# Homework 4

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1. If a binary signal is sent over a 3 kHz channel whose signal-to-noise ratio is 20 dB, waht is the maximum achievable data rate?

   We are given:

   - Number of levels in *binary* signal $\Rightarrow$ 2
   - The signal-to-noise ratio $\Rightarrow$ 20 dB

   So we need to consider the maximum data rate according to Shannon's theorem, and the Nyquist limit for binary signalling over a 3 kHz channel.

   First, Shannon's Theorem gives:

   $$\text{Max Data Rate} = Blog_2(1 + \frac{S}{N})$$

   Since $\frac{S}{N}$ is a power ratio, we must convert from decibels to this power ratio:

   $$\text{Signal-Noise Ratio in dB} = 10 \log_{10}(\frac{S}{N})$$

   $$\frac{S}{N} = 10^{\frac{20}{10}} = 100$$

   Therefore, according to Shannon's theorem, the maximum data rate is:

   $$\text{Max Data Rate} = 3000 \cdot \log_2(1 + 100) \approx 20 \text{ kbps}$$

   The Nyquist limit for binary signalling over a 3kHz channel, however, is only

   $$\text{Max Data Rate} = 2 \cdot B \log_2(V \text{ levels}) = 2 \cdot 3000 \cdot \log_2 2 = 6 \text{ kbps}$$

   Therefore the maximum *achievable* data rate for a 3 kHz channel is $\boxed{6 \text{ kbps}}$.

2. How much bandwidth is there in 0.1 micron of spectrum at a wavelength of 1 micron?

   Since we are dealing with a spectrum for a fiber optic cable, bandwidth is given by

   $$B = \frac{c \cdot \delta_\lambda}{\lambda}$$

   where $c = 300000000$ m/s, or the speed of light, $\lambda = 1$ micron, and $\delta_\lambda = 0.1$ microns. We therefore have approximately $\boxed{\frac{3 \cdot 10^{13}}{1^2} \text{ Hz} = 30 \text{ THz}}$ of bandwidth.

3. Radio antennas often work best when the diameter of the antenna is equal to the wavelength of the radio wave. Reasonable antennas range from 1 cm to 5 meters in diameter. What frequency range does this cover?

   The frequency of any wave is given by the velocity (the speed of light here) over the wavelength:

   $$f = \frac{c}{\lambda}$$

   Thus our range will be the frequency for antennas 5 m in length to the frequency for antennas 1 cm in length:

   $$f_l = \frac{3 \cdot 10^8}{5} = 30\text{GHz}$$

   $$f_h = \frac{3 \cdot 10^8}{0.01} = 60\text{MHz}$$

   This covers the frequency range of $\boxed{60 \text{ MHz to } 30 \text{ GHz}}$.

4. Ten signals, each requiring 4000 Hz, are multiplexed on to a single channel using FDM. How much minimum band- width is required for the multiplexed channel? Assume that the guard bands are 400 Hz wide.

   To avoid interference, we need a guard band to separate the upper and lower edges of the 10 signals, meaning 11 guard bands are needed total.

   $$\boxed{4000 \cdot 10 + 400 \cdot 11 = 44400 \text{ Hz}}$$

5. Suppose that x bits of user data are to be transmitted over a k-hop path in a packet-switched network as a series of packets, each containing p data bits and h header bits. Assume x¿¿p+h. The line speed is b bits per sec and the propagation delay is negligible. What value of p minimizes the total delay?

   The total number of packets necessary for the transmission is $\frac{x}{p}$.
   The total bits to transter including data and the headers which are not included in data is $\frac{(p+h)\cdot x}{p}$ bits.
   To transmit all these bits takes $\frac{(p+h)\cdot x}{bp}$ sec.
   The last packet takes more time to travel by intermediate routing: $\frac{(k-1)(p+h)}{b}$ sec. The total time is thus:

$$\text{Total Time} = \frac{(p+h)\cdot x}{bp} + \frac{(k-1)(p+h)}{b}$$

   To minimize this, we differentiate with respect to p, then set it equal to zero and solve for p:

$$\frac{(k-1)p^2 - hx}{bp^2} = 0$$

$$\boxed{p = \sqrt{\frac{hx}{k-1}}}$$