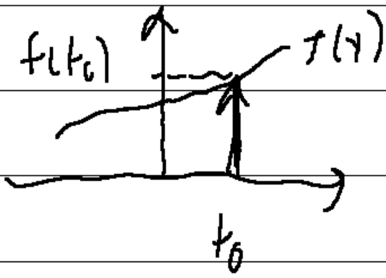
① Definition:

$$\delta(t) = 0 \quad (t \neq 0)$$

$$\int_{0^-}^{0^+} \delta(t)\,dt = 1 \quad (\delta(t) \neq 1 : \text{unit area, strength})$$

② Sifting/Sampling Ft:

$$\int_{-\infty}^{\infty} f(t)\,\delta(t)\,dt = f(0)$$

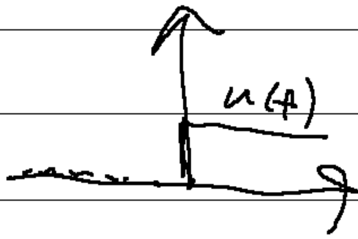


$$\int_{-\infty}^{\infty} f(t)\,\delta(t-t_0) = f(t_0)$$

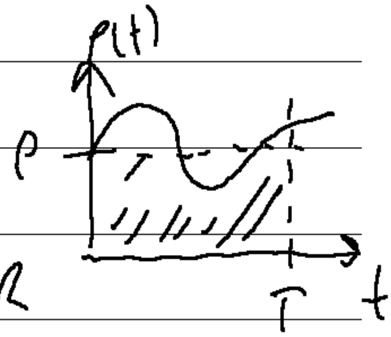
③ integ/diff:

$$\frac{du(t)}{dt} = \delta(t)$$

$$\int_{-\infty}^{t} \delta(\lambda)\,d\lambda = u(t)$$



11.2 Instantaneous and Average Power



(1) Instantaneous power

$$p(t) = i(t)v(t) = v^2(t)/R = i^2(t)R$$

(2) Average power (DC component)

$$P = \frac{1}{T}\int_{t_0}^{t_0+T} p(t)\,dt = \frac{1}{T}\int_{t_0}^{t_0+T} \frac{v^2(t)}{R}\,dt = \frac{1}{R}\frac{1}{T}\int_{t_0}^{t_0+T} v^2(t)\,dt = \frac{V_{rms}^2}{R}$$

$$\left(\text{OR } \frac{1}{T}\int_{t_0}^{t_0+T} i^2(t)\,dt = R\cdot I_{rms}^2\right)$$

11.4 RMS (root mean square)

(1) "Effective" Value

$$V_{rms}\ (= V_{eff}) = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} v^2(t)\,dt}$$

$$I_{rms}\ (= I_{eff}) = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} i^2(t)\,dt}$$

(2) Sinusoidal signals

$V_m \cos \omega t$: $$V_{rms} = \sqrt{\frac{1}{T}\int_0^T (V_m^2 \cos^2 \omega t)\,dt}$$

$$= \sqrt{\frac{1}{T}\int_0^T V_m^2 \frac{(1+\cos 2\omega t)}{2}\,dt} = \frac{V_m}{\sqrt{2}}$$

$$\left(\text{OR } V_{rms}^2 = \frac{V_m^2}{2}\right)$$

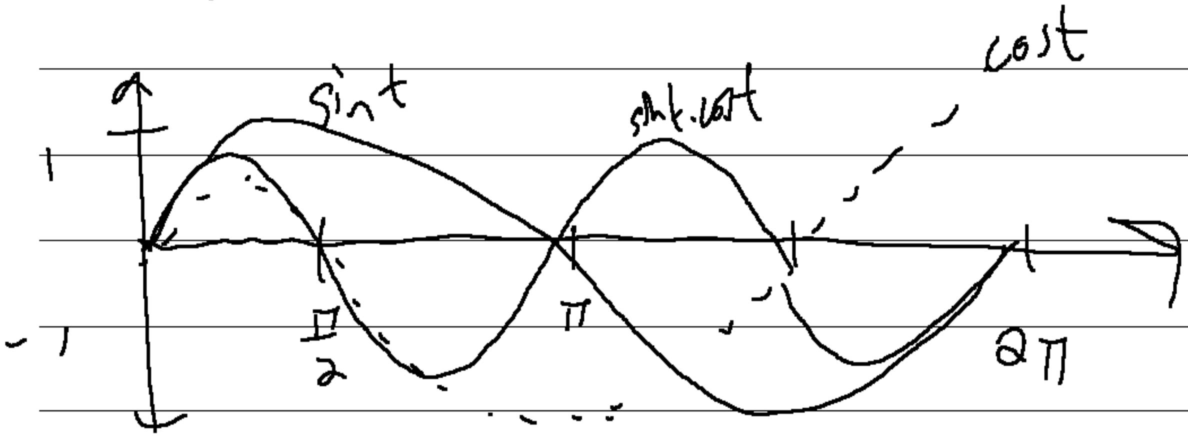
Power Superposition

(1) Orthogonality

① Avg of the product of two fts is zero

② Sinusoids satisfy the orthogonality with 3 conditions

- sin & cos of same freq/phase are orthogonal
- cos is orthogonal to sin & cos of different freq.
- sin is orthogonal to sin & cos of different freq.



(2) RMS Superposition (orthogonal signals)

① $V_{rms}^2 = \frac{1}{T}\int_0^T [v_1(t) + v_2(t)]^2 dt = \frac{1}{T}\int_0^T v_1^2 dt + \frac{1}{T}\int_0^T v_2^2 dt$

$+ \frac{1}{T}\int_0^T \overset{0}{\cancel{2v_1}v_2}\, dt = V_{rms_1}^2 + V_{rms_2}^2$

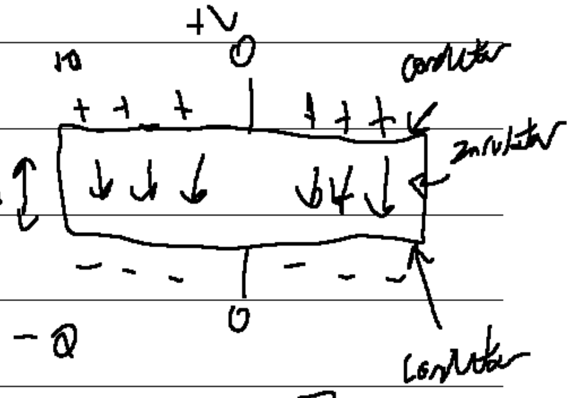
② Generalization: $V_{rms}^2 = V_{rms_1}^2 + V_{rms_2}^2 + \ldots$

$= \sum_1^n V_{rms_n}^2$ (n: integer)

③ Avg power from N orthogonal signals:

$P = \frac{1}{R} V_{rms}^2 = \frac{1}{R}\sum_1^N V_{rms\,n} = \sum_1^N P_n$

6.2 Capacitance



(1) Capacitor

① A device to store energy in the form of electric field

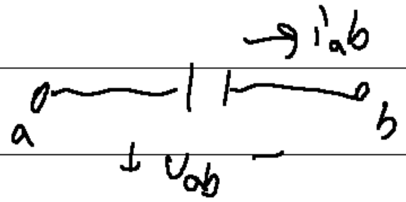
② Capacitance C:

Capacity of charge storage [Farad = coulomb/volt]

$Q = CV$ or $q(t) = C \cdot v(t)$ $\left(C = \frac{\varepsilon A}{d}, \right.$ ε = dielectricity, A: area, d: distance$\left. \right)$

③ Displacement current

$i(t) = \frac{dq(t)}{dt} = \frac{d}{dt} C \cdot v(t)$

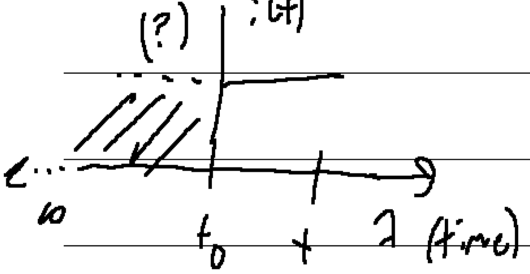


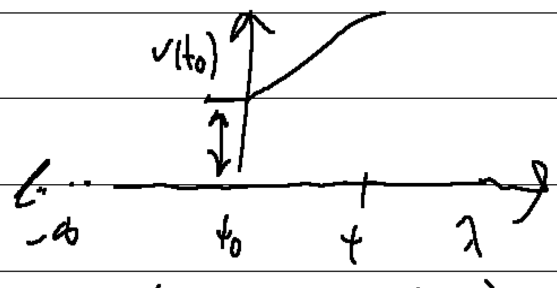
(No actual voltage crosses insulator)

