CS4120: Analysis of Algorithms

Homework 2

***Due date**: Monday, April 09 11:59 pm*

Submit your solutions to Canvas. You may type your answers in a digital file, or upload a scanned/photographed copy of your handwritten homework

1. Describe a $O(n)$ time algorithm that, given a set S of n distinct numbers and a positive integer $k \le n$, determines the k numbers in S that are closest to the median of S. (15 points)
2. Write a complete pseudocode to find the predecessor of a node in a binary search tree. (10 points)
3. Show the red-black trees that result after successively inserting the keys 41, 38, 31, 12, 19, 8 into an initially empty red-black tree. Show all steps. (15 points)
4. Consider a connected weighted graph where the edge weights are all distinct. Show that for every cycle of the graph, the edge of maximum weight on the cycle does not belong to any minimum spanning tree of the graph. (15 points)
5. A certain professor thinks that he has worked out a simpler proof of correctness for Dijkstra's algorithm. He claims that Dijkstra's algorithm relaxes the edges of every shortest path in the graph in the order in which they appear on the path, and therefore the path-relaxation property applies to every vertex reachable from the source. Show that the professor is mistaken by constructing a directed graph for which Dijkstra's algorithm could relax the edges of a shortest path out of order. (15 points)
6. In all the shortest paths algorithms we've learned in class we break ties arbitrarily. Discuss how to modify these algorithms such that, if there are several different paths of the same length, then the one with the minimum number of edges will be chosen. (15 points)
7. In a directed graph a set of paths is edge-disjoint if their edge sets are disjoint, i.e. no two paths share an edge (although they may share vertices). Given a directed graph G = (V, E) with two distinguished vertices s and t, give an efficient algorithm to find the maximum number of edge-disjoint $s - t$ (directed) paths in G. (Hint: Use a flow network.). (15 points)