(2) Initial condition $v(t_0)$

(1) $v(t) = \frac{1}{C}\int_{t_1}^{t_2} i(t)\,dt = \frac{1}{C}\int_{-\infty}^{t} i(\lambda)\,d\lambda$

$$= \underbrace{\frac{1}{C}\int_{-\infty}^{t_0} i(\lambda)\,d\lambda}_{v(t_0)} + \frac{1}{C}\int_{t_0}^{t} i(\lambda)\,d\lambda$$





(t_0: typically 0)

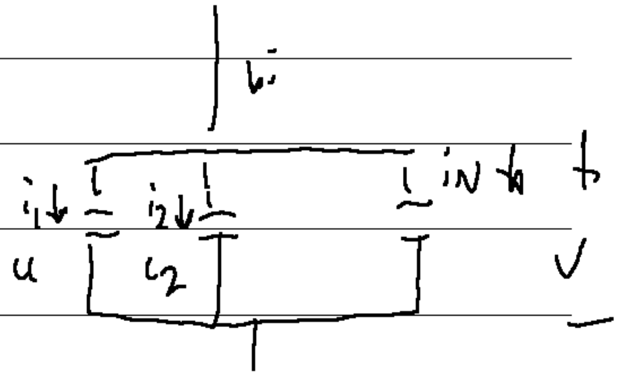
(3) Energy Storage

$$w(t) = \int_{-\infty}^{t} p(\lambda)\,d\lambda = \int_{-\infty}^{t} i(\lambda)v(\lambda)\,d\lambda = \frac{1}{2}Cv^2(t)$$

(always positive value)

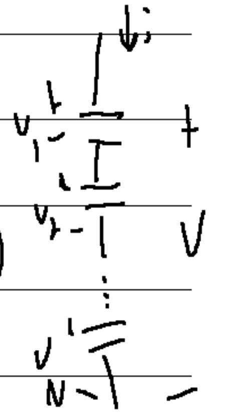
6.3 series/parallel capacitors

(1) Parallel



$$i = i_1 + i_2 + \cdots + i_N$$
$$= C_1\frac{dv}{dt} + C_2\frac{dv}{dt} + \cdots + C_N\frac{dv}{dt}$$
$$= C_{eq}\frac{dv}{dt}$$
$$\rightarrow C_{eq} = C_1 + C_2 + \cdots + C_N$$

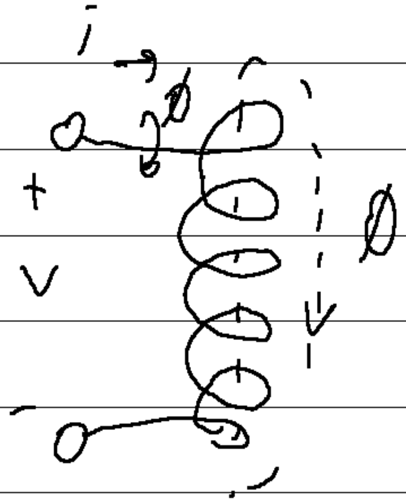
(2) Series



$$v = v_1 + v_2 + \cdots + v_N$$
$$= \frac{1}{C_1}\int_{t_0}^{t} i(\lambda)\,d\lambda + v_1(t_0) + \frac{1}{C_2}\int_{t_0}^{t} i(\lambda)\,d\lambda + v_2(t_0)$$
$$+ \cdots + \frac{1}{C_N}\int_{t_0}^{t} i(\lambda)\,d\lambda + v_2(t_0)$$
$$= \frac{1}{C_{eq}}\int_{t_0}^{t} i(\lambda)\,d\lambda + v(t_0)$$
$$\rightarrow \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_N}$$

6.4 Inductance

(1) Inductor



① A device to store energy in the form of magnetic field

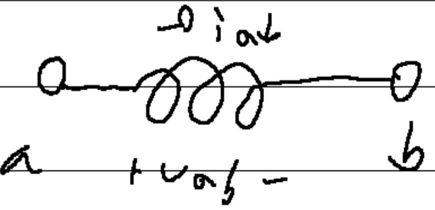
② Inductance L:

Capacity of magnetic field induction [Henry = volt·sec/Amp]

$L = N\frac{\phi}{i}$ $\begin{cases} \phi : \text{magnetic field intensity} \\ N : \text{\# of coil turn} \end{cases}$

③ Faraday's Law

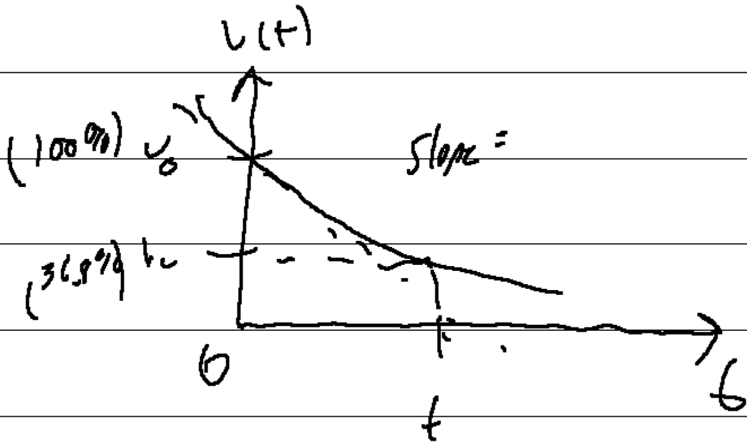
$$v(t) = N\frac{d\phi(t)}{dt} = \frac{d}{dt}\left(\frac{N\phi}{i}\, i\right) = L\frac{di(t)}{dt}$$



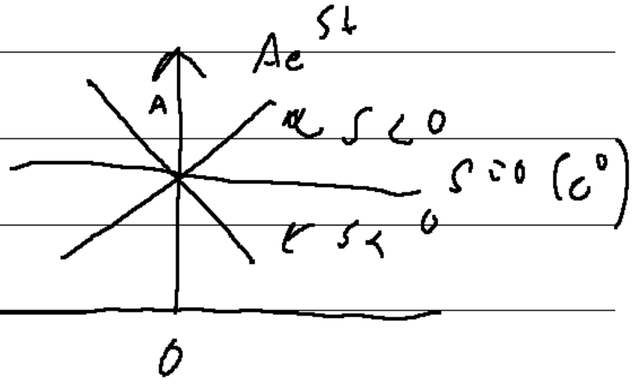
A1 Exponential ft.

(1) Decay constant, τ

$v(t) = V_0 e^{-t/\tau}$



(2) Generalization of constant fts



(3) Multiplication analogy of differentiation
("s - operator")

① $\frac{d}{dt}(Ae^{st}) = s(Ae^{st})$

② Diff of expon ft is "equiv" to multiplication by s

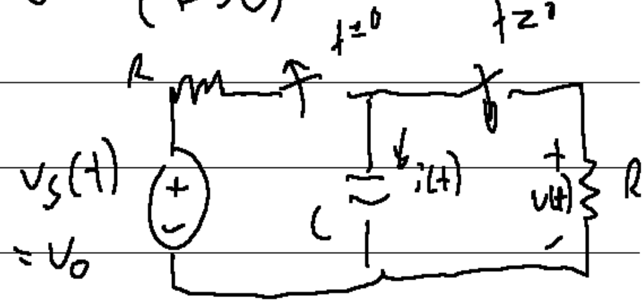
③ $\frac{d}{dt} e^{(a+iwt)} = \frac{d}{dt} e^{st} = se^{t}$

Chapter 7 First-Order Circuits

7.2 Source-free RC

(1) diff eq

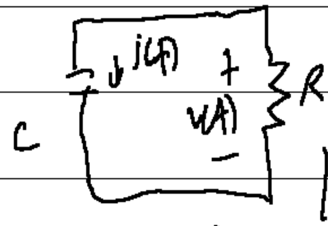
① (KCL) $C\dfrac{dv(t)}{dt} + \dfrac{v}{R} = 0 \quad (t>0)$



$$\frac{dv}{dt} + \frac{v}{RC} = 0$$

$$\frac{dv}{v} = -\frac{dt}{RC}$$

⇒



$(t>0)$

$v(t=0) = V_0$

$$\int_{v(0)}^{v(t)} \frac{dv}{v} = -\frac{1}{RC}\int_0^t dt$$

$$\ln v(t) - \ln v(0) = \frac{-t}{RC}$$

$$\ln \frac{v(t)}{v(0)} = \frac{-t}{RC}$$

$$\ln \frac{v(t)}{v(0)} = \frac{-t}{RC}$$

$$\frac{v(t)}{v(0)} = e^{-t/RC} \quad (t>0)$$

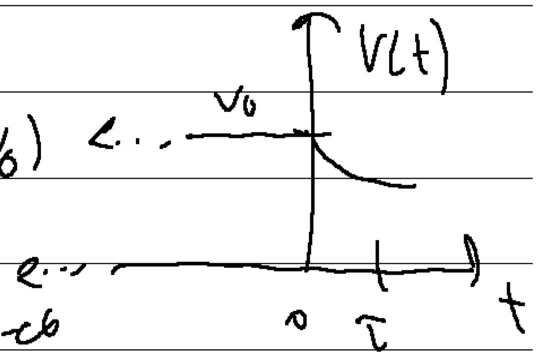
② Natural response: $v(t) = v(0)e^{-t/RC} = V_0 e^{-t/RC}$

General form: $v(t) = Ke^{st} \quad (t>0) \quad \left(s = -\frac{1}{RC} = -\frac{1}{\tau}\right)$

(2) Capacitor behavior

① initial condition:

$v(0^-) = v(0^+) = v(0) = K = (= V_0)$

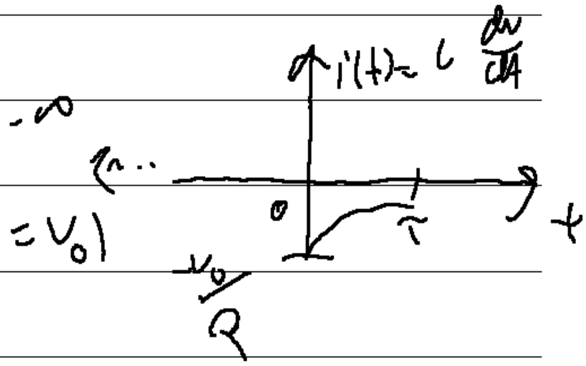


$\left(i = C\frac{dv}{dt} \neq \infty\right.$; capacitor voltage $\left.\right)$

(2) Capacitor behavior

① initial condition:

$v(0^-) = v(0^+) = v(0) = K \ (= V_0)$



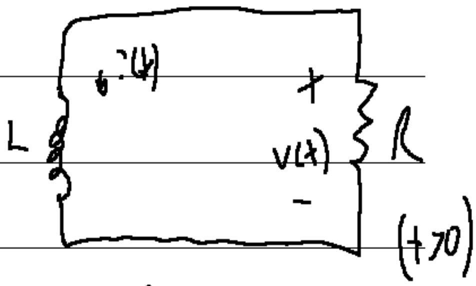
② $i(0^-) \neq i(0^+) = i(0)$

$i = 0$ (open-ckt) with a const. voltage for a very long time.

7.3 Source-Free RL

(1) Diff eq

① (KVL) $L\frac{di(t)}{dt} + iR = 0$ (t>0)



$i(t=0) = I_0$

$$\frac{di}{dt} + \frac{R}{L}i = 0$$

$$\frac{di}{i} = -\frac{R}{L}dt$$

$$\int_{i(0)}^{i(t)} \frac{di}{i} = -\frac{R}{L}\int_0^t dt$$

$$\ln i(t) - \ln i(0) = -\frac{R}{L}t$$

$$\ln\frac{i(t)}{i(0)} = -\frac{R}{L}t$$

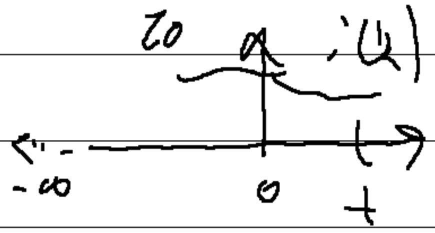
$$\frac{i(t)}{i(0)} = e^{-\frac{R}{L}t} \quad (t>0)$$

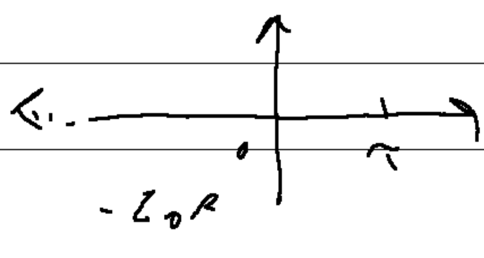
② Natural response: $i(t) = i(0)e^{-\frac{R}{L}t} = I_0 e^{-\frac{R}{L}t}$

General form: $i(t) = Ke^{st}$ (t>0) ($s = -\frac{R}{L} = -\frac{1}{\tau}$)

(2) Inductor Behavior

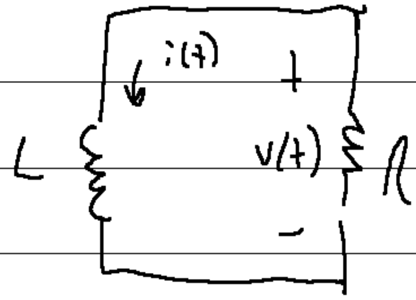
① initial condition:





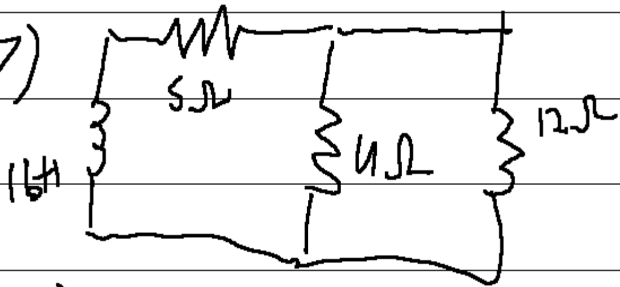
(J) "S − Operator" approach

(1) (KVL)



$i(0^-) = i(0^+)$

Sample Test #3 - Answers

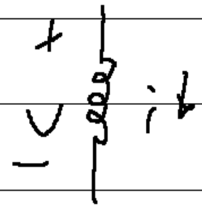
7) 

$i(0^-) = i(0^+) = 16\ (A)$

① $i(t) = ?$ ② $v(t) = ?$ $(t>0)$

③ $\tau = ?$

$(v = L\frac{di}{dt}$



① (KVL) $16\frac{di}{dt} + 5i + (4//12)i = 0$

$\frac{di}{dt} + \frac{1}{2}i = 0$

$si + \frac{1}{2}i = 0 \quad (s = -\frac{1}{2})$

$i(0) = Ke^{-\frac{1}{2}t} =$

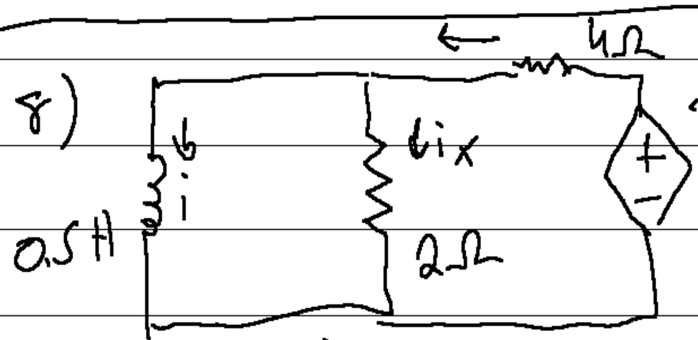
$Ke^0 = 16 \quad (K = 16)$

$\rightarrow i(t) = 16e^{-\frac{1}{2}t}$ [A] $(t>0)$

② $v(t) = -L\frac{di}{dt} = 128e^{-\frac{1}{2}t}$ [V] $(t>0)$

③ $L_{eq} = 16$, $R_{eq} = 5 + (4//12) = 8 \rightarrow \tau = \frac{L_{eq}}{R_{eq}} = 2$ (sec)

---

8) 

$i(0^-) = i(0^+) = 10\ (A)$

① $i(t) = ?$ $(t>0)$ ② $R_{eq} = ?$

① (KVL) $0.5\frac{di}{dt} - 2i_x = 0$

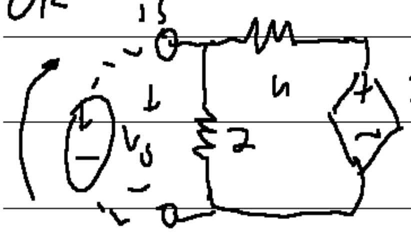
(KVL) $2i_x - 3i + 4(i + i_x) = 0$

$0.5\frac{di}{dt} + \frac{1}{3}i = 0$

$\frac{di}{dt} + \frac{2}{3}i = 0$

$si + \frac{2}{3}i = 0 \quad (s = -\frac{2}{3})$

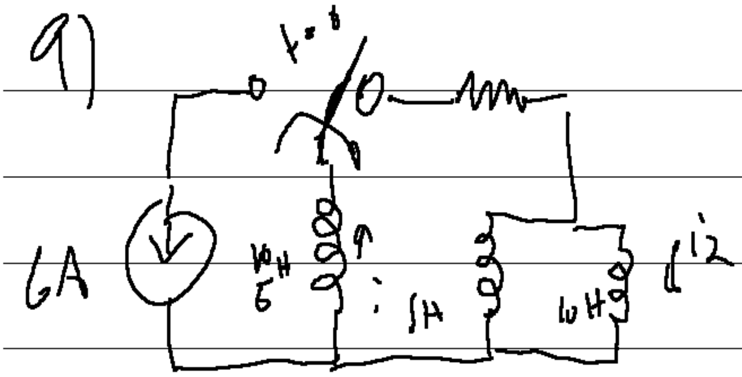
OR



$i(0) = Ke^0 = 10 \rightarrow i(t) = 10e^{-2/3t}$ [A] $(t>0)$

② $\tau = \frac{3}{2}$ [sec] $\rightarrow$ ($R_{eq} = 1/3\ (\Omega)$, $L_{eq} = 0.5$ [H])

9)



① $i(0^-) = i(0^+) = ?$

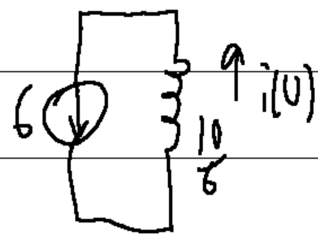
② $i(t) = ?$ ③ $i_2(t) = ?$ $(t>0)$

④ $\tau = ?$

$$L_{eq} = \frac{10}{6} + (5 // 10) = 5(H)$$

$$R_{eq} = 200\ \Omega$$

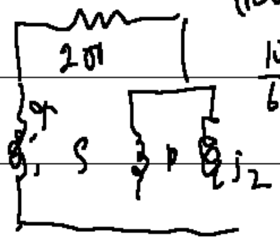
①



$(t<0)$

$$i(0^-) = 6 = i(0^+)$$
(A)

② (KVL)



$$\frac{10}{6}\frac{di}{dt} + 200i + (5//10)\frac{di}{dt} = 0$$

$$\frac{di}{dt} + 40i = 0$$

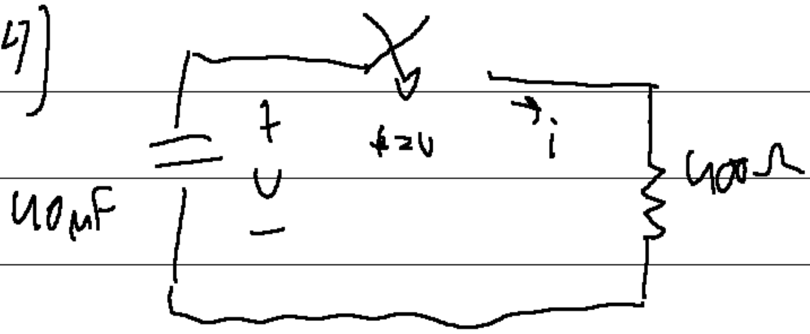
$(t>0)$ $si + 40i = 0$ $(s = -40)$

$i(0) = 1ke^0 = 6 \rightarrow i(t) = 6e^{-40t}$ [A] $(t>0)$

③ $i_2(t) = \left(\frac{\frac{1}{10}}{\frac{1}{5} + \frac{1}{10}}\right) i(t) =$

$$2e^{-40t}\ [A]\quad (t>0)$$

4)



① (KCL)

$40\mu \frac{dv}{dt} + \frac{v}{400} = 0$

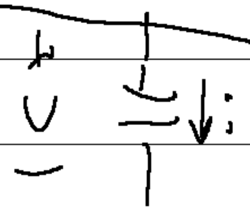
$\frac{dv}{dt} + \frac{v}{1.6\times10^{-2}} = 0$

$s + \frac{1}{1.6\times10^{-2}} = 0 \quad (s = -62.5)$

$v(0^-) = v(0^+) = 100\,[V]$

① $v(t) = ?$ ② $i(t) = ?$ $(t>0)$

$i = C\frac{dv}{dt}$



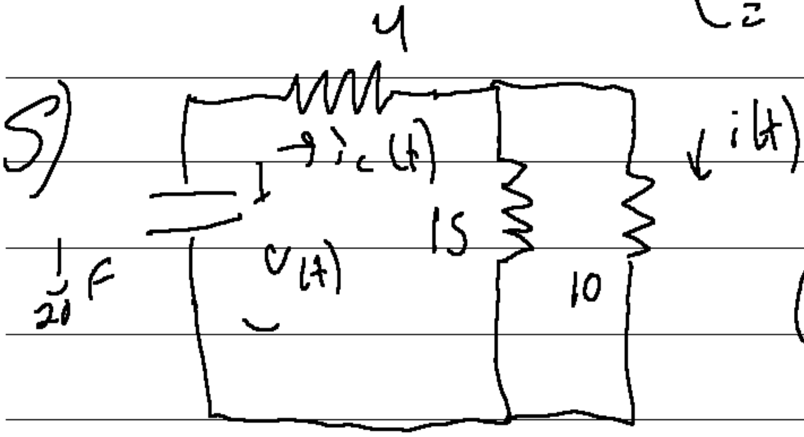
$v(0) = ke^0 = 100 \rightarrow v(t) = \underline{100e^{-62.5(t)}\,[V]}$

$t>0$

② $i(t) = -C\frac{dv}{dt} = 0.25e^{-62.5t}\,(A)\,(t>0)$

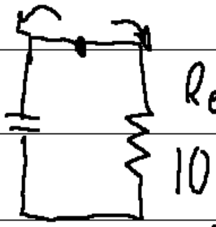
$i = \frac{v}{R} = \underline{\quad ''\quad}$

5)



$\downarrow i(t)$

①



$R_{eq} = 4 + (15//10) =$

(KCL) $\frac{1}{20}\frac{dv}{dt} + \frac{v}{10} = 0$

$\frac{dv}{dt} + 2v = 0$

$s + 2 = 0 \quad (s = -2)$

$v(0^-) = v(0^+) = 100\,[V]$

① $v(t)$ ② $i_c(t)$ ③ $i(t)$ $t>0$

$v(0) = 100e^0 \ (k = 100) \rightarrow$

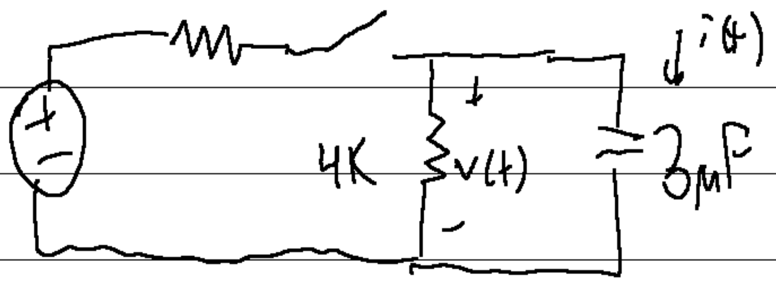
$v(t) = \underline{100e^{-2t}\,(V)}$

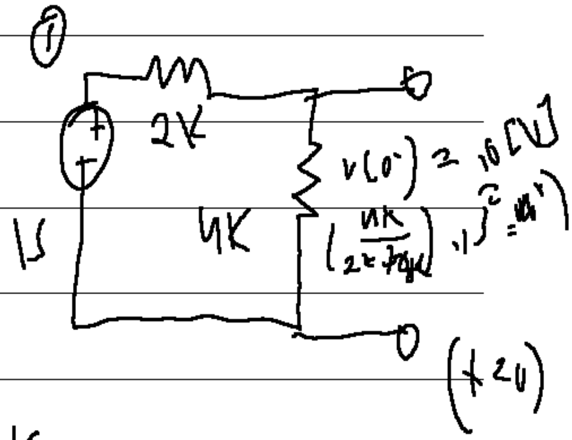
$(t>0)$

② $i_c(t) = -C\frac{dv}{dt} = \underline{10e^{-2t}\,(A)}$ $t>0$

③ $i(t) = \left(\frac{\frac{1}{10}}{\frac{1}{15}+\frac{1}{10}}\right) i_c(t) = 6Ae^{-2t}\,(A)$

6)





① $v(0^-) = v(0^+) = ?$

② $v(t)$ ③ $i(t) = ?$ $(t>0)$

②

(KCL) $3m \frac{dv}{dt} + \frac{v}{4k} = 0$

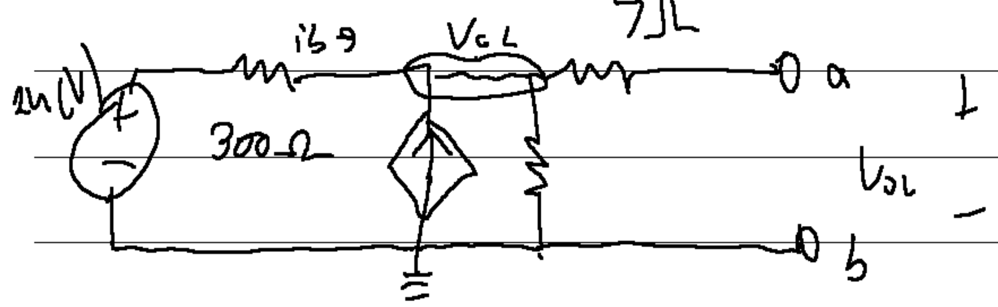
$(t>0)$ $\frac{dv}{dt} + \frac{v}{12} = 0$

$s + \frac{1}{12} = 0$ $s = -\frac{1}{12}$

$v(0) = Ke^0 = 10 \rightarrow$ $v(t) = 10 \cdot e^{-1/12t}$ [V] $t>0)$

③ $i(t) = C \frac{dv}{dt} = -2.5m e^{-1/12t}$ [A] $(t>0)$

Test #3 Solutions

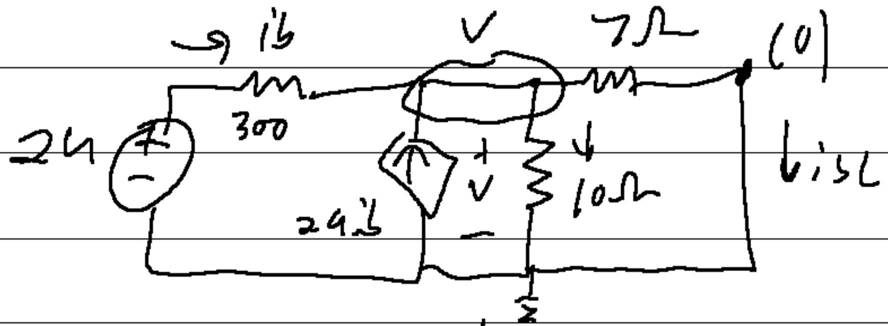


1a) (KCL) $-i_b - 29 i_b + \frac{V_{oc}}{10} = 0$

$$\left(i_b = \frac{(24 - V_{oc})}{300} \right)$$

$\rightarrow$ $\underline{V_{oc} = 12\,[V]}$

1b)



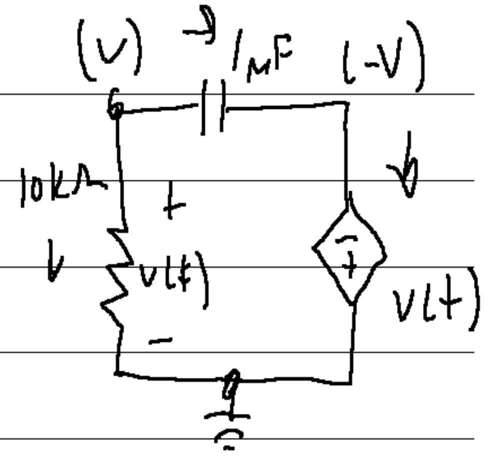
(KCL) $-i_b - 29 i_b + \frac{v}{10} + i_{sc} = 0$

$$\left(i_b = \frac{(24 - v)}{300} \right)$$

(Ohm) $v = 7 i_{sc}$

$\rightarrow$ $\underline{i_{sc} = 1\,[A]}$

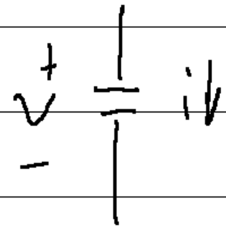
2)



c) (KCL)

$$(1\mu)\frac{d}{dt}[v-(-v)] + \frac{v}{10k} = 0$$

$$(2\mu)\frac{dv}{dt} + \frac{v}{10k} = 0$$



$$\frac{dv}{dt} + \frac{v}{2\cdot 10^{-2}} = 0$$

$i = C\frac{dv}{dt}$

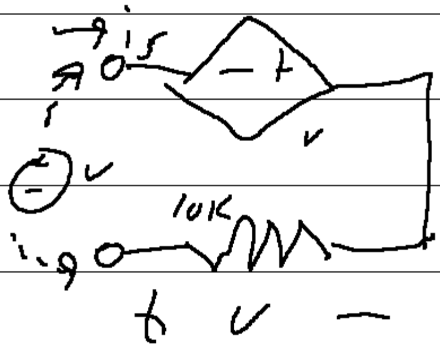
$$\rightarrow s + \frac{1}{2\cdot 10^{-2}} = 0 \quad \left(s = \frac{-1}{2\cdot 10^{-2}} = -50\right)$$

$v(0) = Ke^{0} \rightarrow v(t) = 10e^{-50t}$ [V] $(t>0)$

d) $i(t) = \frac{-v(t)}{R} = -10^{-3}e^{-50t}$ [A] $(t>0)$

(or $= C\frac{d}{dt}(2v)$ = //)

e)



(KVL) $-v_s - v - v = 0$

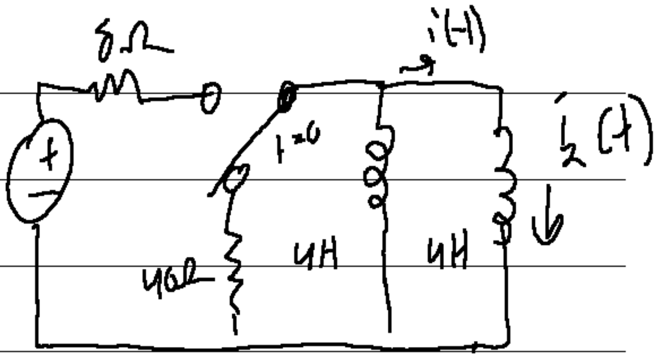
(ohm) $v = -10k\cdot i_s$

$\rightarrow R_{eq} = \frac{v_s}{i_s} = 20[k\Omega]$

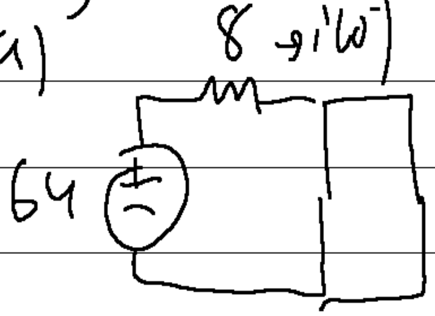
$R_{eq} = \frac{v_s}{i_s}$

$(\tau = C_{eq}\cdot R_{eq} = 1\mu\cdot 20K$
$= 2\cdot 10^{-2}$ [sec]$)$

3)



a)

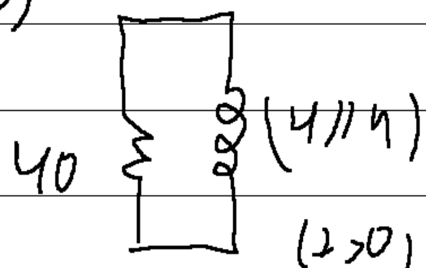


$i(0^-) = \frac{64}{8}$

$= 8\,(A) = i(0^+)$

$v = L\frac{di}{dt}$

b)



$(4//4)\frac{di}{dt} + 40i = 0$

$\frac{di}{dt} + 20i = 0$

$s + 20 = 0 \quad (s = -20)$

$i(0) = Ke^{0} = 8 \rightarrow \underline{i(t) = 8e^{-20t}\,[A] \quad (t>0)}$

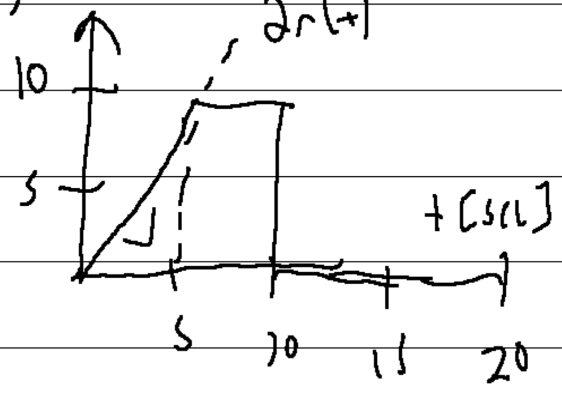
c) $i_2(t) = \left(\frac{1/4}{1/4 + 1/4}\right) i(t) = 4e^{-20t}\,[A] \quad (t>0)$

d) $v(t) = L\frac{di}{dt} = (4//4)\frac{d}{dt}(8e^{-20t})$

$= \underline{-320e^{-20t}\,(A) \quad t>0}$

or $-i(t)R = \underline{\quad ''\quad}$

4)



a) $i(t) = 2r(t) - 2r(t-5) - 10u(t-10)$ [A]

b) $I_{DC_{avg}} = \frac{1}{T}\int_0^T i(t)\,dt = \frac{1}{15}\left[\int_0^5 2(t) + \int_5^{10} 10\,dt\right] =$

5 [A]

c) $I_{rms} = \sqrt{\frac{1}{T}\int_0^T i(t)^2} = \sqrt{\frac{1}{15}\left[\int_0^5 (2t)^2 + \int_5^{10} (10)^2\,dt\right]}$

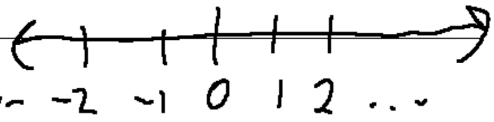
$= 6.67$ [A]

A2 - Complex Numbers

(1) Real numbers:

1-D system (linear)

$$\begin{cases} 1 \times -1 = -1 \\ -1 \times -1 = 1 \end{cases}$$

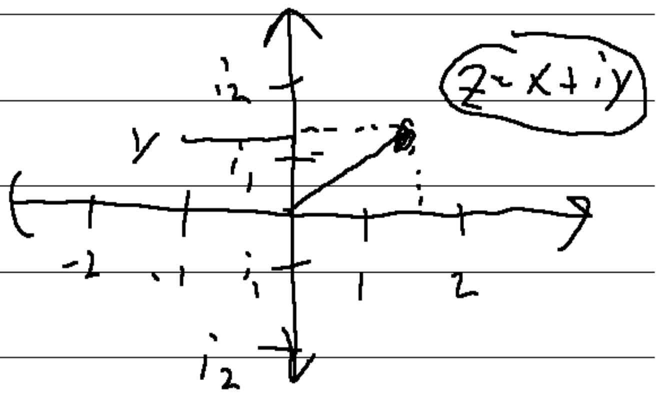


(2) Imaginary Numbers:

$(i)^2 = -1$ or $i = \sqrt{-1}$

2-D system (circular)

$$\begin{cases} 1 \times i = i \\ i \times i = -1 \\ -1 \times i = -i \\ -i \times i = 1 \end{cases}$$



(3) Complex numbers:

$$z = x + iy \rightarrow \begin{cases} |z| = \sqrt{x^2 + y^2} \\ x = |z|\cos\theta, \; y = |z|\sin\theta, \; \theta = \tan^{-1}\frac{y}{x} \end{cases}$$

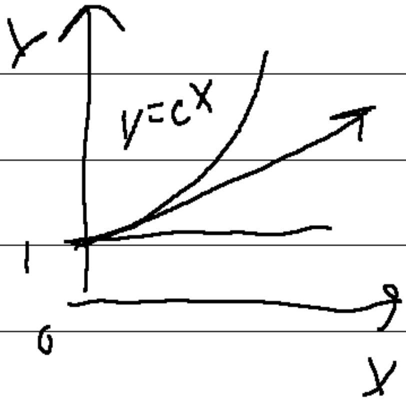
$$z = |z|(\cos\theta + i\sin\theta) = |z| e^{i\theta}$$

$$z_1 \pm z_2 = (x_1 \pm x_2) + i(y_1 \pm y_2)$$

$$z_1 \cdot z_2 = |z_1||z_2| e^{i(\theta_1 + \theta_2)}$$

$$z_1 / z_2 = |z_1| / |z_2| \, e^{i(\theta_1 - \theta_2)}$$

A3 Euler's identity



(1) Taylor's Expansion

① Expression of a ft. by indefinite sums of many terms
(e.g.) $e^x = 1 + x + x^2/2! + x^3/3! + \dots$)

② e^a = 1st + 2nd + 3rd + ...
e^{ia} = (1st Re + 1st Im) + (2nd Re + 2nd Im) + (3rd Re + 3rd Im) + ... = $\cos a + i\sin a$

(2) Euler's identity (sin ↔ exp)

① $e^{i\omega t} = \cos\omega t + i\sin\omega t$ (i.e. $\cos\omega t = Re\{e^{i\omega t}\}$, $\sin\omega t = Im\{e^{i\omega t}\}$)
$e^{-j\omega t} = \cos\omega t - j\sin\omega t$

② $\cos\omega t = (e^{i\omega t} + e^{-i\omega t})/2$
$\sin\omega t = (e^{i\omega t} - e^{-i\omega t})/i2$

③ magnitude (length): $|e^{i\omega t}| = \sqrt{\sin^2\omega t + \cos^2\omega t} = 1$

7.5 Step Response (driven RC ckt)

(1) Complete Response

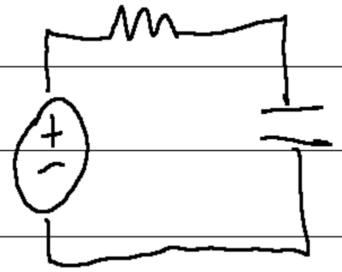
① Natural response (source free), $v_n(t)$

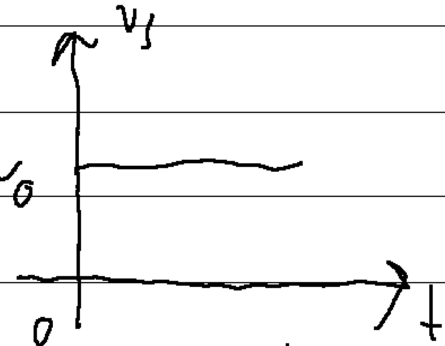
② Particular response (driven, forced) $v_p(t)$

③ Complete response:

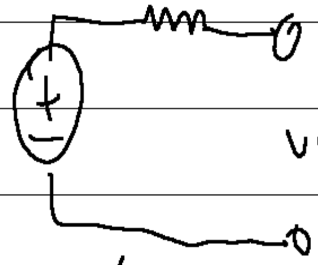
$$v(t) = v_p(t) + v_n(t) = v_p(t) + Ke^{st}$$
$$= v_p(t) + v_n(0^+)e^{st}, \text{ where } K = v_n(0^+)$$
$$= v_p(t) + [v(0^+) - v_p(0^+)]e^{st}, \text{ where}$$
$$v(0^+) = v_n(0^+) + v_p(0^+)$$

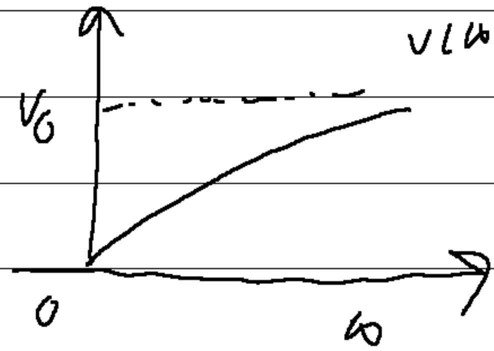
(2) Constant Sources





⇒





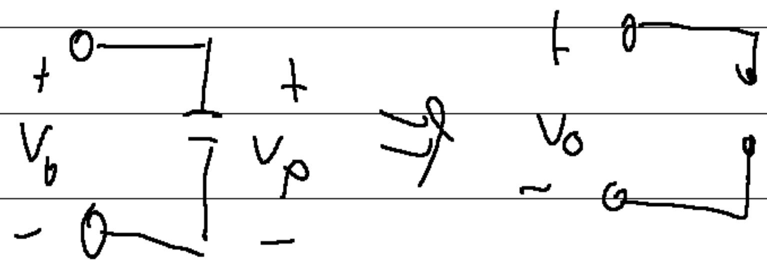
$v(\infty) = V_p = V_0$

=

① $V_p = v(\infty) = V_0$

② $v(t) = v(\infty) + [v(0^+) - v(\infty)]e^{st}$

③ Capacitor behavior @ $t = \infty$



$v(\infty) = V_p = V_0$

only v_p survived @ $t = \infty$, $i(\infty) = C\frac{dv_p}{dt} = 0$ (open-ckt)
(v_n dies off soon)

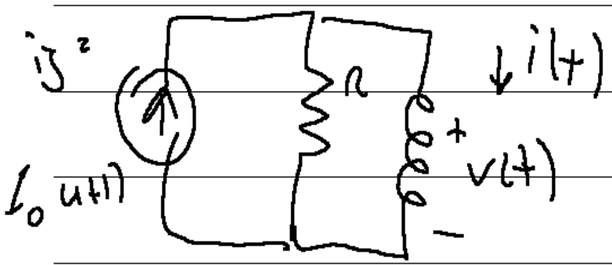
(13) Diff. eq (constant, expon, sine sources)

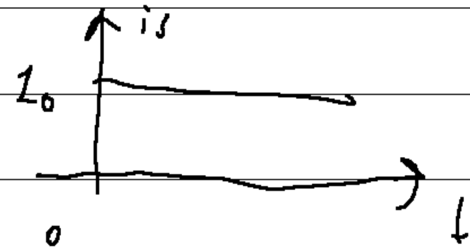
① General form of natural response
$v_n(t) = Ke^{st}$

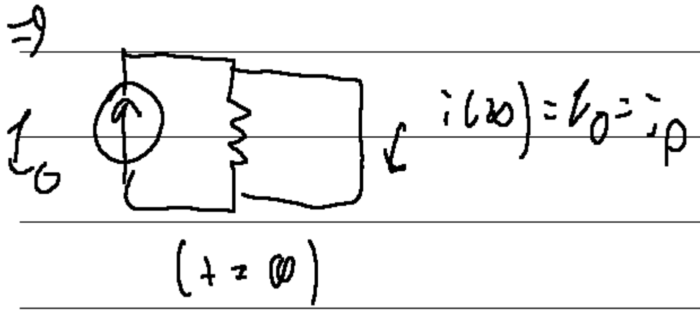
② General form of natural response
$v_p(t)$
$= Ae^0$ (for constant sources; Ae^{s_pt}, $s_p = 0$)
$= Ae^{s_pt}$ (for expon. sources: $s_p \neq s$)
$= At \cdot e^{s_pt}$ (for expon. source: $s_p = s$)
$= Re\{Ae^{js_pt}\}$ (for sine sources)

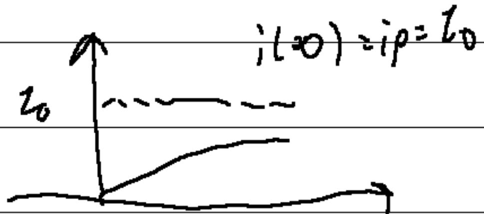
7.6

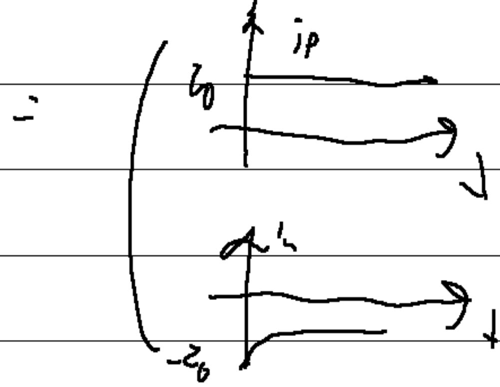
7.6 Step Response (Driven RL Ckt)







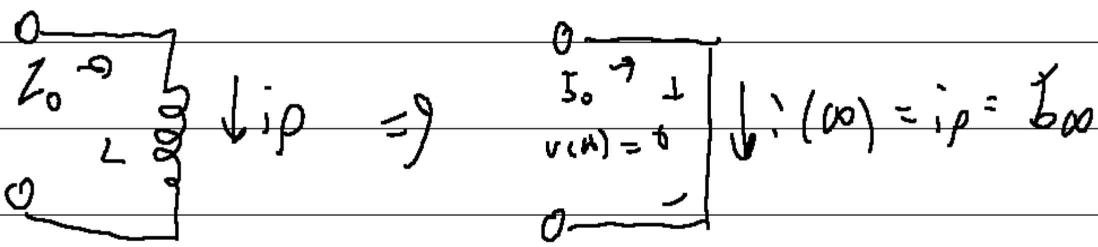




① $i_p = i(\infty)$

② $i(t) = i(\infty) + [i(0^+) - i(\infty)]e^{st}$

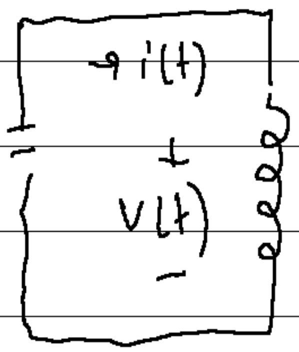
③ Inductor behavior @ $t = \infty$



Ch. 8 Second Order Circuits

(1) Two kinds of energy storage devices (L, C) in a network

$\left(i = C\frac{dv}{dt}\right)$



L $\left(V = L\frac{di}{dt}\right)$

(LC resonant tank)

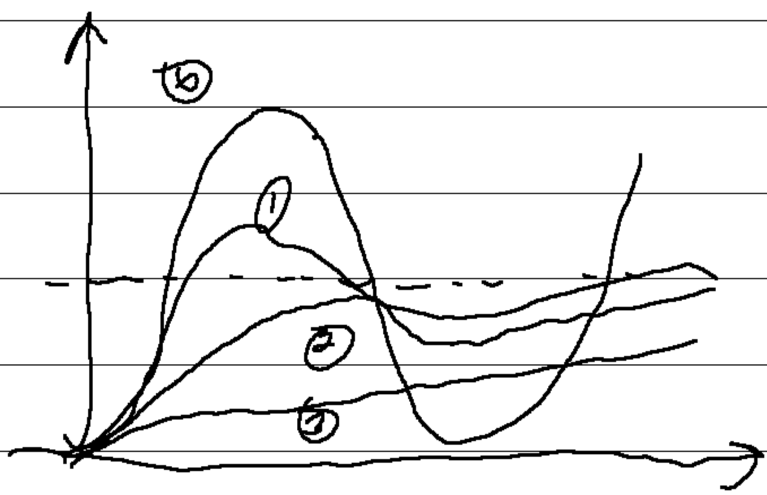
(2) Diff. eq with two roots (s_1, s_2 : real, complex)

⓪ Undamped : sinusodial (oscillation)

① Underdamped : alternating oscillation

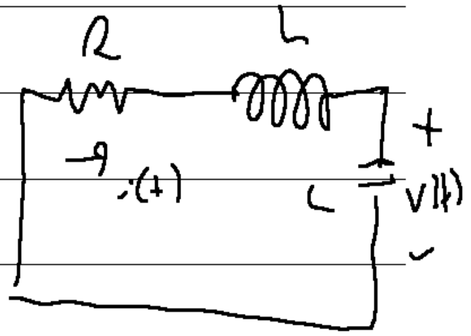
② Critically damped : intermediate

③ overdamped : asymptotic



8.3 Source-Free Series RLC

(1) Diff eq



(KVL)

$$L\frac{di}{dt} + Ri + \frac{1}{C}\int_{t_0}^{t} i(\lambda)\,d\lambda = 0 \quad (t>0)$$

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC} i = 0$$

$i(0^-) = i(0^+) = I_0$

$v(0^-) = v(0^+) = V_0$

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0 \rightarrow s^2 + 2\alpha s + \omega_0^2 = 0$$

$$s_1, s_2 = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$= -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

$$\left(= -\alpha \quad \sqrt{-(\omega_0^2 - \alpha^2)} \right.$$
$$\left. = -\alpha \pm j\omega_d \right)$$

s_1, s_2 : natural freq.

$\alpha = \frac{R}{2L}$: damping factor

$\omega_0 = \frac{1}{\sqrt{LC}}$: resonant freq. (undamped natural freq.)

$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$: damping freq. (damped natural freq.)

(2) General solution form

$i(t) = Ae^{st} \rightarrow = A_1 e^{s_1 t} + A_2 e^{s_2 t}$ (linear combination)

① overdamped ($\alpha > \omega_0$; $C > 4L/R^2$)

$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$ (s_1, s_2 : real numbers)

② Critically damped ($\alpha = \omega_0$; $C = 4L/R^2$)

$i(t) = (A_1 + A_2 t)e^{-\alpha t}$ ($s_1 = s_2 = -\alpha$: one real number)

③ underdamped ($\alpha < \omega_0$: $C < 4L/R^2$)

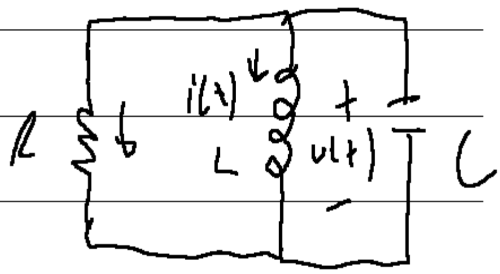
8.4 Source-free parallel RLC

(1) Diff eq.

(KCL) $C\frac{dv}{dt} + \frac{v}{R} + \frac{1}{L}\int_{-\infty}^{t} v(\lambda)\, d\lambda = 0 \ (t>0)$

$$\frac{d^2v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{1}{LC}v = 0$$

$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0 \rightarrow s^2 + 2\alpha s + \omega_0^2 = 0$$



$i(0^-) = i(0^+) = I_0$

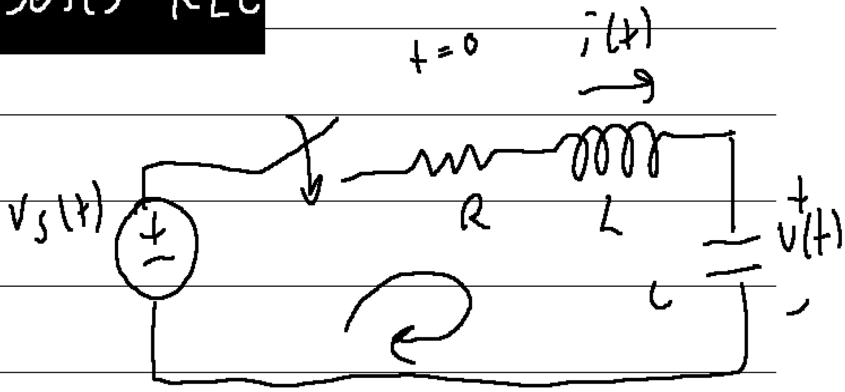
$v(0^-) = v(0^+) = 0$

(2) General Solution form

(same as those of source-free series RLC)

(3)

8.5 Step Responses of Series RLC



(1) Diff eq

(KVL) $L\frac{di}{dt} + Ri + v = v_s(t)$

$\left(i = C\frac{dv}{dt}\right)$ $\left(v(t) = \frac{1}{C}\int_{-\infty}^{t} i(\lambda)\,d\lambda\right)$

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = \frac{v_s}{L} \quad (i)$$

$$\frac{d^2 v}{dt^2} + \frac{R}{L}\frac{dv}{dt} + \frac{v}{LC} = \frac{v_s}{LC} \quad (v)$$

(2) Complete Responses

① Overdamp: $v(t) = v_p(t) + A_1 e^{s_1 t} + A_2 e^{s_2 t}$ $\left(s = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}\right)$

② Critical damp: $v(t) = v_p(t) + (A_1 + A_2 t)e^{-\alpha t}$ $(s = -\alpha)$

③ underdamp: $v(t) = v_p(t) + (A_1\cos\omega_d t + A_2\sin\omega_d t)e^{-\alpha t}$

$(s = -\alpha \pm j\omega_d)$

(3) Particular responses

$v_p(t) = A$ (const. input)

$= Ae^{s_p t}$ (expon. input : $s \neq s_p$)

$= Ate^{s_p t}$ (expon. input : $s = p$)

$= \mathrm{Re}\{ee^{js_p t}\}$

Exam 4 Answer Key

1a)

$v(0^-) = v(0^+) = 0$ [V]

b) (KCL) $\frac{1}{10}\frac{dv}{dt} - 4i_b - i_b = 0$ $\left(i_b = \frac{v_s - v}{10}\right)$

$\frac{dv}{dt} + 10v = 10v_s = 10 \times 100e^{-5t}$

c) $s + 10 = 0$ $(s = -10)$

$\rightarrow v_n(t) = ke^{-10t}$ [V]

$v_p = Ae^{-5t}$, $v_p' = -5Ae^{-5t}$

d) Diff eq $-5Ae^{-5t} + 10Ae^{-5t} = 1{,}000e^{-5t}$

$A = 200$

$\rightarrow v_p(t) = 200e^{-5t}$ [V]

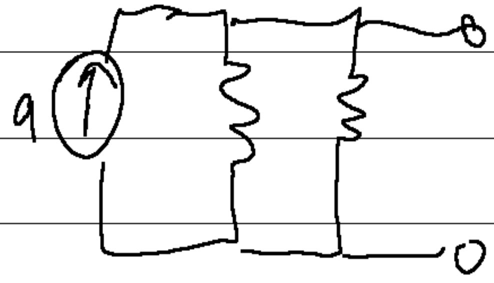
e) $v(t) = v_n + v_p = ke^{-10t} + 200e^{-5t}$

$v(0^+) = ke^0 + 200e^0 = 0$ $(k = -200)$

$\rightarrow v(t)$

2) a)

1



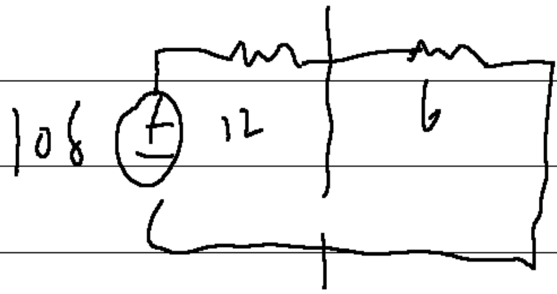
$i_s = 216/24 = 9$

$R_p = 24 // 24 = 12$

$V_s = 9 \times 12 = 108$

$R_s = 12$

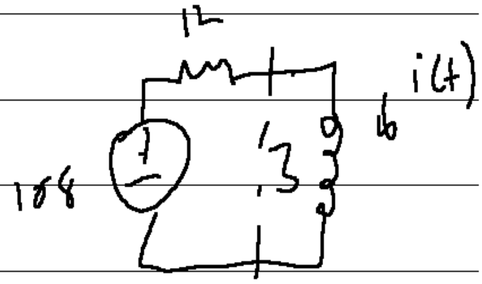
b)



$i(0^-) = \dfrac{108}{(12+6)} = 6 = i(0^+)$ [A]

c) (KVL) $3\dfrac{di}{dt} + 12i = 108$

$\dfrac{di}{dt} + 4i = 36$

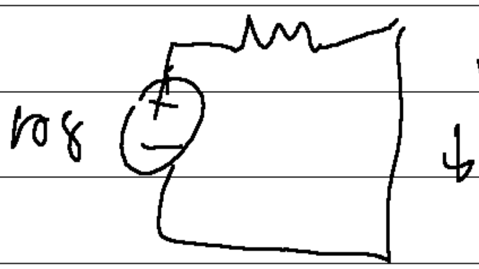


$s + 4 = 0 \rightarrow s = -4;\ \tau = -\dfrac{1}{s} = \dfrac{1}{4}$ (sec)

d)

(OR $\tau = L_{eq}/R_{eq} = 3/12 = \dfrac{1}{4}$)

e)



$i(\infty) = \dfrac{108}{12} = \underline{9 = i_f}$ [A]

3a)

(KCL) $2\frac{dv}{dt} + dv + 8\int_{-\infty}^{t} v(\lambda)\, d\lambda = 0$

$$\frac{d^2v}{dt^2} + 4\frac{dv}{dt} + 4v = 0$$

b) $s^2 + 4s + 4 = 0 \quad (s = -2)$

$\rightarrow v_n(t) = (A_1 + A_2 t)e^{-2t}$ [V] $\left(= v(t)\right)$

c) critically damped

d) $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1/8 \times 2}} = 2$ [Hz]

e) $v(0^+) = (A_1 + 0)e^0 = 2 \quad \rightarrow A_1 = 2$

$v'(t) = A_2 e^{-2t} - 2(A_1 + A_2 t)e^{-2t}$

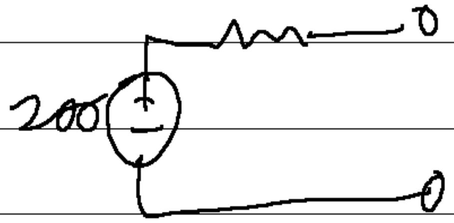
$v'(0^+) = A_2 e^0 - 2(A_1 + 0)e^0 = A_2 - 2A_1 = A_2 - 4$

Diff eq @ $t = 0^+$, $v'(0^+) + \ldots(v(0^+)) + i(0) = 0$

$2(A_2 - 4) + 8 \times 2 \cdot 1 \cdot 4 = 0$

$\rightarrow \underline{A_2 = -6}$

4 a)



b) (KVL) $5\frac{di}{dt} + 50i + 250\int_{-\infty}^{t} i(\lambda)\,d\lambda = V_s$

$$\frac{d^2i}{dt^2} + 10\frac{di}{dt} + 50i = \frac{1}{5}\frac{dV_s}{dt} = 0$$

$s^2 + 10s + 50 = 0$ ($s = -5 \pm j5$)

c) $\Rightarrow i_n(t) = (A_1 \cos 5t + A_2 \sin 5t)e^{-5t}$ [A]

d) $i_f = A \quad i_f' = 0 \quad i_f'' = 0$

2nd order diff eq $\qquad 0 + 10 \times 0 + 50A = 0$

$A = 0 = i_f(t)$ [A]

Final Exam Tips

- 8 of 10 problems